AIMS Mathematics

http://www.aimspress.com/journal/Math

AIMS Mathematics, 9(10): 26446–26461.
DOI: 10.3934/math.20241287
Received: 26 July 2024
Revised: 30 August 2024
Accepted: 06 September 2024
Published: 12 September 2024

Research article

Input-to-state stability of nonlinear systems with delayed impulse based on event-triggered impulse control

Linni Li and Jin-E Zhang*

School of Mathematics and Statistics, Hubei Normal University, Huangshi 435002, China

* **Correspondence:** Email: zhang86021205@163.com.

Abstract: This paper investigates input-to-state stability (ISS) of nonlinear systems with delayed impulse under event-triggered impulse control, where external inputs are different in continuous and impulse dynamics. First, an event-triggered mechanism (ETM) is proposed to avoid Zeno behavior. In order to ensure ISS of the considered system, the relationship among event triggering parameters, impulse intensity, and impulse delay is constructed. Then, as an application, ETM and impulse control gain for a specific kind of nonlinear systems are presented based on linear matrix inequalities (LMI). Finally, two examples confirm the feasibility and usefulness of the proposed strategy.

Keywords: input-to-state stability; nonlinear delayed impulse; event-triggered impulse control; nonlinear systems; Zeno behavior
Mathematics Subject Classification: 93C30

1. Introduction

For systems subject to external disturbances, the requirement to maintain robust dynamic behavior under the influence of exogenous perturbations is particularly important. Therefore, the proposed concept of input-to-state stability [1] proves to be very effective in characterizing the effects of external disturbances on the considered system. Input-to-state stability means that when input is bounded, the state of the system remains bounded. If there is no external disturbance, input-to-state stability indicates that the system is asymptotically stable in the sense of Lyapunov. Based on this thought, input-to-state stability results have been investigated for different types of systems, such as switched systems [2, 3], impulsive systems [4, 5], and stochastic systems [6, 7]. Additionally, it has also been extended to finite time control problems and finite time input-to-state stability [8].

Impulsive systems are a special class of hybrid systems containing both continuous and discrete dynamics, which are widely used in the fields of communication networks, control technology, and image encryption; see [9–12]. From the perspective of impulse effect, work on the stability of impulsive systems primarily focuses on two major areas: impulse perturbation and impulse control. Impulse perturbation considers the robustness of the system under unstable impulses. Whereas impulse control [13] considers stabilization of systems containing stable impulses. By using discrete impulse signals as control inputs, satisfactory performance can be obtained, breaking through the limitations of traditional continuous control methods. As a discontinuous control method, impulse control has advantages of low cost, low energy consumption, high efficiency, and the ability to describe sudden change phenomena of systems. Thus, impulse control has attracted a wide range of attention in different fields [14–16]. In this literature, impulse controllers adopt a time-triggered mechanism, i.e., signal transmission is independent of system state, but impulse moments are pre-scheduled. However, this may lead to overuse of resources in the process of information transmission, resulting in unnecessary depletion of communication resources. Therefore, in order to better save network resources and overcome the drawbacks of time-triggered control methods, the event-triggered mechanism has been introduced; transmission occurs solely when the mechanism is activated; otherwise, the control signals remain updated. In recent years, there have been extensive research on event-triggered mechanisms; for example, reference [17] developed dynamic event-triggered schemes for uncertain nonlinear strict-feedback systems, and reference [18] proposed event-triggered asymptotic tracking control for multi-input and multi-output nonlinear systems.

Event-triggered impulse control combines the characteristics of event-triggered and impulse control, so that impulse control only acts on the considered system at the event-triggered instant, and there is no longer any control effect within two adjacent event-triggered intervals. This control mode is characterized by the fact that the control signal is released only when a specific state-dependence criterion is satisfied, thus greatly reducing the resource consumption. In practical application, this control strategy needs to exclude infinite triggering behaviors occurring within a limited time. A typical example is Zeno behavior, i.e., there exist an infinite number of triggering instants that converge to a positive constant [19]. It has been shown in [20] that Zeno behavior can occur in external perturbation or measurement noise, which gives caution when designing event-triggered control. Another circumstance is that trigger intervals tend to 0 as trigger instants tend to infinity. From a practical perspective, a trigger interval should have a minimum lower bound of a normal number. And in recent years, event-triggered impulse control has been applied in various control issues, such as consensus problems of multiagent systems [21], asymptotic stability of impulsive systems [22], synchronization of multiple neural networks [23], etc. It is noted that the results of the literature just mentioned can only be applied to some specific dynamical systems, but the influence of exogenous disturbances has not been considered, leading to certain limitations. Therefore, ISS under event-triggered impulse control has received more and more attention [24–28]. For example, ISS characteristics of nonlinear systems under continuous and discrete event-triggered impulse control were investigated in [25]. Based on event-triggered impulse control strategy, the ISS of nonlinear impulsive systems was developed, and Zeno behavior was excluded in [26, 27], but impulse delay was not considered in this literature. In practical applications, time delay is inevitable during transmission of impulses; that is, the transient of impulse depends not only on the current state of the system but also on the historical state of the system. In [28], by designing three levels of event triggering schemes, the influence of event-triggered impulse control with time delay on ISS stabilization was discussed. Although delayed impulse is taken into account, these works ignore interference of external input in discrete dynamics. On the other hand, there is some ISS work on impulse control based on the assumption [29], that is, the ratio coefficient of the Lyapunov function is constant. Recently, reference [30] explores the ISS properties of nonlinear impulse systems under event-triggered impulse control. The Lyapunov rate coefficients considered are all constant, but this may not be achieved in practical applications. Therefore, it is necessary to consider the case where the Lyapunov rate coefficient is time-varying. In [31], ISS results of time-varying nonlinear impulsive systems are obtained, but event-triggered impulse control is not considered, resulting in certain conservatism. ISS criteria obtained from [27] are applicable to the nonlinear rate of the Lyapunov functions, but impulse delay and hybrid inputs are not considered, which makes its application limited. In view of this, when delayed impulse is involved, related work to ensure ISS of nonlinear impulsive systems via event-triggered impulse control needs to be further enriched.

