Mathematics

AIMS Mathematics, 9(11): 29736–29772.
DOI: 10.3934/math.20241441
Received: 26 August 2024
Revised: 23 September 2024
Accepted: 09 October 2024
Published: 21 October 2024

https://www.aimspress.com/journal/Math

Review

Recent advances in mechanism/data-driven fault diagnosis of complex engineering systems with uncertainties

Chong Wang[1,2,*], Xinxing Chen[1,2], Xin Qiang[1,2], Haoran Fan[1,2] and Shaohua Li[3]

[1] Tianmushan Laboratory, Yuhang District, Hangzhou 311115, China
[2] National Key Laboratory of Strength and Structural Integrity, Institute of Solid Mechanics, School of Aeronautic Science and Engineering, Beihang University, Beijing 100191, China
[3] Beijing Institute of Spacecraft Environment Engineering, Beijing 100094, China

*** Correspondence:** Email: chongwang@buaa.edu.cn.

Abstract: The relentless advancement of modern technology has given rise to increasingly intricate and sophisticated engineering systems, which in turn demand more reliable and intelligent fault diagnosis methods. This paper presents a comprehensive review of fault diagnosis in uncertain environments, focusing on innovative strategies for intelligent fault diagnosis. To this end, conventional fault diagnosis methods are first reviewed, including advances in mechanism-driven, data-driven, and hybrid-driven diagnostic models and their strengths, limitations, and applicability across various scenarios. Subsequently, we provide a thorough exploration of multi-source uncertainty in fault diagnosis, addressing its generation, quantification, and implications for diagnostic processes. Then, intelligent strategies for all stages of fault diagnosis starting from signal acquisition are highlighted, especially in the context of complex engineering systems. Finally, we conclude with insights and perspectives on future directions in the field, emphasizing the need for the continued evolution of intelligent diagnostic systems to meet the challenges posed by modern engineering complexities.

Keywords: fault diagnosis; multi-source uncertainty; intelligent strategy; information fusion; complex engineering systems
Mathematics Subject Classification: 65C20, 68T10

1. Introduction

In the realm of complex engineering systems—such as those found in aerospace, automotive, and power generation—ensuring reliability and safety is absolutely critical. These systems are distinguished by their intricate architectures and interdependent components, where the failure of even a single part can trigger a cascade of issues, resulting in substantial downtime, significant repair costs, and, in extreme cases, catastrophic outcomes [1]. Consequently, it is essential to conduct thorough research into fault diagnosis methods tailored for these sophisticated engineering environments. Such investigations not only enhance operational efficiency but also safeguard against potential hazards, ultimately contributing to the longevity and resilience of these vital systems.

Fault diagnosis plays a pivotal role in the realm of engineering systems, serving as a vital mechanism for the timely and accurate detection and identification of faults. This process is essential for enabling appropriate maintenance measures, thereby ensuring the reliable operation of the system and minimizing the risk of unexpected failures [2]. At its core, fault diagnosis encompasses a systematic approach to identifying, analyzing, and resolving faults that may arise within a system. This multifaceted process employs a variety of techniques and methodologies aimed at determining both the cause and location of faults. It involves the meticulous collection, analysis, and interpretation of operational data, which provides insights into the system's performance and health.

Fault diagnosis can be categorized into three primary strategies: mechanism-driven, data-driven, and hybrid-driven approaches. Mechanism-driven diagnosis relies heavily on a deep understanding of the system's design and operational principles. Conversely, data-driven diagnosis operates independently of mechanistic models of the system, leveraging relevant analytical techniques to identify anomalies and patterns in the operational data. Hybrid-driven approaches combine elements from both mechanism-driven and data-driven approaches, offering a more comprehensive diagnostic framework that may have unexpected results in complex systems where traditional methods may fall short. As engineering systems continue to grow in complexity, the importance of advanced fault diagnosis techniques will only increase, underscoring the need for ongoing research and innovation in this critical area.

Real-world systems frequently encounter a multitude of uncertainties that can stem from a variety of sources, including model inaccuracies, external disturbances, and measurement noise [3]. The generation of uncertainty and strategies for addressing it are garnering increasing attention from researchers within the engineering domain [4]. Uncertainty can be categorized based on its origin and nature into two primary types: aleatoric uncertainty, which is inherent and stochastic, and epistemic uncertainty, which arises from a lack of knowledge or information [5,6]. The existence of these uncertainties can obscure the behavioral characteristics of the system, complicating the identification and analysis of faults. Therefore, it is imperative to consider and address these uncertainties to ensure reliable and effective fault diagnosis.

In addition to the challenges posed by uncertainty, traditional fault diagnosis is often hindered by issues such as incomplete and inconsistent information. With the continuous development and wide application of intelligent technologies, intelligent fault diagnosis methods have emerged. For the past few years, the development of advanced sensor technologies, powerful computational tools, and sophisticated algorithms has paved the way for intelligent fault diagnosis strategies. These strategies encompass the entire process, from signal acquisition and feature extraction to fault diagnosis and information fusion. Intelligent fault diagnosis leverages artificial intelligence and

machine learning to enhance the detection and identification of faults, providing a more holistic and adaptive approach. This integration not only improves accuracy but also enables systems to learn from past experiences, thereby continuously refining their diagnostic capabilities in dynamic environments.

This paper seeks to offer a comprehensive review of the recent advancements in the field of fault diagnosis of complex engineering systems. The structure of the paper is organized as follows: In Section 2, we delve into general strategies of fault diagnosis, examining mechanism-driven, data-driven, and hybrid approaches. Section 3 explores the origins of uncertainties, detailing techniques for their quantification, as well as methods for robust fault diagnosis. Subsequently, Section 4 presents cutting-edge techniques in intelligent fault diagnosis strategies, highlighting their application across different stages of the fault diagnosis process. Finally, Section 5 closes the paper, encapsulating main points and concluding remarks.

2. General strategies of fault diagnosis

In this section, general fault diagnosis strategies, including mechanism-driven, data-driven, and hybrid-driven fault diagnosis [7], as illustrated in Figure 1, will be reviewed with a focus on their recent advances. These strategies are classified according to the basic principles of diagnosis. Mechanism-driven fault diagnosis methods require the establishment of internal operation mechanisms or dynamic behavior models as the basis. Data-driven methods, on the other hand, can directly use the data generated during system operation for comprehensive analysis. Hybrid mechanism/data-driven strategy, which has been mainly proposed based on the idea of complementary advantages, has just emerged in recent years.

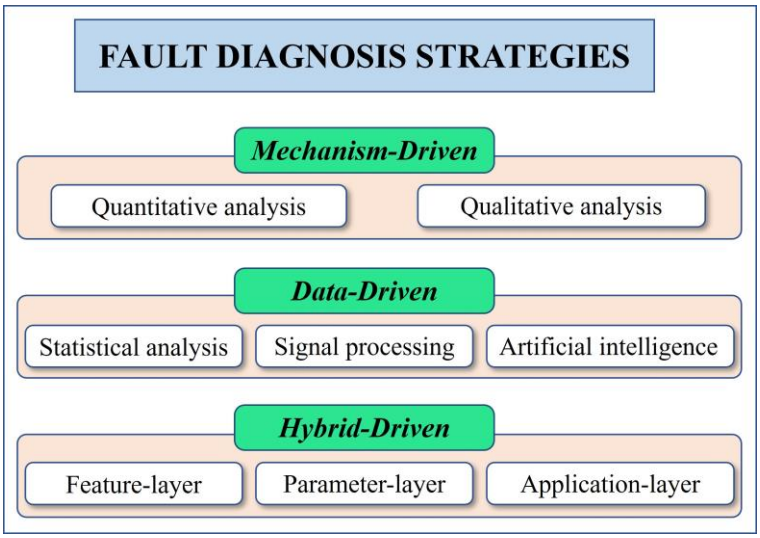

Figure 1. General strategies of fault diagnosis.

2.1. Mechanism-driven fault diagnosis

Mechanism-driven fault diagnosis is based on the internal operating mechanisms or dynamic behavior models of the system. By analyzing the system operation data and combining it with the

mechanism model, the system operation state is inferred so as to discover potential faults or hidden dangers. It can also provide scientific and effective guidance for system maintenance and management. According to the different modeling perspectives, mechanism-driven fault diagnosis can be further classified into quantitative analysis-based fault diagnosis and qualitative analysis-based fault diagnosis.

The quantitative analysis-based fault diagnosis method is formed under the idea of analytical redundancy [8]. It relies on constructing a mathematical model of the system to identify and diagnose faults by comparing the difference between the actual observed system behavior and the behavior predicted by the model. The method provides important support for fault diagnosis by accurately detecting anomalies through the model's interpretation of system behavior. Fault diagnosis methods based on quantitative analysis can be classified into three types: parameter estimation [9], state estimation [10], and parity space method [11], whose basic principles are shown in Figure 2.

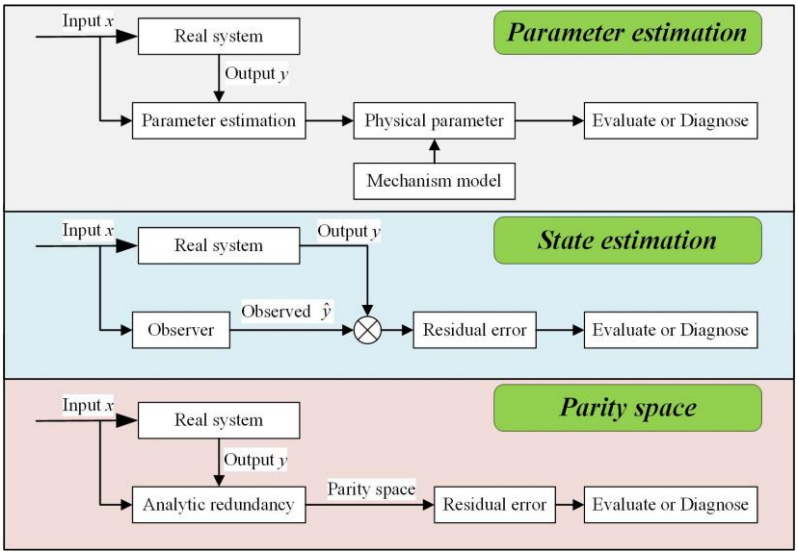

Figure 2. Fault diagnosis methods based on quantitative analysis.

Parameter estimation method: The basic idea of the parameter estimation method is to combine theoretical modeling with parameter identification, and then to judge the faults according to the deviation between the parameter estimates and the normal values [12]. Compared with the state estimation method, the parameter estimation method is more conducive to the isolation of faults. For non-measurable parameters, such as process parameters and state variables, estimation can be performed based on measurable input and output variables. This estimation can be performed by algorithms based on measured data, such as KF, EKF [13], particle filters [14], Bayesian methods [15], and other derived methods. For instance, consider a system where the relationship between the measured output y and the estimated state $\hat{x}$ is given by:

$$\hat{x}(k+1) = A\hat{x}(k) + Bu(k) + K\left(y(k) - C\hat{x}(k)\right), \tag{1}$$

where A, B, and C are system matrices, K is the Kalman gain, $u(k)$ is the control input, and $y(k)$ is the measured output. The deviation between the actual and estimated parameters is then used for fault detection.

Since the parameter estimation method requires finding out the one-to-one correspondence between the model parameters and the physical parameters, and the controlled process needs to be sufficiently stimulated, the parameter estimation method is usually used in combination with other methods to obtain better fault detection and separation performance.

State estimation method: The basic idea of the state estimation method is to detect and isolate system faults through the residuals between the estimated and measured values of a measurable variable, which is reconstructed from the analytical model of the system and the measured signal. Since the method often utilizes observers (filters) to generate the residuals, it is also referred to as the observer or filter method. Various fault detection and diagnosis observers have been proposed such as the Luenberger in a deterministic setting [16], the Kalman Filter in a stochastic setting [17], and the H_∞ based observer, which enables the involvement of frequency specifications as additional criteria for better fault discrimination [18]. The residual $r(t)$ is typically computed as:

$$r(t) = y(t) - \hat{y}(t), \tag{2}$$

where $y(t)$ is the actual measured output, and $\hat{y}(t)$ is the estimated output generated by the observer. This residual is then analyzed to detect and isolate faults.

Among the quantitative analysis-based methods, the observer-based method has been a hot research topic because of its flexible design method, relatively easy to obtain strong robustness, simple algorithms, and fast response speed.

Parity space method: The basic idea of the parity space method is to test the equivalence (i.e., consistency) of the system's mathematical model through the actual measured values of the system's inputs and outputs in order to detect and isolate faults. The main methods include constrained optimization-based equivalent equation method [19], generalized residual generator scheme [20], and decoupled parity space method based on approximate perturbation [21]. Parity space method is structurally equivalent to state estimation method.

Qualitative analysis-based fault diagnosis aims to reveal the deep-seated causes and mechanisms of fault generation through in-depth description and analysis of fault types, causes, and effects. It usually does not rely on systematic and precise mathematical models or analytical models, and thus is more flexible in practical application and can be adapted to a variety of complex fault situations. Common ones are expert knowledge system and fault tree analysis [22,23].

Expert knowledge system: Expert knowledge-based fault diagnosis is a method of fault analysis based on human experience in studying things. It mainly relies on the existing knowledge base or experience base, which is derived from the principles of system physics or summarized from numerous cases. This method does not need to establish a precise mathematical model of the system, and the empirical knowledge it requires basically covers the internal operating laws of the system.

Fault tree analysis: The fault diagnosis method based on the fault tree is a systematic technique used to analyze and diagnose faults in complex systems. By constructing the fault tree of a system, various fault events and their possible causes are systematically decomposed, helping engineers to understand the logical relationships behind fault occurrences. Through step-by-step analysis of the fault tree, specific reasons leading to system failures can be identified, enabling targeted solutions to be implemented. This method provides a structured and systematic approach to fault diagnosis, effectively improving the reliability and safety of systems while minimizing the impact of faults on system operations.

Mechanism-driven fault diagnosis methods are widely used in many fields, especially in

aerospace, automotive, and industrial automation [24–26]. Although mechanism-driven fault diagnosis methods have the advantages of high accuracy and interpretability, there are some limitations. For example, building accurate physical models may require a large amount of system test data and expertise. Especially for complex systems, it is basically impossible. Therefore, it is necessary to weigh their advantages and disadvantages in practical applications and select the appropriate fault diagnosis method according to the specific situation.

2.2. Data-driven fault diagnosis

Data-driven fault diagnosis techniques do not rely on a priori knowledge of the system, such as mathematical models and expert experience [27]. The technique is based on monitoring data collected from different sources and types and uses various data mining techniques to obtain the useful information implied therein. Thus, it can characterize the normal mode and fault mode of system operation and then achieve the purpose of detection and diagnosis, which has become a more practical diagnostic technology at present. It mainly includes statistical analysis [28], signal processing [29], and artificial intelligence methods [30].

