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Research article

Asymptotic stability of impulsive stochastic switched system with double state-dependent delays and application to neural networks and neural network-based lecture skills assessment of normal students

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Abstract: This article investigates the stability problem of impulsive stochastic switched systems with double state-dependent delays. In the designed system, unstable and stable impulses are taken into consideration, respectively, and they do not need to function simultaneously with switching behavior. Additionally, two new ideas, i.e., mode-dependent switching density and mode-dependent impulsive density, are developed. Based on the Lyapunov function method and comparison principle, the asymptotic stability criteria for an impulsive stochastic switched system with state-dependent delays are given. Moreover, the application of theoretical results to neural networks and the neural network-based lecture skills assessment of normal students is analyzed. Finally, two numerical examples are provided to illustrate the effectiveness and reliability of the theoretical criteria.

Keywords: switched system; hybrid model; state-dependent delay; mode-dependent impulsive density; mode-dependent switching density
Mathematics Subject Classification: 92B20, 93C30

1. Introduction

Switched systems and impulsive systems, as two forms of hybrid systems, have attracted increased attention in recent years due to their significance in theoretical research and practical applications. A switched system, in general, consists of numerous subsystems and a switching rule. Additionally, the preset performance of switched system is realized by switching between subsystems. Impulsive systems are hybrid dynamic systems that combine instantaneous state jumps and continuous development processes. Switched systems and impulsive systems are often encountered in many practical engineering applications, such as the switching of operating point of aircrafts [1], the switching of power system networks [2] and circuit modeling [3].

In practical systems such as circuit systems, switched mechanisms and impulse jumps often coexist [4]. Thus, switching and impulses should be considered concurrently, resulting in an impulsive switched system. There are multiple notable results concerning the stability and performance of impulsive switched systems; see [5–7]. In addition, the actual system will be influenced by external interference signals or receive external control signals [8]. These disturbance signals or control inputs will directly or indirectly have a certain impact on the dynamic performance of the system. Furthermore, delay [9, 10], as a common and unavoidable disturbance element affecting system performance, cannot be overlooked. For example, in the process of signal encoding and transmission in network control systems, there is transmission delay. It is necessary to explore the impact of delay on system performance. Therefore, on the basis of switching systems and impulsive systems, stochastic systems and time-delay systems have been generated for the purpose of modeling more practical dynamic systems. It is of great significance to keep striving to improve and develop the qualitative theory of impulsive stochastic switched systems (ISSSs) with time delay.

In recent years, it has been proposed, in many models that delay depends on the state or its explicit or unknown function. These equations are called state-dependent delay differential equations. State-dependent delay differential equations were first used to solve the classic two-body problem in electrodynamics; see [11]. Due to the limited speed of propagation of electrical effects, differential equations involve time delays that are dependent on unknown trajectories. Prior to this, electrodynamics was an unknown mathematical area. In addition, state-dependent delay has been widely employed in practical applications, including drilling engineering [12], megacaryocyte modeling [13], age structured models [14], and virus infection model [15]. Moreover, many intriguing and significant results on systems with state-dependent delay have recently been published; see [16–19]. In [16], Zhang and Huang investigated the stability of stochastic delayed nonlinear systems based on impulses with state-dependent delay by using average impulsive interval (AII), the comparison principle, and differential inequalities. In [17], He et al. explored the finite-time stability of nonlinear systems with state-dependent delay through utilizing the Razumikhin technique. Using Lyapunov stability theory, Li and Yang established weak local exponential stability criteria for nonlinear systems with state-dependent delay in [18]. In [19], Zhang et al. conducted analysis of stochastic networks with state-dependent delay by combining the Lyapunov method and stochastic analysis techniques. Note that these results only focused on state-dependent delay involving continuous subsystems or discrete difference systems. Actually, state-dependent delay always occurs in both continuous subsystems and the impulsive functions of hybrid systems, i.e., double state-dependent delays (DSDDs) occur [20]. However, when time delay coexists in a continuous system and the impulsive function while being state-dependent, it is highly challenging to explore the stability of an ISSS with DSDDs by using a unified paradigm. In fact, because of the existence of state-dependent delay in impulse, it is unfathomable to know how much information is required in history a priori, and it is also difficult to establish the historical state under impulsive conditions.

Use of the AII is one of effective strategies for describing the extent of impulsive occurrence. The average dwell time (ADT) is currently often employed to determine the amount of switches. However, the AII pays little attention to distinctions among impulsive functions. The ADT ignores distinctions among subsystems. Taking this into account, Zhao et al. [21] suggested an idea called mode-dependent AII (MDAII), which allows each impulsive function to have its own AII. Xie et al. [22] introduced a dwell-time idea known as the mode-dependent average dwell time (MDADT), which allows every subsystem to possess its own ADT. The linear connection, nevertheless, limits the numbers of impulses and switches. More recently, in [23], impulse density, which breaks away from linear constraints, is introduced to more properly portray the activated impulse amount. The impulse density establishes a linear/nonlinear relation among time intervals and the amount of impulse occurrences, eliminating the problem of an inadequate/excessive amount of impulses characterized by the AII. Accordingly, control synthesis and stability analysis of impulsive stochastic neural networks with impulse density are also reported; see [24].

However, in switching sequences and impulsive sequences, the characteristic of any two switches or impulses with the same density function with the same time interval may still not be expected, as it independent of the system mode. In addition, it has been plainly demonstrated in literature that impulse density is generated by two mode-independent parameters: impulsive intensity at impulsive instants and the rate coefficient for the Lyapunov function. It is unambiguous that setting the two identical parameters for all subsystems in a mode-independent way will result in a certain conservatism. Nevertheless, so far, the problem of ISSSs with time-varying characteristics has not received sufficient attention, and their dynamic analysis is still an open topic, especially in the case of time-varying impulsive intensity. The key difficulties can be summarized as follows: How can one construct appropriate mode-dependent switching mechanisms and mode-dependent impulsive mechanisms under a time-varying impulsive intensity, time-varying switching frequency and time-varying impulsive number and switching number so as to achieve stability in an ISSS? Furthermore, sliding mode control [25] is also a typical control approach. It has some parallels but also distinctions as compared to impulse density and switching density. Sliding mode control is a nonlinear control approach that uses a sliding mode surface to guarantee system stability [26]. The controller modifies the control input to shift the system state to the sliding mode surface and keep it there, accomplishing the control goal [27]. The sliding mode control method has the advantages of strong robustness and certain resistance to parameter changes and disturbances. Impulse density control is a digital control approach that accomplishes control goals by varying the density of output impulses. Impulse density control technologies are widely employed in disciplines such as switching system control and motor control. The main idea behind the impulse density control method is to transform the control signal into a sequence of impulses and then control the output by adjusting the frequency of the impulses. The impulse density control method has the advantages of a fast response speed, high accuracy, and excellent reliability. Although sliding mode control and impulse density control have some similarities, their application fields and implementation methods are different. The sliding mode control method is suitable for the control of nonlinear systems and strong interference environments, while the impulse density control method is suitable for digital control systems and switching system fields.

Stochastic switched neural networks, which have been effectively used to model circuits, constitute one of the most vital applications of stochastic switched differential equations. On the other hand, in some physical models, switching events and impulsive effects constantly coexist. Thus, it is critical to evaluate the impact of these impulses on stochastic switched networks. The impact of impulses on the stability of stochastic neural network has been extensively studied. Jiao et al. proposed certain global asymptotic stability criteria and sufficient conditions of stability for switching neural networks with impulses by applying dell-time strategy and the discrete Lyapunov function approach in [28]. However, the stability of stochastic switched neural networks in the impulsive sense, and as based on mode-dependent impulsive density and mode-dependent switching density has not been explored so far. Additionally, distributed delay, time-varying delay, constant delay and so on, are currently the main areas addressed when discussing neural networks. In actuality, it is crucial to investigate impulsive stochastic switched neural networks (ISSNNs) with DSDDs.

Inspired by the above discussion, this article serves to analyze the stability of ISSSs with DSDDs, where switches are asynchronous with impulses. The main contributions of this article are as follows: (1) A more generic ISSS with a mode-dependent switching signal and mode-dependent impulsive effects is developed, where the synchronization of switches and impulses is not required. Both the destabilizing and stabilizing instances of impulses with state-dependent delay are completely explored. (2) Two new concepts, mode-dependent switching density and mode-dependent impulsive density, are presented to define switching sequences and impulsive sequences, where the switches and impulses of every mode have their own switching density and impulse density. This helps to remove the linear restrictions on the ADT or MDADT as well as the limitations on the AII or MDAII, making the conclusion more adaptable and practical for the stability of ISSSs. (3) DSDDs, which means that state-dependent delay occurs in both subsystem and impulsive function, is considered in the system. Based on the mode-dependent switching density and mode-dependent impulsive density approaches, some criteria for the stability of ISSSs with DSDDs are derived by employing Lyapunov functions and the comparison; the relationships among the system mode, mode-dependent switching density, mode-dependent impulsive density, state-dependant delay and impulse strength are established, and corollaries are given for particular conditions. (4) Applying theoretical conclusions for ISSNNs, we have derived some results on the mean square asymptotic stability of the considered systems, where we explored specific cases of unstable and stable impulses, respectively.

Notations: Let $\mathbb{R}$ stand for the set of real numbers. $\mathbb{R}^+ = (0, +\infty)$. $\mathbb{N}$ denotes a collection of natural integers that includes 0. $\mathbb{N}^+ = \mathbb{N} \setminus 0$. $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq t_0}, P)$ represents a complete probability space with a natural filtration $\{\mathcal{F}_t\}_{t\geq t_0}$. C refers to a class of nonnegative functions $V_{r(t)}(t, x)$ taking values on $[t_0 - \bar{\tau}, +\infty) \times \mathbb{R}^n$ which are continuously once differentiable and twice differentiable with respect to t and x respectively. Let $\omega(t)$ be an m-dimensional $\mathcal{F}_t$-adapted Brownian motion. $|\cdot|$ indicates a Euclidean norm. The superscript T indicates the transposition of a matrix or vector. $\mathcal{PC}([-\bar{\tau}, 0]; \mathbb{R}^n)$ is the set which contains piecewise continuous functions from $[-\bar{\tau}, 0]$ to $\mathbb{R}^n$ and ϕ is defined on $[-\bar{\tau}, 0]$ with the norm $\|\phi\| = \sup_{-\bar{\tau}\leq\theta\leq 0} |\phi(\theta)|$. For $t \geq t_0$, $\mathcal{PL}^p_{\mathcal{F}_t}$ is the family of all $\mathcal{F}_t$-measurable $\mathcal{PC}([-\bar{\tau}, 0]; \mathbb{R}^n)$-valued processes $\phi = \{\phi(\theta) : -\bar{\tau} \leq \phi \leq 0\}$ such that $\|\phi\|_{L^p} = \sup_{-\bar{\tau}\leq\theta\leq 0} \mathbb{E}|\phi(\theta)|^p < +\infty$, where the operator $\mathbb{E}$ aims to calculate the mathematical expectation. $\Upsilon = \{\alpha_1, \alpha_2, \cdots, \alpha_L\}$, where $L \in \mathbb{N}^+$, $\alpha_q > 1$, $q \in \{1, 2, \cdots, L\}$. $\Theta = \{\mathcal{U}_1, \mathcal{U}_2, \cdots, \mathcal{U}_S\}$, where $S \in \mathbb{N}^+$, $\mathcal{U}_p \in \mathbb{R}^+$, $p \in \{1, 2, \cdots, S\}$. $\mathcal{E} = \{\vartheta(t) : \int_{t_0}^{t} \vartheta(u)du \leq -k_1(t - t_0) + k_2\}$, where $k_1, k_2 \in \mathbb{R}^+$. Moreover, let $\mathcal{K}$ indicate a family of continuous strictly increasing functions $\kappa : \mathbb{R}^+ \to \mathbb{R}^+$ with $\kappa(0) = 0$, and let $\mathcal{K}_\infty$ be the set of unbounded functions in $\mathcal{K}$. $C\mathcal{K}_\infty$ and $\mathcal{VK}_\infty$ are formed by all concave functions and all convex functions of $\mathcal{K}_\infty$ respectively. $\mathcal{K}_0 = \{\lambda$: continuous function from $\mathbb{R}^+$ to $\mathbb{R}, \lambda(t_0) = 0\}$. D^+ represents Dini upper right derivative.

