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Research article

Study of optical stochastic solitons of Biswas-Arshed equation with multiplicative noise

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Abstract: In many nonlinear partial differential equations, noise or random fluctuation is an inherent part of the system being modeled and have vast applications in different areas of engineering and sciences. This objective of this paper is to construct stochastic solitons of Biswas-Arshed equation (BAE) under the influence of multiplicative white noise in the terms of the Itô calculus. Bright, singular, dark, periodic, singular and combined singular-dark stochastic solitons are attained by using the Sardar subequation method. The results prove that the suggested approach is a very straightforward, concise and dynamic addition in literature. By using Mathematica 11, some 3D and 2D plots are illustrated to check the influence of multiplicative noise on solutions. The presence of multiplicative noise leads the fluctuations and have significant effects on the long-term behavior of the system. So, it is observed that multiplicative noise stabilizes the solutions of BAE around zero.

Keywords: Sardar subequation method (SSM); Biswas-Arshed equation (BAE); nonlinear evolution equations (NLEEs); multiplicative noise
Mathematics Subject Classification: 35C05, 35C07, 35R11

1. Introduction

NLEEs have made remarkable contributions in scientific fields including hydrodynamics, biology, chemistry, fluid mechanics, fiber optics, mathematical physics, elasticity theory, plasma physics and engineering. The investigation of exact solutions for NLEEs through different techniques plays an energetic part in physics and has become thrilling subject for researchers. The soliton solutions have many applications in different fields of neural physics, mathematical biology, optical fibers chaos and solid state physics [1–12]. Nowadays, many techniques are used for finding optical soliton solutions such as the He's semi-inverse variational principle [13], F-expansion technique [14], the auxiliary equation approach [15], Kudryashov's method [16,17], the extended tanh technique [18], the semi-inverse variational method [19], the improved generalized Riccati's equation mapping method [20], the simplest equation method [21], the trial equation method [22], the first integral method [23,24], the quasi-stationary solution approach [25], the asymmetric method [26], modified simple equation scheme [27,28], the sine-cosine method [29] and Lie symmetry approach [30].

On the other hand, many stochastic NLEEs are commonly used to study noise in many phenomenons such as predication and simulation in the fields of climate dynamics, physics, atmosphere, biology and other fields [31–33]. The aim of these models is to capture the impact of random or stochastic factors on the behavior of the system. One area of significant interest in this field is the study of stochastic waves, which have been the subject of extensive research by many authors [34–36]. Optical solitons are a particular type of wave that is used to transfer the data over long distances with minimum loss or error [37]. However, the transmission of signals over long distances can be affected by natural conditions such as atmospheric turbulence, which can lead to stochastic distortions that have a significant impact on the quality of the transmitted data. Recently, stochastic NLEEs are gaining attention and have capability of describing complex physical phenomenons with stochastic behavior. Many authors used different techniques to extract various forms of stochastic solutions such as Khan et al. [38] who conducted a study on the stochastic perturbation of solitons along anti-cubic nonlinearity using multi-photon absorption. Secer [39] studied the stochastic solutions with multiplicative noise in itô sense. Other researchers have also explored the stochastic soliton solutions of Broer-Kauf equations in fluid or plasma and (2+1) dimension nonlinear Schrödinger equation [40–41].

As a result, this paper studies the BAE with multiplicative noise in the terms Itô calculus. Recently, mathematicians have proposed a lot of optical structure depends on the Schrödinger equation [42–45]. In recent times, Biswas and Arshad [46] developed a model from the nonlinear Schröodinger equation called BAE. The BAE is a NLPDE used to model the transmission of solitons, taking into the account the various nonlinear effects that occur in the system. A large variety of techniques are used to find optical solutions of BAE [47–50]. The BAE along higher order dispersion is described as

$$\iota y_t + g_1 y_{xx} + c y_{xt} + \iota(h_1 y_{xxx} + h_2 y_{xxt}) = \iota[\lambda(|y|^2 y)_x + u((|y|^2)_x)y + \theta|y|^2 y_x], \tag{1.1}$$

where the wave behavior is described by $y(x,t)$, which depends on variables: x (spatial) and t (temporal). The first term, g_1 and c indicate the temporal evolution of the wave, group velocity dispersion (GVD) and spatio-temporal dispersion (STD) respectively. Next, h_1 represents the 3rd order dispersion (3OD) and spatio-temporal 3OD is provided by the coefficient of h_2. These effects compensate for the low count of GVD. Self-phase modulation effect is absent while the right side of equation shows the nonlinear effects. λ is coefficient of steeping dispersion while u and θ represent the effects of nonlinear dispersion.

The current work deals with newly constructed following BAE with the having multiplicative noise by Itô calculus [51].

$$\begin{aligned} &\iota y_t + g_1 y_{xx} + c y_{xt} + \iota(h_1 y_{xxx} + h_2 y_{xxt}) + \varrho(y - \iota g_2 y_x + h_2 y_{xx})\frac{d\mathrm{W(t)}}{dt} \\ &= \iota[\lambda(|y|^2 y)_x + u((|y|^2)_x)y + \theta|y|^2 y_x], \end{aligned} \tag{1.2}$$

where $y(x,t)$ represents the wave profile, g_1, g_2, h_1, h_2, λ show the chromatic dispersion, STD, 3OD, 3rd order chromatic dispersion, coefficient of steeping term respectively, W(t) represents the Wiener process and $\frac{dW(t)}{dt}$ is white noise. u and θ are due to the effects of nonlinear dispersion. By taking $\varrho = 0$, the model in (1.2) reduces into (1.1).

