Mathematics

https://www.aimspress.com/journal/Math

AIMS Mathematics, 9(11): 31636–31657.
DOI: 10.3934/math.20241521
Received: 19 September 2024
Revised: 16 October 2024
Accepted: 25 October 2024
Published: 07 November 2024

Research article

Applications of soliton solutions of the two-dimensional nonlinear complex coupled Maccari equations

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Abstract: This study investigates the two-dimensional nonlinear complex coupled Maccari equations, which are significant in describing solitary waves concentrated in small spatial regions. These equations have applications across various fields, including hydrodynamics, nonlinear optics, and the study of sonic Langmuir solitons. Using the Bäcklund transformation, we explore a broad range of soliton solutions for this system, focusing on their spectral properties. The proposed method stands out for its simplicity and comprehensive results compared to traditional approaches. The obtained solutions are expressed in rigorous, trigonometric, and hyperbolic forms, providing deeper insights into the dynamics of the system. To enhance understanding, we present contour and three-dimensional graphical representations of the solutions. This study has potential applications in energy and industry by advancing the understanding of nonlinear wave phenomena, which are crucial in optimizing energy transfer processes and designing efficient systems in hydrodynamic and optical engineering. Additionally, the soliton solutions obtained here contribute to technologies in power transmission and high-speed optical communications, offering a foundation for innovations in sustainable energy systems and industrial applications.

Keywords: two-dimensional nonlinear complex coupled Maccari equations; soliton solutions; Bäcklund transformation; nonlinear wave phenomena; trigonometric and hyperbolic functions; solitary waves
Mathematics Subject Classification: 34G20, 35A20, 35A22, 35R11

1. Introduction

Nonlinear wave phenomena are extremely important in many scientific fields, including mathematics, biology, and particularly different aspects of physics [1–4]. These phenomena are highly evident in the fields of nonlinear optics, condensed matter physics, plasma physics, chemical physics, solid-state physics, and fluid dynamics [5–7]. The implications of these phenomena are fascinating within the field of natural sciences. Examples of mathematical expressions include the Schrödinger equation, the Phi-4 equation, the $2D$ Ginzburg–Landau equation, the extended Zakharov system, equations describing blood flow in arteries, diffusive predator–prey systems, and shallow water equations, among others [8–10].

Due to the crucial significance of these equations, a multitude of scholarly articles have demonstrated a strong inclination towards finding solutions for them, employing both numerical and analytical methods [11–13]. Solving nonlinear partial differential equations analytically is an extremely difficult task. Numerous mathematical strategies have been devised to overcome this challenge, highlighting the significance of analytical solutions in academic research. Analytical solutions offer an accurate description of the system under study and aid in a greater understanding of basic physical processes. As a result, research into analytical solutions for nonlinear partial differential equations is very important and continues today. To achieve this goal, scientists have developed a variety of exact analytical methods since these answers are frequently crucial for examining different physical processes. As a result, researchers have worked hard to develop a variety of techniques for solving nonlinear partial differential equations. These methods are essential for enhancing our understanding of intricate nonlinear wave events in various academic fields. These techniques encompass the tanh–coth method [15], exp–function method [16], F–expansion method [17], variational iteration method [18], extended rational sin–cosine and sinh-cosh method [19], simplest equation method [20], trial equation method [21], sine–Gordon method [22], and Kudryashov method [23], among other approaches. These methodologies empower academics to address intricate issues and augment our comprehension of nonlinear wave phenomena in various fields of science.

The Maccari system, which was initially designed for hydraulic systems, has become a vast tool for the analysis of complex non-linear systems in different fields of knowledge. The extensive usage of this model in its various forms in mechanical, civil, and aerospace engineering and in physics results from its flexibility to describe complex structures and nonlinear responses as found in fluid dynamics and heat conveyance. For instance, when analyzing flows of viscoelastic fluids with nonlinear dependencies on the shear rate, like shear-thinning or shear-thickening, the Maccari system is most beneficial. These fluids do not follow the behavior of Newtonian fluids, which have a direct proportionality between the applied shear stress and the resulting shear rate, and thus need more complex models to describe their behavior to an applied stress or shear rate. The non-linear coupled Maccari system is defined by a set of equations [24]:

$$\begin{cases} iF_t + F_{xx} + HF = 0 \\ iG_t + G_{xx} + HG = 0 \\ H_t + H_y + |F + G|_x^2 = 0. \end{cases} \tag{1}$$

Where $F = F(x, y, t), G = G(x, y, t)$ and $H = H(x, y, t)$. In many scientific fields, the Maccari system is an extremely useful tool for precisely depicting nonlinear systems, especially when describing the behaviour of confined waves inside restricted spatial bounds.

In this work, we introduce a new idea, which is the combined application of the Riccati-Bernoulli sub-ODE method and the Bäcklund transformation to obtain analytical solutions to the travelling wave solutions of the coupled nonlinear Maccari system. The Bäcklund transformation, which has been established for its capacity to generate transformation relating to solutions of differential equations, helps to make improvement to the effectiveness of the Riccati–Bernoulli method to solve different differential equations. This combination allows to receive a wide spectrum of trigonometric, hyperbolic, and rational solutions, thereby greatly expanding understanding of the system based on analytical considerations. In addition, our work distinguishes itself by depicting periodic and lump-type kinks structures in the solution plots that are unique characteristics of the behavior of the model under the used methodology. Despite the fact that previous researchers have worked on applying such methods like the sine–Gordon expansion approach [25], the generalized Kudryashov method, and extended trial equation methodology [26], we are establishing a new method by integrating the Bäcklund transformation with the Riccati–Bernoulli sub-ODE. This novel approach not only provides a wider set of solutions but also provides a richer understanding of the behavior of the Maccari system compared to prior studies.

2. Methodology

Take into consideration the following nonlinear partial differential equation (NLPDE) scenario:

$$F\left(f, f'(t), f'(\zeta), f''(t), f''(\zeta), f'''(t), \ldots\right) = 0. \tag{2}$$

The reliance of the polynomial F on the variable function $f(\zeta, t)$ and its partial derivatives defines the function. Its structure includes the highest-order derivatives as well as nonlinear terms. After that, we shall methodically explain the main steps in this approach.

We propose the following complicated wave transformations to investigate potential solutions for Eq (1):

$$\begin{aligned} F &= F(x, y, t) = e^{i\vartheta} F(x, y, t), \\ G &= G(x, y, t) = e^{i\vartheta} G(x, y, t), \\ H &= H(x, y, t). \end{aligned} \tag{3}$$

$$\vartheta = ax + by + Qt. \tag{4}$$

For each real constant a, b, and Q in the equation, the phase component is represented by ϑ. The ordinary differential equation (ODE) that follows can be derived from Eq (1) by means of a particular transformation.

$$P\left(f, f'(\eta), f''(\eta), f'''(\eta), \ldots\right) = 0. \tag{5}$$

After looking at the formal answer for Eq (2), we look at the expression that works to satisfy the specified equation.

$$f(\eta) = \sum_{j=-n}^{n} c_j \psi(\eta)^j. \tag{6}$$

With the constraint that $c_n \neq 0$ and $c_{-n} \neq 0$, the constants c_j must be found. The resultant function is obtained from the ensuing Bäcklund transformation operation.

$$\psi(\eta) = \frac{-\chi B + A\phi(\eta)}{A + B\phi(\eta)}. \tag{7}$$

Allow χ, A, and B to be constants; $B \neq 0$, however. Moreover, take into account the function $\phi(\eta)$, which has the following definition:

$$\frac{d\phi}{d\eta} = \chi + \phi(\eta)^2. \tag{8}$$

The following kinds of answers to Eq (8) are well-known and generally accepted [27]:

(i) If $\chi < 0$, then $\phi(\eta) = -\sqrt{-\chi}\tanh(\sqrt{-\chi}\eta)$, or $\phi(\eta) = -\sqrt{-\chi}\coth(\sqrt{-\chi}\eta)$.

(ii) If $\chi > 0$, then $\phi(\eta) = \sqrt{\chi}\tan(\sqrt{\chi}\eta)$, or $\phi(\eta) = -\sqrt{\chi}\cot(\sqrt{\chi}\eta)$.

(iii) If $\chi = 0$, then $\phi(\eta) = \frac{-1}{\eta}$.

The positive integer (n) is balanced so that the coefficient of the highest order of derivative equals to the coefficient of the nonlinear terms in Eq (5). To represent the order of $f(\eta)$, we write $D[f(\eta)] = n$. In these conditions, the two distinct expression disciplines can be distinguished as follows:

$$D\left[\frac{d^p f}{d\eta^p}\right] = n + p, \quad D\left[f^q \frac{d^p f}{d\eta^p}\right]^s = nq + s(p + n). \tag{9}$$

We create an algebraic system of equations by combining Eqs (5) and (7) with Eq (6), and then grouping terms with the same powers of $u(\eta)$ and setting them to zero. This system may be solved effectively by using the Maple tool to determine the right values for c_i, a, b, Q, and v. This in turn makes it possible to precisely compute the travelling wave solutions for Eq (2), which can be obtained by computational approach.

