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Research article

Fixed-time synchronization control of fuzzy inertial neural networks with mismatched parameters and structures

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Abstract: This research addressed the issue of fixed-time synchronization between random neutral-type fuzzy inertial neural networks and non-random neutral-type fuzzy inertial neural networks. Notably, it should be emphasized that the parameters of the drive and reaction systems did not correspond. Initially, additional free parameters were introduced to reduce the order of the error system. Subsequently, considering the influence of memory on system dynamics, a piecewise time-delay fixed time controller was developed to compensate for the influence of the time delay on the system. Utilizing stochastic analysis techniques and Lyapunov functions, sufficient conditions were derived to ensure the random fixed-time synchronization of the two neural networks. Furthermore, the settling time for system synchronization was assessed using stochastic finite-time inequalities. As a particular case, the necessary criteria for achieving fixed-time synchronization were established when the strength of the random disturbances was equal to zero. Finally, simulation results were provided to demonstrate the effectiveness of the proposed approach.

Keywords: inertial neural network; mismatched parameters; fixed-time synchronization
Mathematics Subject Classification: 93D09, 93D20, 93D23

1. Introduction

Neural networks (NNs) are artificial intelligence models based on the organizations of neurons in the human brain. With the development of computer technology and the renewed attention to neural network research, neural networks became a research hotspot in the 1980s, and a series of important algorithms and models appeared. In recent years, with the rise of deep learning, neural networks have achieved great success in image recognition, natural language processing, speech recognition, financial prediction, and other fields [1–4]. Moreover, neural networks have gradually become one of the core technologies in the field of artificial intelligence, which has a profound impact on various fields.

NNs are often used to describe the dynamic behaviors of neurons and the process of information transmission, where the relationships between the output and input of neurons can be described by first-order differential equations. However, most neural network models described by first-order differential equations usually only involve the adjustment of the output and connection weights of neurons. This may lead to incomplete characterization of the dynamic characteristics of neurons, so it is necessary to further consider the higher-order NNs. Babcock and Westervelt first proposed a NN with second-order derivative terms in [5] and [6], i.e., an inertial neural network (INN), where the second-order derivative terms are called inertial terms. This kind of neural network model described by second-order differential equations is often used to study the vibration characteristics, resonance phenomena, and dynamic modes of neural networks. In addition, INNs are more flexible in modeling complex systems. By introducing more variables and parameters, the modeling requirements of complex systems can be better adapted, thereby improving the accuracy and applicability of the model. In recent years, INN has attracted a lot of attention and achieved some meaningful results. For example, Arbi et al. explored the stability of INNs in an almost anti-periodic environment in [7], whereas Han et al. [8] analyzed project synchronization between inertial neural networks using a direct method.

Synchronization analysis represents a critical dimension in the investigation of dynamic behaviors within neural network systems. Synchronization is defined as the establishment of a relationship among one or more NN models, whereby their state variables tend to exhibit time-consistent behavior or maintain a specific correlation. In recent years, the study of synchronization phenomenon has received extensive attention [9–11]. In [9], the polynomial synchronization of INN models with proportional delays has been explored. Dong et al. [10] investgated the exponential synchronization of discrete NN models and applied the results on the problem of multi-channel audio encryption. Wang et al. explored the global h-synchronization of delayed INNs via the direct second-order response system (SORS) method in [11]. However, in practical modeling scenarios, various sources of uncertainty can impede synchronization among NN models. These uncertainties may include time-varying delays and fuzziness. Time-varying delays typically emerge from physical limitations associated with signal transmission, processing, and propagation. Moreover, the inclusion of time-varying delays within differential terms is referred to as neutral terms. Such neutral terms can give rise to oscillations and chaotic behavior, rendering them a significant consideration in synchronization research. In addition, to effectively capture the uncertainties and fuzziness inherent in models, fuzzy logic operations are often integrated into system modeling and analysis, resulting in the development of fuzzy neural networks (FNNs) [12, 13]. The addition of fuzziness, primarily through fuzzy logic and fuzzy set theory, helps to address a variety of problems that are inherently uncertain, imprecise, or complex. These approaches are particularly beneficial in dealing with systems where binary logic is inadequate for capturing the nuances of real-world phenomena. Based on this characteristic, Liu et al. have applied it to the study of image encryption in [14]. Consequently, the incorporation of fuzzy logic is essential for a comprehensive examination of synchronization phenomena. Therefore, it is imperative to account for both delay and fuzzy factors in the analysis of synchronization within INNs. Recently, many interesting results of neutral inertial fuzzy neural networks (NIFNNs) have been obtained. Jian et al. [15] established the coefficient conditions for the finite-time synchronization of NIFNNs. Duan and Li [16] further explored the FTS of bidirectional associative memory (BAM) type NIFNNs. Han et al. [17] discussed the anti-synchronization between fuzzy INNs.

In addition, synchronization is usually obtained on the basis of infinite time. However, in the practical application process, we hope that the synchronization between systems can be reached as quickly as possible, so the concept of fixed-time synchronization (FTS) is proposed. Based on this concept, many articles have analyzed the fixed-time synchronization between neural network models. Zheng et al. [18] investigated the FTS between competitive neural networks with the same structural parameters. Ping et al. [19] achieved the FTS between memristive neural networks by designing an event-trigger controller. Guo et al. [20] discussed the FTS between stochastic neural networks.

In the studies of FTS mentioned above [14–16, 18–20], they all have predominantly focused on examining two NN models with varying initial conditions. Typically, these systems are characterized by identical structures and system parameters. However, practical modeling endeavors must account for the unavoidable environmental interference and internal deviations that preclude the drive system and response system from being precisely identical in structure or parameters. Therefore, in practical application, the actual controlled disturbed system is usually synchronized with the ideal undisturbed model by control means, which also allows the dynamic properties of the actual controlled disturbed system to be obtained by the ideal undisturbed system model. Consequently, it is more realistic and theoretically meaningful to investigate synchronization phenomena within the context of structurally distinct drive and response systems.

Based on the above discussion, the innovations of this paper are as follows:

1) In this paper, the problem of fixed-time synchronization of INN models with different structures and mismatched parameters are considered. Different from the existing literature [9], this paper chooses two INN models with different structure, the NIFNN and stochastic NIFNN (SNIFNN), as the drive system and the response system.
2) Unlike the studies presented in [21, 22], two variable parameters are introduced when using the reduced order method, which allows us to obtain a less conservative synchronization condition by adjusting the parameters.
3) In addition, in the controller design, we consider the memory effect of the system and design a class of state feedback controller with time delay, which also enhances the robustness of the system to a certain extent.

Finally, the organizational structure of this paper is given. In Section 2, the models, assumptions, and lemmas are given. In Section 3, the main results of this paper are given. An example is given in Section 4.

Notations: $\mathbb{R} = (-\infty, +\infty)$, $\mathbb{R}^+ = (0, +\infty)$, and $\mathbb{Z}^+ = \{a|a = 1, \ldots, n\}$. $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ are the spaces which constitute the n-dimensional real vector and $n \times m$-dimensional real matrices, respectively. $\|A(t)\| = \sum_{i=1}^n |a_i(t)|$, where $A(t) = [a_1(t), \ldots, a_n(t)]^T$. $(\hbar, \mathfrak{F}, \{\mathfrak{F}_t\}_{t\geq 0}, \mathcal{P})$ represents the complete probability space, where filtration $\{\mathfrak{F}_t\}_{t\geq 0}$ is right continuous and includes all $\mathcal{P}$-null sets. $\Phi(t)$ is a Brownian movement which is defined in $(\hbar, \mathfrak{F}, \{\mathfrak{F}_t\}_{t\geq 0}, \mathcal{P})$. $\mathbb{E}$ stands for mathematical expectation operator. $\sum_i = \sum_{i=1}^n$, $\bigwedge_i = \bigwedge_{i=1}^n$, and $\bigvee_i = \bigvee_{i=1}^n$.

