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Research article

Confidence intervals for coefficient of variation of Delta-Birnbaum-Saunders distribution with application to wind speed data

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Abstract: The delta-Birnbaum-Saunders distribution is considered a relatively new distribution that combines the Birnbaum-Saunders distribution with data that include zero values. Furthermore, the coefficient of variation is important because it provides a standardized measure of relative variability that can be calculated from the ratio of the standard deviation to the mean. Consequently, this study focuses on constructing confidence intervals for the coefficient of variation of the delta-Birnbaum-Saunders distribution. We have proposed three methods for constructing confidence intervals: the generalized confidence interval based on the variance-stabilized transformation, the generalized confidence interval based on the Wilson score method, and the normal approximation compared with the bootstrap confidence interval. The performance of all these methods was compared using coverage probabilities and expected lengths through Monte Carlo simulations using the R statistical software, and various parameters were comprehensively specified. The study results revealed that the generalized confidence interval based on the variance stabilized transformation and the generalized confidence interval based on the Wilson score method provided similar results and were the best-performing methods. Additionally, the study results show that as the sample size increases, all methods tend to become more effective. Finally, we applied all the methods presented to wind speed data from Ubon Ratchathani province and Si Sa Kat province in Thailand.

Keywords: bootstrap confidence interval; coefficient of variation; delta-Birnbaum-Saunders; generalized confidence interval; normal approximation
Mathematics Subject Classification: 62F25, 62P12

1. Introduction

In science, wind speed is a fundamental quantity that results from the movement of air from high-pressure areas to low-pressure areas, primarily caused by temperature changes. Wind speed has a diverse impact on life and the economy and is important, such as in renewable energy production, aviation operations, and crop production. Additionally, monitoring and predicting wind speed contributes to preparedness and disaster prevention. Mohammadi, Alavi, and McGowan [1] studied the estimation of wind speed distributions and demonstrated that the Birnbaum-Saunders distribution is the most suitable. However, on days or times with no wind, where the wind speed is zero, the Birnbaum-Saunders distribution cannot be used for analysis since it is positively skewed. Therefore, the delta-Birnbaum-Saunders distribution is a more suitable option. The delta-Birnbaum-Saunders distribution combines both zero and positive values. The zero observations follow the binomial distribution with binomial proportion δ, whereas the positive observations with the probability $1-\delta$ follow the Birnbaum-Saunders (BS) distribution. It is well known that the BS distribution is widely applied across various fields such as environmental research, agriculture, business, industry, and medical sciences [2–5]. Since the Birnbaum-Saunders distribution is positively skewed and cannot be applied when zero values are present, it is not suitable for datasets containing zeros. However, real-world data may include zeros, making the delta-Birnbaum-Saunders distribution a more appropriate choice. The concept of the delta-Birnbaum-Saunders distribution originates from Aitchison's research [6]. Subsequently, several researchers have applied the concept of incorporating zero values into various positive distributions, providing more diverse and accurate approaches for statistical analysis, such as the delta-lognormal distribution. Hasan and Krishnamoorthy [7] used the delta-lognormal distribution to construct confidence intervals for the mean, employing both the fiducial approach and the method of variance estimate recovery (MOVER). Maneerat, Niwitpong, and Niwitpon [8] constructed confidence intervals for the difference between variances using the delta-lognormal distribution. They compared the highest posterior density (HPD) method with the normal approximation (NA), parametric bootstrap (PB), and fiducial generalized confidence interval (FGCI) methods. Singhasomboon, Panichkitkosolkul, and Volodin [9] proposed methods for constructing confidence intervals for the ratio of medians in lognormal distributions. The methods they introduced include the NA, the MOVER, and the generalized confidence interval (GCI). Their findings indicate that GCI performs well in terms of coverage probabilities, and they recommend using the NA method for moderate to large sample sizes when the mean and variance are small. For the delta-Birnbaum-Saunders distribution, Ratasukharom, Niwitpong, and Niwitpong [10] used the GCI, bootstrap confidence interval, generalized fiducial confidence interval (GFCI), and NA to estimate the proportion of zeros using the variance-stabilized transformation (VST), Wilson, and Hannig methods. These approaches were applied to construct confidence intervals for the variance of the delta-Birnbaum-Saunders distribution. They found that the GFCI based on the Wilson method is most suitable for small sample sizes, the GFCI based on the Hannig method is optimal for medium sample sizes, and the GFCI based on the VST method performs best for large sample sizes. For the delta-gamma distribution, Guo et al. [11] proposed GCIs based on fiducial inference, Box-Cox transformation, PB, and MOVER to construct confidence intervals for the difference between coefficients of variation in delta-gamma distributions. They found that all four GCI methods provided satisfactory results in terms of coverage probabilities. For the delta-two-parameter exponential distribution, Khooriphan, Niwitpong, and Niwitpong [12] proposed methods for constructing confidence intervals for the mean of the delta-two-parameter exponential distribution using PB, standard bootstrapping, the GCI, and the MOVER. They found that GCI is recommended for small to moderate sample sizes, while PB is more suitable for large sample sizes.

The coefficient of variation is a statistical measure of relative dispersion used to compare the variability of distinct datasets. The coefficient of variation is defined as the ratio of the standard deviation to the mean. The coefficient of variation value is typically expressed as a percentage. A higher coefficient of variation indicates greater relative variability, while a lower coefficient of variation suggests less relative variability. Moreover, the coefficient of variation is a useful tool and is applied in various real-world scenarios, for example, investment analysis, healthcare, education, and economics. Importantly, environmental scientists use coefficients of variation to study the variability in environmental data, such as rainfall patterns, temperature fluctuations, or pollutant levels [13–15]. Furthermore, numerous researchers have conducted studies on confidence intervals for the coefficient of variation in various distributions. In the normal distribution, Vangel [16] created the confidence intervals for the coefficient of variation. Buntao and Niwitpong [17] used delta-lognormal and lognormal distributions to create the confidence intervals for the coefficient of variation. D'Cunha and Rao [14] described a method for calculating the coefficient of variation of the lognormal distribution using Bayesian inference. Sangnawakij and Niwitpong [18] examined the Gamma distribution's ratio coefficient of variation. Yosboonruang, Niwitpong, and Niwitpong [19] suggested confidence intervals for the difference between two independent coefficients of variation of the two delta-lognormal distributions. Puggard, Niwitpong, and Niwitpong [20] proposed confidence intervals for the coefficient of variation in the Birnbaum-Saunders distribution. La-ongkaew, Niwitpong, and Niwitpong [21] presented the confidence intervals for the ratio of the coefficients of variation between the two Weibull distributions.

Many researchers have studied and developed confidence intervals for parameters in various probability distributions. From the study on constructing confidence intervals for parameters in various positive distributions that include zero values, it was found that the generalized confidence interval and normal approximation methods are effective. Additionally, the bootstrap confidence interval is recognized as a fundamental technique for constructing confidence intervals. Many researchers recommend these methods after comparing them with other methods. However, to date, there has been no research conducted on confidence intervals for parameters of the delta-Birnbaum-Saunders distribution. As a result, the purpose of this study is to construct confidence intervals for the coefficients of variation in the delta-Birnbaum-Saunders distribution. This study proposes three methods for constructing confidence intervals: the normal approximation, the generalized confidence interval that estimates the proportion of zero using variance-stabilizing transformation, as proposed by Wu and Hsieh [22], and the generalized confidence interval that estimates the proportion of zero using the Wilson score method, as proposed by Li, Zhou, and Tian [23]. These three methods are then compared with the bootstrap confidence interval. Furthermore, to validate the accuracy of these methods, all four of them will be applied to real-world data, specifically wind speed data collected in Ubon Ratchathani and Si Sa Kat, Thailand.