3. Mathematical formulation

The methods described in Part 2 will be applied in this part to determine the exact travelling wave solutions for the Maccari system. We use the wave transformation given in Eq (3) to simplify Eq (1) in order to achieve this goal.

$$\begin{aligned} &i(F_t + 2aF_x) + F_{xx} - (a^2 + Q)F + FH = 0, \\ &i(G_t + 2aG_x) + G_{xx} - (a^2 + Q)G + GH = 0 \\ &H_t + H_y + (|F + N|^2)_x = 0. \end{aligned} \tag{10}$$

Consider the transformation represented as:

$$\begin{aligned} &F = f(\eta), G = g(\eta), H = h(\eta) \\ &\eta = x + y + vt. \end{aligned} \tag{11}$$

Upon substitution of this transformation into Eq (10), the resulting expression is produced.

$$
\begin{aligned}
&f'' + i(2a + v)f' - (a^2 + Q)f + fh = 0,\\
&g'' + i(2a + v)g' - (a^2 + Q)g + gh = 0,\\
&(1 + v)h' + 2(f + g)(f' + g') = 0.
\end{aligned} \tag{12}
$$

Equation (12) can be integrated with respect to η, and the following result can be obtained by setting the integration constant to zero:

$$
h = -\frac{(f + g)^2}{v + 1}. \tag{13}
$$

Equation (13) can be used as a substitute in Eq (12) to achieve

$$
\begin{aligned}
&f'' + i(2a + v)f' - (a^2 + Q)f - f\frac{(f + g)^2}{v + 1} = 0,\\
&g'' + i(2a + v)g' - (a^2 + Q)g - g\frac{(f + g)^2}{v + 1} = 0.
\end{aligned} \tag{14}
$$

In order to tackle Eq (14), we will reframe the intricate problem by constructing a more streamlined equation.

$$
g(\eta) = kf(\eta). \tag{15}
$$

Where $k \in R$, by substituting Eq (15) into Eq (14), we obtain:

$$
\begin{aligned}
&f'' + i(2a + v)f' - (a^2 + Q)f - (f)^3\frac{(k + 1)^2}{v + 1} = 0,\\
&g'' + i(2a + v)g' - (a^2 + Q)g - (g)^3\frac{(k + 1)^2}{k^2(v + 1)} = 0.
\end{aligned} \tag{16}
$$

By decomposing Eq (16) into its imaginary and real parts and subsequently equating them to zero, we obtain:

$$
\begin{aligned}
&2a + v = 0,\\
&v = -2a,
\end{aligned} \tag{17}
$$

$$
f'' - (a^2 + Q)f - (f)^3\frac{(k + 1)^2}{v + 1} = 0. \tag{18}
$$

Moreover, Eq (18) can be rearranged as follows:

$$
(k + 1)^2(f(\eta))^3 + (v + 1)(a^2 + Q)f(\eta) - (v + 1)f''(\eta) = 0. \tag{19}
$$

Equation (19) denotes the nonlinear ordinary differential equation (NODE) form of Eq (1), whereas Eq (17) functions as the constraint equation. By applying the homogeneous balance principle, we can determine that the balancing constant, denoted as $n = 1$, is obtained while analyzing the terms f^3 and f'' in Eq (19).

In the present study, Eq (6) is substituted along with Eq (7) and Eq (8) to incorporate them into Eq (19). By carefully collecting the coefficients related to $\psi^j(\eta)$ and setting them to zero, an algebraic

system of equations is formed. We effectively solve the previously described system of algebraic equations by using Maple software as a computational tool, producing the following results:
Case 1.

$$c_1 = \sqrt{8\chi - Q}, c_{-1} = 1, c_0 = c_0, v = v, k = -1, a = \sqrt{-Q}, B = \sqrt{-\chi^{-1}}A. \tag{20}$$

Case 2.

$$\begin{aligned} &c_1 = -\frac{c_{-1}}{\chi}, c_{-1} = c_{-1}, c_0 = 0, v = -1/2\,\frac{2\chi^2 - k^2{c_{-1}}^2 - {c_{-1}}^2 - 2\,k{c_{-1}}^2}{\chi^2},\\ &k = k, a = \sqrt{8\chi - Q}, B = \sqrt{\chi^{-1}}A. \end{aligned} \tag{21}$$

Case 3.

$$c_1 = c_1, c_{-1} = -\chi\, c_1, c_0 = 0, v = k{c_1}^2 + 1/2\,k^2{c_1}^2 - 1 + 1/2\,{c_1}^2, k = k, a = \sqrt{8\chi - Q}, B = B. \tag{22}$$

Case 4.

$$c_1 = c_1, c_{-1} = \chi\, c_1, c_0 = 0, v = k{c_1}^2 + 1/2\,k^2{c_1}^2 - 1 + 1/2\,{c_1}^2, k = k, a = \sqrt{-Q - 4\chi}, B = B. \tag{23}$$

Case 5.

$$c_1 = -\left(-c_{-1} + bc_{-1}\right)\frac{1}{\sqrt{-b^{-1}}}b^{-1}, c_{-1} = c_{-1}, c_0 = 0, v = -1, k = -1, a = a, B = B. \tag{24}$$

Assuming Case 1, we derive the families of solutions listed below:
Solution set 1: Equation (1) elegantly unveils a series of solitary wave solutions pertaining to the parameter $\chi < 0$.

$$\begin{aligned} f_1(x, y, t) = &\frac{\left(A - \sqrt{-\chi^{-1}}A\sqrt{-\chi}\tanh\left(\sqrt{-\chi}\,(x + y + vt)\right)\right)e^{i\left(\sqrt{-Q}x + by + Qt\right)}}{\left(-\chi\sqrt{-\chi^{-1}}A - A\sqrt{-\chi}\tanh\left(\sqrt{-\chi}\,(x + y + vt)\right)\right)}\\ &+ \frac{c_0 e^{i\left(\sqrt{-Q}x + by + Qt\right)} + \left(-\chi\sqrt{-\chi^{-1}}A - A\sqrt{-\chi}\tanh\left(\sqrt{-\chi}\,(x + y + vt)\right)\right)e^{i\left(\sqrt{-Q}x + by + Qt\right)}}{\left(A - \sqrt{-\chi^{-1}}A\sqrt{-\chi}\tanh\left(\sqrt{-\chi}\,(x + y + vt)\right)\right)}, \end{aligned} \tag{25}$$

or

$$\begin{aligned} f_2(x, y, t) = &\frac{\left(A - \sqrt{-\chi^{-1}}A\sqrt{-\chi}\coth\left(\sqrt{-\chi}\,(x + y + vt)\right)\right)e^{i\left(\sqrt{-Q}x + by + Qt\right)}}{\left(-\chi\sqrt{-\chi^{-1}}A - A\sqrt{-\chi}\coth\left(\sqrt{-\chi}\,(x + y + vt)\right)\right)}\\ &+ \frac{c_0 e^{i\left(\sqrt{-Q}x + by + Qt\right)} + \left(-\chi\sqrt{-\chi^{-1}}A - A\sqrt{-\chi}\coth\left(\sqrt{-\chi}\,(x + y + vt)\right)\right)e^{i\left(\sqrt{-Q}x + by + Qt\right)}}{\left(A - \sqrt{-\chi^{-1}}A\sqrt{-\chi}\coth\left(\sqrt{-\chi}\,(x + y + vt)\right)\right)}. \end{aligned} \tag{26}$$

Solution set 2: Equation (1) elegantly unveils a series of solitary wave solutions pertaining to the parameter $\chi > 0$.

$$f_3(x,y,t) = \frac{\left(A + \sqrt{-\chi^{-1}}A\sqrt{\chi}\tan\left(\sqrt{\chi}(x+y+vt)\right)\right)e^{i\left(\sqrt{-Q}x+by+Qt\right)}}{\left(-\chi\sqrt{-\chi^{-1}}A + A\sqrt{\chi}\tan\left(\sqrt{\chi}(x+y+vt)\right)\right)} + \frac{c_0 e^{i\left(\sqrt{-Q}x+by+Qt\right)} + \left(-\chi\sqrt{-\chi^{-1}}A + A\sqrt{\chi}\tan\left(\sqrt{\chi}(x+y+vt)\right)\right)e^{i\left(\sqrt{-Q}x+by+Qt\right)}}{\left(A + \sqrt{-\chi^{-1}}A\sqrt{\chi}\tan\left(\sqrt{\chi}(x+y+vt)\right)\right)}, \tag{27}$$

or

$$f_4(x,y,t) = \frac{\left(A - \sqrt{-\chi^{-1}}A\sqrt{\chi}\cot\left(\sqrt{\chi}(x+y+vt)\right)\right)e^{i\left(\sqrt{-Q}x+by+Qt\right)}}{\left(-\chi\sqrt{-\chi^{-1}}A - A\sqrt{\chi}\cot\left(\sqrt{\chi}(x+y+vt)\right)\right)} + \frac{c_0 e^{i\left(\sqrt{-Q}x+by+Qt\right)} + \left(-\chi\sqrt{-\chi^{-1}}A - A\sqrt{\chi}\cot\left(\sqrt{\chi}(x+y+vt)\right)\right)e^{i\left(\sqrt{-Q}x+by+Qt\right)}}{\left(A - \sqrt{-\chi^{-1}}A\sqrt{\chi}\cot\left(\sqrt{\chi}(x+y+vt)\right)\right)}. \tag{28}$$

