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Research article

A novel analysis of the time-fractional nonlinear dispersive K(m,n,1) equations using the homotopy perturbation transform method and Yang transform decomposition method

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Abstract: The main features of scientific effort in physics and engineering are the development of models for various physical issues and the development of solutions. In this paper, we investigate the numerical solution of time-fractional non-linear dispersive K(m,n,1) type equations using two innovative approaches: the homotopy perturbation transform method and Yang transform decomposition method. Our suggested approaches elegantly combine Yang transform, homotopy perturbation method (HPM) and adomian decomposition method (ADM). With the help of the Yang transform, we first convert the problem into its differential partner before using HPM to get the He's polynomials and ADM to get the Adomian polynomials, both of which are extremely effective supports for non-linear issues. In this case, Caputo sense is used for defining the fractional derivative. The derived solutions are shown in series form and converge quickly. To ensure the effectiveness and applicability of the proposed approaches, the examined problems were analyzed using various fractional orders. We analyze and demonstrate the validity and applicability of the solution approaches under consideration with given initial conditions. Two and three dimensional graphs reflect the outcomes that were attained. To verify the effectiveness of the strategies, numerical simulations are presented. The numerical outcomes demonstrate that only a small number of terms are required to arrive at an approximation that is exact, efficient, and trustworthy. The results of this study demonstrate that the studied methods are effective and strong in solving nonlinear differential equations that appear in science and technology.

Keywords: analytical methods; time-fractional nonlinear dispersive K(m,n,1) equation; Caputo operator
Mathematics Subject Classification: 26A33, 35A20, 35L05

Nomenclature

The following abbreviations are used in this article:

FC	Fractional calculus
PDEs	Partial differential equations
FPDEs	Fractional-order partial differential equations
YT	Yang transform
YTDM	Yang transform decomposition method
HPTM	Homotopy perturbation transform method
Ψ	Independent variable
$\mathcal{T}$	Time
$\Theta(\Psi, \mathcal{T})$	Dependent function representing the physical quantity
$\wp$	Fractional order
Y	Yang transform
Y^{-1}	Inverse Yang transform
ϵ	Perturbation parameter

1. Introduction

Fractional calculus (FC) offers a more simple representation of reality. FC offers a better explanation of the fundamental nature of reality that makes this subject as a common interest in the world of science and engineering. Since FC is the language that nature knows, communicating with it in this way is effective. The FC problem has been with mathematicians for the past three centuries, and only in the last few years has it been brought into more practical sectors like engineering, science and economics. Integrals and derivatives are addressed by FC to an arbitrary real or complex order. Recently, a number of fractional operators, including Caputo, Caputo Fabrizio, Atangana-Baleanu, Katugampola, Hilfer, etc, have been proposed and implemented for dealing with real-world applications [1–6]. Fractional calculus has demonstrated that it is the best tool for studying problems in the actual world. This area of applied analysis has been utilised in a number of technological, engineering, and scientific fields. In order to optimise the wave equation, Bulut et al. [7] examined the time-fractional generalised Burger equation and trial equations. In 1998, He [8] examined the compact solution for the seepage flow equation in porous media. In 2020, Dubey et al. investigated the computer virus propagation model with fractional order [9]. The mathematical model for the chemical system was presented by Kumar et al. [10]. The fractional order multi-dimensional diffusion issues were studied by Singh et al. [11]. The Caputo fractional derivative is often used in practical applications, as it enables one to include the traditional initial and boundary conditions in formulating mathematical models. Moreover, as in the integer-order derivative, the Caputo fractional derivative of a constant is zero [12].

Partial differential equations (PDEs) can be used to express a specific relationship between an unknown function and its partial derivatives. PDEs are widely used in all branches of engineering and science. PDE usage has grown significantly in recent years in fields like biology, economics, image processing and graphics, and social sciences. As a result, suitable functions in these fields can be recognized when some independent variables interact with one another in each of the aforementioned fields, allowing for the modeling of a number of processes by generating equations for the corresponding functions. The study of PDEs has many aspects. Numerous applications can be made from PDE theoretical analysis. It should be mentioned that there are extremely hard equations that even supercomputers cannot solve. All that can be done in such instances is try to collect accurate data on the solution. Additionally, a significantly key issue is the formulation of the equation and its associated side conditions. Usually, a model of a physical or engineering problem serves as the foundation for the equation and it is not immediately obvious that the model is consistent with regard to the results in a solved PDE. Furthermore, it is preferable in the majority of circumstances for the solution to be distinct and stable against minor data disruptions. To determine whether these requirements are met, it is helpful to have a theoretical understanding of the equation. There have been many different approaches for solving classical PDEs proposed, as well as several solutions revealed [13–20].

Due to their numerous applications in diverse scientific domains, the idea of fractional partial differential equations (FPDEs) has been the subject of several studies and is an important topic in computational mathematics [21–27]. The diffusion process can be described more precisely using the fractional derivative by taking into account the long-range interactions, memory effects, and other physics-related phenomena as well as the majority of biological systems. Researchers have recently shown that many phenomena can be accurately modeled by non-integer order mathematical models using mathematical tools, such as the diffusion wave equations, Keller-Segel model for chemotaxis [28], fractional Radhakrishnan-Kundu-Lakshmanan equation [29] and fractional Riccati differential equations [30]. Despite their extensive use and applications, there is a significant problem with the numerical approaches that are presently available for finding solutions to FPDEs. The present study was encouraged by the need for a comprehensive approach that could be applied to problems such as homogeneous, nonhomogeneous, linear, nonlinear and multivariable FPDEs, without requiring important adjustments. The growing academic interest in the numerical solutions of fractional PDEs has led to significant advancements in the study of nonlinear PDEs. However, some of the main drawbacks of numerical approaches are the restrictions in accuracy, mesh construction, transformations, stability, convergence, and the difficulty of applying them to complex geometries.

Mathematicians have created a variety of numerical and analytical strategies to solve FPDEs in light of the aforementioned applications. For example, in order to find series form solutions to various partial differential equations (PDEs) and FPDEs with initial and boundary conditions, Duan et al. [31] implemented the Adomian decomposition method (ADM). Natural transform decomposition method has been applied by Botmart et al. for solving fractional approximate long wave and the modified Boussinesq equations [32] and fractional-order kaup-kupershmidt equation [33]. Similarly, in [34], Fathima et al. have applied the natural transform decomposition method for solving fractional Caudrey-Dodd-Gibbon equations. In [35], Jafari et al. have used the fractional sub-equation method for the solution of the fractional generalized reaction Duffing model and the nonlinear fractional Sharma-Tasso-Olver equation. The analytical solution of the seventh-order Lax's Korteweg-de Vries equation has been found by Mishra et al. in [36]. With the help of the tanh method, Wazwaz in [37] found the exact solutions of the sine-Gordon and the sinh-Gordon equations. The numerical solution of fractional phi-four equation has been found in [38], using the Yang transform decomposition method. Deng implemented the finite element method for solving the space and time fractional Fokker-Planck equation [39]. Mohammed Kbiri Alaoui et al. in [40] investigated the time-fractional Belousov-Zhabotinsky reaction analytically.

This work examines the nonlinear dispersive K(m,n,1) equation with fractional time derivatives:

$$D_{\mathcal{T}}^{\wp}\Theta(\Psi,\mathcal{T}) + (\Theta^2(\Psi,\mathcal{T}))_{\Psi} - (\Theta^2(\Psi,\mathcal{T}))_{\Psi\Psi\Psi} + (\Theta(\Psi,\mathcal{T}))_{\Psi\Psi\Psi\Psi\Psi} = 0, \quad 0 < \wp \leq 1. \tag{1.1}$$

For specific values of m and n, the classic nonlinear dispersive K(m,n,1) equation exhibits solitary waves that are compactly supported. This equation was first proposed by Rosenau and Hymann in 1993. Many other approaches have recently been proposed for studying the nonlinear dispersive K(m,n) equations, including the Adomian method [41], the Exp-function method [42], the variational iteration method [43], the variational method [44] and the HPM [45]. These types of equations can be solved using a variety of efficient and practical techniques as shown in [46]. Equation (1.1) with fractional order of time derivation occurs in discontinuous time on huge time scales in weather forecasting or the extremely short time scale in high energy physics. The fractional model is most suitable to describe such issues because time is discontinuous (but hierarchical) according to the E-infinity hypothesis. Time-fractional equations always exhibit remarkable behaviour as shown in [47].

