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Research article

Research on the wealth management fees of defined contribution pensions during the pre-retirement stage

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Abstract: In this paper, by enhancing the penalty coefficient, the general square loss criterion was modified into a novel criterion to more precisely identify risks and returns. Then, under this criterion, the ideal asset allocation for pension fund participants was investigated considering wealth management fees before retirement. Then, the Hamilton-Jacobi-Bellman (HJB) equation was formulated through the dynamic programming approach, and both the optimal investment strategy and minimum loss function were determined using calculus methods. Finally, how important parameters affect the initial optimal investment strategy and minimum loss function was analyzed, their economic implications were explained, the rationality of the model was validated, and several recommendations for management were provided.

Keywords: modified squared loss criterion; penalty coefficient; wealth management expense; HJB equation; optimal investment strategy; minimum loss function
Mathematics Subject Classification: 91B05, 91G05

1. Introduction

The data released in the "China Aging Report 2024" shows that the trend of aging in China is becoming more evident. In 2000, the population aged 65 and above in China accounted for 7%, and the country started to transition into an aging society. During the year 2021, the share of the population aged 65 and above exceeded 14%, and the country began to transition into a significantly aging society; in 2022 and 2023, the population aged 65 and above accounted for 14.9% and 15.4%, respectively. According to the "China Population Forecast Report 2023" by Yuwa Population, China is predicted to enter a super aging society with an aging proportion accounting for more than 20% around 2030, then continuing to rise rapidly to approximately 37.4% in 2060, and increasing again to about 46% in 2080. After a period of stabilization, almost half of China's total population of 800 million will be elderly. As the aging trend intensifies, the development of pension insurance will be continuously promoted. A report by Xinhua pointed out that, at the end of March 2024, 1.07 billion people had participated in China's basic pension insurance, an increase of 14.34 million compared with that in the same period last year. After years of stable development, China has established the world's largest pension insurance system, and diverse types of pension plans have been developed for individuals to engage in the pension insurance scheme.

According to the approach of contribution and benefit, pension plans can be classified into two types: defined benefit (DB) and defined contribution (DC) plans. In particular, the former refers to a pension scheme in which the contributions are established based on a predetermined benefit, with the associated risks assumed by the insurance company. The latter refers to a pension scheme that involves a set contribution amount, but the retirement benefits may be modified based on the performance of the venture capital market. In this case, both market risk and longevity risk are assumed by the policyholder. Since this type of pension plan is beneficial for fund managers' investment management and can achieve the preservation and appreciation of pension funds, it is regarded as the main type of pension plan both domestically and internationally [1].

In the current pension account management, most pension plans require charging a certain amount of management fees. Murthi [2] conducted a macro-level analysis of the management fees of UK pension schemes prior to 1999 and discovered that pension management fees could reduce the wealth value of pension accounts by over 40%. However, through a macro study on the comprehensive data of 13 countries worldwide before 2000, Whitehouse [3] found that the terminal wealth of individual pension accounts can be increased by using certain investment portfolios and controlling pension management fees. Despite the contrasting conclusions, it can still be observed that management fees have a dynamic impact on the entire process of pension accounts. Both studies only conducted macroeconomic analyses of data from different countries using macroeconomic approaches and lacked the application of mathematical finance methods to uncover the quantitative variations in the dynamic relationship between management fees and individual pension accounts. In recent years, scholars have started investigating the issue of pension management fees from the perspective of mathematical finance, such as in literature [4–10]. Li et al. [11] studied the equivalent management cost of DC pension accounts based on two different charging methods of management fees: asset wealth ratio and salary ratio. Deng [12] investigated the DC pension plan with two types of management costs under the Heston model. Lai et al. [13] considered the management cost of DC pension plans under the premium return clause. Lv et al. [14] studied the optimal portfolio problem of single management fees and mixed charges (charging both management fees and performance fees) of DC pension under the loss aversion criterion.

In the previously mentioned research on DC pension plans, investigations were limited to two prevalent optimization criteria: the utility function and mean-variance. The utility function is one of the earliest optimization criteria. Its advantage lies in the simple structure of the value function, which facilitates mathematical derivation and calculation. However, this approach focuses only on the benefits at the investment's endpoint while overlooking the associated risks at that moment. The mean-variance criterion can overcome this limitation by considering both the benefits and risks at the end moment, but like the utility criterion, it lacks control of the overall process risk. Given the special livelihood attributes of pension funds, it is necessary to avoid significant losses in pension accounts throughout the venture investment process. In the past few years, minimizing the expected squared loss has increasingly become a novel optimization criterion in pension planning matters.

