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Research article

Almost sure exponential synchronization of multilayer complex networks with Markovian switching via aperiodically intermittent discrete observation noise

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Abstract: This paper is concerned with almost sure exponential synchronization of multilayer complex networks with Markovian switching via aperiodically intermittent discrete observation noise. First, Markovian switching and multilayer interaction factors are taken into account simultaneously, which make our system more general compared with the existing literature. Meanwhile, the network architecture may be undirected or directed, and consequently, the adjacency matrix is symmetrical and asymmetrical. Second, the control strategy is based on aperiodically intermittent discrete observation noise, where the average control rate is integrated to depict the distributions of work/rest intervals of the control strategy from an overall perspective. Third, different from the work about pth moment exponential synchronization of network systems, by utilizing M-matrix theory and various stochastic analysis techniques including the Itô formula, the Gronwall inequality, and the Borel-Cantelli lemma, some criteria on almost sure exponential synchronization of multilayer complex networks with Markovian switching have been constructed and the upper bound of the duration time has been also estimated. Finally, several numerical simulations are exhibited to validate the effectiveness and feasibility of our analytical findings.

Keywords: almost sure exponential synchronization; multilayer complex networks; aperiodically intermittent discrete observation noise; M-matrix theory
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1. Introduction

In recent years, an enormous advance of complex networks (CNS) has been witnessed on account of their extensive applications in many areas such as unmanned aerial vehicles, communication systems, power systems, transportation networks, ecological networks, and so forth [1–4]. A large number of interesting achievements about dynamical properties of CNS have sprung up [5–9]. It can be observed that CNS in the majority of the above findings are supposed to be single-link. In contrast to single-link complex, multilayer complex networks (MCNS) can more accurately simulate the real systems, which are more general and practical due to possessing sophisticated structures. For instance, a transportation network is composed of highways, ships, railways, and airplanes, and it can be seen as one type of multilayer complex network. Apparently, the transmission speed and cost are different among these ways, which signifies that each interaction mode has different weights. On the other hand, great importance can also be attached to social networks. There are various kinds of contact ways for people to communicate with others including phone, mail, and internet, which leads to multilayer networks. Particularly, large group decision-making plays a significant role in social networks. In [10], large group decision-making with a rough integrated asymmetric cloud model under a multi-granularity linguistic environment was considered, and large group emergency decision-making with bi-directional trust in social networks was studied based on a probabilistic hesitant fuzzy integrated cloud approach in [11], where strong uncertainty and randomness were tackled effectively. Since the MCNS may better depict the connection and interaction among different layer networks, the investigation of MCNS has important theoretical and realistic significance.

Synchronization is a typical dynamic characteristic of MCNS, which means all the states of different nodes of networks can evolve in one common mode with various different initial data, and plenty of relevant results about MCNS have been reported. Particularly, in [12], global exponential synchronization of multi-link memristive neural networks with time-varying delays was analyzed by designing state feedback controllers. In [13], the practical fixed-time synchronization issue of MCNS was addressed based on an intermittent event-triggered control strategy by utilizing the proposed new practical fixed-time stability lemma. In [14], bipartite synchronization of multilayer signed networks was investigated under aperiodic intermittent-based adaptive dynamic event-triggered control, where the control gains and the triggering parameters were adjusted with the system states. Furthermore, the discussions of various synchronizations were extended to fractional MCNS with the help of impulsive control [15, 16]. On the other hand, in the real world, due to the existence of random disturbances and abrupt variations in systems' structures and parameters, stochastic Markovian switching systems were introduced to simulate such systems. Recently, stability and synchronization of stochastic MCNS with Markovian switching have aroused scholars' interests [17–23]. In [17], pth moment exponential stability for a class of stochastic complex multi-link networks with Markovian switching and multi-delayed impulsive effects was examined by virtue of the Razumikhin approach. In [18], combining with Kirchhoff's matrix tree theorem, finite-time synchronization of stochastic MCNS with Markovian switching was studied via a novel quantized aperiodically intermittent control. Moreover, in [19], the pth moment exponential synchronization issue of stochastic delayed MCNS with semi-Markov jump under aperiodically intermittent control was solved based on the Lyapunov method and graph theory, and several sufficient criteria were derived. Nevertheless, the preceding existing results mainly focus on pth moment synchronization of stochastic MCNS with Markovian switching rather than almost sure exponential synchronization, which sparks the appearance of this work.

It is worth noting that stochastic noise can be used as the control input [24, 25]. For instance, in social networks, by virtue of public opinion control, which can be viewed as noise control, a certain behavior mode of the entire society can keep the consensus in a special direction [26]. Particularly, white noise was incorporated initially to stabilize the neural networks by Liao and Mao [27]. Soon afterward, stochastic stabilization of nonlinear systems was considered by utilizing an intermittent Brownian noise or aperiodical intermittent Brownian noise in [28, 29]. Meanwhile, when the discrete-time feedback control was introduced to the drift term of systems, mean-square exponential stabilization of stochastic nonlinear systems was discussed in [30]. While the discrete-time feedback control was introduced to the diffusion term of systems, the almost sure exponential stabilization issue was solved in [31]. Subsequently, the studies about discrete feedback control and noise stabilization were further developed in [32–34]. Based on the previous findings, a new periodically intermittent discrete observation control scheme was proposed, by which the issue of mean-square exponential synchronization of stochastic neural networks was coped with in [35]. Furthermore, under the similar control strategy in [35], almost sure exponential stabilization of deterministic neural networks and hybrid neural networks was analyzed in [36, 37]. More recently, aperiodically intermittent discrete observation noise control (AIDONC) as one kind of discontinuous control scheme was designed by adjusting the distributions of work/rest intervals. Since feedback control based on intermittent discrete observations on current states are imposed, the control time and control frequency can be reduced. Consequently, energy consumption and control cost can be greatly saved. Besides, efficiency and feasibility of implementation of the control strategy will be improved significantly. By using aperiodically intermittent discrete observation noise control (AIDONC), synchronization of complex networks was investigated in [38, 39]. For MCNS, it can be observed that in [21, 22], the intermittent control was imposed on the drift term and several criteria on pth moment exponential synchronization were presented. Additionally, in [23], some sufficient conditions on almost sure exponential synchronization of MCNS were given, but the control strategy AIDONC was not adopted.

Inspired by the above discussions, this paper will investigate the almost sure exponential synchronization of MCNS with Markovian switching via AIDONC. The main contributions are summarized below:

1) In this paper, Markovian switching and multilayer interaction factors are taken into account simultaneously, which make our system more general compared with the networks in literature [35–39]. Meanwhile, the network architecture may be undirected or directed, and consequently, the adjacency matrix could be symmetrical and asymmetrical.

2) Different from the work about pth moment exponential synchronization of network systems in [21, 22], by utilizing M-matrix theory and various stochastic analysis techniques including the Itô formula, the Gronwall inequality, and the Borel-Cantelli lemma, almost sure exponential synchronization of multilayer coupled networks with Markovian switching is analyzed and the upper bound of the duration time is also estimated.

3) Compared with the work in [23], one kind of control strategy based on AIDONC is adopted, where the average control rate is integrated to depict the distributions of work/rest intervals of the control strategy from an overall perspective.

The remainder of this article is organized as follows. In Section 2, some necessary preliminaries and model descriptions are presented. The main findings about almost sure exponential synchronization for multilayer coupled networks with Markovian switching via aperiodically intermittent noise are derived in Section 3. In Section 4, a multilayer Chua's circuits network with Markovian switching is considered and some numerical simulations are carried out to validate the effectiveness of our theoretical results, and conclusions are drawn in the last section.

For convenience and simplicity, let us introduce some standard notations utilized in the following context. Let $\mathbb{R}^n$ be the n-dimensional Euclidean space. If $x \in \mathbb{R}^n$, then $|x|$ represents the Euclidean norm of x. Let I_N be an $N \times N$ identity matrix. Besides, the superscript T is defined as the transpose of a vector or a matrix. For any real symmetric matrix, $\lambda_{\max}(\cdot)$ and $\lambda_{\min}(\cdot)$ denote the maximum and minimum eigenvalues of the given matrix, respectively. Let $(\Omega, \mathcal{F}, \mathbb{P})$ be a complete probability space with a filtration $\{\mathcal{F}_t\}_{t\geq 0}$ satisfying the usual condition, and $B(t)$ represents one-dimensional Brownian motion defined on the above probability space.