2. Model description and preliminaries

The following ISSS with DSDDs is discussed in this article

$$\begin{aligned} dx(t) =& f_{r(t)}(t, x(t), x(t - \tau(t, x(t))))dt + g_{r(t)}(t, x(t), x(t - \tau(t, x(t))))d\omega(t), \quad t \notin T^{im}, \\ x(t) =& I_{h(t)}(t^-, x(t^-), x(t^- - \tau(t^-, x(t^-)))), \quad t \in T^{im}, \end{aligned} \tag{2.1}$$

where $x(t) \in \mathbb{R}^n$ and $\tau(t, x(t))$ is the state-dependent delay. The initial value $x_{t_0} \in \mathcal{PL}^p_{\mathcal{F}_{t_0}}$, $f_{r(t)}$, $I_{h(t)}$ are functions from $[t_0, +\infty) \times \mathcal{PL}^p_{\mathcal{F}_t} \times \mathcal{PL}^p_{\mathcal{F}_t}$ to $\mathbb{R}^n$, $g : [t_0, +\infty) \times \mathcal{PL}^p_{\mathcal{F}_t} \times \mathcal{PL}^p_{\mathcal{F}_t} \to \mathbb{R}^{n\times m}$. t^{sw}_k stands for the switching instant, and $T^{sw} = \{t^{sw}_1, t^{sw}_2, \cdots\}$. $r(t) = i_k \in Q = \{1, 2, \cdots, L\}$ when $t \in [t^{sw}_k, t^{sw}_{k+1})$, where $L \in \mathbb{N}^+$ represents the amount of switched subsystems. The j_kth subsystem is engaged if $t \in [t^{im}_k, t^{im}_{k+1})$, where $k \in \mathbb{N}^+$, t^{im}_k is the impulsive time, $j_k \in M = \{1, 2, \cdots, U\}$, U is the quantity of impulsive subsystems, and $T^{im} = \{t^{im}_1, t^{im}_2, \cdots\}$. $T = T^{sw} \cup T^{im}$. $N(a, b)$ refers to the amount of discrete-time across (a, b). Additionally, $g_i(t, 0, 0) \equiv 0$, $f_i(t, 0, 0) \equiv 0$, $I_i(t, 0, 0) \equiv 0$. g_i, f_i and I_i are locally Lipschitz, which implies that ISSS (2.1) with DSDDs has a trivial solution.

Below are some basic definitions required for this article.

Definition 2.1. *[21] For any $t \geq t_0$ and switching signal $r(t)$, let $T_1(t, s)$ and $N_1(t, s)$ respectively symbolize the total running time and quantity of the swithes triggered on $(s, t]$. There exist constants $\mathcal{T}_1 > 0$ and $N_{10} > 0$ satisfying*

$$N_1(t, s) \leq N_{10} + \frac{T_1(t, s)}{\mathcal{T}_1}, \quad t \geq s \geq t_0,$$

where $\mathcal{T}_1$ is known as the ADT.

Definition 2.2. *[17] For any $t \geq t_0$ and impulsive signal $h(t)$, let $T_2(t, s)$ and $N_2(t, s)$ respectively symbolize total running time and quantity of the impulses triggered on $(s, t]$. There exist constants $\mathcal{T}_2 > 0$ and $N_{20} > 0$ satisfying*

$$N_2(t, s) \leq N_{20} + \frac{T_2(t, s)}{\mathcal{T}_2}, \quad t \geq s \geq t_0,$$

where $\mathcal{T}_2$ is known as the AII.

The impulsive occurrence number on the interval $(s, t]$, as represented by the AII may be insufficient, even though the AII is randomly tiny. For instance, consider the following impulsive switched system:

$$\begin{cases} dx(t) = A_i x(t)dt, i = 1, 2, \\ x(t_k) = \iota x(t_k^-), \end{cases} \tag{2.2}$$

where $t \geq t_0$, $0 < \iota < 1$ $A_1 = \frac{t}{2}$, $A_2 = t$. Let $V_i(t, x(t)) = x^T P_i x$, $P_1 = 1$ and $P_2 = \frac{1}{4}$. We can calculate that

$$dV_1(t, x(t)) = tx^2(t) = tV_1(t, x(t)), \tag{2.3}$$

$$dV_2(t, x(t)) = \frac{t}{2}x^2(t) = 2tV_2(t, x(t)). \tag{2.4}$$

It follows from a simple calculation that

$$\begin{aligned} V_1(t, x(t)) =& V_1(t_0, x(t_0))\iota^{2N_2(t,t_0)} \exp\Big\{\int_{t_0}^t u du\Big\} \\ \geq& V_1(t_0, x(t_0)) \exp\Big\{2N_{20} \ln \iota + \int_{t_0}^t (u + \frac{2 \ln \iota}{\mathcal{T}_2})du\Big\}, \end{aligned}$$

$$
\begin{aligned}
V_2(t, x(t)) =& V_2(t_0, x(t_0))\iota^{2N_2(t,t_0)} \exp\Big\{\int_{t_0}^{t} 2u du\Big\} \\
\geq& V_2(t_0, x(t_0)) \exp\Big\{2N_{20} \ln \iota + \int_{t_0}^{t} (2u + \frac{2\ln \iota}{\mathcal{T}_2}) du\Big\}.
\end{aligned}
$$

One can judge that $\lim_{t\to\infty} \int_{t_0}^{t}(u + \frac{2\ln\iota}{\mathcal{T}_2})du = +\infty$ and $\lim_{t\to\infty} \int_{t_0}^{t}(2u + \frac{2\ln\iota}{\mathcal{T}_2})du = +\infty$ hold for any $\mathcal{T}_2 > 0$. Even if the AII is arbitrarily small, the equilibrium point of system (2.2) may not always be stable for an impulsive signal. So, the amount of activated impulses characterized by the AII is incomplete.

Definition 2.3. *Let $N_{\sigma l}(t_2, t_1)$, $l \in Q$ represent the quantity of switches triggered in the lth subsystem over any time interval $(t_1, t_2]$. $\psi_l(t)$ is called the mode-dependent switching density; assume that there are an integrable function $\psi_l(u) > 0$ and a constant $N'_{0l} > 0$ satisfying*

$$
N_{\sigma l}(t_2, t_1) \leq N'_{0l} + \int_{t_1}^{t_2} \psi_l(u) du, \quad t_2 \geq t_1 \geq t_0.
$$

Definition 2.4. *Let $N_{rj}(t_2, t_1)$, $j = r(t) \in Q$ refer to the quantity of generated impulses when the jth subsystem is triggered on $(t_1, t_2]$. $\phi_j(u)$ is known as the mode-dependent impulsive density; assume that there are an integrable function $\phi_j(u) > 0$ and a constant $N_{0j} > 0$ satisfying*

$$
N_{rj}(t_2, t_1) \leq N_{0j} + \int_{t_1}^{t_2} \phi_j(u) du, \quad t_2 \geq t_1 \geq t_0.
$$

Definition 2.5. *Let $N_{rqk}(t_2, t_1)$ indicate the amount of the generated kth impulsive intensity when the qth subsystem is triggered on the interval $(t_1, t_2]$. $\phi_{qk}(u)$ is known as the mode-dependent impulsive density for the kth impulsive intensity; assume that there exist an integrable function $\phi_{qk}(u) > 0$ and a constant $N_{0qk} > 0$ satisfying*

$$
\begin{cases}
N_{rqk}(t_2, t_1) \geq -N_{0qk} + \displaystyle\int_{t_1}^{t_2} \phi_{qk}(u) du, \mathcal{U}_k < 1, \\
N_{rqk}(t_2, t_1) \leq N_{0qk} + \displaystyle\int_{t_1}^{t_2} \phi_{qk}(u) du, \mathcal{U}_k \geq 1.
\end{cases}
$$

Remark 2.1. *When $\psi_l(u) = \frac{1}{\mathcal{T}_{al}}$, $N_{\sigma l}(t_2, t_1) \leq N'_{0l} + \frac{t_2 - t_1}{\mathcal{T}_{al}}$, $\phi_j(u) = \frac{1}{\mathcal{T}_{aj}}$ and $N_{rj}(t_2, t_1) \leq N_{0j} + \frac{t_2 - t_1}{\mathcal{T}_{aj}}$. This indicates that the MDADT and MDAII are special cases of mode-dependent switching density and mode-dependent impulsive density, respectively. Linear correlations described by the ADT and AII, however, constrain the descriptions of the times of switches and impulses activated. Therefore, the switching amount and impulsive appearance quantity stipulated by the MDADT and MDAII may be too small or large too small or large ISSS with DSDDs. Additionally, $\psi_l(u)$ and $\phi_j(t)$ are integrable functions; the switching density and impulse density can yield a connection that is linear or nonlinear.*

Definition 2.6. *[23] The equilibrium point of ISSS (2.1) with DSDDs is said to be pth moment asymptotically stable (p-AS), if for any initial state $x_{t_0} \in \mathcal{PL}^p_{\mathcal{F}_{t_0}}$, there exists a constant $p > 0$ such that*

$$
\lim_{t\to\infty} \mathbb{E}|x(t)|^p = 0.
$$

In addition, there are the constants ε and $\delta = \delta(\varepsilon) > 0$ such that for any $\mathbb{E}|x(t_0)|^p \leq \delta$

$$
\mathbb{E}|x(t)|^p \leq \varepsilon, t > t_0.
$$

We define a differential operator $\mathcal{L}$ for system (2.1):

$$\begin{aligned}\mathcal{L}V(t,x(t)) = {} & V_t(t,x(t)) + V_x(t,x(t))f(t,x(t),x(t-\tau(t,x(t)))) \\ & + \frac{1}{2}trace[g^T(t,x(t),x(t-\tau(t,x(t))))V_{xx}(t,x(t))g(t,x(t),x(t-\tau(t,x(t))))].\end{aligned}$$

3. Main results

For the stability of impulsive nonlinear stochastic systems or switched nonlinear stochastic systems, we usually explore stability by selecting Lyapunov functions reasonably. In this regard, it should be pointed out that most studies use the ADT or AII to trigger the amount of switches or impulses applied to control the system. However, due to the linear relationship, the number of switches or impulses activated under this mechanism may be unreasonable. In addition, the current discriminant criteria regarding state-dependent delay only involve subsystems or impulse functions. Therefore, results are rarely related to the existence of state-dependent delay in both subsystems and impulse functions. In this situation, a tricky question is how to investigate ISSSs with DSDDs based on the mode-dependent switching density and mode-dependent impulsive density within a unified framework. At present, there are few related works to be reported. The next sections will explore the cases of unstable impulses and stable impulses individually. Consider the Lyapunov function $V_i(t,x(t))$ satisfying that $\kappa_1(|x(t)|^p) \le V_i(t,x(t)) \le \kappa_2(|x(t)|^p)$, where $\kappa_1 \in \mathcal{VK}_\infty$, $\kappa_2 \in \mathcal{CK}_\infty$, $i \in Q = \{1,2,\cdots,L\}$.