The ambition of this paper is to employ SSM [52–58] for a BAE to acquire stochastic optical solitons. The method yields a sorts of solutions including stochastic bright, dark, singular, combined dark-singular and periodic soliton solutions. The motivation behind this work is to enhance previous efforts in [51] by offering new solutions such as combined singular-dark stochastic solutions that help in the area of telecommunications for data transmission. Then, by doing some mathematical calculations and plotting graphs, some physical characteristics of related shape of waves are investigated.

The paper is organized as: Section 2 gives definitions of Wiener process and white noise. In Section 3, narrative of method is given. Section 4 provides mathematical preliminaries. In Section 5, application of SSM for model (1.2) is discussed. Section 6 gives graphical illustration. Lastly, conclusion is discussed in Section 7.

2. Wiener process and white noise [59,60]

(i) The **Wiener process** W(t), $t \geq 0$ have the following properties
• W(t)=0,
• W(t_1)-W(t_2) for $t_1 < t_2$ is independent,
• W(t), $t \geq 0$ is continuous function of t,
• W(t_2)-W(t_1) possesses a Gaussian distribution with variance $t_2 - t_1$ and mean 0.
(ii) The **white noise** is the time derivative of Wiener process and is considered as a mathematical idealization of events such as enormous and sudden fluctuations.

3. Description of SSM

This part explains the SSM [52–58], suppose nonlinear partial differential equation (NLPDE)

$$R(y, y_t, y_x, y_{tt}, y_{xx}, ...) = 0, \tag{3.1}$$

Let the transformation of the form

$$y(x,t) = U(\zeta), \quad \zeta = x - vt, \tag{3.2}$$

switching (3.2) into (3.1), we get

$$T(U, U', U'', U''', ...) = 0, \tag{3.3}$$

where $U = U(\zeta)$, $U' = \frac{dU}{d\zeta}$, $U'' = \frac{d^2U}{d\zeta^2}$.
Let (3.3) gives solution

$$U(\zeta) = \sum_{i=0}^{m} \alpha_i \chi^i(\zeta), \tag{3.4}$$

where α_i, (i=0, 1, 2, ..., m) are coefficients and $\chi(\zeta)$ satisfies the ODE in the form

$$(\chi'(\zeta))^2 = \varsigma + d\chi^2(\zeta) + \chi^4(\zeta), \tag{3.5}$$

The solutions of (3.5) are

Case1: If ς=0 and $d > 0$ then

$$\chi_1^{\pm}(\zeta) = \pm\sqrt{-pqd}\,\mathbf{sech_{pq}}\,(\sqrt{d}\zeta),$$
$$\chi_2^{\pm}(\zeta) = \pm\sqrt{pqd}\,\mathbf{csch_{pq}}\,(\sqrt{d}\zeta),$$

where

$\mathbf{sech_{pq}}(\zeta) = \frac{\mathbf{2}}{\mathbf{pe^{\zeta}+qe^{-\zeta}}}, \quad \mathbf{csch_{pq}}(\zeta) = \frac{\mathbf{2}}{\mathbf{pe^{\zeta}-qe^{-\zeta}}}.$

Case 2: If ς=0 and $d < 0$ then

$$\chi_3^{\pm}(\zeta) = \pm\sqrt{-pqd}\,\mathbf{sec_{pq}}\,(\sqrt{-d}\zeta),$$
$$\chi_4^{\pm}(\zeta) = \pm\sqrt{-pqd}\,\mathbf{csc_{pq}}\,(\sqrt{-d}\zeta),$$

where

$\mathbf{sec_{pq}}(\zeta) = \frac{\mathbf{2}}{\mathbf{pe^{\iota\zeta}+qe^{-\iota\zeta}}}, \quad \mathbf{csc_{pq}}(\zeta) = \frac{\mathbf{2\iota}}{\mathbf{pe^{\iota\zeta}-qe^{-\iota\zeta}}}.$

Case 3: If $d < 0$ and $\varsigma = \frac{d^2}{4}$ then

$$\chi_5^{\pm}(\zeta) = \pm\sqrt{\frac{-d}{2}}\mathbf{tanh_{pq}}(\sqrt{\frac{-d}{2}}\zeta),$$

$$\chi_6^{\pm}(\zeta) = \pm\sqrt{\frac{-d}{2}}\,\mathbf{coth_{pq}}\,(\sqrt{\frac{-d}{2}}\zeta),$$

$$\chi_7^{\pm}(\zeta) = \pm\sqrt{\frac{-d}{2}}\,(\mathbf{tanh_{pq}}\,(\sqrt{-2d}\zeta) \pm \iota\sqrt{pq}\,\mathbf{sech_{pq}}\,(\sqrt{-2d}\zeta)),$$

$$\chi_8^{\pm}(\zeta) = \pm\sqrt{\frac{-d}{2}}\,(\mathbf{coth_{pq}}\,(\sqrt{-2d}\zeta) \pm \sqrt{pq}\mathbf{csch_{pq}}\,(\sqrt{-2d}\zeta)),$$

$$\chi_9^{\pm}(\zeta) = \pm\sqrt{\frac{-d}{2}}\,(\mathbf{tanh_{pq}}\,(\sqrt{\frac{-d}{2}}\zeta) + \mathbf{coth_{pq}}\,(\sqrt{\frac{-d}{2}}\zeta)),$$

where

$\mathbf{tanh_{pq}}(\zeta) = \frac{\mathbf{pe^{\zeta}-qe^{-\zeta}}}{\mathbf{pe^{\zeta}+qe^{-\zeta}}}, \quad \mathbf{coth_{pq}}(\zeta) = \frac{\mathbf{pe^{\zeta}+qe^{-\zeta}}}{\mathbf{pe^{\zeta}-qe^{-\zeta}}}.$