2. Preliminaries

We consider the following NIFNN as the drive system:

$$\ddot{y}_i(t) = - a_i\dot{y}_i(t) - b_i y_i(t) + \sum_j c_{ij}\varrho_j(y_j(t)) + \sum_j h_{ij}\varrho_j(y_j(t - \gamma_j(t)))$$

$$+\sum_j \lambda_{ij}\varrho_j(\dot{y}_j(t-\gamma_j(t))) + \bigwedge_j e_{ij}\varrho_j(y_i(t-\gamma_j(t))) + \bigvee_j k_{ij}\varrho_j(y_j(t-\gamma_j(t))), \tag{2.1}$$

where a_i and b_i are positive constants, c_{ij}, h_{ij}, and λ_{ij} are all connection weights between the ith and jth neuron. Variables e_{ij} and k_{ij} denote the elements corresponding to the fuzzy feedback MIN template and the fuzzy feedback MAX template, respectively. $\varrho_j(\cdot)$ is the activation function. $\gamma_j(\cdot)$ is the time-varying delay function. $y_i(t)$, $\dot{y}(t)$, and $\ddot{y}(t)$ are the state, the rate of state change, and the inertial term of the drive system (2.1), respectively.

Consider the following SNIFNN as the response system:

$$\begin{aligned} d\dot{x}_i(t) = \Big\{&-a_i\dot{x}_i(t) - b_i x_i(t) + \sum_j \hat{c}_{ij}\varrho_j(x_j(t)) + \sum_j \hat{h}_{ij}\varrho_j(x_j(t-\gamma_j(t))) \\ &+ \sum_j \hat{\lambda}_{ij}\varrho_j(\dot{x}_j(t-\gamma_j(t))) + \bigwedge_j \hat{e}_{ij}\varrho_j(x_i(t-\gamma_j(t))) \\ &+ \bigvee_j \hat{k}_{ij}\varrho_j(x_j(t-\gamma_j(t))) + u_i\Big\}dt + \sum_j \varsigma_{ij}(t, x_j(t))d\Phi(t), \end{aligned} \tag{2.2}$$

where a_i and b_i are the same as defined in (2.1), $x_i(t)$, $\dot{x}(t)$, and $\ddot{x}(t)$ are the state, the rate of state change, and the inertial term of the response system (2.2), respectively. $\hat{c}_{ij} \neq c_{ij}$, $\hat{h}_{ij} \neq h_{ij}$, $\hat{\lambda}_{ij} \neq \lambda_{ij}$, $\hat{e}_{ij} \neq e_{ij}$, and $\hat{k}_{ij} \neq k_{ij}$. $\varsigma_{ij}(\cdot)$ is the intensity function of stochastic disturbances.

Let $Q_i(t) = x_i(t) - y_i(t)$, and then,

$$\begin{aligned} d\dot{Q}_i(t) = \Big\{&-a_i\dot{Q}_i(t)(t) - b_i Q_i(t) + \sum_j \hat{c}_{ij}\tilde{\zeta}_j(Q_j(t)) + \sum_j \hat{h}_{ij}\tilde{\zeta}_j(Q_j(t-\gamma_j(t))) \\ &+ \sum_j \hat{\lambda}_{ij}\tilde{\zeta}_j(\dot{Q}_j(t-\gamma_j(t))) + \bigwedge_j \hat{e}_{ij}\tilde{\zeta}_j(Q_j(t-\gamma_j(t))) \\ &+ \bigvee_j \hat{k}_{ij}\tilde{\zeta}_j(Q_j(t-\gamma_j(t))) + \mathscr{Q}_i(t) + u_i\Big\}dt + \sum_j \varsigma_{ij}(Q_j(t))d\Phi(t), \end{aligned} \tag{2.3}$$

where $\tilde{c}_{ij} = \hat{c}_{ij} - c_{ij}$, $\tilde{h}_{ij} = \hat{h}_{ij} - h_{ij}$, $\tilde{\lambda}_{ij} = \hat{\lambda}_{ij} - \lambda_{ij}$, $\tilde{e}_{ij} = \hat{e}_{ij} - e_{ij}$, $\tilde{k}_{ij} = \hat{k}_{ij} - k_{ij}$, $\tilde{\zeta}_j(Q_i(t)) = \varrho_i(x_i(t)) - \varrho_i(y_i(t))$, and

$$\begin{aligned} \mathscr{Q}_i(t) = &\sum_j \tilde{c}_{ij}\varrho_j(y_j(t)) + \sum_j \tilde{h}_{ij}\varrho_j(y_j(t-\gamma_j(t))) + \sum_j \tilde{\lambda}_{ij}\varrho_j(\dot{y}_j(t-\gamma_j(t))) \\ &+ \bigwedge_j \tilde{e}_{ij}\varrho_j(y_j(t-\gamma_j(t))) + \bigvee_j \tilde{k}_{ij}\varrho_j(y_j(t-\gamma_j(t))). \end{aligned} \tag{2.4}$$

Remark 2.1. *In the modeling process, the parameters of the system are usually obtained by fitting the data in the ideal state. However, the parameters may change during the operation of the system due to various realistic factors, so there often exists mismatched or unequal parameters between the model and the actual operating systems. For example, in industrial systems, due to system aging, wear, manufacturing errors, and other factors, it is easy to cause the ideal system parameters and the actual system parameters to not match or be equal, and therefore, it is more practical to consider the synchronization between the systems with mismatched parameters.*

Remark 2.2. *Due to the existence of of the parameter-matching phenomenon, the error system obtained will have a redundant term, that is,* (2.4)*, which is also a major obstacle for the realization of fixed-time stability. In addition, if* $\mathscr{Q} = 0$*, then the parameter mismatch between the models will disappear and degenerate into a general synchronization problem of the driving response system. In addition, this paper further considers the influence of random interference on synchronization, and how to transform a random process into a general process is also a difficulty we will consider.*

By using the reduced-order method (RoM), let $\bar{Q}_i(t) = \eta_i \frac{dQ_i(t)}{dt} + \xi_i Q_i(t)$, and one has

$$\begin{cases} dQ_i(t) = \Big\{\dfrac{1}{\eta_i}\bar{Q}_i(t) - \dfrac{\xi_i}{\eta_i}Q_i(t)\Big\}dt, \\ d\bar{Q}_i(t) = \Big\{(\dfrac{\xi_i}{\eta_i} - a_i)\bar{Q}_i(t) + (a_i\xi_i - b_i\eta_i - \dfrac{\xi_i^2}{\eta_i})Q_i(t) \\ \qquad + \eta_i\Big[\sum_j \hat{c}_{ij}\tilde{\zeta}_j(Q_j(t)) + \sum_j \hat{h}_{ij}\tilde{\zeta}_j(Q_j(t-\gamma_j(t))) \\ \qquad + \sum_j \hat{\lambda}_{ij}\tilde{\zeta}_j(\dot{Q}_j(t-\gamma_j(t))) + \bigwedge_j \hat{e}_{ij}\tilde{\zeta}_j(Q_i(t-\gamma_j(t))) \\ \qquad + \bigvee_j \hat{k}_{ij}\tilde{\zeta}_j(Q_j(t-\gamma_j(t))) + \mathscr{Q}_i(t) + u_i\Big]\Big\}dt + \sum_j \varsigma_{ij}(Q_j(t))d\Phi(t). \end{cases}$$