In practice, the criterion of minimizing the expected square loss aims to minimize the expected square of the distinction between the actual wealth level and the target wealth level, and it provides a more precise evaluation of the substantial losses within pension accounts. It was initially introduced into the pension plan by Vigna and Habermans [15]. However, it is a risk-averse criterion, which contradicts the existing operational philosophy that insurance companies favor profits. Later, Habermans and Vigna [16] proposed a modified expectation squared loss criterion that considers loss aversion and earning preference based on this criterion, and they studied the DC pension plan with multidimensional assets based on different risk measures. This standard leverages a positive penalty factor to hedge risks by treating the portion of wealth exceeding the expectation as profit and the portion below the expectation as risk. When the penalty coefficient is set to zero, it becomes the general expected square loss criterion, and in this case, managers are generally called risk-averse. When this coefficient is above zero, managers are generally called surplus preference. Under this criterion, Sun Jingyun et al. [6] considered the situation where participants of DC pension have random salaries; Sun Jingyun et al. [7] also considered the situation with multidimensional interdependent risks. Different from the above research conducted only in the preretirement fund accumulation stage, Sun et al. [17] investigated the pension plan under the modified expectation squared loss criterion in the post-retirement stage. However, the abovementioned literature [6–7, 15–17] did not consider the management costs of the fund account.

This paper studies the management cost of the DC pension plan under the modified expectation squared loss criterion, the innovations are as follows:

1) By integrating the penalty coefficient, the general criterion of square loss is modified into a new optimization criterion that considers both loss aversion and earning preference, and the goal of accurately identifying earnings and risks is achieved to a certain extent.

2) Under the modified expectation square loss criterion, this paper considers the pension plan with participants having random salaries in the preretirement stage and receiving management fees proportionally to the wealth of the fund account, assuming a diffusion model with a drift term of the pension payment rate. Finally, the impact of the change in the management fee rate on the optimal investment strategy and the minimum loss function is compared, and some management suggestions are provided.

2. The model assumption

In our study, it is assumed that every stochastic process is established within a complete probability space $(\Omega,\{\mathcal{F}_t\}_{t\in[0,T]},\mathrm{P})$. The σ-filtration flow $\{\mathcal{F}_t\}_{0\le t\le T}$ is both complete and right-continuous. $\{\mathcal{F}_t\}_{t\in[0,T]}$ represents the aggregate of all market information at time t, and every stochastic process functions as an adapted process in $(\Omega,\{\mathcal{F}_t\}_{t\in[0,T]},\mathrm{P})$. T stands for a limited duration, indicating both the investment period for pensions and the benefit period for participants' pensions. In addition, it is assumed that continuous trading and assets can be divided without restriction, there are no frictions involved, self-financing conditions can be applied, and arbitrage opportunities do not exist.

ω_0, ω_0+T, and ω_0+T_d (in units of years) respectively denote the age at which the participant enters the pension plan, the retirement age, and the time when the pension in the life table stops to pay. T also represents the duration of pension investment, T_d denotes the stop time in the pension plan, and the period of the subsequent pension accumulation stage (in units of years) satisfies $0 \le t \le T$. In the numerical example section, the settings of these parameter values refer to the "The China Life Insurance Industry Experience Life Table (2010-2013)", which was published by the China Insurance Regulatory Commission on December 21, 2016.

Fund managers of pension insurance companies operate pension accounts through assert portfolios to minimize losses at the investment termination time ω_0+T. The following presents the price process of investment assets, the wealth process, and the loss function.

The insurance firm deducts a specific percentage $\pi(t,x)$ of the wealth from participants' pension fund accounts to allocate toward high-risk investments, like stocks. The price $P(t)$ (measured in tens of thousands of yuan) follows a diffusion process.

$$dP(t) = P(t)(\mu dt + \sigma dB_1(t)),$$

Here, μ means the annual expected investment return for each unit of risk asset; σ stands for the annual price volatility per unit of risk asset; $B_1(t)$ represents the standard Brownian motion, which indicates the stochastic elements influencing the value of risky assets. The insurance company will invest the remaining portion of wealth from the fund account in a zero-risk asset (e.g., a savings account), for which the price $P_0(t)$ (measured in tens of thousands of yuan) satisfies

$$dP_0(t) = P_0(t) r_0 dt,$$

where r_0 represents the yearly interest rate of a zero-risk asset. This paper operates under the assumption that the stochastic wage payment rate is $L(t)$ (in units of ten thousand yuan) of the participant's random salary before retirement conforms to the following diffusion process:

$$dL(t) = L(t)[(\mu_b + \xi r_0)dt + \sigma_b dB_2(t)],\, L(0) = l_0.$$

Here, $(\mu_b + \xi r_0)$ denotes the growth rate of the stochastic wages, μ_b represents the basic growth rate of the wages, ξr_0 stands for the growth rate of the pension in relation to the general economic context, and $\xi(>0)$ denotes the growth rate coefficient pertaining to the general economic context. When the economic situation is favorable (r_0 increases), the increase in the pension payment rate is large; when the economic situation is poor (r_0 decreases), the increase in the pension payment rate is small. $B_2(t)$ represents a standard Brownian motion, which indicates stochastic elements that influence the pension payout rate. Clearly, random pension payments are closely related to the capital market. This paper assumes that $\langle dB_1(t), dB_2(t)\rangle = \rho dt$, where ρ is the correlation coefficient. The contribution to pension is determined by the proportion of wages q, and the contribution rate is $qL(t)$. The fund company charges a certain percentage of management fees based on the wealth of the fund account. The annual management rate is δ. At the time t, the wealth of the fund account is $X(t)$ (measured in tens of thousands of yuan). It can be inferred from the above assumptions that

$$dX(t) = X(t)\pi(t,x)(\mu dt + \sigma dB_1(t)) + X(t)(1-\pi(t,x))r_0 dt + qL(t)dt - \delta X(t)dt,\, \omega_0 \le t \le \omega_0 + T$$

i.e.,

$$dX(t) = [X(t)\pi(t,x)(\mu - r_0) + X(t)(r_0 - \delta) + qL(t)]dt + X(t)\pi(t,x)\sigma dB_1(t),\ X(0) = x_0. \tag{1}$$

Definition 1. Define $\pi(t,x)$ as the unique strong solution to equation (1). If $\pi(t,x)$ is sequentially measurable by $\{\mathrm{F}_t\}_{t\in[0,T]}$ and there is $E[\int_0^\infty \pi^2(t,x)dt] < \infty a.e$ for $\forall x \in R^+$, then $\pi(t,x)$ is referred to as the allowable strategy. The set Π composed of all permissive policies is called the permissive policy set.

Definition 2. Let $R(t, X^\pi(t)) = (G(t) - X(t))^2 + \eta(G(t) - X(t))$ be the revenue recovery loss function, where $G(t)$ represents the expected wealth level of the fund account at the moment, $\eta(\geq 0)$ is the correction coefficient, and the deviation between the expected wealth level and the actual wealth level is denoted as $G(t) - X(t)$.

If $\eta > 0$, when the actual wealth level $X(t)$ exceeds the expected wealth level $G(t)$, the correction coefficients will reduce the loss of misjudgment and provide compensation for the loss of item $(G(t) - X(t))^2$; when the actual wealth level $X(t)$ is lower than the expected wealth level $G(t)$, the correction coefficient will increase the penalty for the loss. $\eta = 0$ is the case of a general square loss function in literature [15], and it can be seen that this paper presents an improvement of the loss function. Therefore, according to the size relationship between η and 0, pension fund managers can be classified into risk-averse type ($\eta = 0$) and earning preference type ($\eta > 0$). This paper conducts research from the perspective of earning preference-type managers.

Definition 3. For pairs of $\forall \pi \in \Pi$ and let $J^\pi(t,x,l) = E[R(T, X^\pi(T)) | X(t) = x, L(t) = l]$ be the preference function, $V(t,x,l) = \min_{\pi\in\Pi} J^\pi(t,x,l)$ is the value function or the minimum loss function.

As a result, the problem considered in this study may be reduced to the following stochastic optimal control problem:

$$\begin{cases} V(t,x,l) = \min_{\pi\in\Pi} J^\pi(t,x,l), \\ dX(t) = [X(t)\pi(t,x)(\mu + r_0) - X(t)(r_0 + \delta) + qL(t)]dt + X(t)\pi(t,x)\sigma dB_1(t) - L(t)\sigma_b dB_1(t). \end{cases} \tag{2}$$

Solution to the problem

Differentiate the valued function $V(t.x,l)$ as follows

$$\mathrm{A}^\pi V(t,x,l) = V_t + [x\pi(\mu - r_0) + x(r_0 - \delta) + ql]V_x + \frac{1}{2}x^2\pi^2\sigma^2 V_{xx} + (\mu_b + \xi r_0)lV_l + \frac{1}{2}l^2\sigma_b{}^2 V_{ll} + x\pi\sigma l\sigma_b\rho V_{xl},$$