2. Preliminaries and model descriptions

In this section, we consider the multilayer coupled networks with Markovian switching and M nodes as follows:

$$d\delta_k(t) = [(\delta_k(t), t, r(t)) + \sum_{h=1}^{H}\sum_{m=1}^{M}\rho_h(r(t))b_{km}^h(r(t))\Upsilon\delta_m(t)]dt + u_k(t, r(t)), \qquad k = 1, 2, \cdots\cdots, M, \quad (2.1)$$

where $\delta_k(t) = ((\delta_{k1}(t), \delta_{k2}(t), \ldots, \delta_{kn}(t))^T \in \mathbb{R}^n$ denotes the vector of the kth node at time t. $u_i(t, r(t))$ is the controller to be devised. $G(\delta_k(t), t, r(t)) \in \mathbb{R}^n : \mathbb{R}^n \times \mathbb{R}^+ \times \mathbb{S} \to \mathbb{R}^n$ represents the nonlinear activation function with $G(\delta_k(0), r(0), 0) = 0$. $\rho_h(r(t)) > 0$ is the coupling strength of the hth layer. Υ stands for an inner coupling matrix. The adjacency matrix $(b_{km}^h)_{M\times M}$ is the outer coupling configuration matrix, where $b_{km}^h > 0$ holds if node m directs links to k, and otherwise, $b_{km}^h = 0$ $(k \neq m)$. Meanwhile, the diagonal elements $b_{kk}^h = -\sum_{m=1, m\neq k}^{M} b_{km}^h$, which leads to $\sum_{m=1}^{M} b_{km}^h = 0$. Let $r(t), t \geq 0$ signify a right-continuous Markov chain on the probability space $(\Omega, \mathcal{F}, \mathbb{P})$ with initial date $r(0) = r_0 \in \mathbb{S} = \{1, 2, \cdots\cdots, N\}$, and generator $\Psi = (\psi_{ij})_{N\times N}$ given by

$$P(r(t + \Delta) = j|r(t) = i) = \begin{cases} \psi_{ij}\Delta + o(\Delta), & \text{if} \quad i \neq j, \\ 1 + \psi_{ij}\Delta + o(\Delta), & \text{if} \quad i = j, \end{cases} \quad (2.2)$$

where $\Delta > 0$, $\psi_{ii} = -\sum_{j\neq i}\psi_{ij}$, and $\psi_{ij} \geq 0$ denotes the transition rate from i to j.

Subsequently, the isolated node of the network is expressed as

$$ds(t) = G(s(t), t, r(t))dt, \quad (2.3)$$

where $s(t) = (s_1(t), s_2(t), \ldots, s_n(t))^T \in \mathbb{R}^n$. Apparently, the isolated node is independent of other nodes. $s(t)$ can be considered as the equilibrium state or stable objective trajectory, and in multi-agent dynamics, it stands for the leader. Let $\eta_k(t) = \delta_k(t) - s(t)$ be the synchronization error. Accordingly, the error system can be depicted as

$$d\eta_k(t) = [\hat{G}(\eta_k(t), t, r(t)) + \sum_{h=1}^{H}\sum_{m=1}^{M}\rho_h(r(t))b_{km}^h(r(t))\Upsilon\eta_i(t)]dt + u_k(t, r(t)), \qquad k = 1, 2, \cdots\cdots, M, \quad (2.4)$$

where $\hat{G}(\eta_k(t), t, r(t)) = G(\eta_k(t), t, r(t)) - G(s(t), t, r(t))$. In order to realize synchronization, the aperiodically intermittent feedback control based on discrete-time state observations is designed below:

$$u_k(t) = \begin{cases} \beta_k(\eta_k(\mu(t)), t, r(t))dw(t), t \in [t_i, s_i), \\ 0, t \in [s_i, t_{i+1}), \end{cases} \quad (2.5)$$

where $\mu(t) = [t/\tau]\tau, \tau > 0$, τ denotes the duration between two consecutive observations, and $\beta_k(\eta_k(\mu(t), t, r(t)) = C_k(r(t))\eta_k(\mu(t))$ represents the diffusion coefficient column vector. Time interval $[t_i, s_i)$ is the working time of the kth period while $[s_i, t_{i+1})$ stands for the rest time. Furthermore, under aperiodically intermittent noise control, error system (2.4) can be rewritten by

$$\begin{cases} d\eta(t) = \Big[\tilde{G}(\eta(t), t, r(t)) + \sum_{h=1}^{H} \rho_h(r(t))\Big(B^h(r(t)) \otimes \Upsilon\Big)\eta(t)\Big]dt + \tilde{\beta}(\eta(\mu(t)), t, r(t))dw(t), \\ \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad t \in [t_i, s_i), \quad i = 0, 1, \cdots\cdots \\ d\eta(t) = \tilde{G}(\eta(t), t, r(t)) + \sum_{h=1}^{H} \rho_h(r(t))\Big(B^h(r(t)) \otimes \Upsilon\Big)\eta(t), \quad t \in [s_i, t_{i+1}), \quad i = 0, 1, \cdots\cdots \end{cases} \tag{2.6}$$

where $\eta(t) = \left(\eta_1^T(t), \eta_2^T(t), \cdots\cdots, \eta_M^T(t)\right)^T$, $\hat{G}(\eta(t)) = \left(\tilde{G}^T(\eta_1(t)), \tilde{G}^T(\eta_2(t)), \cdots\cdots, \tilde{G}^T(\eta_M(t))\right)^T \in \mathbb{R}^{nM}$, and $\tilde{\beta}(\eta(\mu(t)), t, r(t)) = \left(\eta_1^T(\mu(t))C_1^T(r(t)), \eta_2^T(\mu(t))C_2^T(r(t)), \cdots\cdots, \eta_M^T(\mu(t))C_M^T(r(t))\right)^T$. Since multilayer networks, Markovian switching, and aperiodically intermittent controller based on discrete-time observation noise exist simultaneously, this makes them more complex to analyze the synchronization feature. In order to overcome the difficulties, the auxiliary systems with continuous-time observations are presented below,

$$\begin{cases} dz(t) = \Big[\tilde{G}(z(t), t, r(t)) + \sum_{h=1}^{H} \rho_h(r(t))\Big(B^h(r(t)) \otimes \Upsilon\Big)z(t)\Big]dt + \tilde{\beta}(z(t), t, r(t))dw(t), \\ \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad t \in [t_i, s_i), \quad i = 0, 1, \cdots\cdots \\ dz(t) = \tilde{G}(z(t), t, r(t)) + \sum_{h=1}^{H} \rho_h(r(t))\Big(B^h(r(t)) \otimes \Upsilon\Big)z(t), \quad t \in [s_i, t_{i+1}), \quad i = 0, 1, \cdots\cdots \end{cases} \tag{2.7}$$

where $z(t) = \delta(t) - s(t)$.

In what follows, several necessary definitions and assumptions are provided, which play a fundamental role in acquiring the theoretical results.

Definition 1. *The multilayer coupled networks (2.1) can be almost surely exponentially synchronized if for any initial data $\eta(t_0) = \eta_0$, there exists a positive constant satisfying the following inequality*

$$\limsup_{t\to\infty} \frac{1}{t} \log|\eta(t)| < -\kappa, \quad a.s. \tag{2.8}$$

Assumption 1. *Suppose that there exist two positive constants $\alpha \in (0, 1)$ and N_0 such that*

$$N(t, s) \geq \alpha(t - s) - N_0, \forall t > s \geq t_0, \tag{2.9}$$

where $N(t, s)$ denotes the total control time length on $[s, t)$, and N_0 is the elasticity number.

Assumption 2. *Suppose that there are positive constants l_i and σ_{ki} such that following inequalities hold*

$$|G(u_1, t, i) - G(u_2, t, i)| \leq l_i |u_1 - u_2|, \tag{2.10}$$

for $\forall u_1, u_2 \in \mathbb{R}^n$, and $i \in \mathbb{S}$.

3. Main results

In this section, some novel lemmas are first established by utilizing stochastic analysis techniques. Based on the derived lemmas, almost sure exponential synchronization of MCNS with Markovian switching are further examined via AIDONC, and some special cases are also discussed.

Lemma 3.1. *Let* $Q = \text{diag}\{\chi_1, \chi_2, \cdots, \chi_N\} - \Psi$ *denote an M-matrix, where* $q \in (0, 1)$*,* $\chi_i = 0.5q\Big[(2 - q)d_i - \sigma_i\Big] - ql_i - q\Bigg(\sum_{h=1}^{H} \rho_h(i)\lambda_{\max}\Big(\Big[B^h(i) \otimes \Upsilon\Big]_s\Big)\Bigg)$*, and* $\Psi = (\psi_{ij})_{M\times M}$*. Under Assumptions 1 and 2, if* $\epsilon = \tilde{\lambda}\alpha - \tilde{\zeta}(1 - \alpha) > 0$*, then the solution* $z(t)$ *of Eq (2.7) satisfies that*

$$\mathbb{E}|z(t)|^q \leq K_0\mathbb{E}|z_0|^q e^{-\epsilon t}, \tag{3.1}$$

where $\sigma_i = \max\Big\{\lambda_{\max}\Big(C_1^T(i)C_1(i)\Big), \lambda_{\max}\Big(C_2^T(i)C_2(i)\Big), \cdots\cdots, \lambda_{\max}\Big(C_M^T(i)C_M(i)\Big)\Big\}$, $d_i = \min\Big\{\lambda^2_{\min}\Big(C_1(i)\Big), \lambda^2_{\min}\Big(C_2(i)\Big), \cdots\cdots, \lambda^2_{\min}\Big(C_M(i)\Big)\Big\}$, $Q\theta = Q(\theta_1, \theta_2, \cdots\cdots, \theta_N)^T = (\lambda_1, \lambda_2, \cdots\cdots, \lambda_N)^T > 0$, $\tilde{\lambda} = \min_{1\leq i\leq N}\left\{\frac{\lambda_i}{\theta_i}\right\} > 0$, $\zeta_i = 0.5q\Big[(2 - q)d_i - \sigma_i\Big]\theta_i - \lambda_i$, $\tilde{\zeta} = \max_{1\leq i\leq N}\left\{\frac{\zeta_i}{\theta_i}\right\} > 0$, $\theta_m = \min_{i\in S}\{\theta_i\}$, $\theta_M = \max_{i\in S}\{\theta_i\}$, and $K_0 = \frac{\theta_M e^{N_0(\tilde{\lambda}+\tilde{\zeta})}}{\theta_m}$. In other words, the trivial solution of Eq (2.7) is qth moment exponentially stable.

