Mathematics

Research article

Synchronization robustness analysis of memristive-based neural networks with deviating arguments and stochastic perturbations

Tao Xie*, **Xing Xiong** and **Qike Zhang**

School of Mathematics and Statistics, Hubei Normal University, Huangshi, 435002, China

* **Correspondence:** Email: Xt0216@hbnu.edu.cn.

Abstract: In this article, we investigate the robustness of memristive-based neural networks (MNNs) with deviating arguments (DAs) and stochastic perturbations (SPs). Based on the set-valued mapping method, differential inclusion theory and Gronwall inequalities, we derive the upper bounds for the width of DAs and the intensity of SPs. When the DAs and SPs are smaller than these upper bounds, the MNNs maintains exponential synchronization. Finally, several specific simulation examples demonstrate the effectiveness of the results.

Keywords: memristive-based neural networks; deviating arguments; stochastic perturbations; exponential synchronization
Mathematics Subject Classification: 93B35, 93D23

1. Introduction

The memristors were initially introduced by Chua in 1971 to describe the relationship between charge and magnetic flux [1]. It was predicted to be the fourth fundamental circuit element, distinct from resistors, capacitors and inductors. In 2008, the research team at HP laboratories successfully created a practical memristor device with valuable applications [2]. Similar to conventional resistors, the memristors can handle safe currents through the device. Furthermore, its value changes based on the amount of charge passing through it, therefore the memristors have memory functionality [3–7]. As a result, an increasing number of researchers have been using memristors instead of traditional resistors to serve as connection weights between neurons and for self-feedback connection weights, forming a state-dependent nonlinear switching system known as a memristive neural networks (MNNs). Compared to conventional artificial neural networks (NNs), MNNs possess stronger computational capabilities and information capacity, thereby enhancing the applications of NNs in associative memory, signal processing and image processing [8–13].

The dynamic behavior of MNNs is fundamental to their applications; therefore, it necessary to

analyze their dynamic characteristics [10, 14]. Among the various dynamic behaviors of MNNs, synchronization is an important and fundamental feature. The synchronization of MNNs has garnered extensive attention from researchers due to its numerous potential applications in artificial intelligence, information science, secure communications and various other fields [15–17]. In [5], Du et al. derived finite-time (FNT) synchronization criteria for fractional-order MNNs with delays using the fractional-order Gronwall inequality. The FNT/fixed time (FXT) stability of MNNs was tudied in [18], by designing a synovial membrane controller, the MNNs reaches the sliding-mode surface in FNT/FXT. In [10–18], synchronization criteria were obtained for MNNs with either delays or random interference. The above discussions mostly focus on the synchronization of MNNs with time delays, and few have considered the exponential synchronization (ESy) of MNNs with deviating arguments (DAs).

The theory of DAs differential equations was proposed by Shah and Wiener in 1983 [19]. In [20], by transforming these equations into equivalent integral equations, new stability conditions were obtained. These equations involve DAs, combining the properties of discrete and continuous equations [21–23]. During the system's operation, the relevant arguments characteristics can be altered, allowing the system to become a combination of lag and advance equations [24, 25]. As a result, systems with DAs have broader applications compared to systems with time delays. Reference [26] investigates recurrent neural networks with DAs and establishes criteria for the global exponential stability. In order to further explore the impact of DAs on the exponential stability (ESt) of the systems, the robustness analysis of a fuzzy cellular neural networks with DAs and stochastic disturbance is discussed in [23]. The signals transmitted between MNNs are inevitably subject to stochastic perturbations (SPs) caused by environmental uncertainties [27–31]. For systems with SPs, the feature can significantly impact the dynamic behavior of the system, leading to either synchronization or desynchronization under certain levels of SPs [32–34]. For example, for the following simple linear systems $dx(t) = ax(t)dt$ and $dy(t) = ay(t)dt$, the error system is $de(t) = ae(t)dt$. The system is stable only when $a < 0$. However, the stability of the system is affected by SPs. Therefore, consider the following system $de(t) = ae(t)dt + be(t)dB(t)$. The system is almost surely ESt if and only if the condition $b^2 > 2a$ is satisfied [35]. Then, the error system is ESt, it implies that $x(t)$ and $y(t)$ are exponential synchronization (ESy). Based on the above discussions, we reach the following conclusion: SPs can disrupt the ESy of a system that was synchronized or facilitate the ESy of a system that was initially unsynchronized. If a MNNs with SPs are ESy, can we obtain upper bounds such that the MNNs remains ESy when the SPs are smaller than the bounds?

Based on the discussion above, MNNs can lose synchronization when subjected to disturbances from external perturbations and DAs, provided that the intensity of perturbations and the width of arguments exceed certain limits. In [27, 30–34, 36, 37], there are important results regarding the synchronization of MNNs under external disturbances. In [21–23], scholars research on MNNs with DAs. It is important to note that the aforementioned literature primarily focuses on the synchronization of MNNs than its robustness. Therefore, an interesting question arises: Under the control strategy, how much argument length and perturbation intensity can MNNs with ESy endure without losing synchronization?

The major contributions of this paper include the following aspects:

• Compared to the references [5–7, 11, 16], we focus on the synchronization of MNNs with DAs. The systems with DAs have broader applications compared to traditionally time-delayed systems.

• The references [8–17] extensively investigated MNNs with time delays, providing various stability and synchronization criteria. In references [28, 29, 33, 34], the robustness of ESt in systems with both time delays and SPs was further explored. In references [23, 27], Fang et al. studied the robustness of ESt in fuzzy cellular neural networks with DAs. In contrast to the aforementioned literatures, we focus on the robustness of ESy in MNNs, utilizing the set-valued mapping method, differential inclusion theory and Gronwall inequalities, we derive the upper bounds for the DAs and SPs.

• Compared to the references [22–25, 32–34]. The MNNs with state switching that we consider and results in a more complex system structure.

The paper is organized as follows. In Section II, we introduce the model, assumptions and some preliminary lemmas. In Section III, we present the theorems and lemmas derived in this paper. In Section VI, we provide several examples to validate the feasibility of our results. Finally, in Section V, we have summarized the work carried out in this paper.

2. Preliminaries

2.1. Notation

In the paper, $\mathbb{R}^n$ is Euclidean space, $\mathbb{N}$ represents integers, $\|\chi(t)\|$ is the norm of vector $\chi(t)$, where $\chi(t) \in \mathbb{R}^n$ and $\|\chi(t)\| = \sum_{p=1}^n |\chi(t)|$, the norm $\|A\|$ of the matrix A is given by $\|A\|$, where $A = \max_{1 \le q \le n} \sum_{p=1}^n |a_{pq}|$. For two real-valued sequences ρ_k, η_k, where $k \in \mathbb{N}$, it holds that $\rho_k < \rho_{k+1}$, $\rho_k \le \eta_k \le \rho_{k+1}$ for all $k \in \mathbb{N}$ with $\eta_k \to \infty$ as $k \to \infty$.

Let $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t \ge t_0}, P)$ is a complete probability space with a filtration $\{\mathcal{F}_t\}_{t \ge t_0}$ (the filtration contains all P-null sets and is right continuous). $L^P_{\mathcal{F}_0}([-\tau, 0]; \mathbb{R}^n)$ is the family of all $\mathcal{F}_0$-measurable $\mathbb{C}([-\tau, 0]; R^n)$, the state variable $\xi = \{\xi(s); \tau \le \xi(s) \le 0\}$ satisfies $\sup_{-\tau \le s \le 0} \mathbb{E}(\|\xi(s)\|^P) \le \infty$. $\mathbb{E}(\cdot)$ is the mathematical expectation in the probability space.

2.2. Preparation

Consider the MNNs as the derive system with the SPs,

$$dw_p(t) = [-d_p w_p(t) + \sum_{q=1}^n a_{pq}(w_q(t)) f_q(w_q(t)) + I_p(t)]dt + \sigma w_p(t) d\omega(t), \qquad (2.1)$$

where $p = 1, 2, \cdots, n$, $w_p(t)$ are the state variables, and $f_q\big(w_q(t)\big)$ are the activation functions; $d_p > 0$ is a self-feedback connection weights and $I_p(t)$ is the external inputs, $a_{pq}\big(w_q(t)\big)$ is the memristive connection weights, σ is the interference intensity. $\omega_i(t)$ represents Brownian motion on the compete space.

For convenience, we use $w_p, w_q, a_{pq}\big(w_q\big), u_p, \omega$ to replace $w_p(t), w_q(t), a_{pq}\big(w_q(t)\big), u_p(t), \omega(t)$, respectively. The initial conditions of (2.1) is

$$w_p(t_0) = \varphi_p.$$

The corresponding response system,

$$dv_p(t) = [-d_p v_p(t) + \sum_{q=1}^n a_{pq}(v_q(t)) f_q(v_q(t)) + I_p(t) + u_p(t)]dt + \sigma v_p(t) d\omega(t). \qquad (2.2)$$

For convenience, we use $v_p, v_q, a_{pq}(v_q), u_p$ to replace $v_p(t), v_q(t), a_{pq}(v_q(t)), u_p(t)$, respectively. Then, the memristive parameter of (2.1) and (2.2) are expressed as

$$a_{ij}(w_q) = \begin{cases} á_{pq}, & |w_q| \le T_q, \\ à_{pq}, & |w_q| > T_q, \end{cases} \quad a_{ij}(v_q) = \begin{cases} á_{pq}, & |v_q| \le T_q, \\ à_{pq}, & |v_q| > T_q, \end{cases}$$

where $i, j \in \mathbb{N}$, weights $á_{pq}, à_{pq}$ and switching jumps $T_q > 0$. The initial value of (2) are

$$v_p(t_0) = \phi_p.$$

The linear feedback controller u_p is designed as follows

$$u_p(t) = -\xi_p(v_p(t) - w_p(t)).$$

The error system between the drive system (2.1) and the response system (2.2) is defined as

$$e_p = v_p - w_p, \ \psi_p^1 = \phi_p - \varphi_p, \ p \in \mathbb{N}. \tag{2.3}$$

Remark 1. We can observe that MNNs can be categorized as discontinuous switched systems, which necessitates considering the solution to MNNs (1) using the Filippov's sense. In the following, we will introduce certain definitions pertaining to set-valued maps and the Filippov solution.

Definition 1. (Set-valued map [38]) Consider a set $E \in \mathbb{R}^n$. A set-valued map is defined as follows: For each point x in the set E, there exists a nonempty set $F(x) \in \mathbb{R}^n$ such that x is mapped to $F(x)$.