2. Preliminary

Let $Y = (Y_1, Y_2, \dots, Y_n)$ be a random sample from the delta-Birnbaum-Saunders (DBS) distribution with the proportion of zero δ, shape parameter α, and scale parameter β, denoted by

$Y \sim DBS(\delta, \alpha, \beta)$, the probability density function for the delta-Birnbaum-Saunders population is expressed as

$$f(y;\delta,\alpha,\beta) = \delta \mathrm{I}_0[y] + (1-\delta)\frac{1}{2\alpha\sqrt{2\pi}}\left[\left(\frac{\beta}{y}\right)^{1/2} + \left(\frac{\beta}{y}\right)^{3/2}\right]\exp\left[-\frac{1}{2}\left(\frac{y}{\beta}+\frac{\beta}{y}-2\right)\right]\mathrm{I}_{(0,\infty)}[y],$$

where I is an indicator function, with $\mathrm{I}_0[y] = \begin{cases} 1; & y=0, \\ 0; & otherwise, \end{cases}$ and $\mathrm{I}_{(0,\infty)}[y] = \begin{cases} 0; & y=0, \\ 1; & y>0. \end{cases}$ Then the distribution function of Y is given by

$$G(y;\delta,\alpha,\beta) = \begin{cases} \delta; & y=0, \\ \delta + (1-\delta)F(y;\alpha,\beta); & y>0, \end{cases} \tag{1}$$

where $F(y;\alpha,\beta)$ is the Birnbaum-Saunders distribution function. For $Y=0$, the number of zero observations is distributed according to the binomial distribution denoted by $n_{(0)} \sim Binomial(n,\delta)$. Given $n = n_{(1)} + n_{(0)}$, where $n_{(1)}$ and $n_{(0)}$ represent the numbers of positive and zero values, respectively, the maximum likelihood estimate of δ is $\hat{\delta} = \frac{n_{(0)}}{n}$. According to the Aitchison [6] concept, the population mean, variance, and coefficient of variation can be calculated as follows:

$$E(Y) = (1-\delta)\beta\left(1+\frac{\alpha^2}{2}\right),$$

$$V(Y) = (1-\delta)(\alpha\beta)^2\left(1+\frac{5\alpha^2}{4}\right) + \delta(1-\delta)\beta^2\left(1-\frac{\alpha^2}{2}\right)^2,$$

and

$$\theta = \frac{E(Y)}{V(Y)} = \frac{1}{2+\alpha^2}\sqrt{\frac{\alpha^2(4+5\alpha^2)+\delta(2+\alpha^2)^2}{1-\delta}}. \tag{2}$$

The method for constructing confidence intervals for the coefficient of variation of the delta-Birnbaum-Saunders distribution will be presented in the next section.

3. Proposed methods

3.1. Normal approximation

The normal approximation (NA) method is a technique that depends on the sample size, becoming more accurate as the sample size increases. A statistical approach used to derive an estimator with an asymptotically normal distribution is the delta method. Let

$$\theta = g(\alpha,\delta) = \frac{1}{2+\alpha^2}\sqrt{\frac{\alpha^2(4+5\alpha^2)+\delta(2+\alpha^2)^2}{1-\delta}}.$$

Using the delta method, the asymptotic distribution of the estimator based on the Taylor series of $g(\hat{\alpha},\hat{\delta})$ at α and δ is calculated as follows:

$$g(\hat{\alpha},\hat{\delta}) = g(\alpha,\delta) + \frac{\partial g(\alpha,\delta)}{\partial \alpha}(\hat{\alpha}-\alpha) + \frac{\partial g(\alpha,\delta)}{\partial \delta}(\hat{\delta}-\delta) + Remainder. \quad (3)$$

Consider that it is possible to demonstrate that the probability of $\sqrt{n}Remainder$ converges to 0 as the sample size n approaches infinity. Since $\hat{\alpha} \sim N\left(\alpha, \frac{\alpha^2}{2n_{(1)}}\right)$ and $\hat{\delta} \sim N\left(\delta, \frac{\delta(1-\delta)}{n}\right)$, following computations, we can obtain

$$g(\hat{\alpha},\hat{\delta}) \approx \frac{1}{2+\alpha^2}\sqrt{\frac{\alpha^2(4+5\alpha^2)+\delta(2+\alpha^2)^2}{1-\delta}}$$
$$+\frac{8\alpha(1-\alpha^2)}{(2+\alpha^2)^2\sqrt{(1-\delta)[\alpha^2(4+5\alpha^2)+\delta(2+\alpha^2)^2]}}(\hat{\alpha}-\alpha)$$
$$+\frac{2+\alpha^2(4-3\alpha^2)}{(2+\alpha^2)\sqrt{(1-\delta)^3[\alpha^2(4+5\alpha^2)+\delta(2+\alpha^2)^2]}}(\hat{\delta}-\delta).$$

Subsequently, we can calculate the asymptotic mean and variance of the estimator as follows:

$$E[g(\hat{\alpha},\hat{\delta})] \approx \frac{1}{2+\alpha^2}\sqrt{\frac{\alpha^2(4+5\alpha^2)+\delta(2+\alpha^2)^2}{1-\delta}}$$

and

$$V[g(\hat{\alpha},\hat{\delta})] \approx \frac{1}{O(1-\delta)[\alpha^2(4+5\alpha^2)+O\delta]}\left\{\frac{32\alpha^4(1+2\alpha^2)^2}{n_{(1)}O} + \frac{\delta[2+\alpha^2(4+3\alpha^2)]^2}{n(1-\delta)}\right\},$$

where $O = (2+\alpha^2)^2$. Detailed procedures for deriving the asymptotic mean and variance are provided in the appendix. Assume that $\hat{\alpha}$ and $\hat{\delta}$ are independent; then the maximum likelihood estimator of θ can be determined as

$$\hat{\theta} = \frac{1}{2+\hat{\alpha}^2}\sqrt{\frac{\hat{\alpha}^2(4+5\hat{\alpha}^2)+\hat{\delta}(2+\hat{\alpha}^2)^2}{1-\hat{\delta}}}. \quad (4)$$

where $\hat{\alpha} = \left\{2\left[\left(\sum_{i=1}^{n_{(1)}}\frac{y_i}{n_{(1)}}\left(\sum_{i=1}^{n_{(1)}}\frac{y_i^{-1}}{n_{(1)}}\right)\right)^{1/2} - 1\right]\right\}^{1/2}$ is the modified moment estimator of α proposed by Ng, Kundu, and Balakrishnan [24]. Then, the estimated variance of $\hat{\theta}$ can be written as

$$\hat{V}[\hat{\theta}] \approx \frac{1}{H(1-\hat{\delta})[\hat{\alpha}^2(4+5\hat{\alpha}^2)+H\hat{\delta}]}\left\{\frac{32\hat{\alpha}^4(1+2\hat{\alpha}^2)^2}{n_{(1)}H} + \frac{\hat{\delta}[2+\hat{\alpha}^2(4+3\hat{\alpha}^2)]^2}{n(1-\hat{\delta})}\right\}, \quad (5)$$

where $H = (2+\hat{\alpha}^2)^2$. A random variable $Z = \frac{\hat{\theta}-\theta}{\sqrt{\hat{V}(\hat{\theta})}} \sim N(0,1)$ according to the central limit theorem. Therefore, the $(1-\upsilon)100\%$ CI for θ based on NA is given by

$$[L_{NA}, U_{NA}] = \left[\hat{\theta} - z_{\upsilon/2}\sqrt{\hat{V}(\hat{\theta})}, \hat{\theta} + z_{\upsilon/2}\sqrt{\hat{V}(\hat{\theta})}\right], \tag{6}$$

where $z_{\upsilon/2}$ is the $(\upsilon/2)$th quantile value from the standard normal distribution.

3.2. Generalized confidence interval

The concept of the generalized confidence interval (GCI) method proposed by Weerahandi [25] provides a general framework for constructing confidence intervals by considering the generalized pivotal quantity (GPQ). In constructing confidence intervals for θ based on GCI, the GPQs of β and α are taken into consideration. Let $T \sim t(n_{(1)} - 1)$. Sun [26] recommended that the GPQ of the β should be provided by

$$R_{\beta}(y;T) = \begin{cases} \max(\beta_1, \beta_2); & T \leq 0, \\ \min(\beta_1, \beta_2); & T > 0, \end{cases} \tag{7}$$

$\beta_1$and β_2 are the two solutions of the quadratic equation,

$$\left[(n_{(1)} - 1)A^2 - \frac{1}{n_{(1)}}BT^2\right]\beta^2 - 2[(n_{(1)} - 1)AC - (1 - AC)T^2]\beta + (n_{(1)} - 1)C^2 - \frac{1}{n_{(1)}}DT^2 = 0, \tag{8}$$

where $A = \frac{1}{n_{(1)}}\sum_{i=1}^{n_{(1)}}\frac{1}{\sqrt{Y_i}}$, $B = \sum_{i=1}^{n_{(1)}}\left(\frac{1}{\sqrt{Y_i}} - A\right)^2$, $C = \frac{1}{n_{(1)}}\sum_{i=1}^{n_{(1)}}\sqrt{Y_i}$, and $D = \sum_{i=1}^{n_{(1)}}\left(\sqrt{Y_i} - C\right)^2$, while the GPQ of α should be provided by

$$R_{\alpha}(y;U,T) = \sqrt{\frac{E_1 + E_2 R_{\beta}^2 - 2n_{(1)}R_{\beta}}{R_{\beta}U}}, \tag{9}$$

where $E_1 = \sum_{i=1}^{n_{(1)}} Y_i$, $E_2 = \sum_{i=1}^{n_{(1)}}\frac{1}{Y_i}$, and $U \sim \chi^2_{n_{(1)}}$.