Solution set 3: Equation (1) elegantly unveils a series of solitary wave solutions pertaining to the parameter $\chi = 0$.

$$f_5(x,y,t) = \frac{\left(A - \sqrt{-\chi^{-1}}A(x+y+vt)^{-1}\right)e^{i\left(\sqrt{-Q}x+by+Qt\right)}}{\left(-\chi\sqrt{-\chi^{-1}}A - \frac{A}{x+y+vt}\right)} + \frac{c_0 e^{i\left(\sqrt{-Q}x+by+Qt\right)} + \left(-\chi\sqrt{-\chi^{-1}}A - \frac{A}{x+y+vt}\right)e^{i\left(\sqrt{-Q}x+by+Qt\right)}}{\left(-\chi\sqrt{-\chi^{-1}}A - \frac{A}{x+y+vt}\right)}. \tag{29}$$

Where, $g(x,y,t) = kf(x,y,t)$ and $h(x,y,t) = -\frac{((1+k)f(x,y,t))^2}{v+1}$.

Assuming Case 2, we derive the families of solutions listed below:

Solution set 4: Equation (1) elegantly unveils a series of solitary wave solutions pertaining to the parameter $\chi < 0$.

$$f_6(x,y,t) = \frac{c_{-1}\left(A - \sqrt{\chi^{-1}}A\sqrt{-\chi}\tanh\left(\sqrt{-\chi}\left(x+y-1/2\,\frac{\left(2\chi^2-k^2c_{-1}{}^2-c_{-1}{}^2-2\,kc_{-1}{}^2\right)t}{\chi^2}\right)\right)\right)e^{i\left(\sqrt{8\chi-Q}x+by+Qt\right)}}{\left(-\chi\sqrt{\chi^{-1}}A - A\sqrt{-\chi}\tanh\left(\sqrt{-\chi}\left(x+y-1/2\,\frac{\left(2\chi^2-k^2c_{-1}{}^2-c_{-1}{}^2-2\,kc_{-1}{}^2\right)t}{\chi^2}\right)\right)\right)} - \frac{c_{-1}\left(-\chi\sqrt{\chi^{-1}}A - A\sqrt{-\chi}\tanh\left(\sqrt{-\chi}\left(x+y-1/2\,\frac{\left(2\chi^2-k^2c_{-1}{}^2-c_{-1}{}^2-2\,kc_{-1}{}^2\right)t}{\chi^2}\right)\right)\right)e^{i\left(\sqrt{8\chi-Q}x+by+Qt\right)}}{\chi^{-1}\left(A - \sqrt{\chi^{-1}}A\sqrt{-\chi}\tanh\left(\sqrt{-\chi}\left(x+y-1/2\,\frac{\left(2\chi^2-k^2c_{-1}{}^2-c_{-1}{}^2-2\,kc_{-1}{}^2\right)t}{\chi^2}\right)\right)\right)}, \tag{30}$$

or

$$f_7(x,y,t)=\frac{c_{-1}\left(A-\sqrt{\chi^{-1}}A\sqrt{-\chi}\coth\left(\sqrt{-\chi}\left(x+y-1/2\,\frac{(2\chi^2-k^2c_{-1}{}^2-c_{-1}{}^2-2\,kc_{-1}{}^2)t}{\chi^2}\right)\right)\right)e^{i\left(\sqrt{8\chi-Q}x+by+Qt\right)}}{\left(-\chi\sqrt{\chi^{-1}}A-A\sqrt{-\chi}\coth\left(\sqrt{-\chi}\left(x+y-1/2\,\frac{(2\chi^2-k^2c_{-1}{}^2-c_{-1}{}^2-2\,kc_{-1}{}^2)t}{\chi^2}\right)\right)\right)}$$
$$-\frac{c_{-1}\left(-\chi\sqrt{\chi^{-1}}A-A\sqrt{-\chi}\coth\left(\sqrt{-\chi}\left(x+y-1/2\,\frac{(2\chi^2-k^2c_{-1}{}^2-c_{-1}{}^2-2\,kc_{-1}{}^2)t}{\chi^2}\right)\right)\right)e^{i\left(\sqrt{8\chi-Q}x+by+Qt\right)}}{\chi^{-1}\left(A-\sqrt{\chi^{-1}}A\sqrt{-\chi}\coth\left(\sqrt{-\chi}\left(x+y-1/2\,\frac{(2\chi^2-k^2c_{-1}{}^2-c_{-1}{}^2-2\,kc_{-1}{}^2)t}{\chi^2}\right)\right)\right)}. \tag{31}$$

Solution set 5: Equation (1) elegantly unveils a series of solitary wave solutions pertaining to the parameter $\chi > 0$.

$$f_8(x,y,t)=\frac{c_{-1}\left(A+\sqrt{\chi^{-1}}A\sqrt{\chi}\tan\left(\sqrt{\chi}\left(x+y-1/2\,\frac{(2\chi^2-k^2c_{-1}{}^2-c_{-1}{}^2-2\,kc_{-1}{}^2)t}{\chi^2}\right)\right)\right)e^{i\left(\sqrt{8\chi-Q}x+by+Qt\right)}}{\left(-\chi\sqrt{\chi^{-1}}A+A\sqrt{\chi}\tan\left(\sqrt{\chi}\left(x+y-1/2\,\frac{(2\chi^2-k^2c_{-1}{}^2-c_{-1}{}^2-2\,kc_{-1}{}^2)t}{\chi^2}\right)\right)\right)}$$
$$-\frac{c_{-1}\left(-\chi\sqrt{\chi^{-1}}A+A\sqrt{\chi}\tan\left(\sqrt{\chi}\left(x+y-1/2\,\frac{(2\chi^2-k^2c_{-1}{}^2-c_{-1}{}^2-2\,kc_{-1}{}^2)t}{\chi^2}\right)\right)\right)e^{i\left(\sqrt{8\chi-Q}x+by+Qt\right)}\chi^{-1}}{\left(A+\sqrt{\chi^{-1}}A\sqrt{\chi}\tan\left(\sqrt{\chi}\left(x+y-1/2\,\frac{(2\chi^2-k^2c_{-1}{}^2-c_{-1}{}^2-2\,kc_{-1}{}^2)t}{\chi^2}\right)\right)\right)}, \tag{32}$$

or

$$f_9(x,y,t)=\frac{c_{-1}\left(A-\sqrt{\chi^{-1}}A\sqrt{\chi}\cot\left(\sqrt{\chi}\left(x+y-1/2\,\frac{(2\chi^2-k^2c_{-1}{}^2-c_{-1}{}^2-2\,kc_{-1}{}^2)t}{\chi^2}\right)\right)\right)e^{i\left(\sqrt{8\chi-Q}x+by+Qt\right)}}{\left(-\chi\sqrt{\chi^{-1}}A-A\sqrt{\chi}\cot\left(\sqrt{\chi}\left(x+y-1/2\,\frac{(2\chi^2-k^2c_{-1}{}^2-c_{-1}{}^2-2\,kc_{-1}{}^2)t}{\chi^2}\right)\right)\right)}$$
$$-\frac{c_{-1}\left(-\chi\sqrt{\chi^{-1}}A-A\sqrt{\chi}\cot\left(\sqrt{\chi}\left(x+y-1/2\,\frac{(2\chi^2-k^2c_{-1}{}^2-c_{-1}{}^2-2\,kc_{-1}{}^2)t}{\chi^2}\right)\right)\right)e^{i\left(\sqrt{8\chi-Q}x+by+Qt\right)}\chi^{-1}}{\left(A-\sqrt{\chi^{-1}}A\sqrt{\chi}\cot\left(\sqrt{\chi}\left(x+y-1/2\,\frac{(2\chi^2-k^2c_{-1}{}^2-c_{-1}{}^2-2\,kc_{-1}{}^2)t}{\chi^2}\right)\right)\right)}. \tag{33}$$

Solution set 6: Equation (1) elegantly unveils a series of solitary wave solutions pertaining to the parameter $\chi = 0$.

$$f_{10}(x,y,t)=\frac{c_{-1}\left(A-\sqrt{\chi^{-1}}A\left(x+y-1/2\,\frac{(2\chi^2-k^2c_{-1}{}^2-c_{-1}{}^2-2\,kc_{-1}{}^2)t}{\chi^2}\right)^{-1}\right)e^{i\left(\sqrt{8\chi-Q}x+by+Qt\right)}}{\left(-\chi\sqrt{\chi^{-1}}A-A\left(x+y-1/2\,\frac{(2\chi^2-k^2c_{-1}{}^2-c_{-1}{}^2-2\,kc_{-1}{}^2)t}{\chi^2}\right)^{-1}\right)}$$
$$-\frac{c_{-1}\left(-\chi\sqrt{\chi^{-1}}A-A\left(x+y-1/2\,\frac{(2\chi^2-k^2c_{-1}{}^2-c_{-1}{}^2-2\,kc_{-1}{}^2)t}{\chi^2}\right)^{-1}\right)e^{i\left(\sqrt{8\chi-Q}x+by+Qt\right)}\chi^{-1}}{\left(A-\sqrt{\chi^{-1}}A\left(x+y-1/2\,\frac{(2\chi^2-k^2c_{-1}{}^2-c_{-1}{}^2-2\,kc_{-1}{}^2)t}{\chi^2}\right)^{-1}\right)}. \tag{34}$$

Where, $g(x,y,t) = kf(x,y,t)$ and $h(x,y,t) = -\frac{((1+k)f(x,y,t))^2}{v+1}$.

