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Research article

Research on low-carbon closed-loop supply chain strategy based on differential games-dynamic optimization analysis of new and remanufactured products

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Abstract: With the current increasing global demand for low-carbon and environmentally friendly products, promoting the sustainability of closed-loop supply chains has become one of the key measures. However, consumers often do not regard remanufactured products as equivalent to new products. Therefore, this paper proposes a dynamic closed-loop supply chain that incorporates consumers' purchasing preferences to model a long-term game with product differentiation. Moreover, to enhance consumer acceptance of remanufactured products and reduce manufacturers' costs, low-carbon technologies and cost-sharing mechanisms are introduced. In this way, we construct a differential game in which the manufacturer sells new and remanufactured products through a retailer and makes decisions about the level of low-carbon technology in the remanufacturing process. Based on the theory of differential games, this paper analyzes three different power structures: the manufacturer-dominated Stackelberg game, the Nash game, and the retailer-dominated Stackelberg game. The optimal low-carbon technology level and pricing strategy are obtained by applying Pontryagin's maximum principle. The study shows that the retailer-led Stackelberg game helps retailers maximize profits, while the Nash game enables the entire closed-loop supply chain system to achieve the highest overall profits. This paper innovatively integrates low-carbon technologies into the dynamic game model of the remanufacturing process and reveals how the game behavior of supply chain participants affects the application of low-carbon technologies and the overall profit of the supply chain by comparing the cost-sharing mechanisms under different power structures. The results provide important theoretical support and practical references for closed-loop supply chain management with product differentiation.

Keywords: closed-loop supply chain; differential game; power structure; cost-sharing
Mathematics Subject Classification: 91A23

1. Introduction

The rapid development of e-commerce platforms in recent years has led to more and more large online capital flows and offline commodity flows, which have resulted in not only an increasingly rapid rate of product updating but also environmental pollution, resource waste, and other problems. With the advancement of the global low-carbon economy, there is an increasing demand for remanufacturing and low-carbon technologies by enterprises. As a result, closed-loop supply chain is gradually becoming an important mode of operation in the context of a low-carbon circular economy. The closed-loop supply chain (CLSC) is developed from the traditional forward supply chain by adding the reverse logistics of recycling and remanufacturing of used products [1]. As an important operation mode of enterprises, CLSC can save resources through the recycling of used products, and is an important way to achieve the development of a green economy with a low carbon cycle [2]. The CLSC operation mode can bring good economic benefits for enterprises [3]. Specifically, an enterprise can establish a good image and improve market competitiveness by recycling used products. In addition, the recycling and remanufacturing activities can reduce carbon emissions and save costs. For example, Bosch can save 40% of costs annually through recycling and remanufacturing activities, and reduce 23,000 tons of carbon dioxide emissions [4,5]. Through recycling and remanufacturing activities, Kodak can save 40%–60% [6] and IBM saves up to 80% [7] of production costs.

In essence, CLSC involves the incorporation of a reverse feedback process into the conventional forward supply chain, thereby establishing a complete, closed-loop system. At its core, CLSC operates as a feedback system that encompasses the entire lifecycle of products, spanning from resource extraction and production to consumption, and ultimately to the recycling and renewal of resources, through both forward delivery and reverse recycling processes [8]. CLSC members are committed to recycling and reusing used products, obtaining the surplus value of the used products, reducing production cost, and achieving the cyclic flow of resources, which will help indirectly reduce negative impacts on the environment [9]. Since pricing decisions are also pivotal in shaping the overall operational efficiency of CLSC, it has received extensive attention from scholars [10]. Vorasayan et al. analyzed the effect of changes in the returned product quality and refurbishment cost on the pricing decision [11]. Thereafter, the pricing strategies of the CLSC have been extensively explored [12–14], focusing on various aspects such as incentive strategies, product quality, power asymmetry, and demand uncertainty. Existing studies have discussed pricing strategies, reverse logistics, and remanufacturing in CLSC, but most of them have ignored a key reality: there is a significant difference in consumer acceptance of new and remanufactured products, especially in the context of the gradual increase in low-carbon awareness. To enhance the acceptance of remanufactured products in the market [15,16], this study introduces low-carbon technologies in the hope of increasing consumer demand for remanufactured products by reducing carbon emissions.

While it is true that low-carbon technologies can increase the demand for remanufactured products, many companies are reluctant to actively participate in carbon reduction activities due to concerns about high investment costs [17]. To address this issue, this paper proposes a program where retailers and manufacturers share the investment costs of low-carbon technologies. Based on the literature discussing recycling activities [18,19], a differential response is constructed in which the manufacturer is responsible for product recycling, and the retailer and the manufacturer share the costs of recycling and low-carbon technology development. Unlike traditional studies, this paper focuses on analyzing cost-sharing mechanisms and pricing strategies under different power structures to find out the optimal decision-making of supply chain members with product differentiation and the way to maximize the overall profit.

Based on the above discussion, this paper addresses the following questions for a dynamic supply chain with product differentiation:

(1) What are the equilibrium pricing strategies and low-carbon technology levels of CLSC members under different power structures?

(2) How does the retailer's optimal cost-sharing ratio affect the pricing decision under different power structures?

(3) How do substitutability coefficients and low-carbon preferences affect pricing strategies? Under which power structure is the system profit maximized?

To solve the above questions, we construct a CLSC consisting of a manufacturer and a retailer, since most of the literature assumes no difference between new and used products and does not consider the dynamic characteristics in the selling and recycling activities of used and end-of-life products. Therefore, this study presents a differential game model that captures the dynamic demand patterns specific to both new and remanufactured products. The focus of this study is a CLSC comprising a manufacturer and a retailer, which jointly bear the costs associated with remanufacturing and the low-carbon technology investment. With the differential game model, this study investigates the pricing strategies within three distinct game scenarios: the manufacturer-dominated Stackelberg game, the Nash game, and the retailer-dominated Stackelberg game. In addition, the effects of substitutability coefficient, low-carbon preference, and power structures on pricing decisions are analyzed. Furthermore, the optimal profits attainable within the CLSC system are examined under varying power structures. The main contributions of this study are as follows:

(1) For the first time, we incorporate a dynamic perspective into a CLSC study of low-carbon technology R&D and cost-sharing mechanisms under different power structures.

(2) We innovatively construct a differential response model that covers the dynamic demand for both new and remanufactured products. This model accurately determines the optimal pricing strategy, the level of low-carbon technology upgrading, and predicts market demand for new and remanufactured products under the three different power structures.

(3) We reveal how substitutability coefficients and consumers' low-carbon preferences profoundly affect the formulation of pricing strategies in the context of different power structures. In addition, we clarify how the retailer optimizes its cost-sharing ratio to maximize its own benefits under different power structures.

(4) In this paper, we explore in depth the optimal power structure configuration corresponding to the maximization of the overall profit of the CLSC system.

The above contributions not only fill the research gap in resource recycling and reuse in supply chain management but also provide a new perspective on the research of the low-carbon economy. Moreover, they show how enterprises can achieve a balance between economic and environmental benefits through a CLSC. This is a key challenge under the current dual pressures of market competition and environmental protection.

This study is organized as follows: Section 2 is the literature review; Section 3 describes the problems and basic assumptions; Section 4 shows the model construction and solution under different power structures; Section 5 is about numerical simulation; Section 6 presents the conclusions and prospects.

2. Literature review

The literature related to this study focuses on three areas: the pricing decision problem, the cost sharing and power structure problem, and the application of differential games in CLSC. Table 1 sets out a comparison between previous literature and our work.

Currently, many scholars have profoundly studied pricing decisions in CLSC. Liu et al. explored the pricing strategies that took into account different reverse channels, quality uncertainty, and socially responsible investment behaviors [20–22]. But, the above studies do not consider the recovery cost, and all assume that the remanufacturing cost is zero. In fact, these costs affect the pricing decisions of CLSC members. Herein, we add the investment cost of low-carbon technology to the above costs, and study the pricing strategies of CLSC under the sharing of the recycling cost of the remanufacturing process and the investment cost of low-carbon technology between the manufacturer and the retailer.

Meanwhile, the current society presents a diversified CLSC dominance situation that involves a model where large-scale manufacturers like Volkswagen and first automobile works dominate the market, and a model where large-scale retailers such as Wal-Mart, Carrefour, and Gome dominate the market. There is also a model in which electronic equipment manufacturers and Jingdong, Suning, and other retailers dominate the market simultaneously, which is also known as an evenly matched model [23]. Different power structures determine different decision-making orders of CLSC members, which will impact the pricing decision-making results of the whole CLSC and the benefit distribution structure of CLSC members in different ways [24]. Wei et al. studied the optimal decision-making problem of CLSC with symmetric and asymmetric information structures under manufacturer-led or retailer-led scenarios [25]. Results show that firms in the leadership of the supply chain have the advantage of obtaining higher profits under both symmetric and asymmetric information. Under varying power structures, some researchers studied the optimal pricing strategies, remanufacturing strategies, and coordination mechanisms within CLSC [17,26,27]. Nevertheless, the dynamic factors in product sales and waste product recycling activities were not considered in their research. In reality, the favorable reputation of an enterprise throughout its operations can be translated into consumers' positive perception of the product as a potential value, thereby influencing the sales and recycling of the product in subsequent periods [28]. If it is seen as a static state, the resulting strategy is only the company's local short-term optimal strategy, rather than the global optimal strategy. Therefore, given the dynamic influence of members' decisions, it is more suitable to investigate the effects of low-carbon technology and cost burdens under varying power structures from a dynamic perspective.