By applying the principle of stochastic optimal control, Eq (2) is rewritten as the Hamilton-Jacobi-Bellman (HJB) equation.

$$\begin{cases} \inf_{\pi\in\Pi}\{\mathcal{A}^\pi V(t.x,l)\} = 0, \\ V(T, X(T), L(T)) = (G(T) - X(T))^2 + \eta(G(T) - X(T)). \end{cases} \tag{3}$$

Equation (3) can be solved by using the test theorem similar to Theorem 1 in literature [17], and the solving process will not be repeated here. Assume the equation is something like

$$V(t,x,l) = A_1(t)x^2 + A_2(t)x + A_3(t)l^2 + A_4(t)l + A_5(t)xl + A_6(t) \tag{4}$$

Combined with the boundary condition $V(T, X(T), L(T)) = (G(T) - X(T))^2 + \eta(G(T) - X(T))$ of Eq (3), it is easy to obtain

$$A_1(T)=1,\ A_2(T)=-(2G(T)+\eta),\ A_3(T)=0,\ A_4(T)=0,\ A_5(T)=0,\ A_6(T)=G^2(T)+\eta G(T). \tag{5}$$

The partial derivative with respect to $V(t,x,l)$ is

$$V_t = A_1'(t)x^2 + A_2'(t)x + A_3'(t)l^2 + A_4'(t)l + A_5'(t)xl + A_6'(t),$$

$$V_x = 2A_1(t)x + A_2(t) + A_5(t)l,\ V_{xx} = 2A_1(t),\ V_{xl} = A_5(t),$$

$$V_l = 2A_3(t)l + A_4(t) + A_5(t)x,\ V_{ll} = 2A_3(t).$$

By substituting the aforementioned partial derivatives into Eq (3) and applying the first-order optimal condition, we obtain

$$\pi^*(t,x) = -\frac{(\mu-r_0)(2A_1(t)x + A_2(t) + A_5(t)l) + \sigma l \sigma_b \rho A_5(t)}{2x\sigma^2 A_1(t)} \tag{6}$$

Then, substitute $\pi^*(t,x)$ into Eq (3) and sort it out

$$\{A_1'(t) + [2(r_0-\delta) - \frac{(\mu-r_0)^2}{\sigma^2}]A_1(t)\}x^2 + \{A_2'(t) + [(r_0-\delta) - \frac{(\mu-r_0)^2}{\sigma^2}]A_2(t)\}x$$

$$+\{A_3'(t) + [2(\mu_b+\xi r_0) + \sigma_b^2]A_3(t) - \frac{1}{4}(\frac{\mu-r_0}{\sigma} + \sigma_b\rho)^2 \frac{A_5^2(t)}{A_1(t)} + qA_5(t)\}l^2$$

$$+\{A_4'(t) + (\mu_b+\xi r_0)A_4(t) - \frac{1}{2}[\frac{(\mu-r_0)^2}{\sigma^2} + \sigma_b\rho\frac{\mu-r_0}{\sigma}]\frac{A_2(t)A_5(t)}{A_1(t)} + qA_2(t)\}l$$

$$+\{A_5'(t) + [(\mu_b+\xi r_0) + (r_0-\delta) - \frac{(\mu-r_0)^2}{\sigma^2} - \sigma_b\rho\frac{\mu-r_0}{\sigma}]A_5(t) + 2qA_1(t)\}xl$$

$$+[A_6'(t) - \frac{1}{4}\frac{(\mu-r_0)^2}{\sigma^2}\frac{A_2^2(t)}{A_1(t)}] = 0.$$

Given the arbitrary nature of x and l, the subsequent differential equations can be obtained

$$\begin{cases} A_1'(t) + [2(r_0-\delta) - \dfrac{(\mu-r_0)^2}{\sigma^2}]A_1(t) = 0, \\ A_1(T) = 1, \end{cases} \tag{7}$$

$$\begin{cases} A_2'(t) + [(r_0-\delta) - \dfrac{(\mu-r_0)^2}{\sigma^2}]A_2(t) = 0, \\ A_2(T) = -(2G(T)+\eta), \end{cases} \tag{8}$$

$$
\begin{cases}
A_3'(t)+[2(\mu_b+\xi r_0)+\sigma_b^2]A_3(t)-\dfrac{1}{4}(\dfrac{\mu-r_0}{\sigma}+\sigma_b\rho)^2\dfrac{A_5^2(t)}{A_1(t)}+qA_5(t)=0, \\
A_3(T)=0,
\end{cases}
\tag{9}
$$

