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Research article

Neural networks-based adaptive command filter control for nonlinear systems with unknown backlash-like hysteresis and its application to single link robot manipulator

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Abstract: In this paper, an adaptive neural network control problem for nonstrict-feedback nonlinear systems with an unknown backlash-like hysteresis and bounded disturbance was presented. Radial basis function neural networks (RBFNN) were used to approximate the unknown functions and the problem of the explosion of complexity problem was handled by utilizing the command filter method. Furthermore, the influence of an unknown backlash-like hysteresis input was addressed by approximating an intermediate variable. Based on the backstepping method and the command filter technique, an adaptive neural network controller was designed via the approximation abilities of RBFNN. With the help of the Lyapunov stability theory, the proposed controller ensures that all of the signals in closed-loop systems are bounded and that the tracking error fluctuates close to the origin within a bounded area. Finally, a real-world example based on the single-link manipulator was shown to demonstrate the viability of the presented approach.

Keywords: nonlinear systems; backlash-like hysteresis; command filter; backstepping method; Lyapunov stability; single link robot manipulator
Mathematics Subject Classification: 92B20, 93C10, 93C40

1. Introduction

One of the main areas of system science research is the control of nonlinear systems [1, 2]. There have been several potential nonlinear system control methods over the years, such as sliding mode-based controls and backstepping design-based controls [3–8]. Additionally, the backstepping control method is one of the best methods for designing a controller for nonlinear systems. The backstepping algorithm is based on the idea that a complex nonlinear system can be broken down into several smaller subsystems, each of which is given a virtual controller and a Lyapunov function until the design process for the entire controller is complete [9]. The backstepping technique has been used successfully to achieve several notable accomplishments. It should be noted that these works assume that nonlinearities are either directly restricted by constant variables or known functions, or are a linear combination of unknown parameters or known functions.

Fortunately, these assumptions can be removed because of the universal approximation of fuzzy logic systems (FLSs) and neural networks (NNs), which offer useful tools for developing control schemes of uncertain nonlinear systems because of their effectiveness of nonlinear approximation (see, [10, 11]). For instance, for a strict-feedback nonlinear system, an adaptive fuzzy controller has been reported in [12] by using a fuzzy system and a backstepping process. For a class of nonlinear strict-feedback systems, an adaptive fuzzy tracking control approach has been proposed in [13] based on an observer. An adaptive neural control strategy has been investigated for nonlinear systems with nonstrict feedback that are exposed to input delay [14]. There has been an analysis of a fixed-time adaptive control problem for a class of uncertain nonstrict nonlinear systems [15]. For switched nonlinear systems, a fixed-time fault-tolerant control problem has been addressed in [16]. An NN-based finite-time adaptive control problem for switched nonlinear systems with time-varying delay has been presented in [17]. Furthermore, for switched stochastic nonlinear systems utilizing time-varying delay, a fault-tolerant control problem has been developed in [18].

However, implementing backstepping becomes more challenging as the order of states rises, leading to the explosion of complexity problems caused by the calculation of the derivative of virtual controllers [19]. Dynamic surface control (DSC), which is applicable as a first-order filter to the virtual signal to avoid its repeated differentiations, was introduced to deal with the complexity explosion. The DSC can also be easily combined with NNs or FLSs approximation techniques to address nonlinear systems tracking problems. The DSC, however, ignores the issue of filtering errors produced by a filtering procedure, which may negatively affect system performance [20]. Following that, the command-filtered backstepping approach, one of the major nonlinear control innovations, was developed to address the same problem. It not only solves the complexity explosion issue, but it also provides an error compensation method to compensate for filter errors. Subsequently, numerous novel adaptive command filter control techniques have been put forth for nonlinear systems including switched nonlinear systems, nonlinear multi-agent systems, and stochastic nonlinear systems [21, 22]. An observer-based adaptive fuzzy output feedback control scheme has been developed using the command filter technique [23]. For a class of multi-input multi-output (MIMO) nonlinear systems with an unknown control direction and input saturation, a command-filtered adaptive neural tracking control approach has been presented in [24]. A command-filtered adaptive finite-time control for nonlinear systems using immersion and invariance has been presented in [25]. For nonlinear systems with quantized input signals through a command filter, a control issue has been reported in [26].

Many real-world systems, including electrical power systems, piezoelectric actuators and bilateral teleoperation systems, exhibit backlash-like hysteresis as one type of input nonlinearity [27]. It's vital to note that the nonlinear hysteresis input reduces system performance and can potentially make the systems unstable [28]. To mitigate the effects of unidentified hysteresis, there are two distinct approaches. The first approach involves building an inverse model of hysteresis to eliminate the effects of hysteresis in controller design. The other approach is to use a differential equation to model the hysteresis and then consider the effects as a bounded disturbance [29]. Therefore, a lot of effort has been put into the analysis of the backlash-like hysteresis. An adaptive control technique for a nonlinear system has been reported with backlash-like hysteresis without designing the inverse of the hysteresis [30]. An adaptive fuzzy control problem has been identified for stochastic nonlinear systems with unmeasured states and unknown backlash-like hysteresis [31]. An adaptive finite-time control strategy is suggested for switched nonlinear systems with hysteresis that approximates backlash [32]. The neural approximation has been employed to report a finite-time adaptive controller for nonlinear systems with unknown backlash-like hysteresis [33].

Inspired by the above discussions, an adaptive neural control problem via the command filter technique is presented in this paper for a class of nonlinear systems with unknown backlash-like hysteresis. The primary contribution of this work is summed up as follows:

- In contrast to previous findings [2, 4, 6], the nonstrict-feedback nonlinear system with unknown hysteresis input is taken into consideration in this study. Additionally, differently from previous findings [8–10], the influence of the unknown hysteresis input is compensated by estimating an intermediate variable, and this method can avoid the singularity problem.
- The explosion of the complexity problem with the traditional backstepping design is resolved using command filters and error compensation signals, suggesting that command filter control is more suitable in some real applications.
- For the controller design, the associated adaptive parameters are reduced to only one, which could reduce the computational load and improve the control performance. Based on the Lyapunov stability theory, the suggested control strategy assures that all of the signals in closed-loop systems are bounded and that the tracking error varies close to the origin within a small region.