Assuming Case 3, we derive the families of solutions listed below:

Solution set 7: Equation (1) elegantly unveils a series of solitary wave solutions pertaining to the parameter $\chi < 0$.

$$\begin{aligned} f_{11}(x,y,t) = &-\frac{\chi c_1 \left(A - B\sqrt{-\chi}\tanh\left(\sqrt{-\chi}\left(x+y+kc_1{}^2t\right)\right)\right)e^{i\left(\sqrt{8\chi-Q}x+by+Qt\right)}}{-\chi B - A\sqrt{-\chi}\tanh\left(\sqrt{-\chi}\left(x+y+kc_1{}^2t\right)\right)} \\ &+\frac{c_1\left(-\chi B - A\sqrt{-\chi}\tanh\left(\sqrt{-\chi}\left(x+y+kc_1{}^2t\right)\right)\right)e^{i\left(\sqrt{8\chi-Q}x+by+Qt\right)}}{A - B\sqrt{-\chi}\tanh\left(\sqrt{-\chi}\left(x+y+kc_1{}^2t\right)\right)}, \end{aligned} \tag{35}$$

or

$$\begin{aligned} f_{12}(x,y,t) = &-\frac{\chi c_1 \left(A - B\sqrt{-\chi}\coth\left(\sqrt{-\chi}\left(x+y+kc_1{}^2t\right)\right)\right)e^{i\left(\sqrt{8\chi-Q}x+by+Qt\right)}}{-\chi B - A\sqrt{-\chi}\coth\left(\sqrt{-\chi}\left(x+y+kc_1{}^2t\right)\right)} \\ &+\frac{c_1\left(-\chi B - A\sqrt{-\chi}\coth\left(\sqrt{-\chi}\left(x+y+kc_1{}^2t\right)\right)\right)e^{i\left(\sqrt{8\chi-Q}x+by+Qt\right)}}{A - B\sqrt{-\chi}\coth\left(\sqrt{-\chi}\left(x+y+kc_1{}^2t\right)\right)}. \end{aligned} \tag{36}$$

Solution set 8: Equation (1) elegantly unveils a series of solitary wave solutions pertaining to the parameter $\chi > 0$.

$$\begin{aligned} f_{13}(x,y,t) = &-\frac{\chi c_1 \left(A + B\sqrt{\chi}\tan\left(\sqrt{\chi}\left(x+y+kc_1{}^2t\right)\right)\right)e^{i\left(\sqrt{8\chi-Q}x+by+Qt\right)}}{-\chi B + A\sqrt{\chi}\tan\left(\sqrt{\chi}\left(x+y+kc_1{}^2t\right)\right)} \\ &+\frac{c_1\left(-\chi B + A\sqrt{\chi}\tan\left(\sqrt{\chi}\left(x+y+kc_1{}^2t\right)\right)\right)e^{i\left(\sqrt{8\chi-Q}x+by+Qt\right)}}{A + B\sqrt{\chi}\tan\left(\sqrt{\chi}\left(x+y+kc_1{}^2t\right)\right)}, \end{aligned} \tag{37}$$

or

$$\begin{aligned} f_{14}(x,y,t) = &-\frac{\chi c_1 \left(A - B\sqrt{\chi}\cot\left(\sqrt{\chi}\left(x+y+kc_1{}^2t\right)\right)\right)e^{i\left(\sqrt{8\chi-Q}x+by+Qt\right)}}{-\chi B - A\sqrt{\chi}\cot\left(\sqrt{\chi}\left(x+y+kc_1{}^2t\right)\right)} \\ &+\frac{c_1\left(-\chi B - A\sqrt{\chi}\cot\left(\sqrt{\chi}\left(x+y+kc_1{}^2t\right)\right)\right)e^{i\left(\sqrt{8\chi-Q}x+by+Qt\right)}}{A - B\sqrt{\chi}\cot\left(\sqrt{\chi}\left(x+y+kc_1{}^2t\right)\right)}. \end{aligned} \tag{38}$$

Solution set 9: Equation (1) elegantly unveils a series of solitary wave solutions pertaining to the parameter $\chi = 0$.

$$f_{15}(x,y,t) = \frac{-\chi c_1\left(A - \frac{B}{x+y+kc_1{}^2t}\right)e^{i\left(\sqrt{8\chi-Q}x+by+Qt\right)}}{\left(-\chi B - \frac{A}{x+y+kc_1{}^2t}\right)} + \frac{c_1\left(-\chi B - \frac{A}{x+y+kc_1{}^2t}\right)e^{i\left(\sqrt{8\chi-Q}x+by+Qt\right)}}{\left(A - \frac{B}{x+y+kc_1{}^2t}\right)}. \tag{39}$$

Where, $g(x,y,t) = kf(x,y,t)$ and $h(x,y,t) = -\frac{((1+k)f(x,y,t))^2}{v+1}$.

Assuming Case 4, we derive the families of solutions listed below:

Solution set 10: Equation (1) elegantly unveils a series of solitary wave solutions pertaining to the parameter $\chi < 0$.

$$f_{16}(x,y,t) = \frac{\chi\, c_1 \left(A - B\sqrt{-\chi}\tanh\left(\sqrt{-\chi}\left(x + y + \left(kc_1{}^2 + 1/2\,k^2c_1{}^2 - 1 + 1/2\,c_1{}^2\right)t\right)\right)\right) e^{i\left(\sqrt{-Q-4\chi}x+by+Qt\right)}}{-\chi B - A\sqrt{-\chi}\tanh\left(\sqrt{-\chi}\,(x + y + (kc_1{}^2 + 1/2\,k^2c_1{}^2 - 1 + 1/2\,c_1{}^2)\,t)\right)}$$
$$+ \frac{c_1 \left(-\chi B - A\sqrt{-\chi}\tanh\left(\sqrt{-\chi}\left(x + y + \left(kc_1{}^2 + 1/2\,k^2c_1{}^2 - 1 + 1/2\,c_1{}^2\right)t\right)\right)\right) e^{i\left(\sqrt{-Q-4\chi}x+by+Qt\right)}}{A - B\sqrt{-\chi}\tanh\left(\sqrt{-\chi}\,(x + y + (kc_1{}^2 + 1/2\,k^2c_1{}^2 - 1 + 1/2\,c_1{}^2)\,t)\right)}, \tag{40}$$

or

$$f_{17}(x,y,t) = \frac{\chi\, c_1 \left(A - B\sqrt{-\chi}\coth\left(\sqrt{-\chi}\left(x + y + \left(kc_1{}^2 + 1/2\,k^2c_1{}^2 - 1 + 1/2\,c_1{}^2\right)t\right)\right)\right) e^{i\left(\sqrt{-Q-4\chi}x+by+Qt\right)}}{-\chi B - A\sqrt{-\chi}\coth\left(\sqrt{-\chi}\,(x + y + (kc_1{}^2 + 1/2\,k^2c_1{}^2 - 1 + 1/2\,c_1{}^2)\,t)\right)}$$
$$+ \frac{c_1 \left(-\chi B - A\sqrt{-\chi}\coth\left(\sqrt{-\chi}\left(x + y + \left(kc_1{}^2 + 1/2\,k^2c_1{}^2 - 1 + 1/2\,c_1{}^2\right)t\right)\right)\right) e^{i\left(\sqrt{-Q-4\chi}x+by+Qt\right)}}{A - B\sqrt{-\chi}\coth\left(\sqrt{-\chi}\,(x + y + (kc_1{}^2 + 1/2\,k^2c_1{}^2 - 1 + 1/2\,c_1{}^2)\,t)\right)}. \tag{41}$$

Solution set 11: Equation (1) elegantly unveils a series of solitary wave solutions pertaining to the parameter $\chi > 0$.