On the basis of the above motivation, the main work of this paper is to explore the ISS of nonlinear systems with delayed impluse in the framework of event-triggered impulse control. A Lyapunov-based ETM containing forced impulse sequence is proposed to realize ISS of the considered system, and Zeno behavior is ruled out. Subsequently, design criteria of impulse control gain and ETM are derived by solving LMI. The contribution of this paper can be summarized in three points:

(i) External inputs of continuous and discrete parts are the same in [27, 30, 32], leading to certain restrictions. This paper considers hybrid inputs, that is, external inputs can be different for the continuous and impulsive parts, which broadens the existing conclusions.

(ii) Compared to [25, 27], the impulse part of this paper involves time delay, and delay information is incorporated into the dynamic analysis of the considered system to establish relationships among event- triggering parameters, impulse intensity, and impulse delay.

(iii) ISS criteria in this paper are derived on the premise that the Lyapunov rate coefficient is time-varying, rather than constant, which makes the results relax restrictions in [26, 28, 30].

2. Preliminaries

Notations: R_+, N^+, R are sets of non-negative real numbers, positive integers, and real numbers. R^m is m-dimensional space. $PC([e, f]; R^m) : [e, f] \to R^m$ denotes piecewise continuous function. v_0 denotes a given category of local Lipschtiz function. Symbol $\star$ represents a symmetric block in a symmetric matrix. $\lambda_{max}(\varrho)$, ϱ^{-1} and ϱ^T represent the maximum eigenvalue, inverse, and transpose of matrix ϱ, respectively. $I > 0$ denotes a positive definite matrix I. $\mathcal{K}$ is said to be a class of continuous strictly increasing function $c : R^+ \to R^+$ with $c(0) = 0$. $\mathcal{K}_\infty$ is a radially unbounded subset of $\mathcal{K}$. A function $d : R^+ \times R^+ \to R^+$ is defined to be class $\mathcal{KL}$ if $d(\bullet, t)$ is a kind of $\mathcal{K}$ for every fixed $t \geq 0$, $d(\bullet, t) \to 0$ as $t \to +\infty$. $A \vee B = max\{A, B\}$.

Consider the following delayed impulsive systems:

$$\begin{aligned} \dot{z}(t) &= g(z(t), v_c(t)),\ t \neq t_r,\ t \geq t_0, \\ z(t) &= h_r(z(t^- - \tau), v_d(t^-)),\ t = t_r,\ r \in N^+, \\ z(n - t_0) &= \epsilon_n,\ n \in [t_0 - \tau, t_0], \end{aligned} \tag{1}$$

where $z(t) \in R^m$ is system state. ϵ_n is the initial state. $\tau > 0$ is constant delay. $v_c(t)$, $v_d(t) \in R^m$ are locally bounded exogenous perturbation and impulsive perturbation input. g, $h : R^m \times R^n \to R^m$ satisfy $\beth(0, 0) = \S(0, 0) = 0$ and some appropriate conditions such that existence and uniqueness of solution of system (1) are guaranteed. Impulse instant $\{t_r\}_{r\in N^+}$ satisfies $0 = t_0 < t_1 < \cdots < t_r < \cdots$ and $\lim_{r\to+\infty} t_r = +\infty$. Assume the solution of system (1) is right continuous, that is, $z(t^+) = z(t)$.

Definition 1 ([33]). *If there exist functions $\xi \in \mathcal{KL}$ and $\beta,\ \Gamma_c,\ \Gamma_d \in \mathcal{K}_\infty$, system (1) is ISS if*

$$\beta(|z(t)|) < \xi(\|\epsilon\|_\tau, t - t_0) + \sup_{t_0\le s\le t} \Gamma_c(|v_c(s)|) + \Gamma_d(\max_{t_0\le t_r\le t} \{|v_d(t_r^- - \tau)|\}),\ t \ge t_0,$$

where $\|\epsilon\|_\tau = sup_{[t_0-\tau,t_0]}|\epsilon|$.

Lemma 1 ([34]). *Let continuous functions $\Delta(t),\ \Theta(t),\ v_1(t),\ v_2(t) \in pc([t_0, +\infty); R^m)$ for $t \in [t_{r-1}, t_r),\ \forall r \in N^+,\ \Delta \in R_+$ satisfy*

$$\begin{cases} D^+v_1(t) \le \Delta(t)v_1(t) + \Theta(t), & t \ne t_r,\ t \ge t_0, \\ v_1(t_r) \le \Delta(t)v_1(t_r) + \Theta(t_r^-), & t = t_r, \end{cases}$$

and

$$\begin{cases} D^+v_2(t) > \Delta(t)v_2(t) + \Theta(t), & t \ne t_r,\ t \ge t_0, \\ v_2(t_r) \ge \Delta(t)v_2(t_r) + \Theta(t_r^-), & t = t_r, \end{cases}$$

then $v_1(t) \le v_2(t)$ for $t \ge t_0$.

Lemma 2 ([25]). *There exist real matrices $T > 0$, Ψ, $\varPsi$, and constant $c > 0$, and the following inequality holds:*

$$\Psi^T\varPsi + \varPsi^T\Psi \le c\Psi^T T\Psi + c^{-1}\varPsi^T T^{-1}\varPsi.$$

3. Main results

In this section, in the framework of the event-triggered impulse control approach, considering the effect of delayed impulses, some conditions to ensure ISS of system (1) are established and Zeno behavior is eliminated. First, the following ETM is considered:

$$\begin{aligned} &t_r = min\{t_r^*, t_{r-1} + \triangle r\},\ r \in N^+, \\ &t_r^* = inf\{t \ge t_{r-1} : V(t, z(t)) - exp(\iota_r - \varsigma(t - t_{r-1}))V(t_{r-1}, z(t_{r-1})) - exp(\bar{\iota}_r - \bar{\varsigma}(t - t_{r-1}))\phi_1(\|v_c\|_{[t_{r-1},t]}) \ge 0\}, \end{aligned} \tag{2}$$

where $\phi_1 \in K_\infty$, $V(t, z(t))$ is Lyapunov function depending on solution $z(t)$ of system (1) at time t. Event-triggering parameters $\iota,\ \bar{\iota},\ \varsigma,\ \bar{\varsigma} \in R_+$ and $\triangle r \in R_+$ satisfy

$$\sum_{r=1}^{s} \iota \to +\infty, \sum_{r=1}^{s} \bar{\iota} \to +\infty,\ s \to +\infty, \tag{3}$$

and

$$\inf_{r\in N^+} \{\triangle r\} > 0. \tag{4}$$

In order to exclude Zeno behavior, we give the following result based on designed ETM (2).