$$
\begin{cases}
A_4'(t)+(\mu_b+\xi r_0)A_4(t)-\dfrac{1}{2}[\dfrac{(\mu-r_0)^2}{\sigma^2}+\sigma_b\rho\dfrac{\mu-r_0}{\sigma}]\dfrac{A_2(t)A_5(t)}{A_1(t)}+qA_2(t)=0, \\
A_4(T)=0,
\end{cases}
\tag{10}
$$

$$
\begin{cases}
A_5'(t)+[(\mu_b+\xi r_0)+(r_0-\delta)-\dfrac{(\mu-r_0)^2}{\sigma^2}-\sigma_b\rho\dfrac{\mu-r_0}{\sigma}]A_5(t)+2qA_1(t)=0, \\
A_5(T)=0,
\end{cases}
\tag{11}
$$

$$
\begin{cases}
A_6'(t)-\dfrac{1}{4}\dfrac{(\mu-r_0)^2}{\sigma^2}\dfrac{A_2^2(t)}{A_1(t)}=0, \\
A_6(T)=G^2(T)+\eta G(T),
\end{cases}
\tag{12}
$$

$$
V(t,x,l)=A_1(t)x^2+A_2(t)x+A_3(t)l^2+A_4(t)l+A_5(t)xl+A_6(t)
$$

By introducing an initial substitution with $k_1=\dfrac{\mu-r_0}{\sigma}$, $k_2=r_0-\delta$, $k_3=\mu_b+\xi r_0$, and $k_4=\sigma_b\rho$, the differential equations can be solved in the following manner:

$$
A_1(t)=e^{(k_1^2-2k_2)(t-T)}
\tag{13}
$$

$$
A_2(t)=-(2G(T)+\eta)e^{(k_1^2-k_2)(t-T)}
\tag{14}
$$

$$
\begin{aligned}
A_3(t)=\frac{q^2e^{(2k_3+\sigma_b^2)(T-t)}}{(k_2-k_3+k_1k_4)^2}&\Big[\frac{k_1^2+2k_1k_4+k_4^2}{k_1^2+2k_1k_4+\sigma_b^2}(e^{(k_1^2+2k_1k_4+\sigma_b^2)(t-T)}-1)+\frac{2k_2-2k_3-2k_1^2-2k_4^2-2k_1k_4}{k_1^2-k_2+k_3+k_1k_4+\sigma_b^2}(e^{(k_1^2-k_2+k_3+k_1k_4+\sigma_b^2)(t-T)}-1) \\
&+\frac{k_1^2-2k_2+2k_3+k_4^2}{k_1^2-2k_2+2k_3+\sigma_b^2}(e^{(k_1^2-2k_2+2k_3+\sigma_b^2)(t-T)}-1)\Big]
\end{aligned}
\tag{15}
$$

$$
A_4(t)=\frac{q(2G(T)+\eta)}{k_2-k_3+k_1k_4}(e^{(k_1^2+k_1k_4-k_3)(t-T)}-e^{(k_1^2-k_2)(t-T)})
\tag{16}
$$

$$
A_5(t)=\frac{2q}{k_2-k_3+k_1k_4}e^{(k_1^2-2k_2)(t-T)}(1-e^{(k_2-k_3+k_1k_4)(t-T)})
\tag{17}
$$

$$
A_6(t)=\frac{1}{4}(2G(T)+\eta)^2(e^{k_1^2(t-T)}-1)+(G^2(T)+\eta G(T))
\tag{18}
$$

At this point, by substituting the above solutions into Eqs (6) and (4), the optimal investment strategy $\pi^*(t,x)$ and the value function $V(t,x,l)$ can be explicitly solved, which will not be repeated here.

3. Mathematical and numerical analysis

First, let's ascertain the investment objective $G(T)$ of the pension account at the end of the investment period. Assume that upon the participant's retirement, the present value of the life annuity paid per unit is $G(T)=ka(T)$, where $a(T)$ is the annuity payment rate. To facilitate the calculation, the commonly used death force model De Moivre is used. Then, the survival probability of ω_0+s at the moment after retirement is ${}_{s-T}P_{\omega_0+T}=\dfrac{T_d-s}{\omega_0+T_d}, T\le s\le T_d$. To ensure that the participants' annuities maintain the same purchasing power after retirement, this paper considers the existence of inflation and assumes that the inflation rate is i, then

$$a(T)=\int_T^{T_d} {}_{s-T}P_{\omega_0+T}e^{i(s-T)}e^{-r_0(s-T)}ds=\frac{T_d-T}{(r_0-i)(\omega_0+T_d)}+\frac{1}{(r_0-i)^2(\omega_0+T_d)}(e^{(i-r_0)(T_d-T)}-1) \tag{19}$$