Remark 2.3. *The investigation of the dynamical behaviors of INNs can be approached through two methodologies: The reduced-order method (RoM) and the non-reduced order method (nRoM). The RoM facilitates system analysis by effectively reducing the system's order while increasing its dimensionality. Consequently, the RoM is widely adopted in the literature [14, 23, 24]. However, this approach introduces additional variables that necessitate the validation of the system's stability conditions in relation to these extra parameters. Conversely, substantial research has also been devoted to non-reductive methods [25, 26]. This approach employs the construction of Lyapunov functionals to analyze the properties of INN, yielding a well-defined structure, albeit one that is complex to derive. As this method does not simplify the system, the resulting stability conditions tend to be more intricate.*

Remark 2.4. *In this paper, the controller is placed in the original and undeformed INN model to study the synchronization between INNs. In previous studies [23, 24], synchronization of INNs are investigated using the order reduction method. The controller is placed in the response system after deformation, and synchronization is achieved by controlling the transformation system with high dimensions. However, using this method will make the system with a controller difficult to restore to the original system, and the high dimension of the controller is not easy to achieve. Therefore, this method is not suitable in the actual modeling process.*

The following are the assumptions and lemmas that the model needs to satisfy.

Assumption 2.1. *There exist positive constants* M_i *and* $\bar{M}_i$ *such that*

$$|\varrho_i(u)| \le M_i, \text{ and } |\varrho_i(u) - \varrho_i(v)| \le \bar{M}_i|u - v|. \tag{2.5}$$

Remark 2.5. *Since* $Q_i(t) = x_i(t) - y_i(t)$ *and* $\tilde{\zeta}_j(Q_i(t)) = \varrho_i(x_i(t)) - \varrho_i(y_i(t))$*, Assumption 2.1 implies that* $|\tilde{\zeta}_j(Q_i(t))| \le \bar{M}_i|Q_i(t)|$.

Assumption 2.2. *There exist $\Gamma_{ij} > 0$ such that*

$$\Big|\sum_j \varsigma_{ij}(Q_j(t))\Big|^2 \le \sum_j \Gamma_{ij}|Q_j(t)|^2.$$

Lemma 2.1. *[27] Assume μ, ω, $\bar{\mu}$, and $\bar{\omega}$ are the states of the drive system* (2.1) *and the response system* (2.2)*, respectively. Then,*

$$\begin{aligned}
\Big|\bigwedge_j e_{ij}\varrho_j(\mu) - \bigwedge_j e_{ij}\varrho_j(\omega)\Big| &\le \sum_j |e_{ij}||\varrho_j(\mu) - \varrho_j(\omega)|,\\
\Big|\bigvee_j k_{ij}\varrho_j(\mu) - \bigvee_j k_{ij}\varrho_j(\omega)\Big| &\le \sum_j |k_{ij}||\varrho_j(\mu) - \varrho_j(\omega)|,\\
\Big|\bigwedge_j \hat{e}_{ij}\varrho_j(\bar{\mu}) - \bigwedge_j \hat{e}_{ij}\varrho_j(\bar{\omega})\Big| &\le \sum_j |\hat{e}_{ij}||\varrho_j(\bar{\mu}) - \varrho_j(\bar{\omega})|,\\
\Big|\bigvee_j \hat{k}_{ij}\varrho_j(\bar{\mu}) - \bigvee_j \hat{k}_{ij}\varrho_j(\bar{\omega})\Big| &\le \sum_j |\hat{k}_{ij}||\varrho_j(\bar{\mu}) - \varrho_j(\bar{\omega})|.
\end{aligned}$$

Lemma 2.2. *[28] If $a_1, \dots, a_n \ge 0$, $\hbar > 1$, and $\hbar \in (0,1)$, then*

$$\sum_i a_i^{\hbar} \ge n^{1-\hbar}\Big(\sum_i a_i\Big)^{\hbar}, \quad \sum_i a_i^{\hbar} \ge \Big(\sum_i a_i\Big)^{\hbar}. \tag{2.6}$$

Definition 2.1. *[24] For a nonlinear stochastic system*

$$dR(t) = \Psi_1(R(t))dt + \Psi_2(R(t))d\Phi(t),\ R(0) = R_0, \tag{2.7}$$

where $R \in \mathbb{R}^+$, and $\Psi_i \in \mathbb{R}^+ \times \mathbb{R}^n \mapsto \mathbb{R}^n,\ i = 1, 2$.

(1) The system (2.7) *is called globally stochastically finite-time stable (GSFnS) if there exists a positive function $\mathscr{T}(\cdot)$ such that $\lim_{t\to T(R_0)} \mathbb{E}\|H(t)\|^2 = 0$ and $\mathbb{E}\|H(t)\|^2 = 0$, when $t > \mathscr{T}(R_0) \in \mathbb{R}^+$ hold.*

(2) The system (2.7) *is called globally stochastically fixed-time stable if sytem* (2.7) *is GSFnS and there exists a positive constant $\mathscr{T}^{\max}$ such that $\mathscr{T}(R_0) \le \mathscr{T}^{\max}$ and $\mathbb{E}\|H(t)\|^2 = 0,\ t > \mathscr{T}^{\max}$ hold.*

Definition 2.2. *[29] For system* (2.7) *and given function $V : \mathbb{R}^n \to \mathbb{R}^+$, infinitesimal generator $\mathscr{L}$ is defined as the following:*

$$\mathscr{L}V(t, H(t)) = V_t(t, H(t)) + V_\Omega(t, H(t))\Psi_1(H(t)) + \frac{1}{2}\text{trace}[\Psi_2^T(H(t))V_{\Omega\Omega}\Psi_2(H(t))],$$

where

$$\begin{aligned}
V_t(t, H(t)) &= \frac{\partial V(t, H(t))}{\partial t},\\
V_\Omega(t, H(t)) &= \Big(\frac{\partial V(t, H(t))}{\partial \Omega_1}, \dots, \frac{\partial V(t, H(t))}{\partial \Omega_n}\Big),\\
V_{\Omega\Omega}(t, H(t)) &= \Big(\frac{\partial^2 V(t, H(t))}{\partial \Omega_i(t)\partial \Omega_j(t)}\Big)_{n\times n}.
\end{aligned}$$

Lemma 2.3. *[30] Let $V : \mathbb{R}^n \to \mathbb{R}^+$ be a positive definite and radially unbounded function, then systems* (2.1) *and* (2.2) *can achieve FTS if*

$$\mathscr{L}V(H(t)) \leq -\Phi_1 V(H(t))^\iota - \Phi_2 \tag{2.8}$$

holds, where $H(t) \in \mathbb{R}^n$, $\iota > 1$, and $\Phi_1,\ \Phi_2 > 0$. In addition, the settling time (ST) is estimated by

$$\mathbb{E}[\mathscr{T}(s, B)] \leq \mathscr{T}^{\max} = \frac{1}{\Phi_2}(1 + \frac{1}{\iota - 1})\Big(\frac{\Phi_2}{\Phi_1}\Big)^{\frac{1}{\iota}}. \tag{2.9}$$

When there is no stochastic disturbance in the system, Lemma 2.3 will degenerate to the following lemma.