There is an extensive body of research on dynamic characteristics-based analyses of CLSC [29,30]. For instance, Ma et al. constructed a dynamic differential equation for product goodwill and explored its influence on the decision-making processes of each participant within a CLSC [31–33]. Yang et al. built a dynamic differential equation for carbon emission reduction and thereby investigated the emission reduction decision-making of the manufacturer [34–36]. Ma et al. established a dynamic differential equation for carbon emission reduction by considering the product quality and the dynamic differential equation of product greenness and compared the dynamic pricing model with the static pricing model [37]. Nonetheless, the existing literature fails to differentiate between new and remanufactured products. In contrast, this study proposes a differential game model with product differentiation that captures dynamic demand patterns specific to new and remanufactured products. Moreover, the supply chain includes a manufacturer and a retailer who share the costs of remanufacturing and investing in low-carbon technologies. Finally, we give the optimal cost-sharing ratio for the retailer under different power structures to maximize its own benefit.

Table 1. Most relevant literature.

Literature	Cost-sharing	Power structure	Low-carbon technology	Distinguish between new and remanufactured products	Differential game
[26]	√	√	×	×	×
[35]	×	×	√	×	√
[38]	√	×	×	×	√
[32]	×	√	×	×	√
[28]	×	√	√	×	√
[24]	√	√	×	√	×
This study	√	√	√	√	√

3. Model description and assumptions

This study focuses on the continuous time $t \to [0,+\infty)$, and is aimed to analyze the decisions related to pricing and low-carbon technology within CLSC under distinct power structures (Figure 1). The production of new products and the recycling and remanufacturing activities of used products are handled by the manufacturer. Additionally, the manufacturer conducts research and development to enhance the low-carbon technology used in remanufacturing processes. Simultaneously, the retailer assumes the role of selling both new and remanufactured products to consumers and collaborates with the manufacturer in sharing the recycling price and the investment cost associated with low-carbon technology within the recycling and remanufacturing activities.

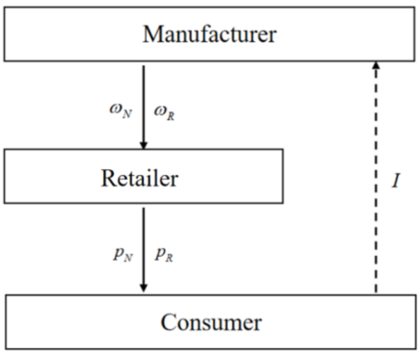


Figure 1. CLSC model.

Table 2 shows the specific symbols and corresponding meanings of relevant parameters. The notation π_i^j denotes the profit by member i within the CLSC. $i \in \{m,r,T\}$ represents the manufacturer, the retailer, and the CLSC system, respectively. $j \in \{MLM, NM, RLM\}$ represents the manufacturer-dominated Stackelberg game, the Nash game, and the retailer-dominated Stackelberg game, respectively.

Table 2. Control variables and parameters.

Category	Symbol	Definitions
Control variables	$p_N(t)$	retail price of new products
	$p_R(t)$	retail price of remanufactured products
	$\omega_N(t)$	wholesale price of new products
	$\omega_R(t)$	wholesale price of remanufactured products
	$E(t)$	Manufacturer's low-carbon technology level
Parameters	γ_1	natural growth factor of new products
	γ_2	natural growth factor of remanufactured products
	a	substitutability coefficient
	b	low-carbon preference coefficient of consumers
	η	investment cost coefficient of low-carbon technology
	I	recycling price
	c_N	production cost of new products
	c_R	production cost of remanufactured products
	ρ	discount rate, $\rho > \gamma_1, \gamma_2$
	φ	cost-sharing ratio

The basic assumptions of the CLSC model are specified below:

Assumption 1. Both parties are rational economic people, and all decisions are made from rational principles.

Assumption 2. Under the assumption of zero inventory cost and out-of-stock cost for both the manufacturer and the retailer, and considering an infinite time horizon, it is assumed that they share the same discount rate at any given point in time.

4. Model construction and analysis

We develop three models for decentralized decision-making: the Nash game (a market game without a leader), the Stackelberg game where the manufacturer dominates, and the Stackelberg game where the retailer dominates. Within these three decentralized decision-making models, both the retailer and the manufacturer exhibit risk-neutral and rational behavior, and their objectives are to maximize their profits throughout the decision-making procedure.

The demands for new and remanufactured products are denoted as Q_N and Q_R, respectively, and satisfy the following differential equations

$$\begin{aligned} Q_N^{'}(t) &= \gamma_1 Q_N(t) - p_N(t) - \omega_N(t) + a p_R(t), \\ Q_R^{'}(t) &= \gamma_2 Q_R(t) - p_R(t) - \omega_R(t) + a p_N(t) + bE(\mathrm{t}). \end{aligned} \tag{1}$$

Where the demand naturally decreases over time. Meanwhile, the prices of both new and remanufactured products influence their respective demands. Specifically, as the price of a particular product increases, its demand tends to be suppressed. Correspondingly, when the price of new products rises, consumers are more likely to purchase remanufactured goods, and vice versa. Additionally, higher wholesale prices negatively affect retailers' procurement of similar products. An increase in the level of low-carbon technology can increase the recognition of remanufactured products. Therefore, it positively affects the change in demand for remanufactured products. At the same time, the research and development cost of low-carbon technology is highly correlated with its level, which is set as a quadratic function in this paper, i.e.,

$$c_m = \frac{1}{2}\eta E^2 .$$

For convenience, the time variable t is omitted below. The necessary theorem on the existence of equilibrium solutions for differential games (Theorem 2.4.2) please refer to [39].

4.1. Manufacture-led model (MLM)

Manufacturer, as the core company in a CLSC, enjoys leadership over the supply chain. In this scenario, the manufacturer first decides on the wholesale price of products ω_N and ω_R, and low-carbon technology level E. The retailer then gives the retail price of products p_N and p_R, based on the manufacturer's decisions. The profit functions for each side are as follows:

$$\max_{\omega_N,\omega_R,E} \pi_m^{MLM} = \max\int_0^{\infty} e^{-\rho t}[(\omega_N - c_N)Q_N + (\omega_R - c_R)Q_R - (1-\varphi)(IQ_R + \frac{1}{2}\eta E^2)]dt,$$

$$\max_{p_N, p_R} \pi_r^{MLM} = \max\int_0^{\infty} e^{-\rho t}[(p_N - \omega_N)Q_N + (p_R - \omega_R)Q_R - \varphi(IQ_R + \frac{1}{2}\eta E^2)]dt.$$

Based on the above decision sequence, each equilibrium solution is found according to the principle of backward derivation, as in Proposition 1.

Proposition 1. *The optimal equilibrium strategies under the MLM model are given by*

$$\begin{aligned}
p_N^{MLM^*} &= (z_1 + z_5)Q_N^{MLM^*} + (z_2 + z_6)Q_R^{MLM^*} + c_N,\\
p_R^{MLM^*} &= (z_3 - z_7)Q_N^{MLM^*} + (z_4 + z_8)Q_R^{MLM^*} + c_R + I,\\
\omega_N^{MLM^*} &= z_5 Q_N^{MLM^*} + z_6 Q_R^{MLM^*} + c_N,\\
\omega_R^{MLM^*} &= (-z_7)Q_N^{MLM^*} + z_8 Q_R^{MLM^*} + c_R + (1-\varphi)I,\\
E^{MLM^*} &= z_9 Q_R^{MLM^*},
\end{aligned}$$

where

$$Q_N^{MLM^*} = \frac{(z_2 - az_4 + 2z_6 - az_8)(2c_R + 2I - \varphi I - ac_N) + (\gamma_2 + az_2 - z_4 + az_6 - 2z_8 + bz_9)(2c_N - aa_R - aI)}{(z_2 - az_4 + az_6 - az_8)(az_1 - z_3 + az_5 + 2z_7) + (\gamma_1 - z_1 + az_3 - 2z_5 - az_7)(\gamma_2 + az_2 - z_4 + az_6 - 2z_8 + bz_9)},$$

$$Q_R^{MLM^*} = \frac{(z_3 - az_1 - az_5 - 2z_7)(2c_N - ac_R - aI) + (\gamma_1 - z_1 + az_3 - 2z_5 - az_7)(2c_R + 2I - \varphi I - ac_N)}{(z_2 - az_4 + az_6 - az_8)(az_1 - z_3 + az_5 + 2z_7) + (\gamma_1 - z_1 + az_3 - 2z_5 - az_7)(\gamma_2 + az_2 - z_4 + az_6 - 2z_8 + bz_9)},$$

and

$$z_1=\frac{\rho-\gamma_1}{1-a^2},\ z_2=\frac{a(\rho-\gamma_1)}{1-a^2}, z_3=\frac{a(\rho-\gamma_2)}{1-a^2}, z_4=\frac{\rho-\gamma_2}{1-a^2}, z_7=z_6, z_9=\frac{b}{2\eta(1-\varphi)},$$

$$z_5=\frac{\{[(\rho-\gamma_1)(2-a^2)-a^2(\rho-\gamma_2)][(\rho-\gamma_2)(2-a^2)-a^2(\rho-\gamma_1)]+a^2(\gamma_1-\gamma_2)^2\}(1-a^2)[(\rho-\gamma_1)(2-a^2)-a^2(\rho-\gamma_2)]}{2\{(1-a^2)^2[(\rho-\gamma_2)(2-a^2)-a^2(\rho-\gamma_1)]+[a(1-a^2)(\gamma_1-\gamma_2)]^2\}},$$

$$z_6=\frac{\{[(\rho-\gamma_1)(2-a^2)-a^2(\rho-\gamma_2)][(\rho-\gamma_2)(2-a^2)-a^2(\rho-\gamma_1)]+a^2(\gamma_1-\gamma_2)^2\}[a(1-a^2)(\gamma_1-\gamma_2)]}{2\{(1-a^2)^2[(\rho-\gamma_2)(2-a^2)-a^2(\rho-\gamma_1)]+[a(1-a^2)(\gamma_1-\gamma_2)]^2\}},$$

$$z_8=\frac{\{[(\rho-\gamma_1)(2-a^2)-a^2(\rho-\gamma_2)][(\rho-\gamma_2)(2-a^2)-a^2(\rho-\gamma_1)]+a^2(\gamma_1-\gamma_2)^2\}(1-a^2)[(\rho-\gamma_2)(2-a^2)-a^2(\rho-\gamma_1)]}{2\{(1-a^2)^2[(\rho-\gamma_2)(2-a^2)-a^2(\rho-\gamma_1)]+[a(1-a^2)(\gamma_1-\gamma_2)]^2\}}.$$

