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Research article

Fuzzy tracking control of singular multi-agent systems under switching topology

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Abstract: The consensus tracking problem of leader-follower multi-agent systems (MASs) with singular structures on jointly connected topology is studied in this paper. To achieve the objective of consensus tracking, a distributed adaptive control protocol is formulated to adjust the coupling weights among the agents using the adaptive rate, where the adaptive protocol can be implemented by each agent in a fully distributed manner without using any global information. A fuzzy logic system method is used to deal with the nonlinear terms in response to the limitations of nonlinear system analysis. The consensus tracking problem is transformed into an error system stability analysis, and two sufficient conditions are provided to guarantee the control objective based on Lyapunov stability theory and singular system theory. Finally, the effectiveness of this method is verified through a simulation example.

Keywords: consensus tracking; singular multi-agent systems; swiching topology; fuzzy logic system
Mathematics Subject Classification: 93D50, 05Cxx

1. Introduction

The consensus of MASs has attracted widespread attention in the last few years [1, 2], among which the leader-follower consensus problem is to design corresponding control protocols through the interaction information between agents so that multiple parameters of state, such as the speed and position of all followers, can be tracked to the leader multi-agent. Its advantage is that it only requires specifying the leader's motion and designing tracking control strategies, which are small in workload and easy to implement [3, 4].

In [5], in an undirected graph, as agents move in the plane, consensus can be achieved if the two-way interaction between agents is frequent enough. [6] emphasizes that the balanced directed graph is a key factor in solving the average consensus problem and discusses the consensus of multi-agent networks under fixed and switched topologies. Reference [7] builds upon the findings of [5], extending the discussion to directed graphs. It delineates the requirement for information consensus in dynamically changing interactive topological structures. It emphasizes the significance of the topological structure of interactions among agents (including directed spanning trees) in achieving asymptotic consensus. In the study [8], the scenario where the graph maintains frequent connectivity is expanded to a joint connected topology, and it achieves linear leader-follower consensus by using Riccati-inequality to calculate the feedback gain matrix. Some less restrictive conditions for the leader-follower consensus (LFC) problem in MASs under switching topology were obtained in [9], which not only proved that weak connections can achieve consensus but also extended to the case of disconnection, ensuring a short disconnection time.

Given that a singular system is a class of dynamical systems described by differential-algebraic or difference-algebraic equations, the system is hierarchical. It encapsulates both the dynamic properties of entities, articulated through differential or difference equations, and the static characteristics of constraints, delineated by algebraic equations [10]. Especially in multi-agent systems, there are indeed constraints between certain physical quantities characterized by algebraic equations. After more than 40 years of development, research on singular systems has made great progress and gradually evolved into one of the most important branches of contemporary control theory. A multitude of theoretical conclusions derived from general systems have been successfully extrapolated and applied to the domain of singular systems [11–14]. Considering that the state response of a singular system does not only include exponential solutions, it perhaps leads to impulsive behavior. When generalizing the results of general systems to singular systems, the control protocol needs to ensure that singular systems are regular and impulse-free [15]. Therefore, delving deeper into the consensus and associated characteristics of singular multi-agent systems (SMASs) holds profound significance in the academic and applied spheres.

Yang et al. [16] studied the consensus of continuous linear SMASs and first proposed the concept of SMASs. From then on, the research on the consensus of SMASs entered the public eye. The LFC problem for a class of SMASs was studied in [17]. The system in question incorporates nonlinear dynamical behaviors and is characterized by a topology that is represented by a signed directed graph. [18] added interference on the basis of [16] and studied the consensus tracking control problem of SMASs with Lipschitz nonlinearity. The research presented in [19] focused on the guaranteed cost consensus issue for high-order SMASs with switching topology. [20] extended the switching topology requirement in [19] that the topology transitions from connected graphs to jointly connected graphs.

The adaptive control can automatically adjust the parameters or structure of the controller to adapt to constantly changing working conditions, thereby achieving more stable and reliable control effects. Therefore, the development of adaptive technologies capable of autonomously adjusting to fluctuations in system performance has emerged as a significant research area [21, 22]. The consensus problem in SMASs usually requires distributed control strategies, and adaptive control can quickly respond to changes in system state and adjust control inputs based on the local information of each agent to achieve global consensus. Thus, the use of adaptive control to study singular multi-agent systems is of great significance, which is a factor that prompted this study.

Unlike [17, 23, 24], which necessitate nonlinear functions to adhere to the Lipschitz condition, for unknown nonlinear terms, intelligent modeling methods such as fuzzy logic systems are employed to overcome the challenge of unknown nonlinearity in the system [25–28]. Although fuzzy methods were used in the above studies, they were mostly applied in general multi-agent systems. There are not many studies on the application of fuzzy control schemes to the consensus of SMASs, which is another factor that prompted this study.

The main contributions of this manuscript are delineated as follows: Considering the communication topology is a switching joint connected topology with non-connected graphs, which means that even when there is a short-term communication failure between some agents, the system can still operate normally and achieve consensus. A distributed adaptive control protocol is designed using local information to study the consensus and impulse-free of a leader-follower SMAS. In nonlinear analysis, fuzzy logic systems are introduced to approximate these unknown nonlinear terms. By transforming the consensus of SMASs into an asymptotic stability analysis of the associated error dynamics, this study ensures the stability of the closed-loop system.

The subsequent structure of this paper is meticulously organized as follows: Section 2 lays the groundwork with preliminaries and the problem formulation. The main results are articulated in Section 3. Section 4 is dedicated to illustrating the simulation results. Finally, Section 5 offers a brief conclusion.

Notations: $\mathbb{R}^{m\times n}$ represents the set of $m\times n$ dimensional matrices over the real number set $\mathbb{R}$. $\mathbb{C}$ denotes the field of complex numbers, while $\mathbb{C}^-$ signifies the open left-half plane of the complex domain. I_n is utilized to represent an identity matrix of dimension $n \times n$. The superscript T indicates the transpose of a real-valued matrix. For a matrix U, the notation $U > 0$ (< 0) is used to express that U is positive (negative) definite. The symbol $\sigma(U)$ denotes the non-zero singular values of matrix U. U^* refers to the Hermitian transpose of matrix U. The Kronecker product of matrices P and Q is represented by $P \otimes Q$. The function $\deg(\cdot)$ is used to describe the degree of a polynomial. $\operatorname{diag}(c_1, \cdots, c_n)$ is used to define a diagonal matrix with diagonal entries c_i, $i = 1, 2, \cdots, n$. The topology of SMASs is defined by the graph $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$, where $\mathcal{V} = \{1, 2, \cdots, N\}$ represents the set of nodes (agents) and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ represents the set of edges (communication links). The adjacency matrix of the graph $\mathcal{G}$ is denoted by $\mathcal{A} = \left[a_{ij}\right] \in R^{n\times n}$, which is defined such that if the pair (i, j) is an element of $\mathcal{E}$, then $a_{ij} = a_{ji} = 1$; otherwise, $a_{ij} = 0$, with the diagonal elements a_{ii} being equal to zero. The Laplace matrix $\mathcal{L}$ of $\mathcal{G}$ is characterized by the elements $l_{ij} = -a_{ij}$ for $i \neq j$ and $l_{ii} = \sum_{k=1}^{N} a_{ik}$.