Statistical analysis method: The method based on statistical analysis mainly analyzes the historical process data and calculates the corresponding monitoring statistics for each sample. The operational status of the current sample is analyzed based on the confidence limits of the monitoring indicators estimated for the normal sample. It is mainly divided into univariate-based statistical monitoring techniques and multivariate-based statistical monitoring techniques. Although univariate-based statistical monitoring methods are easy to implement, they ignore the correlation information between variables and can only be used for monitoring when the data dimension is small. Multivariate-based statistical analysis, on the other hand, can better portray and make use of the correlation between variables, and is suitable for fault detection and diagnosis of high-dimensional systems. The most relevant techniques in recent literature include principal component analysis (PCA), independent component analysis (ICA), partial least square (PLS), canonical correlation analysis (CCA), and Gaussian mixture model (GMM) [28,31–33].

Signal processing method: When an industrial process fails, the amplitude, phase and frequency of the signal will change unpredictably. Fault diagnosis technology based on signal processing makes a comprehensive evaluation of the system operation condition by analyzing the signals and combining them with the expert knowledge base, fault database, etc. The current fault diagnosis techniques based on signal processing mainly include wavelet transform method [34], empirical pattern decomposition method [35], morphological signal processing method [36], and spectral analysis method [37].

Artificial intelligence method: For complex systems, since there is no simple correspondence between fault types and fault signs, artificial intelligence-based fault diagnosis techniques are more suitable. This technology can learn and discover hidden patterns and laws from massive data, assisting engineers to predict, detect, and locate faults, thus improving the reliability and stability of the system. It mainly includes machine learning and deep learning methods [38–41].

Data-driven fault diagnosis can effectively identify and locate faults in complex engineering systems. The approach is widely applicable to various engineering fields and is a hot topic in current research. However, the data-driven fault diagnosis approach also has some limitations. For example, a large amount of labeled data is required for model training, and it may be difficult to obtain sufficient data in some scenarios. In addition, the low interpretability of the models limits their

application in some critical areas [42].

2.3. Hybrid mechanism/data-driven fault diagnosis

The hybrid mechanism/data-driven fault diagnosis method is an integration of mechanism-driven and data-driven, which can effectively avoid the limitations of both in engineering applications. Hybrid methods enhance diagnostic accuracy by utilizing empirical data to refine theoretical models, thereby allowing for a more nuanced understanding of system behavior. This integration also improves the generalization capabilities of the diagnostic process, enabling it to adapt to new conditions and variations that may not be adequately represented in traditional models. Furthermore, hybrid methods can mitigate the reliance on high-quality data by leveraging theoretical insights to guide data analysis, which is particularly beneficial in situations where obtaining extensive datasets is challenging. The method first uses a mechanism model to describe the structure and operation principle of the system, as well as the relationship between different components. Then, by collecting data during system operation, the data-driven technique is used to monitor and analyze the system state. This leads to more accurate identification and localization of faults, while improving the system's ability to detect abnormal behavior. This hybrid-driven method integrates the strengths of both methodologies, being able to model and understand the system using physical mechanisms as well as processing complex data using data-driven techniques. Therefore, it has high applicability and operability in practical applications [43].

An et al. [44] summarized and compared common mechanism-driven and data-driven modeling and prediction methods. It was pointed out that the establishment of hybrid-driven methods based on a full understanding of the two types of methods is the future development trend of the related technology. Guo et al. [45] introduced common data-driven, mechanism-driven, and hybrid-driven methods. It was also pointed out that hybrid-driven methods can both expand the application scope of mechanism-driven methods and improve the interpretability of data-driven methods. Depending on the level of fusion, hybrid mechanism/data-driven strategies can be classified into three categories, as shown in Table 1.

Table 1. Hybrid mechanism/data-driven strategies for fault diagnosis.

Hybrid strategy	Descriptions	Reference papers
Feature-layer hybrid	On the basis of mechanism analysis, the observed data are preprocessed and eigenvalues with physical significance are extracted, and then fault diagnosis is carried out by data analysis methods.	[46,47]
Parameter-layer hybrid	A mechanism-driven approach is used to construct the structural form or constraints of the model, which in turn leads to a data-driven approach to model selection, identification, and parameter estimation.	[48–52]
Application-layer hybrid	Observations under various operating conditions are simulated by a mechanism-driven model, which is then fused with actual observations for modeling.	[53,54]

Overall, the involvement of mechanism-driven models in the three types of fusion-driven

strategies described above has gradually increased, along with the demand for accuracy and reliability of the mechanism models. However, regardless of the method used, mechanism-driven approaches serve as the foundation, while data-driven methods act as the means. In the process of practical application, the two must be organically combined for specific problems in order to achieve the best results.

3. Uncertainty treatment in fault diagnosis

Various strategies for fault diagnosis problems have been described in the previous section, but their application is limited by various factors, uncertainty being one of them. Uncertainty problems are prevalent in engineering systems and arise from a variety of sources. These uncertainties lead to serious impacts on the accuracy and reliability of fault diagnosis, leading to losses and risks in production. In this section, the generation of uncertainty in fault diagnosis and its treatment will be first described. Next, the current research status of uncertainty-based fault diagnosis methods will be presented.

3.1. Uncertainty generation

Uncertainty is the lack of certainty of knowledge and information about certain events or outcomes. It is widespread in various scientific and engineering fields, especially in the analysis and decision-making process of complex systems [55]. The impact of uncertainty in fault diagnosis is multifaceted. First, it affects the detection and identification of fault signals. In the presence of noise and random variations, the diagnostic system may produce false alarms or omissions, reducing the reliability of fault diagnosis. Second, uncertainty complicates the predictability of system behavior, increasing the complexity and computational burden of the diagnostic model. In addition, data and model incompleteness can lead to inaccurate diagnostic results, affecting the maintenance and decision-making process of the system. Depending on the source and nature, uncertainty can be categorized into aleatoric uncertainty and epistemic uncertainty [5,56]. Figure 3 represents their difference and connection with a simple signal curve.

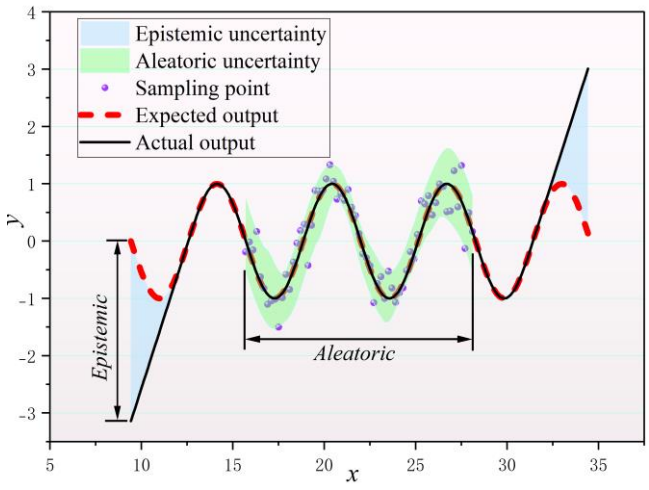

Figure 3. Schematic diagram of aleatoric and epistemic uncertainty.

Aleatoric uncertainty: Aleatoric uncertainty, also known as stochastic uncertainty, arises from random fluctuations within and outside the system [5]. This type of uncertainty is typically manifested as random variations in data and unpredictability in system behavior. For example, in industrial environments, sensor measurement data are often disturbed by environmental noise, which may originate from external factors such as temperature variations, electromagnetic interference, or mechanical vibration. This noise not only affects the accuracy of the data but may also mask fault signals, making fault detection more difficult. In addition, random changes in the operating state of the system are also an important source of aleatoric uncertainty. For example, equipment may operate differently under varying load, temperature, and pressure conditions, and these working condition variations are often random. Random operating condition variations may cause the system to exhibit different fault characteristics, making it difficult to adapt fixed-mode-based diagnostic methods, thus affecting the stability and reliability of the diagnostic results.

Epistemic uncertainty: Epistemic uncertainty stems mainly from a lack of understanding of systems and faults, including imperfect models, incomplete data, and knowledge limitations [5,57]. Uncertainty is introduced by the fact that due to the limited understanding of the mechanisms of complex systems, the mathematical and diagnostic models obtained are often simplified and approximate and do not accurately reflect the true behavior of the system. For example, certain coupling effects in the system may be ignored, resulting in a limited predictive capability of the model. In addition, the introduction of expert experience and knowledge is essential in the fault diagnosis process. However, different experts with different knowledge levels and experience backgrounds may arrive at differing conclusions on the same problem, which is also a kind of epistemic uncertainty. Epistemic uncertainty is more dependent on the assessor's level of knowledge. Hence, it is also known as subjective uncertainty.

In basic lexical terms, uncertainty is simply the lack of certainty, and there is no inherent need to categorize it into different types. It adds to the agreement that there is no fixed boundary between aleatoric uncertainty and epistemic uncertainty [58]. If new knowledge emerges that allows us to explain a situation or phenomenon in greater depth, then the corresponding aleatoric uncertainty is reduced. Ultimately, it may be that all uncertainty is epistemic uncertainty. However, the vast majority of analysts have found the use of the categorization presented above helpful and consistent with practical engineering needs [59].

3.2. Uncertainty quantification

In order to improve the performance of fault diagnostic systems in uncertainty conditions, it is important to quantify uncertainty. Quantifying uncertainty not only helps to describe the state and behavior of the system more accurately but also provides an important basis for diagnosis and decision-making. Many theories of uncertainty quantification have been proposed for this purpose, and Figure 4 illustrates a schematic of the quantization models of some of these theories [60,61].

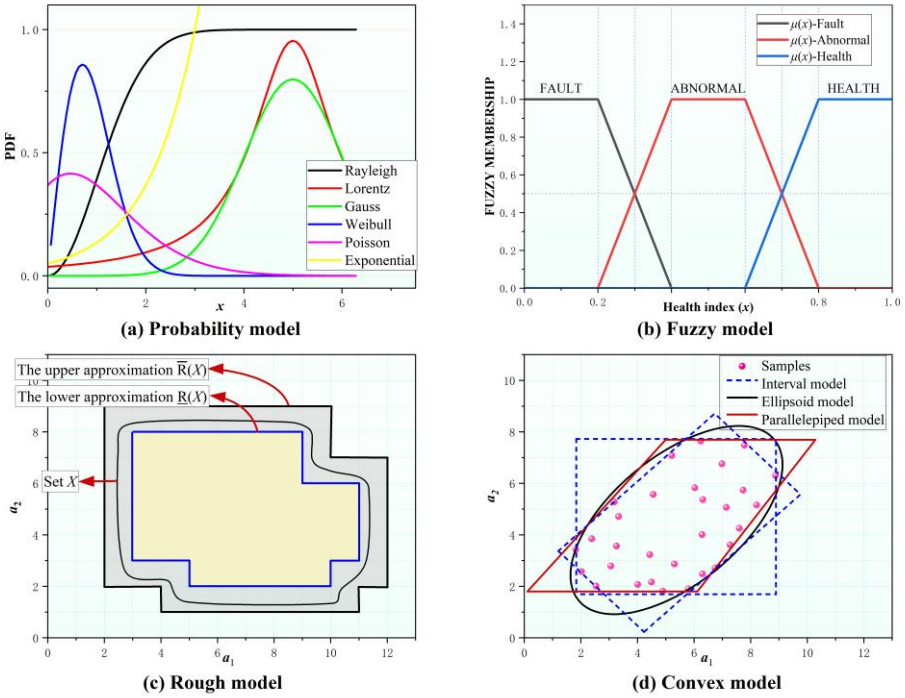

Figure 4. Common uncertainty quantification models.

Probability theory: Probability theory, the most classical and widely used tool, provides a systematic approach to quantifying and analyzing uncertainty [62,63]. Probability theory is based on a probability space, consisting of a sample space Ω, an event set $\mathcal{F}$, and a probability measure $\mathcal{P}$. The sample space $\Omega = \{x_1, x_2, \cdots\}$ is the set of all possible outcomes in an experiment. The event set $\mathcal{F}$ is a subset of the sample space Ω. Moreover, the probability measure $\mathcal{P}$ is a function that assigns a probability to each event, i.e., $P(x)$, satisfying the following conditions:

$$P(x) \in [0,1], \forall x \in \Omega \text{; and } P(\Omega) = 1. \tag{3}$$

A random variable X is a function that maps elements of the sample space to the set of real numbers and is used to describe the amount of uncertainty in a real problem [64]. Probability distributions describe the likelihood that a random variable will take on a value and commonly include discrete distributions (e.g., binomial and Poisson) and continuous distributions (e.g., normal, uniform, and exponential). Expected value and variance are important characteristics of random variables, with the former indicating the mean value of a random variable and the latter indicating its degree of dispersion. In addition, Bayes' theorem [65], the law of large numbers [66], and the central limit theorem [67] are also important theorems in probability theory, providing powerful tools for uncertainty quantification and analysis.

Dempster-Shafer theory: Dempster-Shafer evidence theory is a theory of imprecise reasoning [68,69]. Unlike traditional probability theory, the evidence theory allows for the direct representation and manipulation of uncertain information without relying exclusively on precise probability distributions [70,71]. The core concepts of evidence theory include the belief function Bel, the plausibility function Pl, and the basic probability assignment BPA. These concepts

provide a flexible approach to quantifying and dealing with uncertainty by manipulating and reasoning about sets of evidence.

The belief function Bel represents the degree of belief in a given proposition. For a proposition A, the belief function $Bel(A)$ indicates the total amount of evidence that completely supports A. The belief function's domain is a set called the frame of discernment Θ, which contains all possible propositions. The plausibility function Pl represents the plausibility of a proposition. For a proposition A, the plausibility function $Pl(A)$ indicates the total amount of evidence that does not refute A. The relationship between Bel and Pl is given by $Pl(A) = 1 - Bel(\neg A)$, where $\neg A$ denotes the complement of proposition A. The basic probability assignment (BPA, denoted as m) is a critical concept in evidence theory, assigning a probability value to each element of evidence, reflecting the degree to which the evidence supports a particular proposition. The BPA satisfies the following conditions:

$$\sum_{A \subseteq \Theta} m(A) = 1 \text{ and } m(\varnothing) = 0. \tag{4}$$

In addition, the evidence theory provides methods for combining different pieces of evidence, the most notable of which is Dempster's rule of combination. A synthetic probability assignment function m_C can be obtained by using this rule, so as to improve the overall decision accuracy.

Fuzzy set theory: Fuzzy set theory is a mathematical framework designed to handle uncertainty and imprecision [72,73]. Unlike classical set theory, which dictates that elements either belong to a set or not, fuzzy set theory allows for varying degrees of membership, offering a more flexible approach to representing and manipulating vague and ambiguous information [74].