The aim of this study is to extend the use of the Yang transform decomposition technique (YTDM) and homotopy perturbation transform method (HPTM) to get analytical and approximative solutions to the nonlinear dispersive K(m,n,1) equations with fractional time derivatives. In order to decompose the nonlinear terms, the Adomian and homotopy polynomials are used, and the Yang transform is used to convert differential equations into algebraic equations. Our methods produced infinite series as the results in the numerical examples. When we write the series in closed form, it gives precise solutions to the relevant equations. Researchers can use this study as a fundamental reference to examine these strategies and employ it in many applications to get accurate and approximative results in a few easy steps. The unique aspect of this work is the description of two novel techniques for fractional nonlinear dispersive K(m,n,1) equations with minimal and consecutive steps.

The current article follows the following format: We begin with the basic concept of FC in Section 2. The main ideas of the recommended approaches are covered in Sections 3 and 4. These techniques are applied to solve the time-fractional nonlinear dispersive K(m,n,1) problem in Section 5 with the provided initial condition. In Section 6, the conclusion is offered.

2. Preleminaries

In order to give more analysis and define the solution methods, we will present some illustrative definitions about the fractional calculus.

Definition 2.1. The fractional operator in Caputo mean is as [48]

$$D_{\mathcal{T}}^{\wp}\Theta(\Psi,\mathcal{T}) = \frac{1}{\Gamma(k-\wp)}\int_0^{\mathcal{T}} (\mathcal{T}-\gamma)^{k-\wp-1}\Theta^{(k)}(\Psi,\gamma)d\gamma, \quad k-1 < \wp \leq k, \quad k \in N. \tag{2.1}$$

Definition 2.2. The Yang transform (YT) of the given function is as [49]

$$Y\{\Theta(\mathcal{T})\} = M(u) = \int_0^{\infty} e^{\frac{-\mathcal{T}}{u}}\Theta(\mathcal{T})d\mathcal{T}, \;\; \mathcal{T} > 0, \;\; u \in (-\mathcal{T}_1, \mathcal{T}_2), \tag{2.2}$$

illustrating inverse YT as

$$Y^{-1}\{M(u)\} = \Theta(\mathcal{T}). \tag{2.3}$$

Definition 2.3. The YT associated with nth order derivative is as [49]

$$Y\{\Theta^n(\mathcal{T})\} = \frac{M(u)}{u^n} - \sum_{k=0}^{n-1} \frac{\Theta^k(0)}{u^{n-k-1}}, \ \forall n = 1, 2, 3, \cdots. \tag{2.4}$$

Definition 2.4. The YT associated with the fractional order derivative is as [49]

$$Y\{\Theta^\wp(\mathcal{T})\} = \frac{M(u)}{u^\wp} - \sum_{k=0}^{n-1} \frac{\Theta^k(0)}{u^{\wp-(k+1)}}, \ \ 0 < \wp \leq n. \tag{2.5}$$

3. Basic procedure of HPTM

To illustrate the basic idea of this approach, we discuss a nonlinear FPDE.

$$D_{\mathcal{T}}^\wp \Theta(\Psi, \mathcal{T}) = \mathcal{J}_1[\Psi]\Theta(\Psi, \mathcal{T}) + \mathcal{K}_1[\Psi]\Theta(\Psi, \mathcal{T}), \ \ 0 < \wp \leq 1, \tag{3.1}$$

associated with initial guess

$$\Theta(\Psi, 0) = \xi(\Psi).$$

By plugging in the YT, we obtain

$$Y[D_{\mathcal{T}}^\wp \Theta(\Psi, \mathcal{T})] = Y[\mathcal{J}_1[\Psi]\Theta(\Psi, \mathcal{T}) + \mathcal{K}_1[\Psi]\Theta(\Psi, \mathcal{T})], \tag{3.2}$$

$$\frac{1}{u^\wp}\{M(u) - u\Theta(0)\} = Y[\mathcal{J}_1[\Psi]\Theta(\Psi, \mathcal{T}) + \mathcal{K}_1[\Psi]\Theta(\Psi, \mathcal{T})]. \tag{3.3}$$

We have by simplifying

$$M(\Theta) = u\Theta(0) + u^\wp Y[\mathcal{J}_1[\Psi]\Theta(\Psi, \mathcal{T}) + \mathcal{K}_1[\Psi]\Theta(\Psi, \mathcal{T})]. \tag{3.4}$$

By employing inverse YT, we get

$$\Theta(\Psi, \mathcal{T}) = \Theta(0) + Y^{-1}[u^\wp Y[\mathcal{J}_1[\Psi]\Theta(\Psi, \mathcal{T}) + \mathcal{K}_1[\Psi]\Theta(\Psi, \mathcal{T})]]. \tag{3.5}$$

Now in terms of HPM, we obtain

$$\Theta(\Psi, \mathcal{T}) = \sum_{k=0}^{\infty} \epsilon^k \Theta_k(\Psi, \mathcal{T}), \tag{3.6}$$

with $\epsilon \in [0, 1]$.

Let

$$\mathcal{K}_1[\Psi]\Theta(\Psi, \mathcal{T}) = \sum_{k=0}^{\infty} \epsilon^k H_n(\Theta), \tag{3.7}$$

with

$$H_n(\Theta_0, \Theta_1, ..., \Theta_n) = \frac{1}{\Gamma(n+1)} D_\epsilon^k \left[\mathcal{K}_1\left(\sum_{k=0}^{\infty} \epsilon^i \Theta_i\right)\right]_{\epsilon=0}, \tag{3.8}$$

where $D_\epsilon^k = \frac{\partial^k}{\partial \epsilon^k}$.

Now, we substitute Eqs (3.6) and (3.7) in Eq (3.5) to get

$$\sum_{k=0}^{\infty}\epsilon^k\Theta_k(\Psi,\mathcal{T}) = \Theta(0) + \epsilon \times \left(Y^{-1}\left[u^{\wp}Y\{\mathcal{J}_1\sum_{k=0}^{\infty}\epsilon^k\Theta_k(\Psi,\mathcal{T}) + \sum_{k=0}^{\infty}\epsilon^k H_k(\Theta)\}\right]\right). \tag{3.9}$$

On comparing the ϵ coefficients, we conclude

$$\begin{aligned}
&\epsilon^0 : \Theta_0(\Psi,\mathcal{T}) = \Theta(0),\\
&\epsilon^1 : \Theta_1(\Psi,\mathcal{T}) = Y^{-1}\left[u^{\wp}Y(\mathcal{J}_1[\Psi]\Theta_0(\Psi,\mathcal{T}) + H_0(\Theta))\right],\\
&\epsilon^2 : \Theta_2(\Psi,\mathcal{T}) = Y^{-1}\left[u^{\wp}Y(\mathcal{J}_1[\Psi]\Theta_1(\Psi,\mathcal{T}) + H_1(\Theta))\right],\\
&.\\
&.\\
&.\\
&\epsilon^k : \Theta_k(\Psi,\mathcal{T}) = Y^{-1}\left[u^{\wp}Y(\mathcal{J}_1[\Psi]\Theta_{k-1}(\Psi,\mathcal{T}) + H_{k-1}(\Theta))\right],\ k > 0,\ k \in N.
\end{aligned} \tag{3.10}$$

Finally, the analytical solution is given by

$$\Theta(\Psi,\mathcal{T}) = \lim_{M\to\infty}\sum_{k=1}^{M}\Theta_k(\Psi,\mathcal{T}). \tag{3.11}$$

4. Basic procedure of YTDM

To illustrate the basic idea of this approach, we discuss a nonlinear YTDM.

$$D_{\mathcal{T}}^{\wp}\Theta(\Psi,\mathcal{T}) = \mathcal{J}_1(\Psi,\mathcal{T}) + \mathcal{K}_1(\Psi,\mathcal{T}),\quad 0 < \wp \leq 1, \tag{4.1}$$

associated with initial guess

$$\Theta(\Psi,0) = \xi(\Psi).$$

By plugging in the YT, we obtain

$$\begin{aligned}
&Y[D_{\mathcal{T}}^{\wp}\Theta(\Psi,\mathcal{T})] = Y[\mathcal{J}_1(\Psi,\mathcal{T}) + \mathcal{K}_1(\Psi,\mathcal{T})],\\
&\frac{1}{u^{\wp}}\{M(u) - u\Theta(0)\} = Y[\mathcal{J}_1(\Psi,\mathcal{T}) + \mathcal{K}_1(\Psi,\mathcal{T})].
\end{aligned} \tag{4.2}$$