Lemma 2.4. *[31] Let $V : \mathbb{R}^n \to \mathbb{R}^+$ be a positive definite and radially unbounded function, and then systems* (2.1) *and* (2.2) *can achieve FTS in the absence of stochastic interference if*

$$\frac{dV(H(t))}{dt} \leq -\Phi_1 V(H(t))^\iota - \Phi_2 \tag{2.10}$$

holds, where $H(t) \in \mathbb{R}^n$, $\iota > 1$, and $\Phi_1,\ \Phi_2 > 0$. In addition, the ST is estimated by

$$\mathscr{T}^{\max} = \frac{1}{\Phi_2}(1 + \frac{1}{\iota - 1})\Big(\frac{\Phi_2}{\Phi_1}\Big)^{\frac{1}{\iota}}. \tag{2.11}$$

3. Main results

For simplicity, we introduce the following notations: $Q_i = Q_i(t)$, $\bar{Q}_i = \bar{Q}_i(t)$, and $Q_i^\gamma = Q_i(t - \gamma_i(t))$.

Theorem 3.1. *Under Assumptions 2.1 and 2.2 and the following controller:*

$$u_i = \begin{cases} -\, sign(\bar{Q}_i)\Big(K_{i1}\dfrac{|Q_i|^{\ell+1} + |Q_i|^2}{|\bar{Q}_i|} + K_{i2}|\bar{Q}_i|^\ell + K_{i3}(|Q_i| + |\bar{Q}_i|) + \dfrac{\vartheta_i}{|\bar{Q}_i|} \\ +\, \displaystyle\sum_j \bar{K}_{ij}|Q_j^\gamma| + \delta_i\Big), \quad |\bar{Q}_i(t)| \neq 0,\ \ell > 1, \\ 0, \qquad |\bar{Q}_i(t)| = 0. \end{cases} \tag{3.1}$$

FINNs (2.1) *and* (2.2) *can achieve FTS, where*

$$K_{i1} \geq \Big|\sum_j \Gamma_{ij}^2 - \frac{\xi_i}{\eta_i}\Big|, \tag{3.2}$$

$$K_{i2} \geq 0, \tag{3.3}$$

$$K_{i3} \geq \max_{i=1,2,\ldots,n}\Big\{\frac{1}{\eta_i} + |a_i\xi_i - b_i\eta_i - \frac{\xi_i^2}{\eta_i}|, \frac{\xi_i}{\eta_i} - a_i\Big\}, \tag{3.4}$$

$$\bar{K}_{ij} \geq (|\hat{h}_{ij}| + |\hat{e}_{ij}| + |\hat{k}_{ij}|)\bar{M}_j, \tag{3.5}$$

$$\delta_i \geq \sum_j (|\tilde{c}_{ij}| + |\tilde{h}_{ij}| + |\tilde{\lambda}_{ij}| + |\tilde{e}_{ij}| + |\tilde{k}_{ij}| + 2|\hat{\lambda}_{ij}|)M_j. \tag{3.6}$$

In addition, the settling time can be estimated by

$$\mathbb{E}[\mathscr{T}(s,B)] \leq \mathscr{T}^{\max} = \frac{1}{\Phi_2}(1+\frac{2}{\ell-1})\Big(\frac{\Phi_2}{\Phi_1}\Big)^{\frac{2}{\ell+1}}, \tag{3.7}$$

where

$$\Phi_1 = (2n)^{\frac{1-\ell}{2}} \min_{i=1,2,\ldots,n}\Big\{K_{i1}, K_{i2}\Big\}, \tag{3.8}$$

$$\Phi_2 = \min_{i=1,2,\ldots,n}\{\vartheta_i\}. \tag{3.9}$$

Proof. Take the following Lyapunov functional

$$V(t) = V_1(t) + V_2(t), \tag{3.10}$$

where $V_1(t) = \frac{1}{2}\sum_i |Q_i|^2$ and $V_2(t) = \frac{1}{2}\sum_i |\bar{Q}_i|^2$. Hence,

$$\mathscr{L}V_1(t) = \sum_i sign(Q_i)|Q_i|(\frac{1}{\eta_i}\bar{Q}_i - \frac{\xi_i}{\eta_i}Q_i), \tag{3.11}$$

and

$$\begin{aligned}\mathscr{L}V_2(t) = &\sum_i sign(\bar{Q}_i)|\bar{Q}_i|\Big\{(\frac{\xi_i}{\eta_i} - a_i)\bar{Q}_i + (a_i\xi_i - b_i\eta_i - \frac{\xi_i^2}{\eta_i})Q_i \\ &+ \eta_i\Big[\sum_j \hat{c}_{ij}\tilde{\zeta}_j(Q_j) + \sum_j \hat{h}_{ij}\tilde{\zeta}_j(Q_j^{\gamma}) + \sum_j \hat{\lambda}_{ij}\tilde{\zeta}_j(\dot{Q}_j^{\gamma}) \\ &+ \bigwedge_j \hat{e}_{ij}\tilde{\zeta}_j(Q_j^{\gamma}) + \bigvee_j \hat{k}_{ij}\tilde{\zeta}_j(Q_j^{\gamma})\Big] + \mathscr{Q}_i(t) \\ &- sign(\bar{Q}_i)\Big(K_{i1}\frac{|Q_i|^{\ell+1} + |Q_i|^2}{|\bar{Q}_i|} + K_{i2}|\bar{Q}_i|^{\ell} \\ &+ K_{i3}(|Q_i| + |\bar{Q}_i|) + \sum_j \bar{K}_{ij}|Q_j^{\gamma}| + \delta_i\Big)\Big\} + \frac{1}{2}\sum_i\Big\{\sum_j \varsigma_{ij}(t, Q(t))\Big\}^2.\end{aligned} \tag{3.12}$$

By combining Lemma 2.1 and Assumption 2.1, we have

$$\bigwedge_j \hat{e}_{ij}\tilde{\zeta}_j(Q_j^{\gamma}) \leq \sum_j |\hat{e}_{ij}|\bar{M}_j|Q_j^{\gamma}|, \tag{3.13}$$

$$\bigvee_j \hat{k}_{ij}\tilde{\zeta}_j(Q_j^{\gamma}) \leq \sum_j |\hat{k}_{ij}|\bar{M}_j|Q_j^{\gamma}|, \tag{3.14}$$

$$\sum_j \lambda_{ij}\tilde{\zeta}_j(\dot{Q}_j^{\gamma}) \leq \sum_j 2|\hat{\lambda}_{ij}|M_j, \tag{3.15}$$

$$\mathscr{Q}_i(t) \leq \sum_j\Big(|\tilde{c}_{ij}| + |\tilde{h}_{ij}| + |\tilde{\lambda}_{ij}| + |\tilde{e}_{ij}| + |\tilde{k}_{ij}|\Big)M_j. \tag{3.16}$$

According to Cauchy's inequality as well as (3.13)–(3.16), we can obtain

$$
\begin{aligned}
\mathscr{L}V \leq & \sum_i\Big(\frac{1}{\eta_i} + |a_i\xi_i - b_i\eta_i - \frac{\xi_i^2}{\eta_i}| - K_{i3}\Big)|Q_i||\bar{Q}_i| + \Big(\frac{\xi_i}{\eta_i} - a_i - K_{i3}\Big)|\bar{Q}_i|^2 \\
& + \sum_i\Big[\sum_j\Big((|\hat{h}_{ij}| + |\hat{e}_{ij}| + |\hat{k}_{ij}|)\bar{M}_j - \bar{K}_{ij}\Big)\Big]|\bar{Q}_i||Q_j^{\gamma}| \\
& - \sum_i\Big(K_{i1}|Q_i|^{\ell+1} + K_{i2}|\bar{Q}_i|^{\ell+1}\Big) + \sum_i\Big(-\delta_i + \sum_j(|\tilde{c}_{ij}| + |\tilde{h}_{ij}| + |\tilde{\lambda}_{ij}| \\
& + |\tilde{e}_{ij}| + |\tilde{k}_{ij}| + 2|\hat{\lambda}_{ij}|)M_j\Big)|\bar{Q}_i| + \sum_i(\sum_j \Gamma_{ij}^2 - \frac{\xi_i}{\eta_i} - K_{i1})|Q_i|^2 - \vartheta_i.
\end{aligned} \tag{3.17}
$$