Theorem 1. *If functions $\Upsilon(t) \in PC([t_0, +\infty); R),\ \phi_1 \in K_\infty,\ V \in v_0$ satisfy:*

$$D^+V(t, z(t)) \le \Upsilon(t)V(t, z(t)) + \phi_1(|v_c(t)|),$$

and

$$\int_s^t \Upsilon(u)du \le c(t-s), \forall s,t \ge 0,$$

where $c \ge 0$ is constant, then system (1) has no Zeno phenomenon via ETM (2), impulse sequence $\{t_r\}_{r\in N^+}$ satisfies

$$t_r - t_{r-1} \ge t^* = max\left\{\frac{\iota_r}{c+\varsigma}, \frac{\bar{\iota}_r}{c+\bar{\varsigma}}\right\}. \tag{5}$$

Proof. According to the definition of ETM (2), three scenarios will be considered.

Case 1. Impulse instant t_r is made up entirely of forced impulse instant, i.e., $t_r = t_{r-1} + \triangle r$. Based on $t_r - t_{r-1} = \triangle r$ and assumption condition (4), it is possible to know that there is no Zeno behavior.

Case 2. Forced impulse instant $t_{r-1} + \triangle r$ and event-triggered impulse instant t_r^* occur simultaneously. First, assume that forced impulse instants are finite and satisfy $t_1 + \triangle r < t_2 + \triangle r < \cdots < t_{n+1} + \triangle r$. It clearly holds that impulse instant t_r is composed entirely of event-triggered impulse instant t_r^* after the last forced impulse instant $t_{n+1} + \triangle r$, thus $t_{n+1+r} = t^*_{n+1+r}, r \in N^+$. By Lemma 1, we derive

$$V(t,z(t)) \le v(t) = exp\left(\int_{t_{n+r}}^t \Upsilon(u)du\right)V(t_{n+r}, z(t_{n+r})) + \int_{t_{n+r}}^t exp\left(\int_s^t \Upsilon(v)dv\right)\phi_1(|v_c(u)|)du.$$

Based on ETM (2), we obtain

$$\begin{aligned}V(t^-_{n+r+1}, z(t^-_{n+r+1})) &= exp(\iota_{n+r+1} - \varsigma(t_r - t_{r-1}))V(t_{n+r}, z(t_{n+r})) + exp(\bar{\iota}_{n+r+1} - \bar{\varsigma}(t_r - t_{r-1}))\phi_1(\|v_c\|_{[t_{n+r},t_{n+r+1}]})\\ &\le exp\left(\int_{t_{n+r}}^{t_{n+r+1}} \Upsilon(u)du\right)V(t_{n+r}, z(t_{n+r})) + \int_{t_{n+r}}^{t_{n+r+1}} exp\left(\int_{t_{n+r}}^{t_{n+r+1}} \Upsilon(v)dv\right)\phi_1(|v_c(u)|)du\\ &\le exp(c(t_{n+r+1} - t_{n+r}))V(t_{n+r}, z(t_{n+r})) + exp(c(t_{n+r+1} - t_{n+r}))\phi_1(\|v_c\|_{[t_{n+r},t_{n+r+1}]}),\end{aligned} \tag{6}$$

which yields that

$$t_r - t_{r-1} \ge \frac{\iota_{n+r+1}}{c+\varsigma},$$
$$t_r - t_{r-1} \ge \frac{\bar{\iota}_{n+r+1}}{c+\bar{\varsigma}},$$

hence

$$t_r - t_{r-1} \ge max\left\{\frac{\iota_{n+r+1}}{c+\varsigma}, \frac{\bar{\iota}_{n+r+1}}{c+\bar{\varsigma}}\right\},$$

then, according to condition (3), we know $t_{n+r+1} \to +\infty$ as $r \to +\infty$, which means that Zeno behavior is excluded under this circumstance.

Second, forced impulse instants are infinite. Supposing that under ETM (2) Zeno behavior occurs in system (1), which indicates that there are countless impulse moments in the interval $[t_0, T^*]$, and T^* represents the accumulated time of impulse instants. Then, impulse instants tend to T^*, that is, forced impulse instants also tend to T^*, which is inconsistent with expression (4). Thus, Zeno behavior is also ruled out.

Case 3. Impulse instant t_r consists absolutely of event-triggered impulse instant t_r^*, that is, $t_r = t_r^*,\ r \in N^+$. Proof is similar to case 2, we find

$$V(t,z(t)) \le v(t) = exp\left(\int_{t_{r-1}}^t \Upsilon(u)du\right)V(t_{r-1}, z(t_{r-1})) + \int_{t_{r-1}}^t exp\left(\int_s^t \Upsilon(v)dv\right)\phi_1(|v_c(u)|)du,$$

and

$$
\begin{aligned}
V(t_r, z(t_r)) &= exp(\iota_r - \varsigma(t_r - t_{r-1}))V(t_{r-1}, z(t_{r-1})) + exp(\bar{\iota}_r - \bar{\varsigma}(t_r - t_{r-1}))\phi_1(\|v_c\|_{[t_{r-1},t_r]}) \\
&\leq exp\left(\int_{t_{r-1}}^{t_r} \Upsilon(u)du\right) V(t_{r-1}, z(t_{r-1})) + \int_{t_{r-1}}^{t_{r1}} exp\left(\int_{t_{r-1}}^{t_r} \Upsilon(v)dv\right) \phi_1(|v_c(u)|)du \qquad (7) \\
&\leq exp(c(t_r - t_{r-1}))V(t_{r-1}, z(t_{r-1})) + exp(c(t_r - t_{r-1}))\phi_1(\|v_c\|_{[t_{r-1},t_r]}).
\end{aligned}
$$

Similarly,

$$
t_r - t_{r-1} \geq \frac{\iota_r}{c + \varsigma}, \qquad t_r - t_{r-1} \geq \frac{\bar{\iota}_r}{c + \bar{\varsigma}},
$$

then

$$
t_r - t_{r-1} \geq max\left\{\frac{\iota_r}{c + \varsigma}, \frac{\bar{\iota}_r}{c + \bar{\varsigma}}\right\}.
$$