For the GPQ of δ, we use two concepts: the variance-stabilized transformation (VST) and the Wilson score method (WS). The details are explained in the following subsections.

3.2.1. GCI based on the VST: G.VST

According to Wu and Hsieh [22], the GPQ of δ is defined as

$$R_{\delta}^{VST} = \sin^2\left[\arcsin\sqrt{\hat{\delta}} - \frac{V}{2\sqrt{n}}\right], \tag{10}$$

where $V = 2\sqrt{n}\left(\arcsin\sqrt{\hat{\delta}} - \arcsin\sqrt{\delta}\right) \sim N(0,1)$. Hence, the GPQ for θ is

$$R_{\theta}^{VST} = \frac{1}{2+R_{\alpha}^2}\sqrt{\frac{R_{\alpha}^2(4+5R_{\alpha}^2) + R_{\delta}^{VST}(2+R_{\alpha}^2)^2}{1-R_{\delta}^{VST}}}. \tag{11}$$

Consequently, the $(1-\upsilon)100\%$ CI for θ is based on G.VST is given by

$$[L_{G.VST}, U_{G.VST}] = \left[R_\theta^{VST}(\upsilon/2), R_\theta^{VST}(1-\upsilon/2)\right], \tag{12}$$

where $R_\theta^{VST}(\upsilon)$ is the $(\upsilon/2)$th percentile of R_θ^{VST}.

3.2.2. GCI based on the WS: G.WS

In accordance with Li, Zhou, and Tian [23], the GPQ of δ is described as

$$R_\delta^{WS} = \frac{n_{(0)}+\left(W_{\upsilon/2}^2/2\right)}{n+W_{\upsilon/2}^2} - \frac{W_{\upsilon/2}^2}{n+W_{\upsilon/2}^2}\sqrt{n_{(0)}\left(1-\frac{n_{(0)}}{n}\right)+\frac{W_{\upsilon/2}^2}{4}}, \tag{13}$$

where $W = \frac{n_{(0)}-n\delta}{\sqrt{n\delta(1-\delta)}}$. Thus, the GPQ for θ is

$$R_\theta^{WS} = \frac{1}{2+R_\alpha^2}\sqrt{\frac{R_\alpha^2(4+5R_\alpha^2)+R_\delta^{WS}\left(2+R_\alpha^2\right)^2}{1-R_\delta^{WS}}}. \tag{14}$$

Therefore, the $(1-\upsilon)100\%$ CI for θ is based on G.WS is given by

$$[L_{G.WS}, U_{G.WS}] = \left[R_\theta^{WS}(\upsilon/2), R_\theta^{WS}(1-\upsilon/2)\right], \tag{15}$$

where $R_\theta^{WS}(\upsilon)$ is the $(\upsilon/2)$th percentile of R_θ^{WS}.

3.3. Bootstrap confidence interval

The bootstrap method is a resampling technique used to estimate the sampling distribution of a statistic by repeatedly resampling from the observed data with replacement, as proposed by Efron [27]. Let $\hat{\alpha}^{/}$ and $\hat{\delta}^{/}$ be observed values of $\hat{\alpha}$ and $\hat{\delta}$ based on bootstrap samples. Suppose that K bootstrap samples are available. The bootstrap expectation $E(\hat{\alpha})$ can be approximated by using the mean $\hat{\alpha}_{(.)}^{/} = \frac{1}{K}\sum_{j=1}^{K}\hat{\alpha}_j^{/}$, where $\hat{\alpha}_j^{/}$ is sequence of the bootstrap MLEs of α, for $j = 1,2,\dots,K$. The bootstrap bias estimate based on K replications of $\hat{\alpha}$ is given by

$$\widehat{K}(\hat{\alpha},\alpha) = \hat{\alpha}_{(.)}^{/} - \hat{\alpha}.$$

Then, the constant-bias-correcting estimates, as defined by Mackinnon and Smith [28], are used for creating the bias-corrected estimator, which is

$$\hat{\alpha}^{\#} = \hat{\alpha}^{/} - 2\widehat{K}(\hat{\alpha},\alpha). \tag{16}$$

According to Brown, Cai, and DasGupta [29], they proposed the Jeffreys interval for the binomial proportion, which employs the Jeffreys prior and is represented by $Beta(0.5,0.5)$. Therefore, it results in

$$\hat{\delta}^{*} \sim Beta\left(n_{(0)}^{*}+0.5, n_{(1)}^{*}+0.5\right), \tag{17}$$

where $n_{(0)}^{*}=n\hat{\delta}^{/}$and $n_{(1)}^{*}=n\left(1-\hat{\delta}^{/}\right)$. The bootstrap estimator of θ can be written as

$$\hat{\theta}^{(Boot)}=\frac{1}{2+(\hat{\alpha}^{\#})^{2}} \sqrt{\frac{(\hat{\alpha}^{\#})^{2}(4+5(\hat{\alpha}^{\#})^{2})+\hat{\delta}^{*}(2+(\hat{\alpha}^{\#})^{2})^{2}}{1-\hat{\delta}^{*}}}. \tag{18}$$

Consequently, the $(1-\upsilon)100\%$ CI for θ is based on BCI is given by

$$\left[L_{BCI}, U_{BCI}\right]=\left[\hat{\theta}^{(Boot)}(\upsilon/2), \hat{\theta}^{(Boot)}(1-\upsilon/2)\right], \tag{19}$$

where $\hat{\theta}^{(Boot)}(\upsilon)$ is the $(\upsilon/2)$th percentile of $\hat{\theta}^{(Boot)}$.

4. Results and discussion

In this simulation study, we have compared the performance of the proposed methods by considering the coverage probabilities greater than or equal to the nominal confidence level of 0.95, along with the expected lengths of the shortest confidence interval. This comparison was conducted using Monte Carlo simulations and the statistical software R. The overall number of replications was set to generate a simulation with 5,000 replications in total, 1,000 replications for the GCI, and 500 replications for the BCI. In addition, the sample size has been set to n = 30, 50, 100, 150, and 200, and the following parameters have been specified: δ= 0.1, 0.5, and 0.7; α = 0.25, 0.50, 0.75, 1.00, and 1.50; and β = 1. The algorithm presents the steps for estimating the coverage probability and expected length to compare the efficiency of the proposed methods.

The results from Table 1 are as follows: it is evident that the NA and BCI methods have values that are close, both in terms of coverage probabilities and expected lengths. Similarly, the G.VST and G.WS methods also exhibit close values to each other in almost all the cases studied, with coverage probabilities remaining stable and close to 0.95, and they have the shortest expected lengths. This results in the G.VST and G.WS methods being more efficient than the NA and BCI methods. Figure 1 shows a comparison of various methods in terms of shape parameters relative to coverage probability and expected length. It is evident that the coverage probabilities for the G.VST and G.WS methods are consistently greater than and close to the nominal confidence level of 0.95 in almost all cases. The BCI method achieves a coverage probability close to the specified criterion when the shape parameters are 0.25 and 1.00. Meanwhile, for the NA method, the coverage probability meets the required criterion when the shape parameter is small. As the shape parameter increases, the NA method's coverage probability shows a tendency to decrease. When considering the expected lengths, a consistent trend is observed for all methods. As the shape parameter value increases, expected lengths also increase progressively. Figure 2 shows a comparison of various methods in terms of sample sizes relative to coverage probability and expected length. It was found that the coverage probabilities of the G.VST and G.WS methods meet the specified criteria. For the NA method, the coverage probability increases as the sample size increases. The BCI method provides coverage probability close to the specified level only when the sample size is 50. Regarding the expected length, it reveals that as the sample size increases, the expected length for all methods decreases, resulting in improved efficiency. Figure 3 shows a comparison of various methods in terms of the proportion of zero relative to coverage probability and expected length. It demonstrates that the coverage probabilities for the G.VST and G.WS methods consistently align closely with the specified confidence level. However, the NA and BCI methods provide coverage probabilities that fall below the specified confidence level. When examining expected length, a similar trend is observed across all methods: as the proportion of zero increases, the expected length also increases. Nonetheless, the G.VST and G.WS methods yield a shorter expected length compared to the NA and BCI methods.

