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Research article

EWMA control chart using Bayesian approach under paired ranked set sampling schemes: An application to reliability engineering

Imad Khan[1,*], Muhammad Noor-ul-Amin[2], Narjes Turki Khalifa[3,*] and Asma Arshad[4]

[1] Department of Statistics, Abdul Wali Khan University Mardan, Pakistan
[2] Department of Statistics, COMSATS University Islamabad, Lahore Campus, Pakistan
[3] Department of Mathematics, AL-Qunfudhah University college, Umm Al-Qura University, KSA
[4] Department of Statistics, National College of Business Administration and Economics, Lahore, Pakistan

* **Correspondence:** Email: imad_khan@awkum.edu.pk, nbkhalifa@uqu.edu.sa.

Abstract: Control charts are widely used to efficiently detect small to moderate shifts and they include exponentially moving average control charts, named memory type control charts. Today, memory type control charts are a significant tool to assure quality standards and monitor manufacturing goods. The proposed study suggests a novel Bayesian exponentially weighted moving average (EWMA) control chart design utilizing various pair ranked set sampling schemes for posterior and posterior predictive distributions given an informative prior. The proposed chart strategy is evaluated in terms of the small run length characteristic by using Monte Carlo simulation methods. The comparative analysis is also carried out by using a Bayesian EWMA control chart to apply simple random sampling for the respective average and standard deviation of the run length values in the both control chart designs. The results revealed efficient and rapid detection of shifts in process means which proves the success and superiority of the suggested design. A real-life data sets is used to elaborate the efficient application of the suggested Bay-EWMA-PRSS control chart design. The overall research findings support the theoretical and simulation results, which are provided in the form of extensive tables.

Keywords: statistical process control; EWMA; Bayesian approach; loss function; average run length; PRSS
Mathematics Subject Classification: 62A86

1. Introduction

To overcome the problems in the production process in industries, statistical process control (SPC) tools are used because they have the ability to diagnose the system of the production process. The quality control charts are important tools of the SPC to monitor the infrequent variations in the production process and to detect nonconforming/ defective items. The main purpose of SPC is to enhance the quality of finished products by detecting and addressing unusual variations that can impact process stability. SPC is designed to continuously monitor processes and identify areas for improvement (Montgomery [1]). The concept of control charts was first introduced by Walter A. Shewhart [2]. Shewhart's control charts are often referred to as memoryless type control charts, as they rely solely on current sample information to analyze process performance. The memoryless type control charts are easily implemented and interpreted although they are very sensitive to a large shift in the production process. Page [3] and Roberts [4] suggested the techniques of memory type control charts which are useful for detecting the small/ moderate shift in the production process. The memory type control charts not only use the current sample information but also the past sample information is known as a cumulative sum (CUSUM) and exponentially weighted moving average (EWMA) control charts, respectively. Several authors have studied the use of CUSUM and EWMA control charts to monitor production processes, including Sweet [5], Lu and Reynolds [6], Maravelakis and Castagliola [7], Huwang et al. [8], H. Altoum et al. [9], Noor-ul-Amin et al. [10] and Makhlouf et al. [11]. Estimation and hypothesis testing are the two main branches of statistical inference. The procedure which is performed to estimate the unknown population parameter is known as an estimation. The estimation of the unknown parameter is done via two approaches *i.e.* classical approach and the Bayesian approach. The classical method of estimation relies solely on available sample information and does not consider any prior knowledge of population parameters. In contrast, the Bayesian approach to estimation uses both sample and prior information to derive Bayes estimators. Menzefricke [12] suggested a Bayesian control chart for unknown location parameters and also studied a control chart for dispersion. Menzefricke [13] has investigated the combined performance of the process mean and variance in the Bayesian EWMA control chart, which is based on the normal distribution assumption for the population variance. In Bayesian analysis, the loss function (LF) plays a significant role in reducing risk related to the Bayes estimator. The best Bayes estimator utilizes the LF to get maximum precision. Tsui and Woodall [14] proposed a multivariate control chart with different LFs. Wu and Tian [15] proposed the use of the CUSUM control chart, which employs a weighted LF to detect shifts in both the mean and variance of the production process. Elghribi et al. [16] studied the homogenous function using new characterization and their application to improve the detection power. Riaz et al. [17] proposed a Bayesian EWMA control chart that incorporates informative and non-informative priors to obtain posterior and posterior predictive distributions. The authors used three different functions in their study. A Bayesian hybrid EWMA control chart for posterior and posterior predictive distributions given informative and non-informative prior using different LF was studied by Noor et al. [18]. To assess the performance of a control chart, the average run length (ARL) and standard deviation of run length (SDRL) are commonly used. In their study, Noor et al. [19] proposed a Bayesian adaptive EWMA (AEWMA) control chart that employs various LFs to monitor changes in the process mean.

The above-mentioned works were completed for classical and Bayesian approaches using simple random sampling schemes for monitoring the process mean. Paired ranked set sampling (PRSS) is a sampling method commonly used in quality control to reduce the cost of data collection while improving the accuracy and efficiency of the control chart. By selecting paired samples, PRSS can improve the precision of the estimate compared to simple random sampling. Additionally, PRSS is useful for monitoring quality control processes because it can help capture changes in the underlying population distribution over time. Furthermore, the combination of PRSS with a Bayesian approach and an EWMA control chart can improve the performance of the control chart. This approach takes advantage of the efficient sampling method of PRSS and the flexibility of Bayesian updating to adjust the control limits based on the available data. By combining PRSS with the Bayesian approach in the EWMA control chart, it is possible to achieve significant advantages in scenarios where data collection costs are high or the sample size is limited. This approach allows for more precise control limits, which can improve the ability of the control chart to detect small shifts or changes in the process mean. In this regard, we suggest a Bayesian EWMA control chart that utilizes three different PRSS schemes: PRSS, extreme PRSS (EPRSS), and quartile PRSS (QPRSS). The control chart introduced in the study is constructed using informative priors and relies on two distinct LFs using posterior and posterior predictive distributions. To evaluate the efficacy of the proposed Bayesian EWMA control chart, the study measured its ARL and SDRL. The results of the study indicate that the proposed Bayesian EWMA control chart is effective in detecting small shifts or changes in the process mean. Additionally, the use of informative priors and different PRSS schemes can further enhance the performance of the control chart. The rest of paper is structured as follows: In Section 2, the Bayesian theory is introduced, including basic terminologies under different LFs. Section 3 discusses the PRSS, EPRSS, and QPRSS, while Section 4 presents the proposed Bayesian EWMA control chart. Section 5 includes an extensive simulation study, and Section 6 presents the results, discussions, and main findings. Real-life applications of the proposed control chart are presented in Section 7. Finally, Section 8 provides the conclusion and recommendations.