Case 4: If $d > 0$ and $\varsigma = \frac{d^2}{4}$ then

$$\chi_{10}^{\pm}(\zeta) = \pm\sqrt{\frac{d}{2}}\mathbf{tan_{pq}}(\sqrt{\frac{d}{2}}\zeta),$$

$$\chi_{11}^{\pm}(\zeta) = \pm\sqrt{\frac{d}{2}}\,\mathbf{cot_{pq}}\,(\sqrt{\frac{d}{2}}\zeta),$$

$$\chi_{12}^{\pm}(\zeta) = \pm\sqrt{\frac{d}{2}}\,(\mathbf{tan_{pq}}\,(\sqrt{2d}\zeta) \pm \sqrt{pq}\mathbf{sec_{pq}}\,(\sqrt{2d}\zeta)),$$

$$\chi_{13}^{\pm}(\zeta) = \pm\sqrt{\frac{d}{2}}(\mathbf{cot_{pq}}\,(\sqrt{2d}\zeta) \pm \sqrt{pq}\mathbf{csc_{pq}}\,(\sqrt{2d}\zeta)),$$

$$\chi_{14}^{\pm}(\zeta) = \pm\sqrt{\frac{d}{2}}(\mathbf{tan_{pq}}\,(\sqrt{\frac{d}{2}}\zeta) + \mathbf{cot_{pq}}\,(\sqrt{\frac{d}{2}}\zeta)),$$

where

$\mathbf{tan_{pq}}(\zeta) = -\iota\frac{\mathbf{pe^{\iota\zeta}-qe^{\iota-\zeta}}}{\mathbf{pe^{\iota\zeta}+qe^{-\iota\zeta}}}, \quad \mathbf{cot_{pq}}(\zeta) = \iota\frac{\mathbf{pe^{\iota\zeta}+qe^{\iota-\zeta}}}{\mathbf{pe^{\iota\zeta}-qe^{-\iota\zeta}}}.$

The m in (3.4) is determined by using balancing rule. After inserting (3.4) and (3.5) into (3.3), the equations in powers of $\chi(\zeta)$ are obtained. By equating all coefficient of power of $\chi(\zeta)$ equal to zero, the values of unknown parameters are found and then using these parameters with help of (3.4), the solutions of (3.3) are acquired.

4. Mathematical preliminaries

To solve the (1.2), assume the transformation

$$y(x,t) = U(\zeta)e^{\iota(\phi(x,t)+\varrho \mathrm{W(t)}-\varrho^2 t)}, \tag{4.1}$$

and

$$\phi(x,t) = -kx + \varpi t, \qquad \zeta = x - vt, \tag{4.2}$$

where k, ϖ and v are constants while $U(\zeta)$ and $\phi(x,t)$ are functions of real numbers. ζ, ϖ, k, v, $\phi(x,t)$ shows wave variable, wave number, frequency, velocity and phase component.

By inserting (4.1) and (4.2) into (1.2), we get real part

$$\begin{aligned}&[g_1 + 3h_1k - h_2(2kv + \varpi - \varrho^2) - g_2v]U'' - k(\lambda + \theta)U^3\\&-[g_1k^2 + h_1k^3 + (1 - g_2k - h_2k^2)(\varpi - \varrho^2)]U = 0,\end{aligned} \tag{4.3}$$

and imaginary part

$$\begin{aligned}&(h_1 - h_2)vU''' - (3\lambda + 2u + \theta)U^2U'\\&-[2g_1k + 3h_1k^2 - (g_2 - 2kh_2)(\varpi - \varrho^2) + (1 - g_2k - h_2k^2)v]U' = 0.\end{aligned} \tag{4.4}$$

After integrating (4.4), we have

$$\begin{aligned}&(h_1 - h_2)vU'' - \frac{1}{3}(3\lambda + 2u + \theta)U^3\\&-[2g_1k + 3h_1k^2 - (g_2 - 2kh_2)(\varpi - \varrho^2) + (1 - g_2k - h_2k^2)v]U = 0.\end{aligned} \tag{4.5}$$

Equations (4.3) and (4.4) have same form under some restrictions [25]

$$\begin{aligned}\frac{g_1 + 3h_1k - h_2(2kv + \varpi - \varrho^2) - g_2v}{(h_1 - h_2)v} &= \frac{3k(\lambda + \theta)}{3\lambda + 2u + \theta}\\&= \frac{-g_1k^2 + h_1k^3 + (1 - g_2k - h_2k^2)(\varpi - \varrho^2)}{2g_1k + 3h_1k^2 - (g_2 - 2kh_2)(\varpi - \varrho^2) + (1 - g_2k - h_2k^2)v}.\end{aligned} \tag{4.6}$$