At the heart of fuzzy set theory is the concept of a fuzzy set. A fuzzy set $\tilde{A}$ in a universe of discourse X is characterized by a membership function $\mu_{\tilde{A}} : X \to [0,1]$. For each element $x \in X$, the membership function $\mu_{\tilde{A}}(x)$ assigns a degree of membership ranging from 0 to 1, indicating the extent to which x belongs to $\tilde{A}$. Membership functions can take various forms, including triangular, trapezoidal, and Gaussian shapes [75], depending on the specific application and the nature of the data being modeled. Fuzzy set theory extends classical set operations to handle degrees of membership. The primary operations include union, intersection, and complement. For example, the membership function of the union of two fuzzy sets $\tilde{A}$ and $\tilde{B}$ is given by $\mu_{\tilde{A} \cup \tilde{B}}(x) = \max\left(\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(x)\right)$.

Rough set theory: Unlike fuzzy set theory, rough set theory does not require any prior knowledge or membership functions; instead, it analyzes the data itself to discover and express uncertainty [76]. The foundation of rough set theory lies in equivalence relations and partitions. Within a given universe U, an equivalence relation R partitions U into disjoint equivalence classes, each representing a set of elements that are indistinguishable in some sense. Based on these equivalence relations, rough set theory introduces the concepts of lower and upper approximation to approximate a set under incomplete information:

$$\overline{R}(X) = \left\{ x \in U \mid [x]_R \cap X \neq \varnothing \right\}; \underline{R}(X) = \left\{ x \in U \mid [x]_R \subseteq X \right\}, \tag{5}$$

where $\overline{R}(X)$ and $\underline{R}(X)$ represent the upper and lower approximation set of set X; $[x]_R$ is the equivalence class with respect to element x; and the subscript 'R' denotes the equivalence

relationship. An important application of rough set theory is attribute reduction, which involves removing redundant uncertainties while maintaining the classification ability of the data [77].

Convex model: One of the key advantages of convex models is their robustness in the presence of uncertainty. By modeling uncertainties as convex sets, such as uncertainty in input data or parameters, convex models can provide solutions that are not overly sensitive to variations, making them highly reliable in real-world applications. Common convex models include interval model, ellipsoid model, parallelepiped model, and others [78–83]. The concept of irregular convex sets has even been proposed [84]. The following is an example of the ellipsoid model to introduce the use of convex models in uncertainty quantification.

The explicit mathematical formula to represent the ellipsoid model Ω_e is given as follows:

$$\Omega_e = \left\{ \mathbf{x} \middle| (\mathbf{x}-\mathbf{x}^0)^T \mathbf{G}_e (\mathbf{x}-\mathbf{x}^0) \le 1 \right\}, \tag{6}$$

where $\mathbf{G}_e$ and $\mathbf{x}^0$ are the characteristic matrix and the centroid of the ellipsoid, respectively. In uncertainty quantification based on ellipsoid models, it is considered that the optimal ellipsoid model should wrap all sample points in the smallest volume. The relevant construction methods are rotation matrix method, correlation approximation method, and data-driven method [85–88].

In addition to the above, some other methods are also widely used, such as grey theory [89] or credible set [90]. Each of these methods has its own application scenarios in which they are appropriate, and should be used to analyze specific problems. Thus, by systematically analyzing and quantifying uncertainty, they can provide strong support for the final fault diagnosis decision.

3.3. Pattern recognition with uncertainty

Current uncertainty-based fault diagnosis techniques are rapidly evolving and have shown significant results in dealing with fault diagnosis in complex environments [91,92]. However, a unified and systematic approach has not yet been developed. At this stage, the main methods used are the combination of multiple models, including Bayesian inference, fuzzy logic, neural networks, and machine learning, among others. By integrating multiple techniques, the uncertainty of data can be handled more effectively, thereby improving the accuracy and reliability of diagnosis. Depending on how uncertainty is handled, fault pattern recognition methods can be classified into two categories: parallel and embedded.

Parallel approaches prioritize the quantification of uncertainty, which in turn leads to the use of relevant classification models for pattern recognition. This method allows for a more straightforward assessment of uncertainty metrics, facilitating the selection of the most appropriate model based on the specific characteristics of the fault data. In reference [93], Ma et al. explored the problem of diagnosing power transformer faults with measurement origin uncertainty. This uncertainty arises from the effects of imprecision (i.e., due to noise and outliers) and class imbalance (i.e., uneven distribution of samples among different fault types) of the samples in the training dataset used to identify different fault types. Since all samples in the training dataset of SVM are processed uniformly, ordinary SVM lacks the ability to handle data with noise and outliers. In order to solve this problem, Ma et al. adopted fuzzy support vector machine (Fuzzy-SVM). Fuzzy-SVM assigns different weights to different samples in the training dataset by taking into account their fuzzy affiliation with respect to each category. The weights are then incorporated into the normal SVM training process to reduce the effects of noise and outliers. This approach first quantifies the

uncertainty of the training samples and later uses SVM for fault diagnosis, which is a parallel approach.

Conversely, embedded approaches incorporate the handling of uncertainty directly into the classification model itself. This approach allows for a more seamless integration of uncertainty management within the diagnostic process, often resulting in models that are more robust to variations in data quality and noise. As in reference [94], Shi et al. introduced fuzzy set theory into Bayesian network (BN) in the context of fault diagnosis in coal mine drainage systems in response to the uncertainty in the strength of correlation between system faults and symptoms. First, the BN is established based on the mapping relationship between the fault tree (FT) and the BN. Each event in the FT corresponds to each node in the BN, and the logic gates are converted into directed edges and conditional probabilities of the BN. The triangular fuzzy function is used as the criterion, the expert evaluation and the DOUWA operator are used to obtain the data, and the mean area method is used for deblurring, so as to transform the uncertainty of correlation strength into conditional probabilities between nodes. Finally, the a posteriori probability of each root node is obtained through diagnostic reasoning, which provides a basis for the fault diagnosis of the drainage system. This embedded approach utilizes fuzzy set theory to incorporate uncertainty handling into Bayesian networks.

For the parallel versus embedded approach, Table 2 demonstrates some of the fault diagnosis strategies from recent years that take uncertainty into account.

Table 2. Fault diagnosis strategies under uncertainty.

Processing strategy	Uncertainty diagnostic model	Descriptions	Reference papers
Parallel type	Interval-Fault tree-Evidence network	Epistemic uncertainty is quantified by interval models.	[95]
	Fuzzy interval-SNP	Define an interval-valued fuzzy number to quantify uncertainty.	[96]
	Fuzzy-SVM	Assign each training sample a degree of fuzziness.	[93]
Embedded type	Fuzzy-Bayesian network	The conditional probabilities between nodes are determined by the fuzzy algorithm.	[94]
	Interval-GPR	Representation of GPR parameters with interval-valued data.	[7]
	Fuzzy-GWO	Optimization of fuzzy affiliation functions between samples and fault types.	[97]
	Probability-CNN	Generate weights and biases in CNN in the form of probability distributions.	[98]
	Evidence-DNN	Link the Dirichlet distribution parameter to the evidence assigned over classes.	[99]

At present, there is still little research on uncertain fault diagnosis. Most of them consider the uncertainty in fault diagnosis by combining various models. Future research should not only further explore the integration of these technologies but also form a unified uncertainty diagnosis framework to achieve a more comprehensive and efficient fault diagnosis system.

4. Intelligent strategies of fault diagnosis

In addition to the uncertainty problem, there are many challenges in fault information processing such as inconsistency and incompleteness [100–102]. The complex system structure, diverse operating conditions, and large amounts of real-time data make traditional fault diagnosis methods incompetent in dealing with these problems. In response to these challenges, intelligent fault diagnosis strategies have been developed. In this section, the performance of current intelligent fault diagnosis techniques will be demonstrated in terms of the various stages of system fault diagnosis, and the general framework is shown in Figure 5.

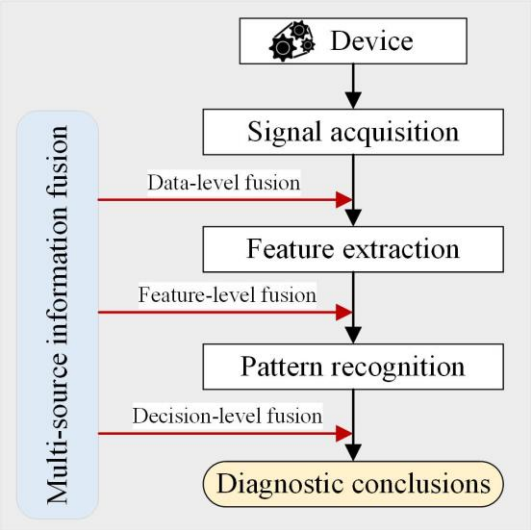

Figure 5. General stages of fault diagnosis.

4.1. Signal acquisition

Signal acquisition is the first part of the fault diagnosis system and the basis of the whole diagnosis process [103]. By accurately and reliably collecting information on the operating status of the equipment, the effectiveness of subsequent analysis and diagnosis can be ensured. The quality of signal acquisition directly affects the accuracy and efficiency of fault diagnosis, making the use of advanced sensor technology particularly important.

A sensor, as a detection device, is responsible for converting various physical or chemical quantities into electrical signals that can be measured and analyzed [104]. By monitoring the environment or the operational status of equipment in real-time, sensors provide accurate and timely data support, assisting engineers and technicians in the effective control and diagnosis of engineering systems. The primary measurement and control parameters in engineering environments include temperature, pressure, force, position, and flow [105].

Temperature sensors can measure the thermal properties of gases, liquids, and solids. Based on different sensing principles, temperature sensors can be categorized into two types: (1) Temperature sensors based on thermoelectric effects, which measure temperature by detecting the influence of temperature on the electrical properties (such as resistance and voltage) of materials. Common examples include thermocouples, thermistors, and resistance temperature detectors (RTD) [106,107]. (2) Temperature sensors based on blackbody radiation, which measure temperature by detecting the

infrared radiation emitted by an object, thus enabling non-contact temperature measurement [108]. A typical example of this type is the infrared thermometer [109]. Pressure sensors detect changes in pressure and convert them into electrical signals, allowing for the identification of forces within gases or liquids. Common types of pressure sensors include piezoresistive, piezoelectric, and capacitive pressure sensors [110–112]. Additionally, resonant pressure sensors and sensors based on optical interference have seen significant development with the advancement of micro-electromechanical systems (MEMS) technology [113]. Force sensors are used to convert applied forces (such as tensile and compressive forces) into electrical signals that reflect the magnitude of the force. Depending on the type of force being measured, force sensors mainly include (1) load cells, which measure compressive force, (2) strain gauges, which measure internal resistance, and (3) force-sensing resistors, which measure the rate of change of the applied force [105,114,115]. Position sensors detect the displacement, orientation, or angle of an object and convert these signals into electrical outputs. The main types of position sensors include potentiometric, capacitive, magnetostrictive linear position, eddy current-based, and optical position sensors [116–119]. Flow sensors detect the movement of gases, liquids, or solids within a pipe or conduit. The main types of flow sensors include differential pressure, turbine, electromagnetic, and ultrasonic flow sensors [105,120]. The above descriptions cover some typical traditional passive sensors. Additionally, there are other types such as accelerometers, humidity sensors, sound sensors, and gas detectors [121]. With the advancement of technology, some novel sensors have also seen significant progress in recent years, such as nuclear sensors (MEMS sensors), micro-sensors, and nano-sensors (NEMS sensors) [122].

In recent years, smart sensors have garnered widespread attention due to their high efficiency and multifunctionality. Compared to traditional sensors, smart sensors integrate various functions such as sensing, data processing, and communication, enabling them to perform complex computations using collected data. Additionally, smart sensors are equipped with signal conditioning, embedded algorithms, and digital interfaces, making the devices capable of detection and self-awareness. With advancements in technology, smart sensors have become increasingly smaller, less power-consuming, and more accurate. As a fundamental component of the Internet of Things (IoT), smart sensors find applications in various fields. For instance, one application of smart sensors is in wireless sensor networks (WSN), where nodes connect with one or more other sensors and sensor hubs, forming a certain communication technology [123]. The main characteristics of smart sensors are illustrated in Figure 6, followed by further explanations regarding their key features.

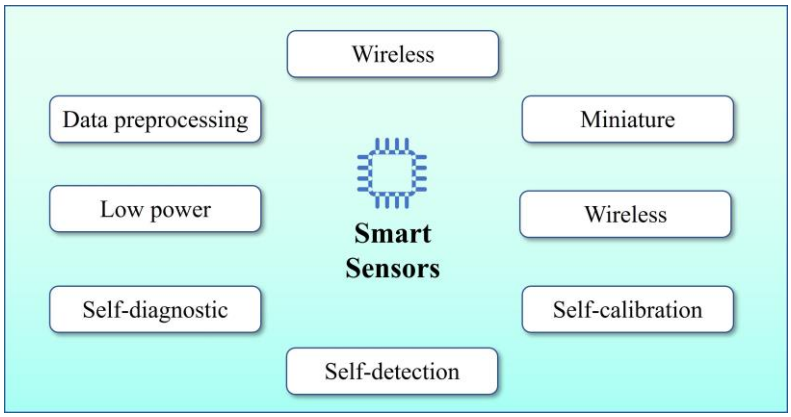

Figure 6. Smart sensor key features.

Smart sensors feature self-calibration capability, allowing them to automatically adjust their measurement parameters to maintain high accuracy and reliability [124]. The self-calibration function reduces reliance on manual intervention, ensuring that the sensor can continuously provide accurate data over long periods of use. Through built-in calibration algorithms, smart sensors can automatically recalibrate based on environmental changes or usage time, enabling them to maintain excellent performance in various complex and changing environments.

The self-diagnostic function enables smart sensors to detect their own faults and abnormal conditions [125]. When a sensor encounters a problem, it can proactively alert users or systems, indicating the need for maintenance or replacement. This functionality significantly enhances system reliability and safety, avoiding downtime or erroneous data caused by sensor failures, thereby ensuring the continuous operation of the system and data accuracy.

The data preprocessing capability allows smart sensors to filter, denoise, calibrate, and perform preliminary analysis on collected data locally [126]. This preprocessing capability not only improves the quality and accuracy of the data but also reduces the burden of data transmission and processing. By optimizing data at the source, smart sensors can provide more valuable real-time information, supporting rapid decision-making and response.

4.2. Feature extraction

Feature extraction plays a crucial role in intelligent fault diagnosis. Its primary purpose is to extract useful information from complex raw signals that effectively reflects the operating status and fault characteristics of equipment, thereby providing strong support for subsequent fault diagnosis algorithms. The effectiveness of feature extraction directly impacts the accuracy and robustness of fault diagnosis.