2. Preliminaries and problem formulation

Consider a MAS represented by linear singular systems with N followers and one leader, and the dynamic of the follower agents is articulated by the following description:

$$E\dot{x}_i(t) = Ax_i(t) + Bu_i(t) \tag{2.1}$$

where $i = 1, 2, \cdots, N$, $x_i \in \mathbb{R}^n$ indicates the state, and $u_i \in \mathbb{R}^p$ denotes the control input of the i-th follower. The leader agent is labeled as $i = 0$, and its dynamic is represented as:

$$E\dot{x}_0(t) = Ax_0(t) \tag{2.2}$$

where x_0 represents the state of the leader, $E, A \in \mathbb{R}^{n\times n}$ are constant matrices of appropriate dimensions, with E possessing singularity and satisfying the condition that $\operatorname{rank}(E) \leq n$, and $B \in \mathbb{R}^{n\times p}$ is full-column rank.

When there are unknown continuous sector nonlinear functions $f(x_i(t)), i = 0, 1, \cdots, N$ in Eqs (2.1) and (2.2), the nonlinear SMASs are obtained:

$$E\dot{x}_0(t) = Ax_0(t) + f(x_0(t)) \tag{2.3}$$

$$E\dot{x}_i(t) = Ax_i(t) + f(x_i(t)) + Bu_i(t) \tag{2.4}$$

Remark 1. *The leader considered in SMASs in this paper is zero input dynamic, which means that the stability of the leader does not depend on control inputs, ensuring that the leader's dynamic is predetermined and providing a stable reference model for followers. Followers then follow the leader through local interactions, which helps to achieve a control structure that combines centralization and decentralization. However, it cannot solve the partial consensus problem of SMASs that may implement non-zero control actions on leaders in order to achieve certain goals in practice.*

The information exchange between SMASs is described by graphs. Firstly, an undirected graph, denoted as $\mathcal{G}$, is employed to articulate the exchange of information among a cohort of N follower agents. A leader-follower multi-agent system communication topology $\bar{\mathcal{G}} = (\bar{\mathcal{V}}, \bar{\mathcal{E}})$, $\bar{\mathcal{V}} = \{0, 1, \cdots, N\}$, consisting of the graph $\mathcal{G}$, leader vertex 0, and edges between 0 and neighboring agents. The information exchange between the follower and the leader is represented by the diagonal array $\mathcal{D} = diag\{d_1, d_2, \cdots, d_N\} \in \mathbb{R}^{n \times n}$. When $d_i = 1$, the follower possesses the capacity to access information from the leader; conversely, $d_i = 0, i = 1, 2, \cdots, N$. It is evident that the leader exclusively utilizes information pertaining to its local state. In contrast, the follower agents depend not only on their local state information but also on the information received from their neighboring agents within the network. Furthermore, it is crucial to recognize that, despite the leader not having a direct link to every agent, the follower agents are nonetheless capable of obtaining the leader's state information indirectly through the intermediary of their neighboring agents. To facilitate the subsequent proof, define the information interaction matrix $H = \mathcal{L} + \mathcal{D}$, the matrix H corresponding to the graph $\bar{\mathcal{G}}$ adheres to the conditions stipulated in the ensuing lemma.

Lemma 1. *([8])*

(1) The eigenvalues of the matrix H are non-negative.
(2) The matrix H is positive definite if and only if the graph $\bar{\mathcal{G}}$ is connected.

In the consensus analysis within leader-follower frameworks, it is imperative to extend our scrutiny beyond merely the characteristics of the Laplacian matrix $\mathcal{L}$. Consequently, ensuring that matrix H adheres to the stipulations of Lemma 1 is of paramount importance. In addition, we introduced another lemma as the cornerstone to confirm our main results.

Lemma 2. *(Barbalat's Lemma) Let $f(t)$ be a consistent continuous function, when t exceeds 0, and $f(t)$ possesses a finite limit value as well as $\dot{f}(t)$ being uniformly continuous, then* $\lim_{t\to\infty}\dot{f}(t) = 0$.

These two lemmas are conditions for ensuring LFC of SMASs, noting that the communication topology graph is time-varying, consider all possible graphs $\left\{\bar{\mathcal{G}}_s : s \in \Theta\right\}$, where Θ is the set of indexes defined on $\bar{\mathcal{V}}$. Define switching signal $\sigma : [0, +\infty) \to \Theta$ with $\bar{\mathcal{G}}_{\sigma(t)} \subseteq \left\{\bar{\mathcal{G}}_s : s \in \Theta\right\}$.

Consider an infinite sequence of time intervals $[t_r, t_{r+1}), r = 0, 1, 2, \cdots$ that are non-empty, bounded, and continuous with $t_0 = 0, t_{r+1} - t_r \leq T$ for some constant $T > 0$. Suppose that in each interval

$[t_r, t_{r+1})$, there exists a series of non-overlapping subintervals $\left[t_r{}^0, t_r{}^1\right), \cdots, \left[t_r{}^i, t_r{}^{i+1}\right), \cdots, \left[t_r{}^{z_r-1}, t_r{}^{z_r}\right)$, $t_r = t_r{}^0, t_{r+1} = t_r{}^{z_r}$ that satisfy $t_r{}^{i+1} - t_r{}^i \geq b_r, 0 \leq i \leq z_r - 1$, for some integer $z_r \geq 0$ and a constant b_k that is specified. The interaction topology is invariant in each subinterval. That is, in each subinterval the topology $\bar{\mathcal{G}}_{\sigma(t)}$ remains constant, and for the purposes of this discussion, it is henceforth referred to as $\bar{\mathcal{G}}_{r_i}$. In each time interval $[t_r, t_{r+1})$, some or all of the graphs $\bar{\mathcal{G}}_{r_i}, i = 0, 1, \cdots, z_r - 1$ may not be connected. It is only necessary to ensure that the graphs are jointly connected, as defined below:

Definition 1. *([8]) An union of graphs $\mathcal{G}_a, \mathcal{G}_b, \cdots, \mathcal{G}_c$ is a graph $\mathcal{G}_U$. The vertex and edge sets of $\mathcal{G}_U$ are unions of the vertex and edge sets of $\mathcal{G}_a, \mathcal{G}_b, \cdots, G_c$. If the union of $\mathcal{G}_a, \mathcal{G}_b, \cdots, \mathcal{G}_c$ forms a connected graph. Graphs $\{\mathcal{G}_{\sigma(s)} : s \in [t, t + T_k]\}$ are said to be jointly connected over time intervals $[t, t + T_k], T_k > 0$ if their union sets are jointly connected, as shown in Figure 1.*

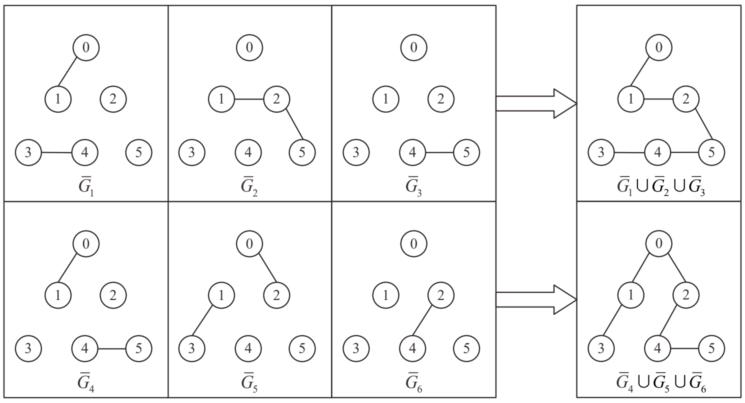

Figure 1. Jointly connected topology graph.