Combining with conditions (3.2)–(3.6), we have

$$
\begin{aligned}
\mathscr{L}V \leq & \mathscr{L}V_1 + \mathscr{L}V_2 \\
\leq & -\sum_i K_{i1}|Q_i|^{\ell+1} - \sum_i K_{i2}|\bar{Q}_i|^{\ell+1} - \vartheta_i \\
\leq & -(2n)^{\frac{1-\ell}{2}} \min_{i=1,2,\ldots,n}\Big\{K_{i1}, K_{i2}\Big\}V^{\frac{\ell+1}{2}} - \min_{i=1,2,\ldots,n}\{\vartheta_i\} \\
= & -\Phi_1 V^{\frac{\ell+1}{2}} - \Phi_2.
\end{aligned} \tag{3.18}
$$

Therefore, the NIFNN (2.1) and SNIFNN (2.2) can achieve FTS with the settling time (3.7). □

Remark 3.1. *First, we design a class of segmented feedback controllers in Theorem 3.1. Compared with the continuous controllers used in the literature [14, 16], the segmented controller can change the control mode in real time according to the system state, and can provide higher control accuracy under the influence of random terms. In addition, the influence of the memory performance of the system on the system performance was considered in the design of the controller. In order to compensate for the delayed response caused by the time delay, we designed a memory feedback controller to reduce the impact of the time delay and improve the robustness of the system.*

When there is no stochastic disturbance in the response system (2.2), we get the following theorem.

Theorem 3.2. *If Assumptions 2.1 and 2.2 hold and stochastic intensity function $\varsigma_{ij}(\cdot) = 0$, NIFNN* (2.1) *and SNIFNN* (2.2) *can achieve FTS under the following controller:*

$$
u_i = \begin{cases} -sign(\bar{Q}_i)\Big(K_{i1}\dfrac{|Q_i|^{\ell+1}}{|\bar{Q}_i|} + K_{i2}|\bar{Q}_i|^{\ell} + K_{i3}(|Q_i| + |\bar{Q}_i|) + \dfrac{\vartheta_i}{|\bar{Q}_i|} \\ + \displaystyle\sum_j \bar{K}_{ij}|Q_j^{\gamma}| + \delta_i\Big), \ \ |\bar{Q}_i(t)| \neq 0, \ \ell > 1, \\ 0, \quad |\bar{Q}_i(t)| = 0, \end{cases} \tag{3.19}
$$

with the settling time (3.7)*, where*

$$
K_{i1} \geq 0, \ K_{i2} \geq 0, \tag{3.20}
$$

$$K_{i3} \geq \max_{i=1,2,\ldots,n}\Big\{\frac{1}{\eta_i} + |a_i\xi_i - b_i\eta_i - \frac{\xi^2}{\eta_i}|, \frac{\xi_i}{\eta_i} - a_i\Big\}, \tag{3.21}$$

$$\bar{K}_{ij} \geq (|\hat{h}_{ij}| + |\hat{e}_{ij}| + |\hat{k}_{ij}|)\bar{M}_j, \tag{3.22}$$

$$\delta_i \geq \sum_j (|\tilde{c}_{ij}| + |\tilde{h}_{ij}| + |\tilde{\lambda}_{ij}| + |\tilde{e}_{ij}| + |\tilde{k}_{ij}| + 2|\hat{\lambda}_{ij}|)M_j. \tag{3.23}$$

Proof. Take the following Lyapunov functional

$$V = \frac{1}{2}\sum_i\Big(|Q_i|^2 + |\bar{Q}_i|^2\Big). \tag{3.24}$$

Then,

$$\begin{aligned}\dot{V} &= \sum_i\Big(sign(Q_i)|Q_i|\dot{Q}_i + sign(\bar{Q}_i)|\bar{Q}_i|\dot{\bar{Q}}_i\Big)\\ &\leq \sum_i\Big(sign(Q_i)|Q_i|(\frac{1}{\eta_i}\bar{Q}_i - \frac{\xi_i}{\eta_i}Q_i)\Big) + \sum_i\Big\{sign(\bar{Q}_i)|\bar{Q}_i|\Big[(\frac{\xi_i}{\eta_i} - a_i)\bar{Q}_i\\ &\quad + (a_i\xi_i - b_i\eta_i - \frac{\xi_i^2}{\eta_i})Q_i + \mathscr{Q}_i + \eta_i\Big[\sum_j \hat{c}_{ij}\tilde{\zeta}_j(Q_j) + \sum_j \hat{h}_{ij}\tilde{\zeta}_j(Q_j^{\gamma})\\ &\quad + \sum_j \hat{\lambda}_{ij}\tilde{\zeta}_j(\dot{Q}_j^{\varphi}) + \bigwedge_j \hat{e}_{ij}\tilde{\zeta}_j(Q_i^{\varphi}) + \bigvee_j \hat{k}_{ij}\tilde{\zeta}_j(Q_j^{\gamma}) + u_i\Big]\Big]\Big\}.\end{aligned} \tag{3.25}$$

Combining this with (3.19) and (3.20)–(3.23), similar to Theorem 3.1, we can obtain

$$\dot{V} \leq -\Phi_1 V^{\frac{\ell+1}{2}} - \Phi_2. \tag{3.26}$$

Hence, NIFNN (2.1) and SNIFNN (2.2) without randomness can achieve FTS with the settling time (3.7). □

When parameters $\hat{c}_{ij}$, $\hat{h}_{ij}$, $\hat{\lambda}_{ij}$, $\hat{e}_{ij}$, and $\hat{k}_{ij}$ are given the same values as c_{ij}, h_{ij}, λ_{ij}, e_{ij}, and k_{ij}, the following corollary can be obtained.

Corollary 3.1. *If Assumptions 2.1 and 2.2 and conditions* (3.2)–(3.5) *hold, then FINN can achieve synchronization under controller* (3.1) *with ST* (3.7)*, where* $\delta_i \geq 0$.

Proof. The proof is similar to Theorem 3.1 and is therefore omitted here. □

4. Example

Example 4.1. *Consider the NIFNN* (2.1) *and the SNIFNN* (2.2) *with the following parameters:* $a_1 = b_2 = 2$, $b_1 = a_2 = 1$, $c_{11} = 0.5377$, $c_{12} = -2.2588$, $c_{21} = 1.8339$, $c_{22} = 0.8622$, $\hat{c}_{11} = 0.3188$, $\hat{c}_{12} = -0.4336$, $\hat{c}_{21} = -1.3077$, $\hat{c}_{22} = 0.3426$, $h_{11} = 3.5784$, $h_{12} = -1.3499$, $h_{21} = 2.7694$, $h_{22} = 3.0349$, $\hat{h}_{11} = 0.7254$, $\hat{h}_{12} = 0.7147$, $\hat{h}_{21} = -0.0631$, $\hat{h}_{22} = -0.2050$, $e_{11} = 0.4889$, $e_{12} = 0.72689$, $e_{21} = 1.0347$, $e_{22} = -0.3034$, $\hat{e}_{11} = 0.2938$, $\hat{e}_{12} = 0.8884$, $\hat{e}_{21} = -0.7873$, $\hat{e}_{22} = -1.1471$, $k_{11} = -1.0689$, $k_{12} = -2.9443$, $k_{21} = -0.8095$, $k_{22} = 1.4384$, $\hat{k}_{11} = 0.3252$, $\hat{k}_{12} = 1.3703$, $\hat{k}_{21} = -0.7549$, $\hat{k}_{22} = -1.7115$, $\lambda_{11} = -0.1241$, $\lambda_{12} = 1.4090$, $\lambda_{21} = 1.4897$, $\lambda_{22} = 1.4172$, $\hat{\lambda}_{11} = 0.6715$, $\hat{\lambda}_{12} = 0.7172$, $\hat{\lambda}_{21} = -1.2075$, *and* $\hat{\lambda}_{22} = 1.6302$.