We can conclude that there is no Zeno phenomenon. Thus, it is clear from the above that Zeno behaviour does not occur under ETM (2) in either case. □

Remark 1. *Zeno behavior implies that an infinite number of continuous trigger instants occur in a finite period of time. ETM (2) consisting of event-triggered impulse instants and forced impulse moments can effectively eliminate Zeno behavior, and it is clear from (4) that there is no upper bound on forced impulse instants in this paper. In addition, condition (5) provides a variable lower bound with respect to parameters ι_r, c, ς, $\bar{\iota}_r$, $\bar{\varsigma}$ on the neighboring event-triggered impulse instants. Whereas literature [28, 35, 36] gives a uniformly positive lower bound, which suggests that conditions in this paper have less conservatism.*

Remark 2. *Due to the existence of the effect of exogenous interference, unlike literature [36], this paper introduces $\phi_1(\|v_c\|_{[t_0,t]})$ to represent the potential effect of exogenous disturbance, leading to a difference from the proof of [37]. It is worth noting that $\phi_1(\|v_c\|_{[t_0,t]})$ cannot be replaced by $\phi_1(\|v_c(t)\|)$. This is because the fact that Theorem 1 effectively rules out Zeno behavior, $\phi_1(\|v_c\|_{[t_0,t]})$ and $\phi_1(\|v_c(t)\|)$ are not comparable in size; therefore, it is necessary to show the value of ϕ_1 on this interval $[t_0, t]$.*

Theorem 2. *Let conditions in Theorem 1 hold, and there exist functions $\alpha_1, \ \alpha_2, \ \phi_2 \in K_\infty, V \in \nu_0$, constants $\iota_r, \ \bar{\iota}_r, \ \varsigma, \ \bar{\varsigma}, \ \aleph_r, \ \varpi \in R_+$, $r \in N^+$ satisfying:*
(i) $\alpha_1(|z|) \leq V(t, z) \leq \alpha_2(|z|)$,
(ii) $V(t_r, h_r(z(t_r^- - \tau), v_d(t_r^- - \tau))) \leq exp(-\aleph_r)V(t_r^- - \tau, z(t_r^- - \tau)) + \phi_2(|v_d(t_r^- - \tau)|)$,
(iii) $\iota_r, \ \bar{\iota}_r, \ \varsigma, \ \bar{\varsigma}_r$, impulse strengths $\aleph_r$ and impulse instant t_r satisfy:

$$
-\aleph_r + \iota_{r+1} + \varsigma\tau > (\varsigma - \bar{\varsigma})(t_{r+1} - t_r),
$$

$$
\sum_{r=1}^{l}(\iota_{m-r} - \aleph_{m-r}) + \bar{\iota}_{m-l} + \varsigma m\tau \leq \varpi, \ l \in \{1, 2, \dots, m-1\}, \ m \geq 2,
$$

then system (1) is ISS under ETM (2).

Proof. It can be seen from ETM (2) that

$$
V(t, z(t)) \leq exp(\iota_1 - \varsigma(t - t_0))V(t_0, z(t_0)) + exp(\bar{\iota}_1 - \bar{\varsigma}(t - t_0))\phi_1(\|v_c\|_{[t_0,t]}), \ t \in [t_0, t_1).
$$

Using condition (ii), for triggering instant t_1, we gain

$$\begin{aligned} V(t_1, z(t_1)) &= h_1(z(t_1^- - \tau), v_d(t_1^- - \tau)) \leq exp(-\aleph_1)V(t_1^- - \tau, z(t_1^- - \tau)) + \phi_2(|v_d(t_1^- - \tau)|) \\ &\leq \begin{cases} exp(-\aleph_1 + \iota_1 - \varsigma(t_1 - \tau - t_0))V^* + exp(-\aleph_1 + \bar{\iota}_1 - \bar{\varsigma}(t_1 - \tau - t_0))\phi_1(\|v_c\|_{[t_0,t_1]}) \\ +\phi_2(|v_d(t_1^- - \tau)|), \ t_0 \leq t_1 - \tau \leq t_1 \\ exp(-\aleph_1)V^* + \phi_2(|v_d(t_1^- - \tau)|), \ t_1 - \tau \leq t_0 \end{cases} \\ &\leq exp(-\aleph_1 + \iota_1 - \varsigma(t_1 - \tau - t_0))V^* \\ &+ exp(-\aleph_1 + \bar{\iota}_1 - \bar{\varsigma}(t_1 - \tau - t_0))\phi_1(\|v_c\|_{[t_0,t_1]}) + \phi_2(|v_d(t_1^- - \tau)|), \end{aligned}$$

where $V^* = sup_{n\in[t_0-\tau,t_0]} V(n, z(n))$, and

$$\begin{aligned} V(t, z(t)) \leq & exp(\iota_2 - \varsigma(t - t_1))V(t_1, z(t_1)) + exp(\bar{\iota}_2 - \bar{\varsigma}(t - t_1))\phi_1(\|v_c\|_{[t_1,t]}) \\ \leq & exp(-\aleph_1 + \iota_1 + \iota_2 - \varsigma(t - \tau - t_0))V^* \\ & + exp(-\aleph_1 + \bar{\iota}_1 + \iota_2 - \varsigma(t - t_1) - \bar{\varsigma}(t_1 - \tau - t_0))\phi_1(\|v_c\|_{[t_0,t_1]}) \\ & + exp(\iota_2 - \varsigma(t - t_1))\phi_2(|v_d(t_1^- - \tau)|) + exp(\bar{\iota}_2 - \bar{\varsigma}(t - t_1))\phi_1(\|v_c\|_{[t_1,t]}), \ t \in [t_1, t_2). \end{aligned}$$