Important assumptions that guarantee that the leader-follower agent systems in the switching topology achieve consensus are given next.

Assumption 1. *The topology is jointly connected at each time interval $[t_i, t_{i+1}), i = 0, 1, \cdots$.*

Assumption 2. *The matrix A is devoid of eigenvalues that possess positive real parts.*

Assumption 3. *Multi-agent systems (2.1) and (2.2) are stabilizable, i.e., (E, A, B) is stabilizable.*

Building upon these assumptions, the goal of this paper is to formulate a control protocol leveraging local information, thereby enabling N follower agents to follow the leader agent. This is made clearer by the following definition.

Definition 2. *The leader-follower multi-agent systems (2.1)–(2.4) achieve consensus if each follower agent $i, i \in \{1, \cdots, N\}$ has a control protocol u_i for any initial state $x_i(0), i = 0, 1, \cdots, N$, such that the closed-loop system satisfys $\lim_{t\to\infty} \|x_i(t) - x_0(t)\| = 0, i = 1, 2, \cdots, N$.*

Remark 2. *Assumptions 1–3 are general assumptions for the consensus problem of SMASs in switching topology. Assumption 1 ensures that the communication topology is jointly connected. Under Assumption 2, multi-agent systems do not diverge at an exponential rate, and are a property instrumental in addressing the consensus issue as delineated in the reference [8, 29, 30]. Assumption 3 affirms the existence of a matrix F that satisfies $\sigma(E, A + BF) \subset \mathbb{C}^-$.*

Remark 3. *Unlike in the consensus study of leaderless SMASs where the positions of trajectories are not specified, the leader-follower consensus trajectories can be specified by the leader. The advantage of having leader consensus is that it only requires specifying the leader's movement and designing tracking control strategies, which are small in workload and easy to implement.*

3. Main results

This section consists of three parts that give state consensus results for singular linear and nonlinear MASs under the switching topology, where the nonlinear terms are handled using fuzzy rules.

3.1. Control protocol design

Consider the control protocol

$$\begin{aligned} u_i(t) &= F\left[\sum_{j=1}^{N} \omega_{ij}(t)a_{ij}(t)(x_i(t)-x_j(t)) + \omega_i(t)d_i(t)(x_i(t)-x_0(t))\right] \\ \dot{\omega}_{ij}(t) &= \psi_{ij}a_{ij}(t)(x_i(t)-x_j(t))^T\Gamma(x_i(t)-x_j(t)) \\ \dot{\omega}_i(t) &= \psi_i d_i(t)(x_i(t)-x_0(t))^T\Gamma(x_i(t)-x_0(t)) \end{aligned} \tag{3.1}$$

where $i, j = 1, \cdots, N$, $F \in \mathbb{R}^{p\times n}, \Gamma \in \mathbb{R}^{n\times n}$ are the state feedback matrices, $\psi_{ij} = \psi_{ji}$ and ψ_i are positive numbers, $\omega_{ij}(t)$ denotes the coupling weight automatically adjusted over time between the follower i and j with $\omega_{ij}(0) = \omega_{ji}(0)$; $\omega_i(t)$ denotes the coupling weight automatically adjusted over time between the leader and the follower i.

Compared to fixed topologies, communication between agents in switching topologies is dynamically adjusted and can adapt to more complex and dynamically changing environments. It is observable that the difference between switching and fixed topology is that $a_{ij}(t)$ and $d_i(t)$ both change over time.

Remark 4. *Linear systems and nonlinear systems are two important concepts in control theory. They each have different characteristics and behaviors, and they are extensively utilized across a spectrum of practical applications. For example, in the field of engineering, many systems have both linear and nonlinear parts. Therefore, considering both linear and nonlinear characteristics comprehensively is crucial for designing and optimizing complex systems. Therefore, studying singular linear and nonlinear systems is of great significance, which helps us to have a more comprehensive understanding and processing of the characteristics and behaviors of various systems.*

3.2. Adaptive consensus protocol design for linear SMASs

Theorem 1. *Consider the linear SMASs (2.1) and (2.2), under Assumptions 1–3, when* $F = -B^TP^{-1}, \Gamma = (P^{-1})^TBB^TP^{-1}$ *and with*

$$\operatorname{rank}\begin{bmatrix} E & 0 \\ A - BB^TP^{-1} & E \end{bmatrix} = n + \operatorname{rank}(E)$$

where $P > 0$ is a solution of $AX + X^T A^T - 2BB^T < 0$, then under the influence of control protocol (3.1), it is ensured that all follower agents are capable of tracking the leader, irrespective of their initial conditions, realizing the LFC of SMASs under the switching topology.

Proof. Since $\operatorname{rank}\begin{bmatrix} E & 0 \\ A - BB^T P^{-1} & E \end{bmatrix} = n + \operatorname{rank}(E)$, then one gets that $(E, A + BF)$ is regular and impulse-free. Let $\varsigma_i(t) = x_i(t) - x_0(t)$, $\varsigma(t) = \left[\varsigma_1{}^T(t), \varsigma_2{}^T(t), \cdots, \varsigma_N{}^T(t)\right]^T$. It can be deduced that

$$\begin{aligned} E\dot{\varsigma}_i(t) &= A\varsigma_i(t) + BF\left[\sum_{j=1}^{N} \omega_{ij}(t)a_{ij}(t)(\varsigma_i(t) - \varsigma_j(t))\right] + BF\omega_i(t)d_i(t)\varsigma_i(t) \\ \dot{\omega}_{ij}(t) &= \psi_{ij}a_{ij}(t)(\varsigma_i(t) - \varsigma_j(t))^T\Gamma(\varsigma_i(t) - \varsigma_j(t)) \\ \dot{\omega}_i(t) &= \psi_i d_i(t)\varsigma_i{}^T(t)\Gamma\varsigma_i(t). \end{aligned} \tag{3.2}$$

Constructing the Lyapunov function as follows:

$$V_1(t) = \sum_{i=1}^{N} \varsigma_i{}^T(t)E^T P^{-1}\varsigma_i(t) + \sum_{i=1}^{N}\sum_{j=1, j\neq i}^{N} \frac{(\omega_{ij}(t) - \chi)^2}{2\psi_{ij}} + \sum_{i=1}^{N} \frac{(\omega_i(t) - \chi)^2}{\psi_i}. \tag{3.3}$$

Here χ represents a positive constant. Clearly, $V_1(t)$ is continuously differentiable except at the switching moment.