The rest of the paper is structured as follows: The problem formulation and preliminaries are presented in section two. The controller design process and stability analysis are presented in section three. A simulation example is provided in section four to demonstrate the viability of the proposed approach. The conclusion is shown in section five.

2. Problem formulation and preliminaries

Consider the following nonlinear systems in nonstrict-feedback form

$$\begin{cases} \dot{\zeta}_i = f_i(\zeta) + g_i(\zeta)\zeta_{i+1} + d_i(t), \quad i = 1, \dots, n-1, \\ \dot{\zeta}_n = f_n(\zeta) + g_n(\zeta)u + d_n(t), \\ y = \zeta_1, \end{cases} \tag{2.1}$$

where ζ represents the state vector, with $\zeta = [\zeta_1, \zeta_2, \dots, \zeta_n]^T$, $f_i(\cdot)$ is an unknown nonlinear function, $g_i(\cdot)$ represents the known nonlinear function, $d_i(t)$ represents unknown bounded disturbances, $y \in \mathbb{R}$

represents the system output and $u \in \mathbb{R}$ represents the system input. The output of the unknown backlash-like hysteresis is described as

$$\frac{du}{dt} = m \mid \frac{dv}{dt} \mid (\gamma v - u) + c\frac{dv}{dt}, \tag{2.2}$$

where v denotes the input of the backlash-like hysteresis, m and c denote the unknown constants, and $\gamma > 0$ represents the slope of the lines with $\gamma \geq c$.

As mentioned in reference [28], (2.2) can be expressed as

$$u(v) = \gamma v(t) + d(v), \tag{2.3}$$

where

$$d(v) = [u_0 - \gamma v_0] e^{-m(v-v_0)} \operatorname{sgn} \dot{v} + e^{-uv \operatorname{sgn} \dot{v}} \int_{v_0}^{v} (c - \gamma) \, e^{m(\operatorname{sgn} \dot{v})} d\eta,$$

where $v_0 = v(0)$, $u_0 = u(0)$ are the initial conditions of u and v and $d(v)$ is bounded, which has been proved in [27] such that $|d(v)| \leq D$ with D being a constant.

Control objective. In this work, the control objective is to design an adaptive control scheme that ensures the system output y tracks a reference signal ζ_d while ensuring that all signals remain bounded within the closed-loop system. Additionally, the goal is to ensure that the tracking error $e_1 = y - \zeta_d$ converges to a bounded set.

In the process of designing a controller, a radial basis function's NN [15] is used to model a continuous nonlinear function $f(X) : \mathbb{R}^n \to \mathbb{R}$, which is represented as

$$f(X) = W^T P(X), \tag{2.4}$$

where, $X \in \Omega_X \subset \mathbb{R}^q$ represents the input vector, $W = [W_1, \ldots, W_l]^T$ is the weight vector with $l > 1$ being the number of nodes, and $P(X) = [p_i(X), ..., p_l(X)]^T \in \mathbb{R}^l$ is the radial basis function vector with $p_i(X)$ selected as a Gaussian function defined as

$$p_i(X) = exp\left(-\frac{(X - \kappa_i)^T (X - \kappa_i)}{\eta^2}\right), \tag{2.5}$$

where $\kappa_i = [\kappa_{i1}, \ldots, \kappa_{iq}]^T$ represents the centers of the receptive field and η represents the width parameter of the Gaussian function.

As described in [15], for any given constant $\epsilon > 0$ and continuous function $f(X)$, there exists a NN $W^{*T} P(X)$ such that

$$f(X) = W^{*T} P(X) + \delta(X), \quad \forall X \in \Omega_X, \tag{2.6}$$

where W^* represents the ideal weight vector defined as

$$W^* = \arg \min_{W \in \mathbb{R}^l} \sup_{X \in \Omega_X} \left| f(X) - W^T P(X) \right|, \tag{2.7}$$

where $\delta(X)$ is the approximation error with $|\delta(X)| < \epsilon$. The following assumptions are considered:

Assumption 1. *[4] For $i = 1, \ldots, n$, the desired signal ζ_d and its ith order time derivatives $\zeta_d^{(i)}$ are continuous and bounded.*

Assumption 2. *[6] The disturbance $d_i(t)$ satisfies $|d_i(t)| \leq \bar{d}_i$ for constants $\bar{d}_i$.*

Assumption 3. *[8] Let $\Omega_d \in \mathbb{R}^n$ be an open set containing the origin, initial condition ζ_0 and reference signal ζ_d. Within system (1), f_i and g_i are bounded within in $\bar{\Omega}_d$. There are positive constants b_m and b_M such that $0 \leq b_m \leq |g_i| \leq b_M$. Without loss of generality, suppose that $g_i > 0$.*

Remark 1. *It's important to highlight that Assumption 3 specifies that g_i is away from zero. Moreover, as demonstrated in [8], such an assumption is both reasonable and commonly accepted.*

Lemma 1. *(Young's Inequality) [34]. For all $(x, y) \in \mathbb{R}^2$, one has*

$$xy \leq \frac{1}{m}|x|^m + \frac{1}{n}|y|^n, \tag{2.8}$$

where $m > 1$, $n > 1$ and $(m-1)(n-1) = 1$.

3. Controller design and stability analysis

In this section, an adaptive control method is presented for nonlinear systems (2.1) using NNs and the backsepping method via the command filter. Now, define the tracking error variable as $e_1 = \zeta_1 - \zeta_d$ and $e_i = \zeta_i - \zeta_{i,c}$ for $i = 2, 3, \ldots, n$. Moreover, the virtual controller α_{i-1} is introduced to represent the control input of the command filters and χ_i is the output of these command filters. From [35], the command filters can be expressed as follows:

$$\dot{\chi}_1 = \omega_n \chi_2, \tag{3.1}$$

$$\dot{\chi}_2 = -2\tau\omega_n\chi_2 - \omega_n(\chi_1 - \alpha_i), \tag{3.2}$$

for $i = 1, 2, \ldots, n-1$. The initial conditions of each filter are given by $\chi_1(0) = \alpha_i(0)$ and $\chi_2(0) = 0$. Furthermore, we can choose the parameters $\omega_n > 0$ and $\tau \in (0, 1]$ such that $|\chi_1 - \alpha_i| \leq \mu$, with $\mu > 0$.

