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Research article

Asymmetric integral barrier function-based tracking control of constrained robots

Tan Zhang and Pianpian Yan*

School of Electrical and Optoelectronic Engineering, West Anhui University, Lu'an 237012, China

* **Correspondence:** Email: 15357187791@163.com; Tel: +8615357187791.

Abstract: In this paper, a new-type time-varying asymmetric integral barrier function is designed to handle the state constraint of nonlinear systems. The barrier Lyapunov function is developed by building an integral upper limit function with respect to transformation errors over an open set to cope with the position constraint of the robotic system. We know that the symmetric time-invariant constraint is only a particular situation of the asymmetric time-variant constraint, and thus compared to existing methods, it is capable of handling more general and broad practical engineering issues. We show that under the integral barrier Lyapunov function combining a disturbance observer-based tracking controller, the position vector tracks a desired trajectory successfully, while the constraint boundary is never violated. It can certify the exponential asymptotic stability of the robotic tracking system by using the given inequality relationship on barrier function and Lyapunov analysis. Finally, the feasibility of the presented algorithm is indicated by completing the simulations.

Keywords: integral barrier function; constraints; tracking control; nonlinear system; robot
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1. Introduction

Over the past few decades, many effective methods has been developed for the tracking control of robotic systems. Lyapunov's direct methods (also called Lyapunov's second method), for instance, were used for designing stable controllers of nonlinear systems by combining the concept of control Lyapunov functions. Control Lyapunov functions generally have a quadratic form; however, more sophisticated Lyapunov functions are required to be constructed for some complex control problems. Adaptive control scheme using Lyapunov's direct method-based two different tracking controllers were developed to improve the convergence speed and transient response of robotic systems [1]. A generalization of Lyapunov's second method was introduced into the control design of nonlinear (controlled) systems for dealing with the following control problem under non-linear and fractional damping [2]. In this study, we construct one new Lyapunov control structure to address practical requirements of robots. In particular, we deal with the tracking control task for robotic systems subject to the state constraint, from the concept that a lot of physical systems was affected by various limitations such as safety specification, control performance requirement, actuator saturation, and physical stoppages [3].

Recently, the methods, which solved constraint problem, include mainly the use of set invariant notions [4], reference governors [5], barrier Lyapunov function [6–9], and prescribed performance control [10, 11], etc. Especially, barrier Lyapunov functions (BLFs) were often applied to the constraint control of robotic arms. A time-invariant logarithmic BLF, for example, was utilized to prevent the destruction of the state restrictions and ensure the uniform ultimate boundedness of closed-loop robotic systems [12]. This BLF applied to constrain the robotic system output [13]. However, time-invariant constraint control methods have limitations in practical applications. Therefore, a time-varying logarithmic BLF was applied to handle output restricting problems of robotic systems [14, 15]. In addition, in [16, 17], the inconstant tangent BLFs were applied to addressing the state and output constraint of the robot, respectively. Existing logarithmic BLFs have been utilized to constrain the states as well as track errors of various systems, and these methods were relatively mature. In order to cope with the development of control theory, we need to explore a new form of BLF that has the same constraint capability as the logarithmic one.

Integral BLFs can be used to constrain system states directly, however, can not deal with transformation errors [18, 19]. The advantage of this method was that it can directly handle the system state and eliminate the conservatism of known error ranges. Neural networks and integral BLF-based adaptive control approaches, for instance, were proposed for a kind of perturbed uncertain nonlinear systems to guarantee the constraint boundary was never violated and address the unknown functions in systems effectively [20]. A control method of nonlinear systems using integral BLF and backstepping method was presented to ensure system states are located within the constraint space [21]. In [22], a tracking strategy combining integral BLF and dynamic surface design was proposed for nonlinear pure-feedback systems to achieve both the solving of the explosion of complexity and the constraint requirement. In [23], integral BLFs were used to deal with full state constraints of nonlinear strict feedback systems. Compared to other types of BLFs, the disadvantage of integral BLF is that it cannot be used to constrain tracking errors directly. Although integral BLF has certain advantages in directly handling system states, it still has some shortcomings in solving some practical problems. Integral BLFs, for instance, cannot deal with the error performance requirements of tracking systems and asymmetric constraint requirements of systems, etc.

Motivated by the above discussions, according to structures of the existing logarithmic and integral BLFs, a new integral BLF is constructed in this paper. Aided by the backstepping design method, the integral BLF-based tracking control strategy for a robot under the time-variant asymmetric position limitations is studied. In view of the published literatures, the main innovations in the paper are summed up as follows:

(I) Unlike existing integral BLFs [18–23], the BLF, which is proposed for the first time, can work for systems with time-variant and asymmetric constraint requirements simultaneously. This integral barrier function is similar to the logarithmic function in [15]. However, the functions proposed in this article are more concise in structure, and the derivation of controllers based on this method is easier.

(II) Different from structures of existing control Lyapunov functions [24–28], the proposed integral

BLF is developed cleverly by constructing an integral upper limit function with respect to transformation errors to cope with the position constraint problem of systems.

(III) Furthermore, under the controller based on the integral BLF, the dissymmetric time-variant position restraint situation are achieved, and all the system error signals are exponentially asymptotically stable.

The organization of this article is as follows. The problem descriptions and preparations are shown in Section 2. Stability analysis and controller design are explained in Section 3 utilizing the presented BLF and disturbance observer. In order to confirm that the presented strategy is effective, the simulation experiment is finished in Section 4. Lastly, Section 5 offers a conclusion of the complete work.

2. Problem descriptions and preparations

2.1. Modeling of an n-link robot

According to [15, 29], dynamics of an n-link robot are depicted as

$$M_0(q)\ddot{q} + C_0(q,\dot{q})\dot{q} + G_0(q) = \tau(t) + f_{s\_un} \tag{2.1}$$

where $M_0(q) \in \mathrm{R}^{n\times n}$, $C_0(q,\dot{q})\dot{q} \in \mathrm{R}^n$ and $G_0(q) \in \mathrm{R}^n$ denote the inertia matrix, Coriolis-centripetal torque and gravitational matrix of the robotic system, respectively. The inertia matrix satisfies

$$M_0(q) = M_0^T(q) > 0.$$

The position, velocity and acceleration of the robotic system are represented by q, $\dot{q}$ and $\ddot{q}$, respectively. System inputs are denoted by $\tau(t)$. System uncertain terms are described by

$$f_{s\_un} = -\Delta C(q,\dot{q})\dot{q} - \Delta M(q)\ddot{q} - \Delta G(q) - J^T(q) f(t),$$

$\Delta\cdot$, $J^T(q)$ and $f(t)$ represent the uncertain part of the system matrix, Jacobian matrix, external force, respectively.