Take $\varrho_j(x) = \tanh(x)$, $\gamma_j(x) = \sin(2x) + 1.5$, and $\varsigma_{ij}(x) = \tanh(\sin(x))$. The states of the NIFNN (2.1) and SNIFNN (2.2) are shown in Figures 1 and 2, respectively. With the increase of running time without the influence of the controller, the drive system gradually exhibits cyclic behavior. However, the response system produces chaos under the influence of noise, and the two systems cannot achieve synchronization. Figure 3 shows the values of the time-varying delay functions and stochastic intensity functions. From Theorem 3.1, we can obtain

$$\begin{aligned} &K_{i1} \geq 1,\ K_{i3} \geq 1,\ \bar{K}_{11} \geq 1.3445,\ \bar{K}_{12} \geq 2.9734,\\ &\bar{K}_{21} \geq 1.6053,\ \bar{K}_{22} \geq 3.0636,\ \delta_1 \geq 15.9031,\\ &\textit{and}\ \delta_2 = 21.3515. \end{aligned}$$

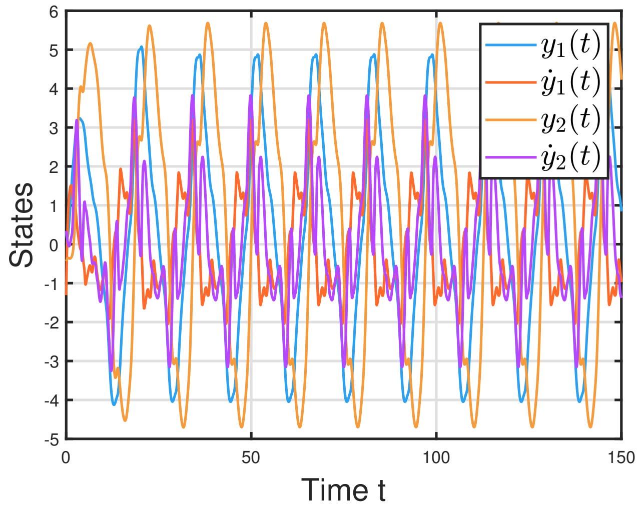

Figure 1. States of the NIFNN (2.1) without control.

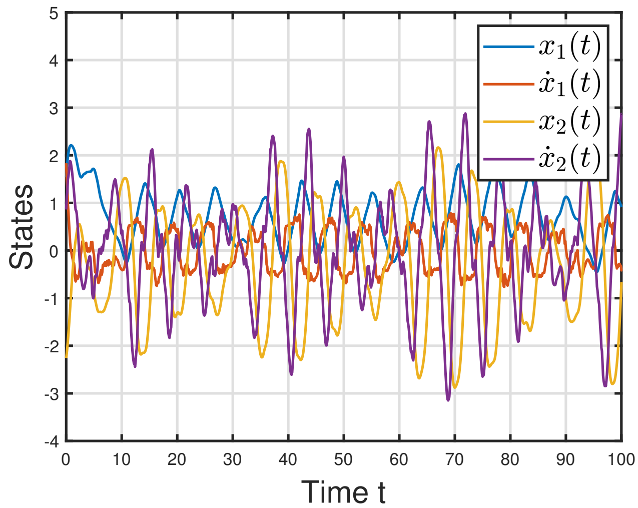

Figure 2. States of the SNIFNN (2.2) without control.

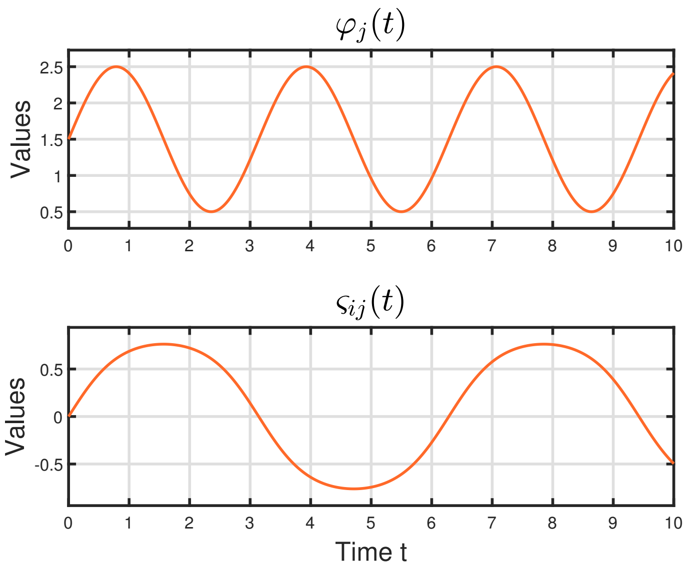

Figure 3. Values of the time-varying delay function $\gamma_j(t)$ and stochastic intensity function $\varsigma_{ij}(t)$.

Based on the above calculation, take the following controller:

$$u_i = \begin{cases} -\,sign(\bar{Q}_i)\Big(\dfrac{|Q_i|^{2.2}+|Q_i|^2}{|\bar{Q}_i|} + 12|\bar{Q}_i|^{1.2} + 2(|Q_i|+|\bar{Q}_i|) + \dfrac{12}{|\bar{Q}_i|} \\ +\sum\limits_j \bar{K}_{ij}|Q_j^{\gamma}| + \delta_i\Big), \quad |\bar{Q}_i(t)| \neq 0, \\ 0, \qquad |\bar{Q}_i(t)| = 0, \end{cases} \tag{4.1}$$

where $\bar{K} = [2.3445, 3.9734; 2.6053, 4.0636]$, $\delta_1 = 16.9031$, and $\delta_2 = 22.3515$.

Therefore, the NIFNN (2.1) and SNIFNN (2.2) can achieve synchronization with the following settling time:

$$\begin{aligned} \mathbb{E}[\mathscr{T}(s,B)] \leq \mathscr{T}^{\max} &= \frac{1}{12}(1+\frac{2}{1.2-1})\Big(\frac{12}{0.8706}\Big)^{\frac{2}{1.2+1}} \\ &= 9.9545. \end{aligned} \tag{4.2}$$

Figure 4 shows the states of systems (2.1) and (2.2) under controller (4.1). The values of the synchronization error are shown in Figure 5. It can be found from Figure 5 that the error system gradually reaches stochastic fixed-time stability, which also means that under the influence of the controller, the state error of NIFNN and SNIFNN further decreases with time, that is, the stochastic FTS is achieved.