Definition 2. (Differential inclusion [38]) For a discontinuous differential system $\dot{r}(t) = \mathbb{F}(t, r_t), t \ge 0$, the function r_t is the solution of the differential equation in the Filippov sense, $t \in [0, t_1], t_1 \ge 0$, if is absolutely continuous and satisfies the following differential inclusion:

$$\dot{r} \in \mathbb{G}(t, r_t),$$

where $t \in [0, +\infty], r \in \mathbb{R}^n$, the initial condition $r(0) = r_0 \in \mathbb{C}([-\tau, 0], \mathbb{R}^n)$. The $\mathbb{G}(t, r_t)$ is a set-valued mapping, satisfies

$$\mathbb{G}(t, r_t) = \bigcap_{\Gamma > 0} \bigcap_{\delta(\mathbb{N})=0} \overline{co}[f(\mathbb{B}(r_t, \Gamma) \backslash \mathbb{N})],$$

where $\overline{co}$ is the convex closure hull of a set, $\mathbb{B}(r_t, \Gamma) = \{x : \|x - r_t\| \le \Gamma\}, \Gamma > 0$ and $\delta(\mathbb{N})$ is Lebesgue measure of set $\mathbb{N}$.

The set value mapps of memristive parameters is as follows:

$$K[a_{pq}(w_q)] = \begin{cases} á_{pq}, & |w_q| < T_q, \\ \overline{co}\{á_{pq}, à_{pq}\}, & |w_q| = T_q, \\ à_{pq}, & |w_q| > T_q, \end{cases} \quad K[a_{pq}(v_q)] = \begin{cases} á_{pq}, & |v_q| < T_q, \\ \overline{co}\{á_{pq}, à_{pq}\}, & |v_q| = T_q, \\ à_{pq}, & |w_q| > T_q, \end{cases}$$

where $p, q \in \mathbb{N}$. $K[a_{pq}(w_q)]$ and $K[a_{pq}(v_q)]$ are all closed, convex and compact about w_q, v_q.

According to Definitions 1 and 2, the Filippov solution of the systems (1) and (2) can be written as:

$$dw_p(t) \in [-d_p w_p + \sum_{q=1}^{n} K[a_{pq}(w_q)] f_q(w_q) + I_p] ds + \sigma w_p d\omega. \tag{2.4}$$

$$dv_p(t) \in [-d_p v_p + \sum_{q=1}^{n} K[a_{pq}(v_q)] f_q(v_q) + I_p + u_p] dt + \sigma w_p(t) d\omega. \tag{2.5}$$

Similarly, there exist

$$\bar{a}_{pq}(w_q) \in K\left[a_{pq}(w_q)\right], \; \check{a}_{pq}(v_q) \in K\left[a_{pq}(v_q)\right],$$

such that

$$dw_p = [-d_p w_p + \sum_{q=1}^{n} \bar{a}_{pq}(w_q) f_q(w_q) + I_p] ds + \sigma w_p d\omega,$$

$$dv_p = [-d_p v_p + \sum_{q=1}^{n} \check{a}_{pq}(v_q) f_q(v_q) + I_p + u_p] dt + \sigma w_p d\omega. \tag{2.6}$$

From (6), let $e_p = v_p - w_p$,

$$de_p = -[(d_p + \xi_p)e_p + \sum_{q=1}^{n} \hat{a}_{pq}(e_q) f_q(e_p)] dt + \sigma e_p d\omega, \tag{2.7}$$

where $\hat{a}_{pq}(e_q) f_q(e_q) = \bar{a}_{pq}(w_q) f_q(w_q) - \check{a}_{pq}(v_q) f_q(v_q)$.

The following error system without random disturbance:

$$\dot{z}_p = -(d_p + \xi_p) z_p + \sum_{q=1}^{n} \hat{a}_{pq}(z_q) f_q(z_q). \tag{2.8}$$

The initial conditions is

$$z_p(t_0) = \psi_p^2, \; p = 1, \cdots, n.$$

The (2.7) and (2.8) can be rewritten as

$$de(t) = [-(D + C)e(t) + \hat{A}F(e(t))] dt + \sigma e(t) d\omega(t)$$
$$\dot{z}(t) = -(D + C)z(t) + \hat{A}F(z(t)), \tag{2.9}$$

where $e(t) = (e_1(t), \cdots, e_n(t))^T$, $z(t) = (z_1(t), \cdots, z_n(t))^T$, $C = \text{diag}\{\xi_1, \xi_2, \cdots, \xi_n\}$, $D = \text{diag}\{d_1, d_2, \cdots, d_n\}$, $\hat{A} = (\hat{a}_{pq})_{n \times n}$, $F(e(t)) = (f_1(e_1(t)), \cdots, f_n(e_n(t)))^T$.

The mian result of the paper are base on the following definitions, assumptions and lemmas.

Definition 3. (Exponential synchronization (ESy)) If the error system $\chi(t)$ is exponential stability (ESt). Then, the MNNs (2.1) and (2.2) are described as ESy, there exsit two nonnegative constant α and β,

$$\|\chi(t)\| \le \alpha \|\psi^1\| \exp(-\beta(t - t_0)),$$

where $\psi^1 = (\psi_1^1, \cdots, \psi_n^1)^T$ is any initial condition, $t \le 0$.

Definition 4. (Mean square exponential synchronization (MSESy)) The state $\chi(t)$ of system (2.7) is said to be MSESt. Then the MNNs (2.1) and (2.2) are described as MSESy, if for any $t_0 \in \mathbb{R}^+$, $\psi^1 \in \mathbb{R}^n$, there exist $\theta > 0$ and $\vartheta > 0$ such that

$$E\|\chi(t)\|^2 \le \theta \|\psi^1\|^2 \exp\{-2\vartheta(t - t_0)\}.$$

Assumption 1. The activation functions $f_q(\cdot)$ and $g_q(\cdot)$ satisfy

$$\|f_q(w_p) - f_q(v_p)\| \le f_q^*\|w_p - v_p\|, \quad \|g_q(w_p) - f_q(v_p)\| \le g_q^*\|w_p - v_p\|,$$

where $f_q^*, g_q^* > 0$ are Lipschitz constants, $w_p, v_p \in \mathbb{R}^n$.

Under Assumption 1, we have the following conclude: The MSESt of system (2.7) implies the almost sure ESt of system (2.7), see [39].

Assumption 2. $f_q(\pm T_q) = g_q(\pm T_q) = 0$.

Lemma 1. [14] From Assumptions 1 and 2, then

$$\left| K\left[a_{pq}(v_q) \right] f_q(v_q) - K\left[a_{pq}(w_q) \right] f_q(w_q) \right| \le a_{pq}^* f_q^* \left| v_q - w_q \right|$$

for $i, j \in \mathbb{N}$.

That is, for any $\bar{a}_{pq}(w_q) \in K\left[a_{pq}(w_q) \right], \check{a}_{pq}(v_q) \in K\left[a_{pq}(v_q) \right]$.

$$\left| \check{a}_j(v_q) f_q(v_q) - \bar{a}_{pq}(w_q) f_q(w_q) \right| \le a_{pq}^* f_q^* \left| v_q - w_q \right|,$$

where $a_{pq}^* = \max\left\{ \left| \acute{a}_{pq} \right|, \left| \grave{a}_{pq} \right| \right\}$.

Lemma 2. [40] (Granwall inequalities) Suppose $\vartheta(t)$, $\theta(t)$ and $u(t)$ are continuous real-valued functions, and $\theta(t)$ is integrable over the interval $I = [t_0, t]$, if $\vartheta(t) \ge 0$ and $u(t)$ satisfies

$$u(t) \le \theta(t) + \int_{t_0}^{t} \vartheta(s)u(s)ds,$$

then,

$$u(t) \le \theta(t) + \int_{t_0}^{t} \theta(s)\vartheta(s) \exp\left(\int_{s}^{t} \vartheta(r)dr \right) ds,$$

where $t > t_0$.

In addition, if $\theta(t)$ is non-decreasing, then

$$u(t) \le \theta(t) \exp\left(\int_{t_0}^{t} \vartheta(s)ds \right).$$

Assumption 3. There exists a nonnegative constant ρ that satisfies, $\rho_{k+1} - \rho_k \le \rho$, for all $k \in \mathbb{N}$.

Assumption 4. $h_1\rho + h_2\rho(1 + h_1\rho) \exp\{h_2\rho\} < 1$,

where $h_1 = \|B^*\|\|G^*\|\rho$, $h_2 = \|C\| + \|D\| + \|A^*\|\|F^*\|$.

Assumption 5. $\rho(3l_1 + 9l_2 + l_1^2 + 3l_1l_2) < 1$,

where $l_1 = 9\rho^2\|B^*\|^2\|G^*\|^2$, $l_2 = 3\rho(2\|C\|^2 + 2\|D\|^2 + \|A^*\|^2\|F^*\|^2) + \theta^2$.

Under the influence of the linear controller, the error system (2.7) can achieve ESt, that is, systems (2.1) and (2.2) can achieve ESy. The next question is how much the SPs and DAs intensities can make the system can maintain ESy.

3. Main results

3.1. Effects of SPs on MNNs synchronization

Theorem 1. Let Assumption 1 hold, MNNs (2.8) is ESt. Then MNNs (2.7) is ESt, that is, the master system MNNs (2.1) and the slave system MNNs (2.2) are ESy. if $|\sigma| \leq \bar{\sigma}$, $\bar{\sigma}$ is the unique nonnegative solution of the transcendental equation below.

$$
\begin{aligned}
(24\mathbb{T}\|A^*\|^2\|F^*\|^2 + 2\sigma^2)\theta/\vartheta \exp\{8\mathbb{T}(4\mathbb{T}(\|D\|^2 \\
+ \|C\|^2 + 3\|A^*\|^2\|F^*\|^2) + \sigma^2)\} + 2\theta \exp\{-4\vartheta\mathbb{T}\} = 1,
\end{aligned}
\tag{3.1}
$$

where $\mathbb{T} > \ln(2\theta)/(4\vartheta) > 0$, $A^* = (a^*_{pq})_{n \times n}$, $F^* = \max_{j \in \mathbb{N}}\{f^*_q\}$, $\mathbb{T}$ is the time interval.