2.2. Basic assumption and coordinate conversion

To contribute to completing the process of control design, we perform the following coordinate conversion. Let

$$\begin{cases} x_1 = q, \\ x_2 = \dot{q}. \end{cases} \tag{2.2}$$

The dynamics of the robot (2.1) can be rewritten as

$$\begin{cases} \dot{x}_1 = x_2, \\ \dot{x}_2 = M_0^{-1}(x_1)(\tau(t) + f_{s\_un} - M_{kn}), \\ y = x_1, \end{cases} \tag{2.3}$$

where

$$M_{kn} = C_0(x_1,x_2)x_2 + G_0(x_1).$$

The control objective tries to design a tracking controller based on a new asymmetric time-varying integral BLF such that system output

$$y = x_1 = q = [q_1, q_2, \cdots, q_n]^T$$

tracks the reference trajectory

$$x_d = [x_{d1}, x_{d2}, \cdots, x_{dn}]^T$$

while guaranteeing that all the signals are exponentially asymptotically stable and the position constraint boundaries are not violated, that is

$$k_{lc}(t) < x_1 < k_{uc}(t), \ \ \forall t \geq 0,$$

where

$$k_{uc}(t) = [k_{uc1}(t), k_{uc2}(t), \cdots, k_{ucn}(t)]^T$$

and

$$k_{lc}(t) = [k_{lc1}(t), k_{lc2}(t), \cdots, k_{lcn}(t)]^T$$

with $k_{uc}(t) > k_{lc}(t) > 0, \forall t \in \mathrm{R}_+, i = 1, 2, \cdots, n$.

Assumption 1. The uncertain term is bounded, differentiable, and slow or fast varying, and thus, $|f_{s\_un}| \leq F_m, F_m > 0$ and $\dot{f}_{s\_un} \approx 0$ or $\dot{f}_{s\_un} < F_{dt}$ with F_{dt} being a positive constant hold.

Assumption 2. [15] There exist the constants K_{lci} and K_{uci} such that $|k_{uci}| \leq K_{uci}$ and $|k_{lci}| \leq K_{lci}$, $\forall t \geq 0, i = 1, 2, \cdots, n$. Furthermore, suppose the upper and lower limitation boundaries of x_d are respectively X_{u1} and X_{l1}, and the conditions $X_{l1} > k_{lc}(t)$ and $X_{u1} < k_{uc}(t)$ hold. The position and velocity tracking errors are defined as

$$e_1 = [e_{11}, e_{12}, \cdots, e_{1n}]^T = x_1 - x_d$$

and

$$e_2 = [e_{21}, e_{22}, \cdots, e_{2n}]^T = x_2 - \alpha,$$

where α denotes desired velocity. Set

$$k_{l\_qi}(t) = x_{di} - k_{lci}(t)$$

and

$$k_{u\_qi}(t) = k_{uci}(t) - x_{di}$$

to be the constraint boundary of the position tracking error e_{1i}, that is $-k_{l\_qi} < e_{1i} < k_{u\_qi}$.

Remark 1. It is easy to know that the uncertain term f_{s_un} is a function containing the variables and their derivatives of the position and velocity. The position and velocity of the robot are bounded and differentiable and the motion trajectory of the robot is smooth, so we make the reasonable assumption that the uncertain term and its derivative are bounded. However, there is conservatism in Assumption 1. In future works, we will focus on issues that do not require consideration of the boundedness of the uncertain term and its derivatives. In addition, the purpose of Assumption 2 is to constrain the system's state by constraining the tracking error. Unfortunately, this method will reduce the range of feasible spaces.

In order to perform the constraint capability of the integral BLF designed in this paper, we perform the following error transformation

$$\begin{cases} \xi_{l\_qi} = \frac{e_{1i}}{k_{l\_qi}(t)}, \quad \xi_{u\_qi} = \frac{e_{1i}}{k_{u\_qi}(t)}, \\ \xi_{qi} = h_1\left(e_{1i}\right)\xi_{u\_qi} + \left(1 - h_1\left(e_{1i}\right)\right)\xi_{l\_qi}, \quad i = 1, 2, \cdots, n, \end{cases} \tag{2.4}$$

where

$$h_1\left(e_{1i}\right) = \begin{cases} 1, e_{1i} > 0, \\ 0, e_{1i} \le 0. \end{cases} \tag{2.5}$$

Lemma 1. Inequality conditions $\left|\xi_{qi}\right| < 1$ and $-k_{l\_qi}(t) < e_{1i}(t) < k_{u\_qi}(t)$ are equivalent.

Proof. Please refer to [30]. □

2.3. A time-varying asymmetric integral BLF

In view of the definition of ξ_{qi}, a new time-varying asymmetric integral BLF over the set $\left|\xi_{qi}\right| < 1$ is constructed as

$$V = \int_0^{\xi_{qi}} \frac{2\sigma}{1 - \sigma^2} d\sigma. \tag{2.6}$$

In light of the definition of V, it is clear that V is positive continuous, differentiable, and radially unbounded as $\left|\xi_{qi}\right| \to 1$ in the open set $\left|\xi_{qi}\right| < 1$.

Remark 2. In terms of the definitions of ξ_{qi} in (2.4) and $h_1\left(e_{1i}\right)$ in (2.5), when $e_{1i} > 0$, we have $h_1\left(e_{1i}\right) = 1$. Then, $\xi_{qi} = \xi_{u\_qi}$ and

$$V = \int_0^{\xi_{u\_qi}} \frac{2\sigma}{1 - \sigma^2} d\sigma = \int_0^{\xi_{qi}} \frac{2\sigma}{1 - \sigma^2} d\sigma$$

hold. $\xi_{qi} = \xi_{l\_qi}$ and

$$V = \int_0^{\xi_{l\_qi}} \frac{2\sigma}{1 - \sigma^2} d\sigma = \int_0^{\xi_{qi}} \frac{2\sigma}{1 - \sigma^2} d\sigma$$

are true when $e_{1i} \le 0$ and $h_1\left(e_{1i}\right) = 0$. Thus, whether $e_{1i} > 0$ or $e_{1i} \le 0$,

$$V = \int_0^{\xi_{qi}} \frac{2\sigma}{1 - \sigma^2} d\sigma$$

is always true.

Theorem 1. The BLF V in (2.6) over the set $\left|\xi_{qi}\right| < 1$ satisfies the inequality

$$\frac{\xi_{qi}^2}{2} \le \int_0^{\xi_{qi}} \frac{2\sigma}{1 - \sigma^2} d\sigma \le \frac{\xi_{qi}^2}{1 - \xi_{qi}^2}. \tag{2.7}$$

Proof. Step 1. In this step, we will verify the inequality on the left side of (2.7) holds. Introducing an auxiliary function

$$f\left(\xi_{qi}\right) = \int_0^{\xi_{qi}} \frac{2\sigma}{1 - \sigma^2} d\sigma - \frac{\xi_{qi}^2}{2}. \tag{2.8}$$

Taking the derivative of (2.8) with respect to ξ_{qi} yields

$$\begin{aligned}\frac{df\left(\xi_{qi}\right)}{d\xi_{qi}} &= \frac{2\xi_{qi}}{1-\xi_{qi}^2} - \xi_{qi} \\ &= \frac{\xi_{qi}\left(1+\xi_{qi}^2\right)}{1-\xi_{qi}^2}.\end{aligned} \tag{2.9}$$

According to derivative of $f\left(\xi_{qi}\right)$, we know that

$$\frac{df\left(\xi_{qi}\right)}{d\xi_{qi}} < 0$$

holds, when $\xi_{qi} < 0$ and

$$\frac{df\left(\xi_{qi}\right)}{d\xi_{qi}} > 0$$

is true when $\xi_{qi} > 0$ in the set $\left|\xi_{qi}\right| < 1$. Furthermore, $f\left(\xi_{qi}\right) = 0$ always holds as $\xi_{qi} = 0$. Thus, we can obtain the inequality

$$\frac{\xi_{qi}^2}{2} \leq \int_0^{\xi_{qi}} \frac{2\sigma}{1-\sigma^2} d\sigma$$

always holds in the set $\left|\xi_{qi}\right| < 1$.