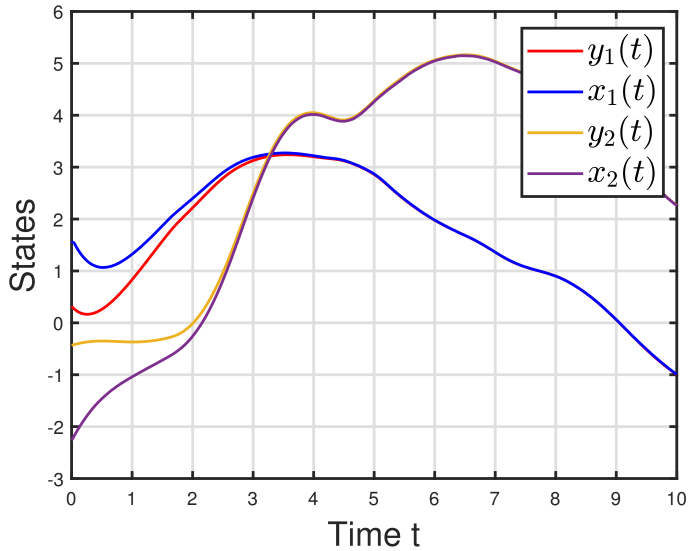

Figure 4. States of systems (2.1) and (2.2) under controller (3.1).

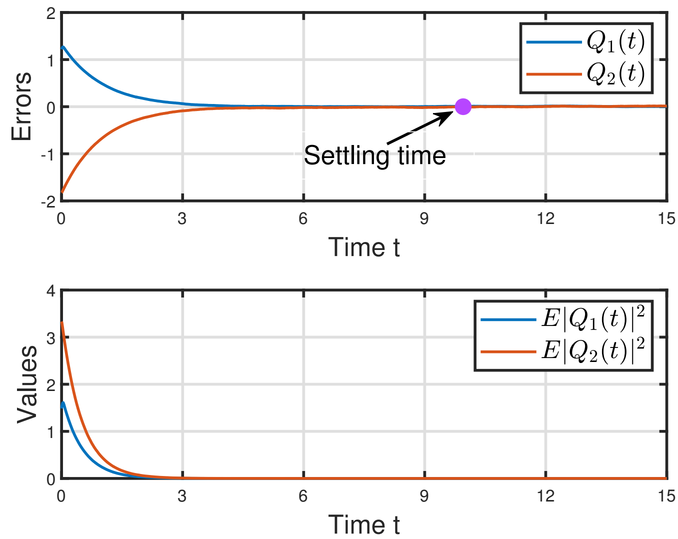

Figure 5. Values of error system (2.5).

Example 4.2. *Take* $\varsigma_{ij}(t) = 0$ *and the same parameters as in Example 4.1. Then, under controller* (3.19)*, NIFNNs* (2.1) *and* (2.2) *can achieve synchronization with ST* (4.2)*. Figure 6 shows the states of NIFNN* (2.1) *and SNIFNN* (2.2) *under* $\varsigma_{ij}(t) = 0$ *without controller* (3.19)*. It can be seen that without the intervention of the controller, the drive system and the response system cannot achieve fixed-time synchronization when* $\varsigma_{ij}(t) = 0$*. Figures 7 and 8 show the synchronization states of NIFNN* (2.1) *and SNIFNN* (2.2) *under* $\varsigma_{ij}(t) = 0$ *and the values of error system* (2.5)*. With the intervention of the controller, the drive system and the response system can obviously achieve FTS before the systems reach the settling time.*

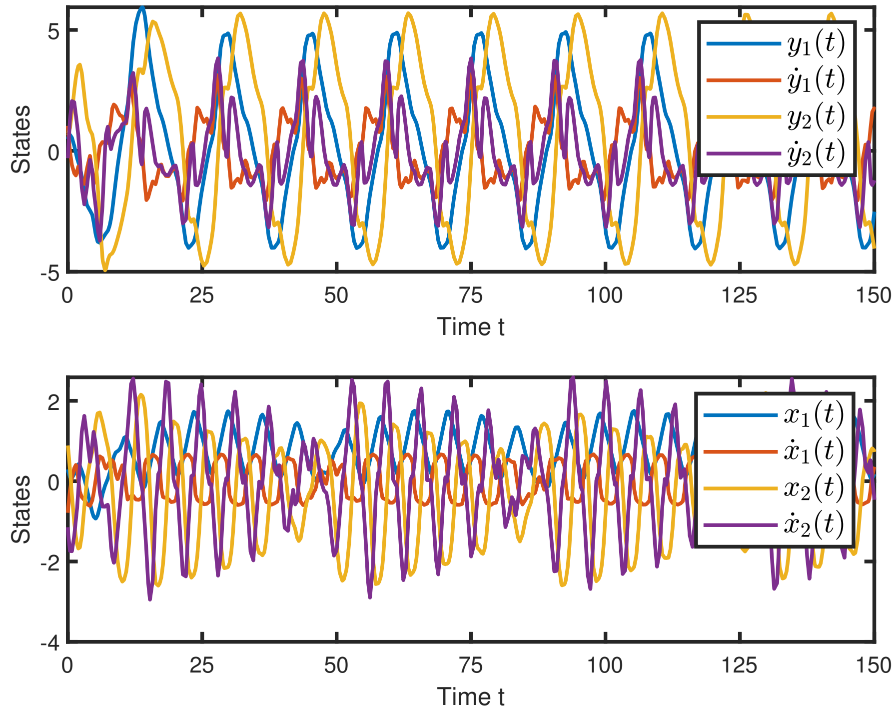

Figure 6. States of systems (2.1) and (2.2) without controller (3.19) with $\varsigma_{ij}(t) = 0$.

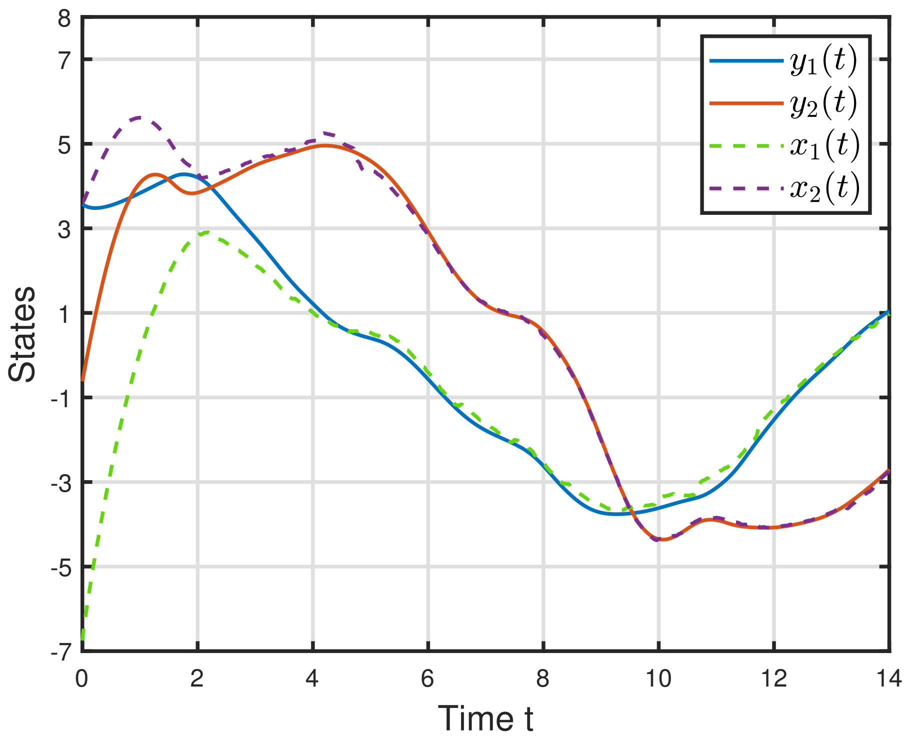

Figure 7. States of systems (2.1) and (2.2) under controller (3.19) with $\varsigma_{ij}(t) = 0$.