First, it is important to show that $\dot{V}_1(t) < 0$ holds at any non-switching moment.

As $F = -B^T P^{-1}, \Gamma = (P^{-1})^T BB^T P^{-1}$ and $\omega_{ij}(t) = \omega_{ji}(t)$, let $\tilde{\delta}_i(t) = P^{-1}\varsigma_i(t)$, $\tilde{\delta}(t) = \left[\tilde{\delta}_1(t)^T, \tilde{\delta}_2(t)^T, \cdots, \tilde{\delta}_N(t)^T\right]^T$, it can be deduced that

$$\begin{aligned} \dot{V}_1(t) &= 2\sum_{i=1}^{N} \varsigma_i(t)^T(P^{-1})^T A\varsigma_i(t) - 2\chi\sum_{i=1}^{N} d_i(t)\varsigma_i(t)^T(P^{-1})^T BB^T P^{-1}\varsigma_i(t) \\ &- 2\chi\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}(t)\varsigma_i(t)^T(P^{-1})^T BB^T P^{-1}(\varsigma_i(t) - \varsigma_j(t)) \\ &= \sum_{i=1}^{N} \tilde{\delta}_i(t)^T\left(AP + P^T A^T\right)\tilde{\delta}_i(t) - 2\chi\sum_{i=1}^{N}\sum_{j=1}^{N} l_{ij}(t)\tilde{\delta}_i(t)^T BB^T\tilde{\delta}_j(t) - 2\chi\sum_{i=1}^{N} d_i(t)\tilde{\delta}_i(t)BB^T\tilde{\delta}_i(t) \\ &= \tilde{\delta}(t)\left[I_N \otimes \left(AP + P^T A^T\right) - 2\chi H_s \otimes BB^T\right]\tilde{\delta}(t). \end{aligned} \tag{3.4}$$

For any $s \in \Theta$, H_s is a symmetric matrix with eigenvalues labeled $\left\{\lambda_s{}^1, \cdots, \lambda_s{}^i, \cdots, \lambda_s{}^N\right\}$, $\lambda_s{}^i \geq 0$. It may be assumed that the number of 0 eigenvalues is $q_s (0 \leq q_s < N)$. Then a unitary matrix M_s can be found such that

$$M_s{}^T H_s M_s = \Lambda_s \triangleq \operatorname{diag}\left(0, \cdots, 0, \lambda_s{}^{q_s+1}, \cdots, \lambda_s{}^N\right) \tag{3.5}$$

holds.

Let $\hat{\delta}_i(t) = M_s{}^T\tilde{\delta}_i(t)$, it can be obtained that

$$\begin{aligned} \dot{V}_1(t) &= \hat{\delta}(t)\left[I_N \otimes \left(AP + P^T A^T\right) - 2\chi\Lambda_s \otimes BB^T\right]\hat{\delta}(t) \\ &\leq \sum_{i=q_s+1}^{N} \hat{\delta}_i(t)^T\left[AP + P^T A^T - 2\chi\lambda_s{}^i BB^T\right]\hat{\delta}_i(t). \end{aligned} \tag{3.6}$$

Choosing a sufficiently large χ such that $\chi\lambda_s{}^i \geq 1, i = q_s + 1, \cdots, N$. The following inequality follows from $AP + P^T A^T - 2BB^T < 0$.

$$AP + P^T A^T - 2\chi\lambda_s{}^i BB^T \leq AP + P^T A^T - 2BB^T < 0. \tag{3.7}$$

This yields $\dot{V}_1(t) < 0$ and hence there exists $\lim_{t\to\infty} V_1(t)$. Next, it is necessary to prove $\lim_{t\to\infty}\varsigma(t) = 0$. By the Cauchy convergence criterion, for any $\varepsilon > 0$, there exists a positive integer Ψ such that $|V_1(t_{k+1}) - V_1(t_k)| < \varepsilon$ holds, i.e., $\left|\int_{t_k}^{t_{k+1}} \dot{V}_1(t)dt\right| < \varepsilon$ for all $k \geq \Psi$.

Representing $[t_k, t_{k+1})$ as subintervals yields the following inequality of integral sum.

$$\int_{t_k{}^0}^{t_k{}^1} (-\dot{V}_1(t))dt + \int_{t_k{}^1}^{t_k{}^2} (-\dot{V}_1(t))dt + \cdots + \int_{t_k{}^{z_k-1}}^{t_k{}^{z_k}} (-\dot{V}_1(t))dt < \varepsilon. \tag{3.8}$$

As $\int_{t_k{}^i}^{t_k{}^{i+1}} (-\dot{V}_1(t))dt < \varepsilon, i = 0, 1, \cdots, z_k - 1$, that means

$$\begin{aligned}
-\varepsilon &< \int_{t_k{}^i}^{t_k{}^{i+1}} \dot{V}_1(t)dt \\
&\leq \int_{t_k{}^i}^{t_k{}^{i+1}} \sum_{i=q_s+1}^{N} \hat{\delta}_i^T(t)\left[AP + P^T A^T - 2\chi\lambda_s{}^i BB^T\right]\hat{\delta}_i(t)dt \\
&\leq \int_{t_k{}^i}^{t_k{}^i+\tau} \sum_{i=q_s+1}^{N} \hat{\delta}_i^T(t)\left[AP + P^T A^T - 2\chi\lambda_s{}^i BB^T\right]\hat{\delta}_i(t)dt.
\end{aligned} \tag{3.9}$$

Let $q_{k_i}, i = 0, 1, \cdots, z_k - 1$ represents the number of eigenvalues 0 in the corresponding matrix H_{k_i} of the topology graph $\bar{\mathcal{G}}_{k_i}$ of the interval $\left[t_k{}^i, t_k{}^{i+1}\right)$, denoted as $\Xi_s{}^i = AP + P^T A^T - 2\chi\lambda_s{}^i BB^T$. From this premise, we can derive the following conclusion:

$$\lim_{t\to\infty}\int_t^{t+\tau}\left[\sum_{i=q_{k_0}+1}^{N} \hat{\delta}_i^T(v)\Xi_s{}^i\hat{\delta}_i(v) + \cdots + \sum_{i=q_{k_{z_k-1}}+1}^{N} \hat{\delta}_i^T(v)\Xi_s{}^i\hat{\delta}_i(v)\right]dv = 0. \tag{3.10}$$

According to Lemma 1, when the communication topology between agents in the interval $[t_k, t_{k+1})$ satisfies jointly connected, it can be obtained that $q_{\sigma(t_k)} = 0$ and

$$\lim_{t\to\infty}\int_t^{t+\tau}\sum_{i=1}^{N}\pi_i\hat{\delta}_i^T(v)\Xi_s{}^i\hat{\delta}_i(v)dv = 0 \tag{3.11}$$

where $\pi_1, \cdots, \pi_N$ are some positive integers.