2. Bayesian inference

This section presents an overview of Bayesian terminologies, specifically the concept of posterior distributions and different types of prior distributions such as informative and non-informative priors, which are used in conjunction with various LF. This article considers the study variable *X*, which follows a normal distribution with a mean of θ and variance of δ^2. So, the details of the basic terminologies are as follows:

2.1. Posterior and prior distribution

Bayesian analysis combines sample information with prior information to obtain a posterior distribution. The choice of prior information for the study parameters is of central importance. If the prior information for the unknown population parameter is available, it is referred to as an informative prior. When the posterior density resembles the informative prior density, the prior is known as a conjugate prior.

In the present study, the conjugate prior (normal prior) with parameter θ considers the hyper-parameters θ_0 and δ_0^2 as

$$p(\theta) = \frac{1}{\sqrt{2\pi\delta_0^2}} exp\left\{-\frac{1}{2\delta_0^2}(\theta - \theta_0)^2\right\} \tag{1}$$

When prior information is not available, a non-informative prior is used, which has minimal influence on the inference. Bayes-Laplace proposed that when there is no prior knowledge about the parameter θ, a prior distribution proportional to a uniform distribution can be considered. In this case, all possible values of the parameter are assigned equal weightage. The expression for the uniform prior for the parameter θ in the case of a normal distribution with an unknown mean θ and a known variance σ^2 is given below:

$$p(\theta) \propto \sqrt{\frac{n}{\delta^2}} = c\sqrt{\frac{n}{\delta^2}} \tag{2}$$

Here, c is the constant of proportionality and n represent the sample size.

While the uniform prior distribution is commonly used in Bayesian analysis, it has been noted that this approach does not always satisfy the invariance property. In other words, different parameterization methods can lead to different posteriors and therefore different conclusions. To address this issue, Jeffreys [20] proposed a prior distribution proportional to Fisher's information, which ensures that prior probabilities are invariant across all possible parameterizations. This approach also results in the same posteriors regardless of the way the model is represented. The $p(\theta)$ is defined as: $p(\theta) \propto \sqrt{I(\theta)}$, where $I(\theta) = -E\left(\frac{\partial^2}{\partial\theta^2} log\, f\,(X/\theta)\right)$ known as Fisher's information, which measures the amount of information that a sample provides about an unknown parameter, and the likelihood function captures this information. To fully characterize the parameter, the prior knowledge and sample information are combined using Bayes' theorem to obtain the posterior distribution. The posterior distribution, denoted as $p(\theta|x)$, represents the updated probability distribution of an unknown parameter θ, given observed data *X* and any prior knowledge about θ. It is expressed mathematically as:

$$p(\theta|x) = \frac{p(x|\theta)\,p(\theta)}{\int p(x|\theta)\,p(\theta)d\theta} \tag{3}$$

The posterior predictive distribution is computed by applying the posterior distribution as the prior distribution. The posterior predictive distribution for a new data point *Y* is mathematized as:

$$p(y|x) = \int p(y|\theta)\,p(\theta|x)d\theta \tag{4}$$

2.2. Loss function

The LF is important to minimize the risk encountered by the estimator when applying Bayesian analysis to estimate the parameter. In the presented study, both symmetric and asymmetric LF values are considered as both play their significant roles in minimizing the risk influencing the estimation. A commonly used LF is the symmetric LF, which yields the same Bayes estimator as the posterior. On the other hand, asymmetric LF lead to different Bayes estimator results, which are treated as the posterior distribution. The Squared Error LF (SELF) is a type of symmetric LF suggested by Gauss [21].

2.2.1. Square error loss function

Gauss [21] proposed a symmetric LF known as the SELF. Suppose that θ is the unknown population parameter for the variable X under study, and $\hat{\theta}$ is an estimator that minimizes the expected LF. Then, the SELF is mathematized as:

$$L\left(\theta,\hat{\theta}\right)=\left(\theta-\hat{\theta}\right)^2 \tag{5}$$

and the Bayesian estimator under SELF becomes:

$$\hat{\theta}=E_{\theta/x}(\theta). \tag{6}$$

2.2.2. LINEX loss function

The LINEX LF (LLF) is an asymmetric LF, introduced by Varian [22]. The LLF is defined as:

$$L\left(\theta,\hat{\theta}\right)=\left(e^{c\left(\theta-\hat{\theta}\right)}-c\left(\theta-\hat{\theta}\right)-1\right) \tag{7}$$

where the estimator $\hat{\theta}$ is used to estimate the unknown population parameter θ, defined as

$$\hat{\theta}=-\frac{1}{c}InE_{\theta/x}\left(e^{-c\theta}\right). \tag{8}$$

3. Paired ranked set sampling

An efficient sampling scheme for estimating the population means named a PRSS scheme proposed by Muttlak [23]. In this sampling scheme, two units are selected from each set instead of a single sampling unit. The procedure to select sampling units under PRSS is as follows:

- For the even set size$l$, the $\left(\frac{l^2}{2}\right)$ sampling units are selected from the population.
- The selected sampling units are allocated at random in $\left(\frac{l}{2}\right)$ sets with the same sample size l. Every set size l is an array in increasing order based on auxiliary information or some sort of expert knowledge.
- After ranking each set the first and lth units are selected from the first set, the second and $(l-1)th$ unit are selected from the second set and the procedure remains to continue until the last set $\left(\frac{l}{2}\right)th$ and $\left(\left(\frac{l}{2}\right)+1\right)th$ sampling units are selected.
- In the case of an odd set size l, the $\left(\frac{l(l+1)}{2}\right)$ sampling units are selected. These units are allocated randomly in $\left(\frac{(l+1)}{2}\right)$ sets, each with the same size l.
- After arranging all values as some ordered sets, the first and lth sampling units are selected to from the first, 2nd and $(l-1)$th units are selected to form the 2nd set, and the sampling continue until the $\left(\frac{(l+1)}{2}\right)th$ unit is selected from the $\left(\frac{(l+1)}{2}\right)th$ set.

If needed, the two aforementioned steps are repeated *r* times to achieve the desired sample size. The PRSS procedure can be exemplified as follows: consider a set of observations labeled as *i, j*= 1, 2, 3, ..., l, and *c* sets of samples identified as 1, 2, 3, ..., c. Let the jth order statistic in the *i*th sample set with cycle *r* be denoted $Z_{i(i)}$.