Theorem 3.1. *Suppose that there exist the functions $\xi_i(t)$ and $\varphi_i(t)$, and constants $\alpha(t_{r(t^-)}) \in \Upsilon = \{\alpha_1,\alpha_2,\cdots,\alpha_L\}$, $\mathcal{U}(t_k) \in \Theta = \{\mathcal{U}_1,\mathcal{U}_2,\cdots,\mathcal{U}_S\}$ and $\lambda(t) \in \mathcal{K}_0$ such that*

(I_1) $\mathbb{E}\mathcal{L}V_i(t,x(t)) \le -\varphi_i(t)\mathbb{E}V_i(t,x(t)) + \xi_i(t)\mathbb{E}V_i(t,x(t-\tau(t,x(t))))$;

(I_2) $\mathbb{E}V_{r(t)}(t,x(t)) \le \alpha(t_{r(t^-)})\mathbb{E}V_{r(t^-)}(t^-,x(t^-)), \quad t \in T^{sw}$;

(I_3) $\mathbb{E}V_{r(t)}(t,I_{h(t)}(t^-,x(t^-\tau(t^-,x(t^-))))) \le \mathcal{U}(t_{h(t^-)})\mathbb{E}V_{r(t^-)}(t^-,x(t^-)), \quad t \in T^{im}$;

(I_4) $\xi_i(t)e^{\lambda(t)-\lambda(t-\tau(t,x(t)))} + D^+\lambda(t) < \varphi_i(t)$;

(I_5) $\lim_{t\to\infty}\int_{t_0}^{t}\Big[\sum_{q=1}^{L}\psi_q(u)\ln\alpha_q + \sum_{p=1}^{S}\sum_{q=1}^{L}\phi_{qp}(u)\ln\mathcal{U}_p\Big]du - \lambda(t) = -\infty$,

where $L \in \mathbb{N}^+$, $\alpha_q > 1$, $q \in \{1,2,\cdots,L\}$, $S \in \mathbb{N}^+$, $\mathcal{U}_p > 1$ and $p \in \{1,2,\cdots,S\}$; then, ISSS (2.1) *with DSDDs is p-AS.*

Proof. Set $\mathcal{W}_{r(t)}(t) = \exp\{\lambda(t)\}V_{r(t)}(t,x(t))$. To investigate the state trajectory of (2.1), we assert that the subsequent inequality is true for $\forall t \ge t_0$:

$$\mathbb{E}\mathcal{W}_{r(t)}(t) \le \prod_{l=1}^{N_\sigma(t,t_0)}\alpha(t_l)\prod_{j=1}^{N_r(t,t_0)}\mathcal{U}(t_j)\kappa_2(\mathbb{E}|x(t_0)|^p), \tag{3.1}$$

where $N_\sigma(t,t_0)$ indicates the total amount of switches that happened in $(t_0,t]$, and $N_r(t,t_0)$ indicates the total amount of impulses that happened in $(t_0,t]$.

Firstly, it is easy to obtain

$$\mathbb{E}\mathcal{W}_{r(t_0)}(t_0) \le \mathbb{E}V_{r(t_0)}(t_0,x(t_0)) \le \kappa_2(\mathbb{E}|x(t_0)|^p),$$

namely, (3.1) holds for $t = t_0$.

Next, (3.1) will be proven to be still valid for any $t > t_0$. We suppose that there is $t^* > t_0$ such that (3.1) is not satisfied. Then two circumstances will be considered, that is, t^* is a discrete instant or continuous instant. First, we discuss that t^* is a discrete time.

If $t^* \in T^{sw}$, let $\hat{t}^*$ be the first switching moment that invalidates (3.1). Then, one can get

$$\begin{aligned}\mathbb{E}\mathcal{W}_{r(\hat{t}^*)}(\hat{t}^*) \leq&\alpha(t_{r(\hat{t}^{*-})})\mathbb{E}\mathcal{W}_{r(\hat{t}^{*-})}(\hat{t}^{*-})\\ \leq&\textstyle\prod_{l=1}^{N_\sigma(\hat{t}^*,t_0)}\alpha(t_l)\prod_{j=1}^{N_r(\hat{t}^{*-},t_0)}\mathcal{U}(t_j)\kappa_2(\mathbb{E}|x(t_0)|^p)\\ \leq&\textstyle\prod_{l=1}^{N_\sigma(\hat{t}^*,t_0)}\alpha(t_l)\prod_{j=1}^{N_r(\hat{t}^*,t_0)}\mathcal{U}(t_j)\kappa_2(\mathbb{E}|x(t_0)|^p).\end{aligned}\tag{3.2}$$

If $t^* \in T^{im}$ and we assume that $\check{t}^*$ is the first impulsive moment that leads to the conditions of (3.1) not being satisfied. Then, one may deduce that

$$\begin{aligned}\mathbb{E}\mathcal{W}_{r(\check{t}^*)}(\check{t}^*) \leq&\mathcal{U}(t_{h(\check{t}^{*-})})\mathbb{E}\mathcal{W}_{r(\check{t}^{*-})}(\check{t}^{*-})\\ \leq&\textstyle\prod_{l=1}^{N_\sigma(\check{t}^{*-},t_0)}\alpha(t_l)\prod_{j=1}^{N_r(\check{t}^*,t_0)}\mathcal{U}(t_j)\kappa_2(\mathbb{E}|x(t_0)|^p)\\ \leq&\textstyle\prod_{l=1}^{N_\sigma(\check{t}^*,t_0)}\alpha(t_l)\prod_{j=1}^{N_r(\check{t}^*,t_0)}\mathcal{U}(t_j)\kappa_2(\mathbb{E}|x(t_0)|^p).\end{aligned}\tag{3.3}$$

According to the above discussion, regardless of whether $t^* \in T^{sw}$ or $t^* \in T^{im}$, it can be judged that (3.1) is valid for any $t \in T$.

Next t^* is considered as a continuous moment. Define $\tilde{t} = \inf\{t \in [t_0, +\infty) \setminus T : \mathbb{E}\mathcal{W}_{r(t)}(t) > \prod_{l=1}^{N_\sigma(t,t_0)}\alpha(t_l)\prod_{j=1}^{N_r(t,t_0)}\mathcal{U}(t_j)\kappa_2(\mathbb{E}|x(t_0)|^p)\}$.

In accordance with the description of $\tilde{t}$, we have the following for $\forall t \in [t_0 - \bar{\tau}, \tilde{t}]$

$$\mathbb{E}\mathcal{W}_{r(t)}(t) \leq \prod_{l=1}^{N_\sigma(t,t_0)}\alpha(t_l)\prod_{j=1}^{N_r(t,t_0)}\mathcal{U}(t_j)\kappa_2(\mathbb{E}|x(t_0)|^p).\tag{3.4}$$

And

$$\mathbb{E}\mathcal{W}_{r(\tilde{t})}(\tilde{t}) = \prod_{l=1}^{N_\sigma(\tilde{t},t_0)}\alpha(t_l)\prod_{j=1}^{N_r(\tilde{t},t_0)}\mathcal{U}(t_j)\kappa_2(\mathbb{E}|x(t_0)|^p).$$

Moreover, for $\forall t \in (\tilde{t}, \tilde{t} + \Delta t)$, one may obtain that

$$\mathbb{E}\mathcal{W}_{r(t)}(t) > \prod_{l=1}^{N_\sigma(t,t_0)}\alpha(t_l)\prod_{j=1}^{N_r(t,t_0)}\mathcal{U}(t_j)\kappa_2(\mathbb{E}|x(t_0)|^p),$$

where the positive number Δt tends to 0.

Then, it follows that

$$\begin{aligned}\mathbb{E}D^+\mathcal{W}_{r(\tilde{t})}(\tilde{t}) =& \limsup_{\Delta t\to 0}\frac{\mathbb{E}\mathcal{W}_{r(\tilde{t}+\Delta t)}(\tilde{t}+\Delta t) - \mathbb{E}\mathcal{W}_{r(\tilde{t})}(\tilde{t})}{\Delta t}\\ \geq& \limsup_{\Delta t\to 0}\frac{1}{\Delta t}\Big[\prod_{l=1}^{N_\sigma(\tilde{t}+\Delta t,t_0)}\alpha(t_l)\prod_{j=1}^{N_r(\tilde{t}+\Delta t,t_0)}\mathcal{U}(t_j) - \prod_{l=1}^{N_\sigma(\tilde{t},t_0)}\alpha(t_l)\prod_{j=1}^{N_r(\tilde{t},t_0)}\mathcal{U}(t_j)\Big]\kappa_2(\mathbb{E}|x(t_0)|^p)\\ \geq&0.\end{aligned}\tag{3.5}$$

It can be generated that through the definition of $\mathcal{W}_{r(t)}(t)$,

$$\begin{aligned}\mathbb{E}V_{r(\tilde{t}-\tau(\tilde{t},x(\tilde{t})))}(\tilde{t}-\tau(\tilde{t},x(\tilde{t})),x(\tilde{t}-\tau(\tilde{t},x(\tilde{t})))) \leq& \exp\{-\lambda(\tilde{t}-\tau(\tilde{t},x(\tilde{t})))\}\mathcal{W}_{r(\tilde{t}-\tau(\tilde{t},x(\tilde{t})))}(\tilde{t}-\tau(\tilde{t},x(\tilde{t})))\\ \leq& \exp\{-\lambda(\tilde{t}-\tau(\tilde{t},x(\tilde{t})))\}\mathcal{W}_{r(\tilde{t})}(\tilde{t})\\ \leq& \exp\{\lambda(\tilde{t})-\lambda(\tilde{t}-\tau(\tilde{t},x(\tilde{t})))\}V_{r(\tilde{t})}(\tilde{t},x(\tilde{t})). \end{aligned} \tag{3.6}$$

According to the Itô formula, one has

$$D^{+}V(t,x(t)) = \mathcal{L}V(t,x(t))dt + V_x(t,x(t))g_{r(t)}(t,x(t-\tau(t,x(t))))d\omega(t).$$

Based on the independent growth property of Brownian motion, i.e. $\mathbb{E}\{d\omega(t)\}=0$, we can acquire

$$D^{+}\mathbb{E}V(t,x(t)) = \mathbb{E}\mathcal{L}V(t,x(t)). \tag{3.7}$$

The following can be generated from conditions (I_1) and (I_4),

$$\begin{aligned}D^{+}\mathbb{E}\mathcal{W}_{r(\tilde{t})}(\tilde{t}) =& D^{+}\lambda(\tilde{t})\mathbb{E}\mathcal{W}_{r(\tilde{t})}(\tilde{t}) + \exp\{\lambda(\tilde{t})\}\mathbb{E}\mathcal{L}V_{r(\tilde{t})}(\tilde{t},x(\tilde{t}))\\ \leq& [-\varphi_{r(\tilde{t})}(\tilde{t}) + \xi_{r(\tilde{t})}(\tilde{t})\exp\{\lambda(\tilde{t})-\lambda(\tilde{t}-\tau(\tilde{t},x(\tilde{t})))\} + D^{+}\lambda(\tilde{t})]\mathbb{E}\mathcal{W}_{r(\tilde{t})}(\tilde{t})\\ <& 0,\end{aligned} \tag{3.8}$$

which is in contradiction with (3.5). Therefore, (3.1) holds for $\forall t \in [t_0,+\infty)$: At the same time, we can find that $\prod_{l=1}^{N_\sigma(t,t_0)}\alpha(t_l) = \prod_{q=1}^{L}\alpha_q^{N_{\sigma q}(t,t_0)}$ and $\prod_{p=1}^{N_r(t,t_0)}\mathcal{U}(t_p) = \prod_{p=1}^{S}\mathcal{U}_p^{\sum_{q=1}^{L}N_{rqp}(t,t_0)}$.