Remark 2. *It is important to emphasize that the use of the filtering command may introduce errors, thereby adding complexity to achieving good tracking performance. To address this concern, an error compensation mechanism is designed to reduce the errors $(\zeta_{i+1,c} - \alpha_i)$ that arise during the filtering process.*

The compensating signals ξ_i $(i = 1, \ldots, n)$ are defined as:

$$\xi_1 = -k_1\xi_1 + g_1\xi_2 + g_1(\zeta_{2,c} - \alpha_1), \tag{3.3}$$

$$\dot{\xi}_i = -k_i\xi_i - g_{i-1}\xi_{i-1} + g_i\xi_{i+1} + g_i(\zeta_{i+1,c} - \alpha_i), \tag{3.4}$$

$$\dot{\xi}_n = -k_n\xi_n - g_{n-1}\xi_{n-1}, \tag{3.5}$$

where $k_i > 0$ is given constants and $\xi(0) = 0$. Also from [36], $\|\xi_i\|$ is bounded with $\lim_{t\to\infty} \|\xi_i\| \leq \frac{\mu\rho}{2k_o}$, where $k_0 = 1/2 \min\{k_i\}$. Define the compensated tracking errors as $\nu_i = e_i - \xi_i$. To achieve the control objective, the virtual control signals and controller are constructed as follows:

$$\alpha_1 = \frac{1}{g_1}\left(-k_1 e_1 - \nu_1 + \dot{\zeta}_d - \frac{\nu_1 \hat{\theta} P_1^T P_1}{2a_1^2}\right), \tag{3.6}$$

$$\alpha_i = \frac{1}{g_i}\left(-k_i e_i - \nu_i + g_{i-1}e_{i-1} + \dot{\zeta}_{i,c} - \frac{\nu_i \hat{\theta} P_i^T P_i}{2a_i^2}\right), \tag{3.7}$$

$$\nu = \frac{1}{g_n \gamma}\left(-k_n e_n - \nu_n + g_{n-1}e_{n-1} + \dot{\zeta}_{n,c} - \frac{\nu_n \hat{\theta} P_n^T P_n}{2a_n^2}\right), \tag{3.8}$$

where $k_i > 0$ and $a_i > 0$ are constants, γ is defined in (2.3) and $\hat{\theta}$ denotes the estimation of the unknown parameter θ, where θ is defined as $\theta = \max(\|W_i\|^2; i = 1, 2, \ldots, n)$ with W_i being the weight vector.

Step 1. Let $V_1 = \frac{1}{2}\nu_1^2$ be a Lyapunov function. Differentiating V_1 yields

$$\dot{V}_1 = \nu_1\left(f_1(\zeta) + g_1(\zeta)\zeta_2 + d_1(t) - \dot{\zeta}_d - \dot{\xi}_1\right). \tag{3.9}$$

Using the approximating ability of radial basis function NNs, one can approximate the unknown function f_1. Thus, for any $\epsilon_1 > 0$, there always exists $W_1^T P_1(X)$ NNs such that f_1 can be approximated as

$$f_1 = W_1^T P_1(X) + \delta_1, \tag{3.10}$$

with δ_1 representing the approximation error and $|\delta_1| \leq \epsilon_1$.

By applying Young's inequality, we find that

$$\nu_1 f_1 \leq \frac{1}{2a_1^2}\nu_1^2\|W_1\|^2 P_1^T P_1 + \frac{1}{2}a_1^2 + \frac{1}{2}\nu_1^2 + \frac{1}{2}\epsilon_1^2, \tag{3.11}$$

$$\nu_1 d_1 \leq \frac{1}{2}\nu_1^2 + \frac{1}{2}\bar{d}_1^2. \tag{3.12}$$

Substituting (3.3), (3.6), (3.11) and (3.12) into (3.9) yields

$$\dot{V}_1 \leq -k_1\nu_1^2 + \frac{1}{2l_1^2}\nu_1^2\left(\|W_1\|^2 - \hat{\theta}\right)P_1^T P_1 + \frac{1}{2}a_1^2 + \frac{1}{2}\epsilon_1^2 + \frac{1}{2}\bar{d}_1^2) + g_1(\zeta)\nu_1\nu_2. \tag{3.13}$$

Step k ($k = 2, 3, \ldots, n-1$)**.** Consider the Lyapunov candidate function as

$$V_k = V_{k-1} + \frac{1}{2}\nu_k^2, \tag{3.14}$$

then, based on the definition of the tracking error variable e_i and the compensating tracking error variable ν_i, the derivative of V_k is given by

$$\begin{aligned}\dot{V}_k &= \dot{V}_{k-1} + \nu_k\dot{\nu}_k = \dot{V}_{k-1} + \nu_k(\dot{e}_k - \dot{\xi}_k)\\ &\leq -\sum_{i=1}^{k-1} k_i\nu_i^2 + \sum_{i=1}^{k-1}\frac{1}{2a_i^2}\nu_i^2\left(\|W_i\|^2 - \hat{\theta}\right)P_i^T P_i\\ &\quad + \frac{1}{2}\sum_{i=1}^{k-1}(a_i^2 + \epsilon_i^2 + \bar{d}_i^2) + g_{k-1}\nu_{k-1}\nu_k + \nu_k(f_k) + g_k x_{k+1} + d_k(t) - \dot{\zeta}_{k,c} - \dot{\xi}_k.\end{aligned} \tag{3.15}$$

By using the approximating ability of radial basis function NNs, one can approximate the unknown function f_k to design the virtual control signal. Thus, for any $\epsilon_k > 0$, there always exists $W_k^T P_k(X)$) NNs such that f_k can be approximated as

$$f_k = W_k^T P_k(X) + \delta_k, \tag{3.16}$$

with $|\delta_k| \leq \epsilon_k$.