If l is even, the mean and variance under PRSS are as follow:

$$\overline{Z}_{(PRSS)e} = \frac{1}{l}\left[\sum_{i=1}^{\frac{l}{2}} Z_{i(i)} + \sum_{i=1}^{\frac{l}{2}} Z_{i(l+1-i)}\right] \tag{9}$$

and

$$var\left(\overline{Z}_{(PRSS)e}\right) = var\left(\overline{Z}_{(RSS)}\right) + \frac{2}{l^2}\sum_{i=1}^{\frac{l}{2}}\sum_{i<l+1-i}^{\frac{l}{2}} cov\left(Z_{(i)}, Z_{(l+1-i)}\right). \tag{10}$$

If l is odd then

$$\overline{Z}_{(PRSS)o} = \frac{1}{l}\left[\sum_{i=1}^{\frac{(l+1)}{2}} Z_{i(i)} + \sum_{i=1}^{\frac{(l-1)}{2}} Z_{i(l+1-i)}\right] \tag{11}$$

and

$$var\left(\overline{Z}_{(PRSS)o}\right) = var\left(\overline{Z}_{(RSS)}\right) + \frac{2}{l^2}\sum_{i=1}^{\frac{(l-1)}{2}}\sum_{i<l+1-i}^{\frac{(l-1)}{2}} cov\left(Z_{(i)}, Z_{(l+1-i)}\right). \tag{12}$$

3.1. Extreme pair ranked set sampling

Balci et al. [24] suggested another modified PRSS scheme called
EPRSS, to selecting a representative sample from the population. The procedure for selecting a sample by EPRSS is as follows:

- if l is even, then $\left(\frac{l^2}{2}\right)$ sampling units are selected from the population and allocated randomly to $\left(\frac{l}{2}\right)$ sets with sample size l.
- Then l units in each set are ordered as in ranked set sampling; then, the smallest and largest ordered sampling units from each set are selected as samples.
- If *l* is an odd number, a total of (*l*(*l*+1)/2) sampling units are picked at random from the population and then divided into ((*l*+1)/2) sets with equal numbers of units.
- Once the population has been ranked or ordered, the first and last units from each of the ((*l*-1)/2) sets are selected, and the median unit from the final set is chosen as the sample.
- If necessary, the EPRSS sampling process can be repeated *r* times in order to achieve the desired sample size of n=*l*r.

If l is even then the mean estimator for EPRSS along with the variance for a single computation is defined as:

$$\overline{Z}_{(EPRSS)e} = \frac{1}{l}\sum_{i=1}^{\frac{l}{2}}\left[Z_{i(1)} + Z_{i(1)}\right] \tag{13}$$

and

$$V\,ar\left(\overline{Z}_{(EPRSS)e}\right) = \frac{1}{2l}\begin{bmatrix} Var\left(Z_{(1)}\right) + Var\left(Z_{(l)}\right) \\ +2Cov\left(Z_{(1)}, Z_{(l)}\right) \end{bmatrix}. \tag{14}$$

If l is odd then

$$\overline{Z}_{(EPRSS)o} = \frac{1}{l}\left[\sum_{i=1}^{\frac{(l-1)}{2}} \left(Z_{i(1)} + Z_{i(l)}\right) + Z_{\frac{l+1}{2}\left(\frac{l+1}{2}\right)}\right] \tag{15}$$

and

$$V\,ar\left(\overline{Z}_{(EPRSS)o}\right) = \frac{l-1}{2l^2}\begin{bmatrix} Var\left(Z_{(1)}\right) + Var\left(Z_{(l)}\right) \\ +2Cov\left(Z_{(1)}, Z_{(l)}\right) \end{bmatrix} + \frac{1}{l^2}\left[Var\left(Z_{\left(\frac{l+1}{2}\right)}\right)\right]. \tag{16}$$

3.2. Quartile pair ranked set sampling

Tayyab et al. [25] proposed a QPRSS scheme for choosing a sample from the population. The complete method of selecting a sample is described as follow:

- if l is even, then $\left(\frac{l^2}{2}\right)$ sampling units are selected from the population and distributed to $\left(\frac{l}{2}\right)$ sets each with sample size l.
- The sampling units in each set are ordered according to cost-effective sources such as auxiliary information or any other source. After ranking, the $\left(\frac{(l+1)}{4}\right)th$ and $\left(\frac{3(l+1)}{4}\right)th$ sampling units are selected from each set.
- If l is odd, then $\left(\frac{l(l+1)}{2}\right)$ sampling units are selected and allocated randomly to $\left(\frac{l+1}{2}\right)$ sets.
- After ranking the population, the sampling procedure selects the (((*l*+1))/4)th and (3(*l*+1)/4)th ordered units from each of the ((*l*-1)/2) sets, and the median units from the final set. If necessary, this process can be repeated *r* times to obtain a sample of size *n*=*lr*.

If l is even then the mean estimator for QPRSS for one complete cycle is defined as

$$\overline{Z}_{(QPRSS)e} = \frac{1}{l}\left[\sum_{i=1}^{\frac{l}{2}} Z_{i(q_1(l+1):l)} + \sum_{i=1}^{\frac{l}{2}} Z_{i(q_3(l+1):l)}\right] \tag{17}$$

and if l is odd then

$$\overline{Z}_{(QPRSS)e} = \frac{1}{l}\left[\sum_{i=1}^{\frac{l}{2}} Z_{i(q_1(l+1):l)} + \sum_{i=1}^{\frac{l}{2}} Z_{i(q_3(l+1):l)} + Z_{\frac{l+1}{2}(q_2(l+1):l)}\right] \tag{18}$$

with the respective variances as

$$V\,ar\left(\overline{Z}_{(QPRSS)e}\right) = \frac{1}{2l}\left[\delta^2_{(q_1(l+1))} + \delta^2_{(q_3(l+1))} + 2\delta_{(q_1(l+1),q_3(l+1))}\right] \tag{19}$$

and

$$V\,ar\left(\overline{Z}_{(QPRSS)o}\right)=\frac{l-1}{2l^2}\begin{bmatrix}\delta^2_{(q_1(l+1))}+\delta^2_{(q_3(l+1))}\\+2\delta_{(q_1(l+1),q_3(l+1))}\end{bmatrix}+\frac{1}{l^2}\delta^2_{(q_1(l+1))}. \tag{20}$$

4. Suggested Bayesian-EWMA control chart design under different under PRSS schemes

This section presents the development of a Bayesian EWMA control chart, which utilizes various PRSS (PRSS, EPRSS, and QPRSS) schemes and symmetric/asymmetric LFs (SELF and LLF) for posterior and productive posterior distributions. The chart is characterized as Bayesian EWMA control chart. The structure of the Bayesian EWMA plotting statistic using PRSS with LFs is defined as:

$$V_t=\lambda\left(\theta_{(PRSS)LF}\right)+(1-\lambda)V_{t-1} \tag{21}$$

and $V_0=E\left(\theta_{(RSS)LF}\right)$; the above Bayesian EWMA statistic used under the prior distribution is given below:

4.1. Posterior distribution utilizing normal prior distribution

In this section, the control limits based on posterior distribution using an informative prior (normal distribution) are discussed. The posterior distribution for the normal prior and likelihood function is defined as

$$P(\theta\mid x)=\frac{1}{\sqrt{2\pi}\sqrt{\dfrac{\delta^2\delta_0^2}{\delta^2+n\delta_0^2}}}\exp\left[-\frac{1}{2}\left(\frac{\theta-\sum_{i=1}^{n}\dfrac{x_i\delta_0^2+\theta_0\delta_0^2}{\delta^2+n\delta_0^2}}{\sqrt{\dfrac{\delta^2\delta_0^2}{\delta^2+n\delta_0^2}}}\right)^2\right] \tag{22}$$

where $\theta_n=\frac{n\overline{x}\delta_0^2+\delta^2\theta_0}{\delta^2+n\delta_0^2}$ and $\delta_n^2=\frac{\delta^2\delta_0^2}{\delta^2+n\delta_0^2}$.