For $t \geq t_0$, we claim that

$$\begin{aligned}&N_{\sigma q}(t,t_0)\ln\alpha_q \leq N'_{0q}\ln\alpha_q + \int_{t_0}^{t}\psi_q(u)\ln\alpha_q du,\\ &\sum_{q=1}^{L}N_{rqp}(t,t_0)\ln\mathcal{U}_p \leq \sum_{q=1}^{L}N_{0qp}|\ln\mathcal{U}_p| + \sum_{q=1}^{L}\int_{t_0}^{t}\phi_{qp}(u)\ln\mathcal{U}_p du\end{aligned} \tag{3.9}$$

hold. One can conclude that from Definition 2.5, and if $\mathcal{U}_p < 1$,

$$\sum_{q=1}^{L}N_{rqp}(t,t_0)\ln\mathcal{U}_p \leq -\sum_{q=1}^{L}N_{0qp}\ln\mathcal{U}_p + \sum_{q=1}^{L}\int_{t_0}^{t}\phi_{qk}(u)\ln\mathcal{U}_p du,$$

if $\mathcal{U}_p > 1$,

$$\sum_{q=1}^{L}N_{rqp}(t,t_0)\ln\mathcal{U}_p \leq \sum_{q=1}^{L}N_{0qp}\ln\mathcal{U}_p + \sum_{q=1}^{L}\int_{t_0}^{t}\phi_{qp}(u)\ln\mathcal{U}_p du.$$

Thus, (3.9) holds.

Combining Definition 2.3 and (3.9), we obtain

$$\prod_{q=1}^{L}\alpha_q^{N_{\sigma q}(t,t_0)}\prod_{p=1}^{S}\mathcal{U}_p^{\sum_{q=1}^{L}N_{rqp}(t,t_0)}\exp\{-\lambda(t)\}$$

$$\leq \exp\Big\{\sum_{q=1}^{L} N'_{0q} \ln \alpha_q + \sum_{p=1}^{S}\sum_{q=1}^{L} N_{0qp}|\ln \mathcal{U}_p|\Big\} \exp\Big\{\int_{t_0}^{t}\Big[\sum_{q=1}^{L} \psi_q(u) \ln \alpha_q + \sum_{p=1}^{S}\sum_{q=1}^{L} \phi_{qp}(u) \ln \mathcal{U}_p\Big] du - \lambda(t)\Big\}.$$

Let $\gamma = \sum_{q=1}^{L} N'_{0q} \ln \alpha_q + \sum_{p=1}^{S}\sum_{q=1}^{L} N_{0qp}|\ln \mathcal{U}_p|$, since $\alpha_q > 1$ and $|\ln \mathcal{U}_p| > 0$, it can be obtained that $\gamma > 0$. Based on the condition (I_5), we can find a constant $\nu > 0$ that satisfies the following inequality for any $t > t_0$

$$\exp\{\gamma\} \exp\Big\{\int_{t_0}^{t}\Big[\sum_{q=1}^{L} \psi_q(u) \ln \alpha_q + \sum_{p=1}^{S}\sum_{q=1}^{L} \phi_{qp}(u) \ln \mathcal{U}_{qp}\Big] du - \lambda(t)\Big\} < \nu. \tag{3.10}$$

Owing to $\kappa_2 \in \mathcal{CK}_\infty$, for any $\varepsilon > 0$, there should be a constant $\breve{\delta}$ such that $\kappa_2(\breve{\delta})\nu < \varepsilon$. Therefore, whenever $\mathbb{E}|x(t_0)|^p \leq \breve{\delta}$, $\mathbb{E}V_{r(t)}(t, x(t)) \leq \varepsilon$. Additionally, because $x(t_0) \in \mathcal{PL}^p_{\mathcal{F}_{t_0}}$, according to ($\mathrm{I}_5$), (3.1), (3.10) and $\kappa_1(\mathbb{E}|x(t)|^p) \leq \mathbb{E}\kappa_1(|x(t)|^p) \leq \mathbb{E}V_i(t, x(t))$, we can get

$$\lim_{t\to\infty} \mathbb{E}|x(t)|^p = 0. \tag{3.11}$$

So ISSS (2.1) with DSDDs is p-AS. □

Remark 3.1. *In this article, the impulsive intensity, switching frequency and impulsive quantity of ISSSs fluctuate with time. $\mathcal{U}(t_{h(t^-)}) \in \Theta$ implies that the intensity of impulses varies throughout time, and impulses might be stable or unstable at various time points.*

Remark 3.2. *Conditions (I_1) and (I_4) are key in coping with state-dependent delay. In previous work, the majority of Lyapunov-type criteria required that $\mathbb{E}\mathcal{L}V(t, x(t)) \leq -\phi\mathbb{E}V(t, x(t)) + \xi\mathbb{E}(t, x(t + \theta))$, where ϕ and ξ are constant and θ is time delay. On the one hand, when we gain the constants ϕ and ξ, constants ϕ and ξ may be excessively large, causing the stability criteria to be overly conservative. To make matters worse, constants ϕ and ξ do not even exist in many actual systems, particularly those with time-varying coefficients.*

Remark 3.3. *Conditions (I_2) and (I_3) represent demand for Lyapunov functions at the switching instant and impulsive instant, respectively. Condition (I_5) is the limit on the switching density and impulse density. Overall, no extremely tight assumptions are required for this article.*

Corollary 3.1. *If there are constants $\alpha(t_{r(t^-)}) \in \Upsilon = \{\alpha_1, \alpha_2, \cdots, \alpha_L\}$, $L \in \mathbb{N}^+$, $\alpha_q > 1$, $q \in \{1, 2, \cdots, L\}$, $\mathcal{U}(t_k) \in \Theta = \{\mathcal{U}_1, \mathcal{U}_2, \cdots, \mathcal{U}_S\}$, $S \in \mathbb{N}^+$, $\mathcal{U}_p > 1$ and $p \in \{1, 2, \cdots, S\}$ such that (I_2), (I_3) and (I_4) are true, and that $\mathbb{E}\mathcal{L}V_i(t, x(t)) \leq -\varphi_i(t)\mathbb{E}V_i(t, x(t)) + \xi_i(t)\mathbb{E}V_i(t, x(t - \tau(t, x(t))))$ and $\lim_{t\to\infty} \int_{t_0}^{t}\Big[\sum_{q=1}^{L} \psi_q(u) \ln \alpha_q + \sum_{p=1}^{S}\sum_{q=1}^{L} \phi_{qp}(u) \ln \mathcal{U}_p\Big] du - \lambda(t) \in \mathcal{E}$ are valid; then, ISSS (2.1) with DSDDs is p-AS.*

Proof. The proof is omitted since it is exactly the same as that for Theorem 3.1. □

Corollary 3.2. *Let $\psi_q(u) \equiv \frac{1}{\mathcal{T}_{\sigma q}}$, $\phi_{qp}(u) \equiv \frac{1}{\mathcal{T}_{rp}}$ and $\varphi > \xi$. If there are constants $\alpha(t_{r(t^-)}) \geq 1$, $\mathcal{U}_i(t_k) \geq 1$ such that (I_2) and (I_3) hold and*

(I_6) $\mathbb{E}\mathcal{L}V_i(t, x(t)) \leq -\varphi\mathbb{E}V_i(t, x(t)) + \xi\mathbb{E}V_i(t, x(t - \tau(t, x(t))))$;
(I_7) $\sum_{q=1}^{L} \frac{\ln \alpha_q}{\mathcal{T}_{\sigma q}} + \sum_{p=1}^{S} \frac{\ln \mathcal{U}_p}{\mathcal{T}_{rp}} < \lambda_0$; where $\lambda_0 \in (0, \tilde{\lambda})$ and $\tilde{\lambda}$ is solution of $\lambda - \varphi + \xi e^{\lambda\tau(t,x(t))} = 0$,

then (2.1) *is p-AS.*

Proof. Take $\lambda(t) = \lambda_0(t - t_0)$ in Theorem 3.1. Owing to this proof being identical to Theorem 3.1, it is simplified here. □

Remark 3.4. *When $g_{r(t)}(t, x(t - \tau(t, x(t)))) = 0$ in ISSS* (2.1) *with DSDDs, condition (I_6) will become $\mathbb{E}D^+V_i(t, x(t)) \leq -\varphi\mathbb{E}V_i(t, x(t)) + \xi\mathbb{E}V_i(t, x(t - \tau(t, x(t))))$. At this point, Corollary 3.2 is the conclusion in [20].*

Theorem 3.2. *Suppose that there exist the functions $\xi_i(t) > 0$ and $\varphi_i(t) > 0$, $\upsilon_i(t) \in \mathcal{K}_\infty$ and constants $\alpha(t_{r(t^-)}) \in \Upsilon = \{\alpha_1, \alpha_2, \cdots, \alpha_L\}$, $L \in \mathbb{N}^+$, $\alpha_q > 1$, $\mathcal{U}(t_{h(t^-)}) \in \Theta = \{\mathcal{U}_1, \mathcal{U}_2, \cdots, \mathcal{U}_S\}$, $S \in \mathbb{N}^+$, $0 < \mathcal{U}_p < 1$ and $p \in \{1, 2, \cdots, S\}$ such that*

(I_1') $\mathbb{E}\mathcal{L}V_i(t, x(t)) \leq \varphi_i(t)\mathbb{E}V_i(t, x(t)) + \xi_i(t)\mathbb{E}V_i(t, x(t - \tau(t, x(t))))$;
(I_2') $\mathbb{E}V_{r(t)}(t, x(t)) \leq \alpha(t_{r(t^-)})\mathbb{E}V_{r(t^-)}(t^-, x(t^-)), \quad t \in T^{sw}$;
(I_3') $\mathbb{E}V_{r(t)}(t, I_{h(t)}(t^-, x(t^-\tau(t^-, x(t^-))))) \leq \mathcal{U}(t_{h(t^-)})\mathbb{E}V_{r(t^-)}(t^-, x(t^-)), \quad t \in T^{im}$;
(I_4') $\varpi\xi_i(t) \int_{t_0}^t \exp\Big\{\int_{u-\tau_x}^u \upsilon_i(s)ds\Big\}du + \exp\Big\{-\int_{t_0}^t [\rho_i(u) - \upsilon_i(u)]du\Big\} - 1 \leq 0,$

where $\tau_x = \tau(u, x(u))$, $\varpi = \prod_{l=1}^L \alpha^{N_{0l}'}(t_l) \prod_{j=1}^S \mathcal{U}^{-\sum_{l=1}^L N_{0lj}}(t_j)$, $\rho_i(t) = -\varphi_i(t) - \sum_{i=1}^L \ln\alpha(t_i)\psi_i(u) - \sum_{j=1}^S \sum_{i=1}^L \ln\mathcal{U}(t_{h(t^-)})\phi_{ij}(u)$ and $\exp\Big\{\int_{t_0}^t [\rho_i(u) - \upsilon_i(u)]du\Big\} > 1$. Then ISSS (2.1) *with DSDDs is p-AS.*

Proof. For any $\epsilon > 0$, let $\zeta_i(t) \equiv \zeta_i(t, x(t))$, $i \in Q$ be a unique solution of the following time-varying system with state-dependent delay:

$$\begin{cases} \mathbb{E}\dot{\zeta}_i(t) = \varphi_i(t)\mathbb{E}\zeta_i(t) + \xi_i(t)\mathbb{E}\zeta_i(t - \tau(t, x(t))) + \epsilon, t \notin T, \quad t \geq t_0, \\ \mathbb{E}\zeta_{r(t)}(t) \leq \alpha(t_{r(t^-)})\mathbb{E}\zeta_{r(t^-)}(t^-), \quad r(t) \in Q, \\ \mathbb{E}\zeta_{r(t)}(t, I_{h(t)}(t^-, x(t^-\tau(t^-, x(t^-))))) \leq \mathcal{U}(t_{h(t^-)})\mathbb{E}\zeta_{r(t^-)}(t^-, x(t^-)), \quad t \in T^{im}, \\ \zeta_{r(t_0)}(t_0) = \kappa_2(|\zeta(t_0)|^p), t \in [t_0 - \bar{\tau}, t_0]. \end{cases} \tag{3.12}$$

Thus, it is easy to deduce that $\mathbb{E}V_{r(t)}(t, x(t)) \leq \mathbb{E}\zeta_{r(t)}(t)$.
Furthermore, similar to the validation procedure of (35) in [29], one can infer that

$$\begin{aligned} \mathbb{E}\zeta_{r(t)}(t) \leq& \varpi\exp\Big\{\int_{t_0}^t -\rho_{r(u)}(u)du\Big\}\kappa_2(|x(t_0)|^p) \\ &+ \int_{t_0}^t \Big[\varpi\exp\Big\{\int_u^t -\rho_{r(s)}(s)ds\Big\}\Big(\xi_{r(t)}(t)\zeta_{r(u-\tau(u,x(u)))}(u - \tau(u, x(u))) + \epsilon\Big)\Big]du. \end{aligned} \tag{3.13}$$

Following that, we shall demonstrate the following inequality

$$\mathbb{E}\zeta_{r(t)}(t) \leq \varpi\exp\Big\{\int_{t_0}^t -\upsilon_{r(u)}(u)du\Big\}\kappa_2(|\zeta_{r(t_0)}(t_0)|^p). \tag{3.14}$$

From condition (I_4'), we know that $\upsilon(t) > 0$.