Time-domain feature extraction is the most intuitive method, primarily analyzing the variations of the signal over time. Common time-domain features include mean, variance, peak value, kurtosis, and skewness [127]. Frequency-domain feature extraction converts time-domain signals into the frequency domain through Fourier transform to analyze the characteristics of the signal at different frequencies. Common frequency-domain features include spectral amplitude, spectral energy, central frequency, and frequency bandwidth [128]. Time-frequency feature extraction combines information from both time and frequency domains to comprehensively describe the characteristics of the signal [129]. Common methods include wavelet transform and short-time Fourier transform. The advantage of time-frequency analysis lies in its ability to handle non-stationary signals, capturing local features of the signal in both time and frequency domains. This is particularly significant for the detection and diagnosis of complex fault patterns.

Traditional feature extraction methods are primarily used for one-dimensional signals, i.e., signals that vary over time. These methods have significant advantages in handling one-dimensional signals such as mechanical vibration signals, motor current signals, and acoustic signals. However, for high-dimensional signals (such as two-dimensional images and video signals), traditional time-domain, frequency-domain, and time-frequency domain feature extraction methods may prove inadequate. With the development of machine learning and deep learning technologies, intelligent feature extraction methods have been widely applied in fault diagnosis. These methods can automatically learn and extract effective features from data, reducing manual intervention and improving the efficiency and accuracy of feature extraction. They show a strong advantage in processing all kinds of signals, especially high-dimensional signals.

PCA is a common technique in feature extraction, aiming to reduce data dimensionality while retaining essential information [130,131]. It works by transforming the original features into a new set of orthogonal components, capturing most of the data variability with fewer dimensions. PCA begins by computing the covariance matrix $\sum$ of the input data, representing feature relationships. Then, it calculates eigenvectors $\mathbf{v}_1, \mathbf{v}_2, \cdots, \mathbf{v}_n$, and corresponding eigenvalues $\lambda_1, \lambda_2, \cdots, \lambda_n$ of $\sum$. Eigenvectors signify directions of maximum variance, while eigenvalues indicate variance magnitudes along these directions. Principal components are obtained by sorting eigenvectors based on corresponding eigenvalues. The first principal component captures the most variance, the second captures the second most, and so on. By selecting the top principal components, data can be projected onto a lower-dimensional space.

Linear discriminant analysis (LDA) is a feature extraction method commonly employed in pattern recognition and classification tasks [130,132]. To begin, LDA calculates the between-class and within-class scatter matrices. The between-class scatter matrix evaluates the dispersion between different classes, while the within-class scatter matrix gauges the dispersion within each class. The objective of LDA is to maximize the ratio of between-class scatter to within-class scatter, resulting in a transformation that enhances class separability. In terms of principle, Figure 7 clearly shows the differences and links between LDA and PCA. The optimization problem for LDA can be expressed as maximizing the ratio:

$$\max_{\mathbf{w}} \frac{\mathbf{w}^T S_b \mathbf{w}}{\mathbf{w}^T S_w \mathbf{w}}. \tag{7}$$

Here, $\mathbf{w}$ represents the projection vector, S_b signifies the between-class scatter matrix, and S_w denotes the within-class scatter matrix. The optimal projection vector $\mathbf{w}$ is then applied to transform the input data into a lower-dimensional space where classes are better differentiated. This transformation substantially reduces data dimensionality while retaining most of the discriminatory information.

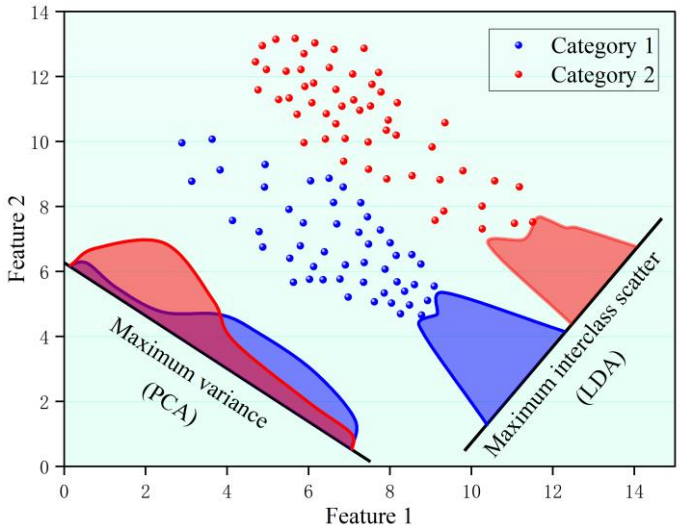

Figure 7. Differences and links between PCA and LDA.

The rise of deep learning technology has brought about revolutionary changes in the field of fault diagnosis. Deep learning methods, by constructing deep neural networks, can automatically extract complex high-dimensional features from large amounts of data, thus avoiding the complex feature engineering involved in traditional methods. Common deep learning models include convolutional neural networks (CNNs), long short-term memory networks (LSTMs), and autoencoders [133]. CNNS employ multiple layers of convolution and pooling operations to automatically learn spatial features of signals, making them particularly suitable for feature extraction from images and time-series data [134]. LSTMs, with their special memory units, capture the temporal dependencies of signals, making them highly effective for processing sequential data. Autoencoders, through unsupervised learning, automatically extract high-dimensional feature representations of signals, which can be utilized for data dimensionality reduction and anomaly detection [135].

4.3. Pattern recognition

In intelligent fault diagnosis, the effective information extracted from features is used to train learning algorithms for intelligent fault recognition. Based on the requirement for labeled data, these learning algorithms can be categorized into supervised learning, unsupervised learning, and semi-supervised learning [136].

Supervised learning involves training models using input data and their corresponding labels to create a system that can predict labels for new data. The basic principle is to learn the mapping relationship between inputs and outputs based on known input-output pairs. The vast majority of machine learning and deep learning techniques fall under supervised learning, and several common methods are described below.

Decision tree: Decision tree-based fault diagnosis methods rely on a tree-structured model, as shown in Figure 8, to classify and identify faults by recursively splitting the dataset based on specific features [137,138]. As a machine learning technique, a decision tree is constructed from a set of training data, where each internal node represents a decision based on a feature, each branch represents the outcome of the decision, and each leaf node represents a fault category or outcome. In decision tree-based fault diagnosis, we start with a root node that contains the entire dataset. The tree is built by selecting the best feature to split the data at each node, typically using criteria such as information gain or Gini impurity. This process continues recursively, creating branches for each possible outcome of the decision, until the data is perfectly classified or a stopping criterion is met. For instance, consider a decision tree diagnosing faults in a mechanical system. The root node might test the vibration level of the system. If the vibration is above a certain threshold, the tree branches out to test another feature, such as temperature or pressure, and so on, until it reaches a leaf node that indicates a specific fault. The decision at each node can be expressed as:

$$Split(D) = \arg\max_{A \in \text{Attributes}} \text{InformationGain}(D, A), \tag{8}$$

where D is the dataset at the node, A is the set of attributes, and $\text{InformationGain}(D, A)$ measures the reduction in entropy after splitting on attribute A.

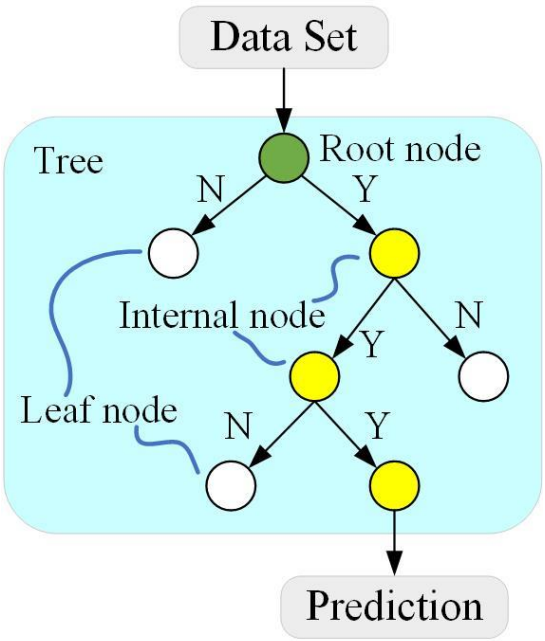

Figure 8. Decision tree model.

Support vector machine (SVM): SVM-based fault diagnosis methods utilize the principles of SVM to classify and identify faults by finding the optimal hyperplane that separates different fault categories in the feature space [139,140], as depicted in Figure 9. In SVM-based fault diagnosis, we begin by mapping the input data into a high-dimensional feature space using a kernel function. The SVM algorithm then identifies the hyperplane that maximizes the margin between different classes, ensuring the best possible separation of faults. The support vectors are the data points closest to the hyperplane, and they are crucial in defining the position and orientation of the hyperplane. For example, in diagnosing electrical system faults, SVM might utilize features such as voltage, current, and frequency. The kernel function transforms these features into a higher-dimensional space where a clear separation between different types of faults, such as short circuits or overloads, can be achieved. The SVM optimization problem is expressed as:

$$\begin{aligned} &\min_{\mathbf{w},b} && \frac{1}{2}\|\mathbf{w}\|^2 \\ &subject\ to && y_i(\mathbf{w}\cdot\mathbf{x}_i + b) \geq 1 \end{aligned}, \tag{9}$$

where $\mathbf{w}$ is the weight vector, b is the bias, $\mathbf{x}_i$ are the feature vectors, and y_i are the class labels.

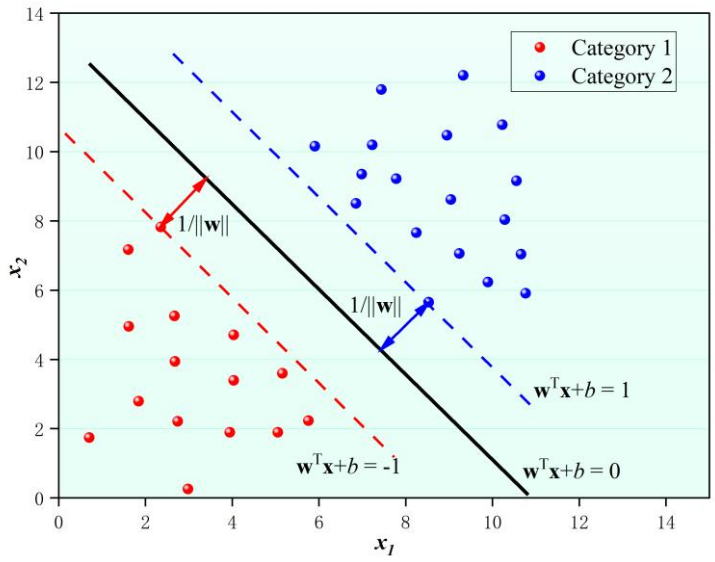

Figure 9. SVM model.

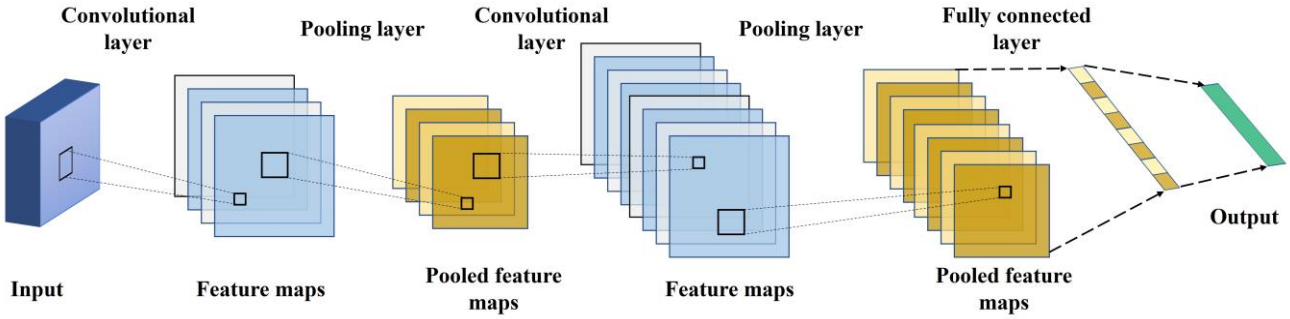

Figure 10. CNN model.

Convolutional neural network (CNN): CNNs have become a powerful tool in fault diagnosis, effectively extracting and learning features from complex datasets [141]. CNNs are composed of several key layers: convolutional layers, pooling layers, and fully connected layers. Figure 10 illustrates the basic architecture of the layers and the relationships between them. The convolutional layers are responsible for detecting local patterns in the input data through convolution operations. Mathematically, a convolution operation can be represented as:

$$\mathbf{f}_{ij} = \sum_{m=1}^{M}\sum_{n=1}^{N}\mathbf{w}_{mn}\cdot\mathbf{x}_{(i+m)(j+n)} + b\,, \tag{10}$$

where $\mathbf{f}_{ij}$ is the output feature map, $\mathbf{w}_{mn}$ are the weights of the filter, $\mathbf{x}_{(i+m)(j+n)}$ is the input data, and b is the bias. Following the convolutional layers, pooling layers perform a downsampling operation to reduce the spatial dimensions of the feature maps. A common pooling operation is max pooling, which can be expressed as:

$$\mathbf{p}_{ij} = \max(\mathbf{f}_{i+k,j+l})\,, \tag{11}$$

for $k,l \in \{0,\ldots,K-1\}$, where $\mathbf{p}_{ij}$ is the pooled feature map, and K is the pooling window size. As the data passes through multiple convolutional and pooling layers, CNNs extract hierarchical features ranging from low-level patterns to high-level abstractions. These features are then flattened and passed to fully connected layers, where the final classification is performed. The optimization of CNN parameters (weights and biases) is typically done using backpropagation and gradient descent algorithms, minimizing a loss function such as cross-entropy:

$$\mathcal{L} = -\sum_i y_i \log(\hat{y}_i), \tag{12}$$

where y_i are the true labels and $\hat{y}_i$ are the predicted probabilities.

Unsupervised learning aims to train models using unlabeled data to discover hidden patterns or intrinsic structures within the data. Unlike supervised learning, there are no predefined output labels in unsupervised learning, and algorithms must infer the structure of the data independently. The most typical approach is the clustering algorithm.

Clustering algorithms group similar data points into clusters based on their features, helping to identify natural groupings or fault patterns within the data [142,143]. Popular clustering algorithms include K-means [144], hierarchical clustering [145], and DBSCAN [146]. For example, in a fault diagnosis scenario for industrial machinery, sensor data such as temperature, pressure, and vibration can be clustered. Each cluster might represent a different operating condition or fault type. The objective of K-means clustering is to minimize the sum of squared distances between data points and their corresponding cluster centroids:

$$\min_{\{C_k\}} \quad \sum_{i=1}^{n} \sum_{k=1}^{K} \| \mathbf{x}_i - \mathbf{c}_k \|^2, \tag{13}$$

where x_i is the i-th data point, $\mathbf{c}_k$ is the centroid of cluster k, and C_k is the set of points in cluster k.

The key to clustering-based fault diagnosis is its ability to reveal inherent structures in the data, allowing for the identification of distinct fault conditions without prior knowledge.