Proof. Noting that Q is an M-matrix, there exists a vector $\theta = (\theta_1, \theta_2, \cdots\cdots, \theta_N)^T > 0$ such that $Q\theta = (\lambda_1, \lambda_2, \cdots\cdots, \lambda_N)^T > 0$. Choose the Lyapunov function $V(z(t), t, i) = \theta_i|z(t)|^q, i \in \mathbb{S}$. Obviously, $\theta_m|z(t)|^q \leq V(z(t), t, i) \leq \theta_M|z(t)|^q$. By utilizing the Itô formula, when $t \in [t_i, s_i), i = 0, 1, 2, \cdots\cdots$, and we have that

$$\begin{aligned}\mathcal{L}V(z(t), t, i) = \ & q\theta_i|z(t)|^{q-2}z^T(t)\Bigg[\tilde{G}(z(t), t, i) + \sum_{h=1}^{H}\rho_h(i)\Big(B^h(i)\otimes\Upsilon\Big)z(t)\Bigg] + \frac{1}{2}q\theta_i|z(t)|^{q-2}|\tilde{\beta}(z(t), t, i)|^2 \\ & - \frac{1}{2}q(2 - q)\theta_i|z(t)|^{q-4}|z^T(t)\tilde{\beta}(z(t), t, i)|^2 + \sum_{j=1}^{N}\psi_{ij}\theta_j|z(t)|^q.\end{aligned} \tag{3.2}$$

According to Assumption 2, the following inequalities are calculated:

$$q\theta_i|z(t)|^{q-2}z^T(t)\hat{G}(z(t), t, i) \leq q\theta_i|z(t)|^{q-1}|\hat{G}(z(t), i, t)| \leq q\theta_i l_i|z(t)|^q, \tag{3.3}$$

$$q\theta_i|z(t)|^{q-2}z^T(t)\Bigg(\sum_{h=1}^{H}\rho_h(i)\Big(B^h(i)\otimes\Upsilon\Big)z(t)\Bigg) \leq q\theta_i\Bigg(\sum_{h=1}^{H}\rho_h(i)\lambda_{\max}\Big(\Big[B^h(i)\otimes\Upsilon\Big]_s\Big)\Bigg)|z(t)|^q, \tag{3.4}$$

$$\begin{aligned}|z(t)|^{q-2}|\tilde{\beta}(z(t), t, i)|^2 &\leq \max\Big(\lambda_{\max}\Big(C_1^T(i)C_1(i)\Big), \lambda_{\max}\Big(C_2^T(i)C_2(i)\Big), \cdots, \lambda_{\max}\Big(C_M^T(i)C_M(i)\Big)\Big)|z(t)|^q \\ &= \sigma_i|z(t)|^q,\end{aligned} \tag{3.5}$$

$$|z(t)|^{q-4}|z^T(t)\tilde{\beta}(z(t), t, i)|^2 \geq \min\Big\{\lambda^2_{\min}\Big(C_1(i)\Big), \lambda^2_{\min}\Big(C_2(i)\Big), \cdots\cdots, \lambda^2_{\min}\Big(C_M(i)\Big)\Big\}|z(t)|^q = d_i|z(t)|^q. \tag{3.6}$$

Substituting inequalities (3.3)–(3.6) into equality (3.2) yields that

$$\mathcal{L}V(z(t),t,i) \leq \left[q\theta_i l_i + q\theta_i\left(\sum_{h=1}^{H}\rho_h(i)\lambda_{\max}\left(\left[B^h(i)\otimes\Upsilon\right]_s\right)\right) + \frac{1}{2}q\theta_i\sigma_i - \frac{1}{2}q(2-q)\theta_i d_i + \sum_{j=1}^{N}\psi_{ij}\theta_j\right]|z(t)|^q. \tag{3.7}$$

Since $Q\theta = (\lambda_1, \lambda_2, \cdots, \lambda_N)^T > 0$, when $t \in [t_i, s_i), i = 0, 1, 2, \cdots\cdots$, we have that

$$\mathcal{L}V(z(t),t,i) \leq -\lambda_i|z(t)|^q \leq -\frac{\lambda_i}{\theta_i}V(z(t),i,t) \leq -\min_{1\leq i\leq N}\left\{\frac{\lambda_i}{\theta_i}\right\}V(z(t),i,t) = -\tilde{\lambda}V(z(t),i,t). \tag{3.8}$$

Moreover, when $t \in [s_i, t_{i+1}), i = 0, 1, 2, \cdots\cdots$, it can be calculated that

$$\begin{aligned}\mathcal{L}V(z(t),t,i) &\leq \left[0.5q\Big((2-q)d(i)-\sigma(i)\Big)\theta_i - \lambda_i\right]|z(t)|^q \leq \frac{\zeta_i}{\theta_i}V(z(t),t,i) \leq \max_{1\leq i\leq N}\left\{\frac{\zeta_i}{\theta_i}\right\}V(z(t),i,t)\\ &= \tilde{\zeta}V(z(t),i,t).\end{aligned} \tag{3.9}$$

Particularly, according to inequality (3.8), if $t \in [t_0, s_0), t_0 = 0$, we can infer that

$$\mathbb{E}V(z(t),t,r(t)) \leq \theta_M\mathbb{E}|z_0|^q e^{-\tilde{\lambda}t}. \tag{3.10}$$

Moreover, combining inequalities (3.9) and (3.10), it can be deduced that

$$\mathbb{E}V(z(t),t,r(t)) \leq \theta_M\mathbb{E}|z(s_0)|^q e^{\tilde{\zeta}(t-s_0)} \leq \theta_M\mathbb{E}|z_0|^q e^{-\tilde{\lambda}(s_0-t_0)+\tilde{\zeta}(t-s_0)}. \tag{3.11}$$

Repeating the iteration leads to the following result,

$$\mathbb{E}V(z(t),t,r(t)) \leq \theta_M\mathbb{E}|z_0|^q e^{-\tilde{\lambda}\sum_{j=0}^{i-1}(s_j-t_j)+\tilde{\zeta}\sum_{j=1}^{i}(t_j-s_{j-1})-\tilde{\lambda}(t-t_i)}, t \in [t_i, s_i), \tag{3.12}$$

and

$$\mathbb{E}V(z(t),t,r(t)) \leq \theta_M\mathbb{E}|z_0|^q e^{-\tilde{\lambda}\sum_{j=0}^{i}(s_j-t_j)+\tilde{\zeta}\sum_{j=1}^{i}(t_j-s_{j-1})+\tilde{\zeta}(t-s_i)}, t \in [s_i, t_{i+1}), \tag{3.13}$$

which means that

$$\mathbb{E}V(z(t),t,r(t)) \leq \theta_M\mathbb{E}|z_0|^q e^{-\tilde{\lambda}N_c(0,t)+\tilde{\zeta}N_r(0,t)}. \tag{3.14}$$

In light of Assumption 1, one has that $N_c(t,0) \geq \alpha t - N_0$ and $N_r(t,0) \geq (1-\alpha)t + N_0$. Consequently,

$$\mathbb{E}V(z(t),t,r(t)) \leq \theta_M\mathbb{E}|z_0|^q e^{-\tilde{\lambda}\left[\alpha t - N_0\right]+\tilde{\zeta}\left[(1-\alpha)t+N_0\right]} \leq e^{N_0(\tilde{\lambda}+\tilde{\zeta})}\theta_M\mathbb{E}|z_0|^q e^{-[\tilde{\lambda}\alpha-\tilde{\zeta}(1-\alpha)]t}. \tag{3.15}$$

Therefore, one has that

$$\mathbb{E}|z(t)|^q \leq K_0\mathbb{E}|z_0|^q e^{-\epsilon t}, \tag{3.16}$$

where $K_0 = \frac{\theta_M e^{N_0(\tilde{\lambda}+\tilde{\zeta})}}{\theta_m}$, and $\epsilon = \tilde{\lambda}\alpha - \tilde{\zeta}(1-\alpha) > 0$.