Algorithm: The coverage probability and expected length

I. For a given α, n, δ, and β.

II. For $m = 1$ to M. Generate sample from the DBS distribution and calculate $\hat{\alpha}$ and $\hat{\delta}$.

III. Construct CIs for θ based on the NA, GCI, and BCI:

For the NA; Calculate $\hat{V}(\hat{\theta})$ and calculate L_{NA} and U_{NA} using Equations (6) and (7).

For the GCI;

1. Calculate A, B, C, D, E_1 and E_2, respectively.
2. At the gth step
   a) Generate $T \sim t(n_{(1)} - 1)$, and then calculate $R_\beta(y; T)$ using Equation (7).
   b) If $R_\beta(y; T) < 0$, regenerate $T \sim t(n_{(1)} - 1)$.
   c) Generate $U \sim \chi^2_{n_{(1)}}$, and then calculate $R_\alpha(y; T)$ using Equation (9)
   d) For *G.VST* method, calculate R_δ^{VST} and R_θ^{VST} using Equations (10) and (11).
   e) For *G.WS* method, calculate R_δ^{WS} and R_θ^{WS} using Equations (13) and (14).
3. Repeat step 2. a number of times, with G = 1,000 times.
4. Calculate $L_{G.VST}$, $U_{G.VST}$, $L_{G.WS}$, and $U_{G.WS}$ using Equations (12) and (15).

For BCI;

1. At the bth step
   a) Generate $y_1^*, y_2^*, \dots, y_n^*$ with replacement from $y_1, y_2, \dots, y_n$.
   b) Calculate $\hat{\alpha}^{/}$ and $\widehat{K}(\hat{\alpha}, \alpha)$.
   c) Calculate $\hat{\alpha}^{\#}$, $\hat{\delta}^*$ and $\hat{\theta}^{(Boot)}$ using Equations (16), (17) and (18).
2. Repeat step 1. a number of times, with B = 500 times.
3. Calculate $L_{BCI}$and U_{BCI} using Equation (19).

IV. If $L[\leq \theta \leq U]$, set $H = 1$; else set $H = 0$. The coverage probability and expected length for each method are obtained by $CP = \frac{1}{M}\sum_{i=1}^{M} H_m$ and $EL = \frac{U-L}{M}$, where U and L are the upper and lower confidence limits, respectively. (End m loop)

Table 1. The coverage probabilities and expected lengths for the 95% CIs for θ.

α	n	δ	Coverage probabilities				Expected lengths			
			NA	G.VST	G.WS	BCI	NA	G.VST	G.WS	BCI
0.25	30	0.1	0.9176	0.9508	0.9498	0.9318	0.3337	0.1001	0.1002	0.3308
		0.3	0.9492	0.9515	0.9589	0.9444	0.5126	0.0934	0.0928	0.5075
		0.5	0.9502	0.9489	0.9522	0.9505	0.7606	0.1195	0.1187	0.7635
	50	0.1	0.9256	0.9526	0.9450	0.9417	0.2646	0.0740	0.0739	0.2590
		0.3	0.9508	0.9520	0.9574	0.9422	0.3963	0.0671	0.0667	0.3919
		0.5	0.9564	0.9552	0.9520	0.9488	0.5818	0.0795	0.0802	0.5791
	100	0.1	0.9522	0.9513	0.9525	0.9597	0.1898	0.0506	0.0506	0.1861
		0.3	0.9561	0.9540	0.9604	0.9549	0.2770	0.0446	0.0442	0.2724
		0.5	0.9532	0.9537	0.9520	0.9502	0.4064	0.0515	0.0513	0.4013
	150	0.1	0.9406	0.9501	0.9484	0.9478	0.1547	0.0408	0.0408	0.1516
		0.3	0.9518	0.9542	0.9552	0.9510	0.2278	0.0356	0.0356	0.2268
		0.5	0.9522	0.9518	0.9532	0.9541	0.3298	0.0410	0.0409	0.3066
	200	0.1	0.9500	0.9520	0.9502	0.9530	0.1341	0.0350	0.0351	0.1337
		0.3	0.9506	0.9512	0.9546	0.9528	0.1968	0.0307	0.0307	0.1824
		0.5	0.9546	0.9548	0.9532	0.9502	0.2856	0.0348	0.0348	0.2589
0.50	30	0.1	0.9341	0.9533	0.9555	0.9523	0.3683	0.2688	0.2678	0.3641
		0.3	0.9512	0.9514	0.9549	0.9487	0.5689	0.2919	0.2932	0.5545
		0.5	0.9524	0.9519	0.9516	0.9422	0.8574	0.3832	0.3848	0.8439
	50	0.1	0.9465	0.9465	0.9516	0.9347	0.2875	0.2005	0.2004	0.2818
		0.3	0.9554	0.9517	0.9484	0.9479	0.4339	0.2142	0.2134	0.4194
		0.5	0.9547	0.9493	0.9497	0.9405	0.6514	0.2696	0.2689	0.6311
	100	0.1	0.9556	0.9484	0.9466	0.9492	0.2035	0.1385	0.1384	0.1981
		0.3	0.9562	0.9525	0.9476	0.9637	0.3078	0.1449	0.1439	0.2945
		0.5	0.9628	0.9504	0.9491	0.9465	0.4615	0.1785	0.1780	0.4433
	150	0.1	0.9506	0.9505	0.9515	0.9042	0.1663	0.1122	0.1122	0.1686
		0.3	0.9566	0.9537	0.9529	0.9174	0.2511	0.1168	0.1167	0.2570
		0.5	0.9544	0.9536	0.9552	0.9294	0.3739	0.1426	0.1425	0.3694
	200	0.1	0.9526	0.9526	0.9543	0.9256	0.1442	0.0968	0.0968	0.1404
		0.3	0.9562	0.9524	0.9560	0.9376	0.2174	0.1005	0.1004	0.2126
		0.5	0.9620	0.9518	0.9520	0.9498	0.3235	0.1220	0.1222	0.1200
0.75	30	0.1	0.9352	0.9511	0.9416	0.9538	0.4527	0.4075	0.4038	0.4480
		0.3	0.9310	0.9509	0.9450	0.9412	0.6593	0.4705	0.4708	0.6473
		0.5	0.9306	0.9530	0.9409	0.9405	0.9800	0.6242	0.6206	0.9646
	50	0.1	0.9475	0.9651	0.9448	0.9547	0.3545	0.3070	0.3081	0.3503
		0.3	0.9526	0.9563	0.9602	0.9510	0.5122	0.3538	0.3533	0.4972
		0.5	0.9459	0.9456	0.9564	0.9395	0.7601	0.4572	0.4583	0.7385
	100	0.1	0.9523	0.9460	0.9615	0.9482	0.2518	0.2144	0.2135	0.2475
		0.3	0.9622	0.9459	0.9536	0.9513	0.3627	0.2436	0.2453	0.3515
		0.5	0.9567	0.9431	0.9487	0.9500	0.5386	0.3132	0.3101	0.5203