Because $V_{r(t_0)}(t_0, x(t_0)) \leq \kappa_2(|x(t_0)|^p)$, one can observe that (3.14) is valid for $t = t_0$. Assume that the above assertion is false for any $t > t_0$; then, there exists an instant t^* such that

$$\mathbb{E}\zeta_{r(t^*)}(t^*) > \varpi\exp\Big\{\int_{t_0}^{t^*} -\upsilon_{r(u)}(u)du\Big\}\kappa_2(|\zeta_{r(t_0)}(t_0)|^p), \tag{3.15}$$

and for all $t_0 \leq t < t^*$

$$\mathbb{E}\zeta_{r(t)}(t) \leq \varpi \exp\Big\{\int_{t_0}^{t} -\upsilon_{r(u)}(u)du\Big\}\kappa_2(|\zeta_{r(t_0)}(t_0)|^p), \tag{3.16}$$

Combining (3.13) and (3.16), one can generate

$$\begin{aligned}
\mathbb{E}\zeta_{r(t^*)}(t^*) \leq & \varpi \exp\Big\{\int_{t_0}^{t^*} -\rho_{r(u)}(u)du\Big\}\kappa_2(|x(t_0)|^p) \\
& + \int_{t_0}^{t^*}\Big[\varpi \exp\Big\{\int_{u}^{t^*} -\rho_{r(s)}(s)ds\Big\}\Big(\xi_{r(t^*)}(t^*)\zeta_{r(u-\tau(u,x(u)))}(u-\tau(u,x(u))) + \epsilon\Big)\Big]du \\
\leq & \varpi \exp\Big\{\int_{t_0}^{t^*} -\rho_{r(u)}(u)du\Big\}\kappa_2(|x(t_0)|^p) \\
& + \int_{t_0}^{t^*}\Big[\varpi \exp\Big\{\int_{u}^{t^*} -\rho_{r(s)}(s)ds\Big\}\Big(\xi_{r(t^*)}(t^*)\varpi \exp\Big\{\int_{t_0}^{u-\tau_x} -\upsilon_{r(s)}(s)ds\Big\}\kappa_2(|\zeta_{r(t_0)}(t_0)|^p) + \epsilon\Big)\Big]du \\
\leq & \varpi \exp\Big\{\int_{t_0}^{t^*} -\rho_{r(u)}(u)du\Big\}\kappa_2(|x(t_0)|^p) + \varpi \exp\Big\{\int_{t_0}^{t^*} -\rho_{r(s)}(s)ds\Big\}\int_{t_0}^{t^*}\Big[\exp\Big\{\int_{t_0}^{u} \rho_{r(s)}(s)ds\Big\} \\
& \times \Big(\xi_{r(t^*)}(t^*)\varpi \exp\Big\{\int_{t_0}^{u-\tau_x} -\upsilon_{r(s)}(s)ds\Big\}\kappa_2(|\zeta_{r(t_0)}(t_0)|^p) + \epsilon\Big)\Big]du.
\end{aligned} \tag{3.17}$$

Moreover, setting $\epsilon \to 0$, one has

$$\begin{aligned}
& \lim_{\epsilon\to 0}\int_{t_0}^{t^*}\Big[\exp\Big\{\int_{t_0}^{u} \rho_{r(s)}(s)ds\Big\}\Big(\xi_{r(t^*)}(t^*)\varpi \exp\Big\{\int_{t_0}^{u-\tau_x} -\upsilon_{r(s)}(s)ds\Big\}\kappa_2(|\zeta_{r(t_0)}(t_0)|^p) + \epsilon\Big)\Big]du \\
= & \int_{t_0}^{t^*} \exp\Big\{\int_{t_0}^{u} \rho_{r(s)}(s)ds\Big\}\xi_{r(t^*)}(t^*)\varpi \exp\Big\{\int_{t_0}^{u-\tau_x} -\upsilon_{r(s)}(s)ds\Big\}\kappa_2(|\zeta_{r(t_0)}(t_0)|^p)du \\
\leq & \int_{t_0}^{t^*} \exp\Big\{\int_{t_0}^{u} \rho_{r(s)}(s) - \upsilon_{r(s)}(s)ds\Big\}\xi_{r(t^*)}(t^*)\varpi \exp\Big\{\int_{u-\tau_x}^{u} \upsilon_{r(s)}(s)ds\Big\}\kappa_2(|\zeta_{r(t_0)}(t_0)|^p)du \\
\leq & \varpi\xi_{r(t^*)}(t^*) \exp\Big\{\int_{t_0}^{t^*} \rho_{r(s)}(s) - \upsilon_{r(s)}(s)ds\Big\}\int_{t_0}^{t^*} \exp\Big\{\int_{u-\tau_x}^{u} \upsilon_{r(s)}(s)ds\Big\}\kappa_2(|\zeta_{r(t_0)}(t_0)|^p)du \\
\leq & \kappa_2(|\zeta_{r(t_0)}(t_0)|^p)\Big[\exp\Big\{\int_{t_0}^{t^*} \rho_{r(s)}(s) - \upsilon_{r(s)}(s)ds\Big\} - 1\Big].
\end{aligned} \tag{3.18}$$

Substituting (3.18) into (3.17) yields

$$\begin{aligned}
\mathbb{E}\zeta_{r(t^*)}(t^*) \leq & \varpi \exp\Big\{\int_{t_0}^{t^*} -\rho_{r(u)}(u)du\Big\}\kappa_2(|x(t_0)|^p) \\
& + \varpi \exp\Big\{\int_{t_0}^{t^*} -\rho_{r(s)}(s)ds\Big\}\kappa_2(|\zeta_{r(t_0)}(t_0)|^p)\Big[\exp\Big\{\int_{t_0}^{t^*} [\rho_{r(s)}(s) - \upsilon_{r(s)}(s)]ds\Big\} - 1\Big] \\
\leq & \varpi\kappa_2(|\zeta_{r(t_0)}(t_0)|^p) \exp\Big\{\int_{t_0}^{t^*} -\upsilon_{r(u)}(u)du\Big\},
\end{aligned} \tag{3.19}$$

which contradicts with (3.15). So, (3.14) holds.

In addition,

$$\mathbb{E}V_{r(t)(t,x(t))} \leq \varpi\kappa_2(|x(t_0)|^p)\exp\Big\{\int_{t_0}^{t^*} -\upsilon_{r(u)}(u)du\Big\}.$$

Therefore, one can obtain that

$$\mathbb{E}|x(t)|^p \leq \mathbb{E}\kappa_1^{-1}\Big(\varpi\kappa_2(|x(t_0)|^p)\exp\Big\{\int_{t_0}^{t^*} -\upsilon_{r(u)}(u)du\Big\}\Big). \tag{3.20}$$

Then ISSS (2.1) with DSDDs is p-AS. □

Remark 3.5. *There are plenty of intriguing conclusions in the literature concerning the control and analysis of nonlinear impulsive stochastic systems. However, the majority of recent studies have simply investigated impulsive stochastic systems without considering stochastic effects; see [15,17,18,20,21]. On the other hand, although [16,23] focused on stochastic impulsive systems, they all presented more conservative constraints on the rate coefficient for the Lyapunov function. Furthermore, their results do not apply to impulsive stochastic systems with switches. Moreover, our approach embraces several current results as exceptional circumstances; for instance, the authors of [20], under the MDADT and MDAII, only analyzed the stability of impulsive switched nonlinear systems with DSDDs. The [20] only explored the stability of switched linear systems under an MDADT.*

4. Application to neural networks

In this section, by applying switching density and impulse density, the restrictions on amounts of switches and impulses are relaxed. Besides, based on Lyapunov functions and the comparison principle, ISSNNs with DSDDs are considered to be mean square asymptotically stable (MSAS). In particular, the derived results fully demonstrate that switching density and impulse density can more accurately describe the switching number and impulsive number, making the system stable.

$$\begin{cases} dx(t) = -A_{r(t)}x(t) + B_{r(t)}f(x(t-\tau(t,x(t))))dt + g(x(t), x(t-\tau(t,x(t))), r(t))d\omega(t), t \notin T^{im}, \\ x(t) = I_{h(t)}(t^-, x(t^-), x(t^- - \tau(t^-, x(t^-)))), t \in T^{im}, \end{cases} \tag{4.1}$$

where $x(t) \in \mathbb{R}^n$ and A_i is an n-dimensional diagonal matrix, representing the self-feedback connection weight matrix. $B_i \in \mathbb{R}^{n\times n}$ is the connection weight matrix among neurons. $\tau(t, x(t))$ is a state-dependent delay. $f(\cdot)$ is the neuron activation function, and we have that $|f_i(x_1) - f_i(x_2)| \leq l_i|x_1 - x_2|$, $L = diag\{l_1, l_2 \cdots l_n\}$ and $f(0) = 0$. $g(\cdot)$ is the noise perturbation. The initial value $\zeta(t) = x(t - \tau(t, x(t))) \in \mathcal{PL}^2_{\mathcal{F}_{t_0}}$. Take the Lyapunov function $V_{r(t)}(t, x(t)) = t^2x^TR_{r(t)}x$, where $R_i \leq v_iI$, $R_i \leq \mu R_j$ and I is the identity matrix; $v_i > 1$ and $\mu > 1$ are constants. For the noise perturbation $g(\cdot)$, we make the following assumption.

Assumption 4.1. *There exist matrices $\Gamma_{1i} > 0$ and $\Gamma_{2i} > 0$ such that*

$$trace[g^T(x(t), x_\tau, i)g((x(t), x_\tau, i))] \leq x^T(t)\Gamma_{1i}x(t) + x_\tau^T\Gamma_{2i}x_\tau,$$

where $x_\tau = x(t - \tau(t, x(t)))$.

Following are several lemmas that will be useful later.

Lemma 4.1. *[30] Let $x, y \in \mathbb{R}^n$; U is a diagonal positive definition matrix with appropriate dimensions; then,*

$$x^T y + y^T x \leq x^T U x + y^T U^{-1} y$$

holds.

Lemma 4.2. *[27] If $V \in \mathbb{R}^{n\times n}$ is a symmetric positive definite matrix and $U \in \mathbb{R}^{n\times n}$ is symmetric matrix, then*

$$\lambda_{min}(V^{-1}U)x^T V x \leq x^T U x \leq \lambda_{max}(V^{-1}U)x^T V x, x \in \mathbb{R}^n$$

holds.