Proof. $z(t)$ and $e(t)$ have the same initial value, $\psi^1 = \psi^2$,

$$
z(t) - e(t) = \int_{t_0}^t [-D(z(s) - e(s)) + \hat{A}F(z(s)) - \hat{A}F(e(s)) - C(z(s) - e(s))]dt - \int_{t_0}^t \sigma e(s)d\omega(t).
$$

The ESt of the MNNs (2.8), when $t > t_0$,

$$
\int_{t_0}^t E\|z(s)\|^2 = \int_{t_0}^t \theta\|\psi^2\| \exp\{-2\vartheta(t - t_0)\} \leq \theta\|\psi^2\|^2/2\vartheta.
$$

When $t \leq t_o + 2\mathbb{T}$, by utilizing the Cauchy-Schwarz inequalities and Lemma 1, we can conclude the following:

$$
\begin{aligned}
E\|z(t) - e(t)\|^2 \leq & 2E\|\int_{t_0}^t \left[-D(z(s) - e(s)) + \hat{A}F(z(s)) - \hat{A}F(e(s)) - C(z(s) - e(s))\right]ds\|^2 \\
& - 2E\|\int_{t_0}^t \sigma e(t)d\omega(s)\|^2 \\
\leq & 2E\int_{t_0}^t 1ds \int_{t_0}^t \left[\| - D(z(s) - e(s)) + \hat{A}F(z(s)) - \hat{A}F(e(s)) - C(z(s) - e(s))\|\right]^2 dt \\
& + 2\sigma^2 \int_{t_0}^t E\|e(s) - z(s) + z(s)\|^2 ds \\
\leq & 4\mathbb{T}E\int_{t_0}^t 4(\|D\|^2\|z(s) - e(s)\|^2 + \|A^*\|^2\|F^*\|^2\|z(s)\|^2 + \|A^*\|^2\|F^*\|^2\|e(s)\|^2 \\
& + \|C\|^2\|z(s) - e(s)\|^2)ds + 2\sigma^2 \int_{t_0}^t E\|e(s) - z(s) + z(s)\|^2 ds.
\end{aligned}
$$

Then,

$$
\begin{aligned}
E\|z(t) - e(t)\|^2 \leq & \left[16\mathbb{T}(\|D\|^2 + \|C\|^2 + 2\|A^*\|^2\|F^*\|^2) + 4\sigma^2\right] \int_{t_0}^t E\|z(s) - e(s)\|^2 ds \\
& + \left(16\mathbb{T}(3\|A^*\|^2\|F^*\|^2) + 4\sigma^2\right) \int_{t_0}^t E\|z(s)\|^2 ds \\
\leq & \left[16\mathbb{T}(\|D\|^2 + \|C\|^2 + 2\|A^*\|^2\|F^*\|^2) + 4\sigma^2\right] \int_{t_0}^t E\|z(s) - e(s)\|^2 ds \\
& + \left(8\mathbb{T}(3\|A^*\|^2\|F^*\|^2) + 2\sigma^2\right) \theta\|\psi^2\|^2/\vartheta,
\end{aligned}
\tag{3.2}
$$

When $t_0 + \mathbb{T} \leq t \leq t_0 + 2\mathbb{T}$, by applying Lemma 2,

$$
\begin{aligned}
E\|z(t) - e(t)\|^2 \leq & \left(8\mathbb{T}(3\|A^*\|^2\|F^*\|^2) + 2\sigma^2\right)\theta\|\psi^2\|^2/\vartheta \exp\left\{(16\mathbb{T}(\|D\|^2 \right. \\
& + \|C\|^2 + 2\|A^*\|^2\|F^*\|^2) + 4\sigma^2)(t - t_0)\Big\} \\
\leq & \left(24\mathbb{T}\|A^*\|^2\|F^*\|^2 + 2\sigma^2\right)\theta/\vartheta \exp\left\{8\mathbb{T}(4(\|D\|^2 + \|C\|^2 + 2\|A^*\|^2\|F^*\|^2) + \sigma^2\right\} \\
& \times \left(\sup_{t_0 \leq t \leq t_0 + 2\mathbb{T}} E\|z(t)\|^2\right).
\end{aligned}
\tag{3.3}
$$

When $t_0 + \mathbb{T} \leq t \leq t_0 + 2\mathbb{T}$,

$$
\begin{aligned}
E\|e(t)\|^2 \leq & 2E\|z(t) - e(t)\|^2 + 2E\|z(t)\|^2 \\
\leq & \left((24\mathbb{T}\|A^*\|^2\|F^*\|^2 + 2\sigma^2)\theta/\vartheta \exp\{8\mathbb{T}(4\mathbb{T}(\|D\|^2 + \|C\|^2 + 2\|A^*\|^2\|F^*\|^2) + \sigma^2)\}\right) \\
& \times \left(\sup_{t_0 \leq t \leq t_0 + \mathbb{T}} E\|z(t)\|^2\right) + 2\theta\|\psi^2\|^2 \exp\{-2\vartheta(t - t_0)\} \\
\leq & \left((24\mathbb{T}\|A^*\|^2\|F^*\|^2 + 2\sigma^2)\theta/\vartheta \exp\{8\mathbb{T}(4\mathbb{T}(\|D\|^2 + \|C\|^2 + 2\|A^*\|^2\|F^*\|^2) + \sigma^2)\}\right. \\
& + 2\theta \exp\{-4\vartheta\mathbb{T}\}\right) \times \left(\sup_{t_0 \leq t \leq t_0 + \mathbb{T}} \|e(t)\|^2\right).
\end{aligned}
\tag{3.4}
$$

From (11), when $|\sigma| < \bar{\sigma}$,

$$
(24\mathbb{T}\|A^*\|^2\|F^*\|^2 + 2\sigma^2)\theta/\vartheta \exp\{8\mathbb{T}(4\mathbb{T}(\|D\|^2 + \|C\|^2 + 3\|A^*\|^2k^2) + \sigma^2)\} + 2\theta \exp\{-4\vartheta\mathbb{T}\} < 1.
$$

Let

$$
\gamma = -\ln\left\{(24\mathbb{T}\|A^*\|^2\|F^*\|^2 + 2\sigma^2)\theta/\vartheta \exp\{8\mathbb{T}(4\mathbb{T}(\|D\|^2 + \|C\|^2 + 3\|A^*\|^2k^2) + \sigma^2)\} + 2\theta \exp\{-4\vartheta\mathbb{T}\}\right\}/\mathbb{T}.
$$

So, $\gamma > 0$,

$$
\sup_{t_0 + \mathbb{T} \leq t \leq t_0 + 2\mathbb{T}} \|e(t)\| \leq \exp(-\gamma\mathbb{T})\left(\sup_{t_0 \leq t \leq t_0 + \mathbb{T}} \|e(t)\|\right).
\tag{3.5}
$$

Then, for any nonnegative integer $\mathbb{M} = 1, 2, \cdots$, when $t \geq t_0 + (\mathbb{M} - 1)\mathbb{T}$,

$$
e(t; t_0, e_0) = e(t; t_0 + (\mathbb{M} - 1)\mathbb{T}, e(t_0 + (\mathbb{M} - 1)\mathbb{T})).
\tag{3.6}
$$

From (3.5) and (3.6)

$$
\begin{aligned}
\sup_{t_0 + \mathbb{M}\mathbb{T} \leq t \leq t_0 + (m+1)\mathbb{T}} \|e(t; t_0, e_0)\| = & \left(\sup_{t_0 + (\mathbb{M}-1)\mathbb{T} + \mathbb{T} \leq t \leq t_0 + (\mathbb{M}-1)\mathbb{T} + 2T} \|e(t; t_0 + (\mathbb{M} - 1)\mathbb{T}, e(t_0 + (\mathbb{M} - 1)\mathbb{T}; t_0, e_0))\|\right) \\
\leq & \exp(-\gamma\mathbb{T})\left(\sup_{t_0 + (\mathbb{M}-1)\mathbb{T} \leq t \leq t_0 + m\mathbb{T}} \|e(t; t_0, e_0)\|\right) \\
\leq & \exp(-\gamma\mathbb{M}\mathbb{T})\left(\sup_{t_0 \leq t \leq t_0 + \mathbb{T}} \|e(t; t_0, e_0)\|\right) \\
= & \aleph \exp(-\gamma\mathbb{M}\mathbb{T}),
\end{aligned}
$$

where

$$\aleph = \sup_{t_0 \le t \le t_0 + \mathbb{T}} \| e(t; t_0, e_0) \|.$$

So for $\forall t > t_0 + \mathbb{T}$, there have a nonnegative integer $\mathbb{M}$ such that $t_0 + \mathbb{M}\mathbb{T} \le t \le t_0 + (\mathbb{M} + 1)\mathbb{T}$,

$$\| e(t; t_0, e_0) \| \le \aleph \exp(-\gamma t + \gamma t_0 + \gamma T) = (\aleph \exp(\gamma T)) \exp(-\gamma(t - t_0)).$$

The condition is also genuine when $t_0 \le t \le t_0 + \mathbb{T}$. So system (2.7) is ESt.

The following analysis considers the influence of the DAs on the ESy of the master-slave system.

3.2. Effects of DAs on MNNs synchronization

Consider the derive system with the DAs,

$$\dot{w}_p(t) = -d_p w_p(t) + \sum_{q=1}^{n} a_{pq}(w_q(t)) f_q(w_q(t)) + \sum_{q=1}^{n} b_{pq}(w_q(\gamma(t))) g_q(w_q(\gamma(t))) + I_p(t) \tag{3.7}$$

$$w_p(t_0) = \varphi_0,$$

where $g_q(w_q(t))$ is the activation functions with DAs, $a_{pq}(w_q(t))$ and $b_{pq}(w_q(\gamma(t)))$ are the memristive connection weights without and with DAs respectively.

The corresponding response system:

$$\dot{v}_p(t) = -d_p v_p(t) + \sum_{q=1}^{n} a_{pq}(v_q(t)) f_q(v_q(t)) + \sum_{q=1}^{n} b_{pq}(v_q(\gamma(t))) g_q(v_q(\gamma(t))) + I_p(t) + u_p(t) \tag{3.8}$$

$$v_p(t_0) = \phi_0.$$

The Filippov solution of the systems (3.7) and (3.8) are

$$\dot{w}_p \in -d_p w_p + \sum_{q=1}^{n} K[a_{pq}(w_q)] f(w_q) + \sum_{q=1}^{n} K[b_{pq}(w_q(\gamma))] g(w_q(\gamma)) + w_p, \tag{3.9}$$

$$\dot{v}_p \in -d_p v_p + \sum_{q=1}^{n} K[a_{pq}(v_q)] f(v_q) + \sum_{q=1}^{n} K[b_{pq}(v_q(\gamma))] g(v_q(\gamma)) + w_p + u_p, \tag{3.10}$$

in which $w(\gamma) = w(\gamma(t))$, $v(\gamma) = v(\gamma(t))$.

The set-valued maps be defined as follows

$$K\left[b_{pq}\left(\tilde{w}_q\right)\right] = \begin{cases} \acute{b}_{pq}, & |\tilde{w}_q| < T_q, \\ \overline{\text{co}}\left\{\acute{b}_{pq}, \grave{b}_{pq}\right\}, & |\tilde{w}_q| = T_q, \\ \grave{b}_{pq}, & |\tilde{w}_q| > T_q, \end{cases} \quad K\left[b_{pq}\left(\tilde{v}_q\right)\right] = \begin{cases} \acute{b}_{pq}, & |\tilde{v}_q| < T_q, \\ \overline{\text{co}}\left\{\acute{b}_{pq}, \grave{b}_{pq}\right\}, & |\tilde{v}_q| = T_q, \\ \grave{b}_{pq}, & |\tilde{v}_q| > T_q, \end{cases}$$

where $\tilde{w}_q, \tilde{v}_q$ to replace $w_q(\gamma), v_q(\gamma)$. $K[a_{pq}(\tilde{w}_q)]$ and $K[a_{pq}(\tilde{v}_q)]$ are all closed, convex and compact about $\tilde{w}_q, \tilde{v}_q$.