Step 2. Similar to step 1, we introduce an auxiliary function for proving the inequality on the right side of (2.7)

$$g\left(\xi_{qi}\right) = \frac{\xi_{qi}^2}{1-\xi_{qi}^2} - \int_0^{\xi_{qi}} \frac{2\sigma}{1-\sigma^2} d\sigma. \tag{2.10}$$

Differentiating (2.10), we get

$$\begin{aligned}\frac{dg\left(\xi_{qi}\right)}{d\xi_{qi}} &= \frac{2\xi_{qi}}{\left(1-\xi_{qi}^2\right)^2} - \frac{2\xi_{qi}}{1-\xi_{qi}^2} \\ &= \frac{2\xi_{qi}^3}{\left(1-\xi_{qi}^2\right)^2}.\end{aligned} \tag{2.11}$$

In the set $\left|\xi_{qi}\right| < 1$,

$$\frac{dg\left(\xi_{qi}\right)}{d\xi_{qi}} < 0 \text{ and } \frac{dg\left(\xi_{qi}\right)}{d\xi_{qi}} > 0$$

hold under the conditions $\xi_{qi} < 0$ and $\xi_{qi} > 0$, respectively. Then, $g\left(\xi_{qi}\right) = 0$ holds as $\xi_{qi} = 0$. Thus, it can be inferred that

$$\int_0^{\xi_{qi}} \frac{2\sigma}{1-\sigma^2} d\sigma \leq \frac{\xi_{qi}^2}{1-\xi_{qi}^2}$$

is always true in the set $\left|\xi_{qi}\right| < 1$. The proof of Theorem 1 is complete. □

3. Controller design and stability analysis

In order to constrain the position of the robot, one new time-varying asymmetric integral BLF with respect to transformation error ξ_{qi} is constructed

$$V_1 = \sum_{i=1}^{n} \int_0^{\xi_{qi}} \frac{2\sigma}{1-\sigma^2} d\sigma. \tag{3.1}$$

In light of the definition of ξ_{qi} and taking the time derivative of V_1 over the set $\left|\xi_{qi}\right| < 1$, we have

$$\begin{aligned} \dot{V}_1 &= \sum_{i=1}^{n} \frac{2\xi_{qi}}{1-\xi_{qi}^2} \dot{\xi}_{qi} \\ &= \sum_{i=1}^{n} \frac{2\xi_{qi}}{1-\xi_{qi}^2} \left(h_1(e_{1i}) \dot{\xi}_{u\_qi} + (1-h_1(e_{1i})) \dot{\xi}_{l\_qi} \right) \\ &= \sum_{i=1}^{n} \frac{2h_1(e_{1i}) \xi_{u\_qi}}{1-\xi_{u\_qi}^2} \dot{\xi}_{u\_qi} + \sum_{i=1}^{n} \frac{2(1-h_1(e_{1i})) \xi_{l\_qi}}{1-\xi_{l\_qi}^2} \dot{\xi}_{l\_qi} \\ &= \sum_{i=1}^{n} \frac{2h_1(e_{1i}) \xi_{u\_qi}}{k_{u\_qi}(t)\left(1-\xi_{u\_qi}^2\right)} \left(\dot{e}_{1i} - e_{1i} \frac{\dot{k}_{u\_qi}(t)}{k_{u\_qi}(t)} \right) \\ &\quad + \sum_{i=1}^{n} \frac{2(1-h_1(e_{1i})) \xi_{l\_qi}}{k_{l\_qi}(t)\left(1-\xi_{l\_qi}^2\right)} \left(\dot{e}_{1i} - e_{1i} \frac{\dot{k}_{l\_qi}(t)}{k_{l\_qi}(t)} \right). \end{aligned} \tag{3.2}$$

Differentiating the position tracking error yields

$$\begin{aligned} \dot{e}_1 &= \dot{x}_1 - \dot{x}_d \\ &= e_2 + \alpha - \dot{x}_d, \\ \dot{e}_{1i} &= e_{2i} + \alpha_i - \dot{x}_{di}. \end{aligned} \tag{3.3}$$

According to (3.3), (3.2) can be rewritten as

$$\begin{aligned} \dot{V}_1 = &\sum_{i=1}^{n} \frac{2h_1(e_{1i}) \xi_{u\_qi}}{k_{u\_qi}(t)\left(1-\xi_{u\_qi}^2\right)} \left(e_{2i} + \alpha_i - \dot{x}_{di} - e_{1i} \frac{\dot{k}_{u\_qi}(t)}{k_{u\_qi}(t)} \right) \\ &+ \sum_{i=1}^{n} \frac{2(1-h_1(e_{1i})) \xi_{l\_qi}}{k_{l\_qi}(t)\left(1-\xi_{l\_qi}^2\right)} \left(e_{2i} + \alpha_i - \dot{x}_{di} - e_{1i} \frac{\dot{k}_{l\_qi}(t)}{k_{l\_qi}(t)} \right). \end{aligned} \tag{3.4}$$

With the help of the backstepping method, the position control law α is devised as

$$\begin{aligned} \alpha &= \dot{x}_d - (K + K_u(t)) e_1, \\ \alpha_i &= \dot{x}_{di} - (k_{1i} + k_{u1i}(t)) e_{1i}, \end{aligned} \tag{3.5}$$

where

$$K = \operatorname{diag}(k_{11}, k_{12}, \cdots, k_{1n}), \tag{3.6}$$

$$K_u(t) = \mathrm{diag}\left(k_{u11}(t), k_{u12}(t), \cdots, k_{u1n}(t)\right), \tag{3.7}$$
$$k_{u1i}(t) = \sqrt{\left(\frac{\dot{k}_{u\_qi}(t)}{k_{u\_qi}(t)}\right)^2 + \left(\frac{\dot{k}_{l\_qi}(t)}{k_{l\_qi}(t)}\right)^2 + o_i}, \quad i = 1, 2, \cdots, n$$

with o_i and k_{1i} being positive constants.