Under the constraint

$$\theta - u = 0,$$

the value of ϖ from (4.6) is

$$\varpi = \frac{(g_2 - 2h_2^2k^3 - 2g_2h_2k^2)\varrho^2 + 2(g_1h_2 + 2g_2h_1)k^3 + 4h_1h_2k^4 + 2(g_1g_2 - h_1)k^2 - g_1k}{(g_2 - 2h_2^2k^3 - 2g_2h_2k^2)}$$

and velocity

$$v = \frac{(g_1 + 2h_1k)(1 - 4h_2k^2)}{(g_2 - 2h_2^2k^3 - 2g_2h_2k^2)}.$$

5. Application of SSM for BAE

By applying the balancing rule on (4.3), the $m = 1$ is obtained and (3.4) becomes

$$U(\zeta) = \alpha_0 + \alpha_1\chi(\zeta), \tag{5.1}$$

Now by putting (5.1) and (3.5) into (4.3), we get equation with different powers of $\chi(\zeta)$. By equating their constants equal to zero, the following equations are extracted

$$\begin{aligned}
&-g_1k^2\alpha_0 - g_2k\varrho^2\alpha_0 + g_2k\varpi\alpha_0 - h_1k^3\alpha_0 - h_2k^2\varrho^2\alpha_0 + h_2k^2\varpi\alpha_0 - \theta k\alpha_0^3\\
&-k\lambda\alpha_0^3 + \varrho^2\alpha_0 - \varpi\alpha_0 = 0,\\
&3dh_1k\alpha_1 + dh_2\varrho^2\alpha_1 - 2dh_2kv\alpha_1 - dh_2\varpi\alpha_1 - g_1k^2\alpha_1 - g_2k\varrho^2\alpha_1 + g_2k\varpi\alpha_1 - dg_2v\alpha_1 +\\
&dg_1\alpha_1 - h_1k^3\alpha_1 - h_2k^2\varrho^2\alpha_1 + h_2k^2\varpi\alpha_1 - 3\theta k\alpha_0^2\alpha_1 - 3k\lambda\alpha_0^2\alpha_1 + \varrho^2\alpha_1 - \varpi\alpha_1 = 0,\\
&-3\theta k\alpha_0\alpha_1^2 - 3k\lambda\alpha_0\alpha_1^2 = 0,\\
&-2g_2v\alpha_1 + 2g_1\alpha_1 + 6h_1k\alpha_1 + 2h_2\varrho^2\alpha_1 - 4h_2kv\alpha_1 - 2h_2\varpi\alpha_1 - \theta k\alpha_1^3 - k\lambda\alpha_1^3 = 0.
\end{aligned}$$

After solving above equations, the following values are obtained

$$\alpha_0 = 0, \quad \alpha_1 = \frac{\sqrt{2(g_2k\varrho^2-g_2k\varpi+g_2k^2v-2h_1k^3+2h_2k^3v-\varrho^2+\varpi)}}{\sqrt{d\theta k+dk\lambda-\theta k^3-k^3\lambda}},$$

$$g_1 = \frac{-3dh_1k-dh_2(\varrho^2+2kv+\varpi)+g_2k(\varrho^2-\alpha)+dg_2v+h_1k^3+(h_2k^2-1)\varrho^2-(g_2k^2+1)\varpi}{d-k^2}.$$

By putting these values along (3.5) in (5.1) and (4.1), the solutions are described as
Case1: If ς=0 and $d > 0$ then

$$U_1 = \frac{\sqrt{2(g_2k\varrho^2 - g_2k\varpi + g_2k^2v - 2h_1k^3 + 2h_2k^3v - \varrho^2 + \varpi)}}{\sqrt{d\theta k + dk\lambda - \theta k^3 - k^3\lambda}}\left(\pm\sqrt{-pqd}\,\mathbf{sech_{pq}}(\sqrt{d}\zeta)\right),$$

$$y_1 = U_1(\zeta)e^{\iota(\phi(x,t)+\varrho\mathrm{W}(\mathrm{t})-\varrho^2t)}.$$

$$U_2 = \frac{\sqrt{2(g_2k\varrho^2 - g_2k\varpi + g_2k^2v - 2h_1k^3 + 2h_2k^3v - \varrho^2 + \varpi)}}{\sqrt{d\theta k + dk\lambda - \theta k^3 - k^3\lambda}}\left(\pm\sqrt{pqd}\,\mathbf{csch_{pq}}(\sqrt{d}\zeta)\right),$$

$$y_2 = U_2(\zeta)e^{\iota(\phi(x,t)+\varrho\mathrm{W}(\mathrm{t})-\varrho^2t)}.$$

Case2: If ς=0 and $d < 0$ then

$$U_3 = \frac{\sqrt{2(g_2k\varrho^2 - g_2k\varpi + g_2k^2v - 2h_1k^3 + 2h_2k^3v - \varrho^2 + \varpi)}}{\sqrt{d\theta k + dk\lambda - \theta k^3 - k^3\lambda}}\left(\pm\sqrt{-pqd}\,\mathbf{sec_{pq}}(\sqrt{-d}\zeta)\right),$$

$$y_3 = U_3(\zeta)e^{\iota(\phi(x,t)+\varrho\mathrm{W}(\mathrm{t})-\varrho^2t)}.$$

$$U_4 = \frac{\sqrt{2(g_2k\varrho^2 - g_2k\varpi + g_2k^2v - 2h_1k^3 + 2h_2k^3v - \varrho^2 + \varpi)}}{\sqrt{d\theta k + dk\lambda - \theta k^3 - k^3\lambda}}\left(\pm\sqrt{-pqd}\,\mathbf{csc_{pq}}(\sqrt{-d}\zeta)\right),$$