There exist

$$\bar{a}_{pq}\left(w_q\right) \in K\left[a_{pq}\left(w_q\right)\right], \quad \bar{b}_{pq}\left(\tilde{w}_q\right) \in K\left[b_{pq}\left(\tilde{w}_q\right)\right],$$

$$\check{a}_{pq}\left(v_q\right) \in K\left[a_{pq}\left(v_q\right)\right], \quad \check{b}_{pq}\left(\tilde{v}_q\right) \in K\left[b_{pq}\left(\tilde{v}_q\right)\right].$$

Let $e_p = v_p - w_p$,

$$\dot{e}_p = -(d_p + \xi_p)e_q + \sum_{q=1}^{n} \hat{a}_{pq}(e_q)f_q(e_q) + \sum_{q=1}^{n} \hat{b}_{pq}(e_q(\gamma))g_q(e_p(\gamma)), \tag{3.11}$$

where $\hat{b}_{pq}(e_q)g_q(e_p) = \bar{b}_{pq}(w_q)g_q(w_p) - \check{b}_{pq}(v_q)g_q(v_p)$.

The following error system without DAs,

$$\dot{z}_p = -(d_p + \xi_p)z_p + \sum_{q=1}^{n} \hat{a}_{pq}(z_q)f_q(z_q) + \sum_{q=1}^{n} \hat{b}_{pq}(z_q)g_q(z_q). \tag{3.12}$$

The (3.11) and (3.12) can be rewritten as

$$\dot{e}(t) = -De(t) + \hat{A}Fe(t) + \hat{B}G(e(\gamma(t))) - Ce(t), \tag{3.13}$$

$$\dot{z}(t) = -Dz(t) + \hat{A}Fz(t) + \hat{B}G(e(t)) - Cz(t), \tag{3.14}$$

where $\hat{B} = \left(\hat{b}_{pq}\right)_{n \times n}$, $G(e(t)) = (g_1(e_1(t)), \cdots, g_n(e_n(t)))^T$.

Lemma 3. Consider the MNNs (3.11) with DAs and the Assumptions 3 and 4 hold, the following inequality is established,

$$e(\gamma(t)) \le \mu e(t). \tag{3.15}$$

Proof. For $\gamma(t) = \eta_k$ and $\eta_k \in [\rho_k, \rho_{k+1}]$,

$$e(t) = e(\eta_k) + \int_{\eta_k}^{t} (-De(s) + \hat{A}F(e(s)) + \hat{B}(G(e(s))) - Ce(s))ds.$$

Utilizing Lemmas 1 and 2,

$$\|e(t)\| \le \|e(\eta_k)\| + \left\| \int_{\eta_k}^{t} (-De(s) + \hat{A}F(e(s)) + \hat{B}Ge(\eta_k) - Ce(s))ds \right\|$$

$$\le (1 + \|B^*\|\|G^*\|\rho)\|e(\eta_k)\| + \int_{\eta_k}^{t} (\|C\| + \|D\| + \|A^*\|\|F^*\|)\|e(s)\|ds$$

$$\le [(1 + \|B^*\|\|G^*\|\rho)\|e(\eta_k)\|] \exp\{(\|C\| + \|D\| + \|A^*\|\|F^*\|)\rho\},$$

where $B^* = (b_{pq}^*)_{n \times n}$, $G^* = \max_{q \in \mathbb{N}}\{g_q^*\}$, then

$$\|e(\eta_k)\| \le \|e(t)\| + \rho\|B^*\|\|G^*\|\|e(\eta_k)\| + \int_{\eta_k}^{t} (\|C\| + \|D\| + \|A^*\|\|F^*\|)\|e(s)\|ds$$

$$\le \|e(t)\| + (h_1\rho + h_2\rho(1 + h_1\rho))\|e(\eta_k)\| \exp\{h_2\rho\},$$

where $h_1 = \|B^*\|\|G^*\|$, $h_2 = \|C\| + \|D\| + \|A^*\|\|F^*\|$.

$$\left(1 - (h_1\rho + h_2\rho(1 + h_1\rho) \exp\{h_2\rho\})\right)\|e(\eta_k)\| \le \|e(t)\|.$$

Therefore, for Assumption 4,

$$\|e(\eta_k)\| \leq \left(1 - (h_1\rho + h_2\rho(1 + h_1\rho)\exp\{h_2\rho\})\right)^{-1} \|e(t)\|$$
$$= \mu\|e(t)\|,$$
$$(3.16)$$

where $\mu = \left(1 - (h_1\rho + h_2\rho(1 + h_1\rho)\exp\{h_2\rho\})\right)^{-1}$, for $t \in [\rho_k, \rho_{k+1}]$. With regards to arbitrary values of t and k, (3.16) holds for $t \in \mathbb{R}^+$.

Remark 2. When considering MNNs (3.11) on the interval $[\rho_k, \rho_{k+1}]$, where $k \in \mathbb{N}$, if $\rho_k \leq t < \eta_k$, MNNs (3.11) behaves as an advanced system. Conversely, if $\eta_k < t \leq \rho_{k+1}$, MMN (3.11) behaves as a retarded system.

Theorem 2. If Assumptions 1–4 hold, MNNs (3.11) is ESt. Then MMNs (3.12) is ESt, that is, the derive system MNNs (3.7) and the response system MNNs (3.8) are ESy. If $|\rho| \leq \min\{\bar{\rho}, \tilde{\rho}\}$, where $\bar{\rho}$ is a unique nonnegative solution of the transcend equation:

$$k_2\alpha/\beta\exp\{2k_1\mathbb{T}\} + \alpha\exp\{-\beta\mathbb{T}\} = 1,$$
$$(3.17)$$

where $\mathbb{T} > \ln(\alpha)/\beta$, $k_1 = \|C\| + \|D\| + \|A^*\|\|F^*\| + \mu\|B^*\|\|G^*\|$, $k_2 = (1 + \mu)\|B^*\|\|G^*\|$.

The $\tilde{\rho}$ is a unique positive solution of the transcend equation:

$$(h_1\rho + h_2\rho(1 + h_1\rho)\exp\{h_2\rho\}) = 1.$$
$$(3.18)$$

Proof. Utilizing Lemmas 1 and 3, initial value $\psi^1 = \psi^2$, we can conclude the following

$$\|z(t) - e(t)\| \leq \left\| \int_{t_0}^t \left[-(C + D)(z(s) - e(s)) + \hat{A}F(z(s)) - \hat{A}F(e(s)) + \hat{B}G(z(s)) - \hat{B}G((\gamma(s))) \right] ds \right.$$

$$\leq \int_{t_0}^t \left((\|C\| + \|D\| + \|A^*\|\|F^*\|)\|z(s) - e(s)\| ds + \|B^*\|\|G^*\|\|z(s)\| + \|B^*\|\|G^*\|\|e(\gamma(s))\| \right) ds$$

$$\leq \int_{t_0}^t \left((\|C\| + \|D\| + \|A^*\|\|F^*\|)\|z(s) - e(s)\| \right.$$
$$+ \|B^*\|\|G^*\|\|z(s)\| + \mu\|B^*\|\|G^*\|\|e(s) - z(s) + z(s)\|) ds$$

$$\leq \int_{t_0}^t (\|C\| + \|D\| + \|A^*\|\|F^*\| + \mu\|B^*\|\|G^*\|)\|z(s) - e(s)\| ds$$
$$+ \int_{t_0}^t ((1 + \mu)\|B^*\|\|G^*\|)\|z(s)\| ds$$

$$\leq \int_{t_0}^t k_1(z(s) - e(s)) ds + k_2\|\psi^2\|\alpha/\beta,$$
$$(3.19)$$

where $k_1 = \|C\| + \|D\| + \|A^*\|\|F^*\| + \mu\|B^*\|\|G^*\|$, $k_2 = (1 + \mu)\|B^*\|\|G^*\|$.

By Lemma 2, when $t_0 + \mathbb{T} - \rho \leq t \leq t_0 + 2\mathbb{T}$,

$$\|e(t) - z(t)\| \leq k_2\alpha/\beta\|\psi^2\|\exp\{2k_1\mathbb{T}\}.$$

So, when $t_0 + \mathbb{T} - \rho \leq t \leq t_0 + 2\mathbb{T}$, from (3.19) and the global exponential stability of (3.12),

$$
\begin{aligned}
\|e(t)\| &= \|e(t) - z(t) + z(t)\| \\
&\leq k_2\alpha/\beta\|\psi^2\|\exp\{2k_1\mathbb{T}\} + \alpha\|\psi^2\|\exp\{-\beta\mathbb{T}\} \\
&\leq \left(k_2\alpha/\beta\exp\{2k_1\mathbb{T}\} + \alpha\exp\{-\beta\mathbb{T}\}\right)\left(\sup_{t_0-\rho\leq t\leq t_0+\mathbb{T}}\|e(t)\|\right).
\end{aligned}
\tag{3.20}
$$

From (3.20), when $|\rho| \leq \bar{\rho}$,

$$
k_2\alpha/\beta\exp\{2k_1T\} + \alpha\exp\{-\beta T\} < 1.
$$

Let $\kappa_1 = -(\ln(k_2\alpha/\beta\exp\{2k_1T\} + \alpha\exp\{-\beta T\}))/\mathbb{T}, \kappa_1 > 0$, when $t_0 - \rho + \mathbb{T} \leq t \leq t_0 + 2\mathbb{T}$,

$$
\sup_{t_0-\rho+\mathbb{T}\leq t\leq t_0+2\mathbb{T}}\|e(t)\| \leq \exp(-\rho\mathbb{T})\sup_{t_0-\rho\leq t\leq t_0+\mathbb{T}}\|e(t)\|.
\tag{3.21}
$$

Consider the existence and uniqueness of the solution $e(t)$ of (8), when $t > t_0 - \xi + (\mathbb{M} - 1)\mathbb{T}$,

$$
e(t, t_0, x_0) = e(t, t_0 - \xi + (\mathbb{M} - 1)\mathbb{T}, e(t_0 - \xi + (\mathbb{M} - 1)\mathbb{T}, t_0, x_0)).
\tag{3.22}
$$

From (3.21) and (3.22),

$$
\begin{aligned}
\sup_{t_0-\rho+m\mathbb{T}\leq t\leq t_0+(m+1)\mathbb{T}}\|e(t, t_0, e_0)\| &= \sup_{t_0-\rho+(\mathbb{M}-1)\mathbb{T}+\mathbb{T}\leq t\leq t_0+(\mathbb{M}-1)\mathbb{T}+2\mathbb{T}}\|e(t, t_0 - \rho + (\mathbb{M} - 1)\mathbb{T}, \\
e(t_0 - \rho + (\mathbb{M} - 1)\mathbb{T}; t_0, e_0))\| &\leq \exp(-\rho\mathbb{T})\sup_{t_0-\rho+(\mathbb{M}-1)\mathbb{T}\leq t\leq t_0+m\mathbb{T}}\|e(t; t_0, e_0)\| \\
&\leq \exp(-\rho m\mathbb{T})\sup_{t_0-\rho\leq t\leq t_0+\mathbb{T}}\|e(t; t_0, e_0)\| \\
&= \kappa_2\exp -\rho m\mathbb{T},
\end{aligned}
$$

where $\kappa_2 = \sup_{t_0-\rho\leq t\leq t_0+\mathbb{T}}\|e(t; t_0, e_0)\|$.