Since $V_1(t)$ is bounded and $\dot{V}_1(t) < 0$, it is established that $\varsigma(t), \omega_{ij}(t), \omega_i(t)$ is bounded. Consider (3.2) to obtain that $\dot{\varsigma}(t)$ is bounded. Therefore, $\sum_{i=1}^{N}\pi_i\hat{\delta}_i^T(t)\Xi_s{}^i\hat{\delta}_i(t)$ is uniformly continuous, and according to Lemma 2, $\lim_{t\to\infty}\sum_{i=1}^{N}\pi_i\hat{\delta}_i^T(t)\Xi_s{}^i\hat{\delta}_i(t) = 0$ can be obtained. Given $\Xi_s{}^i < 0$, this indicates $\lim_{t\to\infty}\sum_{i=1}^{N}\varsigma_i(t) = 0$. Therefore, all follower agents have been implemented to track the leader and achieve consensus. □

Remark 5. *In this section, it is necessary to assume that the interaction between followers is bidirectional, which makes the matrix H symmetric. There is no requirement for the undirected communication topology between the leader and follower agents, which implies that the leader's influence can be isolated from the overall system dynamics. For ease of description, this section directly assumes $\bar{\mathcal{G}}$ an undirected graph. If the interaction between followers is directional, the non-zero eigenvalues of the corresponding Laplace matrix may assume complex values, precluding the possibility of diagonalizing the Laplacian matrix. Therefore, the matrix H_s in Eq (3.4) cannot be reduced to a diagonal form, and the controller in this section cannot achieve stability.*

3.3. Adaptive consensus protocol design for nonlinear SMASs

In this subsection, an introduction is provided for a fuzzy logic-based system that is designed to manage unknown nonlinear functions. The set of IF-THEN rules that underpin the fuzzy model are delineated as follows:

$$\begin{aligned} &R_j \ : \ \textbf{IF}\ l_1\ is\ {Z_1}^j\ and\ \cdots\ and\ l_N\ is\ {Z_N}^j \\ &\textbf{THEN}\ h\ is\ H^j,\ \ j = 1, 2, \cdots, N. \end{aligned}$$

Employing a singleton fuzzifier, product-inference rules, and the center-average defuzzifier, the fuzzy logic system can be articulated mathematically as follows:

$$g(l) = \frac{\sum_{j=1}^{N} \eta_j \prod_{i=1}^{n} \mu_{{Z_i}^j}(l_i)}{\sum_{j=1}^{N} \prod_{i=1}^{n} \mu_{{Z_i}^j}(l_i)} = \eta^T \vartheta(l). \tag{3.12}$$

Within the framework of the fuzzy logic system, $l = [l_1, l_2, \cdots, l_N]^T$ denotes the vector of inputs, and h represents the corresponding output. ${Z_i}^j$ and H^j correspond to the fuzzy sets employed in the model. $\mu_{{Z_i}^j}$ is the corresponding membership function. Additionally, $\eta = [\eta_1, \eta_2, \cdots, \eta_N]^T$ signifies the vector of adjustable parameters that are pivotal in the system's operation. Let $\vartheta(l) = [\vartheta_1(l), \vartheta_2(l), \cdots, \vartheta_N(l)]^T$, where the fuzzy basis function $\vartheta_j(l) = (\prod_{i=1}^{n} \mu_{{Z_i}^j}(l_i)) / \left[\sum_{j=1}^{N} (\prod_{i=1}^{n} \mu_{{Z_i}^j}(l_i))\right], j = 1, 2, \cdots, N$.

Lemma 3. *Define $h(l)$ be a continuous and bounded ($l \in U \subset \mathbb{R}^n$) nonlinear function. For an arbitrarily constant $\varepsilon > 0$, there exists a FLS (3.12) such that*

$$\sup_{X \in U} |h(X) - \eta^T \vartheta(X)| < \varepsilon. \tag{3.13}$$

From this, it can be concluded that $h(l)$ closely approximates $\eta^T \vartheta(l)$. Here, η is the vector of adjustable parameters. By defining $\tilde{\eta}$ as the optimal vector, we can establish the minimum estimation error $\tilde{\gamma} = h(l) - \tilde{\eta}^T \vartheta(l)$ with $|\tilde{\gamma}| \leq \gamma$, and γ is a positive constant.

Assuming $g(l) = \eta^T \vartheta(z)$ satisfies the Lipschitz condition, it implies that there exists a Lipschitz constant $\upsilon > 0$ such that for all $l_a, l_b \in \mathbb{R}^n$, the following inequality holds:

$$\left\|\eta^T \vartheta(l_a) - \eta^T \vartheta(l_b)\right\| \leq \upsilon \left\|l_a - l_b\right\|. \tag{3.14}$$

Theorem 2. *Consider the nonlinear SMASs (2.3) and (2.4), under Assumptions 1 and 2, when $F = -B^T {Q_v}^{-1}, \Gamma = ({Q_v}^{-1})^T B B^T {Q_v}^{-1}$ and with*

$$\text{rank}\begin{bmatrix} E & 0 \\ A - BB^T{Q_v}^{-1} & E \end{bmatrix} = n + \text{rank}(E)$$

where $Q_v > 0$ is a solution of

$$\begin{bmatrix} AQ_v + {Q_v}^T A^T - \kappa BB^T + \upsilon^2 I & Q_v \\ Q_v & -I \end{bmatrix} < 0,$$

where $\kappa > 0$, under the stipulations of the control protocol (3.1), it is posited that each agent within the network is capable of aligning with the leader's trajectory, regardless of their initial states. This means that the LFC of SMASs is achieved under the switching topology.

Proof. Since $\text{rank}\begin{bmatrix} E & 0 \\ A - BB^T{Q_v}^{-1} & E \end{bmatrix} = n + \text{rank}(E)$, then one gets that $(E, A + BF)$ is regular and impulse-free. Let $\varsigma_i(t) = x_i(t) - x_0(t)$, $\varsigma(t) = \left[{\varsigma_1}^T(t), {\varsigma_2}^T(t), \cdots, {\varsigma_N}^T(t)\right]^T$. It can be obtained that

$$\begin{aligned} E\dot{\varsigma}_i(t) &= A\varsigma_i(t) + f(x_i(t)) - f(x_0(t)) \\ &\quad + BF\left[\sum_{j=1}^{N} \omega_{ij}(t)a_{ij}(t)(\varsigma_i(t) - \varsigma_j(t)) + \omega_i(t)d_i(t)\varsigma_i(t)\right] \\ \dot{\omega}_{ij}(t) &= \psi_{ij}a_{ij}(t)(\varsigma_i(t) - \varsigma_j(t))^T\Gamma(\varsigma_i(t) - \varsigma_j(t)) \\ \dot{\omega}_i(t) &= \psi_i d_i(t){\varsigma_i}^T(t)\Gamma\varsigma_i(t). \end{aligned} \tag{3.15}$$

Constructing the Lyapunov function as follows:

$$V_2(t) = \sum_{i=1}^{N} {\varsigma_i}^T(t)E^T{Q_v}^{-1}\varsigma_i(t) + \sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N} \frac{(\omega_{ij}(t) - \chi')^2}{2\psi_{ij}} + \sum_{i=1}^{N} \frac{(\omega_i(t) - \chi')^2}{\psi_i}. \tag{3.16}$$

Here χ' is a positive constant. It is evident that $V_2(t)$ is continuously differentiable except at the switching moment.