Supervised learning excels with ample labeled data but struggles when such data is scarce. Unsupervised learning can identify patterns without labeled data but may not always provide precise fault classifications. To bridge this gap, semi-supervised learning combines the advantages of both methods, using a mix of labeled and unlabeled data to improve model accuracy and robustness [147].

Self-training: Self-training is a common semi-supervised learning technique where a model is initially trained on a small set of labeled data [148,149]. The trained model is then used to predict labels for the unlabeled data. The most confident predictions are added to the labeled dataset, and the model is retrained. This process is iterated until no more unlabeled data is added. In a fault diagnosis scenario, a self-training model might initially be trained on a small set of labeled sensor data from a machine. The model then predicts labels for the remaining unlabeled sensor data. High-confidence predictions are added to the training set, improving the model's performance over time. Mathematically, the self-training process can be described as:

$$\begin{aligned} \mathcal{L} &= \{(\mathbf{x}_i, y_i)\}_{i=1}^{l} \\ \mathcal{U} &= \{\mathbf{x}_i\}_{i=l+1}^{l+u} \end{aligned}, \tag{14}$$

where $\mathcal{L}$ is the labeled dataset, $\mathcal{U}$ is the unlabeled dataset, l is the number of labeled samples, and u is the number of unlabeled samples. The model f is trained on $\mathcal{L}$ and used to predict labels $\hat{y}_i$, for $\mathcal{U}$. The most confident predictions are added to $\mathcal{L}$:

$$\mathcal{L} \leftarrow \mathcal{L} \cup \{(\mathbf{x}_i, \hat{y}_i) \big| \text{confidence}(\hat{y}_i) \geq \tau\}, \tag{15}$$

where τ is a confidence threshold.

Co-Training: Co-training involves training two models on different views of the data [150,151]. Each model is trained on its own labeled dataset and then used to label the unlabeled data for the other model. This method assumes that the views are conditionally independent given the class label. For example, in diagnosing faults in a robotic arm, one view might consist of visual data from cameras, and the other view might consist of force sensor data. Each model trains on its respective view and helps label the unlabeled data for the other model, enhancing overall performance. The datasets for co-training are defined as:

$$\begin{aligned} \mathcal{L}_1 &= \{(\mathbf{x}_i^1, y_i)\}_{i=1}^{l} \\ \mathcal{L}_2 &= \{(\mathbf{x}_i^2, y_i)\}_{i=1}^{l} \\ \mathcal{U} &= \{(\mathbf{x}_i^1, \mathbf{x}_i^2)\}_{i=l+1}^{l+u} \end{aligned}, \tag{16}$$

where x_i^1 and x_i^2 are the two different views of the data. Each model f_1 and f_2 is trained on $\mathcal{L}_1$ and $\mathcal{L}_2$, respectively. The models predict labels $\hat{y}_i^2$ for the unlabeled data. The most confident predictions from each model are added to the labeled datasets of the other model, iteratively improving both models' performance:

$$\begin{aligned} \mathcal{L}_1 &\leftarrow \mathcal{L}_1 \cup \{(\mathbf{x}_i^1, \hat{y}_i^2) |\ \text{confidence}(\hat{y}_i^2) \geq \tau\} \\ \mathcal{L}_2 &\leftarrow \mathcal{L}_2 \cup \{(\mathbf{x}_i^2, \hat{y}_i^1) |\ \text{confidence}(\hat{y}_i^1) \geq \tau\} \end{aligned}. \tag{17}$$

Semi-supervised learning algorithms are being widely studied for their applicability. Successively proposed semi-supervised learning algorithms include SSTI [152], SGAN [153], S3VM [154], etc.

4.4. Multi-source information fusion

In the domain of intelligent fault diagnosis, multi-source information fusion stands out as a pivotal technology. It facilitates the effective integration of information from multiple sensors or systems, thereby enhancing the diagnostic system's capability to handle unreliable or inconsistent data. Information fusion typically encompasses three hierarchical levels: data-level, feature-level, and decision-level, corresponding to the stages of signal acquisition, feature extraction, and pattern recognition in intelligent fault diagnosis [155].

At the data-level fusion stage, the system processes data from diverse sensors, which may be subject to noise, inconsistency, or incompleteness [156]. The primary objective of data-level fusion is to reduce data uncertainty and enhance the system's adaptability to environmental changes.

In feature-level fusion, the system integrates and analyzes features extracted from raw data [157]. These features may originate from different sensors or systems, exhibiting distinct representations and significance. The primary goal of feature-level fusion is to leverage multiple features comprehensively,

enhancing the system's discriminative capability and robustness regarding target state identification.

Decision-level fusion aims to improve the system's fault recognition and localization capability by synthesizing diagnostic results from multiple decision-makers [158]. By employing appropriate decision rules and algorithms, decision-level fusion effectively mitigates the system's misdiagnosis and false-negative rates, thereby augmenting diagnostic accuracy and reliability.

The classic theories of information fusion are described below:

Dempster-Shafer evidence theory: The relevant concepts of Dempster-Shafer evidence theory have been described in Section 3.2, and only its role in information fusion is described here [159]. When there are multiple sources of information for a proposition, information fusion is required so as to effectively deal with complementary and redundant information. Dempster's rule of combination can be used for the fusion of multiple pieces of evidence, denoted by $\oplus$. Let m_1 and m_2 be two sets of basic probabilistic assignments, the corresponding focal elements are $A_1, A_2, \cdots A_k$ and $B_1, B_2, \cdots B_l$. Denote by m the new evidence after the combination of m_1 and m_2, i.e., $m = m_1 \oplus m_2$, which is defined as follows:

$$\begin{cases} m(\varnothing) = 0 \\ m(A) = \dfrac{1}{1-k} \sum\limits_{A_i \cap B_j = A} m_1(A_i) m_2(B_j) \end{cases}, \tag{18}$$

where $k = \sum\limits_{A_i \cap B_j = \varnothing} m_1(A_i) m_2(B_j)$, known as the conflict coefficient, indicates the degree of conflict between the two pieces of evidence. k is larger the greater the degree of conflict. When $k = 1$, it indicates that the two pieces of evidence are completely opposite in their degree of support for the proposition, and the evidence is in complete conflict with each other, at which point the rule no longer applies.

Probabilistic model: Probability-based information fusion methods usually rely on Bayes' rule to combine a priori and observed information to obtain final decision information and a comprehensive description [160]. As a statistical inference method, it uses Bayes' theorem to update the probability estimate of an event H_i by combining the a priori probability with new observations to obtain the posterior probability. In Bayesian inference, we first have a prior belief, i.e., our initial estimate $P(H_i)$ of the probability of an event H_i occurring before any data is observed. Then, when we observe new data or evidence E, we use Bayes' theorem to update our beliefs to obtain the posterior probability $P(H_i|E)$, i.e., a new estimate of the probability of the event occurring after considering the new evidence. Mathematically, Bayesian inference can be expressed as:

$$P(H_i \mid E) = \frac{P(H_i) P(E \mid H_i)}{\sum\limits_j P(H_j) P(E \mid H_j)} = \frac{P(H_i) P(E \mid H_i)}{P(E)}. \tag{19}$$

The key to Bayesian inference is to continually update the prior probabilities, incorporating new evidence into the prior probabilities to obtain more accurate estimates of the posterior probabilities.

Fuzzy set theory: Fuzzy set theory-based information fusion methods rely on the principles of fuzzy logic to handle the uncertainty and imprecision inherent in combining data from multiple sources [161]. As a mathematical framework, fuzzy set theory allows for the representation of

uncertain information by assigning degrees of membership to elements rather than relying on binary true/false classifications. This flexibility makes it particularly useful for situations where information is incomplete, ambiguous, or lacks precise boundaries. In fuzzy set theory, we begin by defining fuzzy sets that represent different possible states or events. Each element in a fuzzy set has a membership value between 0 and 1, indicating the degree to which it belongs to the set. When fusing information from different sources, we combine these membership values to derive a new fuzzy set that represents the aggregated information.

For example, consider two fuzzy sets A and B, representing different information sources about an event E. The membership functions of these sets, denoted as $\mu_A(x)$ and $\mu_B(x)$ express the degree of membership of an element x in sets A and B, respectively. The fusion of these sets can be performed using various operators such as the fuzzy intersection (min operator), fuzzy union (max operator), or other t-norms and s-norms depending on the desired aggregation method. Mathematically, if we use the fuzzy intersection to combine the information, the resulting membership function $\mu_{A\cap B}(x)$ is given by:

$$\mu_{A\cap B}(x) = \min\left(\mu_A(x), \mu_B(x)\right). \tag{20}$$

Alternatively, for fuzzy union, the resulting membership function $\mu_{A\cup B}(x)$ is:

$$\mu_{A\cup B}(x) = \max\left(\mu_A(x), \mu_B(x)\right). \tag{21}$$

In addition to the above, rough set theory and Kalman filter theory are also common methods for information fusion. Table 3 shows representative papers of each fusion strategy from recent years.

Table 3. Representative papers of each fusion strategy.

Model	Reference papers	Fusion level	Topic
Dempster-Shafer evidence theory	[162]	Decision-level	Overview of basic/improved theories
	[159]	Decision-level	Fault diagnosis of rotor system
Probabilistic model	[163]	Data/Feature-level	Overview of basic/improved theories
	[164]	Decision-level	Pumping unit fault detection
Fuzzy set theory	[165]	Data-level	Image fusion for multimodal medical images
Rough set theory	[166]	Feature-level	A review of multi-source information fusion based on rough set theory
Kalman filter theory	[167]	Feature-level	Damage quantification (hole edge crack monitoring)

Some references indicate that AI algorithms are also being widely used for multi-source information fusion, including machine learning algorithms represented by SVM [168], KNN [169], etc., and deep learning algorithms with CNN [170], LSTM [171], etc., as the dataset. But it is not essentially a fusion. Maybe it would be better called a combination. Fusion embedded in AI algorithms is only a simple stitching of multiple sources of information for classification purposes. As in reference

[170], the authors designed an information fusion-based joint CNN (IF-JCNN) to combine US images and RF signals for TN diagnosis. But essentially the idea of the article is to use two CNN branches to extract the features of US images and TN diagnosis and then simply concatenate them to achieve feature fusion. Afterward, the integrated features were processed by three fully connected layers to output the final diagnostic results. This is not consistent with the fusion of multi-source information discussed in this section.

5. Conclusions

This paper has provided a comprehensive overview of fault diagnosis methods for complex engineering systems, which are critical for ensuring the reliability, safety, and efficiency of operations. We examined traditional fault diagnosis methods, including mechanism-driven, data-driven, and hybrid approaches, each with its unique strengths. Additionally, we discussed treatment methods that take into account uncertainties arising from model inaccuracies, external disturbances, and measurement noise, emphasizing the need for robust fault diagnosis under real-world conditions. Finally, we explored intelligent fault diagnosis strategies that utilize advanced sensor technologies and intelligent algorithms to improve fault detection and diagnosis. The following remarks emphasize the prospects and challenges of fault diagnosis in complex engineering systems:

(1) With the development of various advanced sensing technologies, complex engineering systems generate vast amounts of heterogeneous data during operation. Although it provides rich information for health monitoring and fault diagnosis, how to efficiently extract effective information from them remains a major challenge. Compared with analysis methods relying on a single data source, the development of fusion analysis methods for two or more heterogeneous data sources is considered a more effective strategy. For massive, high-dimensional, and heterogeneous data, future research should pay more attention to the combination of data fusion, feature extraction, and dimensionality reduction techniques to enhance the efficiency and accuracy of information acquisition.

(2) Despite the impressive performance of modern machine learning and deep learning techniques in fault detection and diagnosis, their "black box" nature makes the decision-making process difficult to understand. This lack of transparency reduces engineers' trust in the diagnostic results. Therefore, future research could focus on embedding domain knowledge and physical mechanisms to develop interpretable intelligent diagnostic models, which should clearly articulate their decision bases, reveal the root causes of faults, and explain the impact of various factors on system performance, thereby providing strong support for maintenance decisions and manufacturing improvements.

(3) Extensive research has been achieved on intelligent fault diagnosis combining feature extraction, fault diagnosis, and information fusion. But it is mainly based on the support of huge training data, and it is almost impossible to obtain enough fault sample data in complex engineering systems. How to achieve real "intelligent diagnosis" under non-standard conditions such as uncertainty, incompleteness, and small samples, and to be applied in engineering practice, remains a pressing issue for the foreseeable future.

Author contributions

Chong Wang: conceptualization, formal analysis, funding acquisition, methodology, project administration, writing–review & editing; Xinxing Chen: conceptualization, investigation, methodology, writing–original draft and writing–review & editing; Xin Qiang and Haoran Fan: supervision and validation; Shaohua Li: supervision and visualization. All authors have read and approved the final version of the manuscript for publication.

Acknowledgments

This work was supported by the Defense Industrial Technology Development Program (No. JCKY2023601B008).

Conflict of interest

The authors declare no conflicts of interest.