To go a step further, there is the only scalar $m \in \mathbb{N}$ such that $t_0 - \rho + (\mathbb{M} - 1)\mathbb{T} \leq t \leq t_0 + \mathbb{M}\mathbb{T}$,

$$
\|e(t; t_0, x_0)\| \leq \kappa_2\exp(-\rho\mathbb{M}\mathbb{T}) \leq \kappa_2\exp(\rho\mathbb{T})\exp(-\rho(t - t_0)).
\tag{3.23}
$$

Clerly, (3.23) holds for $t_0 - \rho \leq t \leq t_0 + \mathbb{T}$.

3.3. Effects of DAs and SPs on MNNs synchronization

The following consider MNNs with SPs and DAs,

$$
dw_p(t) = [-d_p w_p(t) + \sum_{q=1}^{n} a_{pq}(w_q(t))f_q(w_q(t)) + \sum_{q=1}^{n} b_{pq}(w_q(\gamma(t)))g_q(w_q(\gamma(t))) + w_p(t)]dt + \sigma w_p(t)d\omega(t).
\tag{3.24}
$$

The corresponding response system,

$$
\begin{aligned}
dv_p(t) = &[-d_p v_p(t) + \sum_{q=1}^{n} a_{pq}(v_q(t))f(v_q(t)) + \sum_{q=1}^{n} b_{pq}(v_q(\gamma(t)))g_q(v_q(\gamma(t))) + w_p(t) + u_p(t)]ds \\
&+ \sigma v_p(t)d\omega(t).
\end{aligned}
\tag{3.25}
$$

Let $e_p = v_p - w_p$,

$$de_p = [-d_p e_p + \sum_{q=1}^{n} \hat{a}_{pq}(e_q)f_q(e_p) + \sum_{q=1}^{n} \hat{b}_{pq}(e_q(\gamma))g_q(e_q(\gamma)) - \xi_p e_p]ds + \sigma e_p d\omega. \tag{3.26}$$

The original system is

$$\dot{z}_p = -d_p z_p + \sum_{q=1}^{n} \hat{a}_{pq}(z_q)f_q(z_q) + \hat{b}_{pq}(z_q)g_q(z_q) - \xi_p e_p. \tag{3.27}$$

Further,

$$de(t) = [-De(t) + \hat{A}Fe(t) + \hat{B}G((\gamma(t))) - Ce(t)]dt + \sigma e(t)d\omega(t)$$
$$\dot{z}(t) = -De(t) + \hat{A}Fe(t) + \hat{B}G((t)) - Ce(t).$$

Lemma 4. Let Assumptions 3 and 5 hold, then the following inequality

$$E\|e(\gamma(t))\|^2 \leq \lambda\|e(t)\|^2 \tag{3.28}$$

holds for all $t \in \mathbb{R}^+$, where $\lambda = 3(1 - \varpi)^{-1}$, $\varpi = \rho(3l_1 + 9l_2 + l_1^2 + 3l_1 l_2)$.

Proof. For $\gamma(t) = \eta_k$, $t \in [\rho_k, \rho_{k+1}]$, $\forall t \in \mathbb{R}^+$, $\exists k \in \mathbb{N}$, we have

$$E\|e(t)\|^2 \leq E\|e(\eta_k) + \int_{\eta_k}^{t} \left[-(C + D)e(s) + \hat{A}F(e(s)) + \hat{B}G((\eta_k)) \right]ds + \int_{\eta_k}^{t} \sigma e(s)d\omega(s)\|^2$$

$$\leq 3\left[E\|e(\eta_k)\|^2 + E\| \int_{\eta_k}^{t} \left[-(C + D)e(s) + \hat{A}F(e(s)) + \hat{B}G((\eta_k)) \right]ds\|^2 + E\| \int_{\eta_k}^{t} \sigma e(s)d\omega(s)\|^2 \right]$$

$$\leq 3\left[E\|e(\eta_k)\|^2 + 3\rho E \int_{\eta_k}^{t} 2(\|C\|^2 + \|D\|^2)\|e(s)\|^2 + \|A^*\|^2\|F^*\|^2\|e(s)\|^2 \right.$$

$$\left. + \|B^*\|^2\|G^*\|^2\|e(\eta_k)\|^2 + \sigma^2 \int_{\eta_k}^{t} E\|e(s)\|^2 ds \right]$$

$$\leq 3(1 + 3\rho^2\|B^*\|^2\|G^*\|^2)E\|e(\eta_k)\|^2 + 3(3\rho(2\|C\|^2 + 2\|D\|^2 + \|A^*\|^2\|F^*\|^2) + \sigma^2) \int_{\eta_k}^{t} E\|e(s)\|^2 ds.$$

Applying Lemma 2,

$$E\|e(t)\|^2 \leq (3 + 9\rho^2\|B^*\|^2\|G^*\|^2)E\|e(\eta_k)\|^2 \exp\{3\rho(3\rho(2\|C\|^2 + 2\|D\|^2 + \|A^*\|^2\|F^*\|^2) + \sigma^2)\}$$
$$= (3 + l_1)E\|e(\eta_k)\|^2 \exp\{3\rho l_2\},$$

where $l_1 = 9\rho^2\|B^*\|^2\|G^*\|^2$, $l_2 = 3\rho(2\|C\|^2 + 2\|D\|^2 + \|A^*\|^2\|F^*\|^2) + \sigma^2$.

Similarly, for $t \in [\rho_k, \rho_{k+1}]$,

$$E\|e(\eta_k)\|^2 \leq 3\left[E\|e(s)\|^2 + E\|\int_{eta_k}^t (-(C+D)e(s) + \hat{A}Fe(s) + \hat{B}Ge(s))ds\|^2 + E\|\int_{\eta_k}^t \sigma e(s)d\omega(s)\|^2\right]$$

$$\leq 3\left[E\|e(s)\|^2 + 3\rho E \int_{eta_k}^t (2(\|C\|^2 + \|D\|^2)\|e(s)\|^2 + \|A^*\|^2\|F^2\|\|e(s)\|^2\right.$$

$$+ \|B^*\|^2\|G^*\|^2\|e(\eta_k)\|^2)ds + \sigma^2 \int_{\eta_k}^t E\|e(s)\|^2 ds\right]$$

$$= 3E\|e(s)\|^2 + 9\rho^2\|B^*\|^2\|G^*\|^2\|e(\eta_k)\|^2 + 3(3\rho((2\|C^*\|^2 + 2\|D^*\|^2)$$

$$+ \|A^*\|^2\|F^*\|^2) + \sigma^2) \int_{\eta_k}^t E\|e(s)\|^2 ds$$

$$= 3E\|e(s)\|^2 + \rho(3l_1 + 9l_2 + l_1^2 + 3l_1l_2) \exp\{3\rho l_2\}\|e(\eta_k)\|^2.$$

By the Assumption 5,

$$E\|e(\eta_k)\|^2 \leq 3(1-\varpi)^{-1}E\|e(s)\|^2$$
$$= \lambda E\|e(s)\|^2, \tag{3.29}$$

where $\varpi = \rho(3l_1 + 9l_2 + l_1^2 + 3l_1l_2)$, $\lambda = 3(1-\varpi)^{-1}$. Therefore, (3.29) holds for $t \in [\rho_k, \rho_{k+1}]$. By the randomicities of t and k, (3.29) holds for all $t \in \mathbb{R}$.

In the following, we investigate the effects of DAs and SPs on the robustness of ESy of MNNs (3.26).

Theorem 3. If Assumptions 3–5 and Definition 4 hold, MNNs (3.27) is ESt. Then MMNs (3.26) is ESt, that is, the derive system MNNs (3.24) and the response system MNNs (3.25) are ESy. If $|\sigma| \leq \bar{\sigma}$, $|\rho| \leq \min\{\bar{\rho}, \tilde{\rho}\}$ where $\bar{\rho}$ is a unique nonnegative solution of the transcend equation:

$$\upsilon_2\theta\|\psi^2\|^2/\beta \exp\{2\upsilon_1\mathbb{T}\} + 2\theta\|\psi^2\|^2 \exp\{-2\mathbb{T}\beta\} = 1, \tag{3.30}$$

where $\upsilon_1 = (24\mathbb{T}(\|C\|^2 + \|D\|^2 + 2\|A^*\|^2\|F^*\|^2 + 2\lambda^2\|B^*\|^2\|G^*\|^2) + 4\sigma^2)$, $\upsilon_2 = (12\mathbb{T}(2+2\lambda^2)\|B^*\|^2\|G^*\|^2 + 2\sigma^2)$. The $\tilde{\rho}$ is a unique nonnegative solution of the transcend equation:

$$l_1 + 9\rho l_1 l_2 \exp\{3\rho l_2\} = 1. \tag{3.31}$$

The $\bar{\sigma}$ is a unique nonnegative solution of the transcend equation:

$$(36\mathbb{T}(\|A^*\|^2\|F^*\|^2 + \|B^*\|^2\|G^*\|^2) + 2\sigma^2)\theta/\vartheta \exp\{12\mathbb{T}(4\mathbb{T}(\|D\|^2$$
$$+ \|C\|^2 + 3\|A^*\|^2\|F^*\|^2 + 3\|B^*\|^2\|G^*\|^2) + \sigma^2)\} + 2\theta \exp\{-4\vartheta\mathbb{T}\} = 1. \tag{3.32}$$

Proof. When $t_0 - \rho \leq t \leq t_0 + 2\mathbb{T}$, By applying Lemma 4, initial value $\psi^1 = \psi^2$,

$$E\|z(t) - e(t)\|^2 \leq 2E\|\int_{t_0}^{t}[-(C + D)(z(s) - e(s)) + \hat{A}F(z(s)) - \hat{A}F(e(s)) + \hat{B}G(z(s)) - \hat{B}G((\gamma(s)))]ds\|^2$$

$$+ 2E\|\int_{t_0}^{t}\sigma e(s)d\omega(s)\|^2$$

$$\leq 2E\int_{t_0}^{t}1^2 ds\int_{t_0}^{t}6[(\|C\|^2 + \|D\|^2) + \|A^*\|^2\|F^*\|^2\|z(s)\|^2 + \|A^*\|^2\|F^*\|^2\|e(s)\|^2$$

$$+ \|B^*\|^2\|G^*\|^2\|z(s)\|^2 + \|B^*\|^2\|G^*\|^2\|e(\gamma(s))\|^2]ds + 2\sigma^2\|\int_{t_0}^{t}E\|e(s)\|^2 ds$$

$$\leq 4\mathbb{T}E\int_{t_0}^{t}6[(\|C\|^2 + \|D\|^2 + 2\|A^*\|^2\|F^*\|^2)\|z(s) - e(s)\|^2 + (2\|A^*\|^2\|F^*\|^2$$

$$+ \|B^*\|^2\|G^*\|^2)\|z(s)\|^2 + 2\lambda^2\|B^*\|^2\|G^*\|^2\|e(s) - z(s) + z(s)\|^2]ds$$

$$+ 2\sigma^2\int_{t_0}^{t}E\|e(s) - z(s) + z(s)\|^2 ds$$

$$\leq (24\mathbb{T}(\|C\|^2 + \|D\|^2 + 2\|A^*\|^2\|F^*\|^2 + 2\lambda^2\|B^*\|^2\|G^*\|^2) + 4\sigma^2)\int_{t_0}^{t}E\|z(s) - e(s)\|^2$$

$$+ (24\mathbb{T}(2 + 2\lambda^2)\|B^*\|^2\|G^*\|^2 + 4\sigma^2)\int_{t_0}^{t}E\|e(s)\|^2$$

$$\leq (24\mathbb{T}(\|C\|^2 + \|D\|^2 + 2\|A^*\|^2\|F^*\|^2 + 2\lambda^2\|B^*\|^2\|G^*\|^2) + 4\sigma^2)\int_{t_0}^{t}E\|z(s) - e(s)\|^2$$

$$+ (12\mathbb{T}(2 + 2\lambda^2)\|B^*\|^2\|G^*\|^2 + 2\sigma^2)\theta\|\psi^2\|^2/\vartheta$$

$$= \upsilon_1\int_{t_0}^{t}E\|z(s) - e(s)\|^2 ds + \upsilon_2\theta\|\psi^2\|^2/\vartheta,$$

where $\upsilon_1 = (24\mathbb{T}(\|C\|^2 + \|D\|^2 + 2\|A^*\|^2\|F^*\|^2 + 2\lambda^2\|B^*\|^2\|G^*\|^2) + 4\sigma^2)$, $\upsilon_2 = (12\mathbb{T}(2 + 2\lambda^2)\|B^*\|^2\|G^*\|^2 + 2\sigma^2)$.