Lemma 3.2. *Let $q \in (0,1)$ and Assumption 2 hold. The following estimations are presented:*

$$\mathbb{E}|\eta(t)|^2 \leq |\eta_0|^2 e^{(2\hat{l}+\hat{\sigma})t}, \tag{3.17}$$

$$\mathbb{E}|\eta(t)-\eta(\mu(t))|^2 \leq K_1(\tau) e^{(2\hat{l}+\hat{\sigma})t}, \tag{3.18}$$

$$\mathbb{E}\,|\eta(t)-z(t)|^q \leq |\eta_0|^q \left[\frac{4(\hat{l}+\hat{\sigma})K_1(\tau)}{2\hat{l}+\hat{\sigma}}\right]^{\frac{q}{2}} |\eta_0|^q e^{(\hat{l}+\hat{\sigma})qt}, \tag{3.19}$$

where $\hat{l} = \max\limits_{1\leq i\leq N}\left\{l_i + \sum\limits_{h=1}^{H}\rho_h(i)\lambda_{\max}\left(\left[B^h(i)\otimes\Upsilon\right]_s\right)\right\}$, $\hat{\sigma} = \max\limits_{0\leq i\leq N}\{\sigma_i\}$,
$\sigma_i = \max\left\{\lambda_{\max}\left(C_1^T(i)C_1(i)\right), \lambda_{\max}\left(C_2^T(i)C_2(i)\right), \cdots, \lambda_{\max}\left(C_M^T(i)C_M(i)\right)\right\}$,
$K_1(\tau) = \left\{4\tau\left[\left(\max\limits_{1\leq i\leq N}\{l_i\}\right)^2 + H\max\limits_{1\leq i\leq N}\left\{\sum\limits_{h=1}^{H}\rho_h^2(i)\lambda_{\max}\left(\left(B^h(i)\otimes\Upsilon\right)^T\left(B^h(i)\otimes\Upsilon\right)\right)\right\}\right] + 2\hat{\sigma}\right\}\tau.$

Proof. Error system (2.6) in matrix form can be formulated as

$$d\eta(t) = \left[\tilde{G}(\eta(t),t,r(t)) + \sum_{h=1}^{H}\rho_h(r(t))\Big(B^h(r(t))\otimes\Upsilon\Big)\eta(t)\right]dt + I(t)\hat{\beta}(\eta(\mu(t)),t,r(t))dB(t), \tag{3.20}$$

where $I(t) = \begin{cases}1, t\in[t_i,s_i),\\ 0, t\in[s_i,t_{i+1}), \qquad i=0,1,2,\cdots\cdots.\end{cases}$ By employing the Itô formula, one has that

$$\begin{aligned}\mathbb{E}|\eta(t)|^2 =\ & |\eta_0|^2 + \mathbb{E}\int_0^t\left\{2\eta^T(s)\Big[\tilde{G}(\eta(s),s,r(s)) + \sum_{h=1}^{H}\rho_h(r(s))\left(B^h(r(s))\otimes\Upsilon\right)\eta(s)\Big]\right.\\ & \left.+\left|\hat{\beta}\left(\eta(\mu(s)),s,r(s)\right)\right|^2 I(s)\right\}ds\\ \leq\ & |\eta_0|^2 + \int_0^t 2\max_{1\leq i\leq N}\left\{l_i + \sum_{h=1}^{H}\rho_h(i)\lambda_{\max}\Big(\Big[B^h(i)\otimes\Upsilon\Big]_s\Big)\right\}\mathbb{E}|\eta(s)|^2 ds\\ & + \max_{1\leq i\leq N}\{\sigma_i\}\int_0^t\mathbb{E}|\eta(\mu(s))|^2 ds.\end{aligned} \tag{3.21}$$

Moreover, it can be inferred that

$$\sup_{0\leq s\leq t}\mathbb{E}|\eta(s)|^2 \leq |\eta_0|^2 + (2\hat{l}+\hat{\sigma})\int_0^t\left(\sup_{0\leq s\leq t}\mathbb{E}|\eta(s)|^2\right)ds. \tag{3.22}$$

Applying the Gronwall inequality yields that

$$\sup_{0\leq s\leq t}\mathbb{E}\,|\eta(s)|^2 \leq |\eta_0|^2\, e^{(2\hat{l}+\hat{\sigma})t}. \tag{3.23}$$

Accordingly, it can be concluded that

$$\sup_{0\leq s\leq t}\mathbb{E}\,|\eta(s)|^2 \leq |\eta_0|^2\, e^{(2\hat{l}+\hat{\sigma})t}, \tag{3.24}$$

which indicates that

$$\mathbb{E}\left|\eta(t)\right|^2 \leq \left|\eta_0\right|^2 e^{(2\hat{l}+\hat{\sigma})t}. \tag{3.25}$$

Obviously, it follows from Eq (3.20) that

$$\eta(t)-\eta(\mu(t)) = \int_{\mu(t)}^{t}\left[\tilde{G}(\eta(s),s,r(s)) + \sum_{h=1}^{H}\rho_h(r(s))\left(B^h(r(s))\otimes\Upsilon\right)\right]\eta(s)ds \tag{3.26}$$

$$+\int_{\mu(t)}^{t} I(s)\hat{\beta}(\eta(\mu(s)),s,r(s))dw(s). \tag{3.27}$$

Together with the Hölder inequality, it can be calculated as

$$\begin{aligned}
\mathbb{E}\left|\eta(t)-\eta(\mu(t))\right|^2 \leq{}& 2\mathbb{E}\left|\int_{\mu(t)}^{t}\left[\tilde{G}(\eta(s),s,r(s)) + \sum_{h=1}^{H}\rho_h(r(s))\left(B^h(r(s))\otimes\Upsilon\right)\right]\eta(s)ds\right|^2 \\
&+2\mathbb{E}\int_{\mu(t)}^{t}\left|\hat{\beta}(\eta(\mu(s)),s,r(s))\right|^2 ds \\
\leq{}& 4\tau\left[\left(\max_{1\leq i\leq N}\{l_i\}\right)^2 + H\max_{1\leq i\leq N}\left\{\sum_{h=1}^{H}\rho_h^2(i)\lambda_{max}\left(\left(B^h(i)\otimes\Upsilon\right)^T\left(B^h(i)\otimes\Upsilon\right)\right)\right\}\right]\int_{\mu(t)}^{t}\mathbb{E}|\eta(s)|^2ds \\
&+2\hat{\sigma}\int_{\mu(t)}^{t}\mathbb{E}|\eta(\mu(s))|^2ds \\
\leq{}& \frac{K_1(\tau)}{\tau}\int_{\mu(t)}^{t}\sup_{0\leq s\leq t}\mathbb{E}\left|\eta(s)\right|^2 du \\
\leq{}& K_1(\tau)\left|\eta_0\right|^2 e^{(2\hat{l}+\hat{\sigma})t}.
\end{aligned} \tag{3.28}$$

On the other hand, by utilizing the Itô formula again, one gets that

$$\begin{aligned}
\mathbb{E}\left|\eta(t)-z(t)\right|^2 ={}& 2\mathbb{E}\int_0^t\Big\{[\eta(s)-z(s)]^T\Big[\hat{G}(\eta(s),s,r(s))-\hat{G}(z(s),s,r(s)) \\
&+\sum_{h=1}^{H}\rho_h(r(s))\left(B^h(r(s))\otimes\Upsilon\right)(\eta(s)-z(s))\Big] \\
&+\left|\hat{\beta}(\eta(\mu(s)),s,r(s))-\hat{\beta}(\eta(s),s,r(s))\right|^2 I(s)\Big\}ds \\
\leq{}& 2\left[\max_{1\leq i\leq N}\{l_i\} + \max_{1\leq i\leq N}\left\{\sum_{h=1}^{H}\rho_h(i)\lambda_{max}\left(\left[B^h(i)\otimes\Upsilon\right]_s\right)\right\}\right]\int_0^t\mathbb{E}|\eta(s)-z(s)|^2ds \\
&+\max_{1\leq i\leq N}\left\{\lambda_{\max}\left(C_1^T(i)C_1(i)\right),\lambda_{\max}\left(C_2^T(i)C_2(i)\right),\cdots\cdots,\lambda_{\max}\left(C_M^T(i)C_M(i)\right)\right\} \\
&\times\int_0^t\mathbb{E}|\eta(\mu(s))-z(s)|^2ds \\
\leq{}& 2\hat{l}\int_0^t\mathbb{E}|\eta(s)-z(s)|^2ds + \hat{\sigma}\int_0^t\mathbb{E}|\eta(\mu(s)-z(s)|^2ds
\end{aligned}$$

$$\leq (2\hat{l}+2\hat{\sigma})\int_0^t \mathbb{E}|\eta(s)-z(s)|^2 ds + 2\hat{\sigma}\int_0^t \mathbb{E}|\eta(\mu(s)-\eta(s)|^2 ds. \tag{3.29}$$

Recalling assertion (3.18), it can be deduced that

$$\int_0^t \mathbb{E}|\eta(\mu(s)-\eta(s)|^2 ds \leq K_1(\tau)|\eta_0|^2 \int_0^t e^{(2\hat{l}+\hat{\sigma})s} ds \leq \frac{K_1(\tau)|\eta_0|^2}{2\hat{l}+\hat{\sigma}}\left[e^{(2\hat{l}+\hat{\sigma})t}-1\right]. \tag{3.30}$$

Substituting inequality (3.30) into inequality (3.29) leads to

$$\mathbb{E}|\eta(t)-z(t)|^2 \leq (2\hat{l}+2\hat{\sigma})\int_0^t \mathbb{E}|\eta(s)-z(s)|^2 ds + \frac{2\hat{\sigma}K_1(\tau)|\eta_0|^2}{2\hat{l}+\hat{\sigma}}\left[e^{(2\hat{l}+\hat{\sigma})t}-1\right]. \tag{3.31}$$