By applying Young's inequality, we find that

$$\nu_k f_k \leq \frac{1}{2a_k^2}\nu_k^2 \|W_k\|^2 P_k^T P_k + \frac{1}{2}a_k^2 + \frac{1}{2}\nu_k^2 + \frac{1}{2}\epsilon_k^2, \tag{3.17}$$

$$\nu_k d_k \leq \frac{1}{2}\nu_k^2 + \frac{1}{2}\bar{d}_k^2. \tag{3.18}$$

By substituting equations (3.4), (3.7), (3.17) and (3.18) into (3.15), we have

$$\dot{V}_k \leq -\sum_{i=1}^{k} k_i \nu_i^2 + \sum_{i=1}^{k} \frac{1}{2a_i^2}\nu_i^2 \left(\|W_i\|^2 - \hat{\theta}\right) P_i^T P_i + \frac{1}{2}\sum_{i=1}^{k}(a_i^2 + \epsilon_i^2 + \bar{d}_i^2) + g_k \nu_k \nu_{k+1}. \tag{3.19}$$

Step n. Consider the following Lyapunov function

$$V_n = V_{n-1} + \frac{1}{2}\nu_n^2. \tag{3.20}$$

The time derivative of V_n follows as:

$$\begin{aligned}\dot{V}_n \leq & -\sum_{i=1}^{n-1} k_i \nu_i^2 + \sum_{i=1}^{n-1} \frac{1}{2a_i^2}\nu_i^2 \left(\|W_i\|^2 - \hat{\theta}\right) P_i^T P_i + \frac{1}{2}\sum_{i=1}^{n-1} \frac{1}{n}(l_i^2 + \epsilon_i^2 + \bar{d}_i^2) \\ & + g_{n-1}\nu_{n-1}\nu_n + \nu_n \left(f_n + g_n(\gamma v(t) + d(v)) + d_n(t) - \dot{\zeta}_{n,c} - \dot{\xi}_n\right).\end{aligned} \tag{3.21}$$

Similarly, for a given $\epsilon_n > 0$, we have

$$f_n(\zeta) = W_n^T P_n(X) + \delta_n \tag{3.22}$$

with $|\delta_n| < \epsilon_n$.

Furthermore, one has

$$\nu_n f_n \leq \frac{1}{2a_n^2}\nu_n^2 \|W_n\|^2 P_n^T P_n + \frac{1}{2}a_n^2 + \frac{1}{2}\nu_n^2 + \frac{1}{2}\epsilon_n^2, \tag{3.23}$$

and

$$\nu_n d_n \leq \frac{1}{2}\nu_n^2 + \frac{1}{2}\bar{d}_n^2, \tag{3.24}$$

$$\nu_n d(v) \leq \frac{1}{2}\nu_n^2 + \frac{1}{2}D^2. \tag{3.25}$$

Substituting Eqs (3.5), (3.8) and (3.23)–(3.25) into (3.21), one has

$$\dot{V}_n \leq -\sum_{i=1}^{n} k_i \nu_i^2 + \sum_{i=1}^{n} \frac{1}{2a_i^2}\nu_i^2 \left(\|W_i\|^2 - \hat{\theta}\right) P_i^T P_i + \frac{1}{2}\sum_{i=1}^{n}(a_i^2 + \epsilon_i^2 + \bar{d}_i^2) + \frac{1}{2}D^2. \tag{3.26}$$

Design a parameter $\tilde{\theta}$ as $\tilde{\theta} = \theta - \hat{\theta}$ and choose a Lyapunov function as

$$V = V_n + \frac{1}{2\beta}\tilde{\theta}^T\tilde{\theta}, \tag{3.27}$$

where $\beta > 0$ is a design parameter.

By using (3.26) and taking the time derivative of V defined in (3.27), one has

$$\dot{V} \leq -\sum_{i=1}^{n} k_i v_i^2 + \frac{1}{2}\sum_{i=1}^{n}(a_i^2 + \epsilon_i^2 + \bar{d}_i^2) + \frac{1}{\beta}\tilde{\theta}\left(\sum_{i=1}^{n}\frac{1}{2a_i^2}\beta v_i^2 P_i^T P_i - \dot{\hat{\theta}}\right) + \frac{1}{2}D^2. \tag{3.28}$$

Define the adaptive law as

$$\dot{\hat{\theta}} = \sum_{i=1}^{n}\frac{1}{2a_i^2}\beta v_i^2 P_i^T P_i - \varrho\hat{\theta}, \tag{3.29}$$

where $a_i > 0$, $\varrho > 0$ and $\beta > 0$ are the design parameters.

Theorem 1. *Consider the nonlinear system (2.1) with an unknown hysteresis input (2.2), assuming that the system (2.1) satisfies Assumptions 1–3. Under the virtual controllers (3.6) and (3.7), real controller (3.8) and adaptive law (3.29), the following outcomes can be affirmed: (i) The boundedness of all signals within the closed-loop system can be assured; (ii) the system output y can closely track the reference signal ζ_d.*

Proof. By utilizing (3.29), we rewrite (3.28) as

$$\dot{V} \leq -\sum_{i=1}^{n} k_i v_i^2 + \frac{1}{2}\sum_{i=1}^{n}(a_i^2 + \epsilon_i^2 + \bar{d}_i^2) + \frac{\varrho}{\beta}\tilde{\theta}^T\hat{\theta} + \frac{1}{2}D^2. \tag{3.30}$$

Applying Young's inequality, it is evident that

$$\tilde{\theta}^T\hat{\theta} \leq -\frac{1}{2}\tilde{\theta}^2 + \frac{1}{2}\theta^2. \tag{3.31}$$

Consequently, we can conclude that

$$\dot{V} \leq -\sum_{i=1}^{n} k_i v_i^2 + \frac{1}{2}\sum_{i=1}^{n}(a_i^2 + \epsilon_i^2 + \bar{d}_i^2) - \frac{\varrho}{2\beta}\tilde{\theta}^2 + \frac{\varrho}{2\beta}\theta^2 + \frac{1}{2}D^2 \leq -aV + b, \tag{3.32}$$

where $a = \min\{2k_1, \ldots, 2k_n, \varrho\}$ and $b = \frac{1}{2}\sum_{i=1}^{n}(a_i^2 + \epsilon_i^2 + \bar{d}_i^2) + \frac{\varrho}{2\beta}\theta^2 + \frac{1}{2}D^2$,

then, based on Eq (3.32) one has

$$V(t) \leq \left(V(t_0) - \frac{b}{a}\right)e^{-a(t-t_0)} + \frac{b}{a} \leq V(t_0) + \frac{b}{a}, \forall t \geq t_0. \tag{3.33}$$