When $t_0 - \rho \leq t \leq t_0 + 2\mathbb{T}$, By applying Lemma 2,

$$E\|z(t) - e(t)\|^2 \leq \upsilon_2\theta\|\psi^2\|^2/\vartheta\exp\{2\upsilon_1\mathbb{T}\} \times \sup_{t_0 - \rho \leq t \leq t_0 + \mathbb{T}}E\|z(t)\|^2.$$

Then,

$$E\|e(t)\|^2 \leq 2E\|z(t) - e(t)\|^2 + 2E\|z(t)\|^2$$

$$\leq \upsilon_2\theta\|\psi^2\|^2/\vartheta\exp\{2\upsilon_1\mathbb{T}\}\sup_{t_0 - \rho \leq t \leq t_0 + \mathbb{T}}E\|z(t)\|^2 + 2\theta\|\psi^2\|^2\exp\{-2\vartheta(t - t_0)\}$$

$$\leq (\upsilon_2\theta\|\psi^2\|^2/\vartheta\exp\{2\upsilon_1\mathbb{T}\} + 2\theta\exp\{-2\mathbb{T}\vartheta\})\sup_{t_0 - \rho \leq t \leq t_0 + \mathbb{T}}E\|e(t)\|^2.$$

When $|\rho| \leq \min\{\bar{\rho}, \tilde{\rho}\}$, $|\sigma| \leq \bar{\sigma}$,

$$\upsilon_2\theta\|\psi^2\|^2/\vartheta\exp\{2\upsilon_1\mathbb{T}\} + 2\theta\|\psi^2\|^2\exp\{-2\mathbb{T}\beta\} \leq 1.$$

4. Simulations

We demonstrate the aforementioned theoretical results through three numerical simulations.

Example 1. Consider two dimensional MNNs with SPs.

$$dw_p(t) = [-d_p w_p(t) + \sum_{q=1}^{2} a_{pq}(w_q(t)) f_q(w_q(t)) + w_p(t)] dt + \sigma w_p(t) d\omega(t), \ i = 1, 2, \quad (4.1)$$

where

$$a_{11}(w_1) = \begin{cases} 0.1, & |w_1| \le 1, \\ -0.1, & |w_1| > 1, \end{cases} \quad a_{12}(w_2) = \begin{cases} 0.2, & |w_2| \le 1, \\ -0.2, & |w_2| > 1, \end{cases}$$

$$a_{21}(w_1) = \begin{cases} 0.1, & |w_1| \le 1, \\ -0.1, & |w_1| > 1, \end{cases} \quad a_{22}(w_2) = \begin{cases} 0.2, & |w_2| \le 1, \\ -0.2, & |w_2| > 1, \end{cases}$$

$w_1 = (w_1, w_2)$, $d_1 = d_2 = 1, q = 1, 2, f_q(w_q) = tanh(w_q)$, $w_1 = w_2 = 0$, $\phi_1 = (0.3, 0.35)^T, \phi_2 = (0.2, 0.25)^T$.

The response system is

$$dv_p(t) = [-d_p v_p(t) + \sum_{q=1}^{2} a_{pq}(v_q(t)) f(v_q(t)) + w_p(t) + u_p(t)] dt + \sigma w_p(t) d\omega(t), \ i = 1, 2, \quad (4.2)$$

where $\varphi_1 = (-0.1, -0.15)^T, \varphi_2 = (-0.2, -0.25)^T$.

Let $\theta = 1.1, \vartheta = 0.2, \mathbb{T} \le \ln(\theta)/\vartheta = 0.01, \|A^*\| = 0.1, \|F^*\| = 1, \|C\| = 0.7, \|D\| = 1$.

Solving the following transcedental equation,

$$2.2(24 \times 0.01 \times 0.01 + 2\sigma) \exp\{0.08(0.04(1 + 0.49 + 0.03) + \sigma)\} + 2.2 \exp\{-4.4 \times 0.2\} = 1.$$

We can obtain $\bar{\sigma} = 0.1364$, let $\sigma = 0.04$, $\sigma = 0.06$, $\sigma = 0.1$, The state trajectories of MMNs (4.1) and MNNs (4.2) are shown in Figures 1–3, respectively. It can be seen from the figures that when time tends to infinity, the states of a and b tend to 0 This can show that when the perturbations intensity σ is less than $\bar{\sigma}$, the drive-response systems can achieve Esy. When $\sigma = 0.25$, The states are illustrated in Figure 4.
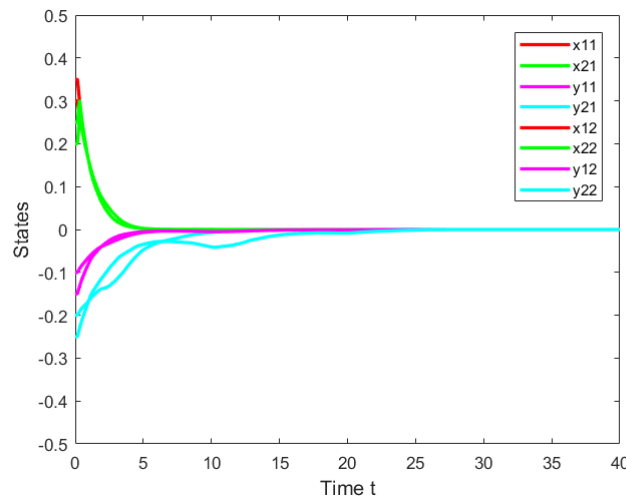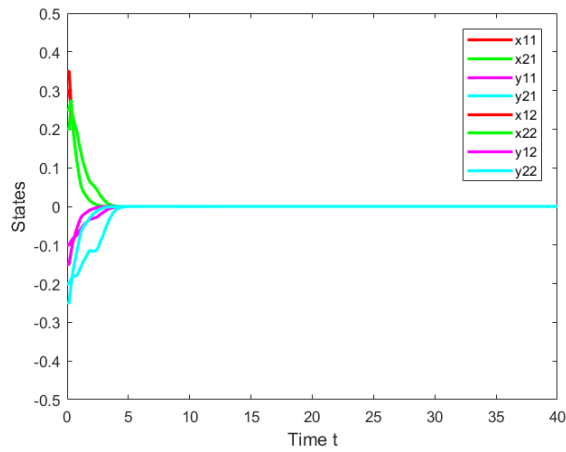

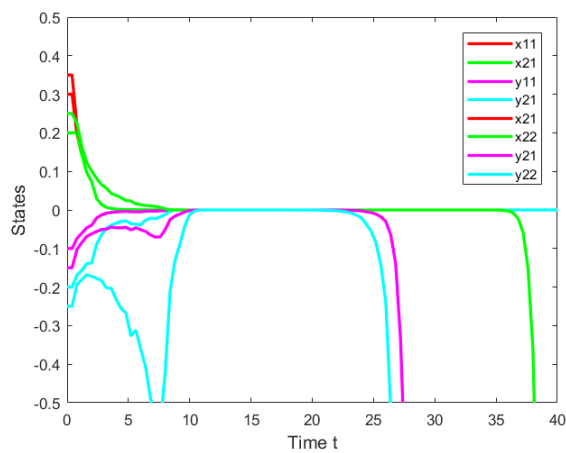**Figure 1.** State of $x(t)$ and $y(t)$ with $\sigma = 0.04$.
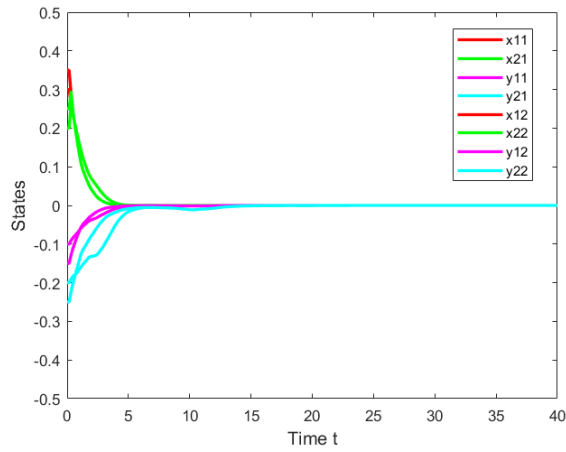
Figure 2. State of $x(t)$ and $y(t)$ with $\sigma = 0.06$.

Figure 3. State of $x(t)$ and $y(t)$ with $\sigma = 0.1$.

Figure 4. State of $x(t)$ and $y(t)$ with $\sigma = 0.25$.