The control limits for the Bayesian EWMA control chart using an informative normal prior and various PRSS schemes under different LFs are defined based on the posterior distribution as follows:

4.1.1. *Utilizing the SELF, Control limits applying PRSS sampling designs*

The construction of Bayesian EWMA control limits under the condition of a symmetric LF (SELF) for different PRSS schemes such as PRSS, EPRSS, and QPRSS is provided. The Bayes estimator $\hat{\theta}$ for the SELF is defined as

$$\hat{\theta}_{SELF(PRSS_i)}=\frac{n\overline{x}_{(PRSS_i)}\delta_0^2+\delta^2\theta_0}{\delta^2+n\delta_0^2}. \tag{23}$$

The properties of $\hat{\theta}_{SELF}$ are mathematized as

$$E\left(\hat{\theta}_{SELF}\right)=\frac{n\theta_1\delta_0^2+\delta^2\theta_0}{\delta^2+n\delta_0^2} \tag{24}$$

and

$$var\left(\hat{\theta}_{SELF}\right)=\frac{n\delta_{(RSS_i)}^2\delta_0^4}{\delta^2+n\delta_0^2}. \tag{25}$$

Using SELF, the asymptotic Bayesian EWMA control limits based on the PRSS designs are expressed as follows:

$$UCL_{PRSS_i}=E\left(\hat{\theta}_{SELF}\right)+L\sqrt{var\left(\hat{\theta}_{SELF}\right)}\sqrt{\frac{\lambda}{2-\lambda}}. \tag{26}$$

$$CL_{PRSS_i}=E\left(\hat{\theta}_{SELF}\right). \tag{27}$$

$$LCL_{PRSS_i}=E\left(\hat{\theta}_{SELF}\right)-L\sqrt{var\left(\hat{\theta}_{SELF}\right)}\sqrt{\frac{\lambda}{2-\lambda}}. \tag{28}$$

Here, $i=1,2,3.$ is elaborated as;

$$PRSS_1=PRSS$$
$$PRSS_2=EPRSS$$
$$PRSS_3=QPRSS.$$

4.1.2. *Implementing LLF, control limits applying PRSS schemes*

In this section, the control limits construction under PRSS schemes for Bayesian EWMA given an asymmetric LF(LLF) Bayes estimator $\hat{\theta}$ is defined as

$$\hat{\theta}_{LLF(PRSS_i)}=\frac{n\overline{x}_{(PRSS_i)}\delta_0^2+\delta^2\theta_0}{\delta^2+n\delta_0^2}-\frac{C'}{2}\delta_n^2 \tag{29}$$

The mean and variance for $\hat{\theta}_{LLF}$ is given below:

$$E\left(\hat{\theta}_{LLF}\right)=\frac{n\theta_1\delta_0^2+\delta^2\theta_0}{\delta^2+n\delta_0^2}-\frac{C'}{2} \tag{30}$$

and

$$var\left(\hat{\theta}_{LLF}\right)=\frac{n\delta_{(RSS_i)}^2\delta_0^4}{\left(\delta^2+n\delta_0^2\right)^2}. \tag{31}$$

The following expressions define the asymptotic control limits for the Bayesian EWMA control chart using the LLF and PRSS

$$UCL_{PRSS_i} = E\left(\hat{\theta}_{LLF}\right) + L\sqrt{var\left(\hat{\theta}_{LLF}\right)}\sqrt{\frac{\lambda}{2-\lambda}}. \tag{32}$$

$$CL_{PRSS_i} = E\left(\hat{\theta}_{LLF}\right). \tag{33}$$

$$LCL_{PRSS_i} = E\left(\hat{\theta}_{LLF}\right) - L\sqrt{var\left(\hat{\theta}_{LLF}\right)}\sqrt{\frac{\lambda}{2-\lambda}}. \tag{34}$$

Here, the respective i values are $i = 1,2,3$.

$$PRSS_1 = PRSS$$
$$PRSS_2 = EPRSS$$
$$PRSS_3 = QPRSS.$$

4.2. Posterior predictive distribution utilizing a normal prior

The construction of the suggested Bayesian EWMA control chart using posterior predictive distribution is outlined below:

Let $y_1, y_2, \ldots., y_k$, be the future observations then, the posterior predictive distribution $y|x$ is expressed as:

$$p(y|x) = \frac{1}{\sqrt{2\pi\delta_1^2}} exp\left\{-\frac{1}{2\delta_1^2}(Y-\theta_n)^2\right\} \tag{35}$$

where $\delta_1^2 = \delta^2 + \frac{\delta^2\delta_0^2}{\delta^2+n\delta_0^2}$, for different LFs; the control limits for Bayesian EWMA using PRSS schemes are mentioned.

Implementing LLF, control limits based on PRSS designs

The proposed Bayesian EWMA control chart with different PRSS schemes given an asymmetric LLF for the Bayes estimator $\hat{\theta}_{LLF}$ is defined as:

$$\hat{\theta}_{LLF(PRSS_i)} = \frac{n\overline{x}_{(PRSS_i)}\delta_0^2+\delta^2\theta_0}{\delta^2+n\delta_0^2} - \frac{C'}{2}\tilde{\delta}_1^2 \tag{36}$$

where $\tilde{\delta}_1^2 = \frac{\delta^2}{k} + \frac{\delta^2\delta_0^2}{\delta^2+n\delta_0^2}$; the properties of $\hat{\theta}_{LLF}$ are mathematized as:

$$E\left(\hat{\theta}_{LLF}\right) = \frac{n\theta_1\delta_0^2+\delta^2\theta_0}{\delta^2+n\delta_0^2} - \frac{C'}{2}\tilde{\delta}_1^2 \tag{37}$$

$$var\left(\hat{\theta}_{SELF}\right) = \frac{n\delta_{(RSS_i)}^2\delta_0^4}{\left(\delta^2+n\delta_0^2\right)^2}. \tag{38}$$

Below are the control limits for the Bayesian EWMA control chart with the LLF and different

PRSS schemes.