The adjusted closing price data of the Power Construction Corporation of China (601669.SH) on 425 valid working days from January 1, 2021 to September 30, 2022 was obtained from the Tushare website and were utilized as research samples after the elimination of null values and outliers. After further processing of the data, the annual average return rate and the annual return volatility of the stock of Power China was calculated to be μ=0.342865 and σ=0.479892, respectively. In the following, by taking the investment in Power Construction of China as an example, the effect of several important parameters on the optimal investment strategy $\pi^*(0,x_0)$ is investigated, the minimum loss function $V(0,x_0,l_0)$ at the initial time t=0 of the pension plan is analyzed using six numerical examples, and the economic importance is discussed. Then, the rationality of the modeling and results is verified, and several management suggestions are provided. First, the following basic parameters are given.

Table 1. Basic parameters.

ω_0	T	T_d	r_0	i	q	μ_b	σ_b	ξ	l_0	ρ
30	30	75	0.03	0.02	0.1	0.02	0.01	0.3	6	0.75

According to the parameters provided above, it is easy to calculate $a(T)=7.03$.

Numerical examples 1. The annual payment rate of pension annuity after retirement is set as $k=8$ million yuan per year, the starting salary level of pension plan participants is set as l_0=6 million yuan per year, and the penalty coefficient is set as η=6.0. Subsequently, by employing MATLAB software, the functional relationship between the initial wealth x_0, the wealth management rate δ of the pension account, and the optimal investment $\pi^*(0,x_0)$ can be obtained, as illustrated in Figure 1.

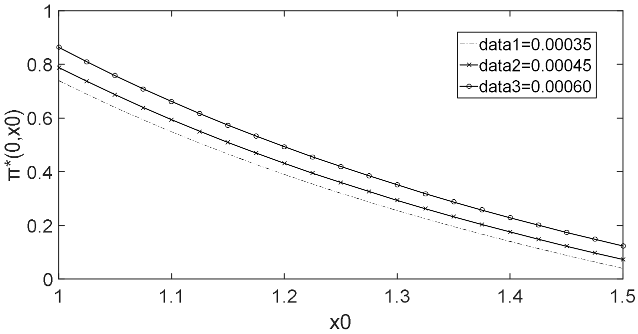

Figure 1. The functional relationship between the initial wealth x_0, the management rate δ, and the optimal investment $\pi^*(0,x_0)$.

It can be observed from Figure 1:

1) The more sufficient the initial wealth x_0, the lower the investment proportion of the risky assets, the more easily the final investment objective can be achieved, which aligns with individuals' basic comprehension;

2) While the wealth management rate δ of the pension account increases, the wealth level of the pension account declines. To fulfill the determined terminal wealth goal, it is essential to raise the share of risky assets to obtain greater returns, which also agrees with people's fundamental understanding.

Numerical examples 2. The annual payment rate of pension annuity after retirement is as $k=8$ million yuan per year, the starting salary level of pension plan participants is set as l_0=6 million yuan per year, and the wealth at the beginning time t=0 of the pension account is set as $x_0=1.0$ million yuan. Then, the functional relationship between the penalty coefficient η, the wealth management rate δ and the optimal investment $\pi^*(0,x_0)$ may be obtained through the utilization of MATLAB, as presented in Figure 2.

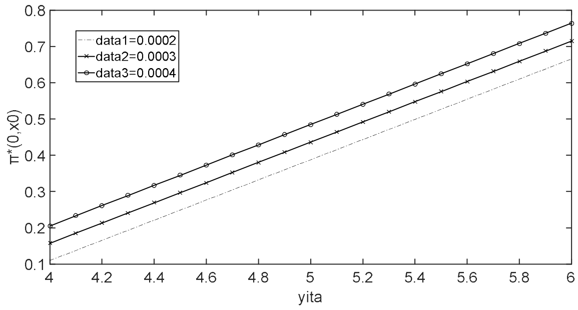

Figure 2. The functional relationship among the wealth management fee δ, the penalty coefficient η and the optimal investment strategy $\pi^*(0,x_0)$.

In Figure 2, when η increases, $\pi^*(0,x_0)$ also increases, which indicates that under more precise risk identification criteria, investors typically transition from risk-averse conservative investors to so-called aggressive investors with earning preference, and the investment proportion will rise to obtain more wealth to fulfill the ultimate wealth goal. This is consistent with people's general comprehension.