Example 2. Consider two-dimensional MNNs with DAs,

$$\dot{w}_p(t) = -d_p w_p(t) + \sum_{q=1}^{2} a_{pq}(w_q(t))f_q(w_q(t)) + \sum_{q=1}^{2} b_{pq}(w_q(\gamma(t)))g_q(w_q(\gamma(t))) + w_p(t), \ i = 1, 2, \quad (4.3)$$

where

$$a_{11}(w_1) = \begin{cases} 0.125, & |w_1| \le 1, \\ -0.125, & |w_1| > 1, \end{cases} \quad a_{12}(w_2) = \begin{cases} 0.15, & |w_2| \le 1, \\ -0.15, & |w_2| > 1, \end{cases}$$

$$a_{21}(w_1) = \begin{cases} 0.125, & |w_1| \le 1, \\ -0.125, & |w_1| > 1, \end{cases} \quad a_{22}(w_2) = \begin{cases} 0.15, & |w_2| \le 1, \\ -0.15, & |w_2| > 1, \end{cases}$$

$$b_{11}(\tilde{w}_1) = \begin{cases} 0.2, & |\tilde{w}_1| \le 1, \\ -0.2, & |\tilde{w}_1| > 1, \end{cases} \quad b_{12}(\tilde{w}_2) = \begin{cases} 0.1, & |\tilde{w}_2| \le 1, \\ -0.1, & |\tilde{w}_2| > 1, \end{cases}$$

$$b_{21}(\tilde{w}_1) = \begin{cases} 0.2, & |\tilde{w}_1| \le 1, \\ -0.2, & |\tilde{w}_1| > 1, \end{cases} \quad b_{22}(\tilde{w}_2) = \begin{cases} 0.1, & |\tilde{w}_2| \le 1, \\ -0.1, & |\tilde{w}_2| > 1, \end{cases}$$

where $\tilde{w}_q = w_q(\gamma)$, $w_1 = (w_1, w_2)$, $d_1 = d_2 = 1$, $q = 1, 2$, $f_q(w_q) = tanh(w_q)$, $g_q(w_q) = |w_q + 1| - |w_q - 1|$, $w_1 = w_2 = 0$, $\phi_1 = (3, 4)^T, \phi_2 = (1, 2)^T$.

The response system is

$$\dot{v}_p(t) = -d_p v_p(t) + \sum_{q=1}^{2} a_{pq}(v_q(t))f_q(v_q(t)) + \sum_{q=1}^{2} b_{pq}(v_q(\gamma(t)))g_q(v_q(\gamma(t))) + w_p(t) + u_p(t), \ i = 1, 2,$$

$$(4.4)$$

where $\varphi_1 = (-3, -4)^T, \varphi_2 = (-1, -2)^T$.

Let $\alpha = 1.1, \beta = 0.5, \mathbb{T} \le \ln(\theta)/\vartheta = 0.01, \|A^*\| = 0.726, \|B^*\|^2 = 0.1 \|F^*\| = 1, \|G^*\| = 1 \|C\| = 0.01, \|D\| = -1$.

Solving the following transcedental equations,

$$0.001\rho + 1.772\rho(1 + 1.772\rho)\exp\{1.772\rho\} = 1,$$

$$0.22(1 + \mu)\exp\{0.02(1.772 + 0.1\mu)\} + 1.1\exp\{-0.5 \times 0.01\} = 1.$$

We can obtain $\bar{\rho} = 0.2506, \tilde{\rho} = 0.3314$, let $\rho = 0.2, \rho = 0.15, \rho = 0.1, \rho = 0.25$, when the length of the DAs in the systems is less than the calculated upper bound, we have that MNNs (4.3) and (4.4) with the controllers is ESy. The states of the MNNs (4.3) and (4.4) are shown in Figures 4–8, respectively.

Example 3. Consider two-dimensional MNNs with DAs and SPs.

$$dw_p(t) = -[d_p w_p(t) + \sum_{q=1}^{2} a_{pq}(w_q(t))f_q(w_q(t)) + \sum_{q=1}^{2} b_{pq}(w_q(\gamma(t)))g_q(w_q(\gamma(t)))$$

$$+ w_p(t)]dt + \sigma w_p d\omega(t). \ i = 1, 2. \quad (4.5)$$

All other parameters remain the same as in Example 2. The response system is,

$$dv_p(t) = -[d_p v_p(t) + \sum_{q=1}^{2} a_{pq}(v_q(t))f_q(v_q(t)) + \sum_{q=1}^{2} b_{pq}(v_q(\gamma(t)))g_q(v_q(\gamma(t)))$$

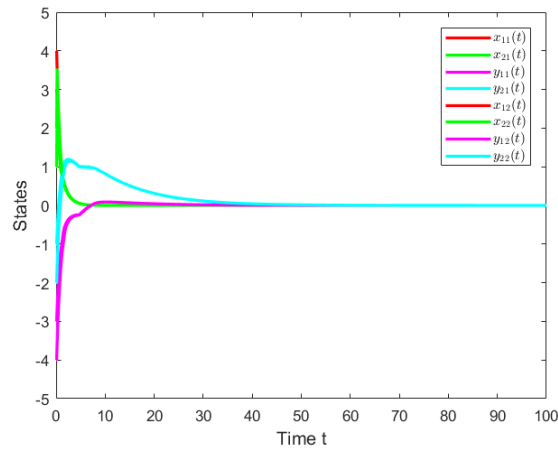$$+ w_p(t) + u_p(t)]dt + \sigma v_p d\omega(t), \ i = 1, 2. \quad (4.6)$$

Solving the transcedental equations. We can obtain $\bar{\rho} = 0.1567$, $\tilde{\rho} = 0.4374$, $\bar{\sigma} = 0.2463$, let $\rho = 0.1$, $\sigma = 0.04$, we have that MNNs (4.5) and (4.6) is ESy, the state trajectories are shown in Figure 9. when $\rho = 0.05$, $\sigma = 0.04$, the state trajectories are shown in Figure 10. When $\rho = 0.9$, $\sigma = 0.1$, the state trajectories are shown in Figure 11.
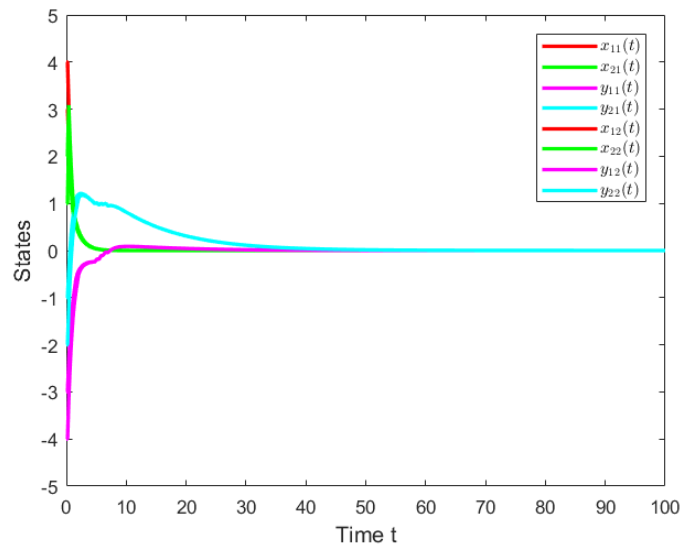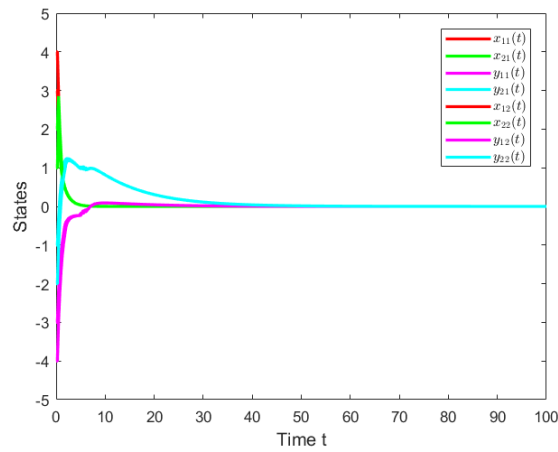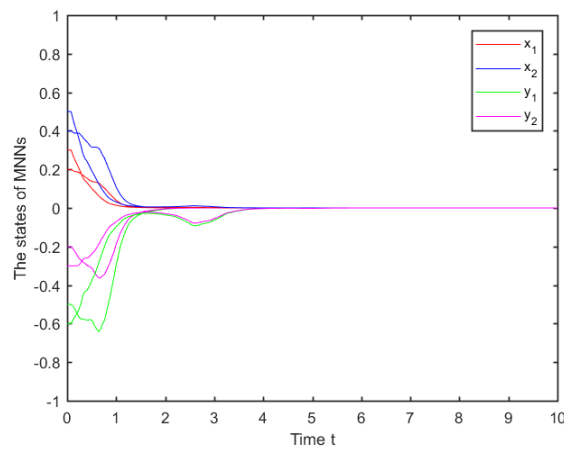

Figure 5. The state with $\rho = 0.2$.
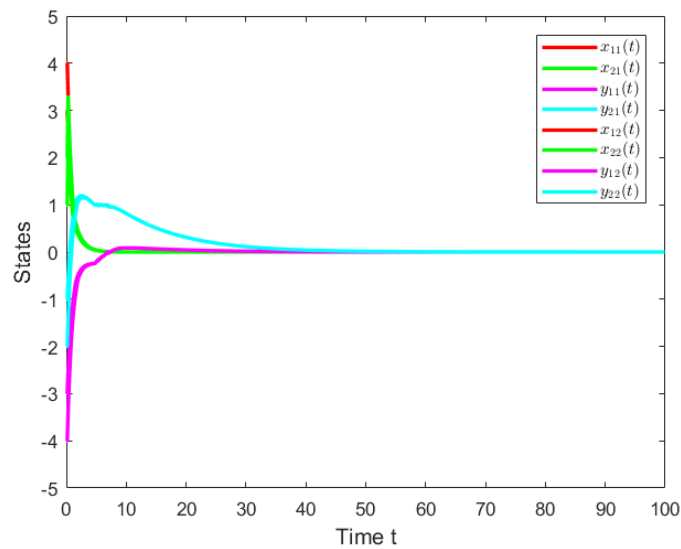
Figure 6. The state with $\rho = 0.15$.
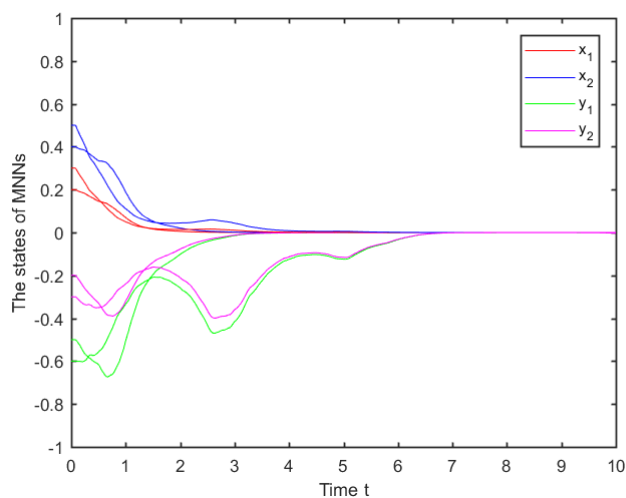
Figure 7. The state with ρ=0.1.

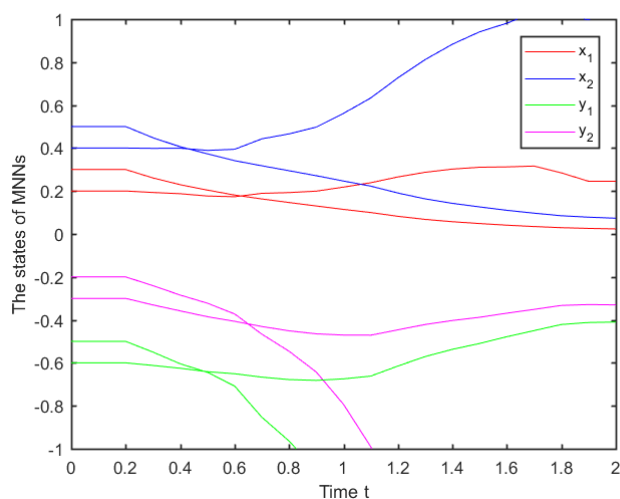**Figure 8.** The state with ρ=0.25.