Following that, we will explore the asymptotic stability of ISSNNs by employing switching density and impulse density. For convenience, take $\lambda_{i2} = \lambda_{\max}(R_i^{-1}L^T L) + v_i\lambda_{\max}(R_i^{-1}\Gamma_{2i})$.

Theorem 4.1. *Under Assumption 4.1, if for any $r(t) = i \in Q$, there exist symmetric $R_i > 0$, constants $v_i > 1$ and $\alpha_i > 1$, $\lambda_{i1} > \lambda_{i2}$, and $\mathcal{U}(t_i) \in \Theta$ such that*

(I_8) *(I_3) and (I_5) are satisfied;*
(I_9) $\lambda_{i2}\frac{t^2}{(\breve{t}-\bar{\tau})}e^{\lambda(\breve{t})-\lambda(\breve{t}-\bar{\tau}(\breve{t},x(\breve{t})))} + D^+\lambda(t) \leq \lambda_{i1} - \frac{2}{\breve{t}}$;
(I_{10}) $-A_i^T R_i - R_i A_i + R_i B_i B_i^T R_i + v_i\Gamma_{1i} + \lambda_{i1}R_i < 0$,

then ISSNN (4.1) with DSDDs is MSAS.

Proof. On the basis of the definition of $\mathcal{L}$, Lemma 4.1 and Lemma 4.2, one has

$$\begin{aligned}
\mathcal{L}V_i(t, x(t)) =& 2tx^T(t)R_i x(t) + 2t^2 x^T(t)R_i[-A_i x(t) + B_i f(x(t - \tau(t, x(t))))] \\
&+ t^2 trace[g^T(x(t), x(t - \tau(t, x(t)), i))R_i g(x(t), x(t - \tau(t, x(t)), i))] \\
\leq& 2tx^T(t)R_i x(t) + t^2\Big[-2x^T(t)R_i A_i x(t) + 2x^T(t)R_i B_i f(x(t - \tau(t, x(t)))) \\
&+ v_i trace[g^T(x(t), x(t - \tau(t, x(t)), i))g(x(t), x(t - \tau(t, x(t)), i))]\Big] \\
\leq& 2tx^T(t)R_i x(t) + t^2\Big[x^T(t)[-A_i R_i - R_i A_i + R_i B_i B_i^T R_i + v_i\Gamma_{1i}]x(t) \\
&+ [\lambda_{\max}(R_i^{-1}L^T L) + v_i\lambda_{\max}(R_i^{-1}\Gamma_{2i})]x^T(t - \tau(t, x(t)))R_i x(t - \tau(t, x(t)))\Big].
\end{aligned} \tag{4.2}$$

Further, we have

$$\mathbb{E}\mathcal{L}V_i(t, x(t)) \leq -(\lambda_{i1} - \frac{2}{t})\mathbb{E}V_i(t, x(t)) + \lambda_{i2}\frac{t^2}{(t - \bar{\tau})^2}\mathbb{E}V_i(t, x(t - \tau(t, x(t)))). \tag{4.3}$$

According to $R_i \leq \mu R_j$, one has

$$\mathbb{E}V_{r(t)}(t, x(t)) \leq \mu\mathbb{E}V_{r(t^-)}(t^-, x(t^-)).$$

Therefore, the remaining discussion is similar to Theorem 3.1, which we will omit here. □

Remark 4.1. *In Theorem 4.1, we develop mean square asymptotic stability criteria for ISSNNs* (4.1) *with DSDDs by implementing mode-dependent impulsive density and mode-dependent switching density under the Lyapunov function. Condition* (I_{10}) *is the concretization of* (I_1).

Remark 4.2. *The range of impulsive leaps in this work is determined by historical state information. This suggests that impulsive conduct is influenced not just by present state information, but also by previous state information. We observe that impulses with state-dependent delay can not only stabilize stochastic switched systems but also disrupt its stability. In order to stress the influence of DSDDs more clearly, different from the results of [18–20, 22, 28], we explore delayed impulses and stochastic noise in the form of* (4.1)*, which makes impulsive behavior reliant on past state information.*

Theorem 4.2. *Under Assumption* 4.1*, if for any* $r(t) = i \in Q$*, there exist symmetric* $R_i > 0$*, constants* $v_i > 1$ *and* $\alpha_i > 1$*,* $\lambda_{i1} > \lambda_{i2}$*, and* $0 < \mathcal{U}(t_i) < 1$ *such that*

(I_{11}) (I_3') *is satisfied;*
(I_{12}) $-A_i^T R_i - R_i A_i + R_i B_i B_i^T R_i + v_i \Gamma_{1i} - \lambda_{i1} R_i < 0$*;*
(I_{13}) $\varpi \lambda_{i2} \frac{t^2}{(t-\bar{\tau})^2} \int_{t_0}^{t} \exp\Big\{ \int_{u-\tau_x}^{u} \upsilon_i(s) ds \Big\} du - 1 + \exp\Big\{ - \int_{t_0}^{t} \rho_i(u) - \upsilon_i(u) du \Big\} \leq 0,$

then ISSNN (4.1) *with DSDDs is MSAS.*

Proof. On the basis of the definition of $\mathcal{L}$, Lemma 4.1 and Lemma 4.2, one has

$$\begin{aligned}
\mathcal{L}V_i(t, x(t)) \leq & 2tx^T(t)R_i x(t) + t^2 \Big[x^T(t)[-A_i R_i - R_i A_i + R_i B_i B_i^T R_i + v_i \Gamma_{1i}]x(t) \\
& + [\lambda_{\max}(R_i^{-1} L^T L) + v_i \lambda_{\max}(R_i^{-1} \Gamma_{2i})] x^T(t - \tau(t, x(t))) R_i x(t - \tau(t, x(t))) \Big] \\
\leq & (\lambda_{i1} + \frac{2}{t}) \mathbb{E} V_i(t, x(t)) + \lambda_{i2} \frac{t^2}{(t - \bar{\tau})^2} \mathbb{E} V_i(t, x(t - \tau(t, x(t)))).
\end{aligned}$$

Thus,

$$\begin{cases}
\mathcal{L}V_i(t, x(t)) \leq (\lambda_{i1} + \frac{2}{t}) \mathbb{E} V_i(t, x(t)) + \lambda_{i2} \frac{t^2}{(t - \bar{\tau})^2} \mathbb{E} V_i(t, x(t - \tau(t, x(t)))), \quad t \notin T, t \geq t_0, \\
\mathbb{E} V_{r(t)}(t, x(t)) \leq \mu \mathbb{E} V_{r(t^-)}(t^-, x(t^-)), \quad r(t) \in Q, \\
\mathbb{E} V_{r(t)}(t, I_{h(t)}(t^-, x(t^- \tau(t^-, x(t^-))))) \leq \mathcal{U}(t_{h(t^-)}) \mathbb{E} V_{r(t^-)}(t^-, x(t^-)), \quad t \in T^{im}.
\end{cases} \tag{4.4}$$

Afterward, in line with Theorem 3.2, ISSNN (4.1) is MSAS. The detailed proof is now omitted. □

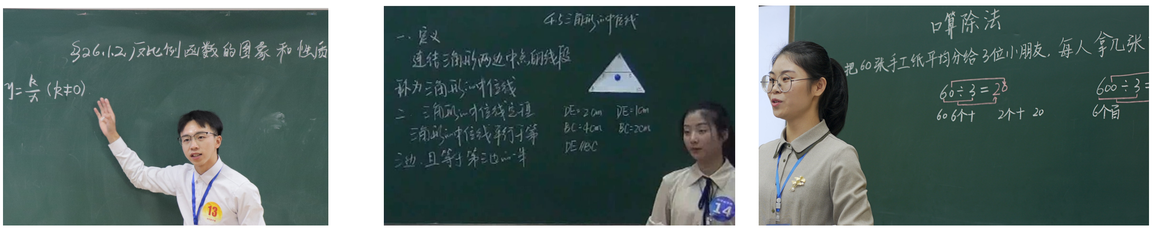
5. Application to neural network-based lecture skills assessment of normal students

In this section, we apply model (4.1) in Section 4 to the neural network-based lecture skills assessment of normal students. This is also an application case of neural networks for multi-class classification.

We know that the comprehensive evaluation hierarchy for the lecture skills of normal students generally includes the following five elements: (a) teaching design ability, (b) educational technology application ability, (c) teaching implementation ability, (d) teaching evaluation ability, and (e) teaching characteristics and innovation. For the comprehensive evaluation of the lecture skills of a single normal student, how to objectively and scientifically quantify the five elements (a), (b), (c), (d) and (e), which have the most critical core impact on the normal students possesses a positive significance for personalized improvement of the quality of talent cultivation for normal student. It should be noted that these five elements (a), (b), (c), (d) and (e) constitute a dynamic process, and that the degree of influence of these elements is also different. Therefore, evaluating which indicators have an impact on a specific normal student is basically a nonlinear classification problem, which brings great difficulties to comprehensive analysis.

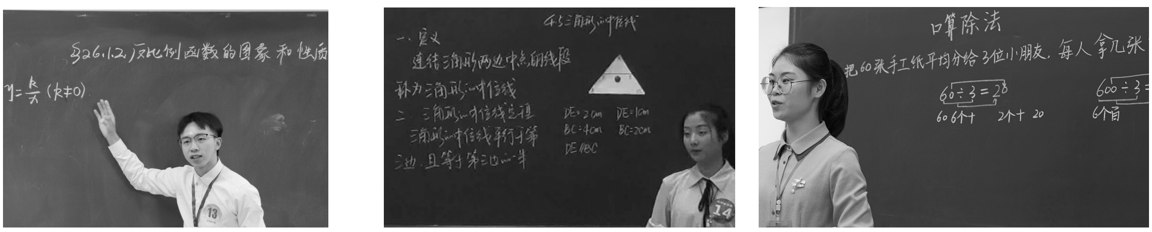
Here, we first adopt a multivariate comprehensive evaluation method based on neural networks [31]. After inputting measurement indexes, the output nodes for network (4.1) in Section 4 determine the five elements (a), (b), (c), (d) and (e) mentioned above (the core idea here is neural network-based multi-class classification). Then, combined with the independent component analysis method in signal processing [32], independent components are separated from the observed signals in the five elements (a), (b), (c), (d) and (e); based on this, we can determine which indicators have an impact on a specific normal student.

Below, we will briefly discuss the use of the simulation software MATLAB to verify the above analysis ideas. Figure 1 shows three classroom teaching scenarios for normal mathematics students, Figure 2 illustrates the normalization of elements for classroom teaching scenarios and Figure 3 depicts the impact of separated independent source signals on a specific normal student.



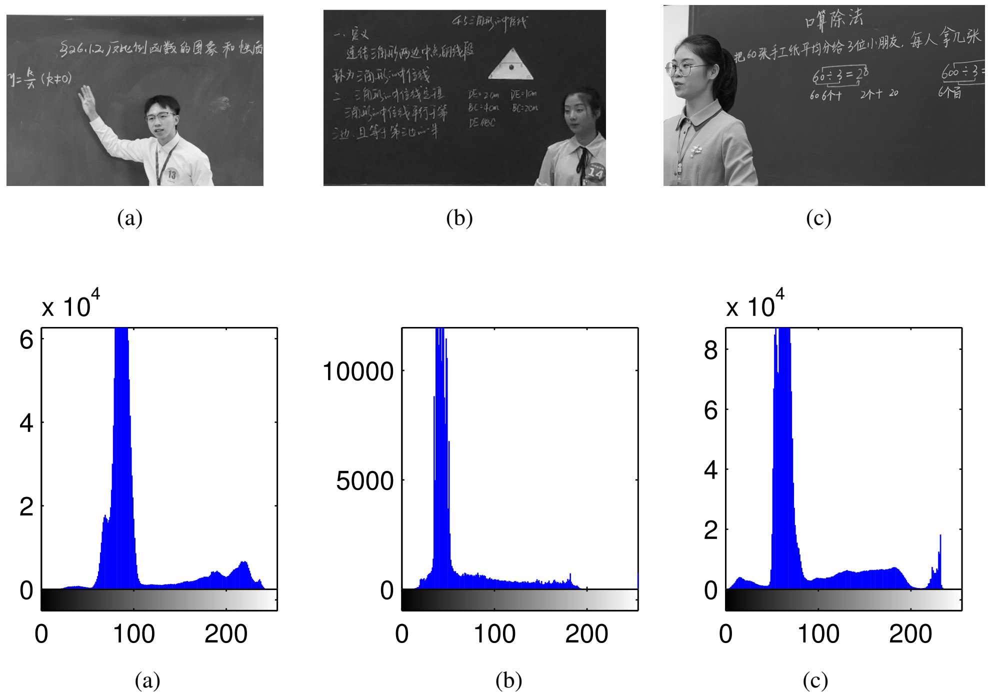
(a) (b) (c)

Figure 1. Three classroom teaching scenarios for normal mathematics students.