In view of (3.5), (3.4) becomes

$$\begin{aligned} \dot{V}_1 &= \sum_{i=1}^{n} H_{u1i}\left(e_{2i} - (k_{1i} + k_{u1i}(t))\, e_{1i} - e_{1i}\frac{\dot{k}_{u\_qi}(t)}{k_{u\_qi}(t)}\right) \\ &\quad + \sum_{i=1}^{n} H_{l1i}\left(e_{2i} - (k_{1i} + k_{u1i}(t))\, e_{1i} - e_{1i}\frac{\dot{k}_{l\_qi}(t)}{k_{l\_qi}(t)}\right) \\ &= \sum_{i=1}^{n}\left(\frac{2h_1(e_{1i})}{k_{u\_qi}^2(t) - e_{1i}^2} + \frac{2(1 - h_1(e_{1i}))}{k_{l\_qi}^2(t) - e_{1i}^2}\right) e_{1i}e_{2i} \\ &\quad - \sum_{i=1}^{n} H_{\zeta i}\left(k_{1i} + k_{u1i}(t) + h_1(e_{1i})\frac{\dot{k}_{u\_qi}(t)}{k_{u\_qi}(t)}\right) \\ &\quad - \sum_{i=1}^{n} H_{\zeta i}\left((1 - h_1(e_{1i}))\frac{\dot{k}_{l\_qi}(t)}{k_{l\_qi}(t)}\right), \end{aligned} \tag{3.8}$$

where

$$H_{u1i} = \frac{2h_1(e_{1i})\,\xi_{u\_qi}}{k_{u\_qi}(t)\left(1 - \xi_{u\_qi}^2\right)}, \quad H_{l1i} = \frac{2(1 - h_1(e_{1i}))\,\xi_{l\_qi}}{k_{l\_qi}(t)\left(1 - \xi_{l\_qi}^2\right)}$$

and

$$H_{\zeta i} = \frac{2\xi_{qi}^2}{1 - \xi_{qi}^2}, \quad i = 1, 2, \cdots, n.$$

According to the design of parameter $k_{u1i}(t)$, the inequality

$$k_{u1i}(t) + h_1(e_{1i})\frac{\dot{k}_{u\_qi}(t)}{k_{u\_qi}(t)} + (1 - h_1(e_{1i}))\frac{\dot{k}_{l\_qi}(t)}{k_{l\_qi}(t)} \geq 0$$

holds. Thus, (3.8) can be rewritten as

$$\begin{aligned} \dot{V}_1 &\leq \sum_{i=1}^{n}\left(\frac{2h_1(e_{1i})}{k_{u\_qi}^2(t) - e_{1i}^2} + \frac{2(1 - h_1(e_{1i}))}{k_{l\_qi}^2(t) - e_{1i}^2}\right) e_{1i}e_{2i} \\ &\quad - \sum_{i=1}^{n} \frac{2k_{1i}\xi_{qi}^2}{1 - \xi_{qi}^2}. \end{aligned} \tag{3.9}$$

Taking the derivative of e_2 yields

$$\dot{e}_2 = M_0^{-1}(x_1)\left(\tau(t) + f_{s\_un} - M_{kn}\right) - \dot{\alpha}. \tag{3.10}$$

Inspired by [31], we design a disturbance observer to estimate the uncertain terms in (3.10)

$$\begin{cases} \hat{f}_{s\_un} = \eta_f + k_f M_0 x_2, \\ \dot{\eta}_f = -k_f\eta_f - k_f\left(\tau(t) - M_{kn} + k_f M_0 x_2\right), \end{cases} \tag{3.11}$$

where

$$k_f = \text{diag}\left(k_{f11}, k_{f22}, \cdots, k_{fnn}\right)$$

denotes the observer parameter, $\eta_f \in \mathrm{R}^n$ presents the observer state variable,

$$\tilde{f}_{s\_un} = f_{s\_un} - \hat{f}_{s\_un}$$

is the estimating error with $\hat{f}_{s\_un}$ denoting the estimated value of f_{s_un}.

Subsequently, the second Lyapunov function candidate is selected as

$$V_2 = V_1 + \frac{1}{2}e_2^T M_0 e_2 + \frac{1}{2}\tilde{f}_{s\_un}^T \tilde{f}_{s\_un}. \tag{3.12}$$

Differentiating (3.12), we have

$$\begin{aligned} \dot{V}_2 = & \sum_{i=1}^{n}\left(\frac{2h_1(e_{1i})}{k_{u\_qi}^2(t) - e_{1i}^2} + \frac{2(1 - h_1(e_{1i}))}{k_{l\_qi}^2(t) - e_{1i}^2}\right)e_{1i}e_{2i} \\ & - \sum_{i=1}^{n}\frac{2k_{1i}\xi_{qi}^2}{1 - \xi_{qi}^2} + e_2^T M_0 \dot{e}_2 + \tilde{f}_{s\_un}^T \dot{\tilde{f}}_{s\_un}. \end{aligned} \tag{3.13}$$

According to Lyapunov stability theory, the controller is designed as

$$\tau(t) = M_{kn} + M_0\dot{\alpha} - \hat{f}_{s\_un} - K_2 e_2 - C_{on}, \tag{3.14}$$

where

$$C_{on} = \begin{bmatrix} \left(\frac{2h_1(e_{11})}{k_{u\_q1}^2(t)-e_{11}^2} + \frac{2(1-h_1(e_{11}))}{k_{l\_q1}^2(t)-e_{11}^2}\right)e_{11} \\ \left(\frac{2h_1(e_{12})}{k_{u\_q2}^2(t)-e_{12}^2} + \frac{2(1-h_1(e_{12}))}{k_{l\_q2}^2(t)-e_{12}^2}\right)e_{12} \\ \cdots \\ \left(\frac{2h_1(e_{1n})}{k_{u\_qn}^2(t)-e_{1n}^2} + \frac{2(1-h_1(e_{1n}))}{k_{l\_qn}^2(t)-e_{1n}^2}\right)e_{1n} \end{bmatrix} \tag{3.15}$$

with

$$K_2 = \text{diag}(k_{21}, k_{22}, \cdots, k_{2n})$$

being the positive definite parameter matrix.

Next, we perform the stable proof of the robotic closed-loop system.

Theorem 2. Consider the robotic system (2.3) subject to Assumptions 1 and 2, with controllers (3.5) and (3.14) and observer (3.11), and suppose the initial position meets $k_{lc}(0) < q(0) < k_{uc}(0)$. Then, the properties listed below are always satisfied:

(I) The position error signals e_{1i}, $i = 1, 2, \cdots, n$ maintain in the open set $(-k_{l\_qi}(t), k_{u\_qi}(t))$.

(II) The position states q_i, $i = 1, 2, \cdots, n$ never break their constraint boundaries, i.e., $k_{lci}(t) < q_i < k_{uci}(t), \forall t \geq 0$.

(III) All the system error signals are exponentially asymptotically stable.