First, it is important to show that $\dot{V}_2(t) < 0$ holds at any non-switching moment.

Based on the fuzzy logic system satisfying the Lipschitz condition, the nonlinear term $f(x_i(t))$ of the nonlinear SMASs (2.3) and (2.4) satisfy

$$\begin{aligned} \left\|f(x_i(t)) - f(x_j(t))\right\| &= \left\|f(x_i(t)) - \eta^T\vartheta(x_i(t)) + \eta^T\vartheta(x_i(t)) + \eta^T\vartheta(x_j(t)) - f(x_j(t)) - \eta^T\vartheta(x_j(t))\right\| \\ &\leq \gamma + \gamma + \left\|\eta^T\vartheta(x_i(t)) - \eta^T\vartheta(x_j(t))\right\| \\ &\leq 2\gamma + \upsilon\left\|x_i(t) - x_j(t)\right\|, \\ &\quad i, j = 0, 1, \cdots, N. \end{aligned} \tag{3.17}$$

As $F = -B^T{Q_v}^{-1}, \Gamma = ({Q_v}^{-1})^T BB^T{Q_v}^{-1}$ and $\omega_{ij}(t) = \omega_{ji}(t)$, let $\tilde{\delta}_i(t) = {Q_v}^{-1}\varsigma_i(t)$, $\tilde{\delta}(t) = \left[\tilde{\delta}_1(t)^T, \tilde{\delta}_2(t)^T, \cdots, \tilde{\delta}_N(t)^T\right]^T$, by using (3.17), and omit '(t)' in the process for simple expression, the following result can be obtained:

$$\dot{V}_2(t) = 2\sum_{i=1}^{N} {\varsigma_i}^T({Q_v}^{-1})^T A\varsigma_i - 2\chi'\sum_{i=1}^{N} d_i{\varsigma_i}^T({Q_v}^{-1})^T BB^T{Q_v}^{-1}\varsigma_i$$

$$
\begin{aligned}
&+2\sum_{i=1}^{N}\varsigma_i{}^T(Q_v{}^{-1})^T\left[f(x_i)-f(x_0)\right]-2\chi'\sum_{i=1}^{N}\sum_{j=1}^{N}a_{ij}\varsigma_i{}^T(Q_v{}^{-1})^TBB^TQ_v{}^{-1}(\varsigma_i-\varsigma_j)\\
&\leq\sum_{i=1}^{N}\tilde{\delta}_i^T\left(AQ_v+Q_v{}^TA^T+\upsilon^2I+Q_v{}^2\right)\tilde{\delta}_i-2\chi'\sum_{i=1}^{N}\sum_{j=1}^{N}l_{ij}\tilde{\delta}_i^TBB^T\tilde{\delta}_j-2\chi'\sum_{i=1}^{N}d_i\tilde{\delta}_iBB^T\tilde{\delta}_i\\
&=\tilde{\delta}(t)^T\left[I_N\otimes\left(AQ_v+Q_v{}^TA^T+\upsilon^2I+Q_v{}^2\right)-2\chi'H_s\otimes BB^T\right]\tilde{\delta}(t).
\end{aligned}\tag{3.18}
$$

Similar to the process in Theorem 1, assuming that the number of 0 eigenvalues of H_s is $q_s(0\leq q_s<N)$, a unitary matrix M_s can be found such that

$$
M_s{}^TH_sM_s=\Lambda_s\triangleq\operatorname{diag}\left(0,\cdots,0,\lambda_s{}^{q_s+1},\cdots,\lambda_s{}^N\right)\tag{3.19}
$$

holds.

Let $\hat{\delta}_i(t)=M_s{}^T\tilde{\delta}_i(t)$, it can be obtained that

$$
\dot{V}_2(t)\leq\sum_{i=q+1}^{N}\hat{\delta}_i(t)^T\left[AQ_v+Q_v{}^TA^T+\upsilon^2I+Q_v{}^2-2\chi'\lambda_s{}^iBB^T\right]\hat{\delta}_i(t).\tag{3.20}
$$

Choosing a sufficiently large χ' such that $\chi'\lambda_s{}^i\geq\kappa, i=q_s+1,\cdots,N$. The following inequality can be obtained from Schur's complement lemma.

$$
\begin{aligned}
&AQ_v+Q_v{}^TA^T+\upsilon^2I+Q_v{}^2-2\chi'\lambda_s{}^iBB^T\\
&\quad\leq AQ_v+Q_v{}^TA^T-\kappa BB^T+\upsilon^2I+Q_v{}^2\\
&\quad<0.
\end{aligned}\tag{3.21}
$$

This yields $\dot{V}_2(t)<0$ and hence there exists $\lim_{t\to\infty}V_2(t)$.

Next, it is necessary to prove $\lim_{t\to\infty}\varsigma(t)=0$. By the Cauchy convergence criterion, for an arbitrary small positive value ε, it is guaranteed that there exists a positive integer Ψ. This integer serves as a threshold such that for all iterations $k\geq\Psi$, $|V_2(t_{k+1})-V_2(t_k)|<\varepsilon$ holds, i.e., $\left|\int_{t_k}^{t_{k+1}}\dot{V}_2(t)dt\right|<\varepsilon$.

Similarly, by representing it as an integral sum, it can be obtained that

$$
\int_{t_k{}^0}^{t_k{}^1}(-\dot{V}_2(t))dt+\int_{t_k{}^1}^{t_k{}^2}(-\dot{V}_2(t))dt+\cdots+\int_{t_k{}^{z_k-1}}^{t_k{}^{z_k}}(-\dot{V}_2(t))dt<\varepsilon.\tag{3.22}
$$

As $\int_{t_k{}^i}^{t_k{}^{i+1}}(-\dot{V}_2(t))dt<\varepsilon, i=0,1,\cdots,z_k-1$, that means

$$
\begin{aligned}
-\varepsilon&<\int_{t_k{}^i}^{t_k{}^{i+1}}\dot{V}_2(t)dt\\
&\leq\int_{t_k{}^i}^{t_k{}^i+\tau}\sum_{i=q_s+1}^{N}\hat{\delta}_i^T(t)\left[AQ_v+Q_v{}^TA^T+\upsilon^2I+Q_v{}^2-2\chi'\lambda_s{}^iBB^T\right]\hat{\delta}_i(t)dt
\end{aligned}\tag{3.23}
$$

Let $q_{\sigma(t_k{}^i)}, i = 0, 1, \cdots, z_k - 1$ represents the number of eigenvalues 0 in the corresponding matrix $H_{\sigma(t_k{}^i)}$ of the topology graph $\bar{G}_{\sigma(t_k{}^i)}$ of interval $\left[t_k{}^i, t_k{}^{i+1}\right)$, denoted as $\hbar_s{}^i = AQ_v + Q_v{}^T A^T + \upsilon^2 I + Q_v{}^2 - 2\chi' \lambda_s{}^i BB^T$. Consequently, this derivation enables us to deduce

$$\lim_{t\to\infty} \int_t^{t+\tau} \left\{ \sum_{i=q_{\sigma(t_k{}^0)}+1}^{N} \hat{\delta}_i^T(v)\hbar_s{}^i\hat{\delta}_i(v) + \cdots + \sum_{i=q_{\sigma(t_k{}^{z_k-1})}+1}^{N} \hat{\delta}_i^T(v)\hbar_s{}^i\hat{\delta}_i(v) \right\} dv = 0. \tag{3.24}$$

According to Lemma 1, when the communication topology between agents in the $[t_k, t_{k+1})$ interval satisfies jointly connected, it can be obtained that $q_{\sigma(t_k)} = 0$ and

$$\lim_{t\to\infty} \int_t^{t+\tau} \sum_{i=1}^{N} \pi_i \hat{\delta}_i^T(v)\hbar_s{}^i\hat{\delta}_i(v)dv = 0 \tag{3.25}$$

where $\pi_1, \cdots, \pi_N$ are some positive integers.