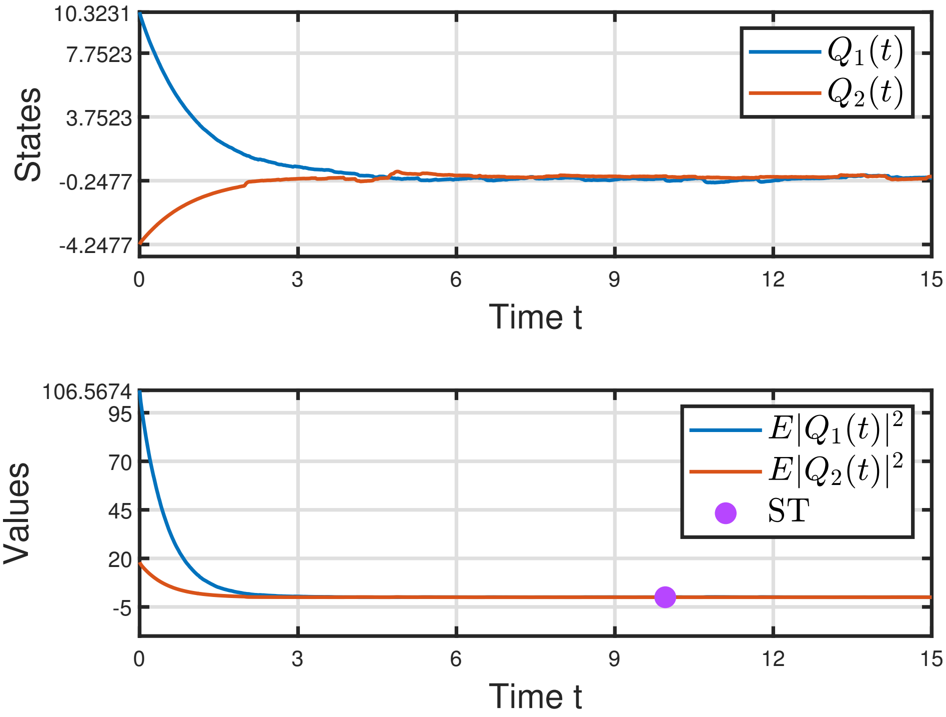

Figure 8. Values of error system (2.5) under $\varsigma_{ij}(t) = 0$.

Remark 4.1. *The synchronization of INNs is addressed in references [14, 23, 25]. Notably, the driver and response systems estabilshed in these studies maintain identical parameter configurations, and it can be seen as a specific case within the framework of the present research. Furthermore, this paper extends the discussion by incorporating a stochastic disturbance term within the response system. This means that there are not only parametric differences between the drive system and the response system but also structural differences, thereby complicating the analysis and enhancing its alignment with real-world modeling processes.*

Remark 4.2. *Table 1 shows the comparison between this paper and the existing literatures. The factors for comparison are as follows: mismatched parameters (MP), different structures of drive and response systems (DSDR), fuzzy terms (FT), FTS, and inertial terms (IT). Although the models considered in [8, 9, 14] are all INN models, it can be seen from Table 1 that the models in the literature can be considered as a special case of the model considered in this paper.*

Table 1. A comparison between the present study and existing literature.

	MP	DSDR	FT	FTS	IT
Zhou et al. (2023) [9]	-	-	-	-	✔
Han et al. (2023) [8]	-	-	-	✔	✔
Liu et al. (2023) [14]	-	-	✔	✔	✔
This paper	✔	✔	✔	✔	✔

Remark 4.3. *The block diagram illustrating the proposed control approach is presented in Figure 9. This diagram provides a visual representation of the various components and interactions that constitute the control strategy. It highlights the relationships between the input, processing, and output stages of the control system. This block diagram serves as a valuable tool for understanding the operational framework and effectiveness of the proposed approach.*

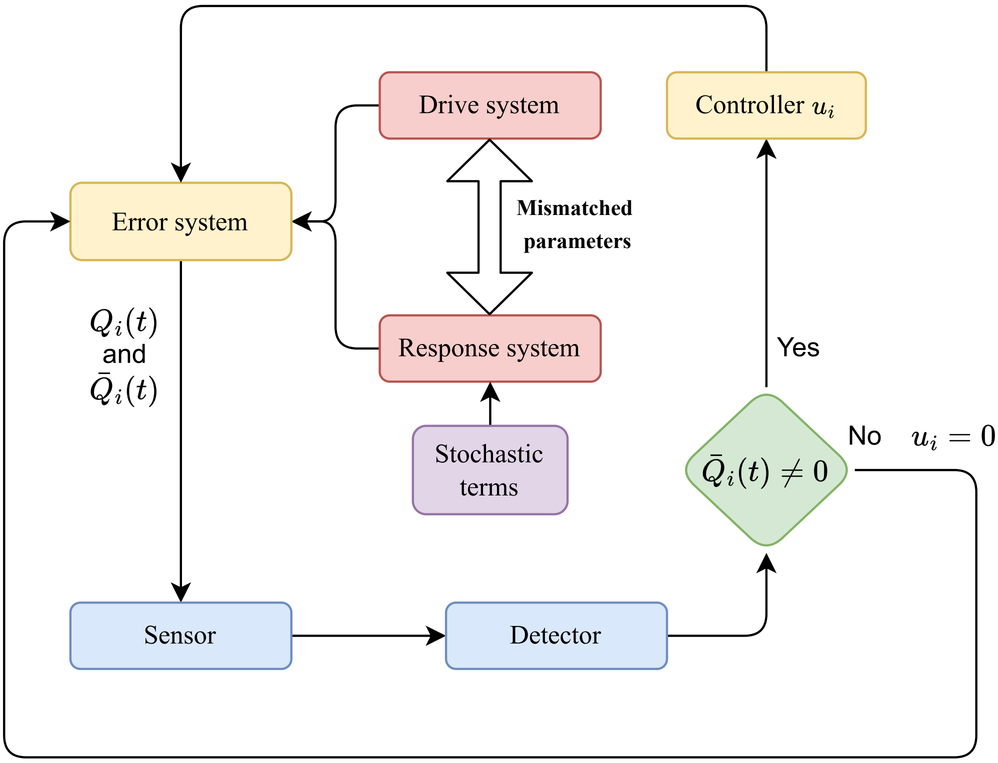

Figure 9. The block diagram of the proposed control approach.

Remark 4.4. *In the process of estimating the settling time, it can be found that the obtained results are related to the dimension of the system. When the size of the system is larger, the error of the deduced settlement time is larger. Therefore, how to derive a more exact settling time in the case of high dimensions is one of our future research priorities.*

5. Conclusions

This paper presents a comprehensive examination of the synchronization of fuzzy inertial neural networks that incorporate neutral terms alongside the complexities introduced by parameter mismatches. In this research, we employ a variety of sophisticated methodologies, including a specially designed controller, stochastic analysis, inequality techniques, and order reduction methods, to systematically investigate the conditions for achieving fixed-time synchronization between a drive system and a response system characterized by differing structures and mismatched parameters.

Additionally, two numerical examples are presented to validate the findings of this study. These examples are carefully chosen to illustrate the effectiveness of the proposed synchronization approach under varying conditions.

Author contributions

Jingyang Ran: Methodology, writing – original draft; Tiecheng Zhang: Project administration, writing – review and editing. All authors have read and approved the final version of the manuscript for publication.

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Conflict of interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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