Proof. 1) When $\dot{f}_{s\_un} \approx 0$, substituting (3.10), (3.14), and $\dot{\hat{f}}_{s\_un}$ into (3.13), we have

$$\begin{aligned}\dot{V}_2 =& \sum_{i=1}^{n}\left(\frac{2h_1(e_{1i})}{k_{u\_qi}^2(t)-e_{1i}^2}+\frac{2(1-h_1(e_{1i}))}{k_{l\_qi}^2(t)-e_{1i}^2}\right)e_{1i}e_{2i}\\&-\sum_{i=1}^{n}\frac{2k_{1i}\xi_{qi}^2}{1-\xi_{qi}^2}+\tilde{f}_{s\_un}^T\dot{\tilde{f}}_{s\_un}\\&+e_2^T\left(\tau(t)+f_{s\_un}-M_{kn}-M_0\dot{\alpha}\right)\\=&\sum_{i=1}^{n}\left(\frac{2h_1(e_{1i})}{k_{u\_qi}^2(t)-e_{1i}^2}+\frac{2(1-h_1(e_{1i}))}{k_{l\_qi}^2(t)-e_{1i}^2}\right)e_{1i}e_{2i}\\&-\sum_{i=1}^{n}\frac{2k_{1i}\xi_{qi}^2}{1-\xi_{qi}^2}-e_2^TK_2e_2-e_2^TC_{on}\\&+e_2^T\tilde{f}_{s\_un}-\tilde{f}_{s\_un}^Tk_f\tilde{f}_{s\_un}\\\le&-\sum_{i=1}^{n}\frac{2k_{1i}\xi_{qi}^2}{1-\xi_{qi}^2}-e_2^T\left(K_2-\frac{1}{2}I_{n\times n}\right)e_2\\&-\tilde{f}_{s\_un}^T\left(k_f-\frac{1}{2}I_{n\times n}\right)\tilde{f}_{s\_un},\end{aligned}\tag{3.16}$$

where $I_{n\times n}\in\mathrm{R}^{n\times n}$ is an identity matrix.

The parameters K_2 and k_f are set to meet the conditions

$$\begin{aligned}&\lambda_{\min}\left(K_2-\tfrac{1}{2}I_{n\times n}\right)>0,\\&\lambda_{\min}\left(k_f-\tfrac{1}{2}I_{n\times n}\right)>0.\end{aligned}\tag{3.17}$$

By means of Theorem 1, (3.16) becomes

$$\begin{aligned}\dot{V}_2\le&-\sum_{i=1}^{n}2k_{1i}\int_0^{\xi_{qi}}\frac{2\sigma}{1-\sigma^2}d\sigma-e_2^T\left(K_2-\frac{1}{2}I_{n\times n}\right)e_2\\&-\tilde{f}_{s\_un}^T\left(k_f-\frac{1}{2}I_{n\times n}\right)\tilde{f}_{s\_un}\\\le&-\rho V_2\le 0,\end{aligned}\tag{3.18}$$

where

$$\rho=\min\left(2k_{1i},\frac{2\lambda_{\min}\left(K_2-\frac{1}{2}I_{n\times n}\right)}{\lambda_{\max}(M_0)},2\lambda_{\min}\left(k_f-\frac{1}{2}I_{n\times n}\right)\right).\tag{3.19}$$

Seeking the solution of the differential Eq (3.18), we get

$$0\le V_2\le V_2(0)\,e^{-\rho t}.\tag{3.20}$$

In terms of (3.1) and (3.12), we have

$$V_2=\sum_{i=1}^{n}\int_0^{\xi_{qi}}\frac{2\sigma}{1-\sigma^2}d\sigma+\frac{1}{2}e_2^TM_0e_2+\frac{1}{2}\tilde{f}_{s\_un}^T\tilde{f}_{s\_un}.\tag{3.21}$$

According to (3.20), we can obtain

$$\int_0^{\xi_{qi}} \frac{2\sigma}{1-\sigma^2} d\sigma \le V_2(0) e^{-\rho t} \le V_2(0). \tag{3.22}$$

Solving the inequality (3.22) yields

$$\xi_{qi}^2 \le \left(1 - e^{-V_2(0)}\right) \text{ and } \left|\xi_{qi}\right| \le \sqrt{\left(1 - e^{-V_2(0)}\right)}.$$

When $e_{1i} > 0$, we have

$$\frac{e_{1i}}{k_{u\_qi}(t)} \le \sqrt{\left(1 - e^{-V_2(0)}\right)},$$

and then

$$e_{1i} \le k_{u\_qi}(t) \sqrt{\left(1 - e^{-V_2(0)}\right)}$$

is true. When $e_{1i} \le 0$,

$$-\frac{e_{1i}}{k_{l\_qi}(t)} \le \sqrt{\left(1 - e^{-V_2(0)}\right)}$$

holds, and then

$$e_{1i} \ge -k_{l\_qi}(t) \sqrt{\left(1 - e^{-V_2(0)}\right)}$$

holds. We arrive at the conclusion that $-k_{l\_qi}(t) < e_{1i}(t) < k_{u\_qi}(t)$ always holds. The proof of property (I) is completed.

Moreover, in light of $e_{1i} = x_{1i} - x_{di}$, we can get

$$x_{di} - k_{l\_qi}(t) < x_{1i} < k_{u\_qi}(t) + x_{di}.$$

Further, according to Assumption 2, we have

$$k_{lci}(t) < x_{1i} = q_i < k_{uci}(t), \forall t \ge 0.$$

The position states q_i, $i = 1, 2, \cdots, n$ never exceed the constraint boundaries. The proof of property (II) is finished.

Finally, considering (3.20), (3.21) and Theorem 1, the following inequalities hold:

$$\begin{aligned} \left|\xi_{qi}\right| &\le \sqrt{2V_2(0) e^{-\rho t}}, \\ \left|e_2\right| &\le \sqrt{\tfrac{2V_2(0)e^{-\rho t}}{\lambda_{\min}(M_0)}}, \\ \left|\tilde{f}_{s\_un}\right| &\le \sqrt{2V_2(0) e^{-\rho t}}. \end{aligned} \tag{3.23}$$

In view of (3.23) and the definition of ξ_{qi}, we can infer that

$$e_{1i} \le k_{u\_qi}(t) \sqrt{2V_2(0) e^{-\rho t}}$$

holds, when $e_{1i} > 0$ and

$$e_{1i} \ge -k_{l\_qi}(t) \sqrt{2V_2(0) e^{-\rho t}}$$

is true when $e_{1i} \leq 0$. Therefore, all the closed-loop signals are exponentially asymptotically stable.

2) When $\dot{f}_{s\_un} < F_{dt}$, substituting (3.10), (3.14) and $\dot{\hat{f}}_{s\_un}$ into (3.13), we have

$$\begin{aligned}
\dot{V}_2 =& \sum_{i=1}^{n}\left(\frac{2h_1(e_{1i})}{k_{u_qi}^2(t)-e_{1i}^2}+\frac{2(1-h_1(e_{1i}))}{k_{l_qi}^2(t)-e_{1i}^2}\right)e_{1i}e_{2i}\\
&-\sum_{i=1}^{n}\frac{2k_{1i}\xi_{qi}^2}{1-\xi_{qi}^2}+\tilde{f}_{s_un}^T\dot{\hat{f}}_{s_un}+e_2^T\left(\tau(t)+f_{s_un}-M_{kn}-M_0\dot{\alpha}\right)\\
=& \sum_{i=1}^{n}\left(\frac{2h_1(e_{1i})}{k_{u_qi}^2(t)-e_{1i}^2}+\frac{2(1-h_1(e_{1i}))}{k_{l_qi}^2(t)-e_{1i}^2}\right)e_{1i}e_{2i}\\
&-\sum_{i=1}^{n}\frac{2k_{1i}\xi_{qi}^2}{1-\xi_{qi}^2}-e_2^TK_2e_2-e_2^TC_{on}+e_2^T\tilde{f}_{s_un}+\tilde{f}_{s_un}^T\left(\dot{f}_{s_un}-\dot{\hat{f}}_{s_un}\right)\\
\leq& -\sum_{i=1}^{n}\frac{2k_{1i}\xi_{qi}^2}{1-\xi_{qi}^2}-e_2^T\left(K_2-\frac{1}{2}I_{n\times n}\right)e_2-\tilde{f}_{s_un}^T\left(k_f-I_{n\times n}\right)\tilde{f}_{s_un}+\frac{1}{2}\left|\dot{f}_{s_un}\right|^2,
\end{aligned} \tag{3.24}$$

where $I_{n\times n} \in \mathrm{R}^{n\times n}$ is an identity matrix.