Subsequently, by using the Gronwall inequality [38], it can be readily obtained that

$$\begin{aligned}\mathbb{E}|\eta(t)-z(t)|^2 &\leq \frac{2\hat{\sigma}K_1(\tau)|\eta_0|^2}{2\hat{l}+\hat{\sigma}}\left[e^{(2\hat{l}+\hat{\sigma})t}-1\right] + \frac{4\hat{\sigma}(\hat{l}+\hat{\sigma})K_1(\tau)|\eta_0|^2}{2\hat{l}+\hat{\sigma}}\int_0^t e^{2(\hat{l}+\hat{\sigma})(t-s)}[e^{(2\hat{l}+\hat{\sigma})s}-1]ds \\ &\leq \left[\frac{4(\hat{l}+\hat{\sigma})K_1(\tau)}{2\hat{l}+\hat{\sigma}}\right]|\eta_0|^2 e^{2(\hat{l}+\hat{\sigma})t}. \end{aligned} \tag{3.32}$$

Hence, in light of the Hölder inequality, one can further acquire the following assertion:

$$\mathbb{E}|\eta(t)-z(t)|^q \leq \left(\mathbb{E}|\eta(t)-z(t)|^2\right)^{\frac{q}{2}} \leq \left[\frac{4(\hat{l}+\hat{\sigma})K_1(\tau)}{2\hat{l}+\hat{\sigma}}\right]^{\frac{q}{2}} |\eta_0|^q e^{q(\hat{l}+\hat{\sigma})t}. \tag{3.33}$$

Theorem 3.1. *Suppose that all of the conditions in Lemma 3.1 are satisfied. Let $\xi \in (0,1)$. $\tau^* > 0$ is the unique positive root of the following equation:*

$$\xi + K_2(\tau)e^{(\hat{l}+\hat{\sigma})q(\tau+\frac{1}{\epsilon}\log(\frac{K_0}{\xi}))} = 1, \tag{3.34}$$

where $\theta_m = \min_{0\leq i\leq N}\{\theta_i\}$, $\theta_M = \max_{0\leq i\leq N}\{\theta_i\}$, $K_2(\tau) = \left[\frac{4(\hat{l}+\hat{\sigma})K_1(\tau)}{2\hat{l}+\hat{\sigma}}\right]^{\frac{q}{2}}$, and ϵ, θ_i, $\hat{\rho}$, $\hat{\sigma}$, K_0, and K_1 are the same as in Lemmas 3.1 and 3.2. If $0 < \tau < \tau^*$, then the multilayer complex network system (2.1) can achieve almost sure exponential synchronization with isolated node (2.3) via aperiodically intermittent noise control, i.e.,

$$\limsup_{t\to\infty}\frac{1}{t}\log|\eta(t)| \leq -\frac{2\hat{\delta}}{5q} < 0, \qquad a.s. \tag{3.35}$$

Proof. For given positive constant $\xi \in (0,1)$, we can choose sufficiently large positive integer m_0 such that

$$\frac{\log(\frac{K_0}{\xi})}{\epsilon\tau} \leq m_0 < 1 + \frac{\log(\frac{K_0}{\xi})}{\epsilon\tau}. \tag{3.36}$$

Combining the above inequality with Lemma 3.1, one gets that

$$K_0 e^{-m_0\epsilon\tau} \leq \xi, \;\; \mathbb{E}|z(m_0\tau)|^q \leq K_0 e^{-m_0\epsilon\tau}|\eta_0|^q \leq \xi|\eta_0|^q. \tag{3.37}$$

Noting that

$$\xi + K_2(\tau)e^{(\hat{l}+\hat{\sigma})qm_0\tau} < \xi + K_2(\tau)e^{(\hat{l}+\hat{\sigma})q(\tau+\frac{1}{\epsilon}\log(\frac{K_0}{\xi}))} = 1, \tag{3.38}$$

there exists a constant $\hat{\delta} > 0$ such that

$$\xi + K_2(\tau)e^{(\hat{l}+\hat{\sigma})qm_0\tau} < e^{-\hat{\delta}m_0\tau}. \tag{3.39}$$

For simplicity, let $z_{m_0} = z(m_0\tau, \eta_0, r_0)$ and $\eta_{m_0} = \eta(m_0\tau, \eta_0, r_0)$. Combing Lemma 3.2 and the elementary inequality $(u+v)^q \le u^q + v^q$, $u \ge 0, v \ge 0, q \in (0,1)$, one can compute that

$$\begin{aligned} E|\eta_{m_0}|^q &\le E|z_{m_0}|^q + E|\eta_{m_0} - z_{m_0}|^q \\ &\le K_0 e^{-m_0\epsilon\tau}|\eta_0|^q + |\eta_0|^q K_2(\tau)e^{(\hat{l}+\hat{\sigma})qm_0\tau} \\ &\le |\eta_0|^q\left[\xi + K_2(\tau)e^{(\hat{l}+\hat{\sigma})qm_0\tau}\right] \\ &\le |\eta_0|^q e^{-\hat{\delta}m_0\tau}. \end{aligned} \tag{3.40}$$

When $t \ge m\tau$, it can be deduced that

$$\mathbb{E}\left(|\eta_{2m_0}|^q|\mathcal{F}_{m_0\tau}\right) \le |\eta_{m_0}|^q e^{-\hat{\delta}m_0\tau} \le |\eta_0|^q e^{-2\hat{\delta}m_0\tau}. \tag{3.41}$$

Similarly, one can derive that

$$\mathbb{E}|\eta_{im_0}|^q \le \mathbb{E}|\eta_{(i-1)m_0}|^q e^{-\hat{\delta}m_0\tau} \le \cdots\cdots \le \mathbb{E}|\eta_{m_0}|^q e^{-\hat{\delta}(i-1)m_0\tau} \le |\eta_0|^q e^{-\hat{\delta}im_0\tau} \qquad i = 1,2,3,\cdots\cdots. \tag{3.42}$$

Let $\tau \in (0, \tau^*)$. It follows from Eq (3.14) that

$$\eta(t) = \eta_0 + \int_0^t \left[\tilde{G}(\eta(s), s, r(s)) + \sum_{h=1}^{H}\rho_h(r(s))\Big(B^h(r(s))\otimes\Upsilon\Big)\eta(s)\right]ds + I(s)\hat{\beta}(\eta(\mu(s)), s, r(s))dB(s).$$

By exploiting the Hölder inequality and the Burkholder-Davis-Gundy inequality, it can be estimated that

$$\begin{aligned} \mathbb{E}\Big(\sup_{0\le t\le m_0\tau}|\eta(t)|^2\Big) \le\ & 3|\eta_0|^2 + 3\mathbb{E}\Big\{\sup_{0\le t\le m_0\tau}|\int_0^t \widetilde{G}(\eta(s), s, r(s)) + \sum_{h=1}^{H}\rho_h(r(s))\Big(B^h(r(s))\otimes\Upsilon\Big)\eta(s)ds|^2\Big\} \\ & + 3\mathbb{E}\Big\{\sup_{0\le t\le m_0\tau}|\int_0^t I(s)\hat{\beta}(\eta(\mu(s)), s, r(s))dB(s)|^2\Big\} \\ \le\ & 3|\eta_0|^2 + 6m\tau\Big[(\max_{1\le i\le N}\{l_i\})^2 + H\max_{1\le i\le N}\Big\{\sum_{i=1}^{H}\rho_h^2(i)\lambda_{\max}\Big(B^h(i)\otimes\Upsilon)^T(B^h(i)\otimes\Upsilon)\Big)\Big\}\Big]. \\ & \mathbb{E}\Big(\sup_{0\le t\le m_0\tau}\int_0^t|\eta(s)|^2ds\Big) + 12\hat{\sigma}\mathbb{E}\Big(\sup_{0\le t\le m_0\tau}\int_0^t|\eta(\mu(s))|^2ds\Big) \\ \le\ & 3|\eta_0|^2 + 6m_0\tau\Big[(\max_{1\le i\le N}\{l_i\})^2 + H\max_{1\le i\le N}\Big\{\sum_{i=1}^{H}\rho_h^2(i)\lambda_{\max}\Big((B^h(i)\otimes\Upsilon)^T(B^h(i)\otimes\Upsilon)\Big)\Big\} \end{aligned}$$

$$+12\hat{\sigma}\Big]\int_0^{m_0\tau}\mathbb{E}\Big(\sup_{0\le t\le s}|\eta(t)|^2\Big)ds. \tag{3.43}$$