$$y_4 = U_4(\zeta)e^{\iota(\phi(x,t)+\varrho \mathrm{W(t)}-\varrho^2 t)}.$$

Case 3: If $d < 0$ and $\varsigma = \frac{d^2}{4}$ then

$$U_5 = \frac{\sqrt{2(g_2 k\varrho^2 - g_2 k\varpi + g_2 k^2 \nu - 2h_1 k^3 + 2h_2 k^3 \nu - \varrho^2 + \varpi)}}{\sqrt{d\theta k + dk\lambda - \theta k^3 - k^3\lambda}}\left(\pm\sqrt{\frac{-d}{2}}\mathbf{tanh_{pq}}(\sqrt{\frac{-d}{2}}\zeta)\right),$$

$$y_5 = U_5(\zeta)e^{\iota(\phi(x,t)+\varrho \mathrm{W(t)}-\varrho^2 t)}.$$

$$U_6 = \frac{\sqrt{2(g_2 k\varrho^2 - g_2 k\varpi + g_2 k^2 \nu - 2h_1 k^3 + 2h_2 k^3 \nu - \varrho^2 + \varpi)}}{\sqrt{d\theta k + dk\lambda - \theta k^3 - k^3\lambda}}\left(\pm\sqrt{\frac{-d}{2}}\mathbf{coth_{pq}}(\sqrt{\frac{-d}{2}}\zeta)\right),$$

$$y_6 = U_6(\zeta)e^{\iota(\phi(x,t)+\varrho \mathrm{W(t)}-\varrho^2 t)}.$$

$$U_7 = \frac{\sqrt{2(g_2 k\varrho^2 - g_2 k\varpi + g_2 k^2 \nu - 2h_1 k^3 + 2h_2 k^3 \nu - \varrho^2 + \varpi)}}{\sqrt{d\theta k + dk\lambda - \theta k^3 - k^3\lambda}}\left(\pm\sqrt{\frac{-d}{2}}\left(\mathbf{coth_{pq}}\left(\sqrt{-2d}\zeta\right)\right.\right.$$

$$\left.\left.\pm\sqrt{pq}\mathbf{csch_{pq}}\left(\sqrt{-2d}\zeta\right)\right)\right),$$

$$y_7 = U_7(\zeta)e^{\iota(\phi(x,t)+\varrho \mathrm{W(t)}-\varrho^2 t)}.$$

$$U_8 = \frac{\sqrt{2(g_2 k\varrho^2 - g_2 k\varpi + g_2 k^2 \nu - 2h_1 k^3 + 2h_2 k^3 \nu - \varrho^2 + \varpi)}}{\sqrt{d\theta k + dk\lambda - \theta k^3 - k^3\lambda)}}\left(\pm\sqrt{\frac{-d}{8}}\left(\mathbf{coth_{pq}}\left(\sqrt{\frac{-d\zeta}{8}}\right)\right.\right.$$

$$\left.\left.+\mathbf{tanh_{pq}}\left(\sqrt{\frac{-d\zeta}{8}}\right)\right)\right),$$

$$y_8 = U_8(\zeta)e^{\iota(\phi(x,t)+\varrho \mathrm{W(t)}-\varrho^2 t)}.$$

Case 4: If $d > 0$ and $\varsigma = \frac{d^2}{4}$ then

$$U_9 = \frac{\sqrt{2(g_2 k\varrho^2 - g_2 k\varpi + g_2 k^2 \nu - 2h_1 k^3 + 2h_2 k^3 \nu - \varrho^2 + \varpi)}}{\sqrt{d\theta k + dk\lambda - \theta k^3 - k^3\lambda}}\left(\pm\sqrt{\frac{d}{2}}\mathbf{tan_{pq}}(\sqrt{\frac{d}{2}}\zeta)\right),$$

$$y_9 = U_9(\zeta)e^{\iota(\phi(x,t)+\varrho \mathrm{W(t)}-\varrho^2 t)}.$$

$$W_{10} = \frac{\sqrt{2(g_2 k\varrho^2 - g_2 k\varpi + g_2 k^2 \nu - 2h_1 k^3 + 2h_2 k^3 \nu - \varrho^2 + \varpi)}}{\sqrt{d\theta k + dk\lambda - \theta k^3 - k^3\lambda}}\left(\pm\sqrt{\frac{d}{2}}\mathbf{cot_{pq}}(\sqrt{\frac{d}{2}}\zeta)\right),$$

$$y_{10} = U_{10}(\zeta)e^{\iota(\phi(x,t)+\varrho \mathrm{W(t)}-\varrho^2 t)}.$$

$$U_{11} = \frac{\sqrt{2(g_2 k\varrho^2 - g_2 k\varpi + g_2 k^2 \nu - 2h_1 k^3 + 2h_2 k^3 \nu - \varrho^2 + \varpi)}}{\sqrt{d\theta k + dk\lambda - \theta k^3 - k^3\lambda}}\left(\pm\sqrt{\frac{d}{2}}\left(\mathbf{tan_{pq}}\left(\sqrt{2d}\zeta\right)\right.\right.$$