$$f_{18}(x,y,t) = \frac{\chi\, c_1 \left(A + B\sqrt{\chi}\tan\left(\sqrt{\chi}\left(x + y + \left(kc_1{}^2 + 1/2\,k^2c_1{}^2 - 1 + 1/2\,c_1{}^2\right)t\right)\right)\right) e^{i\left(\sqrt{-Q-4\chi}x+by+Qt\right)}}{-\chi B + A\sqrt{\chi}\tan\left(\sqrt{\chi}\,(x + y + (kc_1{}^2 + 1/2\,k^2c_1{}^2 - 1 + 1/2\,c_1{}^2)\,t)\right)}$$
$$+ \frac{c_1 \left(-\chi B + A\sqrt{\chi}\tan\left(\sqrt{\chi}\left(x + y + \left(kc_1{}^2 + 1/2\,k^2c_1{}^2 - 1 + 1/2\,c_1{}^2\right)t\right)\right)\right) e^{i\left(\sqrt{-Q-4\chi}x+by+Qt\right)}}{A + B\sqrt{\chi}\tan\left(\sqrt{\chi}\,(x + y + (kc_1{}^2 + 1/2\,k^2c_1{}^2 - 1 + 1/2\,c_1{}^2)\,t)\right)}, \tag{42}$$

or

$$f_{19}(x,y,t) = \frac{\chi\, c_1 \left(A - B\sqrt{\chi}\cot\left(\sqrt{\chi}\left(x + y + \left(kc_1{}^2 + 1/2\,k^2c_1{}^2 - 1 + 1/2\,c_1{}^2\right)t\right)\right)\right) e^{i\left(\sqrt{-Q-4\chi}x+by+Qt\right)}}{-\chi B - A\sqrt{\chi}\cot\left(\sqrt{\chi}\,(x + y + (kc_1{}^2 + 1/2\,k^2c_1{}^2 - 1 + 1/2\,c_1{}^2)\,t)\right)}$$
$$+ \frac{c_1 \left(-\chi B - A\sqrt{\chi}\cot\left(\sqrt{\chi}\left(x + y + \left(kc_1{}^2 + 1/2\,k^2c_1{}^2 - 1 + 1/2\,c_1{}^2\right)t\right)\right)\right) e^{i\left(\sqrt{-Q-4\chi}x+by+Qt\right)}}{A + B\sqrt{\chi}\cot\left(\sqrt{\chi}\,(x + y + (kc_1{}^2 + 1/2\,k^2c_1{}^2 - 1 + 1/2\,c_1{}^2)\,t)\right)}. \tag{43}$$

Solution set 12: Equation (1) elegantly unveils a series of solitary wave solutions pertaining to the parameter $\chi = 0$.

$$f_{20}(x,y,t)=\frac{\chi\,c_1\left(A-\frac{B}{x+y+\left(kc_1{}^2+1/2\,k^2c_1{}^2-1+1/2\,c_1{}^2\right)t}\right)e^{i\left(\sqrt{-Q-4\,\chi}x+by+Qt\right)}}{\left(-\chi\,B-\frac{A}{x+y+\left(kc_1{}^2+1/2\,k^2c_1{}^2-1+1/2\,c_1{}^2\right)t}\right)}+ \\ \frac{c_1\left(-\chi\,B-\frac{A}{x+y+\left(kc_1{}^2+1/2\,k^2c_1{}^2-1+1/2\,c_1{}^2\right)t}\right)e^{i\left(\sqrt{-Q-4\,\chi}x+by+Qt\right)}}{\left(A-\frac{B}{x+y+\left(kc_1{}^2+1/2\,k^2c_1{}^2-1+1/2\,c_1{}^2\right)t}\right)}. \tag{44}$$

Where, $g(x,y,t)=kf(x,y,t)$ and $h(x,y,t)=-\frac{((1+k)f(x,y,t))^2}{v+1}$.

Assuming Case 5, we derive the families of solutions listed below:

Solution set 13: Equation (1) elegantly unveils a series of solitary wave solutions pertaining to the parameter $\chi<0$.

$$f_{21}(x,y,t)=\frac{c_{-1}\left(A-B\sqrt{-\chi}\tanh\left(\sqrt{-\chi}\,(x+y-t)\right)\right)e^{i(ax+by+Qt)}}{-\chi\,B-A\sqrt{-\chi}\tanh\left(\sqrt{-\chi}\,(x+y-t)\right)} \\ +e^{i(ax+by+Qt)}+\frac{\left(-\chi\,B-A\sqrt{-\chi}\tanh\left(\sqrt{-\chi}\,(x+y-t)\right)\right)e^{i(ax+by+Qt)}}{A-B\sqrt{-\chi}\tanh\left(\sqrt{-\chi}\,(x+y-t)\right)} \tag{45}$$

or

$$f_{22}(x,y,t)=\frac{c_{-1}\left(A-B\sqrt{-\chi}\coth\left(\sqrt{-\chi}\,(x+y-t)\right)\right)e^{i(ax+by+Qt)}}{-\chi\,B-A\sqrt{-\chi}\coth\left(\sqrt{-\chi}\,(x+y-t)\right)} \\ +e^{i(ax+by+Qt)}+\frac{\left(-\chi\,B-A\sqrt{-\chi}\coth\left(\sqrt{-\chi}\,(x+y-t)\right)\right)e^{i(ax+by+Qt)}}{A-B\sqrt{-\chi}\coth\left(\sqrt{-\chi}\,(x+y-t)\right)}. \tag{46}$$

Solution set 14: Equation (1) elegantly unveils a series of solitary wave solutions pertaining to the parameter $\chi>0$.

$$f_{23}(x,y,t)=\frac{c_{-1}\left(A+B\sqrt{\chi}\tan\left(\sqrt{\chi}\,(x+y-t)\right)\right)e^{i(ax+by+Qt)}}{-\chi\,B+A\sqrt{\chi}\tan\left(\sqrt{\chi}\,(x+y-t)\right)} \\ +e^{i(ax+by+Qt)}+\frac{\left(-\chi\,B+A\sqrt{\chi}\tan\left(\sqrt{\chi}\,(x+y-t)\right)\right)e^{i(ax+by+Qt)}}{A+B\sqrt{\chi}\tan\left(\sqrt{\chi}\,(x+y-t)\right)}, \tag{47}$$

or

$$f_{24}(x,y,t)=\frac{c_{-1}\left(A-B\sqrt{\chi}\cot\left(\sqrt{\chi}\,(x+y-t)\right)\right)e^{i(ax+by+Qt)}}{-\chi\,B-A\sqrt{\chi}\cot\left(\sqrt{\chi}\,(x+y-t)\right)} \\ +e^{i(ax+by+Qt)}+\frac{\left(-\chi\,B-A\sqrt{\chi}\cot\left(\sqrt{\chi}\,(x+y-t)\right)\right)e^{i(ax+by+Qt)}}{A-B\sqrt{\chi}\cot\left(\sqrt{\chi}\,(x+y-t)\right)}. \tag{48}$$

Solution set 15: Equation (1) elegantly unveils a series of solitary wave solutions pertaining to the parameter $\chi = 0$.

$$f_{25}(x,y,t) = \frac{c_{-1}\left(A - \frac{B}{x+y-t}\right)e^{i(ax+by+Qt)}}{\left(-\chi B - \frac{A}{x+y-t}\right)} + \frac{\left(-\chi B - \frac{A}{x+y-t}\right)e^{i(ax+by+Qt)}}{\left(A - \frac{B}{x+y-t}\right)}. \tag{49}$$

Where, $g(x,y,t) = kf(x,y,t)$ and $h(x,y,t) = -\frac{((1+k)f(x,y,t))^2}{v+1}$.

4. Results and discussion

This work utilizes the Bäcklund transformation approach to expand the scientific community comprehension of the Coupled Maccari system. By doing this, we reveal a wide range of solution categories that were previously uninvestigated. The solutions obtained by our suggested approach utilizing the Bäcklund transformation demonstrate significant improvements compared to conventional analytical solutions. These traditional solutions frequently fail to adequately depict the complete range of physical occurrences. Furthermore, it is important to mention that the Bäcklund transformation approach utilized in this paper has demonstrated its effectiveness as a powerful instrument, having been successfully employed in several physical phenomena. The generated plots helps to visualize the changes in the solution profiles that can be useful to understand behavior of the system. These solutions are particularly useful for studying shock waves in fluid mechanics, modelling solitons in optical fibres, and disentangling solitons in plasma physics. We use Mathematica software to specify specific parameter values, $k = -1$, $v = 0.01$, $Q = 1.5$, $t = 1.5$, $c_{-1} = 1, c_1 = 1$, $A = 1.2$ and $B = 0.5$, within the exact solutions, explaining the unique characteristics of periodic and solitary solutions. The figures visually describe the properties of the solutions, offering a clear and physical depiction of the results. Table 1, comparison of the proposed method with the alternative approach, specifically Sine–Gordon expansion method [25].

The pattern of the coupled Maccari system shown in Figure 1 is an asymmetric oscillation with high amplitude on one side and low amplitudes on the other. This situation of non-symmetry results from the system's non-linearity and coupling. The signal propagates in an uneven manner. Large-amplitude oscillations are useful in practical applications in the modeling of non-homogenous media in fluid dynamics with visco-elastic and non-Newtonian fluids. It also applies in engineering structures under such forces and in physics for the distribution of energy in the optical fibres or plasma systems. This pattern is especially useful in providing understanding of the development of waves in more realistic scenarios.