$$UCL_{PRSS_i} = E\left(\hat{\theta}_{LLF}\right) + L\sqrt{var\left(\hat{\theta}_{LLF}\right)}\sqrt{\frac{\lambda}{2-\lambda}}. \quad (39)$$

$$CL_{PRSS_i} = E\left(\hat{\theta}_{LLF}\right) \quad (40)$$

$$LCL_{PRSS_i} = E\left(\hat{\theta}_{LLF}\right) - L\sqrt{var\left(\hat{\theta}_{LLF}\right)}\sqrt{\frac{\lambda}{2-\lambda}}. \quad (41)$$

Here, the i values are as follows $i = 1,2,3$.

$$PRSS_1 = PRSS$$
$$PRSS_2 = PERSS$$
$$PRSS_3 = QPRSS.$$

5. Simulation study

The Monte Carlo simulation procedure was used to evaluate the performance of the suggested Bayesian EWMA control chart. The ARL and SDRL values were computed for various shift values using the concept of a zero-state ARL. The specified in-control ARL value was 370 for $\lambda = 0.10$ and 0.25 to see the performance of the proposed Bayesian EWMA control chart. The computational findings are provided in Tables 1–6. The simulation procedure was carried out in the following steps:

Step 1: Setting the threshold for in-control ARL

- **i.** The mean and variance of the posterior and posterior predictive distributions are calculated for different LFs using a standard normal distribution as the sampling and prior distribution. i.e., $E\left(\hat{\theta}_{(RSS_i)LF}\right)$ and $\delta_{(RSS_i)LF}$.
- **ii.** Choose the values for λ and L for the initial control limits.
- **iii.** Generate different ranked set sampling schemes for the in-control process and compute the plotting statistic of the proposed Bayesian EWMA control chart.
- **iv.** If the plotting statistic falls between the control limits, repeat steps (iii-iv) to check for any errors or anomalies. These steps are designed to ensure the accuracy and reliability of the data, and they should be repeated as necessary until all issues have been resolved.
- **v.** Once the process has been deemed as in-control, continue to repeat the aforementioned steps until the process displays out-of-control signals. During this time, keep track of the number of consecutive in-control run lengths.

Step 2: Evaluate ARL and SDRL out-of-control limits

- i. To see the influence of the paired ranked-based sampling schemes, the respective data are generated with some shift in the process mean.
- ii. Compute the plotting statistic as per the design of the proposed Bayesian EWMA control chart.
- iii. The respective RL is recorded as the statistic value falls out of the control limits, regarded as an out-of-control process.
- iv. The whole procedure is repeated 10,000 times to compute the ARL and SDRL values.

Table 1. The ARL and SDRL values of the Bayesian EWMA control chart for posterior and predictive posterior distributions given SELF λ = 0.10, n = 5.

Shift	*Bayesian-EWMA SRS*		*Bayesian-AEWMA SRS*		*Bayesian-EWMA PRSS*		*Bayesian-EWMA EPRSS*		*Bayesian-EWMA QPRSS*	
	ARL	*SDRL*	*ARL*	*SDRL*	*ARL*	*SDRL*	*ARL*	*SDRL*	*ARL*	*SDRL*
	$L = 2.7042$		$h = 0.0856$		$L = 2.7123$		$L = 2.7195$		$L = 2.7189$	
0.00	371.82	367.80	372.86	537.77	371.67	368.08	370.93	368.80	370.78	367.61
0.10	250.59	239.54	190.13	188.22	174.06	171.40	179.34	173.95	170.15	166.76
0.20	125.58	114.98	70.61	91.12	65.10	61.89	68.40	63.30	61.81	56.29
0.30	66.576	57.92	35.40	44.53	31.81	26.97	33.11	27.83	29.91	24.65
0.40	41.68	32.78	21.15	26.36	18.92	14.48	19.71	15.26	17.80	13.34
0.50	28.35	20.12	13.55	16.69	12.51	8.69	13.30	9.66	11.91	8.41
0.60	20.98	13.49	9.46	11.23	9.20	6.10	9.69	6.58	8.74	5.87
0.70	16.26	9.59	7.08	7.70	7.08	4.52	7.50	4.88	6.83	4.35
0.75	14.72	8.30	6.15	6.43	6.40	3.97	6.65	4.17	6.04	3.74
0.80	13.41	7.14	5.62	5.82	5.74	3.48	5.91	3.68	5.40	3.31
0.90	11.42	5.69	4.51	4.18	4.79	2.83	4.93	2.96	4.48	2.64
1.00	9.78	4.50	3.85	3.20	3.99	2.30	4.14	2.40	3.79	2.14
1.50	5.79	2.03	2.25	1.29	2.16	1.07	2.23	1.13	2.08	1.04
2.00	4.16	1.20	1.66	0.78	1.46	0.62	1.51	0.66	1.41	0.59
2.50	3.31	0.84	1.36	0.56	1.15	0.37	1.18	0.41	1.13	0.35
3.00	2.76	0.66	1.17	0.39	1.04	0.19	1.05	0.22	1.02	0.16
4.00	2.12	0.38	1.02	0.14	1	0	1	0	1	0

Table 2. The ARL and SDRL values of the Bayesian EWMA control chart for posterior and predictive posterior distributiona given the SELF, for $\lambda = 0.25$, $n = 5$.

Shift	*Bayesian-EWMA SRS*		*Bayesian-AEWMA SRS*		*Bayesian-EWMA PRSS*		*Bayesian-EWMA EPRSS*		*Bayesian-EWMA QPRSS*	
	ARL	*SDRL*	*ARL*	*SDRL*	*ARL*	*SDRL*	*ARL*	*SDRL*	*ARL*	*SDRL*
	$L = 2.8987$		$h = 0.241$		$L = 2.9052$		$L = 2.9179$		$h = 0.0758$	
0.00	369.49	364.82	369.00	367.39	371.34	367.87	372.02	369.08	370.92	367.90
0.10	292.01	288.73	201.12	190.98	227.87	226.17	240.56	236.01	238.65	258.92
0.20	178.20	175.14	97.04	80.91	104.95	101.53	114.37	111.28	99.26	98.83
0.30	104.70	100.95	55.71	42.80	51.65	48.16	53.52	49.92	48.57	45.13
0.40	63.11	58.20	36.15	25.09	28.01	24.80	29.75	26.14	27.15	24.08
0.50	41.21	36.61	25.95	17.04	17.41	14.30	18.45	15.36	16.38	13.40
0.60	28.45	24.57	19.80	12.20	12.11	9.27	12.50	9.51	11.25	8.67
0.70	20.61	16.37	15.41	9.09	8.81	6.21	9.18	6.46	11.43	8.66
0.75	17.97	13.87	14.11	8.17	7.73	5.35	8.13	5.71	8.37	5.84
0.80	15.71	11.75	12.87	7.26	6.77	4.46	7.23	4.80	6.45	4.21
0.90	12.51	8.86	10.76	5.97	5.40	3.36	5.82	3.81	5.25	3.27
1.00	10.22	6.77	9.17	4.96	4.49	2.74	4.73	2.86	4.35	2.58
1.50	5.15	2.51	4.90	2.77	2.37	1.17	2.45	1.25	2.25	1.11
2.00	3.46	1.33	2.98	1.83	1.58	0.69	1.62	0.71	1.53	0.66
2.50	2.66	0.86	1.98	1.15	1.21	0.43	1.25	0.46	1.18	0.40
3.00	2.19	0.61	1.48	0.72	1.06	0.24	1.07	0.26	1.04	0.20
4.00	1.66	0.50	1	0	1	0	1	0	1	0

Table 3. The ARL and SDRL values of the Bayesian EWMA control chart for the posterior distribution given the LLF, for λ = 0.10, n = 5.