It is evident that v_i and $\tilde{\theta}$ are bounded for $i = 1, 2, \ldots, n$. Since θ is a constant, $\hat{\theta}$ is bounded in probability. The norm $\|\xi_i\|$ is bounded, and with $e_i = v_i + \xi_i$ we can ascertain that the signal e_i is also bounded. Consequently, both $\zeta(t)$ and all control signals remain bounded over any time interval. By [36], we can conclude that a solution exists for $t \in [0, \infty)$. Therefore, it can be established that

$$\lim_{t\to\infty}|e_1| \leq \sqrt{\frac{2b}{a}} + \frac{\mu\rho}{2k_0}. \tag{3.34}$$

□

Remark 3. *It is clear from inequality (3.34) and the definitions of a and b that the design parameters* k_i, ϱ *and* a_i *have an impact on the tracking error* $e_1 = y - \zeta_d$. *The tracking error will be significantly reduced by increasing* k_i *and* ϱ *while simultaneously decreasing* a_i.

4. Simulation results

This section gives an example to demonstrate the viability of the proposed control approach.

Example 1. (Single-link robot manipulator system application) Consider the following single-link robot system [37] as depicted in Figure1:

$$M\ddot{q} + \frac{1}{2}m_1 gl\sin(q) = u, \quad y = q, \tag{4.1}$$

where M stands for the moment of inertia with a value of $0.5\,\text{kg}\cdot\text{m}^2$, l represents the length of *one* m, m_1 is the mass with a weight of *one* kg, q denotes the angle between the link and the horizontal ground, g stands for the acceleration due to gravity at $9.8\,\text{m/s}^2$ and u represents the input torque.

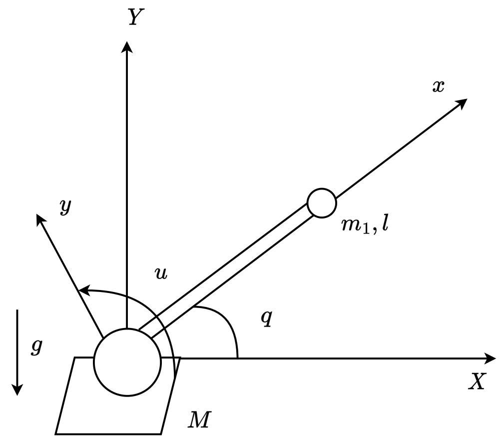

Figure 1. Architecture of single-link robot manipulator system.

Furthermore, (4.1) can be expressed as

$$\begin{cases} \dot{\zeta}_1 = \zeta_2, \\ \dot{\zeta}_2 = \frac{1}{M}u - \frac{1}{2M}m_1 gl\sin(\zeta_1) + \sin(t), \\ y = \zeta_1, \end{cases} \tag{4.2}$$

where $f_1 = 0$, $f_2 = -\frac{1}{2M}m_1 gl\sin(\zeta_1)$, $g_1 = 1$, $g_2 = \frac{1}{M}$, $d_1 = 0$, $d_2 = \sin(t)$ and u represents the output of a backlash-like hysteresis described in (2.2) with $m = 1$, $\gamma = 4.125$ and $c = 0.432$. The reference signal is represented as $\zeta_d = 0.5\sin(2t)$.

The system (4.2) is subjected to control using the command-filtering neural controller proposed in this paper. Error definitions are provided as follows: $e_1 = \zeta_1 - \zeta_d$ and $e_2 = \zeta_2 - \zeta_{2,c}$. The virtual control law is designed as

$$\alpha_1 = \frac{1}{g_1}\left(-k_1 e_1 - \nu_1 + \dot{\zeta}_d - \frac{\nu_1\hat{\theta}P_1^T P_1}{2a_1^2}\right). \tag{4.3}$$

Compensating signals are designed as

$$\dot{\xi}_1 = -k_1\xi_1 + \xi_2 + (\zeta_{1,c} - \alpha_1), \tag{4.4}$$

$$\dot{\xi}_2 = -k_2\xi_2 - \xi_1 + \xi_3 + (\zeta_{2,c} - \alpha_2). \tag{4.5}$$

The compensated error signals are denoted as $v_i = e_i - \xi_i$ for $i = 1, 2$, and the control law v is given by

$$v = \frac{1}{g_2\gamma}\left(-k_2e_2 - v_2 + g_1e_1 + \dot{\zeta}_{2,c} - \frac{v_2\hat{\theta}P_2^TP_2}{2a_2^2}\right). \tag{4.6}$$

The adaptive law is designed as

$$\dot{\hat{\theta}} = \sum_{i=1}^{n}\beta v_i^2P_i^TP_i\frac{1}{2a_i^2} - \varrho\hat{\theta}, \quad i = 1, 2. \tag{4.7}$$

The initial conditions are chosen as $\zeta_1(0) = 0.5$, $\zeta_2(0) = 0$, and $\hat{\theta}(0) = 0$. The design parameters are chosen by using a trial and error method as $k_1 = 10$, $k_2 = 10$, $a_1 = 1$, $a_2 = 1$, $\varrho = 0.5$ and $\beta = 2$.

The simulation results are presented in Figures 2–7. Figures 2 and 3 display the comparative simulation results obtained using the proposed method and the existing control method described in [20], respectively. Observing Figures 2 and 3, it becomes evident that while the control method outlined in [20] yields good tracking performance, the proposed control method exhibits a slight improvement in tracking performance over the existing method [20]. Specifically, the tracking error of the proposed control method is slightly better than that of the existing method in [20], ensuring a more accurate and controlled trajectory for the tracked signal. Furthermore, Figure 4 offers insights into the system state ζ_2. The behavior of the adaptive law $\hat{\theta}$ is illustrated in Figures 5 and 6 for both the proposed control method and the existing control method detailed in [20], demonstrating its bounded nature. The system input u and the control signal v are depicted in Figure 7. These simulation results make it evident that the proposed control method not only ensures the boundedness of all closed-loop signals in the system (4.2), but also achieves impressive tracking performance.