Numerical examples 3. The annual payment rate of pension annuity after retirement is set as $k=8$ million yuan per year, the starting salary level of pension plan participants is set as l_0=6 million yuan per year, and the penalty coefficient is set as η=7.8. Then, the relationship between the wealth management rate δ, the initial wealth x_0 of the pension account, and the function $V(0,x_0,l_0)$ of the minimum loss function can be obtained by using MATLAB, as shown in Figure 3.

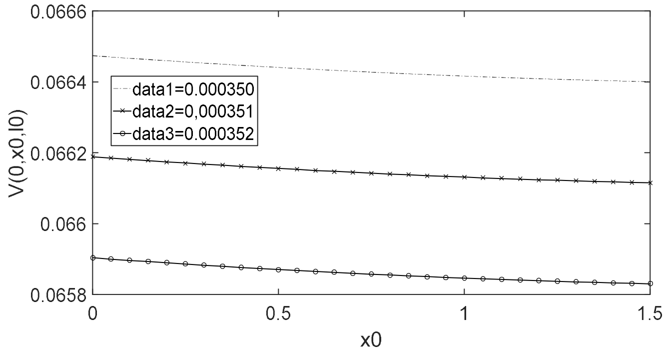

Figure 3. The functional relationship between the wealth management fee δ, the initial wealth x_0 of the pension account, and the minimum loss function $V(0,x_0,l_0)$.

It can be seen from Figure 3 that a relatively sufficient initial reserve of the account is conducive to reducing the minimum loss. Therefore, if possible, the manager should provide a relatively sufficient reserve of the pension account. Meanwhile, charging a certain amount of wealth management fees can help to reduce the minimum loss, which is beneficial for risk management.

Numerical examples 4. The annual payment rate of pension annuity after retirement is set as $k=8$ million yuan per year, the starting salary level of pension plan participants is set as l_0=6 million yuan per year, and the initial wealth of the pension account at the starting moment is set as x_0=1.5 million yuan. Then, the functional relationship between the wealth management rate δ, the penalty coefficient η, and the minimum loss function $V(0,x_0,l_0)$ can be obtained by using MATLAB, as demonstrated in Figure 4.

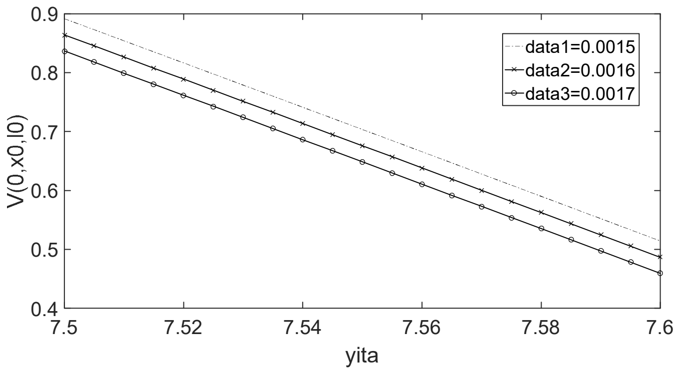

Figure 4. The functional relationship between the wealth management fee δ, the penalty coefficient η, and the minimum loss function $V(0,x_0,l_0)$.

It can be observed from Figure 4 that a larger penalty coefficient can be used to measure a smaller minimum loss. Therefore, introducing the penalty coefficient is conducive to accurately measuring the risk. However, the aggressive managers with earning preference should select an appropriate penalty coefficient for risk measurement and avoid choosing an excessively large penalty coefficient to measure risks unrestricted, so as to prevent risks from being overly reduced in form, leading to aggressive adverse consequences.

Numerical examples 5. With the annual wealth rate set to δ=0.0005 and the penalty coefficient of the pension account set to η=6.0, the functional relationship among x_0, the annual pension payment rate k and $\pi^*(0,x_0)$ may be obtained through the utilization of MATLAB, as illustrated in Figure 5.

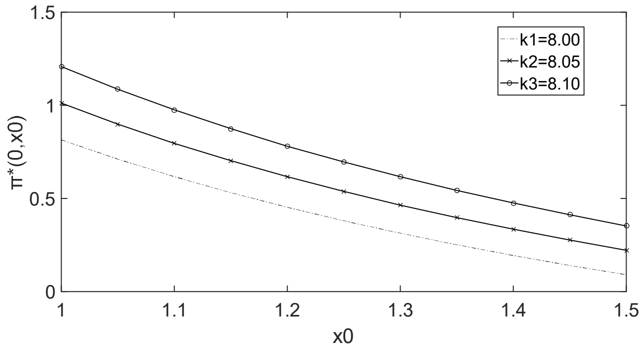

Figure 5. The functional relationship between the initial wealth x_0, the annual pension payment rate k, and the optimal investment $\pi^*(0,x_0)$.