Periodic oscillatory pattern is clearly seen in Figure 2, which is represented as consistent wave form with constant amplitude and frequency. This type of structure is consistent with the periodicity inherent in solutions obtained from the coupled Maccari system, which repeats itself. In practical applications, periodic waves play an invaluable role in analytically modeling different natural and man-made phenomena ranging from the mechanical waves that occur in materials, sound waves, to electromagnetic waves. Periodicity makes the oscillator behavior more predictable and stable, very useful for such applications as signal processing and communications, as well as analysis of vibrations in engineering when there is a need to minimize irregularity and maximize regularity.

Figure 3 illustrates a lump-type kink pattern, the spatial distribution of which is described by a solitary wave with a sharp rising or falling front. This kind of solution is associated with the coupled

Maccari system and characterizes a non-sinusoidal stable wave in which the waveform does not change with propagation. Thus, lump-type kink patterns are essential in explaining processes such as shock wave transmission and soliton and localized disturbance formation in diverse media. These patterns can be used in invasion percolation, fluid dynamics, optical fibers, plasmas, and other systems where transfers of energy are localized and where wavefronts are steep.

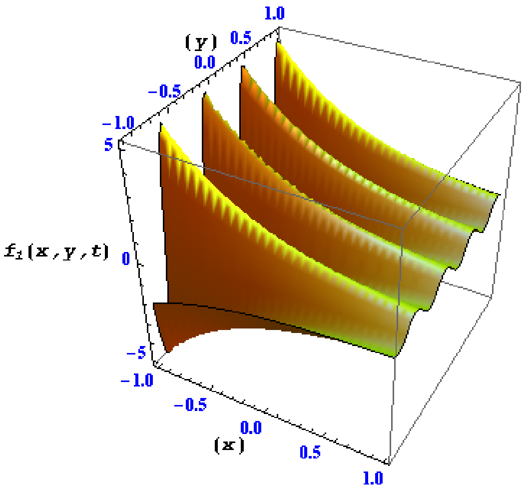
The soliton shown in Figure 4 is a fractal soliton with a self-similar spiky structure having localized wave characterization. In contrast to lump-type kinks, fractal solitons have fine structures that manifest themselves from linear and nonlinear interactions of the coupled Maccari system. Thus, the fractal characteristic of these solitons is useful for the description of wave processes that manifest both localized wave transport and self-similarity, as in nonlinear optics and plasma physics, and fluid dynamics. Fractal solitons self-similar loops could shed light on energy transfer mechanisms in diverse media since soliton persistence is essential in some wave functions.

The lump-type kink pattern is shown in Figure 5 as a solitary wave that represents a single isolated wave with a well-defined peak. This solution from the coupled Maccari system gives us information showing that it can have a steady propagating wave without damaging the waveform, which can be likened to the characteristics of solitons. Solitary waves play an important role across numerous physical applications, for instance in the transport of bounded perturbations in nonlinear structures, by which they deliver energy over substantially large distances with little distortion. Such a single wave can be viewed as illustrative of the intricacies of the dynamics of the coupled Maccari system and the importance of the system for understanding phenomena that require understanding of the features of the propagation of solitary waves.

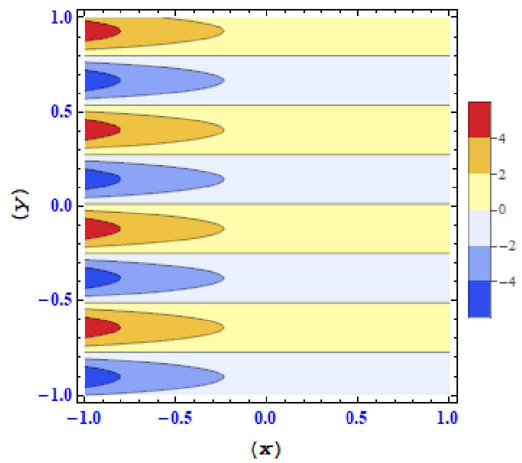
Possible future directions that may be derived from the coupled Maccari system are a study of soliton in higher dimensions so as to analyze multi dimensionality of solitons as well as elaborated discussion on wave interactions. Studying the impact of such external influence or interference in the system's behavior will give a clue as to stability and physical reactions during changes. Adding numerical simulation to analytical solutions may extend understanding of the systems, and studying the prospects of chaos might enlighten relevant nonlinearity areas. In addition, the model should be applied to various professions, and comparison analyses should be made with other nonlinear models, and sensitivity tests should be done to test parameter modifications to increase its applicability.

Table 1. Comparison of the proposed method with the alternative approach, specifically sine–Gordon expansion method [25].

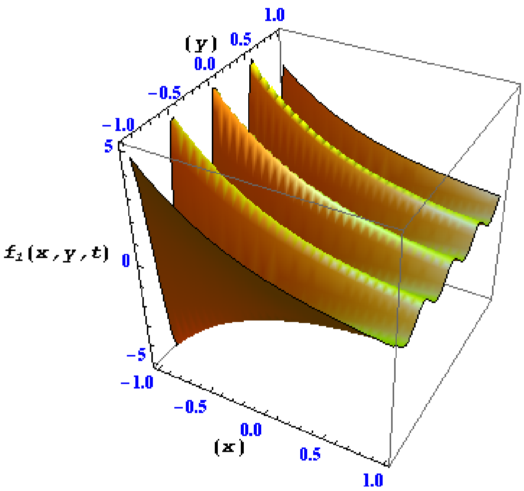
Case I: $\chi < 0$ Present method
$f(x,y,t) = -\frac{\chi c_1\left(A-B\sqrt{-\chi}\tanh\left(\sqrt{-\chi}\left(x+y+kc_1{}^2t\right)\right)\right)e^{i\left(\sqrt{8\chi-Q}x+by+Qt\right)}}{-\chi B-A\sqrt{-\chi}\tanh\left(\sqrt{-\chi}\left(x+y+kc_1{}^2t\right)\right)} + \frac{c_1\left(-\chi B-A\sqrt{-\chi}\tanh\left(\sqrt{-\chi}\left(x+y+kc_1{}^2t\right)\right)\right)e^{i\left(\sqrt{8\chi-Q}x+by+Qt\right)}}{A-B\sqrt{-\chi}\tanh\left(\sqrt{-\chi}\left(x+y+kc_1{}^2t\right)\right)}.$
Case I: $\mu^2 = \lambda + k^2$ Sine-Gordon method
$Q(x,y,t) = -\frac{\sqrt{2}e^{i\left(kx+\alpha y+\left(\mu^2-k^2\right)t+l\right)}\sqrt{(2k-1)\mu^3}\,\mathrm{sech}[\mu(x+y-2kt)]}{\sqrt{(1+r_1+r_2)^2}}.$
Case II: $\chi > 0$ Present method
$f(x,y,t) = -\frac{\chi c_1\left(A-B\sqrt{\chi}\cot\left(\sqrt{\chi}\left(x+y+kc_1{}^2t\right)\right)\right)e^{i\left(\sqrt{8\chi-Q}x+by+Qt\right)}}{-\chi B-A\sqrt{\chi}\cot\left(\sqrt{\chi}\left(x+y+kc_1{}^2t\right)\right)} + \frac{c_1\left(-\chi B-A\sqrt{\chi}\cot\left(\sqrt{\chi}\left(x+y+kc_1{}^2t\right)\right)\right)e^{i\left(\sqrt{8\chi-Q}x+by+Qt\right)}}{A-B\sqrt{\chi}\cot\left(\sqrt{\chi}\left(x+y+kc_1{}^2t\right)\right)}.$
Case II: $\mu^2 = -\sqrt{k^2+\lambda}$ Sine-Gordon method
$Q(x,y,t) = -\frac{\sqrt{2}e^{i\left(kx+\alpha y+\left(\mu^2-k^2\right)t+l\right)}\sqrt{(1-2k)\left(k^2+\lambda\right)^{\frac{3}{2}}}\,\mathrm{sech}\left[\sqrt{k^2+\lambda}(x+y-2kt)\right]}{\sqrt{(1+r_1+r_2)^2}}.$
Case III: $\chi = 0$ Present method
$f(x,y,t) = \frac{c_{-1}\left(A-\frac{B}{x+y-t}\right)e^{i(ax+by+Qt)}}{\left(-\frac{A}{x+y-t}\right)} + \frac{\left(-\frac{A}{x+y-t}\right)e^{i(ax+by+Qt)}}{\left(A-\frac{B}{x+y-t}\right)}.$
Case III: $k = \frac{\sqrt{\mu^6\left(\mu^2-\lambda\right)(1+r_1+r_2)^4}}{\mu^3(1+r_1+r_2)^2}$ Sine-Gordon method
$Q(x,y,t) = \mathrm{sech}\left[\mu\left(x+y-\frac{2t\sqrt{\mu^6\left(\mu^2-\lambda\right)(1+r_1+r_2)^4}}{\mu^3(1+r_1+r_2)^2}\right)\right]e^{i\left(\alpha y+\lambda t+l+\frac{x\sqrt{\mu^6\left(\mu^2-\lambda\right)(1+r_1+r_2)^4}}{\mu^3(1+r_1+r_2)^2}\right)}\sqrt{\frac{-2\mu^3(1+r_1+r_2)^2+4\sqrt{\mu^6\left(\mu^2-\lambda\right)(1+r_1+r_2)^4}}{(1+r_1+r_2)^4}}.$



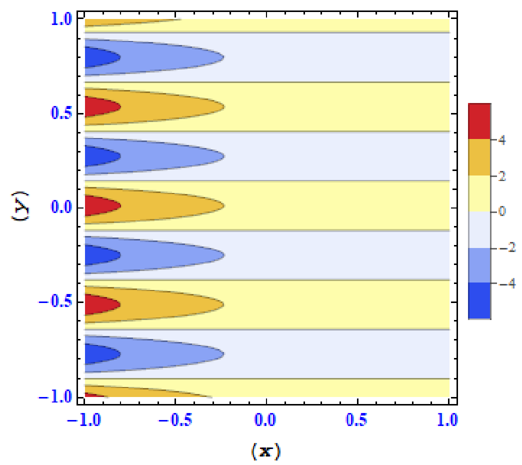
(a) A three-dimensional plot depicting the real part of $f_1(x, y, t)$ is presented.