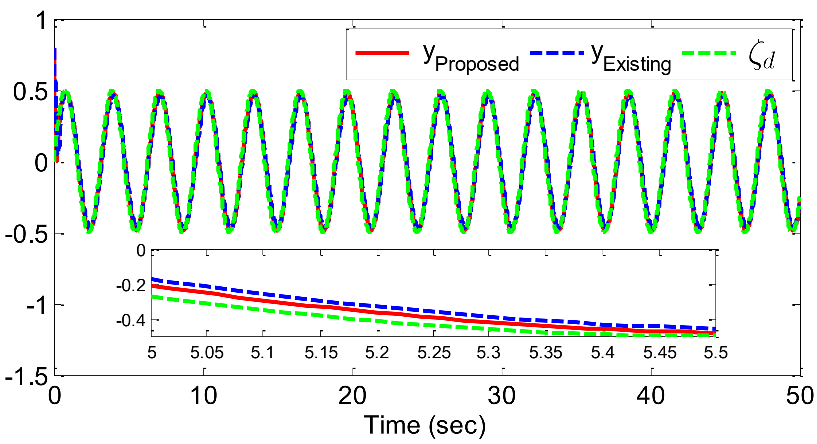

Figure 2. Tracking Performance. ζ_d is the reference signal and y_{Proposed} and y_{Existing} are the system outputs by using the proposed control method and existing control method in [20], respectively.

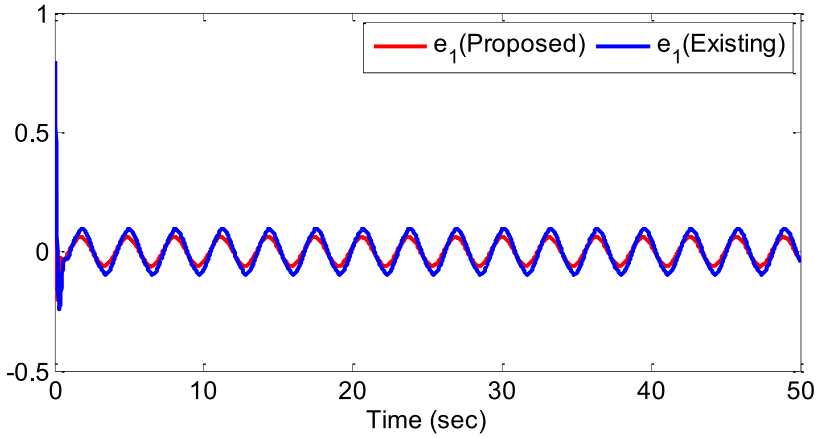

Figure 3. Tracking error e_1. e_1(Proposed) and e_1(Existing) represent the tracking error by using the proposed control method and existing proposed method in [20], respectively.

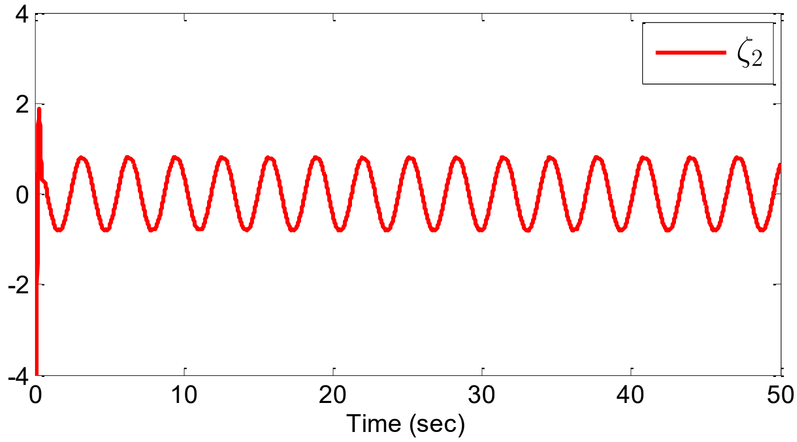

Figure 4. The state variable ζ_2 .

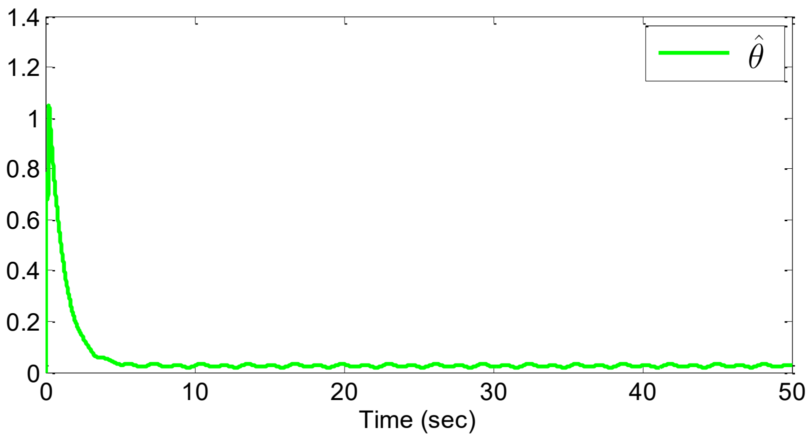

Figure 5. The response of adaptive law $\hat{\theta}$ by using the proposed method.

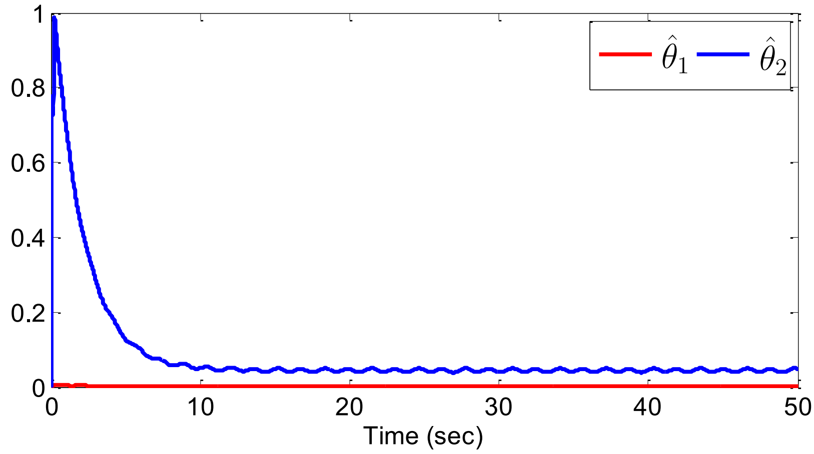

Figure 6. The response of adaptive law $\hat{\theta}_1$ and $\hat{\theta}_2$ by using the existing control method in [20].