References

1. L. Jia, Q. Gao, Z. P. Liu, H. B. Tan, L. W. Zhou, Multidisciplinary fault diagnosis of complex engineering systems: A case study of nuclear power plants, *Int. J. Ind. Ergon.*, **80** (2020), 103060. https://doi.org/10.1016/j.ergon.2020.103060
2. Y. B. Li, B. Li, J. C. Ji, H. Kalhori, Advanced fault diagnosis and health monitoring techniques for complex engineering systems, *Sensors*, **22** (2022), 10002. https://doi.org/10.3390/s222410002
3. C. Wang, H. G. Matthies, Random model with fuzzy distribution parameters for hybrid uncertainty propagation in engineering systems, *Comput. Meth. Appl. Mech. Eng.*, **359** (2020), 112673. https://doi.org/10.1016/j.cma.2019.112673
4. F. Villecco, A. Pellegrino, Evaluation of uncertainties in the design process of complex mechanical systems, *Entropy*, **19** (2017), e19090475. https://doi.org/10.3390/e19090475
5. E. Hüllermeier, W. Waegeman, Aleatoric and epistemic uncertainty in machine learning: An introduction to concepts and methods, *Mach. Learn.*, **110** (2021), 457–506. https://doi.org/10.1007/s10994-021-05946-3
6. H. R. Fan, C. Wang, S. H. Li, Novel method for reliability optimization design based on rough set theory and hybrid surrogate model, *Comput. Meth. Appl. Mech. Eng.*, **429** (2024), 117170. https://doi.org/10.1016/j.cma.2024.117170
7. M. Mansouri, R. Fezai, M. Trabelsi, M. Hajji, M.-F. Harkat, H. Nounou, et al., A novel fault diagnosis of uncertain systems based on interval Gaussian process regression: Application to wind energy conversion systems, *IEEE Access*, **8** (2020), 219672–219679. https://doi.org/10.1109/access.2020.3042101
8. X. X. Liu, Y. T. Ju, X. H. Liu, S. Miao, W. G. Zhang, An imu fault diagnosis and information reconstruction method based on analytical redundancy for autonomous underwater vehicle, *IEEE Sens. J.*, **22** (2022), 12127–12138. https://doi.org/10.1109/jsen.2022.3174340

9. D. Yu, Fault diagnosis for a hydraulic drive system using a parameter-estimation method, *Control Eng. Practice*, **5** (1997), 1283–1291. https://doi.org/10.1016/s0967-0661(97)84367-5
10. G. C. Zhang, L. Chen, K. K. Liang, Fault monitoring and diagnosis of aerostat actuator based on pca and state observer, *Int. J. Model. Identif. Control*, **32** (2019), 145. https://doi.org/10.1504/ijmic.2019.102367
11. Y. Song, M. Y. Zhong, J. Chen, Y. Liu, An alternative parity space-based fault diagnosability analysis approach for linear discrete time systems, *IEEE Access*, **6** (2018), 16110–16118. https://doi.org/10.1109/access.2018.2816970
12. V. Venkatasubramanian, R. Rengaswamy, S. N. Kavuri, A review of process fault detection and diagnosis, *Comput. Chem. Eng.*, **27** (2003), 313–326. https://doi.org/10.1016/s0098-1354(02)00161-8
13. S. W. Pan, D. Xiao, S. T. Xing, S. S. Law, P. Y. Du, Y. J. Li, A general extended kalman filter for simultaneous estimation of system and unknown inputs, *Eng. Struct.*, **109** (2016), 85–98. https://doi.org/10.1016/j.engstruct.2015.11.014
14. E. Walker, S. Rayman, R. E. White, Comparison of a particle filter and other state estimation methods for prognostics of lithium-ion batteries, *J. Power Sources*, **287** (2015), 1–12. https://doi.org/10.1016/j.jpowsour.2015.04.020
15. S. Nolan, A. Smerzi, L. Pezzè, A machine learning approach to Bayesian parameter estimation, *npj Quantum Inform.*, **7** (2021), 169. https://doi.org/10.1038/s41534-021-00497-w
16. R. Tarantino, F. Szigeti, E. Colina-Morles, Generalized luenberger observer-based fault-detection filter design: An industrial application, *Control Eng. Practice*, **8** (2000), 665–671. https://doi.org/10.1016/s0967-0661(99)00181-1
17. L. A. Rusinov, N. V. Vorobiev, V. V. Kurkina, Fault diagnosis in chemical processes and equipment with feedbacks, *Chemometrics Intell. Lab. Syst.*, **126** (2013), 123–128. https://doi.org/10.1016/j.chemolab.2013.03.015
18. F. Pierri, G. Paviglianiti, F. Caccavale, M. Mattei, Observer-based sensor fault detection and isolation for chemical batch reactors, *Eng. Appl. Artif. Intell.*, **21** (2008), 1204–1216. https://doi.org/10.1016/j.engappai.2008.02.002
19. H. M. Odendaal, T. Jones, Actuator fault detection and isolation: An optimised parity space approach, *Control Eng. Practice*, **26** (2014), 222–232. https://doi.org/10.1016/j.conengprac.2014.01.013
20. C. J. Duan, Z. Y. Fei, J. C. Li, A variable selection aided residual generator design approach for process control and monitoring, *Neurocomputing*, **171** (2016), 1013–1020. https://doi.org/10.1016/j.neucom.2015.07.042
21. P. Zhang, S. X. Ding, Disturbance decoupling in fault detection of linear periodic systems, *Automatica*, **43** (2007), 1410–1417. https://doi.org/10.1016/j.automatica.2007.01.005
22. Q. Wang, C. Taal, O. Fink, Integrating expert knowledge with domain adaptation for unsupervised fault diagnosis, *IEEE Trans. Instrum. Meas.*, **71** (2022), 1–12. https://doi.org/10.1109/tim.2021.3127654
23. P. Zhao, X. D. Mu, Z. R. Yin, Z. X. Yi, An approach of fault diagnosis for system based on fuzzy fault tree, *2008 International Conference on MultiMedia and Information Technology*, Three Gorges, China, 2008, 697–700. https://doi.org/10.1109/mmit.2008.142

24. Z. N. Lin, Y. X. Wang, H. Q. Xu, F. R. Wei, A novel reduced-order analytical fault diagnosis model for power grid, *IEEE Access*, **12** (2024), 59521–59532. https://doi.org/10.1109/access.2024.3392905
25. C. Cheng, X. Y. Qiao, H. Luo, W. X. Teng, M. L. Gao, B. C. Zhang, et al., A semi-quantitative information based fault diagnosis method for the running gears system of high-speed trains, *IEEE Access*, **7** (2019), 38168–38178. https://doi.org/10.1109/access.2019.2906976
26. J. P. Shi, W. G. Tong, D. L. Wang, Design of the transformer fault diagnosis expert system based on fuzzy reasoning, *2009 International Forum on Computer Science-Technology and Applications*, Chongqing, China, 2009, 110–114. https://doi.org/10.1109/ifcsta.2009.34
27. A. R. Sahu, S. K. Palei, A. Mishra, Data-driven fault diagnosis approaches for industrial equipment: A review, *Expert Syst.*, **41** (2024), 13360. https://doi.org/10.1111/exsy.13360
28. G. Wang, J. Y. Zhao, J. H. Yang, J. F. Jiao, J. L. Xie, F. Feng, Multivariate statistical analysis based cross voltage correlation method for internal short-circuit and sensor faults diagnosis of lithium-ion battery system, *J. Energy Storage*, **62** (2023), 106978. https://doi.org/10.1016/j.est.2023.106978
29. Z. Zhang, X. He, Active fault diagnosis for linear systems: Within a signal processing framework, *IEEE Trans. Instrum. Meas.*, **71** (2022), 1–9. https://doi.org/10.1109/tim.2022.3150889
30. R. N. Liu, B. Y. Yang, E. Zio, X. F. Chen, Artificial intelligence for fault diagnosis of rotating machinery: A review, *Mech. Syst. Signal Proc.*, **108** (2018), 33–47. https://doi.org/10.1016/j.ymssp.2018.02.016
31. Y. Q. Liu, B. Liu, X. J. Zhao, M. Xie, A mixture of variational canonical correlation analysis for nonlinear and quality-relevant process monitoring, *IEEE Trans. Ind. Electron.*, **65** (2018), 6478–6486. https://doi.org/10.1109/tie.2017.2786253
32. G. Lee, C. H. Han, E. S. Yoon, Multiple-fault diagnosis of the tennessee eastman process based on system decomposition and dynamic pls, *Ind. Eng. Chem. Res.*, **43** (2004), 8037–8048. https://doi.org/10.1021/ie049624u
33. G. Yu, C. N. Li, J. Sun, Machine fault diagnosis based on Gaussian mixture model and its application, *Int. J. Adv. Manuf. Technol.*, **48** (2010), 205–212. https://doi.org/10.1007/s00170-009-2283-5
34. W. Deng, S. J. Zhang, H. M. Zhao, X. H. Yang, A novel fault diagnosis method based on integrating empirical wavelet transform and fuzzy entropy for motor bearing, *IEEE Access*, **6** (2018), 35042–35056. https://doi.org/10.1109/access.2018.2834540
35. J. B. Guo, Fault diagnosis method of flexible converter valve equipment based on ensemble empirical mode decomposition and temporal convolutional networks, *J. Electr. Syst.*, **20** (2024), 344–352. https://doi.org/10.52783/jes.2386
36. D. J. Yu, M. Wang, X. M. Cheng, A method for the compound fault diagnosis of gearboxes based on morphological component analysis, *Measurement*, **91** (2016), 519–531. https://doi.org/10.1016/j.measurement.2016.05.087
37. L. Ciabattoni, F. Ferracuti, A. Freddi, A. Monteriu, Statistical spectral analysis for fault diagnosis of rotating machines, *IEEE Trans. Ind. Electron.*, **65** (2018), 4301–4310. https://doi.org/10.1109/tie.2017.2762623

38. W. E. Sanders, T. Burton, A. Khosousi, S. Ramchandani, Machine learning: At the heart of failure diagnosis, *Curr. Opin. Cardiol.*, **36** (2021), 227–233. https://doi.org/10.1097/hco.0000000000000833
39. Y. G. Lei, B. Yang, X. W. Jiang, F. Jia, N. P. Li, A. K. Nandi, Applications of machine learning to machine fault diagnosis: A review and roadmap, *Mech. Syst. Signal Proc.*, **138** (2020), 106587. https://doi.org/10.1016/j.ymssp.2019.106587
40. Z. N. An, F. Wu, C. Zhang, J. H. Ma, B. Sun, B. H. Tang, et al., Deep learning-based composite fault diagnosis, *IEEE Jour. Emer. Select. Top. Circu. Syste.*, **13** (2023), 572–581. https://doi.org/10.1109/jetcas.2023.3262241
41. D. T. Hoang, H. J. Kang, A survey on deep learning based bearing fault diagnosis, *Neurocomputing*, **335** (2019), 327–335. https://doi.org/10.1016/j.neucom.2018.06.078
42. X. Y. Fan, J. Li, H. Hao, Review of piezoelectric impedance based structural health monitoring: Physics-based and data-driven methods, *Adv. Struct. Eng.*, **24** (2021), 3609–3626. https://doi.org/10.1177/13694332211038444
43. Q. Ni, X. M. Li, Z. W. Chen, Z. L. Zhao, L. L. Lai, A mechanism and data hybrid-driven method for main circuit ground fault diagnosis in electrical traction system, *IEEE Trans. Ind. Electron.*, **70** (2023), 12806–12815. https://doi.org/10.1109/tie.2023.3260356
44. D. An, N. H. Kim, J. H. Choi, Practical options for selecting data-driven or physics-based prognostics algorithms with reviews, *Reliab. Eng. Syst. Saf.*, **133** (2015), 223–236. https://doi.org/10.1016/j.ress.2014.09.014
45. J. Guo, Z. Y. Li, M. Y. Li, A review on prognostics methods for engineering systems, *IEEE Trans. Reliab.*, **69** (2020), 1110–1129. https://doi.org/10.1109/tr.2019.2957965
46. L. Kou, C. Liu, G. W. Cai, J. N. Zhou, Q. D. Yuan, S. M. Pang, Fault diagnosis for open-circuit faults in npc inverter based on knowledge-driven and data-driven approaches, *IET Power Electron.*, **13** (2020), 1236–1245. https://doi.org/10.1049/iet-pel.2019.0835
47. X. X. Xiao, C. H. Li, J. Huang, T. Yu, Fault diagnosis of rolling bearing based on knowledge graph with data accumulation strategy, *IEEE Sens. J.*, **22** (2022), 18831–18840. https://doi.org/10.1109/JSEN.2022.3201839
48. K. Sachin, M. Torres, Y. C. Chan, M. Pecht, A hybrid prognostics methodology for electronic products, *2008 IEEE International Joint Conference on Neural Networks*, Hong Kong, China, 2008, 3479–3485. https://doi.org/10.1109/IJCNN.2008.4634294
49. S. F. Cheng, M. Pecht, A fusion prognostics method for remaining useful life prediction of electronic products, *2009 IEEE International Conference on Automation Science and Engineering*, Bangalore, India, 2009, 102–107. https://doi.org/10.1109/COASE.2009.5234098
50. H. G. Zhang, R. Kang, M. Pecht, A hybrid prognostics and health management approach for condition-based maintenance, *2009 IEEE International Conference on Industrial Engineering and Engineering Management*, Hong Kong, China, 2009, 1165–1169. https://doi.org/10.1109/ieem.2009.5372976
51. M. A. Chao, C. Kulkarni, K. Goebel, O. Fink, Fusing physics-based and deep learning models for prognostics, *Reliab. Eng. Syst. Saf.*, **217** (2022), 107961. https://doi.org/10.1016/j.ress.2021.107961
52. T. T. Li, Y. Zhao, C. B. Zhang, J. Luo, X. J. Zhang, A knowledge-guided and data-driven method for building hvac systems fault diagnosis, *Build. Environ.*, **198** (2021), 107850. https://doi.org/10.1016/j.buildenv.2021.107850

53. L. H. Ye, X. Ma, C. L. Wen, Rotating machinery fault diagnosis method by combining time-frequency domain features and cnn knowledge transfer, *Sensors*, **21** (2021), 8168. https://doi.org/10.3390/s21248168
54. W. Xu, Y. Wan, T. Y. Zuo, X. M. Sha, Transfer learning based data feature transfer for fault diagnosis, *IEEE Access*, **8** (2020), 76120–76129. https://doi.org/10.1109/ACCESS.2020.2989510
55. X. P. Niu, R. Z. Wang, D. Liao, S. P. Zhu, X. C. Zhang, B. Keshtegar, Probabilistic modeling of uncertainties in fatigue reliability analysis of turbine bladed disks, *Int. J. Fatigue*, **142** (2021), 105912. https://doi.org/10.1016/j.ijfatigue.2020.105912
56. M. Valdenegro-Toro, D. S. Mori, A deeper look into aleatoric and epistemic uncertainty disentanglement, *2022 IEEE/CVF Conference on Computer Vision and Pattern Recognition Workshops (CVPRW)*, New Orleans, LA, USA, 2022, 1508–1516. https://doi.org/10.1109/cvprw56347.2022.00157
57. C. Wang, H. G. Matthies, M. H. Xu, Y. L. Li, Dual interval-and-fuzzy analysis method for temperature prediction with hybrid epistemic uncertainties via polynomial chaos expansion, *Comput. Meth. Appl. Mech. Eng.*, **336** (2018), 171–186. https://doi.org/10.1016/j.cma.2018.03.013
58. A. D. Kiureghian, O. Ditlevsen, Aleatory or epistemic? Does it matter?, *Struct. Saf.*, **31** (2009), 105–112. https://doi.org/10.1016/j.strusafe.2008.06.020
59. M. E. Paté-Cornell, Uncertainties in risk analysis: Six levels of treatment, *Reliab. Eng. Syst. Saf.*, **54** (1996), 95–111. https://doi.org/10.1016/s0951-8320(96)00067-1
60. C. Wang, H. R. Fan, X. Qiang, A review of uncertainty-based multidisciplinary design optimization methods based on intelligent strategies, *Symmetry-Basel*, **15** (2023), 1875. https://doi.org/10.3390/sym15101875
61. C. Wang, X. Qiang, M. H. Xu, T. Wu, Recent advances in surrogate modeling methods for uncertainty quantification and propagation, *Symmetry-Basel*, **14** (2022), 1219. https://doi.org/10.3390/sym14061219
62. D. Di Francesco, M. Girolami, A. B. Duncan, M. Chryssanthopoulos, A probabilistic model for quantifying uncertainty in the failure assessment diagram, *Struct. Saf.*, **99** (2022), 102262. https://doi.org/10.1016/j.strusafe.2022.102262
63. P. Manfredi, Probabilistic uncertainty quantification of microwave circuits using Gaussian processes, *IEEE Trans. Microw. Theory Tech.*, **71** (2023), 2360–2372. https://doi.org/10.1109/TMTT.2022.3228953
64. J. S. Wu, G. E. Apostolakis, D. Okrent, Uncertainties in system analysis: Probabilistic versus nonprobabilistic theories, *Reliab. Eng. Syst. Saf.*, **30** (1990), 163–181. https://doi.org/10.1016/0951-8320(90)90093-3
65. B. Hu, Q. M. Gong, Y. Q. Zhang, Y. H. Yin, W. J. Chen, Characterizing uncertainty in geotechnical design of energy piles based on Bayesian theorem, *Acta Geotech.*, **17** (2022), 4191–4206. https://doi.org/10.1007/s11440-022-01535-3
66. K. Yao, J. Gao, Law of large numbers for uncertain random variables, *IEEE Trans. Fuzzy Syst.*, **24** (2016), 615–621. https://doi.org/10.1109/TFUZZ.2015.2466080
67. C. Zhang, Q. Liu, B. Zhou, C. Y. Chung, J. Li, L. Zhu, et al., A central limit theorem-based method for dc and ac power flow analysis under interval uncertainty of renewable power generation, *IEEE Trans. Sustain. Energy*, **14** (2023), 563–575. https://doi.org/10.1109/TSTE.2022.3220567