Similarly, at triggering instant t_2,

$$\begin{aligned} V(t_2, z(t_2)) &= h_2(z(t_2^- - \tau), v_d(t_2^- - \tau)) \leq exp(-\aleph_2)V(t_2^- - \tau, z(t_2^- - \tau)) + \phi_2(|v_d(t_2^- - \tau)|) \\ &\leq \begin{cases} exp(-\aleph_1 - \aleph_2 + \iota_1 + \iota_2 - \varsigma(t_2 - 2\tau - t_0))V^* + exp(-\aleph_1 - \aleph_2 + \bar{\iota}_1 + \iota_2 - \varsigma(t_2 - \tau - t_1) \\ -\bar{\varsigma}(t_1 - \tau - t_0))\phi_1(\|v_c\|_{[t_0,t_1]}) + exp(-\aleph_2 + \iota_2 - \varsigma(t_2 - \tau - t_1))\phi_2(|v_d(t_1^- - \tau)|) \\ +exp(-\aleph_2 + \bar{\iota}_2 - \bar{\varsigma}(t_2 - \tau - t_1))\phi_1(\|v_c\|_{[t_1,t_2]}) + \phi_2(|v_d(t_2^- - \tau)|), \ t_1 \leq t_2 - \tau \leq t_2 \\ exp(-\aleph_2 + \iota_1 - \varsigma(t_2 - \tau - t_0))V^* + exp(-\aleph_2 + \bar{\iota}_1 - \bar{\varsigma}(t_2 - \tau - t_0))\phi_1(\|v_c\|_{[t_0,t_1]}) \\ +\phi_2(|v_d(t_2^- - \tau)|), \ t_0 \leq t_2 - \tau \leq t_1 \\ exp(-\aleph_2)V^* + \phi_2(|v_d(t_2^- - \tau)|), \ t_2 - \tau \leq t_0 \end{cases} \\ &\leq exp(-\aleph_1 - \aleph_2 + \iota_1 + \iota_2 - \varsigma(t_2 - 2\tau - t_0))V^* \\ &\quad + exp(-\aleph_1 - \aleph_2 + \bar{\iota}_1 + \iota_2 - \varsigma(t_2 - \tau - t_1) - \bar{\varsigma}(t_1 - \tau - t_0)) \\ &\quad \phi_1(\|v_c\|_{[t_0,t_1]}) + exp(-\aleph_2 + \iota_2 - \varsigma(t_2 - \tau - t_1))\phi_2(|v_d(t_1^- - \tau)|) \\ &\quad + exp(-\aleph_2 + \bar{\iota}_2 - \bar{\varsigma}(t_2 - \tau - t_1))\phi_1(\|v_c\|_{[t_1,t_2]}) + \phi_2(|v_d(t_2^- - \tau)|). \end{aligned}$$

Analogously,

$$\begin{aligned} V(t, z(t)) \leq & exp(\iota_3 - \varsigma(t - t_2))V(t_2, z(t_2)) + exp(\bar{\iota}_3 - \bar{\varsigma}(t - t_2))\phi_1(\|v_c\|_{[t_2,t]}) \\ \leq & exp(-\aleph_1 - \aleph_2 + \iota_1 + \iota_2 + \iota_3 - \varsigma(t - 2\tau - t_0))V^* + exp(-\aleph_1 - \aleph_2 + \bar{\iota}_1 + \iota_2 + \iota_3 - \varsigma(t - \tau - t_1) \\ & - \bar{\varsigma}(t_1 - \tau - t_0))\phi_1(\|v_c\|_{[t_0,t_1]}) + exp(-\aleph_2 + \iota_2 + \iota_3 - \varsigma(t - \tau - t_1))\phi_2(|v_d(t_1^- - \tau)|) \\ & + exp(-\aleph_2 + \bar{\iota}_2 + \iota_3 - \varsigma(t - t_2) - \bar{\varsigma}(t_2 - \tau - t_1))\phi_1(\|v_c\|_{[t_1,t_2]}) \\ & + exp(\iota_3 - \varsigma(t - t_2))\phi_2(|v_d(t_2^- - \tau)|) \\ & + exp(\bar{\iota}_3 - \bar{\varsigma}(t - t_2))\phi_1(\|v_c\|_{[t_2,t]}), \ t \in [t_2, t_3). \end{aligned}$$

Repeating the above steps, one can derive that

$$
\begin{aligned}
V(t,z(t)) \leq & exp(\iota_k + \sum_{n=1}^{k-1}(\iota_n - \aleph_n) - \varsigma(t-(k-1)\tau) - t_0)V^* \\
& + exp(\iota_k + \sum_{n=2}^{k-1}(\iota_n - \aleph_n) - \aleph_1 + \bar{\iota}_1 - \bar{\varsigma}(t_1 - \tau - t_0) - \varsigma(t-(k-2)\tau - t_1)\phi_1(\|v_c\|_{[t_0,t_1]}) \\
& + exp(\iota_k + \sum_{n=3}^{k-1}(\iota_n - \aleph_n) - \aleph_2 + \bar{\iota}_2 - \bar{\varsigma}(t_2 - \tau - t_1) - \varsigma(t-(k-3)\tau - t_2)\phi_1(\|v_c\|_{[t_1,t_2]}) \\
& + exp(\iota_k + \sum_{n=2}^{k-1}(\iota_n - \aleph_n) - \varsigma(t-(k-2)\tau - t_1)\phi_2(|v_d(t_1^- - \tau)|) \\
& + exp(\iota_k + \sum_{n=3}^{k-1}(\iota_n - \aleph_n) - \varsigma(t-(k-3)\tau - t_2)\phi_2(|v_d(t_2^- - \tau)|) + \cdots \\
& + exp(\iota_k - \aleph_{k-1} + \bar{\iota}_{k-1} - \bar{\varsigma}(t_{k-1} - \tau - t_{k-2}) - \varsigma(t - t_{k-1}))\phi_1(\|v_c\|_{[t_{k-2},t_{k-1}]}) \\
& + exp(\iota_k - \varsigma(t - t_{k-1}))\phi_2(|v_d(t_{k-1}^- - \tau)|) + exp(\bar{\iota}_k - \bar{\varsigma}(t_k - t_{k-1})\phi_1(\|v_c\|_{[t_{k-1},t]})),\ t \in (t_{k-1}, t_k).
\end{aligned}
$$

Together with (i) and (iii), we obtain

$$
\begin{aligned}
\alpha_1(|z(t)|) \leq & exp(\varpi + \iota)\alpha_2(\|\epsilon\|_\tau)exp(-\varsigma(t-t_0)) + exp((\varpi + \iota) \vee \bar{\iota})\phi_1(\|v_c\|_{[t_0,t]}) \\
& + exp(\varpi + \iota)\phi_2(\max_{t_0 \leq t_k \leq t} |v_d(t_k^- - \tau)|), t \in (t_{k-1}, t_k),
\end{aligned}
$$

where $\iota = sup_{k\in N^+}\{\iota_k\}$, $\bar{\iota} = sup_{k\in N^+}\{\bar{\iota}_k\}$, which confirms system (1) is ISS under ETM (2). □