Since $V_2(t)$ is bounded and $\dot{V}_2(t) < 0$, it is evident that $\varsigma(t), \omega_{ij}(t), \omega_i(t)$ is bounded. Consider (3.15) to obtain that $\dot{\varsigma}(t)$ is bounded. Therefore, $\sum\limits_{i=1}^{N} \pi_i \hat{\delta}_i^T(t)\hbar_s{}^i\hat{\delta}_i(t)$ is uniformly continuous, and In light of Lemma 2, $\lim_{t\to\infty} \sum\limits_{i=1}^{N} \pi_i \hat{\delta}_i^T(t)\hbar_s{}^i\hat{\delta}_i(t) = 0$ can be obtained. Given $\hbar_s{}^i < 0$, this indicates $\lim_{t\to\infty} \sum\limits_{i=1}^{N} \varsigma_i(t) = 0$. Therefore, all follower agents have been implemented to track the leader and achieve consensus. □

4. Numerical simulation

In this segment, a detailed numerical illustration is furnished to evidence the effectiveness of the aforementioned algorithm, denoted by (3.1).

In this example, consider the problem of unmanned vehicle formation, which involves one leader unmanned aerial vehicle and four follower unmanned aerial vehicles with SMAS dynamics. The leader system is described as

$$\begin{aligned} \dot{x}_{01} &= -3x_{01} - 2x_{02} + \sin(x_{01}) \\ 0 &= 2x_{01} + x_{02} + \cos(x_{02}). \end{aligned} \tag{4.1}$$

The follower agent system is represented as

$$\dot{x}_{i1} = -3x_{i1} - 2x_{i2} + \sin(x_{i1}) + u_{i1} \tag{4.2}$$

$$0 = 2x_{i1} + x_{i2} + \cos(x_{i2}) + u_{i2} \tag{4.3}$$

where $i = 1, 2, 3, 4, 5$, and $x_{i1}, x_{i2} \in \mathbb{R}$, $u_{i1}, u_{i2} \in \mathbb{R}$ represent state and control input, respectively.

Assuming all possible topologies are $\left\{\bar{\mathcal{G}}_1, \bar{\mathcal{G}}_2, \bar{\mathcal{G}}_3, \bar{\mathcal{G}}_4, \bar{\mathcal{G}}_5, \bar{\mathcal{G}}_6\right\}$, as shown in Figure 1.

The communication topology switches in sequence $\bar{\mathcal{G}}_1 \to \bar{\mathcal{G}}_2 \to \bar{\mathcal{G}}_3 \to \bar{\mathcal{G}}_4 \to \bar{\mathcal{G}}_5 \to \bar{\mathcal{G}}_6 \to \bar{\mathcal{G}}_1 \to \bar{\mathcal{G}}_2 \to \cdots$, with a dwell time of 1/3 second for each graph. The switching signal is shown in Figure 2. Since $\left\{\bar{\mathcal{G}}_1 \cup \bar{\mathcal{G}}_2 \cup \bar{\mathcal{G}}_3\right\}$ and $\left\{\bar{\mathcal{G}}_4 \cup \bar{G}_5 \cup \bar{\mathcal{G}}_6\right\}$ are connected, if $t_k = k$ and $t_{k+1} = k + 1$ are selected, then $t_k{}^0 = k, t_k{}^1 = k + \frac{1}{3}, t_k{}^2 = k + \frac{2}{3}, t_k{}^3 = k + 1, k = 0, 1, \cdots$.

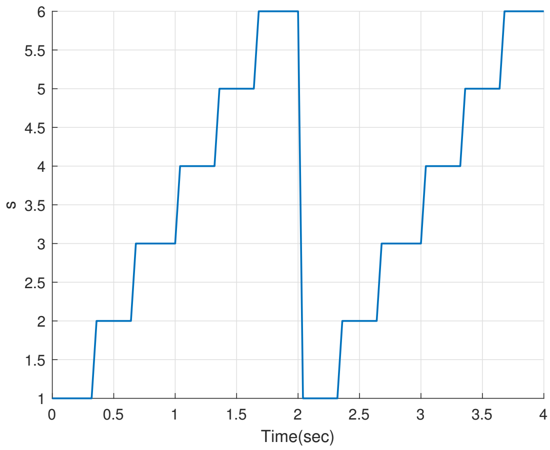

Figure 2. A switching signal describing time-vary interaction topology.

Fuzzy logic multi-agent system: By using fuzzy modeling method, the premise variables are defined as $z_1 = \cos(x_{i2}) = (\theta_{i1}(z_1)+\theta_{i2}(z_1)2/\pi)x_{i2}$, $z_2 = \sin(x_{i1}) = (\theta_{i1}(z_2)+\theta_{i2}(z_2)2/\pi)x_{i1}$, with $i = 0, 1, 2, 3, 4, 5$. The membership functions are described as follows:

$$\theta_{i1}(z_1) = \begin{cases} \frac{z_1-2\cos^{-1}(z_1)/\pi}{(1-2/\pi)\cos^{-1}(z_1)}, z_1 \neq 0 \\ 1, otherwise \end{cases}$$
$$\theta_{i2}(z_1) = \begin{cases} \frac{\cos^{-1}(z_1)-z_1}{(1-2/\pi)\cos^{-1}(z_1)}, z_1 \neq 0 \\ 0, otherwise \end{cases} \tag{4.4}$$

$$\Upsilon_{i1}(z_2) = \begin{cases} \frac{z_2-2\sin^{-1}(z_2)/\pi}{(1-2/\pi)\sin^{-1}(z_2)}, z_2 \neq 0 \\ 1, otherwise \end{cases}$$
$$\Upsilon_{i2}(z_2) = \begin{cases} \frac{\sin^{-1}(z_2)-z_2}{(1-2/\pi)\sin^{-1}(z_2)}, z_2 \neq 0 \\ 0, otherwise. \end{cases} \tag{4.5}$$

Therefore, the fuzzy rules of the SMAS are represented as follows:
Fuzzy Rule 1 if z_1 is θ_{i1}, and z_2 is Υ_{i1}, then

$$\begin{aligned} \dot{x}_{1i} &= -2x_{i1} - 2x_{i2} + u_{i1} \\ 0 &= 2x_{i1} + 2x_{2i} + u_{i2}. \end{aligned} \tag{4.6}$$