68. C. Wang, Z. K. Song, H. R. Fan, Novel evidence theory-based reliability analysis of functionally graded plate considering thermal stress behavior, *Aerosp. Sci. Technol.*, **146** (2024), 108936. https://doi.org/10.1016/j.ast.2024.108936
69. C. Wang, Evidence-theory-based uncertain parameter identification method for mechanical systems with imprecise information, *Comput. Meth. Appl. Mech. Eng.*, **351** (2019), 281–296. https://doi.org/10.1016/j.cma.2019.03.048
70. F. Arévalo, M. P. C. Alison, M. T. Ibrahim, A. Schwung, Adaptive information fusion using evidence theory and uncertainty quantification, *IEEE Access*, **12** (2024), 2236–2259. https://doi.org/10.1109/ACCESS.2023.3348270
71. H. R. Bae, R. V. Grandhi, R. A. Canfield, Uncertainty quantification of structural response using evidence theory, *AIAA J.*, **41** (2003), 2062–2068. https://doi.org/10.2514/2.1898
72. Y. He, M. Mirzargar, R. M. Kirby, Mixed aleatory and epistemic uncertainty quantification using fuzzy set theory, *Int. J. Approx. Reasoning*, **66** (2015), 1–15. https://doi.org/10.1016/j.ijar.2015.07.002
73. C. Wang, H. G. Matthies, Hybrid evidence-and-fuzzy uncertainty propagation under a dual-level analysis framework, *Fuzzy Sets Syst.*, **367** (2019), 51–67. https://doi.org/10.1016/j.fss.2018.10.002
74. R. M. Rodríguez, L. Martínez, V. Torra, Z. S. Xu, F. Herrera, Hesitant fuzzy sets: State of the art and future directions, *Int. J. Intell. Syst.*, **29** (2014), 495–524. https://doi.org/10.1002/int.21654
75. S. H. Khairuddin, M. H. Hasan, M. A. Hashmani, M. H. Azam, Generating clustering-based interval fuzzy type-2 triangular and trapezoidal membership functions: A structured literature review, *Symmetry-Basel*, **13** (2021), 239. https://doi.org/10.3390/sym13020239
76. C. Wang, H. R. Fan, T. Wu, Novel rough set theory-based method for epistemic uncertainty modeling, analysis and applications, *Appl. Math. Model.*, **113** (2023), 456–474. https://doi.org/10.1016/j.apm.2022.09.002
77. X. Y. Zhang, Y. Y. Yao, Tri-level attribute reduction in rough set theory, *Expert Syst. Appl.*, **190** (2022), 116187. https://doi.org/10.1016/j.eswa.2021.116187
78. F. Y. Li, Z. Luo, G. Y. Sun, N. Zhang, An uncertain multidisciplinary design optimization method using interval convex models, *Eng. Optimiz.*, **45** (2013), 697–718. https://doi.org/10.1080/0305215x.2012.690871
79. H. Lü, K. Yang, X. T. Huang, W.-B. Shangguan, K. G. Zhao, Uncertainty and correlation propagation analysis of powertrain mounting systems based on multi-ellipsoid convex model, *Mech. Syst. Signal Proc.*, **173** (2022), 109058. https://doi.org/10.1016/j.ymssp.2022.109058
80. X. Qiang, C. Wang, H. R. Fan, Hybrid interval model for uncertainty analysis of imprecise or conflicting information, *Appl. Math. Model.*, **129** (2024), 837–856. https://doi.org/10.1016/j.apm.2024.02.014
81. C. Wang, X. Qiang, H. R. Fan, T. Wu, Y. L. Chen, Novel data-driven method for non-probabilistic uncertainty analysis of engineering structures based on ellipsoid model, *Comput. Meth. Appl. Mech. Eng.*, **394** (2022), 114889. https://doi.org/10.1016/j.cma.2022.114889
82. C. Wang, H. G. Matthies, A modified parallelepiped model for non-probabilistic uncertainty quantification and propagation analysis, *Comput. Meth. Appl. Mech. Eng.*, **369** (2020), 113209. https://doi.org/10.1016/j.cma.2020.113209

83. C. Wang, L. Hong, X. Qiang, M. H. Xu, Novel numerical method for uncertainty analysis of coupled vibro-acoustic problem considering thermal stress, *Comput. Meth. Appl. Mech. Eng.*, **420** (2024), 116727. https://doi.org/10.1016/j.cma.2023.116727
84. L. X. Cao, J. Liu, L. Xie, C. Jiang, R. G. Bi, Non-probabilistic polygonal convex set model for structural uncertainty quantification, *Analog Integr. Circuits Process.*, **89** (2021), 504–518. https://doi.org/10.1016/j.apm.2020.07.025
85. L. P. Zhu, I. Elishakoff, J. H. Starnes, Derivation of multi-dimensional ellipsoidal convex model for experimental data, *Math. Comput. Model.*, **24** (1996), 103–114. https://doi.org/10.1016/0895-7177(96)00094-5
86. C. Jiang, X. Han, G. Y. Lu, J. Liu, Z. Zhang, Y. C. Bai, Correlation analysis of non-probabilistic convex model and corresponding structural reliability technique, *Comput. Meth. Appl. Mech. Eng.*, **200** (2011), 2528–2546. https://doi.org/10.1016/j.cma.2011.04.007
87. J. Liu, Z. B. Yu, D. Q. Zhang, H. Liu, X. Han, Multimodal ellipsoid model for non-probabilistic structural uncertainty quantification and propagation, *Int. J. Mech. Mater. Des.*, **17** (2021), 633–657. https://doi.org/10.1007/s10999-021-09551-z
88. Z. Kang, W. B. Zhang, Construction and application of an ellipsoidal convex model using a semi-definite programming formulation from measured data, *Comput. Meth. Appl. Mech. Eng.*, **300** (2016), 461–489. https://doi.org/10.1016/j.cma.2015.11.025
89. L. Wang, J. X. Liu, Dynamic uncertainty quantification and risk prediction based on the grey mathematics and outcrossing theory, *Appl. Sci.-Basel*, **12** (2022), 5389. https://doi.org/10.3390/app12115389
90. Y. H. Yan, X. J. Wang, Y. L. Li, Non-probabilistic credible set model for structural uncertainty quantification, *Structures*, **53** (2023), 1408–1424. https://doi.org/10.1016/j.istruc.2023.05.011
91. T. Zhang, J. Y. Jiao, J. Lin, H. Li, J. D. Hua, D. He, Uncertainty-based contrastive prototype-matching network towards cross-domain fault diagnosis with small data, *Knowledge-Based Syst.*, **254** (2022), 109651. https://doi.org/10.1016/j.knosys.2022.109651
92. J. Chen, D. Zhou, Z. Guo, J. Lin, C. Lyu, C. Lu, An active learning method based on uncertainty and complexity for gearbox fault diagnosis, *IEEE Access*, **7** (2019), 9022–9031. https://doi.org/10.1109/ACCESS.2019.2890979
93. H. Ma, C. Ekanayake, T. K. Saha, Power transformer fault diagnosis under measurement originated uncertainties, *IEEE Trns. Dielectr. Electr. Insul.*, **19** (2012), 1982–1990. https://doi.org/10.1109/tdei.2012.6396956
94. X. J. Shi, H. B. Gu, B. Yao, Fuzzy Bayesian network fault diagnosis method based on fault tree for coal mine drainage system, *IEEE Sens. J.*, **24** (2024), 7537–7547. https://doi.org/10.1109/jsen.2024.3354415
95. R. X. Duan, Y. N. Lin, Y. N. Zeng, Fault diagnosis for complex systems based on reliability analysis and sensors data considering epistemic uncertainty, *Eksploat. Niezawodn.*, **20** (2018), 558–566. https://doi.org/10.17531/ein.2018.4.7
96. J. Wang, H. Peng, W. P. Yu, J. Ming, M. J. Pérez-Jiménez, C. Y. Tao, et al., Interval-valued fuzzy spiking neural p systems for fault diagnosis of power transmission networks, *Eng. Appl. Artif. Intell.*, **82** (2019), 102–109. https://doi.org/10.1016/j.engappai.2019.03.014
97. A. Hoballah, D. E. A. Mansour, I. B. M. Taha, Hybrid grey wolf optimizer for transformer fault diagnosis using dissolved gases considering uncertainty in measurements, *IEEE Access*, **8** (2020), 139176–139187. https://doi.org/10.1109/access.2020.3012633

98. K. Zhou, J. Tang, Probabilistic gear fault diagnosis using Bayesian convolutional neural network, *IFAC-PapersOnLine*, **55** (2022), 795–799. https://doi.org/10.1016/j.ifacol.2022.11.279
99. H. T. Zhou, W. H. Chen, L. S. Cheng, J. Liu, M. Xia, Trustworthy fault diagnosis with uncertainty estimation through evidential convolutional neural networks, *IEEE Trans. Ind. Inform.*, **19** (2023), 10842–10852. https://doi.org/10.1109/TII.2023.3241587
100. S. Huang, R. Duan, J. He, T. Feng, Y. Zeng, Fault diagnosis strategy for complex systems based on multi-source heterogeneous information under epistemic uncertainty, *IEEE Access*, **8** (2020), 50921–50933. https://doi.org/10.1109/ACCESS.2020.2980397
101. S. X. Liu, S. Y. Zhou, B. Y. Li, Z. H. Niu, M. Abdullah, R. R. Wang, Servo torque fault diagnosis implementation for heavy-legged robots using insufficient information, *ISA Transactions*, **147** (2024), 439–452. https://doi.org/10.1016/j.isatra.2024.02.004
102. T. Zhang, S. He, J. Chen, T. Pan, Z. Zhou, Toward small sample challenge in intelligent fault diagnosis: Attention-weighted multidepth feature fusion net with signals augmentation, *IEEE Trans. Instrum. Meas.*, **71** (2022), 1–13. https://doi.org/10.1109/TIM.2021.3134999
103. A. Kulkarni, J. Terpenny, V. Prabhu, Sensor selection framework for designing fault diagnostics system, *Sensors*, **21** (2021), 6470. https://doi.org/10.3390/s21196470
104. C. Herrojo, F. Paredes, J. Mata-Contreras, F. Martín, Chipless-rfid: A review and recent developments, *Sensors*, **19** (2019), 3385. https://doi.org/10.3390/s19153385
105. T. Kalsoom, N. Ramzan, S. Ahmed, M. Ur-Rehman, Advances in sensor technologies in the era of smart factory and industry 4.0, *Sensors*, **20** (2020), 6783. https://doi.org/10.3390/s20236783
106. A. Leal, J. Casas, C. Marques, M. J. Pontes, A. Frizera, Application of additive layer manufacturing technique on the development of high sensitive fiber bragg grating temperature sensors, *Sensors*, **18** (2018), 4120. https://doi.org/10.3390/s18124120
107. G. D. Lewis, P. Merken, M. Vandewal, Enhanced accuracy of cmos smart temperature sensors by nonlinear curvature correction, *Sensors*, **18** (2018), 4087. https://doi.org/10.3390/s18124087
108. H. Landaluce, L. Arjona, A. Perallos, F. Falcone, I. Angulo, F. Muralter, A review of iot sensing applications and challenges using rfid and wireless sensor networks, *Sensors*, **20** (2020), 2495. https://doi.org/10.3390/s20092495
109. S. L. Wei, W. B. Qin, L. W. Han, F. Y. Cheng, The research on compensation algorithm of infrared temperature measurement based on intelligent sensors, *Cluster Comput.*, **22** (2019), 6091–6100. https://doi.org/10.1007/s10586-018-1828-5
110. M. Tessarolo, L. Possanzini, E. G. Campari, R. Bonfiglioli, F. S. Violante, A. Bonfiglio, et al., Adaptable pressure textile sensors based on a conductive polymer, *Flex. Print. Electron.*, **3** (2018), 034001. https://doi.org/10.1088/2058-8585/aacbee
111. K. A. Mathias, S. M. Kulkarni, Investigation on influence of geometry on performance of a cavity-less pressure sensor, *IOP Conf. Ser.: Mater. Sci. Eng.*, **417** (2018), 012035. https://doi.org/10.1088/1757-899x/417/1/012035
112. W. P. Eaton, J. H. Smith, Micromachined pressure sensors: Review and recent developments, *Smart Mater. Struct.*, **6** (1997), 30–41. https://doi.org/10.1117/12.276606
113. M. Mousavi, M. Alzgool, S. Towfighian, A mems pressure sensor using electrostatic levitation, *IEEE Sens. J.*, **21** (2021), 18601–18608. https://doi.org/10.1109/JSEN.2021.3091665
114. A. P. Cherkun, G. V. Mishakov, A. V. Sharkov, E. I. Demikhov, The use of a piezoelectric force sensor in the magnetic force microscopy of thin permalloy films, *Ultramicroscopy*, **217** (2020), 113072. https://doi.org/10.1016/j.ultramic.2020.113072