We have by simplifying

$$M(\Theta) = u\Theta(0) + u^{\wp}Y[\mathcal{J}_1(\Psi,\mathcal{T}) + \mathcal{K}_1(\Psi,\mathcal{T})]. \tag{4.3}$$

By employing inverse YT, we obtain

$$\Theta(\Psi,\mathcal{T}) = \Theta(0) + Y^{-1}[u^{\wp}Y[\mathcal{J}_1(\Psi,\mathcal{T}) + \mathcal{K}_1(\Psi,\mathcal{T})]. \tag{4.4}$$

Now, the solution in terms of infinite series is

$$\Theta(\Psi,\mathcal{T}) = \sum_{m=0}^{\infty}\Theta_m(\Psi,\mathcal{T}). \tag{4.5}$$

Also, the nonlinear term is illustrated as

$$\mathcal{K}_1(\Psi,\mathcal{T}) = \sum_{m=0}^{\infty} \mathcal{A}_m, \tag{4.6}$$

with

$$\mathcal{A}_m = \frac{1}{m!}\left[\frac{\partial^m}{\partial \ell^m}\left\{\mathcal{K}_1\left(\sum_{k=0}^{\infty} \ell^k \Psi_k, \sum_{k=0}^{\infty} \ell^k \mathcal{T}_k\right)\right\}\right]_{\ell=0}. \tag{4.7}$$

Now, we substitute Eqs (4.5) and (4.6) in Eq (4.4) to get

$$\sum_{m=0}^{\infty} \Theta_m(\Psi,\mathcal{T}) = \Theta(0) + Y^{-1}u^{\wp}\left[Y\left\{\mathcal{J}_1(\sum_{m=0}^{\infty}\Psi_m, \sum_{m=0}^{\infty}\mathcal{T}_m) + \sum_{m=0}^{\infty}\mathcal{A}_m\right\}\right]. \tag{4.8}$$

On comparison of both sides, we conclude

$$\Theta_0(\Psi,\mathcal{T}) = \Theta(0), \tag{4.9}$$

$$\Theta_1(\Psi,\mathcal{T}) = Y^{-1}\left[u^{\wp}Y\{\mathcal{J}_1(\Psi_0,\mathcal{T}_0) + \mathcal{A}_0\}\right].$$

Finally, the analytical solution is given by

$$\Theta_{m+1}(\Psi,\mathcal{T}) = Y^{-1}\left[u^{\wp}Y\{\mathcal{J}_1(\Psi_m,\mathcal{T}_m) + \mathcal{A}_m\}\right].$$

5. Applications

Example 1. Assume the fractional K(2, 2, 1) equation is

$$D^{\wp}_{\mathcal{T}}\Theta(\Psi,\mathcal{T}) + (\Theta^2(\Psi,\mathcal{T}))_{\Psi} - (\Theta^2(\Psi,\mathcal{T}))_{\Psi\Psi\Psi} + (\Theta(\Psi,\mathcal{T}))_{\Psi\Psi\Psi\Psi\Psi} = 0, \quad 0 < \wp \leq 1, \tag{5.1}$$

associated with initial guess

$$\Theta(\Psi,0) = \frac{16\rho - 1}{12}\cosh^2\left(\frac{\Psi}{4}\right).$$

By plugging in the YT, we obtain

$$Y\left(\frac{\partial^{\wp}\Theta}{\partial \mathcal{T}^{\wp}}\right) = Y\Bigg((\Theta^2(\Psi,\mathcal{T}))_{\Psi\Psi\Psi} - (\Theta^2(\Psi,\mathcal{T}))_{\Psi} - (\Theta(\Psi,\mathcal{T}))_{\Psi\Psi\Psi\Psi\Psi}\Bigg). \tag{5.2}$$

We have by simplifying

$$\frac{1}{u^{\wp}}\{M(u) - u\Theta(0)\} = Y\Bigg((\Theta^2(\Psi,\mathcal{T}))_{\Psi\Psi\Psi} - (\Theta^2(\Psi,\mathcal{T}))_{\Psi} - (\Theta(\Psi,\mathcal{T}))_{\Psi\Psi\Psi\Psi\Psi}\Bigg), \tag{5.3}$$

$$M(u) = u\Theta(0) + u^{\wp}\Bigg((\Theta^2(\Psi,\mathcal{T}))_{\Psi\Psi\Psi} - (\Theta^2(\Psi,\mathcal{T}))_{\Psi} - (\Theta(\Psi,\mathcal{T}))_{\Psi\Psi\Psi\Psi\Psi}\Bigg). \tag{5.4}$$

By employing inverse YT, we obtain

$$\Theta(\Psi,\mathcal{T}) = \Theta(0) + Y^{-1}\left[u^{\wp}\left\{Y\left((\Theta^2(\Psi,\mathcal{T}))_{\Psi\Psi\Psi} - (\Theta^2(\Psi,\mathcal{T}))_{\Psi} - (\Theta(\Psi,\mathcal{T}))_{\Psi\Psi\Psi\Psi\Psi}\right)\right\}\right],$$
$$\Theta(\Psi,\mathcal{T}) = \frac{16\rho-1}{12}\cosh^2\left(\frac{\Psi}{4}\right) + Y^{-1}\left[u^{\wp}\left\{Y\left((\Theta^2(\Psi,\mathcal{T}))_{\Psi\Psi\Psi} - (\Theta^2(\Psi,\mathcal{T}))_{\Psi} - (\Theta(\Psi,\mathcal{T}))_{\Psi\Psi\Psi\Psi\Psi}\right)\right\}\right]. \tag{5.5}$$

Now in terms of HPM, we get

$$\sum_{k=0}^{\infty}\epsilon^k\Theta_k(\Psi,\mathcal{T}) = \frac{16\rho-1}{12}\cosh^2\left(\frac{\Psi}{4}\right) + \epsilon\Bigg(Y^{-1}\Bigg[u^{\wp}Y\Bigg[\left(\sum_{k=0}^{\infty}\epsilon^k H_k^1(\Theta)\right) - \left(\sum_{k=0}^{\infty}\epsilon^k H_k^2(\Theta)\right)$$
$$- \left(\sum_{k=0}^{\infty}\epsilon^k\Theta_k(\Psi,\mathcal{T})\right)_{\Psi\Psi\Psi\Psi\Psi}\Bigg]\Bigg]\Bigg). \tag{5.6}$$

Assume the He's polynomial $H_k(\mathbb{U})$ as

$$\sum_{k=0}^{\infty}\epsilon^k H_k^1(\Theta) = (\Theta^2(\Psi,\mathcal{T}))_{\Psi\Psi\Psi},$$
$$\sum_{k=0}^{\infty}\epsilon^k H_k^2(\Theta) = (\Theta^2(\Psi,\mathcal{T}))_{\Psi}. \tag{5.7}$$

The first components are illustrated as

$$\begin{aligned}
H_0^1(\Theta) &= (\Theta_0^2)_{\Psi\Psi\Psi},\\
H_1^1(\Theta) &= (2\Theta_0\Theta_1)_{\Psi\Psi\Psi},\\
H_2^1(\Theta) &= (\Theta_1^2 + 2\Theta_0\Theta_2)_{\Psi\Psi\Psi},\\
&\vdots\\
H_0^2(\Theta) &= (\Theta_0^2)_{\Psi},\\
H_1^2(\Theta) &= (2\Theta_0\Theta_1)_{\Psi},\\
H_2^2(\Theta) &= (\Theta_1^2 + 2\Theta_0\Theta_2)_{\Psi}.
\end{aligned}$$