Proof. The Hamiltonian function for the follower is obtained as

$$\begin{aligned}H_r^{MLM}(t,p_N,p_R)=(p_N-\omega_N)Q_N+(p_R-\omega_R)Q_R-\varphi(IQ_R+\frac{1}{2}\eta E^2)\\+\beta_1[\gamma_1Q_N-p_N-\omega_N+ap_R]\\+\beta_2[\gamma_2Q_R-p_R-\omega_R+ap_N+bE],\end{aligned}\tag{2}$$

where β_1,β_2 are the adjoint variables of the corresponding state variables. The HJB equations are obtained by taking a first-order partial derivative for p_N, p_R, respectively,

$$\begin{aligned}\frac{\partial H_r^{MLM}}{\partial p_N}=Q_N-\beta_1+a\beta_2=0,\\\frac{\partial H_r^{MLM}}{\partial p_R}=Q_R+a\beta_1-\beta_2=0.\end{aligned}\tag{3}$$

The adjoint variables satisfy the following adjoint equations

$$\begin{aligned}\beta_1^{'}=\rho\beta_1-\frac{\partial H_r^{MLM}}{\partial Q_N}=(\rho-\gamma_1)\beta_1+\omega_N-p_N,\\\beta_2^{'}=\rho\beta_2-\frac{\partial H_r^{MLM}}{\partial Q_R}=(\rho-\gamma_2)\beta_2+\omega_R-p_R+\varphi I.\end{aligned}$$

And the limiting transversality conditions

$$\begin{aligned}\lim_{t\to\infty}e^{-\rho t}Q_N\beta_1=0,\\\lim_{t\to\infty}e^{-\rho t}Q_R\beta_2=0.\end{aligned}$$

Then, in order not to violate the transversality condition, we choose a set of stable solutions

$$\begin{aligned}\beta_1=\beta_1(0)=\frac{p_N-\omega_N}{\rho-\gamma_1},\\\beta_2=\beta_2(0)=\frac{p_R-\omega_R-\varphi I}{\rho-\gamma_2}.\end{aligned}\tag{4}$$

The definition of a stable solution is one solution that satisfies

$$\beta_i'(t) = 0$$

and is itself a solution of the adjoint equation. Substituting Eq (4) into Eq (3), we obtain

$$Q_N - \frac{p_N - \omega_N}{\rho - \gamma_1} + a\frac{p_R - \omega_R - \varphi I}{\rho - \gamma_2} = 0,$$
$$Q_R + a\frac{p_N - \omega_N}{\rho - \gamma_1} - \frac{p_R - \omega_R - \varphi I}{\rho - \gamma_2} = 0.$$

We can obtain

$$\begin{aligned} p_N &= z_1 Q_N + z_2 Q_R + \omega_N, \\ p_R &= z_3 Q_N + z_4 Q_R + \omega_R + \varphi I, \end{aligned} \tag{5}$$

where

$$z_1 = \frac{\rho - \gamma_1}{1 - a^2}, \quad z_2 = \frac{a(\rho - \gamma_1)}{1 - a^2}, \quad z_3 = \frac{a(\rho - \gamma_2)}{1 - a^2}, \quad z_4 = \frac{\rho - \gamma_2}{1 - a^2}.$$

Substituting Eq (5) into the leader's objective function yields the leader's Hamiltonian function

$$\begin{aligned} H_m^{MLM}(t, \omega_N, \omega_R, E) = {} & (\omega_N - c_N)Q_N + (\omega_R - c_R)Q_R - (1-\varphi)(IQ_R + \frac{1}{2}\eta E^2) \\ & + \lambda_1[(\gamma_1 - z_1 + az_3)Q_N(t) + (az_4 - z_2)Q_R(t) + a\omega_R(t) - 2\omega_N(t) + a\varphi I] \\ & + \lambda_2[(az_1 - z_3)Q_N(t) + (\gamma_2 - z_4 + az_2)Q_R(t) - 2\omega_R(t) + a\omega_N(t) + bE(\mathrm{t}) - \varphi I]. \end{aligned} \tag{6}$$

The HJB equations are obtained by taking a first-order partial derivative for ω_N, ω_R, E, respectively

$$\begin{aligned} \frac{\partial H_m^{MLM}}{\partial \omega_N} &= Q_N - 2\lambda_1 = 0, \\ \frac{\partial H_m^{MLM}}{\partial \omega_R} &= Q_R - 2\lambda_2 = 0, \\ \frac{\partial H_m^{MLM}}{\partial E} &= -(1-\varphi)\eta E + b\lambda_2 = 0. \end{aligned} \tag{7}$$

Similarly, in order not to violate the transversality conditions, we choose a set of stable solutions. Then

$$\lambda_1' = \rho\lambda_1 - \frac{\partial H_m^{MLM}}{\partial Q_N} = 0,$$
$$\lambda_2' = \rho\lambda_2 - \frac{\partial H_m^{MLM}}{\partial Q_R} = 0.$$

According to Eq (6), the constant adjoint variables

$$\lambda_1 = \lambda_1(0)$$

and

$$\lambda_2 = \lambda_2(0)$$

are obtained,

$$\lambda_1 = \frac{\{[(\rho-\gamma_2)(2-a^2)-a^2(\rho-\gamma_1)](\omega_N-c_N)-a(\gamma_1-\gamma_2)(\omega_R-c_R-I+\varphi I)\}(1-a^2)}{[(\rho-\gamma_1)(2-a^2)-a^2(\rho-\gamma_2)][(\rho-\gamma_2)(2-a^2)-a^2(\rho-\gamma_1)]+a^2(\gamma_1-\gamma_2)^2},$$
$$\lambda_2 = \frac{\{[(\rho-\gamma_1)(2-a^2)-a^2(\rho-\gamma_2)](\omega_R-c_R-I+\varphi I)+a(\gamma_1-\gamma_2)(\omega_N-c_N)\}(1-a^2)}{[(\rho-\gamma_1)(2-a^2)-a^2(\rho-\gamma_2)][(\rho-\gamma_2)(2-a^2)-a^2(\rho-\gamma_1)]+a^2(\gamma_1-\gamma_2)^2}. \tag{8}$$

Substituting Eq (8) into Eq (7), we can obtain

$$\begin{aligned} \omega_N^{MLM^*} &= z_5 Q_N^{MLM^*} + z_6 Q_R^{MLM^*} + c_N, \\ \omega_R^{MLM^*} &= (-z_7) Q_N^{MLM^*} + z_8 Q_R^{MLM^*} + c_R + (1-\varphi) I, \\ E^{MLM^*} &= z_9 Q_R^{MLM^*}, \end{aligned} \tag{9}$$

where

$$z_5 = \frac{\{[(\rho-\gamma_1)(2-a^2)-a^2(\rho-\gamma_2)][(\rho-\gamma_2)(2-a^2)-a^2(\rho-\gamma_1)]+a^2(\gamma_1-\gamma_2)^2\}(1-a^2)[(\rho-\gamma_1)(2-a^2)-a^2(\rho-\gamma_2)]}{2\{(1-a^2)^2[(\rho-\gamma_2)(2-a^2)-a^2(\rho-\gamma_1)]+[a(1-a^2)(\gamma_1-\gamma_2)]^2\}},$$

$$z_6 = \frac{\{[(\rho-\gamma_1)(2-a^2)-a^2(\rho-\gamma_2)][(\rho-\gamma_2)(2-a^2)-a^2(\rho-\gamma_1)]+a^2(\gamma_1-\gamma_2)^2\}[a(1-a^2)(\gamma_1-\gamma_2)]}{2\{(1-a^2)^2[(\rho-\gamma_2)(2-a^2)-a^2(\rho-\gamma_1)]+[a(1-a^2)(\gamma_1-\gamma_2)]^2\}},$$

$$z_7 = z_6,$$

$$z_8 = \frac{\{[(\rho-\gamma_1)(2-a^2)-a^2(\rho-\gamma_2)][(\rho-\gamma_2)(2-a^2)-a^2(\rho-\gamma_1)]+a^2(\gamma_1-\gamma_2)^2\}(1-a^2)[(\rho-\gamma_2)(2-a^2)-a^2(\rho-\gamma_1)]}{2\{(1-a^2)^2[(\rho-\gamma_2)(2-a^2)-a^2(\rho-\gamma_1)]+[a(1-a^2)(\gamma_1-\gamma_2)]^2\}},$$

$$z_9 = \frac{b}{2\eta(1-\varphi)}.$$

Plugging Eq (9) into Eq (5)

$$\begin{aligned} p_N^{MLM^*} &= (z_1+z_5) Q_N^{MLM^*} + (z_2+z_6) Q_R^{MLM^*} + c_N, \\ p_R^{MLM^*} &= (z_3-z_7) Q_N^{MLM^*} + (z_4+z_8) Q_R^{MLM^*} + c_R + I. \end{aligned}$$

In order to find the Stackelberg solutions, it is necessary to solve Q_N, Q_R. Therefore, by substituting $p_N^{MLM^*}, p_R^{MLM^*}, \omega_N^{MLM^*}, \omega_R^{MLM^*}$ into (1), we obtain

$$\begin{aligned} Q_N^{'} &= (\gamma_1 - z_1 + az_3 - 2z_5 - az_7) Q_N + (az_4 - z_2 + az_8 - 2z_6) Q_R - (2c_N - aa_R - aI), \\ Q_R^{'} &= (az_1 - z_3 + az_5 + 2z_7) Q_N + (\gamma_2 + az_2 - z_4 + az_6 - 2z_8 + bz_9) Q_R - (2c_R + 2I - \varphi I - ac_N). \end{aligned}$$