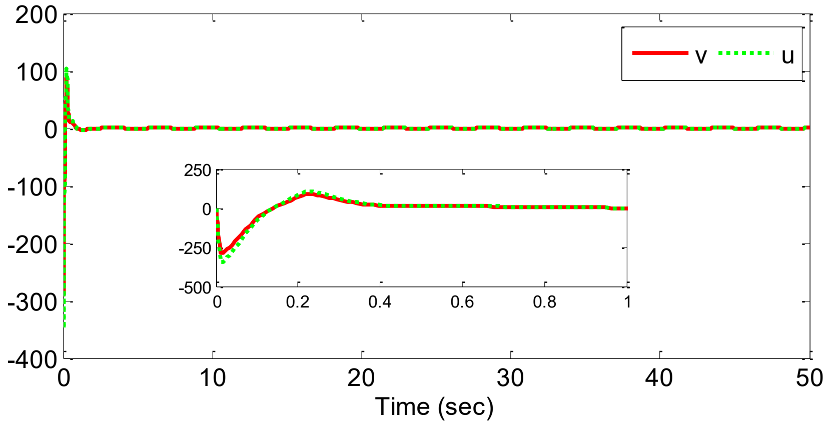

Figure 7. Trajectory of v and u.

5. Conclusions

This paper addressed the adaptive neural control problem for nonstrict-feedback nonlinear systems with unknown backlash-like hysteresis and bounded disturbance. By using the approximation abilities of radial basis function neural networks (RBFNN), the command filter method and the backstepping technique, an adaptive controller was designed, which effectively ensures boundedness of all signals in the closed-loop system. The feasibility of this approach was demonstrated via a single-link manipulator example. Future research will examine the inclusion of state variables that are not directly measurable, improving the applicability for real industrial systems.

Use of AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

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Conflict of interest

The authors confirm no conflicts of interest.

References

1. K. Yu, Y. Li, Adaptive fuzzy control for nonlinear systems with sampled data and time-varying input delay, *AIMS Mathematics*, **5** (2020), 2307–2325. https://doi.org/10.3934/math.2020153
2. P. Li, G. Yang, Backstepping adaptive fuzzy control of uncertain nonlinear systems against actuator faults, *J. Control Theory Appl.*, **7** (2009), 248–256. https://doi.org/10.1007/s11768-009-8074-6
3. L. Zhi, J. Wu, Adaptive constraint control for nonlinear multi-agent systems with undirected graphs, *AIMS Mathematics*, **6** (2021), 12051–12064. https://doi.org/10.3934/math.2021698
4. Y. Liu, X. Liu, Y. Jing, Adaptive neural networks finite-time tracking control for non-strict feedback systems via prescribed performance, *Inform. Sciences*, **468** (2018), 29–46. https://doi.org/10.1016/j.ins.2018.08.029
5. X. Song, L. Shen, F. Chen Adaptive backstepping position tracking control of quadrotor unmanned aerial vehicle system, *AIMS Mathematics*, **8** (2023), 16191–16207. https://doi.org/10.3934/math.2023828
6. X. Yang, W. Deng, J. Yao, Disturbance-observer-based adaptive command filtered control for uncertain nonlinear systems, *ISA T.*, **130** (2022), 490–499. https://doi.org/10.1016/j.isatra.2022.04.007
7. X. Zhang, L. Liu, Y. Liu, Adaptive NN control based on Butterworth low-pass filter for quarter active suspension systems with actuator failure, *AIMS Mathematics*, **6** (2021), 754–771. https://doi.org/10.3934/math.2021046
8. J. Zhang, J. Xia, W. Sun, Z. Wang, H. Shen, Command filter-based finite-time adaptive fuzzy control for nonlinear systems with uncertain disturbance, *J. Frank. I.*, **356** (2019), 711270–11284. https://doi.org/10.1016/j.jfranklin.2019.05.042
9. H. Wang, P. Liu, X. Zhao, X. Liu, Adaptive fuzzy finite-time control of nonlinear systems with actuator faults, *IEEE T. Cybernetics*, **50** (2020), 1786–1797. https://doi.org/10.1109/TCYB.2019.2902868
10. K. Sun, J. Qiu, H. Karimi, Y. Fu, Event-triggered robust fuzzy adaptive finite-time control of nonlinear systems with prescribed performance, *IEEE T. Fuzzy Syst.*, **29** (2021), 1460–1471. https://doi.org/10.1109/TFUZZ.2020.2979129
11. K. Sun, L. Liu, J. Qiu, G. Feng, Fuzzy adaptive finite-time fault-tolerant control for strict-feedback nonlinear systems, *IEEE T. Fuzzy Syst.*, **29** (2021), 786–796. https://doi.org/10.1109/TFUZZ.2020.2965890