Shift	*Bayesian-EWMA SRS*		*Bayesian-AEWMA SRS*		*Bayesian-EWMA PRSS*		*Bayesian-EWMA EPRSS*		*Bayesian-EWMA QPRSS*	
	ARL	*SDRL*	*ARL*	*SDRL*	*ARL*	*SDRL*	*ARL*	*SDRL*	*ARL*	*SDRL*
	L =2.7047		h = 0.086		L = 2.7157		L = 2.7212		L = 2.7165	
0.00	370.63	368.13	370.98	539.06	371.22	366.89	370.51	368.23	371.02	365.29
0.10	250.27	240.24	195.78	190.09	172.08	172.62	179.90	177.01	166.75	162.64
0.20	123.94	115.00	71.98	92.48	63.17	58.87	67.68	62.17	62.94	56.58
0.30	115.00	57.42	36.26	45.49	31.60	25.87	32.87	27.04	30.37	25.27
0.40	41.33	32.49	21.09	26.30	18.82	14.31	19.84	15.29	18.14	13.50
0.50	28.51	20.18	13.71	16.73	12.70	8.92	13.34	9.44	12.13	8.48
0.60	20.95	13.50	9.53	11.25	9.16	6.07	9.68	6.55	7.94	5.45
0.70	16.46	9.64	7.09	7.86	7.10	4.56	7.41	4.76	6.82	4.29
0.75	14.79	8.35	6.20	6.50	6.28	3.98	6.62	4.15	6.02	3.73
0.80	13.38	7.17	5.54	5.54	5.66	3.49	5.90	3.63	5.48	3.37
0.90	11.29	5.57	4.52	4.17	4.67	2.80	4.93	2.98	4.45	2.62
1.00	9.79	4.49	3.83	3.20	3.97	2.28	4.19	2.41	3.80	2.19
1.50	5.82	2.03	2.26	1.27	2.13	1.06	2.21	1.14	2.06	1.03
2.00	4.18	1.20	1.66	0.78	1.47	0.62	1.51	0.66	1.41	0.59
2.50	3.31	0.84	1.34	0.55	1.15	0.37	1.18	0.41	1.13	0.34
3.00	2.75	0.66	1.16	0.39	1.03	0.19	1.04	0.21	1.03	0.18
4.00	2.13	0.38	1.02	0.15	1	0	1	0	1	0

Table 4. The ARL and SDRL values of the Bayesian EWMA control chart for the posterior distribution given the LLF, for $\lambda = 0.25$, $n = 5$.

Shift	*Bayesian-EWMA SRS*		*Bayesian-AEWMA SRS*		*Bayesian-EWMA PRSS*		*Bayesian-EWMA EPRSS*		*Bayesian-EWMA QPRSS*	
	ARL	*SDRL*	*ARL*	*SDRL*	*ARL*	*SDRL*	*ARL*	*SDRL*	*ARL*	*SDRL*
	$L = 2.9050$		$h = 0.242$		$L = 2.9157$		$L = 2.9124$		$L = 2.9098$	
0.00	371.05	368.88	370.14	434.88	371.57	368.09	370.73	368.31	369.57	366.39
0.10	299.03	294.31	245.22	241.12	225.04	212.48	228.76	215.78	215.29	213.67
0.20	179.81	175.30	86.77	83.25	111.62	106.51	107.79	104.87	101.63	100.35
0.30	105.54	101.21	55.44	42.26	52.13	48.13	54.15	51.14	48.72	45.33
0.40	64.00	59.50	36.76	25.98	28.32	24.87	29.65	25.98	26.97	23.60
0.50	41.56	37.30	25.86	16.88	17.89	14.57	19.00	15.81	16.63	13.68
0.60	28.54	24.33	19.65	12.16	12.25	9.20	12.77	9.85	11.29	8.54
0.70	20.96	16.82	15.62	9.17	9.03	6.49	9.33	6.70	8.36	5.83
0.75	18.11	14.02	14.23	8.29	7.83	5.40	8.25	5.76	7.33	4.90
0.80	15.89	11.94	12.83	7.30	6.89	4.67	7.24	4.92	6.48	4.23
0.90	12.61	8.89	10.79	5.90	5.52	3.45	5.84	3.69	5.14	3.20
1.00	10.27	6.77	9.25	5.00	4.60	2.75	4.76	2.90	4.33	2.58
1.50	5.18	2.50	4.95	2.80	2.39	1.19	2.47	1.26	2.27	1.11
2.00	3.46	1.33	2.97	1.81	1.59	0.70	1.63	0.72	1.51	0.65
2.50	2.64	0.85	1.97	1.13	1.22	0.44	1.25	0.46	1.18	0.40
3.00	2.19	0.62	1.48	0.73	1.06	0.24	1.07	0.26	1.04	0.21
4.00	1.66	0.50	1.09	0.30	1	0	1	0	1	0

Table 5. The ARL and SDRL values of the Bayesian EWMA control chart for the posterior predictive distribution given the LLF, for $\lambda = 0.10$, $n = 5$.