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α	n	δ	Coverage probabilities				Expected lengths			
			NA	G.VST	G.WS	BCI	NA	G.VST	G.WS	BCI
0.75	150	0.1	0.9538	0.9510	0.9524	0.8598	0.2061	0.1743	0.1741	0.2027
		0.3	0.9530	0.9504	0.9512	0.8790	0.2967	0.1986	0.1986	0.2670
		0.5	0.9558	0.9546	0.9519	0.9306	0.4399	0.2522	0.2525	0.4332
	200	0.1	0.9518	0.9502	0.9528	0.8724	0.1793	0.1510	0.1509	0.1709
		0.3	0.9518	0.9516	0.9532	0.9012	0.2567	0.1711	0.1709	0.2207
		0.5	0.9530	0.9546	0.9552	0.9386	0.3810	0.2178	0.2177	0.3418
1.00	30	0.1	0.9190	0.9482	0.9506	0.9352	0.5233	0.4901	0.4904	0.5209
		0.3	0.9247	0.9471	0.9505	0.9407	0.7447	0.5875	0.5879	0.7419
		0.5	0.9321	0.9460	0.9500	0.9413	1.1191	0.7878	0.7901	1.1260
	50	0.1	0.9342	0.9478	0.9528	0.9387	0.4100	0.3755	0.3749	0.4065
		0.3	0.9333	0.9506	0.9625	0.9471	0.5742	0.4467	0.4455	0.5743
		0.5	0.9411	0.9576	0.9610	0.9410	0.8602	0.5933	0.5933	0.8628
	100	0.1	0.9472	0.9511	0.9504	0.9548	0.2901	0.2625	0.2616	0.2892
		0.3	0.9490	0.9521	0.9582	0.9522	0.4090	0.3111	0.3127	0.4080
		0.5	0.9489	0.9545	0.9597	0.9514	0.6065	0.4093	0.4101	0.6084
	150	0.1	0.9468	0.9514	0.9516	0.9556	0.2375	0.2142	0.2141	0.2318
		0.3	0.9446	0.9526	0.9508	0.9536	0.3335	0.2538	0.2538	0.3035
		0.5	0.9504	0.9530	0.9502	0.9526	0.4945	0.3328	0.3329	0.4903
	200	0.1	0.9482	0.9540	0.9588	0.9524	0.2061	0.1854	0.1853	0.2059
		0.3	0.9524	0.9510	0.9532	0.9508	0.2892	0.2196	0.2195	0.2768
		0.5	0.9522	0.9524	0.9546	0.9510	0.4279	0.2872	0.2875	0.4209
1.50	30	0.1	0.9209	0.9490	0.9542	0.9433	0.5521	0.5213	0.5235	0.5702
		0.3	0.9072	0.9540	0.9528	0.9570	0.7735	0.6436	0.6442	0.8364
		0.5	0.8931	0.9505	0.9449	0.9308	1.1629	0.8781	0.8739	1.2941
	50	0.1	0.9246	0.9529	0.9548	0.9418	0.4281	0.4016	0.4012	0.4409
		0.3	0.9162	0.9580	0.9522	0.9409	0.5987	0.4948	0.4947	0.6473
		0.5	0.9097	0.9498	0.9560	0.9494	0.8909	0.6693	0.6685	0.9837
	100	0.1	0.9453	0.9571	0.9535	0.9530	0.3028	0.2822	0.2823	0.3135
		0.3	0.9349	0.9498	0.9526	0.9591	0.4231	0.3465	0.3468	0.4583
		0.5	0.9164	0.9520	0.9484	0.9438	0.6292	0.4660	0.4664	0.6997
	150	0.1	0.9338	0.9522	0.9514	0.9006	0.2475	0.2298	0.2301	0.2580
		0.3	0.9272	0.9542	0.9522	0.9086	0.3439	0.2822	0.2822	0.3483
		0.5	0.9188	0.9504	0.9500	0.9230	0.5113	0.3794	0.3795	0.5248
	200	0.1	0.9522	0.9506	0.9510	0.9218	0.2144	0.1991	0.1992	0.2232
		0.3	0.9364	0.9554	0.9542	0.9226	0.2977	0.2441	0.2440	0.3085
		0.5	0.9276	0.9532	0.9500	0.9430	0.4417	0.3277	0.3279	0.3317

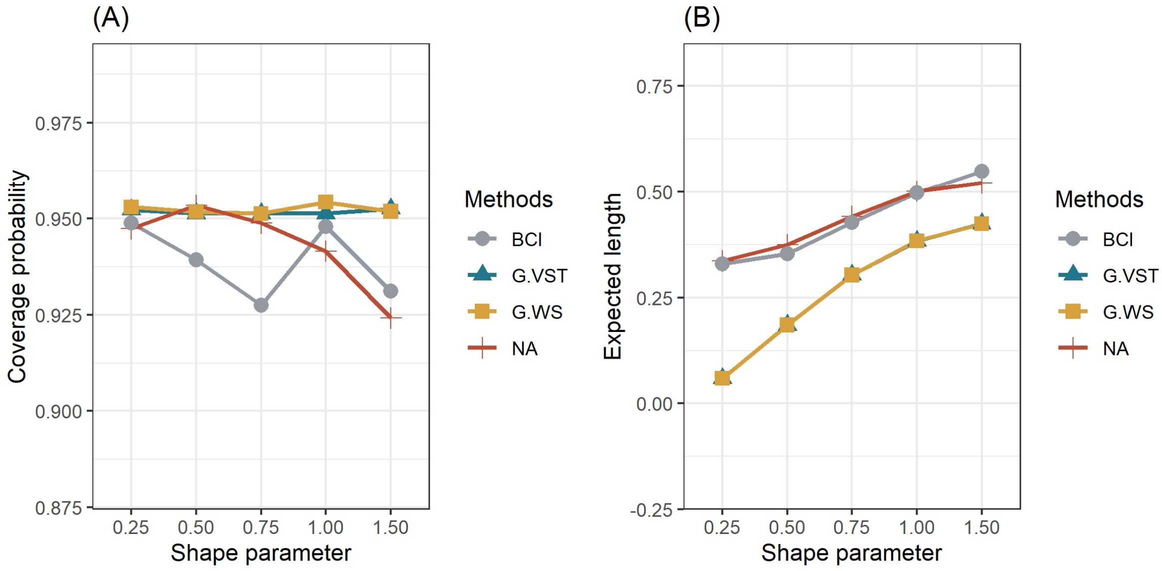

Figure 1. Graphs comparing the performance of the shape parameter with respect to the (A) coverage probability and (B) expected length.

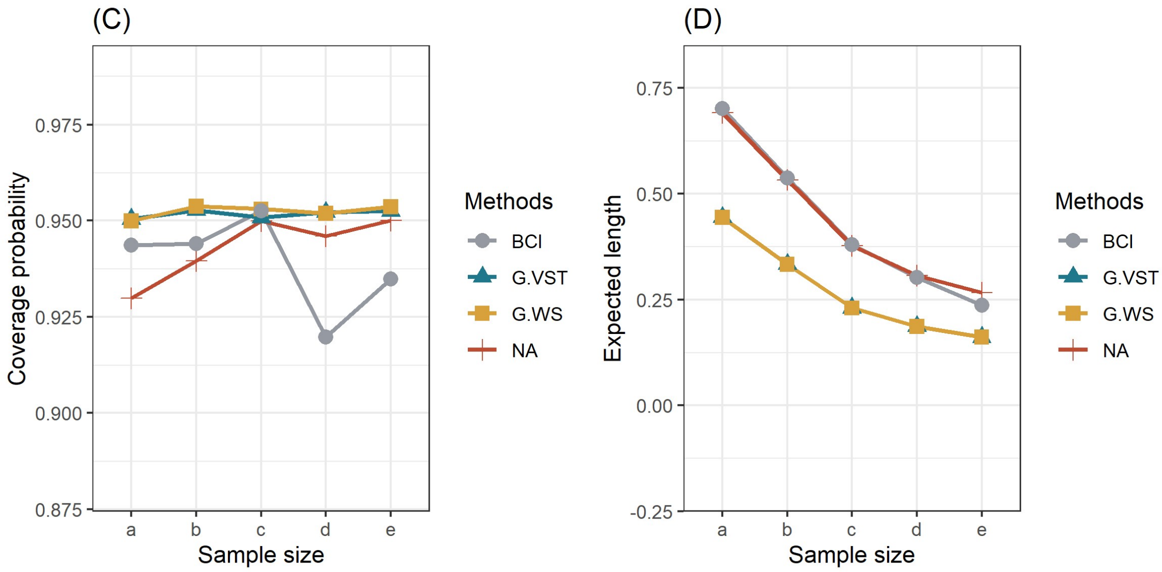

Figure 2. Graphs comparing the performance of the sample sizes with respect to the (C) coverage probability and (D) expected length (a = 30, b = 50, c = 100, d = 150, e = 200).