Moreover, by the Gronwall inequality, it can be attained that

$$\mathbb{E}\Big(\sup_{0\le t\le m_0\tau}|\eta(t)|^2\Big)\le 3|\eta_0|^2e^{K_3(\tau)m_0\tau}, \tag{3.44}$$

where $K_3(\tau)=6m_0\tau\Big[(\max_{1\le i\le N}\{l_i\})^2+H\max_{1\le i\le N}\Big\{\sum_{i=1}^H\rho_h^2(i)\lambda_{\max}\Big((B^h(i)\otimes\Upsilon)^T(B^h(i)\otimes\Upsilon)\Big)\Big\}+12\hat{\sigma}\Big]$. By virtue of the Hölder inequality, one gets that

$$\mathbb{E}\Big(\sup_{0\tau\le t\le m_0\tau}|\eta(t)|^q\Big)\le 3^{\frac{q}{2}}|\eta_0|^qe^{\frac{qK_3(\tau)m\tau}{2}}=K_4(\tau)|\eta_0|^q, \tag{3.45}$$

where $K_4(\tau)=3^{\frac{q}{2}}e^{\frac{qK_3(\tau)m\tau}{2}}$. Accordingly, based on the time stationarity of stochastic differential equations, it can be derived that

$$\mathbb{E}\left(\sup_{im_0\tau\le t\le(i+1)m_0\tau}|\eta(t)|^q|\mathcal{F}_{im_0\tau}\right)\le K_4(\tau)|\eta_{im_0\tau}|^q, i=0,1,2,\cdots\cdots. \tag{3.46}$$

Combining (3.42) and (3.46) gives

$$\mathbb{E}\left(\sup_{im_0\tau\le t\le(i+1)m_0\tau}|\eta(t)|^q\right)\le K_4(\tau)\mathbb{E}|\eta_{im_0\tau}|^q\le K_4(\tau)|\eta_0|^qe^{-\hat{\delta}im_0\tau}. \tag{3.47}$$

According to the Markov inequality, one gets that

$$P\left\{\sup_{im_0\tau\le t\le(i+1)m_0\tau}|\eta(t)|^q\ge e^{-0.4m_0\hat{\delta}i\tau}\right\}\le K_4(\tau)|\eta_0|^qe^{-0.6m_0\hat{\delta}i\tau},\qquad i=1,2,\cdots\cdots. \tag{3.48}$$

As a result, by the Borel-Cantelli lemma, for almost all $\omega\in\Omega$, there exists a stochastic integer $i_0=i_0(\omega)$ satisfying

$$\sup_{i^*m_0\tau\le t\le(i^*+1)m_0\tau}|\eta(t)|^q<e^{-0.4m_0\hat{\delta}i^*\tau}, \tag{3.49}$$

for $i^*>i_0(\omega)$. Hence, it can be concluded that

$$\limsup_{t\to\infty}\frac{1}{t}\log|\eta(t)|\le-\frac{2\hat{\delta}}{5q}<0,\qquad a.s. \tag{3.50}$$

which signifies that the multilayer complex network system (2.1) realizes almost sure exponential synchronization with isolated node (2.3) via aperiodically intermittent noise control.

Remark 1. *Recently, stability and synchronization of multilayer complex networks have been discussed by developing various control approaches such as state feedback control, event-triggered control, impulsive control, and intermittent control in [12, 14, 16, 21]. Compared with the previous findings, this paper has dealt with the almost sure exponential synchronization issue of MCNS by adopting aperiodically intermittent discrete observation noise control, where the stochastic noise can be viewed* *as control input in the diffusion term rather than the drift term [22]. In the considered control scheme, the average control rate has been integrated to depict the distributions of work/rest intervals of the control strategy from an overall perspective. In general, as the average control rate become larger, the control performance will be better.*

Remark 2. *Different from the work about pth moment exponential synchronization of network systems in [21, 22], by utilizing M-matrix theory and various stochastic analysis techniques including the Itô formula, the Gronwall inequality, and the Borel-Cantelli lemma, almost sure exponential synchronization of multilayer coupled networks with Markovian switching has been analyzed via AIDONC. Although some sufficient conditions on almost sure exponential synchronization of MCNS have been given in [23], the control strategy AIDONC has not been adopted. Additionally, in [35, 36], almost sure exponential synchronization and stabilization of neural networks have been investigated under AIDONC and the theoretical work has been extended to hybrid neural networks and complex networks [37–39]. In this paper, Markovian switching and multilayer interaction factors have been taken into account simultaneously, which make our network system more general.*

Remark 3. *In this paper, we utilize the stochastic operator $\mathcal{L}V$ and matrix multiplication to investigate the almost sure exponential synchronization of stochastic multilayer complex networks. Actually, fast algorithms about matrix multiplication are an interesting and hot topic which can be applied in contemporary Intel Xeon microprocessors. Particularly, in [40], a broad series of algorithms taking advantage of the efficiency of fast matrix multiplication algorithms in other mathematical and computer science operations have been reported. Extending the applications of these algorithms to complex networks remains a current challenge.*

In particular, when $\tau = 0$ and $H = 1$, the multilayer networks are reduced to the following single-layered networks.

$$d\delta_k(t) = [G(\delta_k(t), t, r(t)) + \rho(r(t)) \sum_{m=1}^{M} b_{km}(r(t))\Upsilon\delta_m(t)]dt + u_k(t, r(t)), \qquad k = 1, 2, \cdots\cdots, M. \quad (3.51)$$

Corollary 1. *Let $Q = \mathrm{diag}\{\chi_1, \chi_2, \cdots\cdots, \chi_N\} - \Psi$ denote an M-matrix, where $q \in (0, 1)$, $\chi_i = 0.5q\Big((2-q)d(i) - \sigma(i)\Big) - ql_i - q\bigg(\rho(i)\lambda_{\max}\Big(\big[B(i) \otimes \Upsilon\big]_s\Big)\bigg)$, and $\Psi = (\psi_{ij})_{N\times N}$. Under Assumptions 1 and 2, if $\epsilon = \tilde{\lambda}\alpha - \tilde{\zeta}(1-\alpha) > 0$, complex network (3.51) can achieve almost sure exponential synchronization with the isolated node (2.3), where $\sigma_i = \max\Big\{\lambda_{\max}\Big(C_1^T(i)C_1(i)\Big), \lambda_{\max}\Big(C_2^T(i)C_2(i)\Big), \cdots\cdots, \lambda_{\max}\Big(C_M^T(i)C_M(i)\Big)\Big\}$, $d_i = \min\Big\{\lambda_{\min}^2\Big(C_1(i)\Big), \lambda_{\min}^2\Big(C_2(i)\Big), \cdots\cdots, \lambda_{\min}^2\Big(C_M(i)\Big)\Big\}$, $Q\theta = Q(\theta_1, \theta_2, \cdots\cdots, \theta_N)^T = (\lambda_1, \lambda_2, \cdots\cdots, \lambda_N)^T > 0$, $\tilde{\lambda} = \min_{1\le i\le N}\Big\{\frac{\lambda_i}{\theta_i}\Big\} > 0$, $\zeta_i = 0.5q\Big[(2-q)d_i - \sigma_i\Big]\theta_i - \lambda_i$, $\tilde{\zeta} = \max_{1\le i\le N}\Big\{\frac{\zeta_i}{\theta_i}\Big\} > 0$, $\theta_m = \min_{i\in S}\{\theta_i\}$, $\theta_M = \max_{i\in S}\{\theta_i\}$, and $K_0 = \frac{\theta_M e^{N_0(\tilde{\lambda}+\tilde{\zeta})}}{\theta_m}$.*

In particular, when $\theta = (\theta_1, \theta_2, \cdots\cdots, \theta_n)^T = (1, 1, \cdots\cdots, 1)^T$ *and* $H = 1$, immediately, we acquire the following assertion.

Corollary 2. *Let all the conditions in Theorem 1 hold except $(\lambda_1, \lambda_2, \cdots\cdots, \lambda_n)^T = (1, 1, \cdots\cdots, 1)^T$, $\chi_i = 0.5q\Big[(2-q)d_i - \sigma_i\Big] - ql_i - q\bigg(\rho(i)\lambda_{\max}\Big(\big[B(i) \otimes \Upsilon\big]_s\Big)\bigg)$, and $\hat{l} = \max_{1\le i\le N}\{l_i + \rho(i)\lambda_{\max}([B(i) \otimes \Upsilon]_s)\}$. If $0 < \tau < \tau^*$, then the multilayer coupled network system (3.51) can achieve almost sure exponential* *synchronization with isolated node (2.3) via aperiodically intermittent noise control, i.e.,*

$$\limsup_{t\to\infty} \frac{1}{t}\log|\eta(t)| \leq -\frac{2\hat{\delta}}{5q} < 0, \qquad a.s. \tag{3.52}$$

Remark 4. *Fractional differences can be seen as generalization or as an extension of classical calculus while fractional order derivatives are non-locally distributed, which can describe the memory and hereditary effects of complex processes and materials accurately. In recent years, many mathematical models in engineering and biological sciences have been proposed by using discrete or continuous fractional equations [41–45]. Particularly, in [41, 42], some dynamic behaviors such as the existence and stability of solutions of worms in a wireless sensor model in the sense of the fractal fractional and fractional nabla difference COVID-19 model have been analyzed through function analysis and the Ulam-Hyers stability technique. Furthermore, some properties such as existence, uniqueness, controllability, stability of solutions of some equations including Hilfer fractional evolution equations, ABC-fuzzy-Volterra integro-differential equations, and coupled pantograph discrete fractional order difference equations have been discussed in [43–45]. Actually, fractional complex networks have been a current attractive topic. In the future, our work can be further extended to stochastic fractional complex networks.*