The parameters K_2 and k_f are set to meet the following conditions

$$\begin{aligned}
&\lambda_{\min}\left(K_2-\tfrac{1}{2}I_{n\times n}\right)>0,\\
&\lambda_{\min}\left(k_f-I_{n\times n}\right)>0.
\end{aligned} \tag{3.25}$$

By means of Theorem 1, (3.16) becomes

$$\begin{aligned}
\dot{V}_2 \leq& -\sum_{i=1}^{n}2k_{1i}\int_0^{\xi_{qi}}\frac{2\sigma}{1-\sigma^2}d\sigma-e_2^T\left(K_2-\frac{1}{2}I_{n\times n}\right)e_2-\tilde{f}_{s_un}^T\left(k_f-I_{n\times n}\right)\tilde{f}_{s_un}+\frac{1}{2}\left|\dot{f}_{s_un}\right|^2\\
\leq& -\rho_1V_2+\rho_2,
\end{aligned} \tag{3.26}$$

where

$$\rho_1=\min\left(2k_{1i},\frac{2\lambda_{\min}\left(K_2-\frac{1}{2}I_{n\times n}\right)}{\lambda_{\max}(M_0)},2\lambda_{\min}\left(k_f-I_{n\times n}\right)\right),\quad \rho_2=\frac{1}{2}\left|\dot{f}_{s\_un}\right|^2. \tag{3.27}$$

Seeking the solution of the differential Eq (3.18), we get

$$0 \leq V_2 \leq V_2(0)e^{-\rho_1 t}+\frac{\rho_2}{\rho_1} \triangleq \bar{V}_2. \tag{3.28}$$

In terms of (3.1) and (3.12), we have

$$V_2=\sum_{i=1}^{n}\int_0^{\xi_{qi}}\frac{2\sigma}{1-\sigma^2}d\sigma+\frac{1}{2}e_2^TM_0e_2+\frac{1}{2}\tilde{f}_{s\_un}^T\tilde{f}_{s\_un}. \tag{3.29}$$

According to (3.28), we can obtain

$$\int_0^{\xi_{qi}}\frac{2\sigma}{1-\sigma^2}d\sigma \leq \bar{V}_2. \tag{3.30}$$

Solving the inequality (3.30) yields

$$\xi_{qi}^2 \leq \left(1 - e^{-\bar{V}_2}\right) \text{ and } \left|\xi_{qi}\right| \leq \sqrt{\left(1 - e^{-\bar{V}_2}\right)}.$$

When $e_{1i} > 0$, we have

$$\frac{e_{1i}}{k_{u\_qi}(t)} \leq \sqrt{\left(1 - e^{-\bar{V}_2}\right)},$$

and then

$$e_{1i} \leq k_{u\_qi}(t) \sqrt{\left(1 - e^{-\bar{V}_2}\right)}$$

is true. When $e_{1i} \leq 0$,

$$-\frac{e_{1i}}{k_{l\_qi}(t)} \leq \sqrt{\left(1 - e^{-\bar{V}_2}\right)}$$

holds, and then

$$e_{1i} \geq -k_{l\_qi}(t) \sqrt{\left(1 - e^{-\bar{V}_2}\right)}$$

holds. We arrive at the conclusion that $-k_{l\_qi}(t) < e_{1i}(t) < k_{u\_qi}(t)$ always holds. The proof of property (I) is completed.

Moreover, in light of $e_{1i} = x_{1i} - x_{di}$, we can get $x_{di} - k_{l\_qi}(t) < x_{1i} < k_{u\_qi}(t) + x_{di}$. Further, according to Assumption 2, we have $k_{lci}(t) < x_{1i} = q_i < k_{uci}(t), \forall t \geq 0$. The position states q_i, $i = 1, 2, \cdots, n$ never exceed the constraint boundaries. The proof of property (II) is finished. □

Finally, considering (3.28), (3.29) and Theorem 1, the following inequalities hold:

$$\begin{aligned} \left|\xi_{qi}\right| &\leq \sqrt{2\left(V_2(0) e^{-\rho_1 t} + \frac{\rho_2}{\rho_1}\right)}, \\ \left|e_2\right| &\leq \sqrt{\frac{2\left(V_2(0) e^{-\rho_1 t} + \frac{\rho_2}{\rho_1}\right)}{\lambda_{\min}(M_0)}}, \\ \left|\tilde{f}_{s\_un}\right| &\leq \sqrt{2\left(V_2(0) e^{-\rho_1 t} + \frac{\rho_2}{\rho_1}\right)}. \end{aligned} \tag{3.31}$$

In view of (3.31) and the definition of ξ_{qi}, we can infer that

$$e_{1i} \leq k_{u\_qi}(t) \sqrt{2\left(V_2(0) e^{-\rho_1 t} + \frac{\rho_2}{\rho_1}\right)}$$

holds, when $e_{1i} > 0$ and

$$e_{1i} \geq -k_{l\_qi}(t) \sqrt{2\left(V_2(0) e^{-\rho_1 t} + \frac{\rho_2}{\rho_1}\right)}$$

is true when $e_{1i} \leq 0$. Therefore, all the closed-loop signals are exponentially asymptotically stable. This completes the proof of Theorem 2.

Remark 3. According to the proof of property (III) in 1) of Theorem 2, it can be seen that

$$-k_{l\_qi}(t) \sqrt{2V_2(0) e^{-\rho t}} \leq e_{1i} \leq k_{u\_qi}(t) \sqrt{2V_2(0) e^{-\rho t}}$$

always holds. Different from the existing papers [32, 33], the new time-varying asymmetric integral BLF proposed for the first time can guarantee the boundedness and exponential asymptotic stability of the constrained error simultaneously.

Remark 4. In this study, the tracking controller for the robot is designed based on the proposed BLF and disturbance observer. The disturbance observer designed in this paper is inspired by [31], which can ensure the exponential convergence of the estimation error. There are many types of observers in control systems, such as one observer in [34] is applied to the tracking control of switched stochastic uncertain nonlinear systems. With the help of the backstepping approach, the control strategy based on the observer and neural fault-tolerant control is proposed to guarantee that the signal of the system is stable in probability. There are two main methods for solving state constraints, one is the barrier function method, and the other is called the nonlinear mapping method. The barrier function is the most commonly used constraint method. The barrier function proposed in this article can effectively solve the constraint issues of the robot position, but it is not intended for systems without constraint requirements. In [35], a uniform barrier function is used to transform the original constrained nonlinear system into an equivalent "unconstrained" one, enabling it to handle more general systems.