On comparing the ϵ coefficients, we conclude

$$\begin{aligned}
\epsilon^0 &: \Theta_0(\Psi,\mathcal{T}) = \frac{16\rho-1}{12}\cosh^2\left(\frac{\Psi}{4}\right),\\
\epsilon^1 &: \Theta_1(\Psi,\mathcal{T}) = -\frac{(16\rho-1)\rho}{24\cdot 2}\sinh\left(\frac{\Psi}{2}\right)\frac{\mathcal{T}^{\wp}}{\Gamma(\wp+1)},\\
\epsilon^2 &: \Theta_2(\Psi,\mathcal{T}) = \frac{(16\rho-1)\rho^2}{24\cdot 2^2}\cosh\left(\frac{\Psi}{2}\right)\frac{\mathcal{T}^{2\wp}}{\Gamma(2\wp+1)},\\
\epsilon^3 &: \Theta_3(\Psi,\mathcal{T}) = -\frac{(16\rho-1)\rho^3}{24\cdot 2^3}\sinh\left(\frac{\Psi}{2}\right)\frac{\mathcal{T}^{3\wp}}{\Gamma(3\wp+1)},\\
&\vdots
\end{aligned}$$

Finally, the analytical solution is given by

$$\begin{aligned}\Theta(\Psi,\mathcal{T}) =& \Theta_0(\Psi,\mathcal{T}) + \Theta_1(\Psi,\mathcal{T}) + \Theta_2(\Psi,\mathcal{T}) + \Theta_3(\Psi,\mathcal{T}) + \cdots,\\ \Theta(\Psi,\mathcal{T}) =& \frac{16\rho-1}{12}\cosh^2\left(\frac{\Psi}{4}\right) - \frac{(16\rho-1)\rho}{24\cdot 2}\sinh\left(\frac{\Psi}{2}\right)\frac{\mathcal{T}^{\wp}}{\Gamma(\wp+1)} + \frac{(16\rho-1)\rho^2}{24\cdot 2^2}\cosh\left(\frac{\Psi}{2}\right)\frac{\mathcal{T}^{2\wp}}{\Gamma(2\wp+1)}\\ &- \frac{(16\rho-1)\rho^3}{24\cdot 2^3}\sinh\left(\frac{\Psi}{2}\right)\frac{\mathcal{T}^{3\wp}}{\Gamma(3\wp+1)} + \cdots.\end{aligned}$$

Solution by implementing YTDM

By plugging in the YT, we obtain

$$Y\left\{\frac{\partial^{\wp}\Theta}{\partial\mathcal{T}^{\wp}}\right\} = Y\Big((\Theta^2(\Psi,\mathcal{T}))_{\Psi\Psi\Psi} - (\Theta^2(\Psi,\mathcal{T}))_{\Psi} - (\Theta(\Psi,\mathcal{T}))_{\Psi\Psi\Psi\Psi\Psi}\Big). \tag{5.8}$$

We have by simplifying

$$\frac{1}{u^{\wp}}\{M(u) - u\Theta(0)\} = Y\Big((\Theta^2(\Psi,\mathcal{T}))_{\Psi\Psi\Psi} - (\Theta^2(\Psi,\mathcal{T}))_{\Psi} - (\Theta(\Psi,\mathcal{T}))_{\Psi\Psi\Psi\Psi\Psi}\Big), \tag{5.9}$$

$$M(u) = u\Theta(0) + u^{\wp}Y\Big((\Theta^2(\Psi,\mathcal{T}))_{\Psi\Psi\Psi} - (\Theta^2(\Psi,\mathcal{T}))_{\Psi} - (\Theta(\Psi,\mathcal{T}))_{\Psi\Psi\Psi\Psi\Psi}\Big). \tag{5.10}$$

By employing inverse YT, we obtain

$$\begin{aligned}\Theta(\Psi,\mathcal{T}) &= \Theta(0) + Y^{-1}\left[u^{\wp}\left\{Y\Big((\Theta^2(\Psi,\mathcal{T}))_{\Psi\Psi\Psi} - (\Theta^2(\Psi,\mathcal{T}))_{\Psi} - (\Theta(\Psi,\mathcal{T}))_{\Psi\Psi\Psi\Psi\Psi}\Big)\right\}\right],\\ \Theta(\Psi,\mathcal{T}) &= \frac{16\rho-1}{12}\cosh^2\left(\frac{\Psi}{4}\right) + Y^{-1}\left[u^{\wp}\left\{Y\Big((\Theta^2(\Psi,\mathcal{T}))_{\Psi\Psi\Psi} - (\Theta^2(\Psi,\mathcal{T}))_{\Psi} - (\Theta(\Psi,\mathcal{T}))_{\Psi\Psi\Psi\Psi\Psi}\Big)\right\}\right].\end{aligned} \tag{5.11}$$

Now the solution in terms of infinite series is

$$\Theta(\Psi,\mathcal{T}) = \sum_{m=0}^{\infty}\Theta_m(\Psi,\mathcal{T}). \tag{5.12}$$

Also, the nonlinear term is illustrated as $(\Theta^2(\Psi,\mathcal{T}))_{\Psi} = \sum_{m=0}^{\infty}\mathcal{A}_m, (\Theta^2(\Psi,\mathcal{T}))_{\Psi} = \sum_{m=0}^{\infty}\mathcal{B}_m$. So, we get

$$\begin{aligned}\sum_{m=0}^{\infty}\Theta_m(\Psi,\mathcal{T}) &= \Theta(\Psi,0) + Y^{-1}\left[u^{\wp}Y\left[\left(\sum_{m=0}^{\infty}\mathcal{A}_m\right) - \left(\sum_{m=0}^{\infty}\mathcal{B}_m\right) - (\Theta(\Psi,\mathcal{T}))_{\Psi\Psi\Psi\Psi\Psi}\right]\right],\\ \sum_{m=0}^{\infty}\Theta_m(\Psi,\mathcal{T}) &= \frac{16\rho-1}{12}\cosh^2\left(\frac{\Psi}{4}\right) + Y^{-1}\left[u^{\wp}Y\left[\left(\sum_{m=0}^{\infty}\mathcal{A}_m\right) - \left(\sum_{m=0}^{\infty}\mathcal{B}_m\right) - (\Theta(\Psi,\mathcal{T}))_{\Psi\Psi\Psi\Psi\Psi}\right]\right].\end{aligned} \tag{5.13}$$

The first components are illustrated as

$$\begin{aligned}
\mathcal{A}_0 &= (\Theta_0^2)_{\Psi\Psi\Psi},\\
\mathcal{A}_1 &= (2\Theta_0\Theta_1)_{\Psi\Psi\Psi},\\
\mathcal{A}_2 &= (\Theta_1^2 + 2\Theta_0\Theta_2)_{\Psi\Psi\Psi},\\
&\vdots\\
\mathcal{B}_0 &= (\Theta_0^2)_{\Psi},\\
\mathcal{B}_1 &= (2\Theta_0\Theta_1)_{\Psi},\\
\mathcal{B}_2 &= (\Theta_1^2 + 2\Theta_0\Theta_2)_{\Psi}.
\end{aligned}$$

On comparing both sides, we conclude

$$\Theta_0(\Psi, \mathcal{T}) = \frac{16\rho - 1}{12}\cosh^2\left(\frac{\Psi}{4}\right).$$

On $m = 0$,

$$\Theta_1(\Psi, \mathcal{T}) = -\frac{(16\rho - 1)\rho}{24 \cdot 2}\sinh\left(\frac{\Psi}{2}\right)\frac{\mathcal{T}^{\wp}}{\Gamma(\wp + 1)}.$$

On $m = 1$,

$$\Theta_2(\Psi, \mathcal{T}) = \frac{(16\rho - 1)\rho^2}{24 \cdot 2^2}\cosh\left(\frac{\Psi}{2}\right)\frac{\mathcal{T}^{2\wp}}{\Gamma(2\wp + 1)}.$$

On $m = 2$,

$$\Theta_3(\Psi, \mathcal{T}) = -\frac{(16\rho - 1)\rho^3}{24 \cdot 2^3}\sinh\left(\frac{\Psi}{2}\right)\frac{\mathcal{T}^{3\wp}}{\Gamma(3\wp + 1)}.$$