4. Numerical example

In order to exhibit the feasibility of our theoretical findings, a famous Chua's circuits network is considered here, which has been extensively applied to various areas as an essential nonlinear electronic oscillator model. Initially, the single Chua's circuit is described as follows:

$$\begin{cases} \dot{V}_1(t) = -\dfrac{1}{R_1C_1^*}V_1(t) + \dfrac{1}{R_1C_1^*}V_2(t) - \dfrac{1}{C_1^*}\Gamma(V_1(t)); \\ \dot{V}_2(t) = \dfrac{1}{R_1C_2^*}V_1(t) - \dfrac{1}{R_1C_2^*}V_2(t) + \dfrac{1}{C_2^*}I_3(t); \\ \dot{I}_3(t) = -\dfrac{1}{L_0}(V_2(t) + R_2I_3(t)); \end{cases} \tag{4.1}$$

where $V_1(t)$ and $V_2(t)$ denote the voltages across the capacitors C_1^* and C_2^*. Meanwhile, I_3 signifies the current through the inductance L_0, and R_1 and R_2 stands for the linear resistors. $\Gamma(V_1(t))$ denotes the current through the nonlinear resistor N_{R_1}, which can be recasted as $\Gamma(V_1(t)) = \varpi_1 V_1 + 0.5(\varpi_2 - \varpi_1)(|V_1+1| - |V_1-1|), \varpi_1 = -0.05, \varpi_2 = -0.07$. Soon afterward, we consider a multilayer coupled network of Chua's circuits with Markovian switching as follows:

$$\begin{aligned} \begin{pmatrix} d\delta_{k1}(t) \\ d\delta_{k2}(t) \\ d\delta_{k3}(t) \end{pmatrix} &= \begin{pmatrix} \tilde{F}_{11}(r(t)) & \tilde{F}_{12}(r(t)) & 0 \\ \tilde{F}_{21}(r(t)) & \tilde{F}_{22}(r(t)) & \tilde{F}_{23}(r(t)) \\ 0 & \tilde{F}_{32}(r(t)) & \tilde{F}_{33}(r(t)) \end{pmatrix} \begin{pmatrix} \delta_{k1}(t) \\ \delta_{k2}(t) \\ \delta_{k3}(t) \end{pmatrix} dt + \begin{pmatrix} \tilde{\Gamma}_k(r(t)) \\ 0 \\ 0 \end{pmatrix} dt \\ &+ \sum_{h=1}^{H}\sum_{m=1}^{M} \rho_h(r(t)) b_{km}^h(r(t)) \Upsilon \delta_m(t) dt + u_k(t, r(t)), \end{aligned} \tag{4.2}$$

where $\delta_k(t) = (\delta_{k1}(t), \delta_{k2}(t), \delta_{k3}(t))^T = (V_{k1}, V_{k2}, I_{k3})^T$, $(k = 1,2,3,4,5,6)$, $\tilde{F}_{11}(r(t)) = -\frac{1}{R_1(r(t))C_1^*(r(t))}$, $\tilde{F}_{12}(r(t)) = -\tilde{F}_{11}(r(t))$, $\tilde{F}_{21}(r(t)) = \frac{1}{R_1(r(t))C_2^*(r(t))}$, $\tilde{F}_{22}(r(t)) = -\frac{1}{R_1(r(t))C_2^*(r(t))}$, $\tilde{F}_{23}(r(t)) =$

$\frac{1}{C_2^*(r(t))}$, $\tilde{F}_{32}(r(t)) = -\frac{1}{L_0(r(t))}$, $\tilde{F}_{33}(r(t)) = -\frac{R_2(r(t))}{L_0(r(t))}$, and $\tilde{\Gamma}_k(r(t)) = -\frac{\Gamma(\delta_{k1}(t))}{C_1^*(r(t))}$. Accordingly, the isolated node of the network is expressed as

$$ds(t) = G(s(t), t, r(t))dt. \tag{4.3}$$

Meanwhile, controller $u_k(t, r(t))$ is designed as

$$u_k(t) = \begin{cases} C_k(r(t))\eta_k(\mu(t))dw(t), t \in [t_i, s_i), \\ 0, t \in [s_i, t_{i+1}), \end{cases} \tag{4.4}$$

where $\eta_k(t) = \delta_k(t) - s(t)$ and $\mu(t) = [t/\tau]\,\tau, \tau > 0$, and τ denotes the duration between two consecutive observations. It is supposed that Markov jump $r(t) \in \mathbb{S} = \{1, 2\}$ with generator

$$\Psi = \begin{pmatrix} -4 & 4 \\ 1 & -1 \end{pmatrix}.$$

Parameters M and H can be selected as $M = 6$ and $H = 3$. When $r(t) = 1$, let $C_1^*(1) = 2$, $C_2^*(1) = 4$, $R_1^*(1) = 2$, $R_2^*(1) = 0.5$, $L_0^*(1) = 4$, $\rho_1(1) = 0.6$, $\rho_2(1) = 0.5$, and $\rho_3(1) = 0.4$. When $r(t) = 2$, let $C_1^*(2) = 4$, $C_2^*(2) = 2$, $R_1^*(2) = 2.5$, $R_2^*(2) = 1$, $L_0^*(2) = 5$, $\rho_1(2) = 0.4$, $\rho_2(2) = 0.6$, and $\rho_3(2) = 0.5$. The values of six adjacency matrices $B^1(1), B^2(1), B^3(1)$, $B^1(2)$, $B^2(2)$, and $B^3(2)$ are given as

$$B^1(1) = \begin{bmatrix} -0.3 & 0.1 & 0.1 & 0 & 0 & 0.1 \\ 0 & -0.2 & 0.1 & 0 & 0.1 & 0 \\ 0 & 0 & -0.1 & 0.1 & 0 & 0 \\ 0.1 & 0.1 & 0 & -0.3 & 0.1 & 0 \\ 0.1 & 0 & 0 & 0 & -0.2 & 0.1 \\ 0 & 0 & 0.1 & 0.1 & 0 & -0.2 \end{bmatrix}, B^2(1) = \begin{bmatrix} -0.2 & 0.1 & 0 & 0.1 & 0 & 0 \\ 0 & -0.3 & 0.1 & 0 & 0.1 & 0.1 \\ 0.1 & 0 & -0.2 & 0.1 & 0 & 0 \\ 0 & 0.1 & 0 & -0.2 & 0 & 0.1 \\ 0.1 & 0 & 0 & 0 & -0.1 & 0 \\ 0.1 & 0 & 0.1 & 0.1 & 0 & -0.3 \end{bmatrix},$$

$$B^3(1) = \begin{bmatrix} -0.4 & 0.1 & 0.1 & 0 & 0.1 & 0.1 \\ 0.1 & -0.3 & 0 & 0.1 & 0.1 & 0 \\ 0 & 0 & -0.2 & 0.1 & 0 & 0.1 \\ 0 & 0 & 0 & -0.1 & 0.1 & 0 \\ 0.1 & 0 & 0.1 & 0 & -0.2 & 0 \\ 0 & 0.1 & 0 & 0.1 & 0 & -0.2 \end{bmatrix}, B^1(2) = \begin{bmatrix} -0.3 & 0.1 & 0 & 0.1 & 0 & 0.1 \\ 0 & -0.2 & 0.1 & 0 & 0.1 & 0 \\ 0.1 & 0 & -0.3 & 0.1 & 0 & 0.1 \\ 0 & 0.1 & 0 & -0.2 & 0.1 & 0 \\ 0 & 0 & 0.1 & 0 & -0.1 & 0 \\ 0.1 & 0 & 0.1 & 0.1 & 0 & -0.3 \end{bmatrix},$$

$$B^2(2) = \begin{bmatrix} -0.3 & 0.1 & 0 & 0.1 & 0 & 0.1 \\ 0 & -0.1 & 0.1 & 0 & 0 & 0 \\ 0 & 0 & -0.2 & 0.1 & 0.1 & 0 \\ 0 & 0.1 & 0.1 & -0.3 & 0.1 & 0 \\ 0.1 & 0 & 0 & 0 & -0.2 & 0.1 \\ 0 & 0.1 & 0 & 0.1 & 0 & -0.2 \end{bmatrix}, B^3(2) = \begin{bmatrix} -0.2 & 0 & 0.1 & 0 & 0.1 & 0 \\ 0.1 & -0.3 & 0 & 0 & 0.1 & 0.1 \\ 0.1 & 0 & -0.2 & 0.1 & 0 & 0 \\ 0 & 0.1 & 0 & -0.2 & 0.1 & 0 \\ 0.1 & 0 & 0.1 & 0 & -0.3 & 0.1 \\ 0 & 0.1 & 0 & 0.1 & 0 & -0.2 \end{bmatrix}.$$

Furthermore, choose $q = 0.5, \Upsilon = 0.5I, \alpha = 0.8, C_k(r(t)) = 2I, \xi = 0.9$, and $N_0 = 0.01$. By calculation, we can obtain $l_1 = 0.3215, l_2 = 0.3561$, $\hat{l} = 0.3633, \hat{\sigma} = 4$, $\chi_1 = 0.3349, \chi_2 = 0.3184$, $\lambda_1 = \lambda_2 = 1, \theta_1 = 3.1009, \theta_2 = 3.1105, \hat{\lambda} = 0.3215, \hat{\zeta} = 0.1785$, and $\epsilon = \tilde{\lambda}\alpha - \tilde{\zeta}(1-\alpha) = 0.2215 > 0$. According to $\xi + K_2(\tau)e^{(\hat{l}+\hat{\sigma})q(\tau+\frac{1}{\epsilon}\log(\frac{K_0}{\xi}))} = 1$, the upper bound of duration time τ is estimated as $\tau < \tau* = 3.6213 \times 10^{-5}$. Therefore, all of the conditions of Theorem 1 are satisfied, and the almost sure exponential synchronization between network Eq (4.2) and isolated node Eq (4.3) is realized. Figure 1 shows a right continuous Markov chain with initial data $r(0) = 2$. Meanwhile, Figures 2–4 illustrate the synchronization sample trajectories $\eta_{i1}, \eta_{i2}, \eta_{i3}, (i = 1, 2, 3, 4, 5, 6)$ between network Eq (4.2) and isolated node Eq (4.3), respectively. It can be observed from Figures 2–4 that the error system is stable under aperiodically intermittent discrete observation noise control, which means that almost sure exponential synchronization is achieved. Therefore, the numerical simulations validate the effectiveness of the theoretical findings. By increasing the average control rate and the control intensity, the synchronization performance including the exponential convergent rate will be further improved. In comparison, this method is one discontinuous control, which can save cost and promote efficiency.