Figure 9. The state of MMNs (51) and (52) with $\sigma = 0.1$ and ρ=0.04.

Figure 10. The state of MMNs (51) and (52) with $\sigma = 0.05$ and $\rho = 0.04$.

Figure 11. The state of MMNs (51) and (52) with $\sigma = 0.1$ and $\rho = 0.9$.

5. Conclusions

In this paper, the robustness analysis of MNNs exponential synchronization problem with DAs and SPs is studied by using the Granwall inequalities and inequality techniques, and a method different from Linear Matrix Inequality method (LMI) and Lyapunov theory is used to solve the synchronization robustness of MNNs.

Use of AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

Conflict of interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

References

1. L. Chua, Memristor-the missing circuit element, *IEEE T. Circuits-Theor.*, **18** (1971), 507–519. https://doi.org/10.1109/TCT.1971.1083337

2. D. B. Strukov, G. S. Snider, D. R. Stewart, R. S. Williams, The missing memristor found, *Nature*, **453** (2008), 80–83. https://doi.org/10.1038/nature06932

3. X. Huang, Y. Fan, J. Jia, Z. Wang, Y. Li, Quasi-synchronisation of fractional-order memristor-based neural networks with parameter mismatches, *IET Control Theory A.*, **11** (2017), 2317–2327. https://doi.org/10.1049/iet-cta.2017.0196

4. H. Bao, J. H. Park, J. Cao, Adaptive synchronization of fractional-order memristor-based neural networks with time delay, *Nonlinear Dynam.*, **82** (2015), 1343–1354. https://doi.org/10.1007/s11071-015-2242-7

5. F. Du, J. G. Lu, New criteria for finite-time stability of fractional order memristor-based neural networks with time delays, *Neural Comput.*, **421** (2021), 349–359. https://doi.org/10.1016/j.neucom.2020.09.039

6. L. Wang, Y. Shen, Q. Yin, G. Zhang, Adaptive synchronization of memristor-based neural networks with time-varying delays, *IEEE T. Neural Net. Lear.*, **26** (2014), 2033–2042. https://doi.org/10.1109/TNNLS.2014.2361776

7. C. Yang, L. Huang, Z. Cai, Fixed-time synchronization of coupled memristor-based neural networks with time-varying delays, *Neural Networks*, **116** (2019), 101–109. https://doi.org/10.1016/j.neunet.2019.04.008

8. S. Yang, Z. Guo, J. Wang, Robust synchronization of multiple memristive neural networks with uncertain parameters via nonlinear coupling, *IEEE T. Syst. Man Cy.-S.*, **45** (2015), 1077–1086. https://doi.org/10.1109/TSMC.2014.2388199

9. H. B. Bao, J. D. Cao, Projective synchronization of fractional-order memristor-based neural networks, *Neural Networks*, **63** (2015), 1–9. https://doi.org/10.1016/j.neunet.2014.10.007

10. H. Ren, Z. Peng, Y. Gu, Fixed-time synchronization of stochastic memristor-based neural networks with adaptive control, *Neural Networks*, **130** (2020), 165–175. https://doi.org/10.1016/j.neunet.2020.07.002

11. B. Zhang, F. Deng, S. Xie, S. Luo, Exponential synchronization of stochastic time-delayed memristor-based neural networks via distributed impulsive control, *Neurocomputing*, **286** (2018), 41–50. https://doi.org/10.1016/j.neucom.2018.01.051

12. R. Rakkiyappan, G. Velmurugan, J. Cao, Stability analysis of memristor-based fractional-order neural networks with different memductance functions, *Cogn. Neurodynamics*, **9** (2015), 145–177. https://doi.org/10.1007/s11571-014-9312-2

13. L. Wang, H. He, Z. Zeng, Global synchronization of fuzzy memristive neural networks with discrete and distributed delays, *IEEE T. Fuzzy Syst.*, **28** (2019), 2022–2034. https://doi.org/10.1109/TFUZZ.2019.2930032

14. J. Chen, Z. Zeng, P. Jiang, Global mittag-leffler stability and synchronization of memristor-based fractional-order neural networks, *Neural Networks*, **51** (2014), 1–8. https://doi.org/10.1016/j.neunet.2013.11.016

15. S. Liu, Y. Yu, S. Zhang, Robust synchronization of memristor-based fractional-order hopfield neural networks with parameter uncertainties, *Neural Comput. Appl.*, **31** (2019), 3533–3542. https://doi.org/10.1007/s00521-017-3274-3

16. L. Chen, T. Huang, J. T. Machado, A. M. Lopes, Y. Chai, R. Wu, Delay-dependent criterion for asymptotic stability of a class of fractional-order memristic neural networks with time-varying delays, *Neural Networks*, **118** (2019), 289–299. https://doi.org/10.1016/j.neunet.2019.07.006

17. H. Cheng, S. Zhong, Q. Zhong, K. Shi, X. Wang, Lag exponential synchronization of delayed memristor-based neural networks via robust analysis, *IEEE Access*, **7** (2018), 173–182. https://doi.org/10.1109/ACCESS.2018.2885221

18. L. Wang, Z. Zeng, M. F. Ge, A disturbance rejection framework for finite-time and fixed-time stabilization of delayed memristive neural networks, *IEEE T. Syst. Man. Cy.-S.*, **51** (2019), 905–915. https://doi.org/10.1109/TSMC.2018.2888867

19. S. Shah, J. Wiener, Advanced differential equations with piecewise constant argument deviations, *Int. J. Math. Math. Sci.*, **6** (1983), 671–703. https://doi.org/10.1155/S0161171283000599

20. M. U. Akhmet, D. Aruğaslan, E. Yılmaz, Stability analysis of recurrent neural networks with piecewise constant argument of generalized type, *Neural Networks*, **23** (2010), 805–811. https://doi.org/10.1016/j.neunet.2010.05.006

21. A. Wu, L. Liu, T. Huang, Z. Zeng, Mittag-leffler stability of fractional-order neural networks in the presence of generalized piecewise constant arguments, *Neural Networks*, **85** (2017), 118–127. https://doi.org/10.1016/j.neunet.2016.10.002

22. J. E. Zhang, Robustness analysis of global exponential stability of nonlinear systems with deviating argument and stochastic disturbance, *IEEE Access*, **5** (2017), 446–454. https://doi.org/10.1109/ACCESS.2017.2727500

23. W. X. Fang, T. Xie, B. W. Li, Robustness analysis of fuzzy cellular neural network with deviating argument and stochastic disturbances, *IEEE Access*, **11** (2023), 2023. https://doi.org/10.1109/ACCESS.2023.3233946

24. G. Bao, S. Wen, Z. Zeng, Robust stability analysis of interval fuzzy cohen-grossberg neural networks with piecewise constant argument of generalized type, *Neural Networks*, **33** (2012), 32–41. https://doi.org/10.1016/j.neunet.2012.04.003

25. Q. Xi, X. Liu, Finite-time stability and controller design for a class of hybrid dynamical systems with deviating argument, *Nonlineat Anal.-Hybri.*, **39** (2021), 2021. https://doi.org/10.1016/j.nahs.2020.100952

26. M. U. Akhmet, D. Aruğaslan, E. Yılmaz, Method of lyapunov functions for differential equations with piecewise constant delay, *J. Comput. Appl. Math.*, **235** (2011), 4554–4560. https://doi.org/10.1016/j.cam.2010.02.043

27. W. Fang, T. Xie, B. Li, Robustness analysis of fuzzy bam cellular neural network with time-varying delays and stochastic disturbances, *AIMS Math.*, **8** (2023), 9365–9384. https://doi.org/10.3934/math.2023471

28. Q. Zhu, T. Huang, Stability analysis for a class of stochastic delay nonlinear systems driven by g-brownian motion, *Syst. Control. Lett.*, **140** (2020), 104699. https://doi.org/10.1016/j.sysconle.2020.104699

29. Q. Zhu, Stabilization of stochastic nonlinear delay systems with exogenous disturbances and the event-triggered feedback control, *IEEE T. Automat. Contr.*, **64** (2018), 3764–3771. https://doi.org/10.1109/TAC.2018.2882067

30. L. Zhang, X. Yang, C. Xu, J. Feng, Exponential synchronization of complex-valued complex networks with time-varying delays and stochastic perturbations via time-delayed impulsive control, *Appl. Math. Comput.*, **306** (2017), 22–30. https://doi.org/10.1016/j.amc.2017.02.004

31. C. Chen, L. Li, H. Peng, Y. Yang, T. Li, Synchronization control of coupled memristor-based neural networks with mixed delays and stochastic perturbations, *Neural Process. Lett.*, **47** (2018), 679–696. https://doi.org/10.1007/s11063-017-9675-6

32. X. Wang, K. She, S. Zhong, J. Cheng, Exponential synchronization of memristor-based neural networks with time-varying delay and stochastic perturbation, *Neurocomputing*, **242** (2017), 131–139. https://doi.org/10.1016/j.neucom.2017.02.059

33. Y. Shen, J. Wang, Robustness analysis of global exponential stability of recurrent neural networks in the presence of time delays and random disturbances, *IEEE T. Neural Net. Learn*, **23** (2011), 87–96. https://doi.org/10.1109/TNNLS.2011.2178326

34. Y. Shen, J. Wang, Robustness of global exponential stability of nonlinear systems with random disturbances and time delays, *IEEE T. Syst. Man Cy.-S.*, **46** (2015), 1157–1166. https://doi.org/10.1109/TSMC.2015.2497208

35. X. Mao, *Stochastic differential equations and applications*, Elsevier, 2007.

36. Y. Zhang, L. Li, H. Peng, J. Xiao, Y. Yang, M. Zheng, et al., Finite-time synchronization for memristor-based bam neural networks with stochastic perturbations and time-varying delays, *Int. J. Robust Nonlin.*, **28** (2018), 5118–5139. https://doi.org/10.1002/rnc.4302

37. C. Li, J. Lian, Y. Wang, Stability of switched memristive neural networks with impulse and stochastic disturbance, *Neurocomputing*, **275** (2018), 2565–2573. https://doi.org/10.1016/j.neucom.2017.11.031

38. J. P. Aubin, A. Cellina, *Differential inclusions: Set-valued maps and viability theory*, Springer Science & Business Media, 2012. https://doi.org/10.1007/978-3-642-69512-4

39. X. Mao, Stability and stabilisation of stochastic differential delay equations, *IET Control Theory A.*, **1** (2007), 1551–1566. http://dx.doi.org/10.1049/iet-cta:20070006

40. R. Bellman, The stability of solutions of linear differential equations, *Duke Math. J.*, **10** (1943), 643–647.