Finally, the analytical solution is given by

$$\Theta(\Psi, \mathcal{T}) = \sum_{m=0}^{\infty}\Theta_m(\Psi, \mathcal{T}) = \Theta_0(\Psi, \mathcal{T}) + \Theta_1(\Psi, \mathcal{T}) + \Theta_2(\Psi, \mathcal{T}) + \Theta_2(\Psi, \mathcal{T}) + \cdots,$$

$$\begin{aligned}
\Theta(\Psi, \mathcal{T}) =& \frac{16\rho - 1}{12}\cosh^2\left(\frac{\Psi}{4}\right) - \frac{(16\rho - 1)\rho}{24 \cdot 2}\sinh\left(\frac{\Psi}{2}\right)\frac{\mathcal{T}^{\wp}}{\Gamma(\wp + 1)} + \frac{(16\rho - 1)\rho^2}{24 \cdot 2^2}\cosh\left(\frac{\Psi}{2}\right)\frac{\mathcal{T}^{2\wp}}{\Gamma(2\wp + 1)}\\
&- \frac{(16\rho - 1)\rho^3}{24 \cdot 2^3}\sinh\left(\frac{\Psi}{2}\right)\frac{\mathcal{T}^{3\wp}}{\Gamma(3\wp + 1)} + \cdots.
\end{aligned}$$

Taking $\wp = 1$, we get

$$\Theta(\Psi, \mathcal{T}) = \frac{16\rho - 1}{12}\cosh^2\left(\frac{\rho\mathcal{T} - \Psi}{4}\right). \tag{5.14}$$

Example 2. Assume the fractional K(3, 3, 1) equation is

$$D_{\mathcal{T}}^{\wp}\Theta(\Psi, \mathcal{T}) + (\Theta^3(\Psi, \mathcal{T}))_{\Psi} - (\Theta^3(\Psi, \mathcal{T}))_{\Psi\Psi\Psi} + (\Theta(\Psi, \mathcal{T}))_{\Psi\Psi\Psi\Psi\Psi} = 0, \quad 0 < \wp \leq 1, \tag{5.15}$$

associated with initial guess

$$\Theta(\Psi, 0) = \sqrt{\frac{81\rho - 1}{54}}\cosh\left(\frac{\Psi}{3}\right).$$

By plugging in the YT, we have

$$Y\left(\frac{\partial^{\wp}\Theta}{\partial\mathcal{T}^{\wp}}\right)=Y\Big((\Theta^{3}(\Psi,\mathcal{T}))_{\Psi\Psi\Psi}-(\Theta^{3}(\Psi,\mathcal{T}))_{\Psi}-(\Theta(\Psi,\mathcal{T}))_{\Psi\Psi\Psi\Psi\Psi}\Big). \tag{5.16}$$

We have by simplifying

$$\frac{1}{u^{\wp}}\{M(u)-u\Theta(0)\}=Y\Big((\Theta^{3}(\Psi,\mathcal{T}))_{\Psi\Psi\Psi}-(\Theta^{3}(\Psi,\mathcal{T}))_{\Psi}-(\Theta(\Psi,\mathcal{T}))_{\Psi\Psi\Psi\Psi\Psi}\Big), \tag{5.17}$$

$$M(u)=u\Theta(0)+u^{\wp}\Big((\Theta^{3}(\Psi,\mathcal{T}))_{\Psi\Psi\Psi}-(\Theta^{3}(\Psi,\mathcal{T}))_{\Psi}-(\Theta(\Psi,\mathcal{T}))_{\Psi\Psi\Psi\Psi\Psi}\Big). \tag{5.18}$$

By employing inverse YT, we get

$$\begin{aligned}\Theta(\Psi,\mathcal{T})&=\Theta(0)+Y^{-1}\left[u^{\wp}\left\{Y\Big((\Theta^{3}(\Psi,\mathcal{T}))_{\Psi\Psi\Psi}-(\Theta^{3}(\Psi,\mathcal{T}))_{\Psi}-(\Theta(\Psi,\mathcal{T}))_{\Psi\Psi\Psi\Psi\Psi}\Big)\right\}\right],\\ \Theta(\Psi,\mathcal{T})&=\sqrt{\frac{81\rho-1}{54}}\cosh\left(\frac{\Psi}{3}\right)+Y^{-1}\left[u^{\wp}\left\{Y\Big((\Theta^{3}(\Psi,\mathcal{T}))_{\Psi\Psi\Psi}-(\Theta^{3}(\Psi,\mathcal{T}))_{\Psi}-(\Theta(\Psi,\mathcal{T}))_{\Psi\Psi\Psi\Psi\Psi}\Big)\right\}\right].\end{aligned} \tag{5.19}$$

Now in terms of HPM, we get

$$\begin{aligned}\sum_{k=0}^{\infty}\epsilon^{k}\Theta_{k}(\Psi,\mathcal{T})=&\sqrt{\frac{81\rho-1}{54}}\cosh\left(\frac{\Psi}{3}\right)+\epsilon\Bigg(Y^{-1}\Bigg[u^{\wp}Y\Bigg[\left(\sum_{k=0}^{\infty}\epsilon^{k}H_{k}^{1}(\Theta)\right)-\left(\sum_{k=0}^{\infty}\epsilon^{k}H_{k}^{2}(\Theta)\right)\\ &-\left(\sum_{k=0}^{\infty}\epsilon^{k}\Theta_{k}(\Psi,\mathcal{T})\right)_{\Psi\Psi\Psi\Psi\Psi}\Bigg]\Bigg]\Bigg).\end{aligned} \tag{5.20}$$

Assume the He's polynomial $H_k(\mathbb{U})$ as

$$\begin{aligned}\sum_{k=0}^{\infty}\epsilon^{k}H_{k}^{1}(\Theta)&=(\Theta^{3}(\Psi,\mathcal{T}))_{\Psi\Psi\Psi},\\ \sum_{k=0}^{\infty}\epsilon^{k}H_{k}^{2}(\Theta)&=(\Theta^{3}(\Psi,\mathcal{T}))_{\Psi}.\end{aligned} \tag{5.21}$$

The first components are illustrated as

$$\begin{aligned}H_0^1(\Theta)&=(\Theta_0^3)_{\Psi\Psi\Psi},\\ H_1^1(\Theta)&=(3\Theta_0^2\Theta_1)_{\Psi\Psi\Psi},\\ H_2^1(\Theta)&=(3\Theta_2\Theta_0^2+3\Theta_0\Theta_1^2)_{\Psi\Psi\Psi},\\ &\vdots\\ H_0^2(\Theta)&=(\Theta_0^3)_{\Psi},\\ H_1^2(\Theta)&=(3\Theta_0^2\Theta_1)_{\Psi},\\ H_2^2(\Theta)&=(3\Theta_2\Theta_0^2+3\Theta_0\Theta_1^2)_{\Psi}.\end{aligned}$$

On comparing the ϵ coefficients, we conclude

$$\epsilon^0 : \Theta_0(\Psi,\mathcal{T}) = \sqrt{\frac{81\rho-1}{54}}\cosh\left(\frac{\Psi}{3}\right),$$

$$\epsilon^1 : \Theta_1(\Psi,\mathcal{T}) = -\sqrt{\frac{81\rho-1}{54}}\frac{\rho}{3}\sinh\left(\frac{\Psi}{3}\right)\frac{\mathcal{T}^{\wp}}{\Gamma(\wp+1)},$$

$$\epsilon^2 : \Theta_2(\Psi,\mathcal{T}) = \sqrt{\frac{81\rho-1}{54}}\frac{\rho^2}{3^2}\cosh\left(\frac{\Psi}{3}\right)\frac{\mathcal{T}^{2\wp}}{\Gamma(2\wp+1)},$$

$$\epsilon^3 : \Theta_3(\Psi,\mathcal{T}) = -\sqrt{\frac{81\rho-1}{54}}\frac{\rho^3}{3^3}\sinh\left(\frac{\Psi}{3}\right)\frac{\mathcal{T}^{3\wp}}{\Gamma(3\wp+1)},$$

$$\vdots$$

Finally, the analytical solution is given by

$$\Theta(\Psi,\mathcal{T}) = \Theta_0(\Psi,\mathcal{T}) + \Theta_1(\Psi,\mathcal{T}) + \Theta_2(\Psi,\mathcal{T}) + \Theta_3(\Psi,\mathcal{T}) + \cdots,$$

$$\Theta(\Psi,\mathcal{T}) = \sqrt{\frac{81\rho-1}{54}}\cosh\left(\frac{\Psi}{3}\right) - \sqrt{\frac{81\rho-1}{54}}\frac{\rho}{3}\sinh\left(\frac{\Psi}{3}\right)\frac{\mathcal{T}^{\wp}}{\Gamma(\wp+1)} + \sqrt{\frac{81\rho-1}{54}}\frac{\rho^2}{3^2}\cosh\left(\frac{\Psi}{3}\right)\frac{\mathcal{T}^{2\wp}}{\Gamma(2\wp+1)} - \sqrt{\frac{81\rho-1}{54}}\frac{\rho^3}{3^3}\sinh\left(\frac{\Psi}{3}\right)\frac{\mathcal{T}^{3\wp}}{\Gamma(3\wp+1)} + \cdots.$$