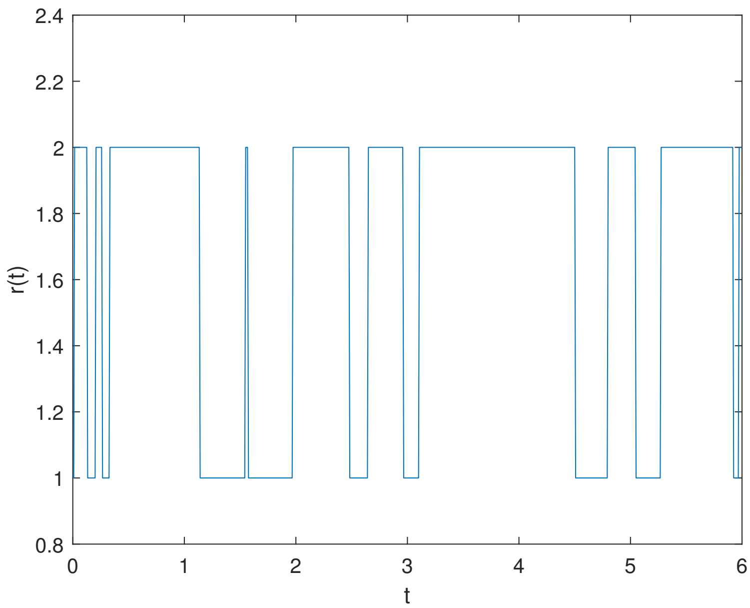

Figure 1. The Markov chain.

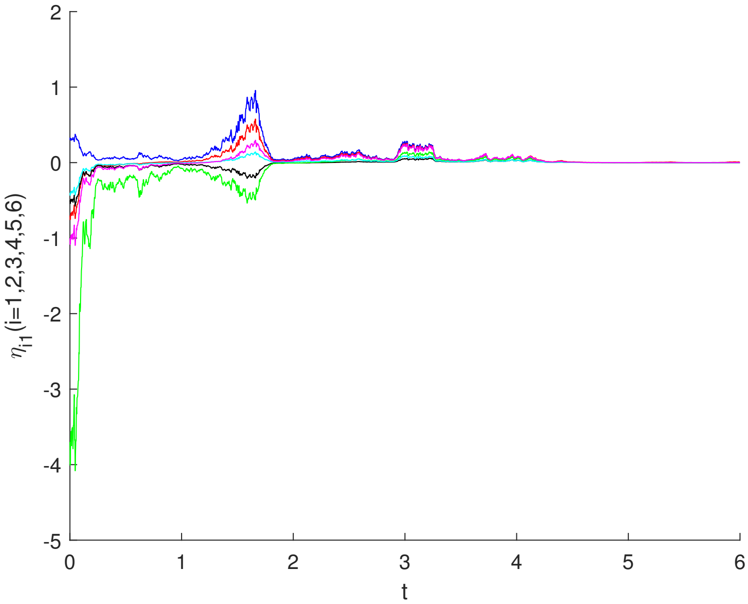

Figure 2. Synchronization errors $\eta_{i1}(t)$ between network Eq (4.2) and isolated node Eq (4.3).

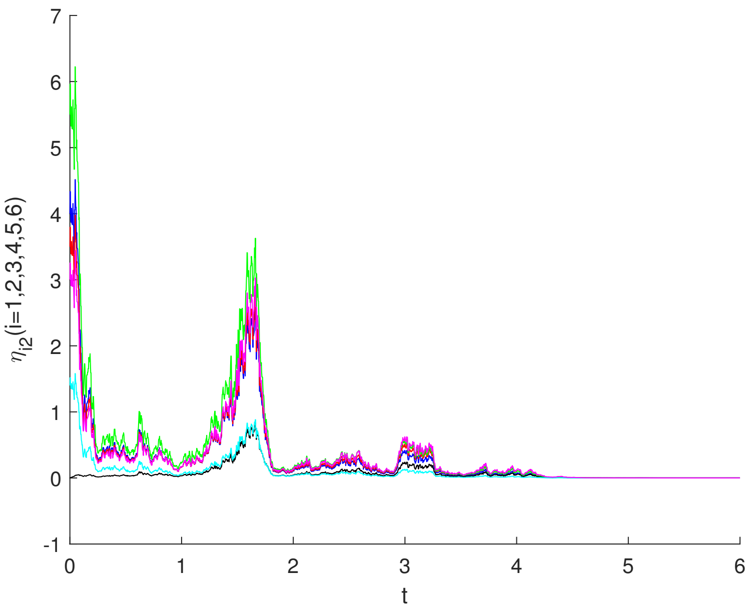

Figure 3. Synchronization errors $\eta_{i2}(t)$ between network Eq (4.2) and isolated node Eq (4.3).

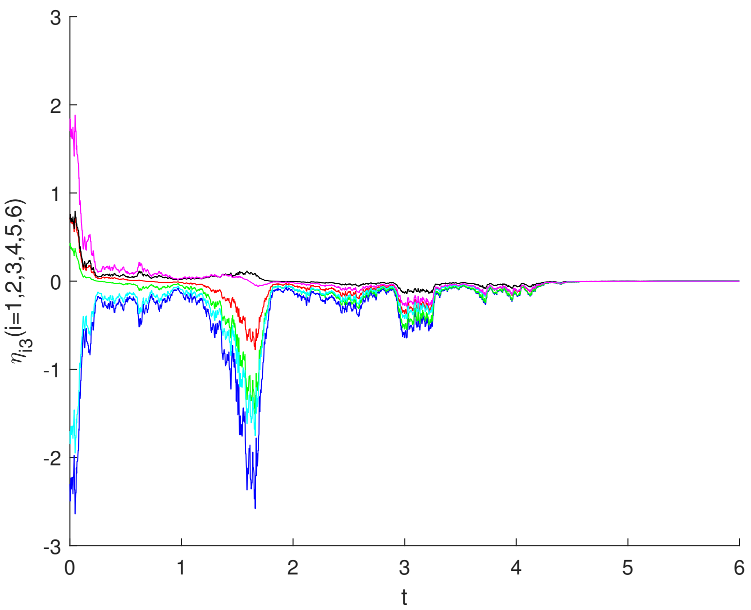

Figure 4. Synchronization errors $\eta_{i3}(t)$ between network Eq (4.2) and isolated node Eq (4.3).

5. Conclusions

In this paper, almost sure exponential synchronization of multilayer complex networks with Markovian switching via AIDONC is investigated. Different from the work about pth moment exponential synchronization of network systems in [21, 22], by utilizing M-matrix theory and various stochastic analysis techniques including the Itô formula, the Gronwall inequality, and the Borel-Cantelli lemma, some criteria on almost sure exponential synchronization of multilayer coupled networks with Markovian switching are constructed and the upper bound of the duration time is also estimated. It is noted that the control strategy is based on aperiodically intermittent discrete observation noise, where the average control rate is integrated to depict the distributions of work/rest intervals of the control strategy from an overall perspective. Finally, some numerical simulations are exhibited to illustrate the effectiveness and feasibility of our analytical findings. Although the aperiodically intermittent discrete observation noise control strategy can reduce energy consumption and save control cost, the derived algebraic criteria on almost sure exponential synchronization generally are comparatively difficult to calculate. Meanwhile, the duration time is comparatively small, and its length needs to be extended by designing several optimization algorithms in order to demonstrate the feasibility in practise. In the future, based on the work in [46, 47], the issue of stabilization and synchronization of coupled networks with semi-Markovian switching based on AIDONC will be further explored.

Author contributions

Li Liu: conceptualization, investigation, writing-original draft; Yinfang Song: funding acquisition, methodology, supervision; Hong Yu: software; Gang Zhang: writing-review and editing. All authors have read and approved the final version of the manuscript for publication.

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Conflict of interest

The authors declare no conflict of interest.

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