Remark 3. *It follows from proof of Theorem 2 that in order to ensure ISS of system (1), it is necessary to introduce forced impulse sequence into ETM (2). In other words, without a forced impulse instant, an event trigger may not occur or occur countless times, so this requires frequent occurrence of stable impulses. The average dwell time is often used to solve this problem in the previous literature, but it causes unnecessary waste. However, forced impulse time in this paper satisfies conditions of ETM (2) and does not necessarily need to occur continuously, reflecting the necessity of its existence.*

Remark 4. *The ISS problem of nonlinear systems without delayed impulse based on event-triggered impulse control has been involved in [25, 27, 32]. When time delay in impulse is taken into account, overdispersion of system causes some trouble in description of delayed impulses. Therefore, the relationship among trigger parameters* ι_r, $\bar{\iota}_r$, ς, $\bar{\varsigma}_r$, *impulse intensity* $\aleph_r$, *and time delay* τ *is established under condition (iii) of Theorem 2 to overcome the influence of time delay.*

4. Applications

In this section, our presented event-triggered impulse control tactics are applied to nonlinear systems to achieve ISS.

Considering the following systems with external disturbance:

$$
\dot{z}(t) = \Lambda z(t) + \Gamma \beth(z(t)) + \Upsilon u(t) + v(t),\ t \neq t_r,\ t \geq t_0, \tag{8}
$$

where $\Lambda,\ \Gamma,\ \Upsilon \in R^n$ are given real matrices. $u(t) \in R^m$ is the locally bounded interference input. $\beth$ satisfies globally Lipschitz with Lipschitz matrix M. The following Dirac control input is considered to stabilize system (8):

$$v(t) = \sum_{r=1}^{\infty} Oz(t)\delta(t - t_r), \tag{9}$$

where $\{t_r\}_{r\in N^+}$ is impulse instant. O is the impulsive control gain matrix; in this circumstance, system (8) can be written in the underlying form:

$$\begin{aligned} \dot{z}(t) &= \Lambda z(t) + \Gamma\beth(z(t)) + \Upsilon u(t),\ t \geq t_0,\ t \neq t_r, \\ z(t) &= (\Xi + O)z(t^- - \tau),\ t = t_r,\ r \in N^+, \end{aligned} \tag{10}$$

where Ξ is the identity matrix.

Theorem 3. *If positive constants Θ, a_1, a_2, $\aleph$ and matrix $\Omega_{n\times n}$, $P_{n\times n} > 0$, diagonal matrix $K_{n\times n} > 0$, real matrix $X_{n\times n}$ satisfy:*

$$\begin{pmatrix} \Lambda^T\Omega + \Omega\Lambda + M^T KM - \Theta\Omega & \Omega\Gamma & \Omega\Upsilon \\ \star & -K & 0 \\ \star & \star & -P \end{pmatrix} \leq 0, \tag{11}$$

$$\begin{pmatrix} -exp(-\aleph)\Omega & \Omega + X \\ \star & -\Omega \end{pmatrix} \leq 0, \tag{12}$$

then, ISS is guaranteed for system (10) under impulsive control gain $O = \Omega^{-1}X^T$ and ETM:

$$\begin{aligned} t_r &= min\left\{t_r^*, t_{r-1} + \triangle r\right\},\ r \in N^+, \\ t_r^* &= inf\left\{t \geq t_{r-1} : \Delta(t) \geq 0\right\}, \end{aligned} \tag{13}$$

with

$$\Delta(t) = z^T(t)\Omega z(t) - a_1 z^T(t_{r-1})\Omega z(t_{r-1}) - a_2\lambda_{max}(P)||u||^2_{[t_{r-1},t]}.$$

Proof. Select $V(t) = z^T(t)\Omega z(t)$. By using the Ito formula, we obtain

$$\begin{aligned} D^+V(t) =& 2z^T(t)\Omega(\Lambda z(t) + \Gamma\beth(z(t)) + \Upsilon u(t)) \\ =& z^T(t)(\Omega\Lambda + \Lambda\Omega^T)z(t) + 2z^T(t)\Omega\Gamma\beth(z(t)) + 2z^T(t)\Omega\Upsilon u(t), \end{aligned}$$

combined with Lemma 2, we conclude

$$\begin{aligned} 2z^T(t)\Omega\Gamma\beth z(t) =& z^T(t)\Omega\Gamma\beth(z(t)) + \beth^T(z(t))\Gamma^T\Omega z(t) \\ \leq& z^T(t)\Omega\Gamma K^{-1}\Gamma^T\Omega z(t) + z^T(t)M^T KMz(t), \end{aligned}$$

$$\begin{aligned} 2z^T(t)\Omega\Upsilon u(t) =& z^T(t)\Omega\Upsilon u(t) + u^T(t)\Upsilon^T\Omega z(t) \\ \leq& z^T(t)\Omega\Upsilon P^{-1}\Upsilon^T\Omega z(t) + u^T(t)Pu(t). \end{aligned}$$

Together with (11) and (12), it can be derived that

$$D^+V(t) \leq \Theta z^T(t)\Omega z(t) + \lambda_{max}(P)|u(t)|^2,$$

and

$$-exp(-\aleph)\Omega + (\Xi + O)^T\Omega(\Xi + O) \leq 0,$$

which implies that

$$\begin{aligned} V(z(t_r)) =& z^T(t_r)\Omega z(t_r) \\ \leq& z^T(t_r^- - \tau)(\Xi + O)^T\Omega(\Xi + O)z(t_r^- - \tau) \\ \leq& exp(-\aleph)z^T(t_r^- - \tau)\Omega z(t_r^- - \tau) \\ =& exp(-\aleph)V(z(t_r^- - \tau)). \end{aligned}$$

Similar to proof of Theorem 2, the ISS of system (10) is shown to hold under ETM (13). □

Remark 5. *The ISS criterion given in Theorem 3 is based on the system being affected by delayed impulse and external interference. Moreover, the impulse control gain matrix is determined by the LMI method from the predetermined constants* Θ *and* $\aleph$.