$$\pm\sqrt{pq}\mathbf{sec_{pq}}\left(\sqrt{2d}\zeta\right)\Big),$$

$$y_{11}=U_{11}(\zeta)e^{\iota(\phi(x,t)+\varrho\mathrm{W(t)}-\varrho^2 t)}.$$

$$U_{12}=\frac{\sqrt{2(g_2k\varrho^2-g_2k\varpi+g_2k^2v-2h_1k^3+2h_2k^3v-\varrho^2+\varpi)}}{\sqrt{d\theta k+dk\lambda-\theta k^3-k^3\lambda}}\left(\pm\sqrt{\frac{d}{2}}\Big(\mathbf{cot_{pq}}\left(\sqrt{2d}\zeta\right)\right.$$

$$\left.\pm\sqrt{-pq}\mathbf{csc_{pq}}\left(\sqrt{2}\sqrt{d}\zeta\right)\Big)\right),$$

$$y_{12}=U_{12}(\zeta)e^{\iota(\phi(x,t)+\varrho\mathrm{W(t)}-\varrho^2 t)}.$$

$$U_{13}=\frac{\sqrt{2(g_2k\varrho^2-g_2k\varpi+g_2k^2v-2h_1k^3+2h_2k^3v-\varrho^2+\varpi)}}{\sqrt{d\theta k+dk\lambda-\theta k^3-k^3\lambda}}\left(\pm\sqrt{\frac{d}{2}}\left(\mathbf{cot_{pq}}\left(\sqrt{\frac{d\zeta}{8}}\right)\right.\right.$$

$$\left.\left.+\mathbf{tan_{pq}}\left(\sqrt{\frac{d\zeta}{8}}\right)\right)\right),$$

$$y_{13}=U_{13}(\zeta)e^{\iota(\phi(x,t)+\varrho\mathrm{W(t)}-\varrho^2 t)}.$$

6. The influence of noise on BAE

In this paper, the novel solutions in form of stochastic optical solitons for BAE using SSM are effectively constructed. This method is not applied to this model earlier and considered as most recent technique to obtain the solutions of NLEEs. In order to scrutinize the influence of noise on solutions, the graphical illustration of these solutions via 3D and 2D plots are represented with appropriate values of parameters. These obtained solutions have many applications in communication because without changing its shapes, solitons transfer the data over long distances. Attained results are distinct and novel from that reported results. Only particular figures are drawn to avoid ambiguity.

• For the parameters of $d=\pm 2$, $\theta=-3$, $h_1=1$, $h_2=1.5$, $k=1$, $\lambda=0.5$, $v=1$, $p=0.5$, $g_2=1.09$, $q=0.98$, $\varpi=1$ and different values of ϱ, the physical behavior of y_1, y_4, y_5, y_6, and y_8 are shown in Figures 1–5 which represent the bright, dark, singular, periodic singular and combined dark-singular stochastic soliton solutions respectively.

• Furthermore, if the multiplicative noise is increased, the surface becomes fluctuated. Hence, it is observed that the solutions of BAE sustain by multiplicative noise around zero.

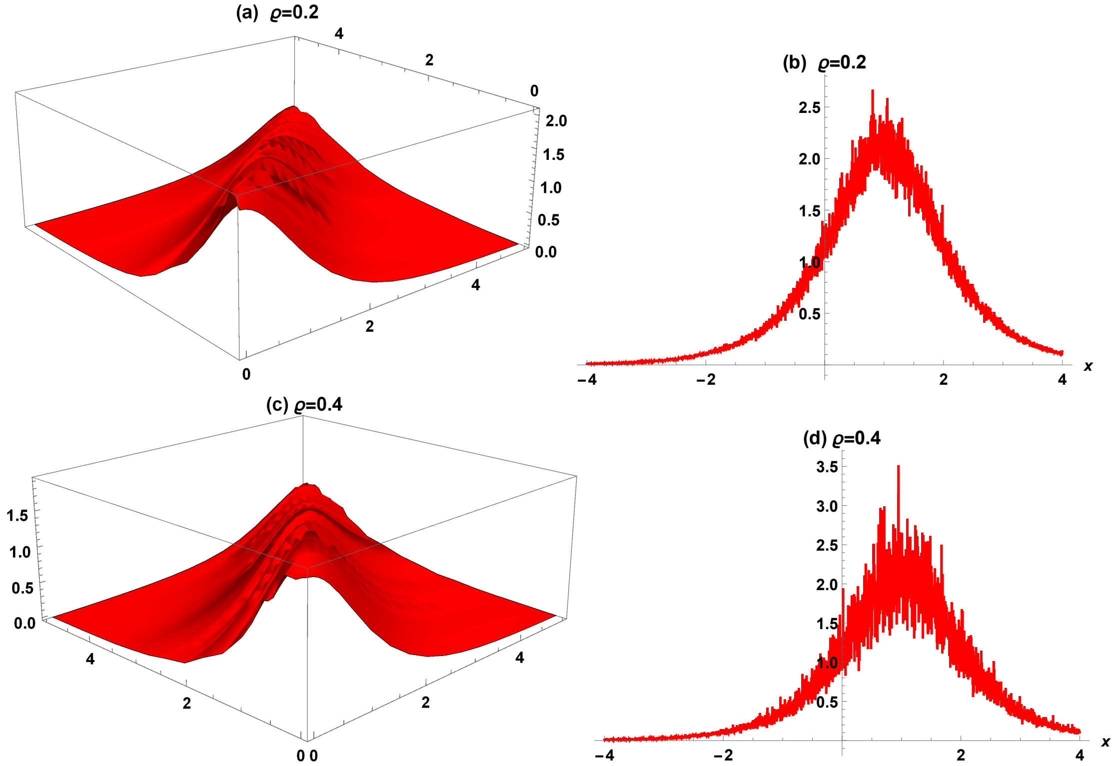

Figure 1. Graphical illustration of bright stochastic soliton to y_1.