Shift	*Bayesian-EWMA SRS*		*Bayesian-AEWMA SRS*		*Bayesian-EWMA PRSS*		*Bayesian-EWMA EPRSS*		*Bayesian-EWMA QPRSS*	
	ARL	*SDRL*	*ARL*	*SDRL*	*ARL*	*SDRL*	*ARL*	*SDRL*	*ARL*	*SDRL*
	L=2.7018		h = 0.0856		L = 2.7137		L = 2.7121		L = 2.7154	
0.00	371.50	368.69	369.58	524.70	371.54	365.58	369.62	366.21	369.82	366.78
0.10	251.98	249.23	220.12	215.67	167.05	162.99	168.67	165.34	165.67	162.95
0.20	122.45	113.13	70.53	91.22	69.55	65.21	65.87	59.13	60.56	55.20
0.30	67.08	57.94	35.71	45.25	31.53	26.09	32.22	27.31	30.58	24.86
0.40	41.41	32.72	21.24	26.29	18.88	14.31	19.52	14.87	17.97	13.41
0.50	28.05	19.84	13.66	16.90	12.59	8.91	13.40	9.58	12.20	8.51
0.60	21.08	13.64	9.46	11.08	9.20	6.12	9.61	6.45	8.82	5.77
0.70	16.27	9.54	6.94	7.70	7.11	4.54	7.38	4.77	6.79	4.38
0.75	14.73	8.23	6.22	6.53	6.29	3.90	6.57	4.11	6.09	3.78
0.80	13.33	7.17	5.50	5.58	5.66	3.44	5.90	3.67	5.45	3.32
0.90	11.22	5.60	4.52	4.15	4.66	2.77	4.86	2.92	4.45	2.66
1.00	9.65	4.44	3.77	3.17	3.92	2.29	4.08	2.38	3.81	2.22
1.50	5.82	2.02	2.26	1.29	2.14	1.07	2.23	1.13	2.06	1.00
2.00	4.18	1.20	1.66	0.78	1.46	0.63	1.49	0.64	1.41	0.59
2.50	3.30	0.84	1.35	0.55	1.17	0.39	1.18	0.40	1.13	0.35
3.00	2.76	0.65	1.16	0.39	1.03	0.19	1.05	0.22	1.03	0.17
4.00	2.13	0.38	1.02	0.15	1	0	1	0	1	0

Table 6. The ARL and SDRL values of the Bayesian EWMA control chart for the posterior productive distribution given the LLFs, for λ = 0.25, n = 5.

Shift	*Bayesian-EWMA SRS*		*Bayesian-AEWMA SRS*		*Bayesian-EWMA PRSS*		*Bayesian-EWMA EPRSS*		*Bayesian-EWMA QPRSS*	
	ARL	*ARL*	*ARL*	*SDRL*	*ARL*	*SDRL*	*ARL*	*SDRL*	*ARL*	*SDRL*
	L = 2.8986		h = 0.2414		L = 2.9137		L = 2.9141		L = 2.9074	
0.00	370.23	368.87	368.67	359.45	369.21	367.42	370.50	364.16	370.77	364.78
0.10	295.14	290.95	266.90	261.23	240.95	236.95	239.12	235.30	235.99	223.47
0.20	177.79	174.80	98.16	83.24	104.38	102.96	109.89	106.69	100.83	96.66
0.30	103.68	99.36	54.92	41.45	48.99	45.91	54.62	51.73	47.68	43.74
0.40	63.21	58.43	36.19	25.48	28.80	25.74	29.99	26.61	26.60	23.77
0.50	41.26	37.00	25.97	17.13	17.05	13.81	18.36	15.16	16.45	13.32
0.60	28.35	24.16	19.68	12.21	12.21	9.44	12.62	9.76	11.52	8.87
0.70	20.68	16.45	15.56	9.19	8.84	6.28	9.41	6.71	8.22	5.77
0.75	18.04	14.01	14.18	8.26	7.77	5.25	7.08	4.68	7.33	4.99
0.80	15.79	11.87	12.79	7.24	6.85	4.54	7.16	4.86	6.48	4.25
0.90	12.52	8.81	10.74	5.93	5.44	3.42	5.74	3.60	5.22	3.23
1.00	10.23	6.71	9.20	4.98	4.55	2.72	4.78	2.89	4.39	2.57
1.50	5.18	2.51	4.94	2.79	2.36	1.19	2.45	1.26	2.23	1.12
2.00	3.46	1.33	2.95	1.81	1.58	0.70	1.63	0.72	1.51	0.66
2.50	2.65	0.86	1.98	1.14	1.21	0.43	1.26	0.46	1.17	0.39
3.00	2.19	0.62	1.48	0.72	1.05	0.23	1.07	0.26	1.04	0.20
4.00	1.66	0.50	1.09	0.30	1	0	1	0	1	0

6. Results, discussions and main findings

In this section, the comparative analysis of the proposed Bayesian EWMA control chart for PRSS schemes with different LFs and informative prior distribution is made with the Bayesian EWMA with SRS (existing). The detailed computations for the existing and the suggested Bayesian-EWMA control chart designs are exclusively mentioned in Tables 1-6 for different shifts and controlling constant values as $\lambda = 0.10$ and 0.25. A smaller value of the ARL_1 depicts the fast detection ability of the proposed Bayesian EWMA control chart in comparison with the existing counterpart.

For instance, Table 1 indicates the ARL_1 $(SDRL_1)$ for shift= 0.3 with $\lambda = 0.10$ the existing Bayesian EWMA control chart gives ARL_1 $(SDRL_1) = 66.576(57.92)$, and for Bayesian AEWMA control chart gives ARL_1 $(SDRL_1) = 35.40(44.53)$, additionally the proposed Bayesian PRSS gives $31.81(26.97)$, the EPRSS gives $33.11(27.83)$, and the QPRSS gives $29.91(24.65)$. Similar behavior can be seen for $\lambda = 0.25$, where the existing control chart gives ARL_1 $(SDRL_1) = 104.70(100.95)$ and ARL_1 $(SDRL_1) = 35.40(44.53)$ for the Bayesian AEWMA control chart, whereas the proposed Bayesian PRSS gives $51.65(48.16)$, the EPRSS gives $53.52(49.92)$, and the QPRSS gives $48.57(45.13)$.

So, it can be seen that for the smaller value of λ all proposed Bayesian EWMA control charts behave much better. Moreover, the proposed Bayesian EWMA control charts give significantly improved results in comparison with the existing SRS-based control chart. The main findings about the efficiency of the proposed Bayesian EWMA control chart are discussed as follows:

- The performance of the proposed Bayesian control chart was observed with changes in the values of the λ smoothing constant; for example, Tables 1 and 2 present the ARL and SDRL values for posterior and posterior predictive distributions under SELF using normal prior distribution. The suggested Bayesian EWMA control chart performs better for smaller values of λ, it can be noticed that from Tables 1-2 for $\lambda = 0.10$ and at $ARL_0 = 370$, $\delta = 0.30$, $ARL_1 = 31.81$ and 51.65 for $\lambda = 0.25$ in the case of PRSS; for EPRSS the ARL_1 values are 33.11 and 53.52, at the same λ the ARL_1 values for QPRSS are 29.91 and 48.57.
- The tabular results revealed that the proposed Bayesian EWMA for the posterior predictive distribution under the condition of the LLF gives much better results for PRSS, EPRSS, and QPRSS at $\lambda = 0.10$ such that: from Table 5 the ARL_1 ($SDRL_1) = 31.53, 32.22, 30.58$ $(26.09, 27.31, 24.86)$ than the proposed Bayesian EWMA control chart for posterior and predictive posterior distribution under SELF: from Table 1 the ARL_1 $(SDRL_1) = 31.81, 33.11, 29.91$ $(26.97, 27.83, 24.65)$, and the EWMA control chart based on Bayesian theory with posterior distribution applying LLF such that: from Table 3 the ARL_1 $(SDRL_1) = 31.60, 32.87, 30.37$ $(25.87, 27.04, 25.27)$.
- From Tables 3 and 4, the ARL_1values based on the LLF for $ARL_0 = 370$, $\lambda = 0.10$ and for $\delta = 0.30$ were 31.60 for PRSS and 32.87 for EPRSS and, the QPRSS the ARL_1 value is 30.37, and for $\lambda = 0.25$, the ARL values for PRSS and EPRSS are 52.13 and 54.15. the ARL_1 value for QPRSS was 48.72, The results suggest that an increase in the smoothing constant λ decreases the effectiveness of the proposed Bayesian EWMA control chart for both the posterior and posterior predictive distributions.