Remark 5. Compared to the existing literature, there are two types of integral BLFs: one is symmetric time-invariant, and the other is symmetric time-varying. The first type is used to address systems with symmetric time-invariant constraint requirements [18, 20–23], and the second type works for systems with symmetric time-varying constraint requirements [19]. The proposed integral BLF can work for systems subject to asymmetric time-varying requirements. We know that the two existing types of integral BLFs are the property of a particular situation of the asymmetric time-variant constraint, therefore, it has more general ability to deal with a engineering practical problem.

4. Simulation example

In order to verify the effectiveness of the presented scheme based on the new integral barrier function, the two-degree robot is utilized to complete the simulation example. Please refer to papers [15, 29] for the main parameters and relevant matrices for the two-degree robot. Moreover, the tracking controller for the two-degree robot based on logarithmic BLF in [15, 30] is used to complete the comparative simulation.

According to Assumption 2, the joint angles' initial values with their desired reference values respectively are set as

$$\begin{cases} q_1(0) = 0.8, & q_2(0) = 0.8, \\ \dot{q}_1(0) = 0, & \dot{q}_2(0) = 0, \end{cases} \tag{4.1}$$

and

$$x_d = [0.14\sin(t) + 0.5, 0.14\cos(t) + 0.5]^T. \tag{4.2}$$

The unknown terms of the system are described as

$$f_{s\_un} = M_0[0.3\sin(t), 0.3\cos(t)]^T + C_0[0.3\cos(0.5t), 0.3\sin(0.5t)]^T. \tag{4.3}$$

The position constraint boundaries are set as

$$k_{lc} = [k_{lc1}, k_{lc2}]^T = [0.2 + 0.14\cos(t), 0.2 + 0.14\sin(t)]^T$$

and

$$k_{uc} = [k_{uc1}, k_{uc2}]^T = [0.9 + 0.14\cos(t), 0.9 + 0.14\sin(t)]^T.$$

According to Assumption 2, the position error constraint boundaries are set as

$$k_{l\_q} = \left[k_{l\_q1}, k_{l\_q2}\right]^T = [0.3 + 0.14\sin(t) - 0.14\cos(t), 0.3 + 0.14\cos(t) - 0.14\ \sin(t)]^T$$

and

$$k_{u\_q} = \left[k_{u\_q1}, k_{u\_q2}\right]^T = [0.4 + 0.14\cos(t) - \ 0.14\sin(t), 0.4 + 0.14\sin(t) - 0.14\cos(t)]^T.$$

The control parameters are set as

$$k_{11} = k_{12} = 2, \quad o_1 = o_2 = 0.1 \ \text{ and } \ K_2 = \mathrm{diag}(20, 20).$$

The observer parameter is set as $k_f = \mathrm{diag}(20, 20)$.

In the comparative simulations, the proposed BLF denotes the method used in this paper, and log-BLF represents the control stratagy utilized in [15, 30]. In Figures 1 and 2, the red and black curves denote the trajectories under the proposed BLF and log-BLF, respectively. Moreover, the magenta and green curves denote the upper and lower boundaries of the position and tracking error, respectively. It can be found from the simulation effects depicted in Figures 1–6 that both the time-varying asymmetric integral BLF and log-BLF-based control scheme is successful in guaranteeing the robotic system tracks the reference trajectory smoothly. However, the control accuracy under the control strategy in this article is slightly higher than that under the comparison method. The joints' tracking effects as well as their tracking errors are described in Figures 1 and 2, which demonstrate that both methods one based on integral BLF and the other on logarithmic BLF, have a satisfactory control effect. Nevertheless, from the partially enlarged image of Figures 1 and 2, it can be seen that the convergence speed of the proposed method is faster than that of the comparative method and in the initial stage of control, the control effects under these two methods are relatively significantly different.

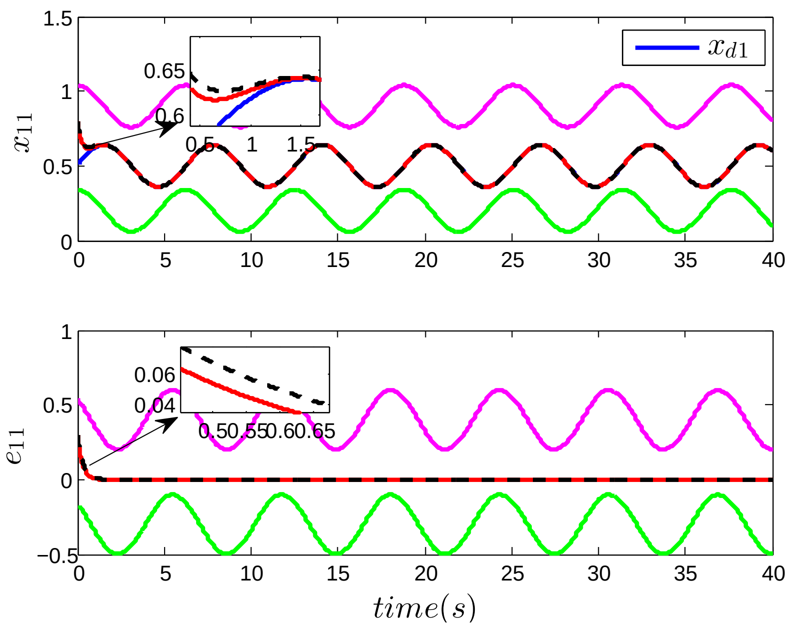

Figure 1. The trajectories of x_{11} and e_{11}.

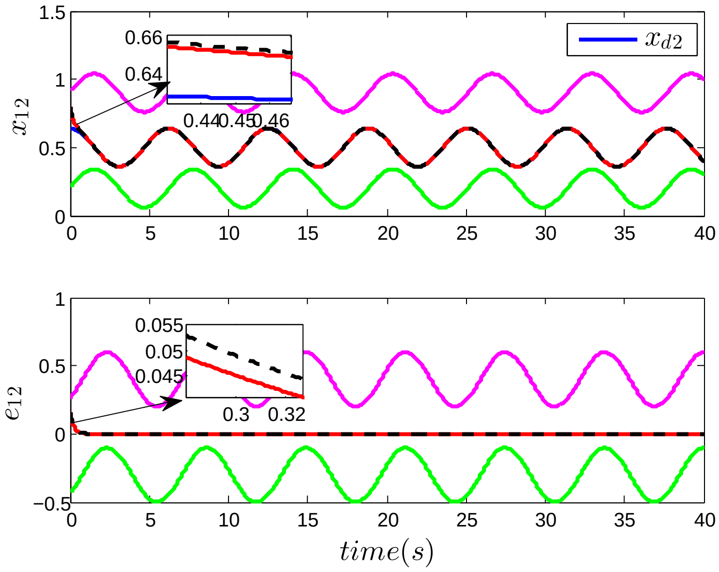

Figure 2. The trajectories of x_{12} and e_{12}.