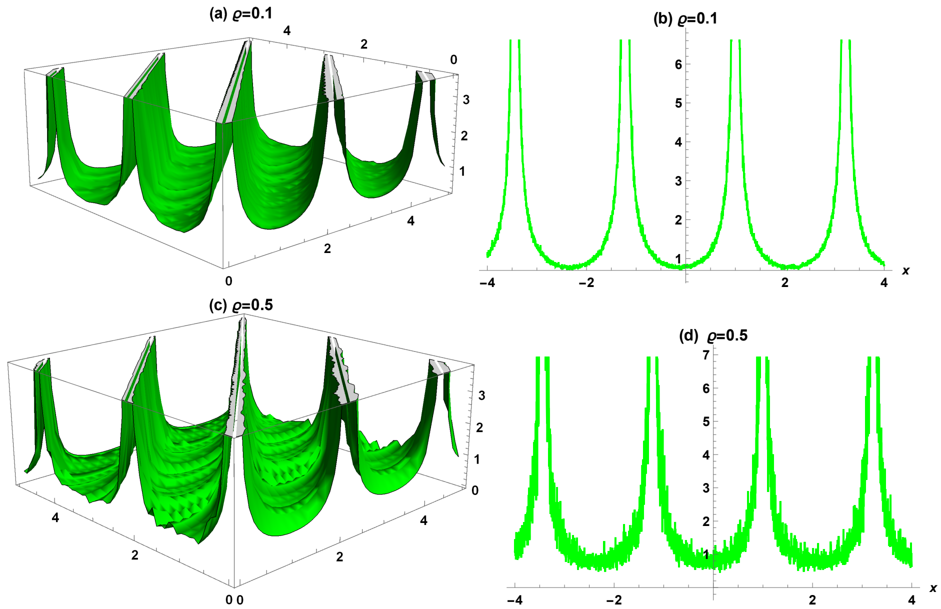

Figure 2. Graphical illustration of periodic stochastic soliton to y_4.

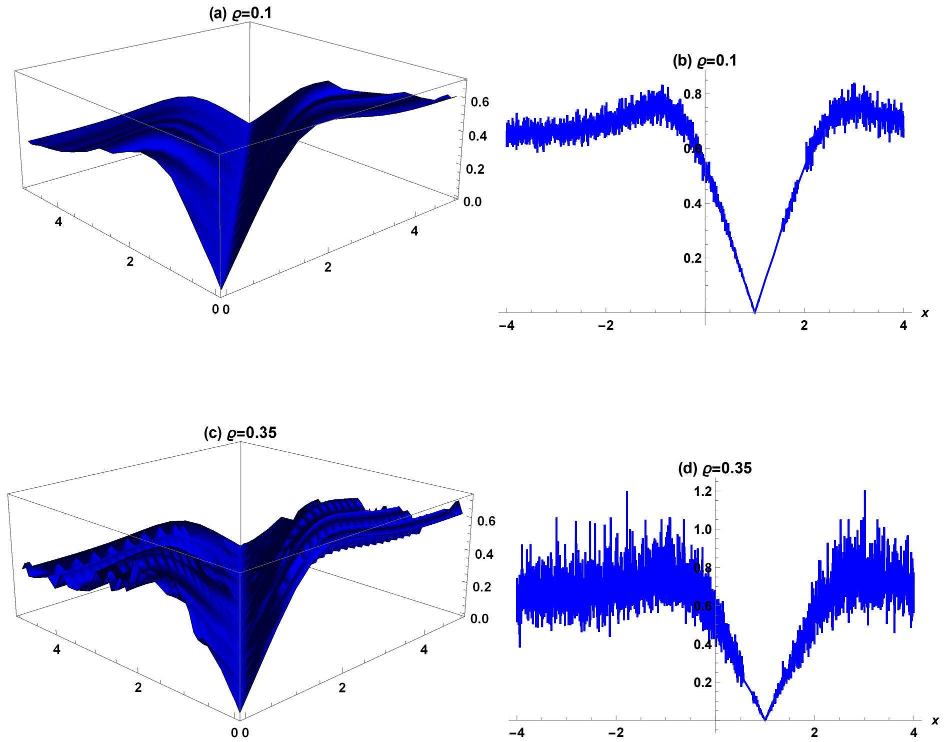

Figure 3. Graphical illustration of dark stochastic soliton to y_5.