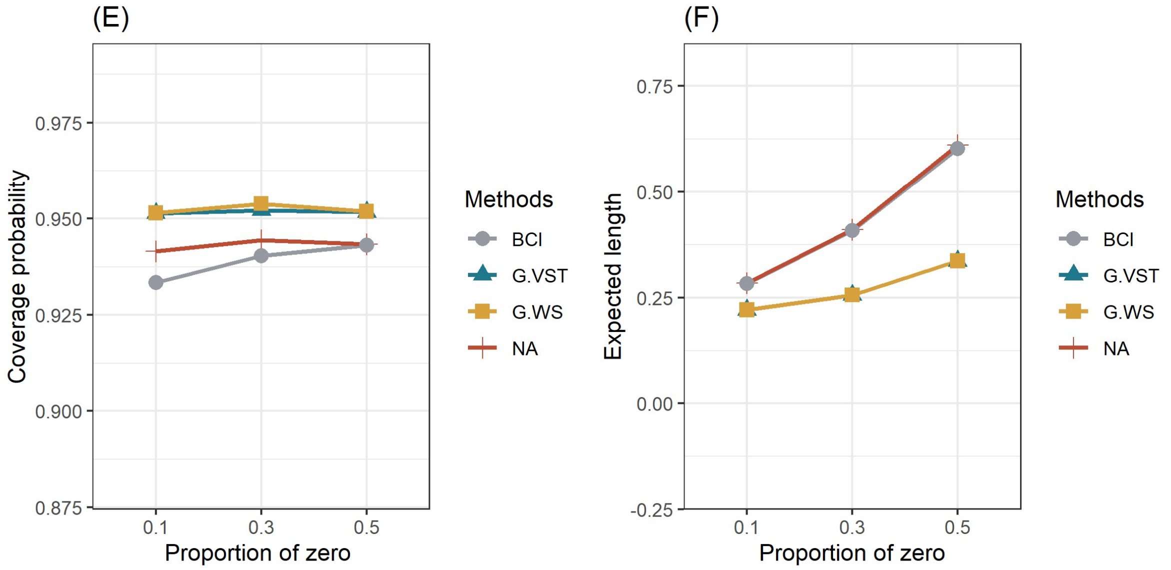

Figure 3. Graphs comparing the performance of the proportion of zero with respect to the (E) coverage probability and (F) expected length.

5. Application

Wind speed plays multiple important roles and has various impacts, particularly in agriculture. It affects plant growth rates, leading to faster growth and increased crop yields. Because Thailand is known as an agricultural country, a large portion of its population has always been engaged in farming or related occupations. Therefore, wind speed is an important factor that affects agriculture in Thailand. In this research, wind speed data from Ubon Ratchathani province for the hourly periods on March 9–10, 2023, and wind speed data from Si Sa Kat province for the hourly periods on April 3–7, 2023, have been applied for analysis, as presented in Tables 2 and 3. The wind speed data for both provinces was obtained from the Automatic Weather System in Thailand (http://www.aws-observation.tmd.go.th/main/main). We have plotted histograms of the wind speed data for Ubon Ratchathani and Si Sa Kat provinces to visualize the data distribution, shown in Figures 4 and 5. Since the wind speed data include both zero values (no wind) and positive values, we examined the suitability of the data distribution for positive values by comparing it to other distributions, including the normal, exponential, Cauchy, logistic, and Birnbaum-Saunders distributions. To assess the suitability of these distributions for the data, we have used the Akaike information criterion (AIC) and the Bayesian information criterion (BIC), calculated as

$$AIC = 2\ln(L) + 2p,$$

and

$$AIC = 2\ln(L) + 2p\ln(o),$$

respectively, where p represents the number of parameters estimated, o represents the number of observations, and L represents the likelihood function. From Table 4, it is evident that the AIC and BIC values for the Birnbaum-Saunders distribution are the lowest compared to other distributions. This suggests that the Birnbaum-Saunders distribution is the most suitable for the positive value of the wind speed data. As a result, the wind speed data, which contains both positive and zero values, is modeled as the delta-Birnbaum-Saunders distribution. Consequently, we have used this distribution to calculate confidence intervals for the coefficients of variation of the wind speed data. In addition, we have presented summary statistics for the wind speed data in Table 5. In the wind speed data, the parameter α represents the shape or skewness of the distribution, reflecting the tendency toward lower or higher-than-normal wind speeds. The parameter β indicates the scale of the wind speed distribution in the area; if β changes, the distribution of the data will also shift. The parameter δ represents the proportion of zero values in the dataset. Point estimates or coefficients of variation for Ubon Ratchathani and Si Sa Kat provinces were found to be 1.2183 and 1.3085, respectively. Table 6 presents the calculated 95% confidence intervals for the coefficient of variation for the wind speed data from Ubon Ratchathani and Si Sa Kat provinces. We compared the wind speed data from Ubon Ratchathani with the parameters from the data simulation, using the sample size of n = 50, parameter α = 0.75, and parameter δ = 0.3, from Table 1. The simulation results indicate that the NA, G.VST, G.WS, and BCI methods achieve coverage probabilities greater than the specified confidence level of 0.95. Additionally, it was found that the G.WS method provides the shortest confidence interval compared to other methods. The confidence interval for the wind speed data from Ubon Ratchathani using the G.WS method is (1.0797, 1.4925), with the confidence interval length of 0.4128, the shortest among the methods. This indicates that the study results are consistent. Subsequently, we compared the wind speed data from Si Sa Ket with the parameters from the data simulation using the sample size of n = 100, parameter α = 1.00, and parameter δ = 0.3, from Table 1. The simulation results show that the G.VST, G.WS, and BCI methods achieve coverage probabilities greater than the specified confidence level, while the NA method has a coverage probability lower than the specified confidence level. Therefore, we considered only the G.VST, G.WS, and BCI methods. It was found that the G.VST method provides the shortest confidence interval. The confidence interval for the wind speed data from Si Sa Ket using the G.VST method is (1.2002, 1.4655), with the confidence interval length of 0.2653, the shortest among all methods. This indicates that the study results are consistent. Consequently, to construct confidence intervals for the coefficient of variation of wind speed data in Thailand, we recommend using the G.WS method for Ubon Ratchathani province and the G.VST method for Si Sa Ket province.

Table 2. Data on the wind speed of Ubon Ratchathani, Thailand.

Data on the wind speed of Ubon Ratchathani (Knots)							
4.9	2.9	2.3	1.0	3.9	0.0	0.6	5.8
0.8	1.6	3.3	2.3	2.1	0.0	0.0	4.7
6.2	3.9	0.8	0.0	1.6	0.0	0.0	1.6
3.3	5.2	1.9	0.0	1.9	0.0	0.6	0.0
3.3	0.6	3.3	0.0	1.6	1.6	0.0	0.6
0.4	3.1	3.9	0.0	0.0	0.0	0.0	0.0

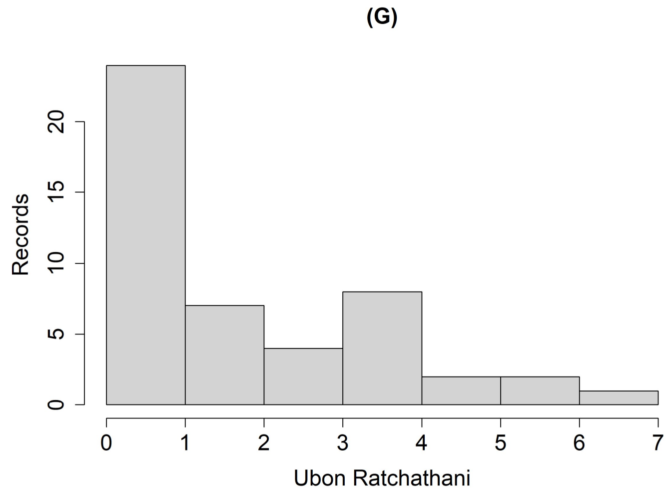

Figure 4. Histogram of wind speed data for Ubon Ratchathani.

Table 3. Data on the wind speed of Si Sa Kat, Thailand.