When the autonomy equations are stable,

$$\begin{aligned} Q_N^{'} &= (\gamma_1 - z_1 + az_3 - 2z_5 - az_7) Q_N + (az_4 - z_2 + az_8 - 2z_6) Q_R - (2c_N - aa_R - aI) = 0, \\ Q_R^{'} &= (az_1 - z_3 + az_5 + 2z_7) Q_N + (\gamma_2 + az_2 - z_4 + az_6 - 2z_8 + bz_9) Q_R - (2c_R + 2I - \varphi I - ac_N) = 0. \end{aligned}$$

Then the equilibrium solutions for the demand for the new and remanufactured products are

$$Q_N^{MLM^*}=\frac{\left(z_2-az_4+2z_6-az_8\right)\left(2c_R+2I-\varphi I-ac_N\right)+\left(\gamma_2+az_2-z_4+az_6-2z_8+bz_9\right)\left(2c_N-aa_R-aI\right)}{\left(z_2-az_4+az_6-az_8\right)\left(az_1-z_3+az_5+2z_7\right)+\left(\gamma_1-z_1+az_3-2z_5-az_7\right)\left(\gamma_2+az_2-z_4+az_6-2z_8+bz_9\right)},$$
$$Q_R^{MLM^*}=\frac{\left(z_3-az_1-az_5-2z_7\right)\left(2c_N-ac_R-aI\right)+\left(\gamma_1-z_1+az_3-2z_5-az_7\right)\left(2c_R+2I-\varphi I-ac_N\right)}{\left(z_2-az_4+az_6-az_8\right)\left(az_1-z_3+az_5+2z_7\right)+\left(\gamma_1-z_1+az_3-2z_5-az_7\right)\left(\gamma_2+az_2-z_4+az_6-2z_8+bz_9\right)}.$$

The Stackelberg solutions for the manufacturer and retailer control variables are obtained

$$\begin{aligned}
p_N^{MLM^*}&=(z_1+z_5)Q_N^{MLM^*}+(z_2+z_6)Q_R^{MLM^*}+c_N,\\
p_R^{MLM^*}&=(z_3-z_7)Q_N^{MLM^*}+(z_4+z_8)Q_R^{MLM^*}+c_R+I,\\
\omega_N^{MLM^*}&=z_5Q_N^{MLM^*}+z_6Q_R^{MLM^*}+c_N,\\
\omega_R^{MLM^*}&=(-z_7)Q_N^{MLM^*}+z_8Q_R^{MLM^*}+c_R+(1-\varphi)I,\\
E^{MLM^*}&=z_9Q_R^{MLM^*}.
\end{aligned}$$

4.2. Nash model (NM)

The Nash game, a leaderless market game, assumes that both parties make decisions at the same time, but the manufacturer decides on the wholesale price of the products ω_N and ω_R, and the level of low-carbon technology E, and the retailer decides on the retail price of the products p_N and p_R. The profit functions for each side are as follows:

$$\begin{aligned}
\max_{\omega_N,\omega_R,E}\pi_m^{NM}&=\max\int_0^\infty e^{-\rho t}[(\omega_N-c_N)Q_N+(\omega_R-c_R)Q_R-(1-\varphi)(IQ_R+\frac{1}{2}\eta E^2)]dt,\\
\max_{p_N,p_R}\pi_r^{NM}&=\max\int_0^\infty e^{-\rho t}[(p_N-\omega_N)Q_N+(p_R-\omega_R)Q_R-\varphi(IQ_R+\frac{1}{2}\eta E^2)]dt.
\end{aligned}$$

Proposition 2. *The optimal equilibrium strategies under the NM model are given by*

$$\begin{aligned}
p_N^{NM^*}&=y_1Q_N^{NM^*}+y_2Q_R^{NM^*}+c_N,\\
p_R^{NM^*}&=y_3Q_N^{NM^*}+y_4Q_R^{NM^*}+c_R+I,\\
\omega_N^{NM^*}&=y_5Q_N^{NM^*}+c_N,\\
\omega_R^{NM^*}&=y_6Q_N^{NM^*}+c_R+(1-\varphi)I,\\
E^{NM^*}&=y_7Q_R^{NM^*},
\end{aligned}$$

where

$$Q_N^{NM^*}=\frac{(\gamma_2-y_4-y_6+ay_2+by_7)(2c_N-ac_R-aI)+(y_2-ay_4)(2c_R+2I-\varphi I-ac_N)}{(\gamma_1-y_1-y_5+ay_3)(\gamma_2-y_4-y_6+ay_2+by_7)+(ay_1-y_3)(y_2-ay_4)},$$
$$Q_R^{NM^*}=\frac{(\gamma_1-y_1-y_5+ay_3)(2c_R+2I-\varphi I-ac_N)+(y_3-ay_1)(2c_N-ac_R-aI)}{(\gamma_1-y_1-y_5+ay_3)(\gamma_2-y_4-y_6+ay_2+by_7)+(ay_1-y_3)(y_2-ay_4)},$$

and

$$y_1=\frac{(\rho-\gamma_1)(2-a^2)}{(1-a^2)},\ y_2=\frac{a(\rho-\gamma_1)}{(1-a^2)},\ y_3=\frac{a(\rho-\gamma_2)}{(1-a^2)},$$
$$y_4=\frac{(\rho-\gamma_2)(2-a^2)}{(1-a^2)},\ y_5=(\rho-\gamma_1),\quad y_6=\frac{b}{\eta(1-\varphi)}.$$

Proof. The Hamiltonian functions for manufacturer and retailer, respectively, are as follows:

$$\begin{aligned}H_m^{NM}(t,\omega_N,\omega_R,E)&=(\omega_N-c_N)Q_N+(\omega_R-c_R)Q_R-(1-\varphi)(IQ_R+\frac{1}{2}\eta E^2)\\&+\lambda_1[\gamma_1Q_N-p_N-\omega_N+ap_R]+\lambda_2[\gamma_2Q_R-p_R-\omega_R+ap_N+bE],\\H_r^{NM}(t,p_N,p_R)&=(p_N-\omega_N)Q_N+(p_R-\omega_R)Q_R-\varphi(IQ_R+\frac{1}{2}\eta E^2)\\&+\beta_1[\gamma_1Q_N-p_N-\omega_N+ap_R]+\beta_2[\gamma_2Q_R-p_R-\omega_R+ap_N+bE].\end{aligned}$$

The HJB equations are obtained by taking a first order partial derivative for ω_N,ω_R,E and p_N,p_R,

$$\begin{aligned}&\frac{\partial H_r^{NM}}{\partial p_N}=Q_N-\beta_1+a\beta_2=0,\ \ \frac{\partial H_r^{NM}}{\partial p_R}=Q_R+a\beta_1-\beta_2=0,\\&\frac{\partial H_m^{NM}}{\partial \omega_N}=Q_N-2\lambda_1=0,\qquad \frac{\partial H_m^{NM}}{\partial \omega_R}=Q_R-2\lambda_2=0,\\&\frac{\partial H_m^{NM}}{\partial E}=-(1-\varphi)\eta E+b\lambda_2=0.\end{aligned}\tag{10}$$

The adjoint variables satisfy the following adjoint equations:

$$\begin{aligned}\lambda_1'&=\rho\lambda_1-\frac{\partial H_r^{NM}}{\partial Q_N}=(\rho-\gamma_1)\lambda_1+c_N-\omega_N,\\\lambda_2'&=\rho\lambda_2-\frac{\partial H_r^{NM}}{\partial Q_R}=(\rho-\gamma_2)\lambda_2+c_R-\omega_R+(1-\varphi)I,\\\beta_1'&=\rho\beta_1-\frac{\partial H_r^{NM}}{\partial Q_N}=(\rho-\gamma_1)\beta_1+\omega_N-p_N,\\\beta_2'&=\rho\beta_2-\frac{\partial H_r^{NM}}{\partial Q_R}=(\rho-\gamma_2)\beta_2+\omega_R-p_R+\varphi I.\end{aligned}$$

And the limiting transversality conditions

$$\begin{aligned}&\lim_{t\to\infty}e^{-\rho t}Q_N\lambda_1=0,\\&\lim_{t\to\infty}e^{-\rho t}Q_R\lambda_2=0,\\&\lim_{t\to\infty}e^{-\rho t}Q_N\beta_1=0,\\&\lim_{t\to\infty}e^{-\rho t}Q_R\beta_2=0.\end{aligned}$$

In order not to violate the transversality condition, we choose a set of stable solutions

$$\begin{aligned}&\lambda_1=\frac{\omega_N-c_N}{\rho-\gamma_1},\quad \lambda_2=\frac{\omega_R-c_R-(1-\varphi)I}{\rho-\gamma_2},\\&\beta_1=\frac{p_N-\omega_N}{\rho-\gamma_1},\quad \beta_2=\frac{p_R-\omega_R-\varphi I}{\rho-\gamma_2}.\end{aligned}\tag{11}$$

Substituting Eq (11) into Eq (10), we obtain

$$\begin{aligned}&-\frac{1}{\rho-\gamma_1}p_N+\frac{a}{\rho-\gamma_2}p_R+\frac{1}{\rho-\gamma_1}\omega_N-\frac{a}{\rho-\gamma_2}\omega_R=\frac{a\varphi I}{\rho-\gamma_2}-Q_N,\\&\frac{a}{\rho-\gamma_1}p_N-\frac{1}{\rho-\gamma_2}p_R-\frac{a}{\rho-\gamma_1}\omega_N+\frac{1}{\rho-\gamma_2}\omega_R=-\frac{\varphi I}{\rho-\gamma_2}-Q_R,\\&\frac{1}{\rho-\gamma_1}\omega_N=\frac{c_N}{\rho-\gamma_1}+Q_N,\\&\frac{1}{\rho-\gamma_2}\omega_R=\frac{(1-\varphi I)}{\rho-\gamma_2}+\frac{c_R}{\rho-\gamma_2}+Q_R,\\&\frac{b}{\rho-\gamma_2}\omega_R-(1-\varphi)\eta E=\frac{b(1-\varphi I)+bc_R}{\rho-\gamma_2}.\end{aligned}$$