12. B. Chen, X. Liu, S. Ge, C. Lin, Adaptive fuzzy control of a class of nonlinear systems by fuzzy approximation approach, *IEEE T. Fuzzy Syst.*, **20** (2012), 1012–1021. https://doi.org/10.1109/TFUZZ.2012.2190048
13. B. Chen, C. Lin, X. Liu, K. Liu, Observer-based adaptive fuzzy control for a class of nonlinear delayed systems, *IEEE T. Syst. Man Cy.*, **46** (2016), 27–36. https://doi.org/10.1109/TSMC.2015.2420543
14. Y. Han, Design of decentralized adaptive control approach for large-scale nonlinear systems subjected to input delays under prescribed performance, *Nonlinear Dyn.*, **106** (2021), 565–582. https://doi.org/10.1007/s11071-021-06843-z
15. D. Ba, Y. Li, S. Tong, Fixed-time adaptive neural tracking control for a class of uncertain nonstrict nonlinear systems, *Neurocomputing*, **363** (2019), 273–280. https://doi.org/10.1016/j.neucom.2019.06.063
16. D. Cui, Z. Xiang, Nonsingular fixed-time fault-tolerant fuzzy control for switched uncertain nonlinear systems, *IEEE T. Fuzzy Syst.*, **31** (2023), 174–183. https://doi.org/10.1109/TFUZZ.2022.3184048
17. D. Cui, W. Zou, J. Guo, Z. Xiang, Neural network-based adaptive finite-time tracking control of switched nonlinear systems with time-varying delay, *Appl. Math. Comput.*, **428** (2022), 127216. https://doi.org/10.1016/j.amc.2022.127216
18. D. Cui, W. Zou, J. Guo, Z. Xiang, Adaptive fault-tolerant decentralized tracking control of switched stochastic uncertain nonlinear systems with time-varying delay, *Int. J. Adapt. Control*, **36** (2022), 2971–2987. https://doi.org/10.1002/acs.3491
19. J. Ding, W. Zhang, Finite-time adaptive control for nonlinear systems with uncertain parameters based on the command filters, *Int. J. Adapt. Control*, **35** (2021), 1754–1767. https://doi.org/10.1002/acs.3287
20. J. Xia, J. Zhang, J. Feng, Z. Wang, G. Zhuang, Command filter-based adaptive fuzzy control for nonlinear systems with unknown control directions, *IEEE T. Syst. Man Cy.*, **51** (2019), 1945–1953. https://doi.org/10.1109/TSMC.2019.2911115
21. J. Ma, J. Park, S. Xu, Command-filter-based finite-time adaptive control for nonlinear systems with quantized input, *IEEE T. Automat. Contr.*, **66** (2021), 2339–2344. https://doi.org/10.1109/TAC.2020.3006283
22. Y. Wang, N. Xu, Y. Liu, X. Zhao Adaptive fault-tolerant control for switched nonlinear systems based on command filter technique, *Appl. Math. Comput.*, **392** (2021), 125725. https://doi.org/10.1016/j.amc.2020.125725
23. J. Yu, P. Shi, W. Dong, H. Yu, Observer and command-filter-based adaptive fuzzy output feedback control of uncertain nonlinear systems, *IEEE T. Ind. Electron.*, **62** (2015), 5962–5970. https://doi.org/10.1109/TIE.2015.2418317
24. J. Yu, P. Shi, C. Lin, H. Yu, Adaptive neural command filtering control for nonlinear MIMO systems with saturation input and unknown control direction, *IEEE T. Cybernetics*, **50** (2020), 2536–2545. https://doi.org/10.1109/TCYB.2019.2901250
25. J. Yu, P. Shi, X. Chen, G. Cui, Finite-time command filtered adaptive control for nonlinear systems via immersion and invariance, *Sci. China Inf. Sci.*, **64** (2021), 192202. https://doi.org/10.1007/s11432-020-3144-6

26. S. Song, J. Park, B. Zhang, X. Song, Z. Zhang, Adaptive command filtered neuro-fuzzy control design for fractional-order nonlinear systems with unknown control directions and input quantization, *IEEE T. Syst. Man Cy.*, **51** (2021), 7238–7249. https://doi.org/10.1109/TSMC.2020.2967425
27. C. Su, Y. Stepanenko, J. Svoboda, T. Leung, Robust adaptive control of a class of nonlinear systems with unknown backlash-like hysteresis, *IEEE T. Automat. Contr.*, **45** (2000), 2427–2432. https://doi.org/10.1109/9.895588
28. L. Liu, L. Tang, Partial state constraints-based control for nonlinear systems with backlash-like hysteresis, *IEEE T. Syst. Man Cy.*, **50** (2020), 3100–3104. https://doi.org/10.1109/TSMC.2018.2841063
29. L. Bai, Q. Zhou, L. Wang, Z. Yu, H. Li, Observer-based adaptive control for stochastic nonstrict-feedback systems with unknown backlash-like hysteresis, *Int. J. Adapt. Control*, **31** (2017), 1481–1490. https://doi.org/10.1002/acs.2780
30. W. Liu, T. Zhao, An active disturbance rejection control for hysteresis compensation based on Neural Networks adaptive control, *ISA T.*, **109** (2021), 81–88. https://doi.org/10.1016/j.isatra.2020.10.019
31. Z. Zhu, Y. Pan, Q. Zhou, C. Lu, Event-triggered adaptive fuzzy control for stochastic nonlinear systems with unmeasured states and unknown backlash-like hysteresis, *IEEE T. Fuzzy Syst.*, **29** (2021), 1273–1283. https://doi.org/10.1109/TFUZZ.2020.2973950
32. C. Fu, Q. Wang, J. Yu, C. Lin, Neural network-based finite-time command filtering control for switched nonlinear systems with backlash-like hysteresis, *IEEE T. Neur. Net. Lear.*, **32** (2021), 3268–3273. https://doi.org/10.1109/TNNLS.2020.3009871
33. Z. Li, F. Wang, R. Zhu, Finite-time adaptive neural control of nonlinear systems with unknown output hysteresis, *Appl. Math. Comput.*, **403** (2021), 126175. https://doi.org/10.1016/j.amc.2021.126175
34. J. Ma, J. Park, S. Xu, Command-filter-based finite-time adaptive control for nonlinear systems with quantized input, *IEEE T. Automat. Contr.*, **66** (2021), 2339–2344. https://doi.org/10.1109/TAC.2020.3006283
35. J. Yu, P. Shi, W. Dong, H. Yu, Observer and command-filter-based adaptive fuzzy output feedback control of uncertain nonlinear systems, *IEEE T. Ind. Electron.*, **62** (2015), 5962–5970. https://doi.org/10.1109/TIE.2015.2418317
36. W. Dong, J. Farrell, M. Polycarpou, V. Djapic, M. Sharma, Command filtered adaptive backstepping, *IEEE T. Contr. Syst. T.*, **20** (2012), 566–580. https://doi.org/10.1109/TCST.2011.2121907
37. A. Wang, L. Liu, J. Qiu, G. Feng, Event-triggered adaptive fuzzy output-feedback control for nonstrict-feedback nonlinear systems with asymmetric output constraint, *IEEE T. Cybernetics*, **52** (2022), 712–722. https://doi.org/10.1109/TCYB.2020.2974775