Data on the wind speed of Si Sa Kat (Knots)											
0.2	0.2	1.2	0.4	1.6	2.5	4.3	2.1	3.3	0.0	1.2	0.0
2.3	0.0	3.3	0.4	1.9	9.9	8.7	0.0	2.9	1.0	0.0	1.6
0.4	0.0	5.8	0.8	1.0	5.2	6.2	1.2	1.2	0.6	0.0	0.8
2.3	0.0	7.6	0.6	0.6	7.2	5.8	0.0	3.9	0.4	0.0	0.0
8.7	0.6	5.8	0.0	1.2	5.6	8.9	0.6	2.1	0.0	1.6	1.2
3.3	2.5	5.2	0.6	0.6	0.8	2.5	0.8	3.5	1.6	0.6	1.0
3.9	1.2	0.8	0.0	0.6	8.4	2.1	0.8	2.9	0.0	0.0	0.0
2.9	0.0	2.9	1.2	0.0	2.5	0.2	0.2	3.9	1.2	0.0	0.0
2.5	0.8	1.2	2.9	0.8	4.3	3.3	0.8	0.4	0.0	1.6	0.6
3.1	0.2	2.3	1.2	1.6	1.6	3.1	0.0	0.6	1.0	0.6	0.0

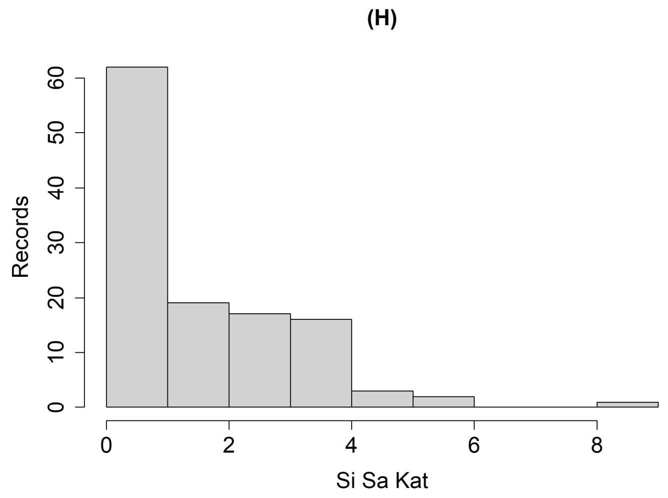

Figure 5. Histogram of wind speed data for Si Sa Kat.

Table 4. The AIC and BIC values of each model for the wind speed data.

Data	Model	Normal	Exponential	Birnbaum-Saunders	Cauchy	Logistic
Ubon Ratchathani	AIC	125.404	125.910	121.308	139.931	127.085
	BIC	128.335	127.378	125.705	142.863	130.016
Si Sa Kat	AIC	321.226	294.814	288.638	345.058	316.721
	BIC	326.135	297.268	296.001	349.967	321.630

Table 5. Summary statistics for the wind speed data.

Data	n	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\delta}$	$\hat{\theta}$
Ubon Ratchathani	48	0.7968	1.9355	0.3333	1.2183
Si Sa Kat	120	0.9682	1.3744	0.2833	1.3085

Table 6. The 95% confidence intervals for the coefficients of variation of the wind speed data.

Data	Methods	Interval	Length
Ubon Ratchathani	NA	(0.9314, 1.5052)	0.5738
	G.VST	(1.0736, 1.4905)	0.4169
	G.WS	(1.0797, 1.4925)	0.4128
	BCI	(0.9795, 1.5470)	0.5675
Si Sa Kat	NA	(1.1290, 1.4881)	0.3591
	G.VST	(1.2002, 1.4655)	0.2653
	G.WS	(1.1992, 1.4783)	0.2791
	BCI	(1.1382, 1.4969)	0.3587

6. Conclusions

In this study, we constructed confidence intervals for the coefficient of variation of the delta-Birnbaum-Saunders distribution. We proposed three methods: NA, G.VST, and G.WS, and compared them with BCI. Then, we compared the performance of the proposed method based on the coverage probabilities greater than or equal to the 0.95 confidence level, along with the expected lengths of the shortest confidence interval. The simulation results indicate that the coverage probabilities of the G.VST and G.WS methods are greater than or close to the nominal confidence level. Meanwhile, the NA method shows coverage probability greater than the nominal confidence level when the shape parameter is small. Additionally, the coverage probability of the BCI method becomes closer to the nominal confidence level as the sample size increases. Considering the expected lengths, the BCI method provides shorter confidence intervals than the NA method, except when the shape parameter is large. However, the G.VST and G.WS methods yield the shortest and most similar confidence intervals, making these two methods the most efficient overall. Moreover, all the proposed methods were applied to wind speed data in Thailand and yielded results consistent with the simulation outcomes. Therefore, the G.VST and G.WS methods are recommended for constructing confidence intervals for the coefficient of variation of the delta-Birnbaum-Saunders distribution. In future research, we will investigate new methods and expand the parameters of interest in the delta-

Birnbaum-Saunders distribution to enhance the effectiveness of constructing confidence intervals.

Author contributions

Usanee Janthasuwan analyzed the data, drafted, and wrote the manuscript. Suparat Niwitpong conceptualized and designed the experiment and revised the manuscript. Sa-Aat Niwitpong proposed analytical tools, approved the final draft, and secured funding.

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Conflict of interest

The authors declare no conflict of interest.

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Appendix

The asymptotic mean and variance

We have used the Delta method to obtain an estimator with an asymptotically normal distribution based on the Taylor series, as follows:

$$g(\hat{\alpha},\hat{\delta}) = g(\alpha,\delta) + \frac{\partial g(\alpha,\delta)}{\partial\alpha}(\hat{\alpha}-\alpha) + \frac{\partial g(\alpha,\delta)}{\partial\delta}(\hat{\delta}-\delta) + Remainder, \tag{A.1}$$

where $g(\alpha,\delta) = \frac{1}{2+\alpha^2}\sqrt{\frac{\alpha^2(4+5\alpha^2)+\delta(2+\alpha^2)^2}{1-\delta}}$. Now, we will calculate the partial derivatives of $g(\alpha,\delta)$ with respect to α as follows:

$$\frac{\partial g(\alpha,\delta)}{\partial\alpha} = \frac{\partial}{\partial\alpha}\left[\frac{1}{2+\alpha^2}\sqrt{\frac{\alpha^2(4+5\alpha^2)+\delta(2+\alpha^2)^2}{1-\delta}}\right]$$

$$\approx \left\{\frac{1}{2+\alpha^2}\frac{1}{2}\left[\frac{\alpha^2(4+5\alpha^2)+\delta(2+\alpha^2)^2}{1-\delta}\right]^{-\frac{1}{2}}\frac{1}{1-\delta}(8\alpha+20\alpha^3+8\alpha\delta+4\alpha^3\delta)\right\}$$

$$+\left\{\left[\frac{\alpha^2(4+5\alpha^2)+\delta(2+\alpha^2)^2}{1-\delta}\right]^{\frac{1}{2}}\left[-\frac{2\alpha}{(2+\alpha^2)^2}\right]\right\}$$

$$\approx \left\{\frac{4\alpha+10\alpha^3+4\alpha\delta+2\ \ {}^3\delta}{(2+\alpha^2)\sqrt{1-\delta[\alpha^2(4+5\alpha^2)+\delta(2+\alpha^2)^2]}}\right\} - \frac{2\alpha}{(2+\alpha^2)^2}\sqrt{\frac{\alpha^2(4+5\alpha^2)+\delta(2+\alpha^2)^2}{1-\delta}}$$

$$\approx \frac{2\alpha}{(2+\alpha^2)\sqrt{1-\delta}}\left[\frac{2+5\alpha^2+2\delta+\alpha^2\delta}{\sqrt{\alpha^2(4+5\alpha^2)+\delta(2+\alpha^2)^2}} - \frac{\sqrt{\alpha^2(4+5\alpha^2)+\delta(2+\alpha^2)^2}}{(2+\alpha^2)}\right]$$

$$\approx \frac{2\alpha}{(2+\alpha^2)\sqrt{1-\delta}}\left\{\frac{[(2+\alpha^2)(2+5\alpha^2+2\delta+\alpha^2\delta)]-\left[\alpha^2(4+5\alpha^2)+\delta(2+\alpha^2)^2\right]}{(2+\alpha^2)\sqrt{\alpha^2(4+5\alpha^2)+\delta(2+\alpha^2)^2}}\right\}$$

$$\approx \frac{2\alpha\left\{[(2+\alpha^2)(2+5\alpha^2+2\delta+\alpha^2\delta)]-\alpha^2(4+5\alpha^2)-\delta(2+\alpha^2)^2\right\}}{(2+\alpha^2)^2\sqrt{(1-\delta)\alpha^2(4+5\alpha^2)+\delta(2+\alpha^2)^2}}$$

$$\approx \frac{8\alpha(1+2\alpha^2)}{(2+\alpha^2)^2\sqrt{(1-\delta)\alpha^2(4+5\alpha^2)+\delta(2+\alpha^2)^2}}.$$