Fuzzy Rule 2 if z_1 is θ_{i2}, and z_2 is Υ_{i1}, then

$$\begin{aligned} \dot{x}_{i1} &= (-3 + \frac{2}{\pi})x_{i1} - 2x_{2i} + u_{i1} \\ 0 &= 2x_{i1} + 2x_{2i} + u_{i2}. \end{aligned} \tag{4.7}$$

Fuzzy Rule 3 if z_1 is θ_{i1}, and z_2 is Υ_{i2}, then

$$\dot{x}_{1i} = -2x_{i1} - 2x_{i2} + u_{i1}$$

$$0 = 2x_{i1} + (1 + \frac{2}{\pi})x_{2i} + u_{i2}. \tag{4.8}$$

Fuzzy Rule 4 if z_1 is θ_{i2}, and z_2 is Υ_{i2}, then

$$\begin{aligned}\dot{x}_{i1} &= (-3 + \frac{2}{\pi})x_{i1} - 2x_{2i} + u_{i1}\\ 0 &= 2x_{i1} + (1 + \frac{2}{\pi})x_{2i} + u_{i2},\end{aligned} \tag{4.9}$$

where $i = 0, 1, 2, 3, 4, 5$. When $i = 0$, there is $u_{i1} = u_{i2} = 0$.

Therefore, the fuzzy model of the SMAS is modeled as follows:

$$\begin{aligned}\dot{x}_{i1} &= \theta_{i1}(-2x_{i1} - 2x_{i2} + u_{i1}) + \theta_{i2}((-3 + \frac{2}{\pi})x_{i1} - 2x_{2i} + u_{i1})\\ 0 &= \Upsilon_{i1}(2x_{i1} + 2x_{2i} + u_{i2}) + \Upsilon_{i2}(2x_{i1} + (1 + \frac{2}{\pi})x_{2i} + u_{i2}),\\ i &= 0, 1, 2, 3, 4, 5.\end{aligned} \tag{4.10}$$

Consider parameters $\psi_{ij} = 0.1, \psi_i = 0.3, i, j = 1, 2, 3, 4, 5, \kappa = 2, l = 0.1$. The initial states are $x_0(0) = [3, -6]^T$, $x_1(0) = [2, -4]^T$, $x_2(0) = [4, -8]^T$, $x_3(0) = [-2, 4]^T$, $x_4(0) = [-4, 8]^T$, $x_5(0) = [1, -2]^T$.

Figures 3 and 4 illustrate the trajectory of the error, denoted as e_i, which is defined as the difference between the follower state $x_i, i = 1, 2, 3, 4, 5$ and the leader state x_0. From this, it is deduced that the trajectories of all follower agents converge towards the state of the leader agent, thus achieving the goal of this paper and achieving consensus. The results indicate that the designed adaptive controller is effective. Figure 5 illustrates the convergence of the control input, as the topology of the model is a switching topology with non-connected graphs, when agent i does not have communication with other agents, u_i is 0. This is because the control protocol contains a_ij and d_i, which change with the topology. Therefore, the control input signal is non-smooth and will appear at the switching signal. But it can be seen that the control input is generally bounded and approaches zero. It can be demonstrated that under the adjustment of the adaptive controller designed in this paper, the nonlinear SMASs can achieve consensus under the switching topology.

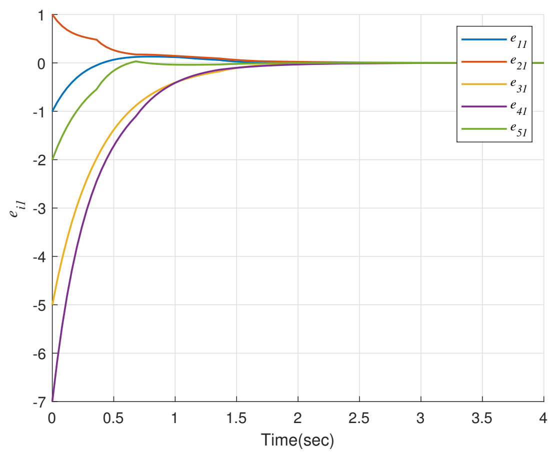

Figure 3. Trajectories of error $x_{i1} - x_{01}$.

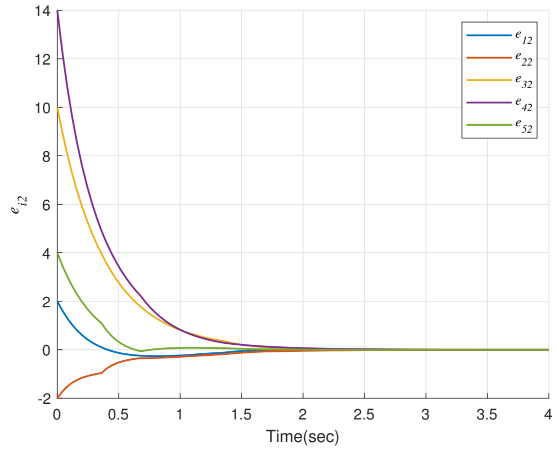

Figure 4. Trajectories of error $x_{i2} - x_{02}$.

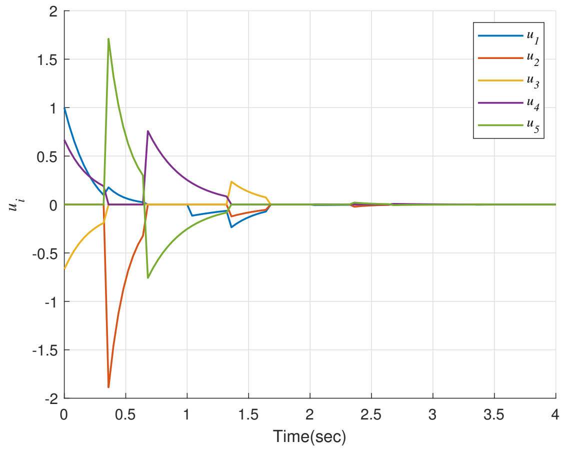

Figure 5. Control inputs u_i.

5. Conclusions

This paper proposes a new distributed adaptive collaborative algorithm for LFC tracking in the SMASs under switching topologies. The method is designed to ensure the regularity and impulse-free nature of the system, culminating in the stabilization of all agent states. First, a tolerance domain for linear SMASs is proposed. A fuzzy logic system provides a universal and adaptive approach to solving nonlinear items in nonlinear SMASs. Subsequently, through a rigorous analysis of the error system's asymptotic stability, the stability of the closed-loop system is obtained. Finally, a simulation example is exhibited, which confirms the effectiveness of the proposed method. This study not only helps to deepen the understanding of the dynamic behavior of SMASs but also provides important theoretical support for the design and control of SMASs in practical applications.

Author contributions

Jiawen Li: Conceptualization, Writing original draft, Methodology, Software, Proof of conclusions; Yi Zhang: Conceptualization, Methodology, Writing-review and editing; Heung-wing Joseph Lee:

Writing review, Supervision, Project administration; Yingying Wang: Validation, Funding acquisition, Resources. All the authors have read and approved the final version of the manuscript for publication.

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Conflict of interest

All authors declare no conflicts of interest in this paper.

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