5. Examples

Example 1. *Let us consider underlying system:*

$$\begin{aligned} &\dot{z}(t) = 1.9sin(t)z(t) + v_c(t),\ t \neq t_r,\ t \geq t_0, \\ &z(t) = exp(-0.11)z(t^- - \tau) + v_d(t^-),\ t = t_r,\ r \in N^+, \end{aligned} \tag{14}$$

where $v_c(t)$ *and* $v_d(t^-)$ *are external inputs for continuous and impulse parts, respectively. We select* $V(z(t)) = z(t)$, $c = 10$, $\iota = 0.9$, $\varsigma = 0.5$, $\bar{\iota}_r = 0.2$, $\bar{\varsigma} = 0.11$, $v_c(t) = sin(t)$, $v_d(t^-) = 1/10cos(t^-)$. *First, when forced impulse instant is not present, ETM is as follows:*

$$t_r^* = inf\left\{t \geq t_{r-1} : |z(t)| \geq exp(0.9 - 0.5(t - t_{r-1}))V(t_{r-1}, z(t_{r-1})) + exp(0.2 - 0.11(t - t_{r-1}))\phi_1(\|v_c\|_{[t_{r-1},t]})\right\}. \tag{15}$$

Through simulation (Figure 1), we find that system (14) fails to satisfy ISS under ETM (15). Hence, we will introduce the forced impulse sequence $t_r = t_{r-1} + 5$, *then ETM will be designed as follows:*

$$\begin{aligned} &t_r = min\left\{t_r^*, t_{r-1} + 5\right\},\ r \in N^+, \\ &t_r^* = inf\left\{t \geq t_{r-1} : |z(t)| \geq exp(0.9 - 0.5(t - t_{r-1}))V(t_{r-1}, z(t_{r-1})) + exp(0.2 - 0.11(t - t_{r-1}))\phi_1(\|v_c\|_{[t_{r-1},t]})\right\}. \end{aligned} \tag{16}$$

It follows from Theorem 1 that system (14) is ISS under ETM (16), and this is verified in Figure 2.

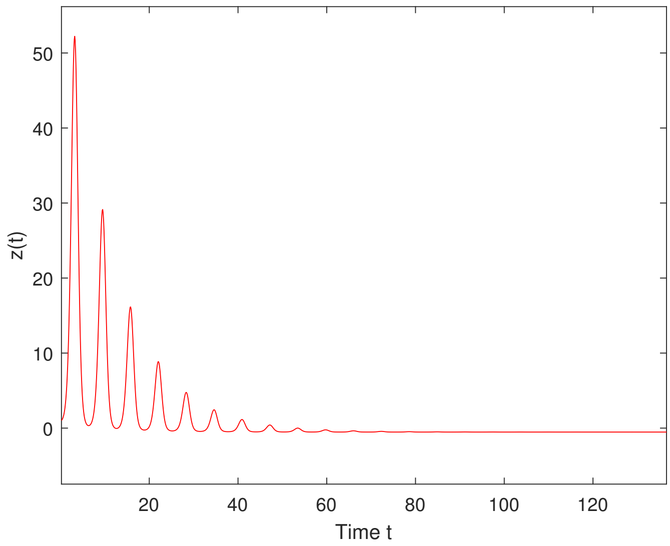

Figure 1. State trajectory of system (14) without forced impulse instant.

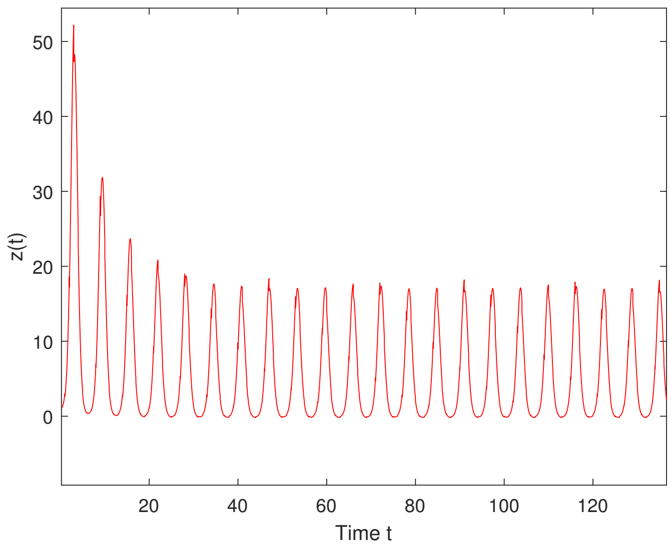

Figure 2. State trajectory of system (14) with forced impulse instant.

Example 2. *Now we consider system (10) with*

$$\mathit{\Lambda} = \begin{pmatrix} 0.69 & -1.1 \\ -0.79 & 0.3 \end{pmatrix}, \ \Gamma = \begin{pmatrix} 0.4 & 1 \\ 0 & 0.18 \end{pmatrix}, \ \Upsilon = \begin{pmatrix} 0.1 & 0.4 \\ 0.2 & 0.3 \end{pmatrix},$$

$\beth_1(z(t)) = \beth_2(z(t)) = tanh(2z(t)),\ u(t) = (sin(t), cos(t))^T$, *Figure 3 shows that when the impulse effect is not present, system (10) cannot reach ISS. So we will design ETM to make system (10) achieve ISS. We select* $a_1 = 1.1052,\ a_2 = 0.075,\ \Theta = 4.765$ *and* $\aleph = 0.31$. *By using Matlab to solve LMI (11) and (12),* *then ETM is presented as follows:*

$$\begin{aligned} t_r &= min\left\{t_r^*, t_{r-1}+3\right\},\ r \in N^+, \\ t_r^* &= inf\left\{t \geq t_{r-1} : z^T(t)\Omega z(t) \geq 1.1052 z^T(t_{r-1})\Omega z(t_{r-1}) - 0.075\lambda_{max}(P)\|u\|^2_{[t_{r-1},t]}\right\}, \end{aligned} \tag{17}$$

where

$$\Omega = \begin{pmatrix} 22.4211 & -4.6996 \\ -4.6996 & 25.1256 \end{pmatrix},\ P = \begin{pmatrix} 35.2366 & 0 \\ 0 & 35.2366 \end{pmatrix},$$

$$K = \begin{pmatrix} 13.7092 & 5.4444 \\ 5.4444 & 19.7417 \end{pmatrix},\ X = \begin{pmatrix} -31.3896 & -5.1481 \\ -5.1481 & -29.9487 \end{pmatrix}$$

and impulsive control gain

$$O = \Omega^{-1}X^T = \begin{pmatrix} -1.5018 & -0.4556 \\ -0.4858 & -1.0782 \end{pmatrix}.$$