(b) A contour plot representing the real part of $f_1(x, y, t)$ is depicted.

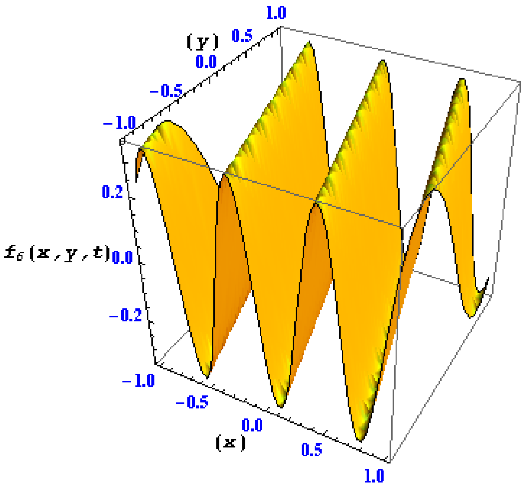


(c) A three-dimensional plot depicting the Img part of $f_1(x, y, t)$ is presented

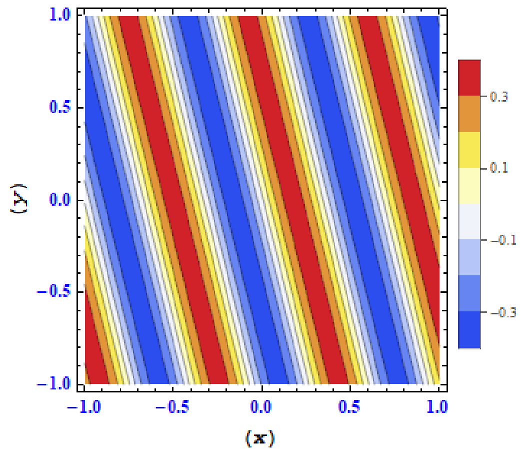


(d) A contour plot representing the Img part of $f_1(x, y, t)$ is depicted.

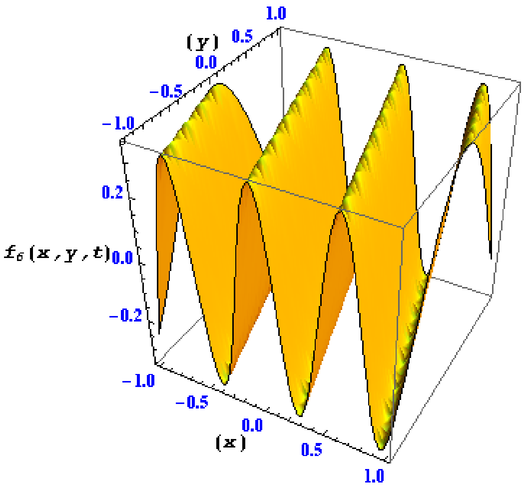
Figure 1. For the amplitude and phase components of $f_1(x, y, t)$, these plots show differing degrees of detail.



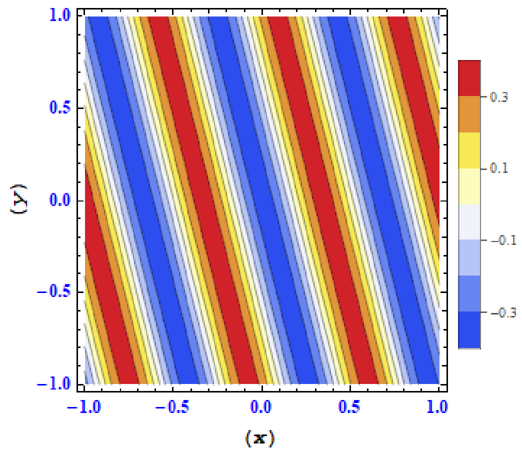
(a) A three-dimensional plot depicting the real part of $f_6(x, y, t)$ is presented.



(b) A contour plot representing the real part of $f_6(x, y, t)$ is depicted.

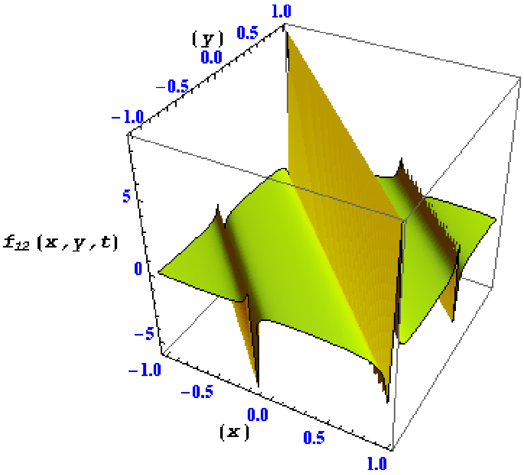


(c) A three-dimensional plot depicting the Img part of $f_6(x, y, t)$ is presented.

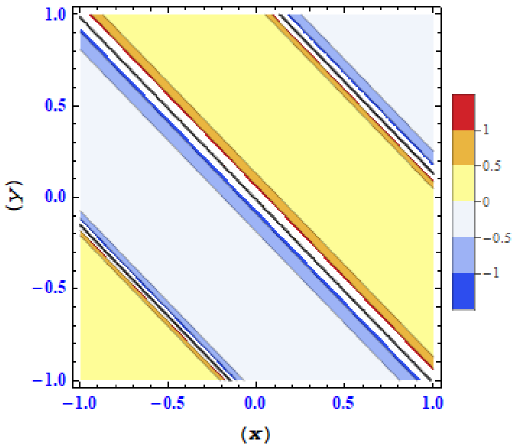


(d) A contour plot representing the Img part of $f_6(x, y, t)$ is depicted.

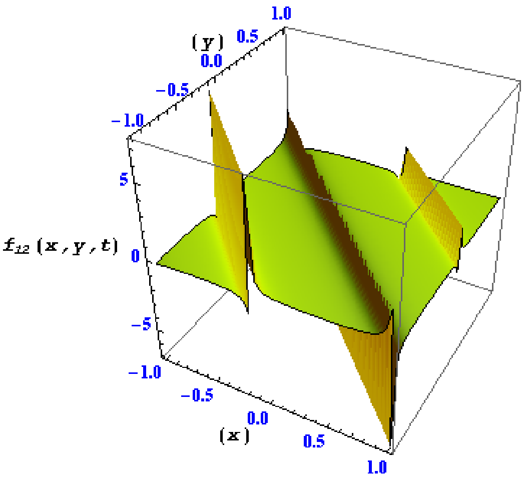
Figure 2. For the amplitude and phase components of $f_6(x, y, t)$, these plots show differing degrees of detail.



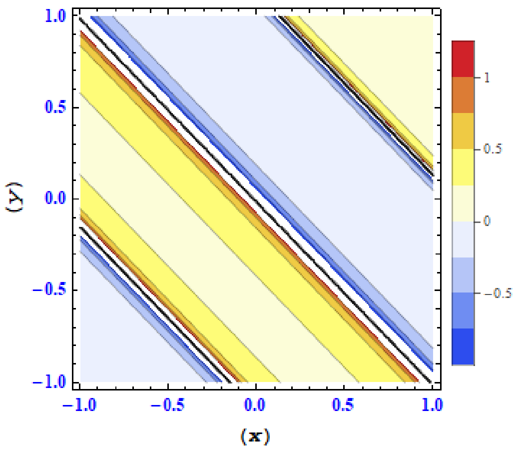
(a) A three-dimensional plot depicting the real part of $f_{12}(x, y, t)$ is presented.



(b) A contour plot representing the real part of $f_{12}(x, y, t)$ is depicted.