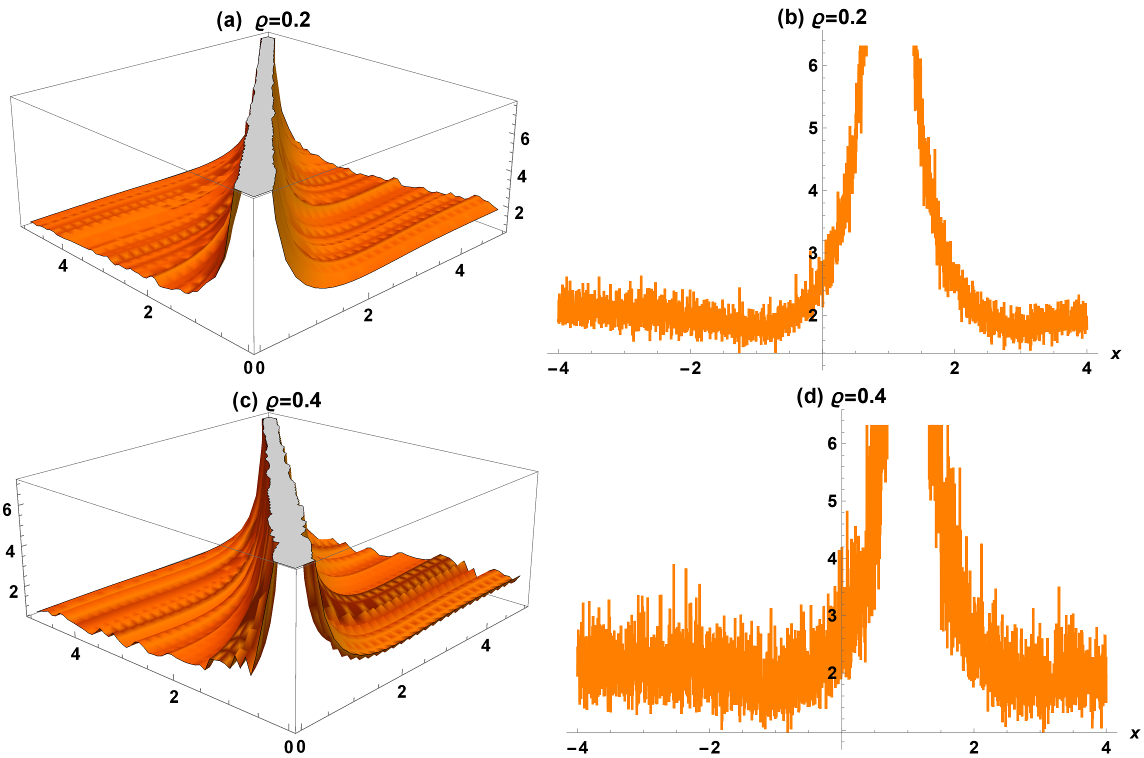

Figure 4. Graphical illustration of singular stochastic soliton to y_6.

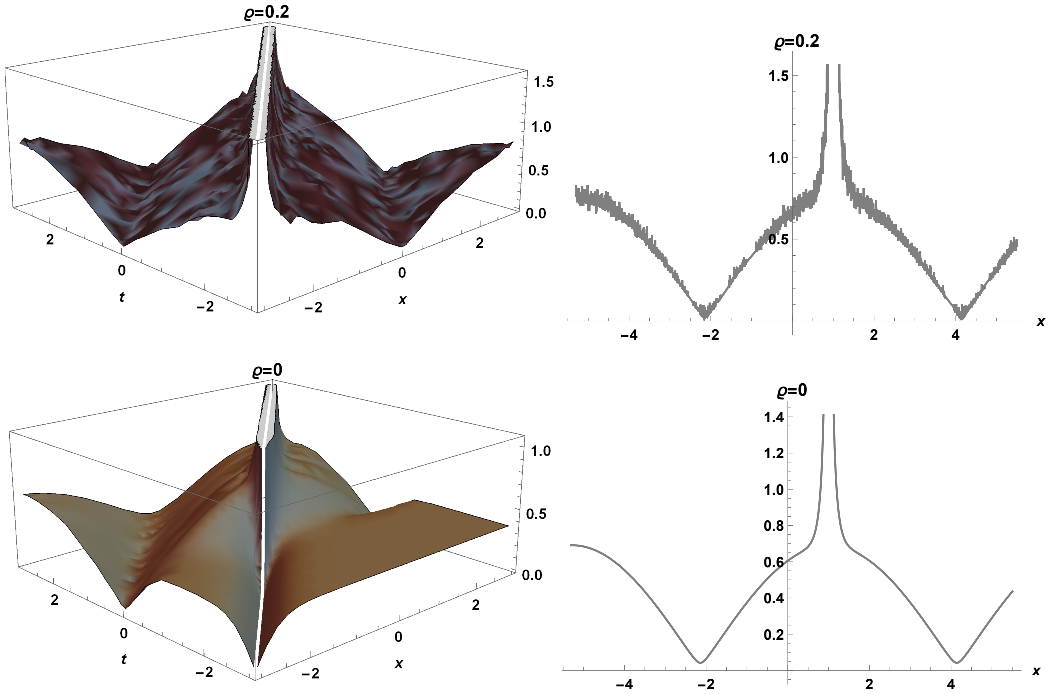

Figure 5. Graphical illustration of combined singular-dark stochastic soliton to y_8.

7. Conclusions

This paper successfully applies the SSM to study the BAE with multiplicative white noise. This analysis yields a variety of stochastic solutions including dark, bright, combined dark-bright, periodic, combined dark-singular and singular solitons. Compared to previous studies [51], our results provide new additional solutions like combined dark-singular solitons. Moreover, to illustrate the features of these solutions, the 3D and 2D plots are also illustrated. In order to check the effect of noise term, Figures 1–5 indicate that an increase in the noise term coefficient causes an increase in fluctuations. The graph without noise demonstrates no fluctuations as shown in Figure 5.

Use of AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

Conflict of interest

The authors declare that they have no competing interest.

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