(a) (b) (c)

Figure 2. The normalization of elements for classroom teaching scenarios.



(a) (b) (c)

Figure 3. The impact of separated independent source signals.

6. Numerical examples

In this part, we will utilize the following system to test the viability of the theoretical results of this article.

Example 1. We consider stochastic noise and DSDDs on the following system described in [33]

$$\begin{cases} \dot{x}_1(t) = \varsigma(x_2(t) - m_{1\sigma(t)}x_1(t) + m_{2\sigma(t)}g_{\sigma(t)}(x_1(t))) - cx_1(t - 0.5x_1(t)), \\ \dot{x}_2(t) = x_1(t) - x_2(t) + x_3(t) - cx_1(t - 0.5x_1(t)), \\ \dot{x}_3(t) = -\pi_{\sigma(t)}x_2(t) + c[2x_1(t - 0.5x_1(t)) - x_3(t - 0.5x_3(t))], \end{cases} \tag{6.1}$$

$\sigma(t) \in \{1, 2\}$; take $\varsigma = 11$, $m_{11} = 1$, $m_{12} = \frac{2}{7}$, $m_{21} = -1$, $m_{22} = 1$, $\pi_1 = 14.87$, $\pi_2 = 14.28$ $c = 0.1$, $g_1(x_1(t)) = ax_1 + \frac{1}{2}(b-a)(|x_1(t)+1| - |x_1(t)-1|)$, $g_2(x_1(t)) = \frac{1}{2}(m_{12} + \frac{1}{7})(|x_1(t)+1| - |x_1(t)-1|)$, $a = -0.68$ and $b = -0.27$. Then (6.1) can be revised as below:

$$\dot{x}(t) = A_{\sigma(t)}x(t) + A_{1\sigma(t)}x(t - 0.5x(t)) + C_{\sigma(t)}f_{\sigma}(x(t)), \tag{6.2}$$

where

$$A_1 = \begin{bmatrix} -\varsigma & 0 & 0 \\ 0 & -6 & 0 \\ 0 & 0 & -\pi_1 \end{bmatrix}, \ A_2 = \begin{bmatrix} -\frac{18}{7} & 0 & 0 \\ 0 & -12.37 & 0 \\ 0 & 0 & -16.28 \end{bmatrix},$$

$$A_{11}=A_{12}=\begin{bmatrix}-c & 0 & 0\\ -c & 0 & 0\\ 2c & 0 & -c\end{bmatrix},\quad C_1=\begin{bmatrix}1 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix},$$
$$C_2=\begin{bmatrix}0.01 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix},$$

$f_1(x(t)) = (-\varsigma g_1(x_1(t)), 0, 0)^T$ and $f_2(x(t)) = (\frac{1}{2}(|x_1(t)+1| - |x_1(t)-1|), 0, 0)^T$. The response system has the same structure as the drive system; however, it takes controller-related and random disturbance into account. For convenience, let $y_\tau = y(t-0.5(y(t)))$ and $x_\tau = x(t-0.5x(t))$.

$$dy(t) =[A_{\sigma(t)}y(t) + A_{1\sigma(t)}y_\tau + C_{\sigma(t)}f_{\sigma(t)}(y(t))]dt + g_3(t, y(t)-x(t), y_\tau - x_\tau)d\omega(t). \tag{6.3}$$

In order to synchronize drive system (6.2) and response system (6.3), an impulse controller is designed:

$$u_2(t) = \sum_{k=1}^{2} B_{2k}(y_\tau - x_\tau)\delta(t-t_k),$$

where $\delta(t)$ is the Dirac function.

Then, synchronization error can be described as follows:

$$\begin{cases} de(t) =A_{\sigma(t)}e(t) + A_{1\sigma(t)}e_\tau + C_{\sigma(t)}\bar{f}_{\sigma(t)}(e(t))dt + g_3(t, e(t), e_\tau)d\omega(t), \\ \Delta(t_k) =B_{2k}e(t_k - 0.5e(t_k)), \end{cases} \tag{6.4}$$

where $e_\tau = e(t-0.5e(t))$, $\bar{f}_{\sigma(t)}(e(t)) = f_{\sigma(t)}(y(t)) - f_{\sigma(t)}(x(t))$,

$$B_{21}=\begin{bmatrix}0.2 & 0 & 0\\ 0 & 0.2 & 0\\ 0 & 0 & 0\end{bmatrix},\quad B_{22}=\begin{bmatrix}0 & 0 & 0\\ 0 & 0.2 & 0\\ 0 & 0 & 0.2\end{bmatrix}.$$

Let $V_1(e(t)) = V_2(e(t)) = t^2e^T(t)e(t)$, and set

$$\Gamma_{11}=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}, \Gamma_{12}=\begin{bmatrix}0.67 & 0 & 0\\ 0 & 0.98 & 0\\ 0 & 0 & 1\end{bmatrix},$$
$$\Gamma_{21}=\Gamma_{22}=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}.$$

Then,

$$\begin{aligned}\mathcal{L}V_1(t, e(t)) =&2te^T(t)e(t) + t^2\Big[e^T(t)[A_1 + A_1^T + A_{11}^TA_{11} + 2C_1\varsigma|b|I + \Gamma_{11}]e(t) + 2e_\tau^Te(t)\Big] \\ \leq&(\frac{2}{t} - 11)t^2e^T(t)e(t) + \frac{2t^2}{(t-0.5e(t))^2}(t-0.5e(t))^2e_\tau^Te_\tau,\end{aligned}$$

$$\mathcal{L}V_2(e(t)) = 2te^T(t)e(t) + t^2\Big[e^T(t)[A_2 + A_2^T + A_{12}^T A_{12} + 2C_2 + \Gamma_{12}]e(t) + 2e_\tau^T e_\tau\Big]$$
$$\leq (\frac{2}{t} - 4.3928)t^2 e^T(t)e(t) + \frac{2t^2}{(t-0.5e(t))^2}(t-0.5e(t))^2 e_\tau^T e_\tau.$$

Let $t_0 = 1$ and $\lambda(t) = -2\ln t + 2t$. Set $\varphi_1(t) = -\frac{2}{t} + 11$, $\xi_1(t) = \frac{2t^2}{(t-0.5e(t))^2}$ and $\varphi_2(t) = -\frac{2}{t} + 4.3928$, $\xi_2(t) = \frac{2t^2}{(t-0.5e(t))^2}$. Then, we have that $\xi_1(t)\exp\{\lambda(t) - \lambda(t-0.5e(t))\} + D^+\lambda(t) - \varphi_1(t) < 0$ and $\xi_2(t)\exp\{\lambda(t) - \lambda(t-0.5e(t))\} + D^+\lambda(t) - \varphi_2(t) < 0$. Take the switching density $\psi_q(t) \equiv \psi(t) = 5.5 - \frac{5}{t}$, $\alpha_q(t) \equiv \alpha(t) = e^{0.2}$, impulse density $\phi_{qp}(t) \equiv \phi(t) = 4 - \frac{7}{2t}$, impulse strength $\mathcal{U}_p \equiv \mathcal{U} = e^{0.2}$; then, we have that $\lim_{t\to\infty}\int_{t_0}^{t}\psi(u)\ln\alpha + \phi(u)\ln\mathcal{U}_p du - \lambda(t) = -\infty$. Then, because all prerequisites in Theorem 3.1 are met, we can infer that system (6.4) is MSAS. Then, Figures 4 and 5 illustrate the state trajectory and mean square trajectory of the system, respectively, and they adequately explain asynchronous occurrence between switches and impulses, validating the efficacy of our findings.

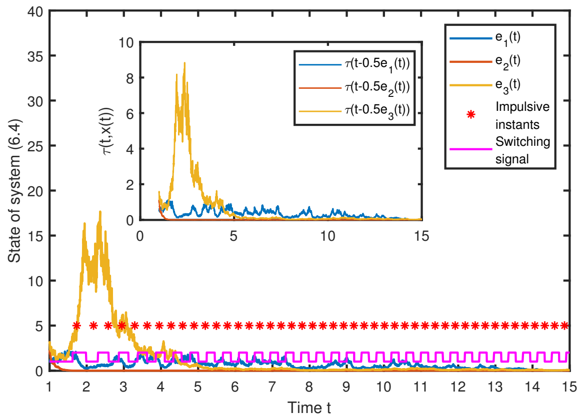


Figure 4. Dynamic behavior of synchronization error system (6.4) with switching density $\psi_q(t) \equiv \psi(t) = 5.5 - \frac{5}{t}$ and impulse density $\phi_p(t) \equiv \phi(t) = 4 - \frac{7}{2t}$.

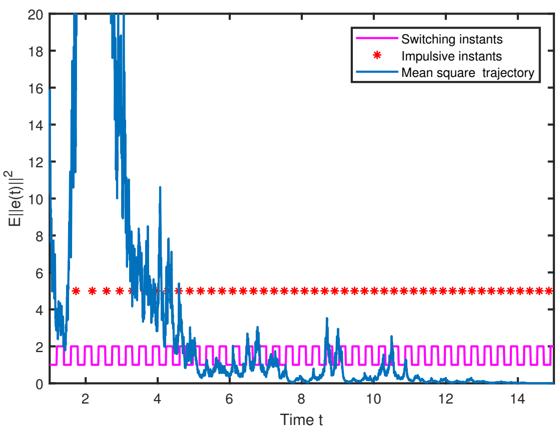


Figure 5. Mean square trajectory of synchronization error system (6.4) with switching density $\psi_q(t) \equiv \psi(t) = 5.5 - \frac{5}{t}$ and impulse density $\phi_p(t) \equiv \phi(t) = 4 - \frac{7}{2t}$.

Remark 6.1. *In order to demonstrate the advantages of the proposed mode-dependent impulsive density, we set the AII parameters for comparison based on the same initial values and switching density; see Figures 6 and 7. It is not difficult to see from comparison that error trajectories under the mode-dependent impulsive density strategy and mode-dependent switching density strategy are synchronized faster and smoother. According to Figures 6 and 7, under the AII method, unstable impulses in system* (6.4) *have shorter impulsive intervals, implying that the impulses occur more frequently. Comparing Figure 8 with Figure 4, it can be observed that the synchronization of system* (6.4) *is slower based on the MDADT and MDAII. This is mainly due to the fact that the linear connections defined by the MDADT and MDAII restrict the depiction of the numbers of switching and impulsive occurrences with time-varying characteristics. Hence, in light of the above description, we conclude that designing an appropriate mode-dependent impulsive density and mode-dependent switching density will be more adaptable and realistic, with fewer constraints to improve system performance.*

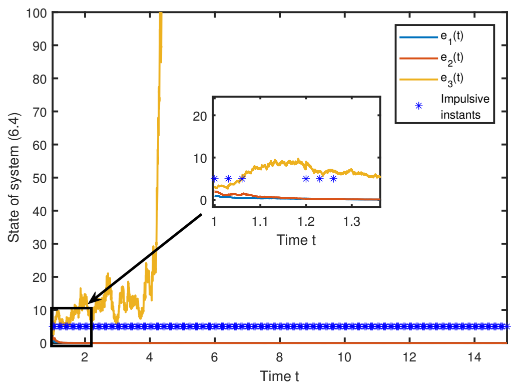


Figure 6. Dynamic behavior of synchronization error system (6.4) with switching density $\psi_q(t) \equiv \psi(t) = 5.5 - \frac{5}{t}$ and with AII $\mathcal{T}_2 = 0.2$.