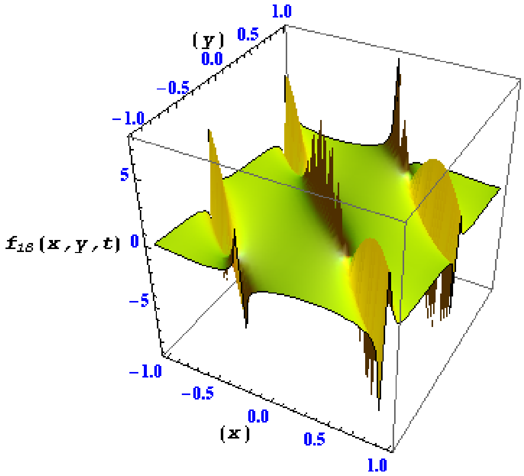


(c) A three-dimensional plot depicting the Img part of $f_{12}(x, y, t)$ is presented.

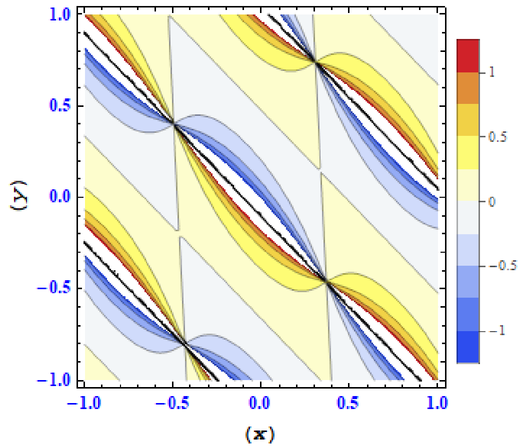


(d) A contour plot representing the Img part of $f_{12}(x, y, t)$ is depicted.

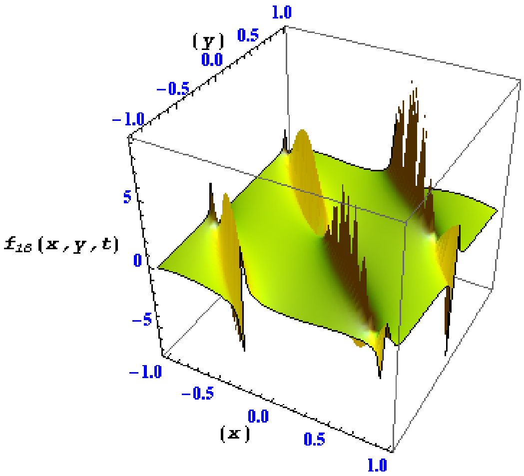
Figure 3. For the amplitude and phase components of $f_{12}(x, y, t)$, these plots show differing degrees of detail.



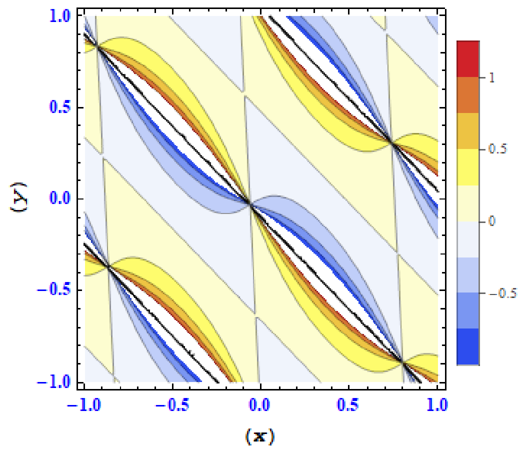
(a) A three-dimensional plot depicting the real part of $f_{18}(x, y, t)$ is presented.



(b) A contour plot representing the real part of $f_{18}(x, y, t)$ is depicted.

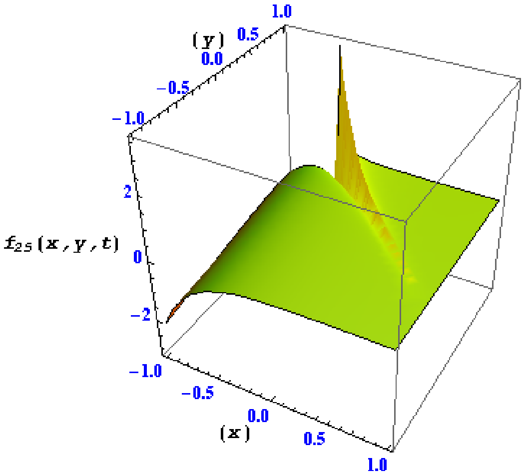


(c) A three-dimensional plot depicting the Img part of $f_{18}(x, y, t)$ is presented.

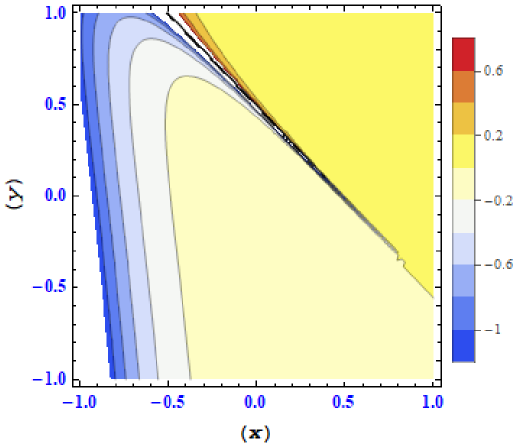


(d) A contour plot representing the Img part of $f_{18}(x, y, t)$ is depicted.

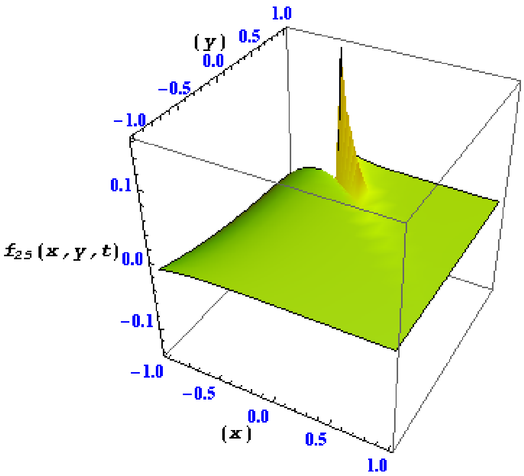
Figure 4. For the amplitude and phase components of $f_{18}(x, y, t)$, these plots show differing degrees of detail.



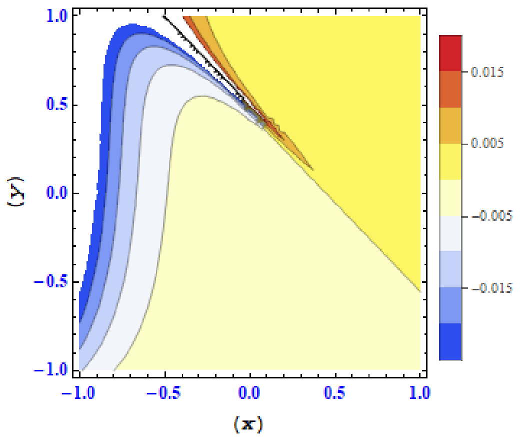
(a) A three-dimensional plot depicting the real part of $f_{25}(x, y, t)$ is presented.



(b) A contour plot representing the real part of $f_{25}(x, y, t)$ is depicted.



(c) A three-dimensional plot depicting the Img part of $f_{25}(x, y, t)$ is presented.



(d) A contour plot representing the Img part of $f_{25}(x, y, t)$ is depicted.

Figure 5. For the amplitude and phase components of $f_{25}(x, y, t)$, these plots show differing degrees of detail.

5. Conclusions

The outcomes of our work present a fresh and efficient method for resolving intricate nonlinear problems and thus make distinctive contributions. Compared to prior works, integrating the presented Riccati–Bernoulli sub-ODE method with the Bäcklund transformation makes it possible to avoid frequent linearization procedures. This helps us to find out three types of traveling wave solutions—periodic, hyperbolic, and rational—of the coupled Maccari system, which enriches the number of solutions and their depth. The Mathematica software used in this study creates a graphical representation of these solutions with special emphasis on the periodic, fractal soliton, and lump-type kink structures, which in turn advances the comprehension of the broader system behavior. These solutions provide a strong basis for the analysis and representation of numerous physical phenomena, including the propagation and dispersion of waves in non-linear media. In addition, the Bäcklund transformation increases the dimensionality of the solution set by creating new solution families and thus enabling a wider investigation of the system behavior. This expansion allows the observation to better describe the behavior of waves and extend some concepts of wave behavior like wave decay and energy distribution over time. In summary, our method offers a useful method for solving nonlinear systems, yielding more accurate and widened solutions that augment the comprehension of the mathematics and the physics of the models and processes modeled.

Author contributions

M. Alqudah: Conceptualization, formal analysis, investigation, project administration, validation, visualization, writing-review & editing; M. Singh: Data curation, resources, validation, software, visualization, resources, project administration, writing-review & editing, funding. All authors have read and agreed to the published version of the manuscript.

Acknowledgments

The authors gratefully acknowledge the funding of the Deanship of Graduate Studies and Scientific Research, Jazan University, Saudi Arabia, through Project Number: GSSRD-24.

Conflict of interest

The authors declare that they have no conflicts of interest.

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