We can obtain

$$\begin{aligned}&p_N^{NM^*}=y_1Q_N^{NM^*}+y_2Q_R^{NM^*}+c_N,\\&p_R^{NM^*}=y_3Q_N^{NM^*}+y_4Q_R^{NM^*}+c_R+I,\\&\omega_N^{NM^*}=y_5Q_N^{NM^*}+c_N,\\&\omega_R^{NM^*}=y_6Q_N^{NM^*}+c_R+(1-\varphi)I,\\&E^{NM^*}=y_7Q_R^{NM^*},\end{aligned}$$

and

$$\begin{aligned}&y_1=\frac{(\rho-\gamma_1)(2-a^2)}{(1-a^2)},\ y_2=\frac{a(\rho-\gamma_1)}{(1-a^2)},\ y_3=\frac{a(\rho-\gamma_2)}{(1-a^2)},\\&y_4=\frac{(\rho-\gamma_2)(2-a^2)}{(1-a^2)},\ y_5=(\rho-\gamma_1),\quad y_6=\frac{b}{\eta(1-\varphi)}.\end{aligned}$$

In order to find the Nash solutions, it is necessary to solve Q_N, Q_R. Therefore, by substituting $p_N^{MLM^*}, p_R^{MLM^*}, \omega_N^{MLM^*}, \omega_R^{MLM^*}, E$ into Eq (1), we get

$$\begin{aligned}&Q_N^{'}=\left(\gamma_1-y_1+ay_3-y_5\right)Q_N+\left(ay_4-y_2\right)Q_R-\left(2c_N-aa_R-aI\right),\\&Q_R^{'}=\left(ay_1-y_3\right)Q_N+\left(\gamma_2+ay_2-y_4-y_6+by_7\right)Q_R-\left(2c_R+2I-\varphi I-ac_N\right).\end{aligned}$$

When the autonomy equations are stable,

$$\left(\gamma_1 - y_1 + ay_3 - y_5\right)Q_N + \left(ay_4 - y_2\right)Q_R - \left(2c_N - aa_R - aI\right) = 0,$$
$$\left(ay_1 - y_3\right)Q_N + \left(\gamma_2 + ay_2 - y_4 - y_6 + by_7\right)Q_R - \left(2c_R + 2I - \varphi I - ac_N\right) = 0.$$

Then the optimal demands for the new and remanufactured products are

$$Q_N^{NM^*} = \frac{(\gamma_2 - y_4 - y_6 + ay_2 + by_7)(2c_N - ac_R - aI) + (y_2 - ay_4)(2c_R + 2I - \varphi I - ac_N)}{(\gamma_1 - y_1 - y_5 + ay_3)(\gamma_2 - y_4 - y_6 + ay_2 + by_7) + (ay_1 - y_3)(y_2 - ay_4)},$$
$$Q_R^{NM^*} = \frac{(\gamma_1 - y_1 - y_5 + ay_3)(2c_R + 2I - \varphi I - ac_N) + (y_3 - ay_1)(2c_N - ac_R - aI)}{(\gamma_1 - y_1 - y_5 + ay_3)(\gamma_2 - y_4 - y_6 + ay_2 + by_7) + (ay_1 - y_3)(y_2 - ay_4)}.$$

And the Nash equilibrium for the manufacturer and retailer control variables are obtained

$$\begin{aligned}
p_N^{NM^*} &= y_1 Q_N^{NM^*} + y_2 Q_R^{NM^*} + c_N, \\
p_R^{NM^*} &= y_3 Q_N^{NM^*} + y_4 Q_R^{NM^*} + c_R + I, \\
\omega_N^{NM^*} &= y_5 Q_N^{NM^*} + c_N, \\
\omega_R^{NM^*} &= y_6 Q_N^{NM^*} + c_R + (1-\varphi)I, \\
E^{NM^*} &= y_7 Q_R^{NM^*}.
\end{aligned}$$

4.3. Retailer-led model (RLM)

The retailer, as the core company in a CLSC, enjoys leadership over the supply chain. In this scenario, the retailer first decides the retail price of the products p_N and p_R. The manufacturer then gives the wholesale price of the product ω_N and ω_R, and the level of low-carbon technology E, based on the retailer's decision. The profit functions for each side are as follows:

$$\max_{\omega_N,\omega_R,E} \pi_m^{RLM} = \max\int_0^{\infty} e^{-\rho t}[(\omega_N - c_N)Q_N + (\omega_R - c_R)Q_R - (1-\varphi)(IQ_R + \frac{1}{2}\eta E^2)]dt,$$
$$\max_{p_N,p_R} \pi_r^{RLM} = \max\int_0^{\infty} e^{-\rho t}[(p_N - \omega_N)Q_N + (p_R - \omega_R)Q_R - \varphi(IQ_R + \frac{1}{2}\eta E^2)]dt.$$

Proposition 3. *The optimal equilibrium strategies under the MLM model are*

$$\begin{aligned}
p_N^{RLM^*} &= x_4 Q_N^{RLM^*} + x_5 Q_R^{RLM^*} + c_N, \\
p_R^{RLM^*} &= x_6 Q_N^{RLM^*} + x_7 Q_R^{RLM^*} + c_R + I, \\
\omega_N^{RLM^*} &= x_1 Q_N^{RLM^*} + c_N, \\
\omega_R^{RLM^*} &= x_2 Q_R^{RLM^*} + c_R + (1-\varphi)I, \\
E^{RLM^*} &= x_3 Q_R^{RLM^*},
\end{aligned}$$

where

$$Q_N^{NM^*}=\frac{(\gamma_2-x_2+bx_3+ax_5-x_7)(2c_N-ac_R-aI)+(x_5-ax_7)(2c_R+2I-\varphi I-ac_N)}{(\gamma_1-x_1-x_4+ax_6)(\gamma_2-x_2+bx_3+ax_5-x_7)+(x_5-ax_7)(ax_4-x_6)},$$
$$Q_R^{NM^*}=\frac{(\gamma_1-x_1-x_4+ax_6)(2c_R+2I-\varphi I-ac_N)+(x_6-ax_4)(2c_N-ac_R-aI)}{(\gamma_1-x_1-x_4+ax_6)(\gamma_2-x_2+bx_3+ax_5-x_7)+(x_5-ax_7)(ax_4-x_6)},$$

and

$$x_1=\rho-\gamma_1,\ x_2=\rho-\gamma_2,\ x_3=\frac{b}{\eta(1-\varphi)},\ x_4=\frac{2(\rho-\gamma_1)(2-a^2)}{(1-a^2)},$$
$$x_5=\frac{2a(\rho-\gamma_1)}{(1-a^2)},\ x_6=\frac{2a(\rho-\gamma_2)(1-\varphi)\eta-ab^2}{(1-a^2)(1-\varphi)\eta},$$
$$x_7=\frac{(2-a^2)[2(\rho-\gamma_2)(1-\varphi)\eta-b^2]}{(1-a^2)(1-\varphi)\eta}+\frac{b^2(1+\varphi)[2(\rho-\gamma_2)(1-\varphi)\eta-b^2]}{[2(\rho-\gamma_2)(1-\varphi)^2\eta-b^2](1-\varphi)\eta}.$$

Proof. Similar to the proof of Proposition 1.

5. Numerical analysis

In order to be more intuitive and thorough for more in-depth research and analysis, we conduct simulations to explore the impacts of cost-sharing coefficient, product substitutability, and consumers' low-carbon preference on pricing decisions. The parameters in this section include

$$\gamma_1=0.9,\quad \gamma_2=0.8,\quad \rho=1,\quad c_N=0.8,\quad c_R=0.3,\quad I=0.2,\quad \eta=1.$$

5.1. Optimal cost-sharing ratios under different power structures

In addition to the manufacturer benefiting from recycling, recycling is also a profitable endeavor for the retailer. Therefore, the retailer is actively involved in cost-sharing, both in terms of recycling costs for the manufacturer and investment costs for low-carbon technology, thus incentivizing recycling. Therefore, it is necessary to investigate the optimal cost-sharing ratio for retailers under different power structures to maximize the retailer's profits. The parameters in this section are

$$a=0.5,\quad b=0.5.$$

As demonstrated in Figure 2, the optimal cost-sharing ratio is exclusively present within the RLM model. Conversely, in both the NM and MLM models, the retailer achieves maximum profit without engaging in cost-sharing measures, and the profits decrease as the cost-sharing ratio increases. Under the RLM model, the retailer's profit and its cost-sharing ratio are in an inverted U-shaped curve, as the retailer has the optimal cost-sharing ratio of 0.8, at which the retailer's profit reaches 14.75, which is much higher than the profit under the NM and MLM models. Therefore, as the retailer's power increases, its profit also rises. To maximize their profits, the retailer must establish the most suitable cost-sharing ratio based on their levels of power within the CLSC.

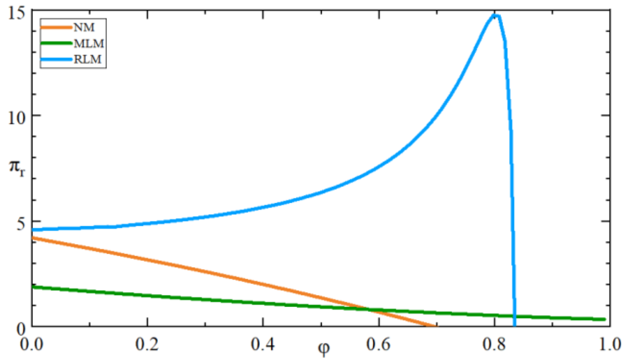


Figure 2. Impact of φ on π_r under different power structures.