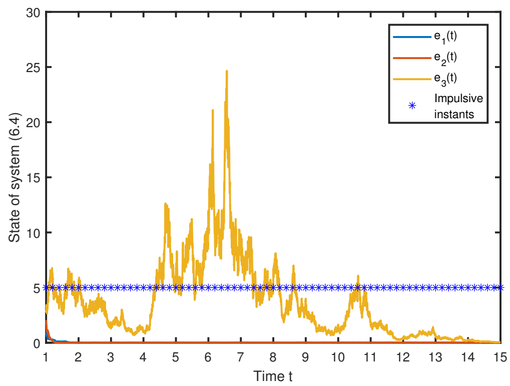


Figure 7. Dynamic behavior of synchronization error system (6.4) with switching density $\psi_q(t) \equiv \psi(t) = 5.5 - \frac{5}{t}$ and with AII $\mathcal{T}_2 = 0.6$.

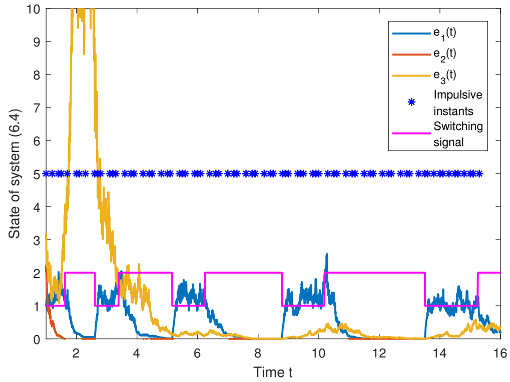


Figure 8. Dynamic behavior of synchronization error system (6.4) with MDADTs $\mathcal{T}_{11} = 0.3$, $\mathcal{T}_{12} = 1$ and MDAIIs $\mathcal{T}_{21} = 0.5$, $\mathcal{T}_{22} = 0.6$.

Remark 6.2. *Figures 9 and 10 respectively represent the state trajectories of subsystems of system* (2.2) *with stable impulses based on an AII strategy, where the amount of impulses is insufficient to stabilize the system. Figures 11 and 12 depict the results of the mode-dependent impulsive density method, where state trajectories of subsystems of system* (2.2) *have attained stability.*

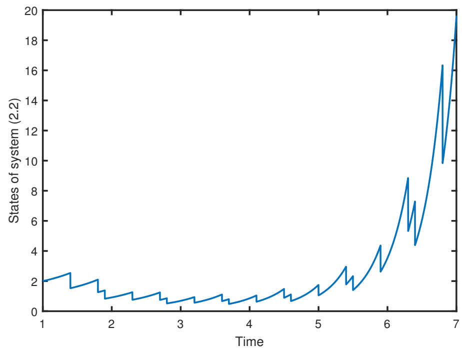


Figure 9. Dynamic behavior of subsystem (1) of system (2.2) with AII $\mathcal{T}_2 = 0.9$.

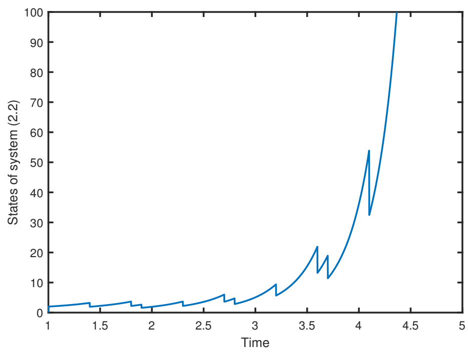


Figure 10. Dynamic behavior of subsystem (2) of system (2.2) with AII $\mathcal{T}_2 = 0.9$.

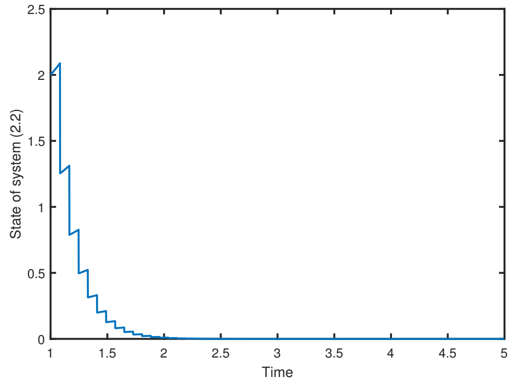


Figure 11. Dynamic behavior of subsystem (1) of system (2.2) with impulse density $\phi_p(t) \equiv \phi(t) = t + 11$.

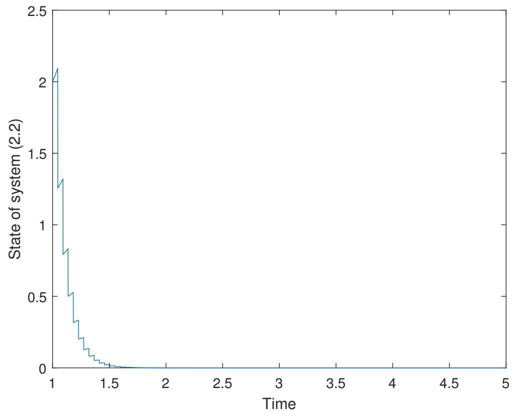


Figure 12. Dynamic behavior of subsystem (2) of system (2.2) with impulse density $\phi_p(t) \equiv \phi(t) = \frac{1}{t} + 21$.

Example 2. In (4.1), set $r(t) \in \{1, 2\}$, $h(t) \in \{1, 2\}$ and $g(x(t), x(t - \tau(t, x(t))), r(t)) = x(t) + |x(t - \tau(t, x(t)))|$. Consider ISSNN (4.1) with the coefficients shown below

$$A_1 = A_2 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},\ B_1 = \begin{bmatrix} 5 & 0 \\ -1 & -3 \end{bmatrix},\ B_2 = \begin{bmatrix} 5 & -2 \\ -3 & -4 \end{bmatrix}.$$

State-dependent delay $\tau(t, x(t)) = 0.5 * |x(t)|$. $R_i = I$. The state trajectories of subsystems of ISSNNs (4.1) are shown in Figures 13 and 14. It can be seen that ISSNNs (4.1) is unstable. Let the switching density $\psi_i(t) = 17 + \frac{2}{t}$ and impulse density $\phi_{ij}(t) = 22 + \frac{5}{t}$. By calculation, it can be concluded that $\lambda_{12} = 4$, $\lambda_{11} = 38$, $\lambda_{21} = 5$, $\lambda_{22} = 39$, $v_1 = 3$, $v_2 = 4$, $\alpha_1 = 1.5, \alpha_2 = 1.2$ and $\upsilon_i(t) \equiv \frac{3}{t}$. The impulses strength $\mathcal{U}_1 = \mathcal{U}_2 = 0.3$.

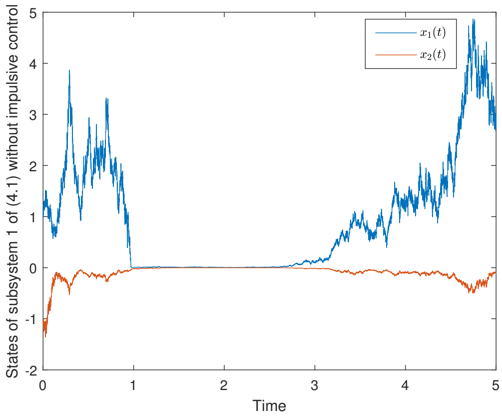


Figure 13. The state of subsystem (1) of (4.1) without impulsive control.

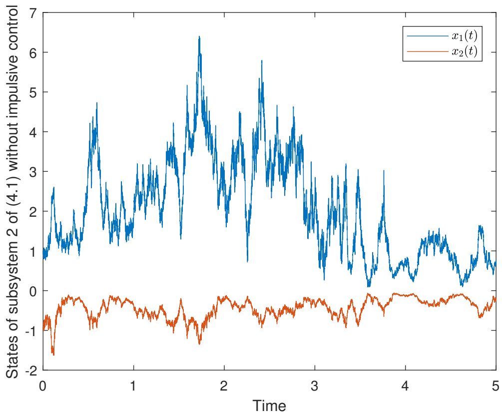


Figure 14. The state of subsystem (2) of (4.1) without impulsive control.

In addition, we can calculate that

$$\lambda_{max}\Big\{-A_1^T R_1 - R_1 A_1 + R_1 B_1 B_1^T R_1 + \nu_1 \Gamma_{11} - \lambda_{11} R_1\Big\} = -10.8197,$$
$$\lambda_{max}\Big\{-A_2^T R_2 - R_2 A_2 + R_2 B_2 B_2^T R_2 + \nu_2 \Gamma_{12} - \lambda_{21} R_2\Big\} = -2.7199,$$
$$\varpi \lambda_{i2} \frac{t^2}{(t-\bar{\tau})^2} \int_{t_0}^{t} \exp\Big\{\int_{u-\tau_x}^{u} \upsilon_i(s)ds\Big\} du - 1 + \exp\Big\{-\int_{t_0}^{t} \rho_i(u) - \upsilon_i(u)du\Big\} \leq 0.$$

Therefore, the conditions of Theorem 4.2 are satisfied. The state trajectory of ISSNN (4.1) stabilized by impulsive control through the impulse density strategy is depicted in Figure 15, which shows that the obtained impulsive control signal can stabilize the considered ISSNNs under switching signals.

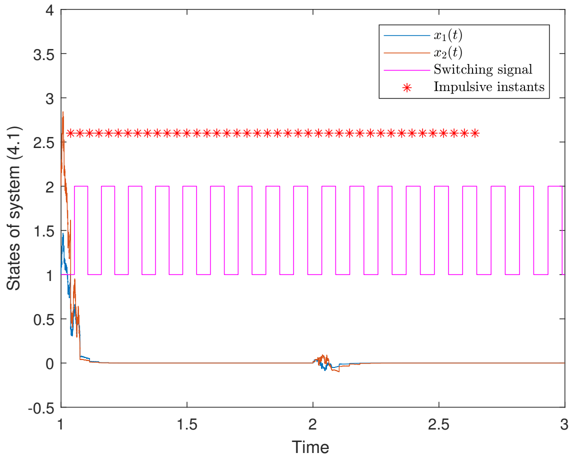


Figure 15. The state of (4.1) with switching density $\psi_q(t) \equiv \psi(t) = 17 + \frac{2}{t}$ and with impulse density $\phi_{ij}(t) \equiv \phi(t) = 22 + \frac{5}{t}$.

7. Conclusions

The article focuses on a type of ISSS with DSDDs. The unstable impulsive dynamics and dynamics of unstable continuous subsystems have been explored independently. In fact, the concept of mode-denependt switching density and mode-dependent impulsive density are extensions of the MDADT and MDAII repectively, with fewer constraints than typical ADT switching and AII impulses. In addition, applying theoretical results for neural networks to the neural network-based lecture skills assessment of normal students has been examined. In the end, two numerical examples were used to show theoretical efficacy and validity.

Use of AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

Acknowledgments

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Conflict of interest

The authors declare that there are no conflicts of interest.

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