It can be seen from Figure 5 that a larger pension payment rate leads to a decrease in the wealth of the pension account, and it is inevitable to increase risky investment to achieve the terminal target wealth.

Numerical examples 6. Under the annual wealth rate δ=0.0005, the penalty coefficient η=6.0, and the penalty coefficient η=7.8, the functional relationship between the annual pension payment rate k, the initial wealth x_0 of the fund account, and the minimum loss function $V(0,x_0,l_0)$ can be obtained by using MATLAB, as illustrated in Figure 6.

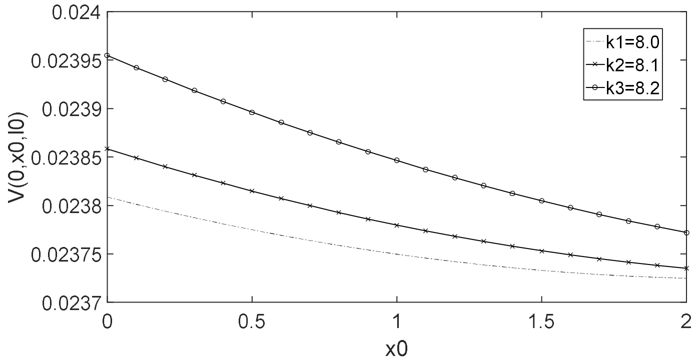

Figure 6. The functional relationship between the pension annual payment rate k, the initial fund account wealth x_0, and the minimum loss function $V(0,x_0,l_0)$.

As shown in Figure 6, the increase in the pension annual payout rate k will cause the minimum loss to increase, so it is necessary to select an appropriate pension annual payout rate to facilitate risk control.

4. Conclusions

By increasing the penalty coefficient, this paper modifies the general square loss criterion into a novel criterion to identify risks and returns more precisely. Then, under this criterion, this study investigates the optimal portfolio for the defined contribution pensions with wealth management fees before retirement. The research discovers that charging wealth management fees will affect the optimal investment strategy and the minimum loss function. As the wealth management fee increases, the minimum loss function will decline to a certain extent. Therefore, imposing certain wealth management fees helps to lower the risk level. As the wealth management fee rises, so does the optimal investment strategy. This is because the wealth level of the pension account is decreased due to the increase in management fees, and the fund manager needs to add a certain amount of risky investment to achieve the target wealth at the terminal.

Meanwhile, it is found that the increase in the penalty coefficient also affects the minimum loss function and the optimal investment strategy. Under the modified squared loss criterion with a larger penalty coefficient, fund managers typically transition from conservative risk-averse managers to aggressive ones with earning preferences. Due to the correct identification of earnings and losses, the minimum loss function will decrease with the increase in the penalty coefficient. Therefore, under a low risk control level, increasing the optimal investment strategy is a financial method to achieve the target wealth as soon as possible. However, it is noteworthy that an appropriate penalty coefficient should be selected because an overly large penalty coefficient will excessively expand profits and reduce losses, leading to an adverse effect on risk prevention and control.

Finally, it is discovered that the pension rate for fund participants after retirement also affects the minimum loss function and the optimal investment strategy. The higher the annual pension rate, the lower the fund account wealth level, and the minimum loss function will increase. As a result, it needs to enhance the optimal investment strategy to achieve a higher fund account wealth level.

It is evident that the model used in this study has certain limitations. There is no definite method to select the appropriate penalty coefficient, and using a certain constant to represent the market environment is not consistent with its random changes. The stochastic market model with the Markov switching of Xu et al. [18–19] has some reference value for our further research. In addition, for the convenience of mathematical deduction, the paper only considers the post-retirement stage and does not consider the preretirement stage regarding inflation factors, which is also inconsistent with the practice of financial markets. The study by Wei et al. [20,21] considered two-stage inflation factors, which provides a valuable reference for our next research.

Author contributions

Zongqi Sun: Conceptualization, idea, methodology, resources, data cu-ration, formal analysis, software, writing; Peng Yang: Supervision, validation; Ying Wang: Project administration; Jing Lu: Investigation. All authors have read and approved the final version of the manuscript for publication.

Use of Generative-AI tools declaration

The authors declar they have not used Artificial Intelligence (AI) tools in the creation of this article.

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Conflict of interest

The authors declare no conflict of interest.

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