5.2. Impact of cost-sharing ratios on pricing decisions

Within this subsection, the impact of cost-sharing ratios on the equilibrium solution of members within a CLSC is analyzed under varying power structures. This analysis is visually represented in Figures 3–5. As shown in Figure 3, there is

$$E^{NM} > E^{MLM} > E^{RLM},$$

and the low-carbon technology level under the NM and MLM models slowly increase as b rises, indicating the cost-sharing mechanism can improve the low-carbon technology level of remanufacturing. However, the low-carbon technology level under the RLM model is decreasing on the whole, and falls to 0 at $\varphi = 0.21$. Therefore, when the retailer possesses greater power within the CLSC, the corresponding consequence is a lower level of low-carbon technology for the manufacturer.

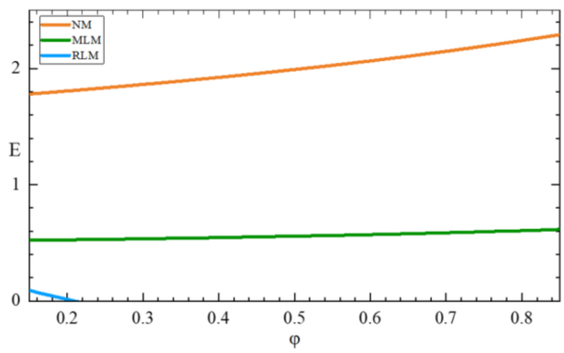


Figure 3. Impact of φ on E under different power structures.

The retail price of new products under the RLM model is consistently higher than under the NM and MLM models regardless of the values of φ (Figure 4). As φ increases, under both the RLM and NM models, the retail price of new products rises, albeit at different rates. Specifically, the retail price rises gradually under the NM model, while it escalates more rapidly under the RLM model. In the MLM model, the retailer's retail prices are slowly decreasing, and there is a threshold point between the RLM and NM models at

$$\varphi = 0.12.$$

When at

$$\varphi < 0.21,$$

the retail price of a new product in the MLM model is slightly higher than in the NM model, while the opposite is true at

$$\varphi > 0.21.$$

This result indicates that as the retailer's power in the CLSC increases, the retail price of the new products rises, because the retailer transfers its inputs for cost-sharing in the recycling and remanufacturing process to the new product. As shown in Figure 5, the retail price of remanufactured products decreases under all three models regardless of the values of φ, and changes in the order of

$$p_R^{NM} > p_R^{MLM} > p_R^{RLM}.$$

When the manufacturer and retailer simultaneously make their pricing strategies, the retail price of the remanufactured product is higher, but the price decreases as power shifts to the retailer (Figure 5).

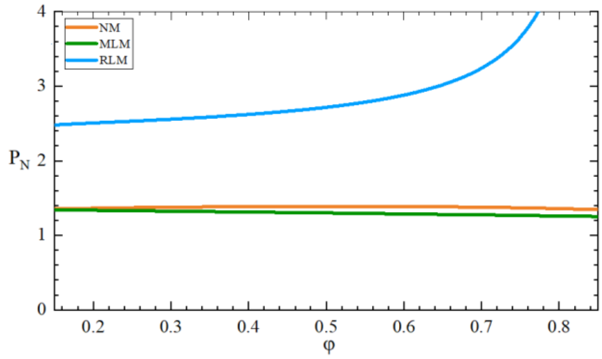


Figure 4. Impact of φ on p_N under different power structures.

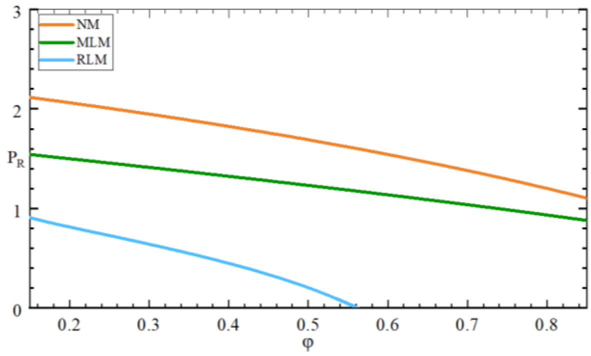


Figure 5. Impact of φ on p_R under different power structures.

The wholesale price of new products is minimally affected by the sharing ratio in both the NM and MLM models (Figure 6). The wholesale price gradually decreases in the MLM model, while it slowly increases in the NM model. At higher sharing ratios, there is

$$\omega_N^{RLM} > \omega_N^{NM} > \omega_N^{MLM} .$$

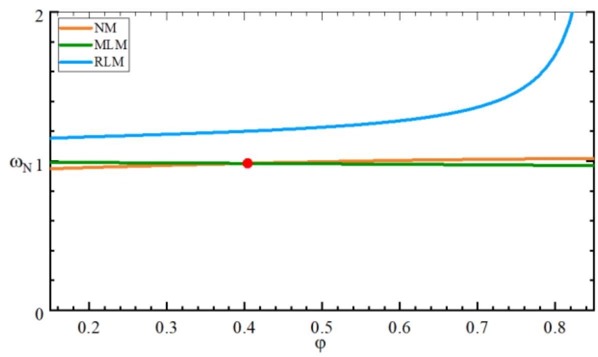


Figure 6. Impact of φ on ω_N under different power structures.

When dominance shifts from the manufacturer to the retailer, the retailer's bargaining power is intensified, while the manufacturer's wholesale price increases. Under all three models, the wholesale price of remanufactured products decreases as the sharing ratio increases:

$$\omega_R^{NM} > \omega_R^{MLM} > \omega_R^{RLM} ,$$

and an increased sharing of investment in recycling activities by retailers, manufacturers demonstrates their willingness to lower the wholesale price of remanufactured products as a means to support retailer (Figure 7).

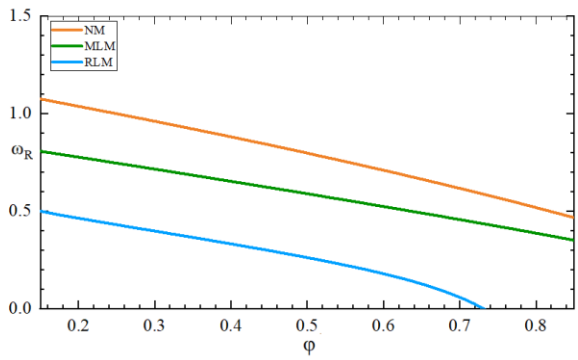
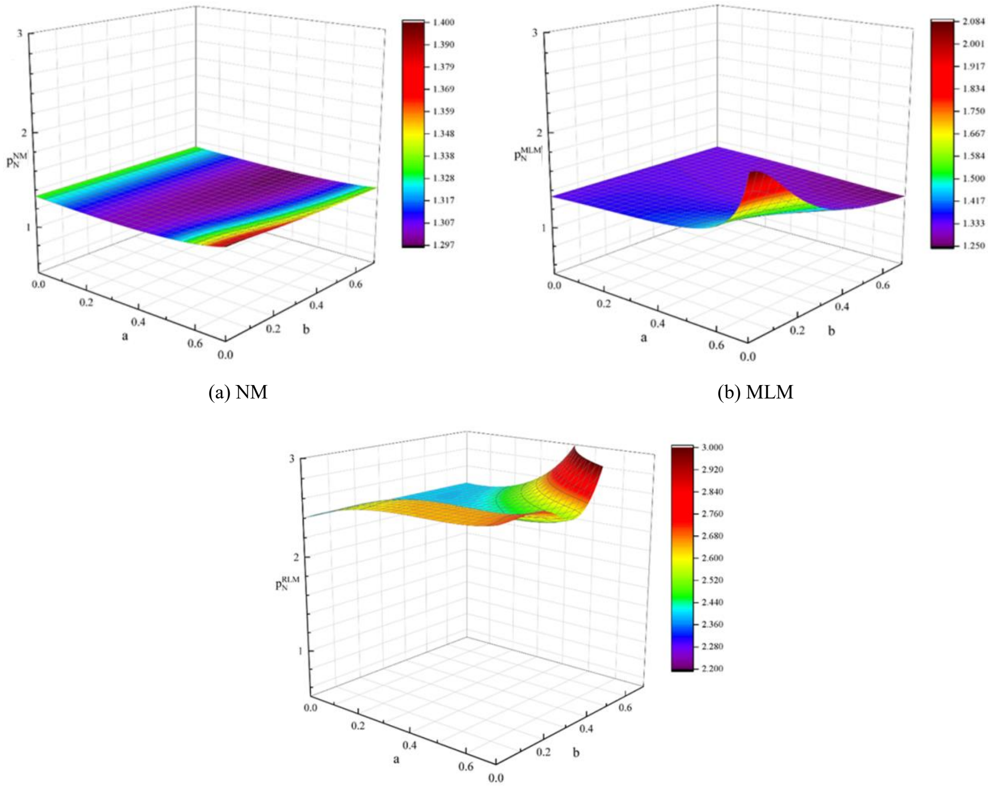


Figure 7. Impact of φ on ω_R under different power structures.

5.3. Impact of substitutability coefficient, and low-carbon preference on pricing decisions

The substitutability coefficient of the product and the consumers' low-carbon preference under the Nash game little affect the retail price of a new product, which fluctuates between 1.297 and 1.400 with a relatively small range as the two parameters change (Figure 8). In the manufacturer-dominated Stackelberg game, the substitutability coefficient positively impacts the retail price of new products, while low-carbon preference has a negative impact, resulting in a decrease. Compared to the previously mentioned games, the retailer-dominated Stackelberg game results in a higher retail price for new products. The highest retail price is attained when both the substitutability coefficient and low-carbon preference are at larger values.