Although the control effects under the two control methods are similar, the integral barrier function proposed in this article is more concise in structure compared to the log-type barrier function. Further, the controller based on the proposed BLF is easier to obtain. Furthermore, these two barrier functions share the same idea in handling asymmetric constraint requirements, as they both establish a piecewise function. In addition, Figures 1 and 2 illustrate that the time-variant asymmetric constraint boundaries k_{lc}, k_{uc} of the positions, as well as boundaries k_{l_q}, k_{u_q} of position tracking errors, are not broken under two controllers. This proves that both control methods can effectively achieve asymmetric constraint control. The velocity tracking error e_2 is depicted in Figure 3.

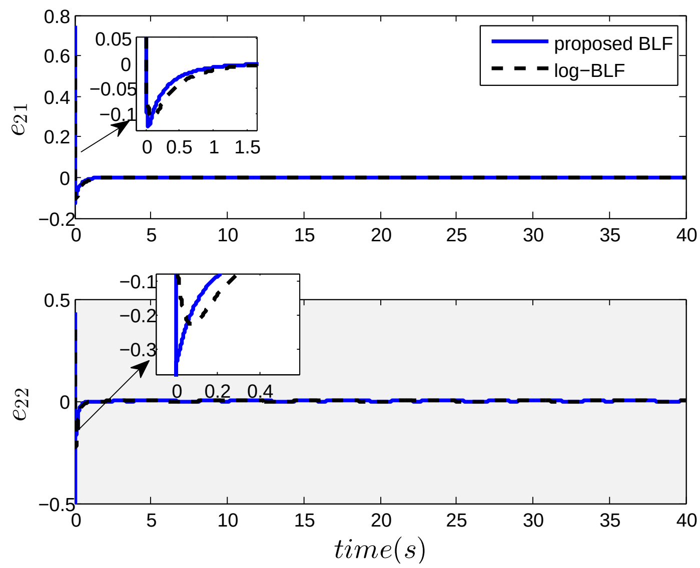

Figure 3. The trajectories of e_{21} and e_{22}.

We can see that the overshoot of the speed error of the proposed control strategy is greater than that of the comparison method in the initial stage to ensure that the constrained position error is better constrained within the set area. The robotic system's uncertain terms with their estimating values are depicted in Figure 4. Overall, error signals e_1 and e_2, and $\tilde{f}_{s\_un}$ of the closed-loop system can quickly trend to a very small neighborhood near zero. The robotic system inputs of the two schemes are shown in Figure 5. The comprehensive effect that two joints track the desired circular trajectory is shown in Figure 6. It can be intuitively felt from Figure 6 that, under the proposed control method, the actual trajectory converges to the prescribed trajectory more quickly and the tracking error is smaller than that under the comparison method.

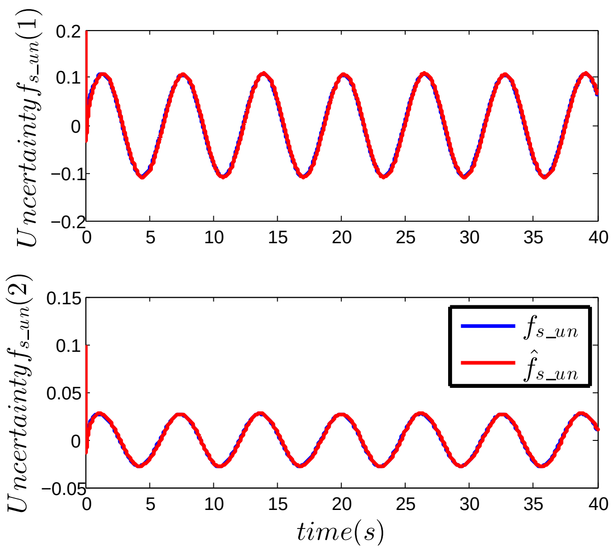

Figure 4. The uncertain terms.

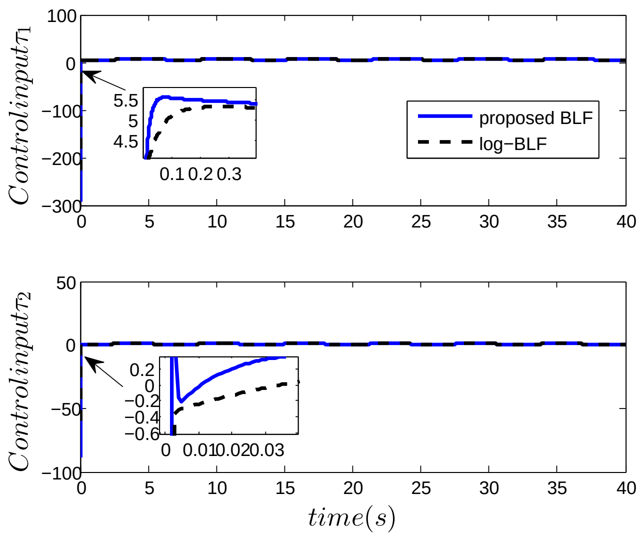

Figure 5. The control inputs.

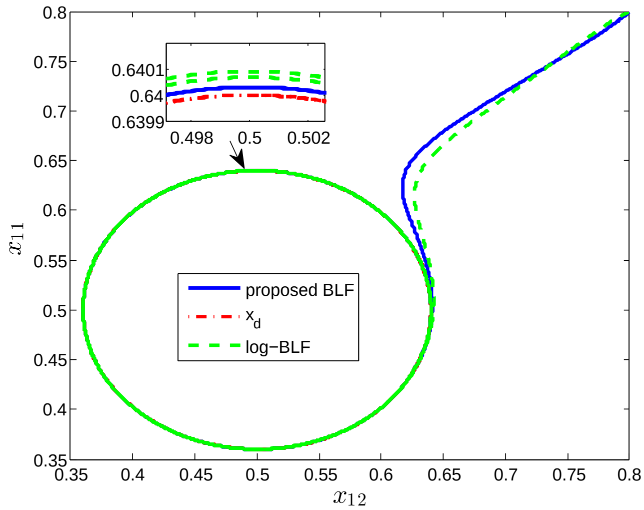

Figure 6. The phase portrait of x_{11} and x_{12}.

5. Conclusions

In this study, one new time-variant dissymmetric integral BLF-based tracking control scheme of a robot with position constraints is proposed. By using the advantages of the proposed integral BLF addressing the constraint problems, the controller is developed with the aid of backstepping control technology. After that, the exponential asymptotic stability of the robotic system's errors can be demonstrated by utilizing Lyapunov analysis, and the given Theorem 1. Finally, a simulation example shows that time-variant dissymmetric restriction boundaries of the positions and of their tracking errors are not violated, and the good tracking performance is obtained. In future works, the proposed BLF will be combined with existing advanced adaptive control technologies to improve the robustness of control strategies. In addition, the actuator saturation issue will be resolved by incorporating saturation functions into the presented BLF.

Use of AI tools declaration

We declare we have not used Artificial Intelligence (AI) tools in the creation of this article.

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Conflict of interest

The authors declare they have no conflicts of interest in this study.

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