Solution by implementing YTDM

By plugging in the YT, we obtain

$$Y\left\{\frac{\partial^{\wp}\Theta}{\partial\mathcal{T}^{\wp}}\right\} = Y\Big((\Theta^3(\Psi,\mathcal{T}))_{\Psi\Psi\Psi} - (\Theta^3(\Psi,\mathcal{T}))_{\Psi} - (\Theta(\Psi,\mathcal{T}))_{\Psi\Psi\Psi\Psi\Psi}\Big). \tag{5.22}$$

We have by simplifying

$$\frac{1}{u^{\wp}}\{M(u) - u\Theta(0)\} = Y\Big((\Theta^3(\Psi,\mathcal{T}))_{\Psi\Psi\Psi} - (\Theta^3(\Psi,\mathcal{T}))_{\Psi} - (\Theta(\Psi,\mathcal{T}))_{\Psi\Psi\Psi\Psi\Psi}\Big), \tag{5.23}$$

$$M(u) = u\Theta(0) + u^{\wp}Y\Big((\Theta^3(\Psi,\mathcal{T}))_{\Psi\Psi\Psi} - (\Theta^3(\Psi,\mathcal{T}))_{\Psi} - (\Theta(\Psi,\mathcal{T}))_{\Psi\Psi\Psi\Psi\Psi}\Big). \tag{5.24}$$

By employing inverse YT, we get

$$\Theta(\Psi,\mathcal{T}) = \Theta(0) + Y^{-1}\left[u^{\wp}\left\{Y\Big((\Theta^3(\Psi,\mathcal{T}))_{\Psi\Psi\Psi} - (\Theta^3(\Psi,\mathcal{T}))_{\Psi} - (\Theta(\Psi,\mathcal{T}))_{\Psi\Psi\Psi\Psi\Psi}\Big)\right\}\right],$$

$$\Theta(\Psi,\mathcal{T}) = \sqrt{\frac{81\rho-1}{54}}\cosh\left(\frac{\Psi}{3}\right) + Y^{-1}\left[u^{\wp}\left\{Y\Big((\Theta^3(\Psi,\mathcal{T}))_{\Psi\Psi\Psi} - (\Theta^3(\Psi,\mathcal{T}))_{\Psi} - (\Theta(\Psi,\mathcal{T}))_{\Psi\Psi\Psi\Psi\Psi}\Big)\right\}\right]. \tag{5.25}$$

Now the solution in terms of infinite series is

$$\Theta(\Psi,\mathcal{T}) = \sum_{m=0}^{\infty}\Theta_m(\Psi,\mathcal{T}). \tag{5.26}$$

Also, the nonlinear term is illustrated as $(\Theta^3(\Psi,\mathcal{T}))_\Psi = \sum_{m=0}^{\infty}\mathcal{A}_m, (\Theta^3(\Psi,\mathcal{T}))_\Psi = \sum_{m=0}^{\infty}\mathcal{B}_m$. So, we get

$$\begin{aligned}\sum_{m=0}^{\infty}\Theta_m(\Psi,\mathcal{T}) &= \Theta(\Psi,0) + Y^{-1}\left[u^{\wp}Y\left[\left(\sum_{m=0}^{\infty}\mathcal{A}_m\right) - \left(\sum_{m=0}^{\infty}\mathcal{B}_m\right) - (\Theta(\Psi,\mathcal{T}))_{\Psi\Psi\Psi\Psi\Psi}\right]\right],\\ \sum_{m=0}^{\infty}\Theta_m(\Psi,\mathcal{T}) &= \sqrt{\frac{81\rho-1}{54}}\cosh\left(\frac{\Psi}{3}\right) + Y^{-1}\left[u^{\wp}Y\left[\left(\sum_{m=0}^{\infty}\mathcal{A}_m\right) - \left(\sum_{m=0}^{\infty}\mathcal{B}_m\right) - (\Theta(\Psi,\mathcal{T}))_{\Psi\Psi\Psi\Psi\Psi}\right]\right].\end{aligned} \tag{5.27}$$

The first components are illustrated as

$$\begin{aligned}\mathcal{A}_0 &= (\Theta_0^3)_{\Psi\Psi\Psi},\\ \mathcal{A}_1 &= (3\Theta_0^2\Theta_1)_{\Psi\Psi\Psi},\\ \mathcal{A}_2 &= (3\Theta_2\Theta_0^2 + 3\Theta_0\Theta_1^2)_{\Psi\Psi\Psi},\\ &\vdots\\ \mathcal{B}_0 &= (\Theta_0^3)_{\Psi},\\ \mathcal{B}_1 &= (3\Theta_0^2\Theta_1)_{\Psi},\\ \mathcal{B}_2 &= (3\Theta_2\Theta_0^2 + 3\Theta_0\Theta_1^2)_{\Psi}.\end{aligned}$$

On comparing both sides, we conclude

$$\Theta_0(\Psi,\mathcal{T}) = \sqrt{\frac{81\rho-1}{54}}\cosh\left(\frac{\Psi}{3}\right).$$

On $m = 0$,

$$\Theta_1(\Psi,\mathcal{T}) = -\sqrt{\frac{81\rho-1}{54}}\frac{\rho}{3}\sinh\left(\frac{\Psi}{3}\right)\frac{\mathcal{T}^{\wp}}{\Gamma(\wp+1)}.$$

On $m = 1$,

$$\Theta_2(\Psi,\mathcal{T}) = \sqrt{\frac{81\rho-1}{54}}\frac{\rho^2}{3^2}\cosh\left(\frac{\Psi}{3}\right)\frac{\mathcal{T}^{2\wp}}{\Gamma(2\wp+1)}.$$

On $m = 2$,

$$\Theta_3(\Psi,\mathcal{T}) = -\sqrt{\frac{81\rho-1}{54}}\frac{\rho^3}{3^3}\sinh\left(\frac{\Psi}{3}\right)\frac{\mathcal{T}^{3\wp}}{\Gamma(3\wp+1)}.$$

Finally, the analytical solution is given by

$$\Theta(\Psi,\mathcal{T}) = \sum_{m=0}^{\infty}\Theta_m(\Psi,\mathcal{T}) = \Theta_0(\Psi,\mathcal{T}) + \Theta_1(\Psi,\mathcal{T}) + \Theta_2(\Psi,\mathcal{T}) + \Theta_2(\Psi,\mathcal{T}) + \cdots,$$

$$\begin{aligned}\Theta(\Psi,\mathcal{T}) =& \sqrt{\frac{81\rho-1}{54}}\cosh\left(\frac{\Psi}{3}\right) - \sqrt{\frac{81\rho-1}{54}}\frac{\rho}{3}\sinh\left(\frac{\Psi}{3}\right)\frac{\mathcal{T}^{\wp}}{\Gamma(\wp+1)}\\ &+ \sqrt{\frac{81\rho-1}{54}}\frac{\rho^2}{3^2}\cosh\left(\frac{\Psi}{3}\right)\frac{\mathcal{T}^{2\wp}}{\Gamma(2\wp+1)} - \sqrt{\frac{81\rho-1}{54}}\frac{\rho^3}{3^3}\sinh\left(\frac{\Psi}{3}\right)\frac{\mathcal{T}^{3\wp}}{\Gamma(3\wp+1)} + \cdots.\end{aligned}$$

Taking $\wp = 1$, we get

$$\Theta(\Psi,\mathcal{T}) = \sqrt{\frac{81\rho-1}{54}}\cosh\left(\frac{\rho\mathcal{T}-\Psi}{3}\right). \tag{5.28}$$

6. Physical interpretation of results

This section's graphical and numerical analysis provides important insights into the behaviour and precision of our suggested non-singular kernel operators and natural transform solution approach for the time-fractional K(m,n,1) equations for a range of fractional parameter $\wp$ values. We examined two nonlinear problems to demonstrate that the suggested technique can handle challenging nonlinear problems. These remarkably positive results demonstrate the efficacy of the strategy being discussed. The surface in Figure 1(a) shows the exact solution and Figure 1(b) shows the approximated solution of the proposed techniques at $\wp = 1$. The surface in Figure 2(a) shows the approximated solution of the proposed techniques at $\wp = 0.8$ and Figure 2(b) shows the approximated solution of the proposed techniques at $\wp = 0.9$. Figure 3(a) illustrates the nature of the proposed techniques solution in terms of absolute error and Figure 3(b) illustrates the 2D nature of the proposed techniques solution at distinct values of fractional order $\wp = 0.7, 0.8, 0.9, 1$. Table 1 presents the accurate solution and computed approximate solution generated from the 4th-order series solution obtained by using the proposed techniques at different orders of $\wp$.