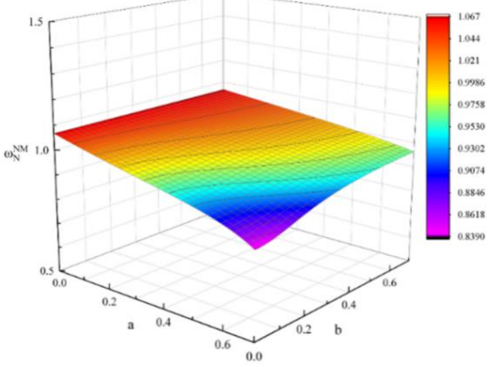
(a) NM

(b) MLM

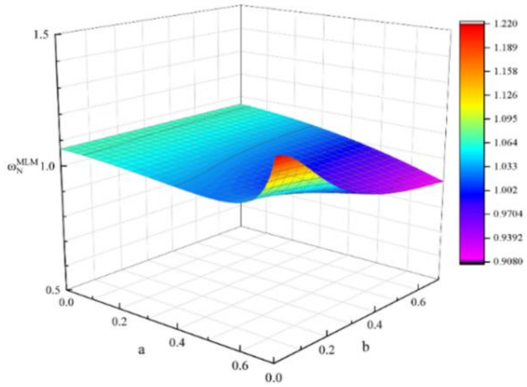
(c) RLM

Figure 8. Impact of a,b on p_N under different power structures.

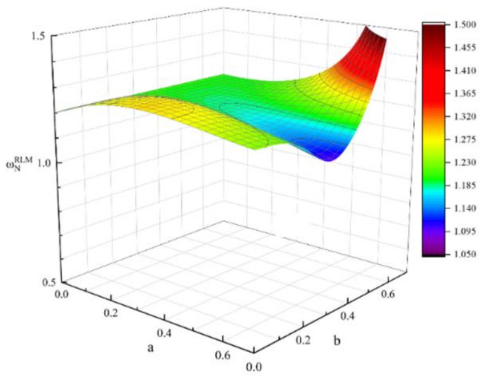
Figure 9 illustrates that the wholesale price of a new product follows a downward trend as the substitutability factor increases under the Nash game. Conversely, when low-carbon preference increases, the wholesale price shows an upward trend. In the manufacturer-dominated Stackelberg game, the impact of both parameters on the wholesale price of a new product is in line with their impact on the retail price. In the context of the retailer-dominated Stackelberg game, the wholesale price of the new products fluctuates between 1.050 and 1.500 with the change of the two parameters, which is a wider range of fluctuation than the two games mentioned above.



(a) NM



(b) MLM



(c) RLM

Figure 9. Impact of a, b on p_R under different power structures.

5.4. Optimal profit under different power structures

To further refine the presentation of our model results and minimize the impact of parameter selection, we conducted additional simulations using randomly generated parameters. Ensuring that all parameter values remained meaningful in practice, we generated 500 different parameter sets to analyze the variations in total supply chain profit under different cost-sharing ratios across the three power structures. The total supply chain profit, defined as the sum of the manufacturer's and retailer's profits, is denoted by the subscript T. The results are presented in Table 3 and Figure 10. The findings reveal that when the cost-sharing ratio is low, the MLM structure has a higher probability of achieving greater total profit compared to NM and RLM, while the likelihood of NM outperforming RLM remains relatively low. As the cost-sharing ratio increases, the advantage of MLM becomes more evident. When the ratio reaches around 0.8, the probability of MLM's total profit exceeding that of RLM reaches its peak. At the same time, the chance of NM achieving higher profits than RLM also increases significantly.

Table 3. Optimal profit under different power structures.

Cost-sharing ratio	Power structures	Condition	Count	Percentage
$\varphi = 0.2$	$\pi_T^{MLM} - \pi_T^{NM}$	>0	317	63.40%
		<0	183	36.60%
	$\pi_T^{MLM} - \pi_T^{RLM}$	>0	311	62.20%
		<0	189	37.80%
	$\pi_T^{NM} - \pi_T^{RLM}$	>0	229	45.80%
		<0	271	54.20%
$\varphi = 0.4$	$\pi_T^{MLM} - \pi_T^{NM}$	>0	357	71.40%
		<0	143	28.60%
	$\pi_T^{MLM} - \pi_T^{RLM}$	>0	361	72.20%
		<0	139	27.80%
	$\pi_T^{NM} - \pi_T^{RLM}$	>0	236	47.20%
		<0	264	52.80%
$\varphi = 0.6$	$\pi_T^{MLM} - \pi_T^{NM}$	>0	343	68.60%
		<0	157	31.40%
	$\pi_T^{MLM} - \pi_T^{RLM}$	>0	362	72.40%
		<0	138	27.60%
	$\pi_T^{NM} - \pi_T^{RLM}$	>0	245	49.00%
		<0	255	51.00%
$\varphi = 0.8$	$\pi_T^{MLM} - \pi_T^{NM}$	>0	319	63.80%
		<0	181	36.20%
	$\pi_T^{MLM} - \pi_T^{RLM}$	>0	433	86.60%
		<0	67	13.40%
	$\pi_T^{NM} - \pi_T^{RLM}$	>0	327	65.40%
		<0	173	34.60%

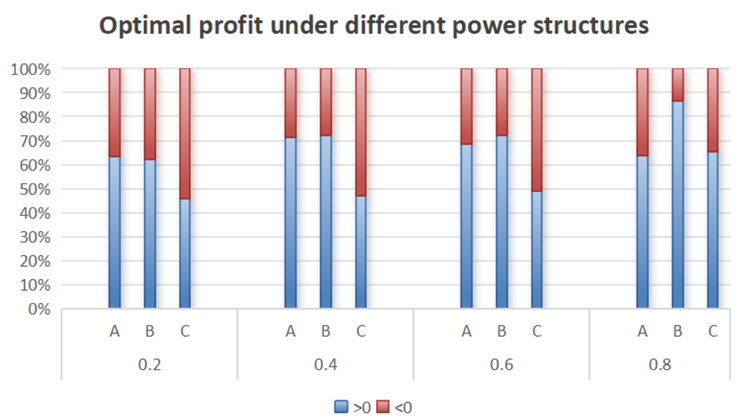


Figure 10. Optimal profit under different power structures, A: $\pi_T^{MLM} - \pi_T^{NM}$, B: $\pi_T^{MLM} - \pi_T^{RLM}$, C: $\pi_T^{NM} - \pi_T^{RLM}$.

This indicates that, with a reduced cost burden, the manufacturer can allocate more resources to low-carbon technology development and product innovation. Their technological advantage in both new and remanufactured products enables the supply chain to maintain high operational efficiency. This insight also highlights that, although the manufacturer bears sole responsibility for the recovery process, their ability to optimize the remanufacturing workflow and control costs is stronger, ensuring a high level of profitability across the entire supply chain.

6. Discussion and conclusions

6.1. Discussion

This study offers valuable insights into the management of CLSC with product differentiation, particularly within the context of a low-carbon economy. Unlike previous research that primarily focused on static models, we combine product differentiation and dynamic game theory to provide a more detailed understanding of long-term strategic interactions among supply chain participants. Our findings demonstrate that integrating low-carbon technologies and cost-sharing mechanisms can significantly enhance both environmental sustainability and economic efficiency. By offering new theoretical perspectives, this study contributes to the field of sustainable supply chain management.

Theoretically, this study advances the development of CLSC management with product differentiation. Our model highlights how different power structures, such as manufacturer-led and retailer-led scenarios, impact the optimal level of low-carbon technology and cost-sharing strategies. This deepens our understanding of strategic decision-making in the supply chain.

From a practical perspective, our findings offer actionable insights for policymakers and industry leaders. By implementing the cost-sharing mechanisms and low-carbon technologies proposed in this paper, companies can strengthen their sustainability practices while simultaneously improving their economic performance. This has the potential to drive significant progress toward achieving environmental goals.

Specifically, the impact of this research extends beyond the specific context of CLSC. By proposing a model that integrates low-carbon technologies and cost-sharing mechanisms, we provide a framework that can be adapted to various industries facing similar sustainability challenges. This approach not only aids in policy development but also offers guidance for practitioners in balancing economic and environmental objectives.

6.2. Conclusions

This paper constructs a differential game model for new and remanufactured products concerning dynamic pricing and low-carbon technology decision-making in CLSC. It fills the gap left by previous studies, which failed to distinguish between new and remanufactured products and primarily relied on static models. By introducing dynamic game analysis, we investigate the pricing strategies and low-carbon technology levels under three power structures: manufacturer-dominated, retailer-dominated, and Nash games.

The results of the study show that there are significant differences in the decision-making behaviors of manufacturers and retailers under different power structures, particularly in terms of cost-sharing mechanisms. Compared with the static model, this study demonstrates, for the first time, the long-term effect of the manufacturer reducing the wholesale price of remanufactured products for the retailer after the retailer bears more costs, using a dynamic model. This plays a positive role in the overall coordination of the supply chain. The problem of “free-riding”, which occurs in some studies, is resolved.

In addition, we find that an increase in the level of low-carbon technology contributes to the growth of the market for remanufactured products. However, if the level of low-carbon technology is too high, it may undermine the competitiveness of new products, especially if it leads to manufacturer-led price reductions for new products. Therefore, the control of low-carbon technology needs to be moderate to balance the market share between old and new products. This finding addresses gaps in existing studies.

Through the quantitative analysis of numerical simulation, we find that the retailer-dominated Stackelberg game maximizes profit for the retailer and that there exists an optimal cost-sharing ratio. Compared with previous studies, we not only extend the applicability of the game model but also provide a new quantitative benchmark for power structures and profit distribution in the supply chain, particularly in the context of low-carbon technologies and cost-sharing mechanisms.

In conclusion, the research in this paper provides a theoretical basis and practical guidance for the strategic management of CLSC with product differentiation in the context of a low-carbon economy. It also proposes an effective cost-sharing mechanism and power structure selection strategy by comparing different game structures.

Author contributions

Jun Wang: designed project, provided figures and revised the manuscript; Dan Wang: designed the project, wrote the first draft and revised the manuscript; Yuan Yuan: conceptualized and supervised the project, revised the manuscript, and acquired funding. All authors have read and approved the final version of the manuscript for publication.

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Conflict of interest

The authors declare no conflicts of interest.