According to Theorem 3, system (10) is ISS; see Figure 4. From the other side, with other parameters fixed, we only change impulsive control gain so that it does not satisfy conditions of Theorem 3, such as $\bar{O} = \begin{pmatrix} 1 & 5 \\ 2 & 3 \end{pmatrix}$*, as can be seen from Figure 5, system (10) is non-ISS, which shows the feasibility of our proposed event-triggered impulse control method.*

Remark 6. *In Example 1, Figure 1 illustrates that when ETM (2) does not contain forced impulse instant, the system is not ISS. But when a forced impulse instant is introduced into ETM (2), the system reaches ISS (i.e., Figure 2). In other words, forced impulse instant plays a key role in ensuring ISS characteristics. In Example 2, Figure 3 demonstrates that when the impulse effect is not present, system (10) is unstable under external perturbations. Therefore, in order to enable system (10) to achieve ISS, ETM (17) and impulse control gain are designed and verified in Figure 3. Figure 4 shows that when impulse control gain changes, that is, conditions (11) and (12) are not satisfied, the system cannot achieve ISS under ETM (17), which shows the feasibility of our proposed event-triggered impulse control method.*

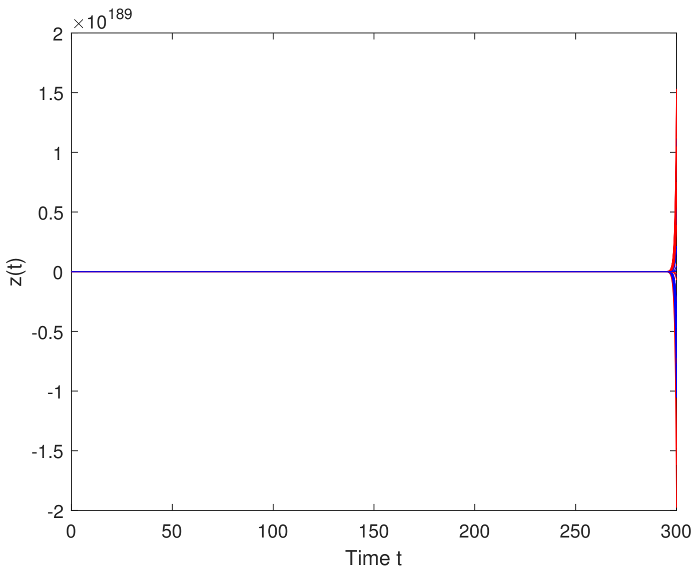

Figure 3. State trajectory of system (10) without impulse.

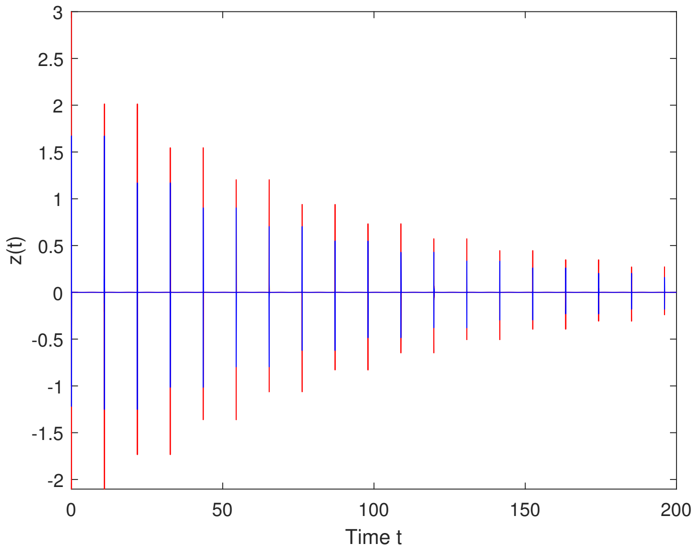

Figure 4. State trajectory of system (10) under (17).

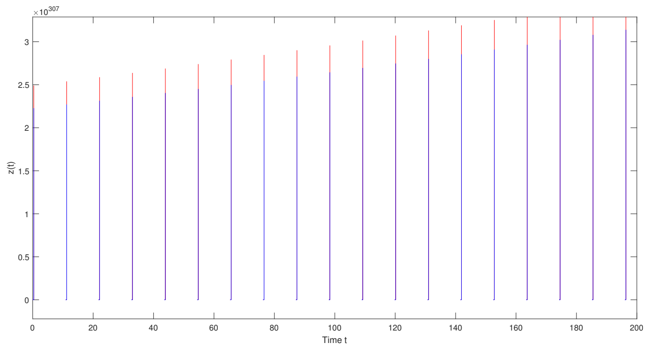

Figure 5. State trajectory of system (10) under (17) with $\bar{O}$.

6. Conclusions

In this paper, ISS properties of nonlinear delayed impulse systems with hybrid inputs in the framework of event-triggered impulse control are investigated, and related criteria of the considered system are derived based on designed ETM, where Zeno behavior is excluded. Then the theoretical results are applied to nonlinear systems, and some sufficient conditions of ETM and impulse control gain are obtained by LMI. Finally, two simulation examples are given to demonstrate the rationality of theoretical results. However, due to time delay in the impulse part, only constant delay is considered, and the control mechanism in this paper is given in advance, which has limitations in application. Then, if it can be extended to multi-agent systems with actuation delay and a self-triggered impulse control method for group consensus of multi-agent systems with sensing/ actuation delays is considered, this is worth further study.

Author contributions

Linni Li: writing-original draft; Jin-E Zhang: supervision, writing-review and editing. Both authors have read and approved the final version of the manuscript for publication.

Acknowledgments

This work is supported by Hubei Province Education Science Planning (Grant No. 2023GA058) and the Research Project on HBNU Education and Teaching Reform (Grant No. 2023002).

Conflict of interest

The authors declare no conflicts of interest.

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