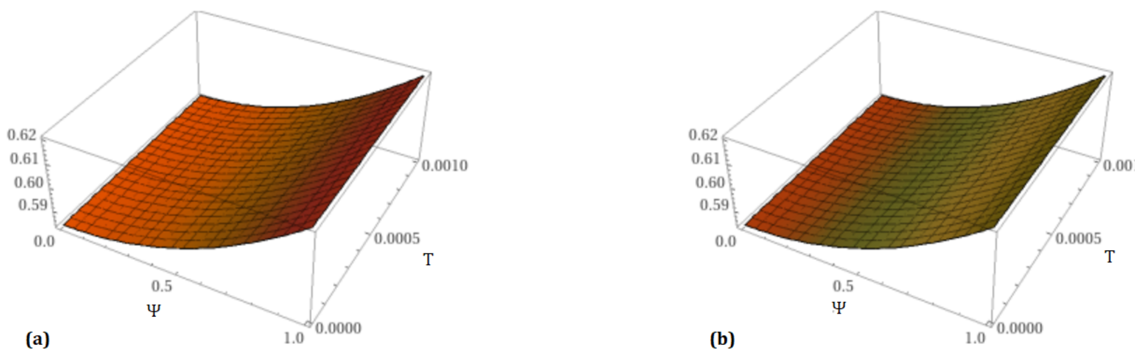

Figure 1. The plot (a) showing the precise solution and (b) showing behavior of the offered techniques at $\wp = 1$.

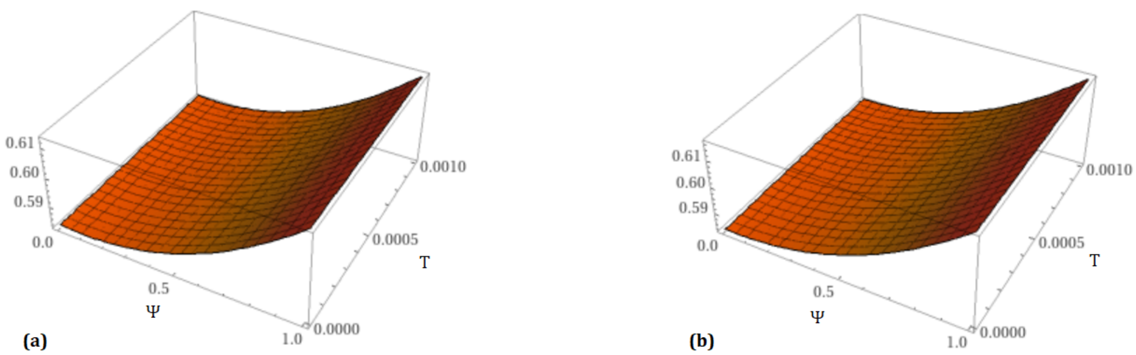

Figure 2. The plot (a) showing behavior of the offered techniques solution at $\wp = 0.8$ and (b) showing behavior of the offered techniques solution at $\wp = 0.9$.

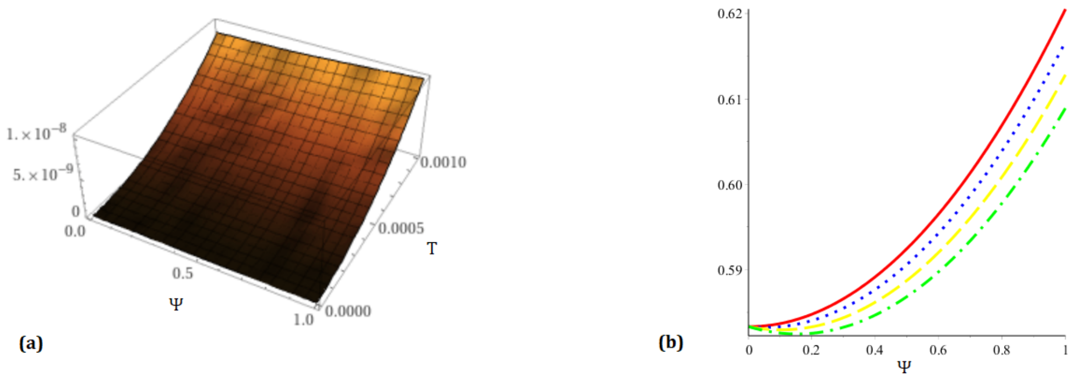

Figure 3. The plot (a) showing the absolute error and (b) showing the behavior of offered techniques solution at $\wp = 0.7, 0.8, 0.9, 1$.

Table 1. Analysis of the accurate solution and our approach's solution at numerous $\wp$ orders.

Ψ	$\mathcal{T}$	$\wp = 0.85$	$\wp = 0.90$	$\wp = 0.95$	$\wp = 1(appro)$	$\wp = 1(exact)$
	0.2	0.58477120	0.58477777	0.58478237	0.58478558	0.58478558
	0.4	0.58914247	0.58915572	0.58916499	0.58917146	0.58917146
0.001	0.6	0.59649098	0.59651104	0.59652507	0.59653487	0.59653487
	0.8	0.60689027	0.60691735	0.60693628	0.60694950	0.60694950
	1	0.62044443	0.62047879	0.62050282	0.62051958	0.62051958
	0.2	0.58475394	0.58476475	0.58477261	0.58477831	0.58477831
	0.4	0.58910757	0.58912945	0.58914532	0.58915681	0.58915681
0.002	0.6	0.59643809	0.59647126	0.59649530	0.59651269	0.59651269
	0.8	0.60681886	0.60686365	0.60689611	0.60691957	0.60691957
	1	0.62035378	0.62041064	0.62045184	0.62048162	0.62048162
	0.2	0.58473810	0.58475246	0.58476314	0.58477105	0.58477105
	0.4	0.58907543	0.58910459	0.58912620	0.58914217	0.58914217
0.003	0.6	0.59638932	0.59643357	0.59646634	0.59649053	0.59649053
	0.8	0.60675298	0.60681276	0.60685701	0.60688967	0.60688967
	1	0.62027013	0.62034604	0.62040221	0.62044367	0.62044367
	0.2	0.58472314	0.58474064	0.58475386	0.58476381	0.58476381
	0.4	0.58904498	0.58908061	0.58910743	0.58912756	0.58912756
0.004	0.6	0.59634308	0.59639720	0.59643789	0.59646840	0.59646840
	0.8	0.60669048	0.60676364	0.60681860	0.60685979	0.60685979
	1	0.62019074	0.62028367	0.62035346	0.62040575	0.62040575
	0.2	0.58470883	0.58472917	0.58474473	0.58475659	0.58475659
	0.4	0.58901575	0.58905729	0.58908893	0.58911296	0.58911296
0.005	0.6	0.59629864	0.59636180	0.59640984	0.59644628	0.59644628
	0.8	0.60663040	0.60671579	0.60678071	0.60682993	0.60682993
	1	0.62011441	0.62022291	0.62030535	0.62036784	0.62036784

The surface in Figure 4(a) shows the exact and Figure 4(b) shows the approximated solution of the proposed techniques at $\wp = 1$. The surface in Figure 5(a) shows the approximated solution of the proposed techniques at $\wp = 0.8$ and Figure 5(b) shows the approximated solution of the proposed techniques at $\wp = 0.9$. Figure 6(a) illustrates the nature of the proposed techniques solution in terms of absolute error and Figure 6(b) illustrates the 2D nature of the proposed techniques solution at distinct values of fractional order $\wp = 0.7, 0.8, 0.9, 1$. Table 2 presents the accurate solution and computed approximate solution generated from the 4th-order series solution obtained by using the proposed techniques at different orders of $\wp$. The accuracy of the findings is ensured by the numerical simulations that are provided. In comparison to the exact solution, tables yield much better outcomes. Finally, we believe that the methods we propose are very dependable and applicable to multidisciplinary study classifications including fractional-order nonlinear scientific techniques, which enhances our comprehension of nonlinear compound phenomena in related disciplines of science and innovation.