References

1. V. D. R. Guide, T. P. Harrison, L. N. Van Wassenhove, The challenge of closed-loop supply chains, *Interfaces*, **33** (2003), 3–6. https://doi.org/10.1287/inte.33.6.3.25182
2. A. Atasu, V. D. R. Guide, L. N. Van Wassenhove, Product reuse economics in closed-loop supply chain research, *Prod. Oper. Manage.*, **17** (2008), 483–496. https://doi.org/10.3401/poms.1080.0051

3. Z. Zhang, L. Yu, Dynamic decision-making and coordination of low-carbon closed-loop supply chain considering different power structures and government double subsidy, *Clean Technol. Environ. Policy*, **25** (2023), 143–171. https://doi.org/10.1016/j.ijpe.2009.02.012
4. X. Wang, Y. Zhu, H. Sun, F. Jia, Production decisions of new and remanufactured products: Implications for low carbon emission economy, *J. Clean. Prod.*, **171** (2018), 1225–1243. https://doi.org/10.1016/j.jclepro.2017.10.053
5. L. Yang, Y. Hu, L. Huang, Collecting mode selection in a remanufacturing supply chain under cap-and-trade regulation, *Eur. J. Oper. Res.*, **287** (2020), 480–496. https://doi.org/10.1016/j.ejor.2020.04.006
6. R. C. Savaskan, S. Bhattacharya, L. N. Van Wassenhove, Closed-loop supply chain models with product remanufacturing, *Manage. Sci.*, **50** (2004), 239–252. https://doi.org/10.1287/mnsc.1030.0186
7. M. Fleischmann, P. Beullens, J. M. Bloemhof-Ruwaard, L. N. Van Wassenhove, The impact of product recovery on logistics network design, *Prod. Oper. Manage.*, **10** (2003), 156–173. https://doi.org/10.1111/j.1937-5956.2001.tb00076.x
8. V. D. R. Guide, V. Jayaraman, J. D. Linton, Building contingency planning for closed-loop supply chains with product recovery, *J. Oper. Manage.*, **21** (2003), 259–279. https://doi.org/10.1016/S0272-6963(02)00110-9
9. T. M. Choi, Y. Li, L. Xu, Channel leadership, performance and coordination in closed loop supply chains, *Int. J. Prod. Econ.*, **146** (2013), 371–380. https://doi.org/10.1016/j.ijpe.2013.08.002
10. Y. Feng, X. Xia, X. Yin, L. Wang, Z. Zhang, Pricing and coordination of remanufacturing supply chain with government participation considering consumers' preferences and quality of recycled products, *Complexity*, 2022, 8378639. https://doi.org/10.1155/2022/8378639
11. J. Vorasayan, S. M. Ryan, Optimal price and quantity of refurbished products, *Prod. Oper. Manage.*, **15** (2006), 69–383. https://doi.org/10.1111/j.1937-5956.2006.tb00251.x
12. P. D. Giovanni, P. V. Reddy, G. Zaccour, Incentive strategies for an optimal recovery program in a closed-loop supply chain, *Eur. J. Oper. Res.*, **249** (2016), 605–617. https://doi.org/10.1016/j.ejor.2015.09.021
13. H. Guo, Z. Liu, B. Hu, H. Lin, Pricing and coordination mechanism of supply chain considering product recycling under asymmetric power, *Math. Probl. Eng.*, **2021** (2021), 5579655. https://doi.org/10.1155/2021/5579655
14. J. H. Ma, L. Xie, The comparison and complex analysis on dual-channel supply chain under different channel power structures and uncertain demand, *Nonlinear Dyn.*, **83** (2016), 1379–1393. https://doi.org/10.1007/s11071-015-2410-9
15. R. Harms, J. D. Linton, Willingness to pay for eco-certified refurbished products: the effects of environmental attitudes and knowledge, *J. Ind. Ecol.*, **20** (2016), 893–904. https://doi.org/10.1111/jiec.12301
16. J. Ni, J. Zhao, L. K. Chu, Supply contracting and process innovation in a dynamic supply chain with information asymmetry, *Eur. J. Oper. Res.*, **288** (2021), 552–562. https://doi.org/10.1016/j.ejor.2020.06.008
17. Q. Xia, M. Jin, H. Wu, C. Yang, A DEA-based decision framework to determine the subsidy rate of emission reduction for local government, *J. Clean. Prod.*, **202** (2018), 846–852. https://doi.org/10.1016/j.jclepro.2018.08.171

18. N. Wang, Q. He, B. Jiang, Hybrid closed-loop supply chains with competition in recycling and product markets, *Int. J. Prod. Econ.*, **217** (2019), 246–258. https://doi.org/10.1016/j.ijpe.2018.01.002
19. W. Wang, Y. Zhang, K. Zhang, T. Bai, J. Shang, Reward-penalty mechanism for closed-loop supply chains under responsibility-sharing and different power structures, *Int. J. Prod. Econ.*, **170** (2015), 178–190. https://doi.org/10.1016/j.ijpe.2015.09.003
20. Z. Liu, J. Chen, C. Diallo, Optimal production and pricing strategies for a remanufacturing firm, *Int. J. Prod. Econ.*, **204** (2018), 290–315. https://doi.org/10.1016/j.ijpe.2018.07.015
21. S. Liu, F. Yao, D. Chen, CSR investment decision and coordination strategy for closed-loop supply chain with two competing retailers, *J. Clean. Prod.*, **310** (2021), 1–10. https://doi.org/10.1016/j.jclepro.2021.127378
22. Y. Zhang, Y. He, J. Yue, Q. Gou, Pricing decisions for a supply chain with refurbished products, *Int. J. Prod. Res.*, **57** (2019), 2867–2900. https://doi.org/10.1080/00207543.2018.1543968
23. X. P. Hong, L. Xu, P. Du, W. J. Wang, Joint advertising, pricing and collection decisions in a closed-loop supply chain, *Int. J. Prod. Econ.*, **167** (2015), 12–22. https://doi.org/10.1016/j.ijpe.2015.05.001
24. H. Chen, Z. Dong, G. Li, K. He, Remanufacturing process innovation in closed-loop supply chain under cost-sharing mechanism and different power structures, *Comput. Ind. Eng.*, **162** (2021), 107743. https://doi.org/10.1016/j.cie.2021.107743
25. J. Wei, K. Govindan, Y. Li, J. Zhao, Pricing and collecting decisions in a closed-loop supply chain with symmetric and asymmetric information, *Comput. Oper. Res.*, **54** (2015), 257–265. https://doi.org/10.1016/j.cor.2013.11.021
26. J. Gao, X. Wang, Q. Yang, Q. Zhong, Pricing decisions of a dual-channel closed-loop supply chain under uncertain demand of indirect channel, *Math. Probl. Eng.*, **2016** (2016), 6053510. https://doi.org/10.1155/2016/6053510
27. Z. J. Ma, N. Zhang, Y. Dai, S. Hu, Managing channel profits of different cooperative models in closed-loop supply chains, *Omega*, **59** (2016), 251–262. https://doi.org/10.1016/j.omega.2015.06.013
28. Z. Zhang, L. Yu, Altruistic mode selection and coordination in a low-carbon closed-loop supply chain under the government's compound subsidy: a differential game analysis, *J. Clean. Prod.*, **336** (2022), 132863. https://doi.org/10.1016/j.jclepro.2022.132863
29. T. Xu, J. H. Ma, Feed-in tariff or tax-rebate regulation? Dynamic decision model for the solar photovoltaic supply chain, *Appl. Math. Modell.*, **89** (2021), 1106–1123. https://doi.org/10.1016/j.apm.2020.08.007
30. J. H. Ma, T. Xu, Optimal strategy of investing in solar energy for meeting the renewable portfolio standard requirement in America, *J. Oper. Res. Soc.*, **74** (2023), 181–194. https://doi.org/10.1080/01605682.2022.2032427
31. Z. Xiang, M. Xu, Dynamic cooperation strategies of the closed-loop supply chain involving the internet service platform, *J. Clean. Prod.*, **220** (2019), 1180–1193. https://doi.org/10.1016/j.jclepro.2019.01.310
32. D. Ma, J. Hu, Research on collaborative management strategies of closed-loop supply chain under the influence of big-data marketing and reference price effect, *Sustainability*, **12** (2020), 1685. https://doi.org/10.3390/su12041685

33. L. Liu, L. Wang, T. Huang, J. Pang, The differential game of a closed-loop supply chain with manufacturer competition considering goodwill, *Mathematics*, **10** (2022), 1795. https://doi.org/10.3390/math10111795
34. L. Guo, Y. Qu, M. L. Tseng, C. Wu, X. Wang, Two-echelon reverse supply chain in collecting waste electrical and electronic equipment: a game theory model, *Comput. Ind. Eng.*, **126** (2018), 187–195. https://doi.org/10.1016/j.cie.2018.09.036
35. Y. Yang, X. Xu, A differential game model for closed-loop supply chain participants under carbon emission permits, *Comput. Ind. Eng.*, **135** (2019), 1077–1090. https://doi.org/10.1016/j.cie.2019.03.049
36. C. H. Wu, A dynamic perspective of government intervention in a competitive closed-loop supply chain, *Eur. J. Oper. Res.*, **294** (2021), 122–137. https://doi.org/10.1016/j.ejor.2021.01.014
37. J. H. Ma, F. Zhang, H. Jiang, Dynamic pricing game under different channel power structures in a closed-loop supply chain, *Int. J. Bifurcation Chaos*, **30** (2020), 2050052. https://doi.org/10.1142/S0218127420500522
38. Z. H. Xiang, M. L. Xu, Dynamic game strategies of a two-stage remanufacturing closed-loop supply chain considering big data marketing, *Technol. Innovation Overconfidence*, **145** (2020), 106538. https://doi.org/10.1016/j.cie.2020.106538
39. D. W. K. Yeung, L. A. Petrosyan, *Cooperative stochastic differential games*, Springer, 2006. https://doi.org/10.1007/0-387-27622-X

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