Next, we have calculated the partial derivatives of $g(\alpha,\delta)$ with respect to δ, and we obtain that

$$\frac{\partial g(\alpha,\delta)}{\partial \delta} = \frac{\partial}{\partial \delta}\left[\frac{1}{2+\alpha^2}\sqrt{\frac{\alpha^2(4+5\alpha^2)+\delta(2+\alpha^2)^2}{1-\delta}}\right]$$

$$\approx \frac{1}{2+\alpha^2}\left\{\frac{1}{2}\left[\frac{\alpha^2(4+5\alpha^2)+\delta(2+\alpha^2)^2}{1-\delta}\right]^{-\frac{1}{2}}\left[\frac{(1-\delta)(2+\alpha^2)^2+\left[\alpha^2(4+5\alpha^2)+\delta(2+\alpha^2)^2\right]}{(1-\delta)^2}\right]\right\}$$

$$\approx \frac{\alpha^2(4+5\alpha^2)+\delta(2+\alpha^2)^2}{2(2+\alpha^2)(1-\delta)^{3/2}\sqrt{\alpha^2(4+5\alpha^2)+\delta(2+\alpha^2)^2}}$$

$$\approx \frac{\alpha^2(4+5\alpha^2)+\delta(2+\alpha^2)^2}{(2+\alpha^2)\sqrt{(1-\delta)^3[\alpha^2(4+5\alpha^2)+\delta(2+\alpha^2)^2]}}.$$

After that, by using the equation above to substitute into Eq A.1, we get that

$$g(\hat{\alpha},\hat{\delta}) = g(\alpha,\delta) + \frac{\partial g(\alpha,\delta)}{\partial \alpha}(\hat{\alpha}-\alpha) + \frac{\partial g(\alpha,\delta)}{\partial \delta}(\hat{\delta}-\delta) + Remainder$$

$$\approx \frac{1}{2+\alpha^2}\sqrt{\frac{\alpha^2(4+5\alpha^2)+\delta(2+\alpha^2)^2}{1-\delta}}$$

$$+\frac{8\alpha(1+2\alpha^2)}{(2+\alpha^2)^2\sqrt{(1-\delta)\alpha^2(4+5\alpha^2)+\delta(2+\alpha^2)^2}}(\hat{\alpha}-\alpha)$$

$$+\frac{\alpha^2(4+5\alpha^2)+\delta(2+\alpha^2)^2}{(2+\alpha^2)\sqrt{(1-\delta)^3[\alpha^2(4+5\alpha^2)+\delta(2+\alpha^2)^2]}}(\hat{\delta}-\delta),$$

as $n \to \infty$. It is well known that the asymptotic distribution of α and δ is given by

$$\sqrt{n_{(1)}}(\hat{\alpha}-\alpha) \xrightarrow{D} N\left(0,\frac{\alpha^2}{2}\right) \text{ and } \sqrt{n}(\hat{\delta}-\delta) \xrightarrow{D} N(0,\delta(1-\delta)),$$

respectively. We have calculated the asymptotic mean of the coefficient of variation of the Delta-Birnbaum-Saunders distribution as follows:

$$E\left(g(\hat{\alpha},\hat{\delta})\right) \approx E\left[\frac{1}{2+\alpha^2}\sqrt{\frac{\alpha^2(4+5\alpha^2)+\delta(2+\alpha^2)^2}{1-\delta}}\right.$$

$$+\frac{8\alpha(1+2\alpha^2)}{(2+\alpha^2)^2\sqrt{(1-\delta)\alpha^2(4+5\alpha^2)+\delta(2+\alpha^2)^2}}(\hat{\alpha}-\alpha)$$

$$\left.+\frac{\alpha^2(4+5\alpha^2)+\delta(2+\alpha^2)^2}{(2+\alpha^2)\sqrt{(1-\delta)^3[\alpha^2(4+5\alpha^2)+\delta(2+\alpha^2)^2]}}\left(\hat{\delta}-\delta\right)\right]$$

$$\approx \frac{1}{2+\alpha^2}\sqrt{\frac{\alpha^2(4+5\alpha^2)+\delta(2+\alpha^2)^2}{1-\delta}}$$

$$+\frac{8\alpha(1+2\alpha^2)}{(2+\alpha^2)^2\sqrt{(1-\delta)\alpha^2(4+5\alpha^2)+\delta(2+\alpha^2)^2}}E(\hat{\alpha}-\alpha)$$

$$+\frac{\alpha^2(4+5\alpha^2)+\delta(2+\alpha^2)^2}{(2+\alpha^2)\sqrt{(1-\delta)^3[\alpha^2(4+5\alpha^2)+\delta(2+\alpha^2)^2]}}E\left(\hat{\delta}-\delta\right)$$

$$\approx \frac{1}{2+\alpha^2}\sqrt{\frac{\alpha^2(4+5\alpha^2)+\delta(2+\alpha^2)^2}{1-\delta}}.$$

In addition, the asymptotic variance of the coefficient of variation of the Delta-Birnbaum-Saunders distribution is given by

$$V\left(g(\hat{\alpha},\hat{\delta})\right) \approx V\left[\frac{1}{2+\alpha^2}\sqrt{\frac{\alpha^2(4+5\alpha^2)+\delta(2+\alpha^2)^2}{1-\delta}}\right.$$

$$+\frac{8\alpha(1+2\alpha^2)}{(2+\alpha^2)^2\sqrt{(1-\delta)\alpha^2(4+5\alpha^2)+\delta(2+\alpha^2)^2}}(\hat{\alpha}-\alpha)$$

$$\left.+\frac{\alpha^2(4+5\alpha^2)+\delta(2+\alpha^2)^2}{(2+\alpha^2)\sqrt{(1-\delta)^3[\alpha^2(4+5\alpha^2)+\delta(2+\alpha^2)^2]}}\left(\hat{\delta}-\delta\right)\right]$$

$$\approx \left[\frac{8\alpha(1+2\alpha^2)}{(2+\alpha^2)^2\sqrt{(1-\delta)\alpha^2(4+5\alpha^2)+\delta(2+\alpha^2)^2}}\right]^2 V(\hat{\alpha}-\alpha)$$

$$+\left[\frac{\alpha^2(4+5\alpha^2)+\delta(2+\alpha^2)^2}{(2+\alpha^2)\sqrt{(1-\delta)^3[\alpha^2(4+5\alpha^2)+\delta(2+\alpha^2)^2]}}\right]^2 V(\hat{\delta}-\delta)$$

$$\approx \frac{1}{(2+\alpha^2)^2(1-\delta)[\alpha^2(4+5\alpha^2)+\delta(2+\alpha^2)^2]}\left\{\frac{64\alpha^2(1+2\alpha^2)^2}{(2+\alpha^2)^2}\left(\frac{\alpha^2}{2n_{(1)}}\right)\right.$$

$$\left.+\frac{[2+\alpha^2(4+3\alpha^2)]^2}{(1-\delta)^2}\left(\frac{\delta(1-\delta)}{n}\right)\right\}$$

$$\approx \frac{1}{(2+\alpha^2)^2(1-\delta)[\alpha^2(4+5\alpha^2)+\delta(2+\alpha^2)^2]}\left\{\frac{32\alpha^4(1+2\alpha^2)^2}{n_{(1)}(2+\alpha^2)^2}\right.$$

$$\left.+\frac{\delta[2+\alpha^2(4+3\alpha^2)]^2}{n(1-\delta)}\right\}.$$

Note that $\hat{\alpha}\sim N\left(\alpha,\frac{\alpha^2}{2n_{(1)}}\right)$ and $\hat{\delta}\sim N\left(\delta,\frac{\delta(1-\delta)}{n}\right)$.