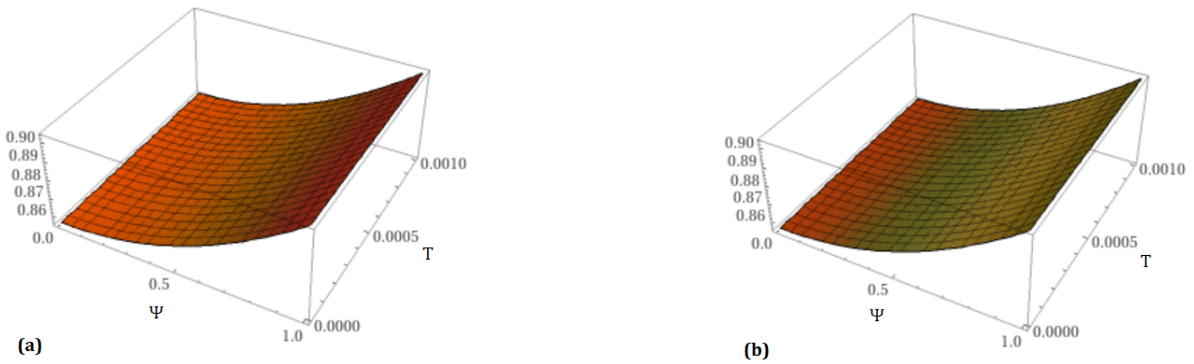

Figure 4. The plot (a) showing the precise solution and (b) showing behavior of the offered techniques at $\wp = 1$.

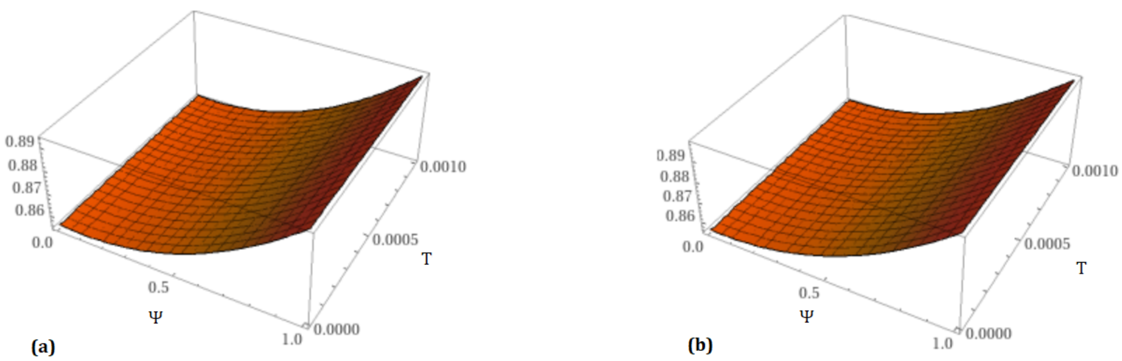

Figure 5. The plot (a) showing behavior of the offered techniques solution at $\wp = 0.8$ and (b) showing behavior of the offered techniques solution at $\wp = 0.9$.

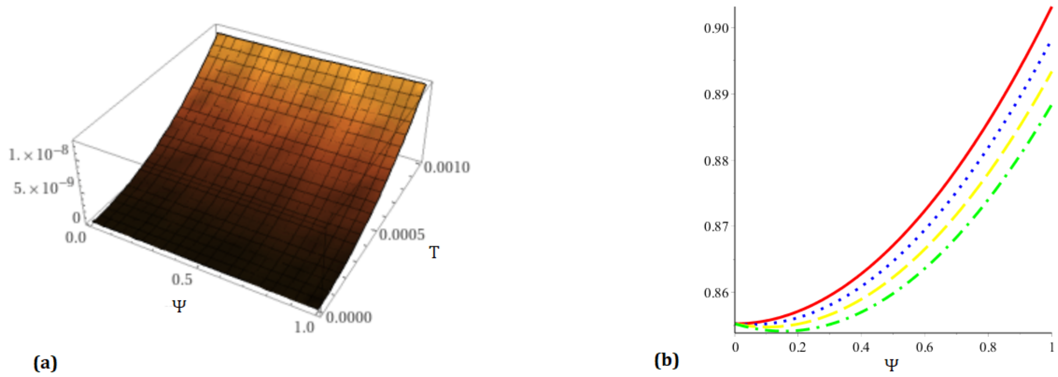

Figure 6. The plot (a) showing the absolute error and (b) showing the behavior of offered techniques solution at $\wp = 0.7, 0.8, 0.9, 1$.

Table 2. Analysis of the accurate solution and our approach's solution at numerous $\wp$ orders.

Ψ	$\mathcal{T}$	$\wp = 0.85$	$\wp = 0.90$	$\wp = 0.95$	$\wp = 1(appro)$	$\wp = 1(exact)$
	0.2	0.85571760	0.85619944	0.85667967	0.85715869	0.85715870
	0.4	0.85997291	0.86093872	0.86190132	0.86286148	0.86286149
0.001	0.6	0.86805173	0.86950582	0.87095506	0.87240063	0.87240065
	0.8	0.87998998	0.88193881	0.88388115	0.88581856	0.88581857
	1	0.89584075	0.89829298	0.90073705	0.90317492	0.90317493
	0.2	0.85569827	0.85618406	0.85666754	0.85714918	0.85714923
	0.4	0.85993415	0.86090790	0.86187700	0.86284242	0.86284247
0.002	0.6	0.86799338	0.86945941	0.87091844	0.87237194	0.87237198
	0.8	0.87991178	0.88187661	0.88383206	0.88578010	0.88578015
	1	0.89574235	0.89821472	0.90067529	0.90312652	0.90312657
	0.2	0.85568040	0.85616947	0.85665572	0.85713967	0.85713978
	0.4	0.85989833	0.86087865	0.86185330	0.86282336	0.86282346
0.003	0.6	0.86793945	0.86941537	0.87088276	0.87234324	0.87234334
	0.8	0.87983951	0.88181759	0.88378425	0.88574163	0.88574174
	1	0.89565141	0.89814045	0.90061512	0.90307812	0.90307823
	0.2	0.85566342	0.85615537	0.85664409	0.85713016	0.85713035
	0.4	0.85986430	0.86085038	0.86183000	0.86280429	0.86280449
0.004	0.6	0.86788821	0.86937281	0.87084768	0.87231454	0.87231473
	0.8	0.87977083	0.88176054	0.88373723	0.88570317	0.88570336
	1	0.89556499	0.89806867	0.90055596	0.90302972	0.90302992
	0.2	0.85564707	0.85614161	0.85663261	0.85712065	0.85712095
	0.4	0.85983153	0.86082281	0.86180700	0.86278523	0.86278553
0.005	0.6	0.86783887	0.86933130	0.87081305	0.87228584	0.87228614
	0.8	0.87970470	0.88170492	0.88369081	0.88566470	0.88566501
	1	0.89548178	0.89799868	0.90049755	0.90298132	0.90298163

7. Conclusions

Many systems and equations in several branches of research, including mathematics, physics, engineering statics, etc., require solutions. Numerous numerical and analytical techniques were developed by mathematicians in order to determine the solution exactly or approximately. In this paper, we have successfully applied two unique methods to solve time fractional non-linear dispersive K(m,n,1)-type equations analytically. The method uses the recurrence relations of the suggested methods to provide a number of successive approximations. We have solved two cases and examined our findings with the precise solution of the problem in order to demonstrate the precision and effectiveness of our methodology. The generated results and the actual solution to the issue are very similar, as shown by the solution graphs and tables. The solutions obtained at each fractional order are found to converge to the problems integer orders. We consequently come to the conclusion that the presented approaches are significant non-sophisticated effective tools that generate good quality approximations for nonlinear partial differential equations using straightforward computations and that achieve convergence with a minimal number of terms. Thus, the expansion will be significantly valued to add other operators and approaches in the future, especially in light of the advantages of the current operator. The offered strategies were determined to be suitable to address any physical problem that arises in engineering and the sciences because of their straightforward operation.

Use of AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

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Conflict of interest

The authors declare that there are no conflicts of interest regarding the publication of this article.

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