115. A. Nastro, M. Ferrari, V. Ferrari, Double-actuator position-feedback mechanism for adjustable sensitivity in electrostatic-capacitive mems force sensors, *Sens. Actuator A-Phys.*, **312** (2020), 112127. https://doi.org/10.1016/j.sna.2020.112127
116. M. L. Gödecke, C. M. Bett, D. Buchta, K. Frenner, W. Osten, Optical sensor design for fast and process-robust position measurements on small diffraction gratings, *Opt. Lasers Eng.*, **134** (2020), 106267. https://doi.org/10.1016/j.optlaseng.2020.106267
117. Y. J. Chan, A. R. Carr, S. Charkhabi, M. Furnish, A. M. Beierle, N. F. Reuel, Wireless position sensing and normalization of embedded resonant sensors using a resonator array, *Sens. Actuator A-Phys.*, **303** (2020), 111853. https://doi.org/10.1016/j.sna.2020.111853
118. J. A. Kim, J. W. Kim, C. S. Kang, J. Y. Lee, J. Jin, On-machine calibration of angular position and runout of a precision rotation stage using two absolute position sensors, *Measurement*, **153** (2020), 107399. https://doi.org/10.1016/j.measurement.2019.107399
119. L. E. Helseth, On the accuracy of an interdigital electrostatic position sensor, *J. Electrost.*, **107** (2020), 103480. https://doi.org/10.1016/j.elstat.2020.103480
120. K. Palmer, H. Kratz, H. Nguyen, G. Thornell, A highly integratable silicon thermal gas flow sensor, *J. Micromech. Microeng.*, **22** (2012), 065015. https://doi.org/10.1088/0960-1317/22/6/065015
121. A. Moreno-Gomez, C. A. Perez-Ramirez, A. Dominguez-Gonzalez, M. Valtierra-Rodriguez, O. Chavez-Alegria, J. P. Amezquita-Sanchez, Sensors used in structural health monitoring, *Arch. Comput. Method Eng.*, **25** (2018), 901–918. https://doi.org/10.1007/s11831-017-9217-4
122. A. M. Shkel, Smart mems: Micro-structures with error-suppression and self-calibration control capabilities, *Proceedings of the 2001 American Control Conference*, Arlington, VA, USA, 2001, 1208–1213. https://doi.org/10.1109/ACC.2001.945886
123. X. Insausti, M. Zárraga-Rodríguez, C. Nolasco-Ferencikova, J. Gutiérrez-Gutiérrez, In-network algorithm for passive sensors in structural health monitoring, *IEEE Signal Process. Lett.*, **30** (2023), 952–956. https://doi.org/10.1109/lsp.2023.3298279
124. B. Jeon, J. S. Yoon, J. Um, S. H. Suh, The architecture development of industry 4.0 compliant smart machine tool system (smts), *J. Intell. Manuf.*, **31** (2020), 1837–1859. https://doi.org/10.1007/s10845-020-01539-4
125. M. H. Zhu, J. Li, W. B. Wang, D. P. Chen, Self-detection and self-diagnosis methods for sensors in intelligent integrated sensing system, *IEEE Sens. J.*, **21** (2021), 19247–19254. https://doi.org/10.1109/JSEN.2021.3090990
126. J. Chen, P. Li, G. B. Song, Z. Ren, Y. Tan, Y. J. Zheng, Feedback control for structural health monitoring in a smart aggregate based sensor network, *Int. J. Struct. Stab. Dyn.*, **18** (2017), 1850064. https://doi.org/10.1142/S0219455418500645
127. C. Wang, Z. M. Peng, R. Liu, C. Chen, Research on multi-fault diagnosis method based on time domain features of vibration signals, *Sensors*, **22** (2022), 8164. https://doi.org/10.3390/s22218164
128. Z. F. Du, R. H. Zhang, H. Chen, Characteristic signal extracted from a continuous time signal on the aspect of frequency domain, *Chin. Phys. B*, **28** (2019), 090502. https://doi.org/10.1088/1674-1056/ab344a
129. Y. Lu, J. Tang, On time-frequency domain feature extraction of wave signals for structural health monitoring, *Measurement*, **114** (2018), 51–59. https://doi.org/10.1016/j.measurement.2017.09.016

130. M. Imani, Modified pca, lda and lpp feature extraction methods for polsar image classification, *Multimed. Tools Appl.*, **83** (2024), 41171–41192. https://doi.org/10.1007/s11042-023-17269-7
131. Z. Xia, Y. Chen, C. Xu, Multiview pca: A methodology of feature extraction and dimension reduction for high-order data, *IEEE T. Cybern.*, **52** (2022), 11068–11080. https://doi.org/10.1109/TCYB.2021.3106485
132. Y. Aliyari Ghassabeh, F. Rudzicz, H. A. Moghaddam, Fast incremental lda feature extraction, *Pattern Recognit.*, **48** (2015), 1999–2012. https://doi.org/10.1016/j.patcog.2014.12.012
133. E. Parsaeimehr, M. Fartash, J. A. Torkestani, Improving feature extraction using a hybrid of cnn and lstm for entity identification, *Neural Process. Lett.*, **55** (2023), 5979–5994. https://doi.org/10.1007/s11063-022-11122-y
134. P. Wang, X. M. Zhang, Y. Hao, A method combining cnn and elm for feature extraction and classification of sar image, *J. Sens.*, **2019** (2019), 6134610. https://doi.org/10.1155/2019/6134610
135. O. İrsoy, E. Alpaydın, Unsupervised feature extraction with autoencoder trees, *Neurocomputing*, **258** (2017), 63–73. https://doi.org/10.1016/j.neucom.2017.02.075
136. Y. Y. Wang, D. J. Song, W. T. Wang, S. X. Rao, X. Y. Wang, M. N. Wang, Self-supervised learning and semi-supervised learning for multi-sequence medical image classification, *Neurocomputing*, **513** (2022), 383–394. https://doi.org/10.1016/j.neucom.2022.09.097
137. W. X. Sun, J. Chen, J. Q. Li, Decision tree and pca-based fault diagnosis of rotating machinery, *Mech. Syst. Signal Proc.*, **21** (2007), 1300–1317. https://doi.org/10.1016/j.ymssp.2006.06.010
138. N. R. Sakthivel, V. Sugumaran, S. Babudevasenapati, Vibration based fault diagnosis of monoblock centrifugal pump using decision tree, *Expert Syst. Appl.*, **37** (2010), 4040–4049. https://doi.org/10.1016/j.eswa.2009.10.002
139. Y. Y. Li, L. Y. Song, Q. C. Sun, H. Xu, X. G. Li, Z. J. Fang, et al., Rolling bearing fault diagnosis based on quantum ls-svm, *EPJ Quantum Technol.*, **9** (2022), 18. https://doi.org/10.1140/epjqt/s40507-022-00137-y
140. A. H. Zhang, D. L. Yu, Z. Q. Zhang, Tlsca-svm fault diagnosis optimization method based on transfer learning, *Processes*, **10** (2022), 362. https://doi.org/10.3390/pr10020362
141. T. Huang, Q. Zhang, X. A. Tang, S. Y. Zhao, X. N. Lu, A novel fault diagnosis method based on cnn and lstm and its application in fault diagnosis for complex systems, *Artif. Intell. Rev.*, **55** (2022), 1289–1315. https://doi.org/10.1007/s10462-021-09993-z
142. H. Fang, H. Liu, X. Wang, J. Deng, J. An, The method based on clustering for unknown failure diagnosis of rolling bearings, *IEEE Trans. Instrum. Meas.*, **72** (2023), 1–8. https://doi.org/10.1109/TIM.2023.3251406
143. A. Rodríguez-Ramos, A. J. da Silva Neto, O. Llanes-Santiago, An approach to fault diagnosis with online detection of novel faults using fuzzy clustering tools, *Expert Syst. Appl.*, **113** (2018), 200–212. https://doi.org/10.1016/j.eswa.2018.06.055
144. L. K. Chang, S. H. Wang, M. C. Tsai, Demagnetization fault diagnosis of a pmsm using auto-encoder and k-means clustering, *Energies*, **13** (2020), 4467. https://doi.org/10.3390/en13174467
145. J. Du, S. P. Wang, H. Y. Zhang, Layered clustering multi-fault diagnosis for hydraulic piston pump, *Mech. Syst. Signal Proc.*, **36** (2013), 487–504. https://doi.org/10.1016/j.ymssp.2012.10.020

146. Y. Y. Li, J. D. Wang, H. Y. Zhao, C. Wang, Q. Shao, Adaptive dbscan clustering and gasa optimization for underdetermined mixing matrix estimation in fault diagnosis of reciprocating compressors, *Sensors*, **24** (2024), 167. https://doi.org/10.3390/s24010167
147. C. X. Jian, K. J. Yang, Y. H. Ao, Industrial fault diagnosis based on active learning and semi-supervised learning using small training set, *Eng. Appl. Artif. Intell.*, **104** (2021), 104365. https://doi.org/10.1016/j.engappai.2021.104365
148. S. Zheng, J. Zhao, A self-adaptive temporal-spatial self-training algorithm for semisupervised fault diagnosis of industrial processes, *IEEE Trans. Ind. Inform.*, **18** (2022), 6700–6711. https://doi.org/10.1109/TII.2021.3120686
149. J. Y. Long, Y. B. Chen, Z. Yang, Y. W. Huang, C. Li, A novel self-training semi-supervised deep learning approach for machinery fault diagnosis, *Int. J. Prod. Res.*, **61** (2023), 8238–8251. https://doi.org/10.1080/00207543.2022.2032860
150. K. Yu, H. Z. Han, Q. Fu, H. Ma, J. Zeng, Symmetric co-training based unsupervised domain adaptation approach for intelligent fault diagnosis of rolling bearing, *Meas. Sci. Technol.*, **31** (2020), 115008. https://doi.org/10.1088/1361-6501/ab9841
151. L. Wang, D. F. Zhou, H. Tian, H. Zhang, W. Zhang, Parametric fault diagnosis of analog circuits based on a semi-supervised algorithm, *Symmetry-Basel*, **11** (2019), 228. https://doi.org/10.3390/sym11020228
152. C. X. Jian, Y. H. Ao, Imbalanced fault diagnosis based on semi-supervised ensemble learning, *J. Intell. Manuf.*, **34** (2023), 3143–3158. https://doi.org/10.1007/s10845-022-01985-2
153. X. Li, F. L. Zhang, Classification of multi-type bearing fault features based on semi-supervised generative adversarial network (gan), *Meas. Sci. Technol.*, **35** (2024), 025107. https://doi.org/10.1088/1361-6501/ad068e
154. L. Wang, H. Tian, H. Zhang, Soft fault diagnosis of analog circuits based on semi-supervised support vector machine, *Analog Integr. Circuits Process.*, **108** (2021), 305–315. https://doi.org/10.1007/s10470-021-01851-w
155. P. Xu, L. X. Fu, K. Xu, W. B. Sun, Q. Tan, Y. P. Zhang, et al., Investigation into maize seed disease identification based on deep learning and multi-source spectral information fusion techniques, *J. Food Compos. Anal.*, **119** (2023), 105254. https://doi.org/10.1016/j.jfca.2023.105254
156. P. F. Zhang, T. R. Li, Z. Yuan, C. Luo, G. Q. Wang, J. Liu, et al., A data-level fusion model for unsupervised attribute selection in multi-source homogeneous data, *Inf. Fusion*, **80** (2022), 87–103. https://doi.org/10.1016/j.inffus.2021.10.017
157. M. B. Song, Y. F. Zhi, M. D. An, W. Xu, G. H. Li, X. L. Wang, Centrifugal pump cavitation fault diagnosis based on feature-level multi-source information fusion, *Processes*, **12** (2024), 196. https://doi.org/10.3390/pr12010196
158. L. L. Liu, X. Wan, J. Y. Li, W. X. Wang, Z. G. Gao, An improved entropy-weighted topsis method for decision-level fusion evaluation system of multi-source data, *Sensors*, **22** (2022), 6391. https://doi.org/10.3390/s22176391
159. Y. W. Liu, Y. Q. Cheng, Z. Z. Zhang, J. J. Wu, Multi-information fusion fault diagnosis based on knn and improved evidence theory, *J. Vib. Eng. Technol.*, **10** (2022), 841–852. https://doi.org/10.1007/s42417-021-00413-8

160. J. Xu, Y. Sui, T. Dai, A Bayesian network inference approach for dynamic risk assessment using multisource-based information fusion in an interval type-2 fuzzy set environment, *IEEE Trans. Fuzzy Syst.*, **32** (2024), 5702–5713. https://doi.org/10.1109/TFUZZ.2024.3425495
161. Y. C. Jie, Y. Chen, X. S. Li, P. Yi, H. S. Tan, X. Q. Cheng, Fufusion: Fuzzy sets theory for infrared and visible image fusion, In: *Pattern recognition and computer vision*, Singapore: Springer, 2024, 466–478. https://doi.org/10.1007/978-981-99-8432-9_37
162. F. Y. Xiao, Multi-sensor data fusion based on the belief divergence measure of evidences and the belief entropy, *Inf. Fusion*, **46** (2019), 23–32. https://doi.org/10.1016/j.inffus.2018.04.003
163. G. Koliander, Y. El-Laham, P. M. Djuric, F. Hlawatsch, Fusion of probability density functions, *Proceedings of the IEEE*, **110** (2022), 404–453. https://doi.org/10.1109/jproc.2022.3154399
164. Y. J. Pan, R. Q. An, D. Z. Fu, Z. Y. Zheng, Z. H. Yang, Unsupervised fault detection with a decision fusion method based on Bayesian in the pumping unit, *IEEE Sens. J.*, **21** (2021), 21829–21838. https://doi.org/10.1109/jsen.2021.3103520
165. K. V. Kumar, A. Sathish, Medical image fusion based on type-2 fuzzy sets with teaching learning based optimization, *Multimed. Tools Appl.*, **83** (2024), 33235–33262. https://doi.org/10.1007/s11042-023-16859-9
166. P. F. Zhang, T. R. Li, G. Q. Wang, C. Luo, H. M. Chen, J. B. Zhang, et al., Multi-source information fusion based on rough set theory: A review, *Inf. Fusion*, **68** (2021), 85–117. https://doi.org/10.1016/j.inffus.2020.11.004
167. Y. S. Wang, M. Y. He, L. Sun, D. Wu, Y. Wang, X. L. Qing, Weighted adaptive kalman filtering-based diverse information fusion for hole edge crack monitoring, *Mech. Syst. Signal Proc.*, **167** (2022), 108534. https://doi.org/10.1016/j.ymssp.2021.108534
168. N. Guenther, M. Schonlau, Support vector machines, *Stata J.*, **16** (2016), 917–937. https://doi.org/10.1177/1536867x1601600407
169. P. Cunningham, S. J. Delany, K-nearest neighbour classifiers-a tutorial, *ACM Comput. Surv.*, **54** (2021), 128. https://doi.org/10.1145/3459665
170. Z. Liu, S. B. Zhong, Q. Liu, C. X. Xie, Y. Z. Dai, C. Peng, et al., Thyroid nodule recognition using a joint convolutional neural network with information fusion of ultrasound images and radiofrequency data, *Eur. Radiol.*, **31** (2021), 5001–5011. https://doi.org/10.1007/s00330-020-07585-z
171. A. Y. Chen, F. Wang, W. H. Liu, S. Chang, H. Wang, J. He, et al., Multi-information fusion neural networks for arrhythmia automatic detection, *Comput. Meth. Programs Biomed.*, **193** (2020), 105479. https://doi.org/10.1016/j.cmpb.2020.105479