According to the results presented in Tables 1–6, the Bayesian EWMA control chart for both posterior and predictive posterior distributions based on SRS, under the condition of both the SELF and LLF, was less efficient in detecting shifts compared to the suggested Bayesian control chart based on QPRSS. In fact, the suggested Bayesian control chart based on QPRSS was found to outperform the proposed method in term of detecting shifts. The QPRSS-based control chart is recommended as a more reliable and effective option for process monitoring and control. These findings are particularly relevant for industries that require accurate detection of shifts and timely intervention to maintain quality control.

7. Real life data applications

Data Set I

An actual dataset of 30 units from Montgomery [1] was used as an example to demonstrate the suggested Bayesian EWMA control chart's practical application. Four control charts were created to evaluate and compare the effectiveness of the existing Bayesian EWMA control chart that used the posterior and posterior predictive distributions with the SELF for SRS (shown in Figure 1) against the proposed EWMA control chart that used posterior and posterior predictive distributions with the SELF for PRSS (shown in Figure 2), QPRSS (shown in Figure 3), and EPRSS (shown in Figure 4). This allowed for a direct comparison of the performance of each method in terms of detecting shifts in the dataset.

The in-control and out-of-control situations are discussed by taking the first 20 units in control with $\mu = 10$ and a unit standard deviation and the remaining 10 units were taken as out-of-control with $\mu = 11$ for a shift $\delta = 1$, $\lambda = 0.10$ and subgroup size are $n = 5$. The mean of the Bayes estimator using the SELF and standard normal prior distribution is estimated as 9.9819. so, the respective control limits for SRS (existing chart) are $LCL = 9.700$ and $UCL = 10.2635$; for the proposed Bayesian EWMA PRSS are $UCL = 10.1781$ and $LCL = 9.79831$, for EPRSS $UCL = 10.18382$ and $LCL = 9.78006$ and QPRSS resulted in $UCL = 10.1958$ and $LCL = 9.7680$.

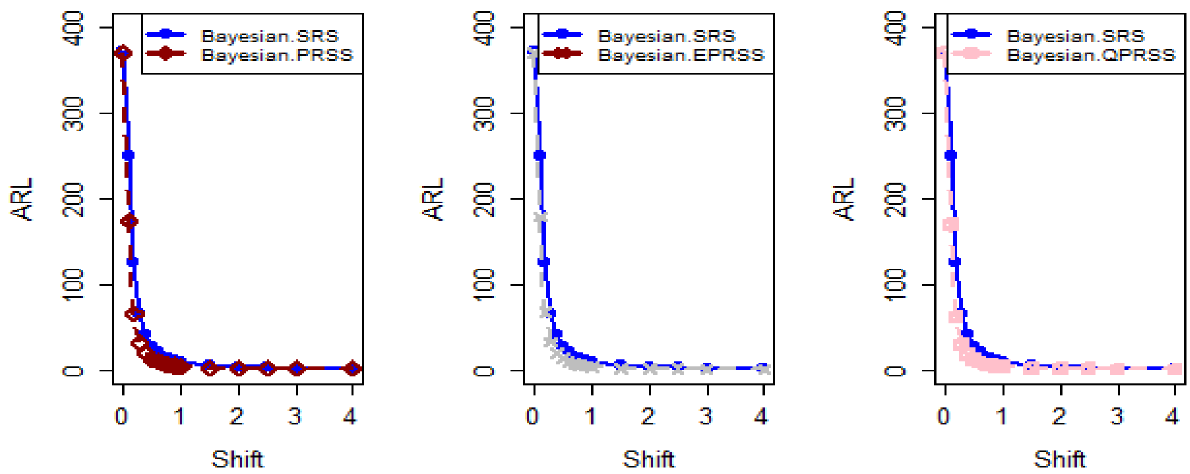

Figure 1. Results obtained under the conditions of the SELF, plots using posterior and posterior predictive distributions applying various ranked-based sampling schemes.

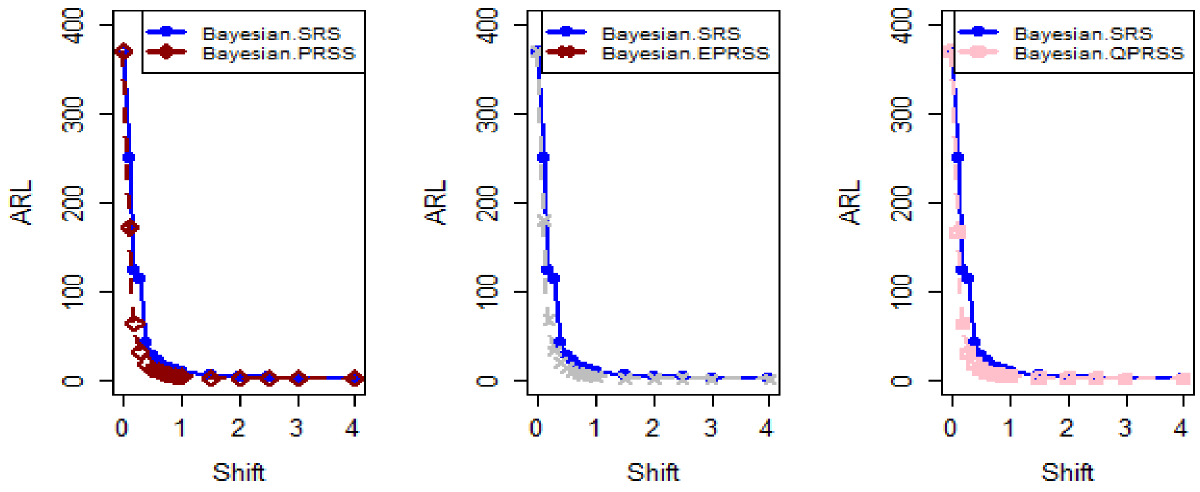

Figure 2. Results of applying LLF, ARL to obtain the suggested Bayesian EWMA control chart for the PRSS, EPRSS, and QPRSS schemes.

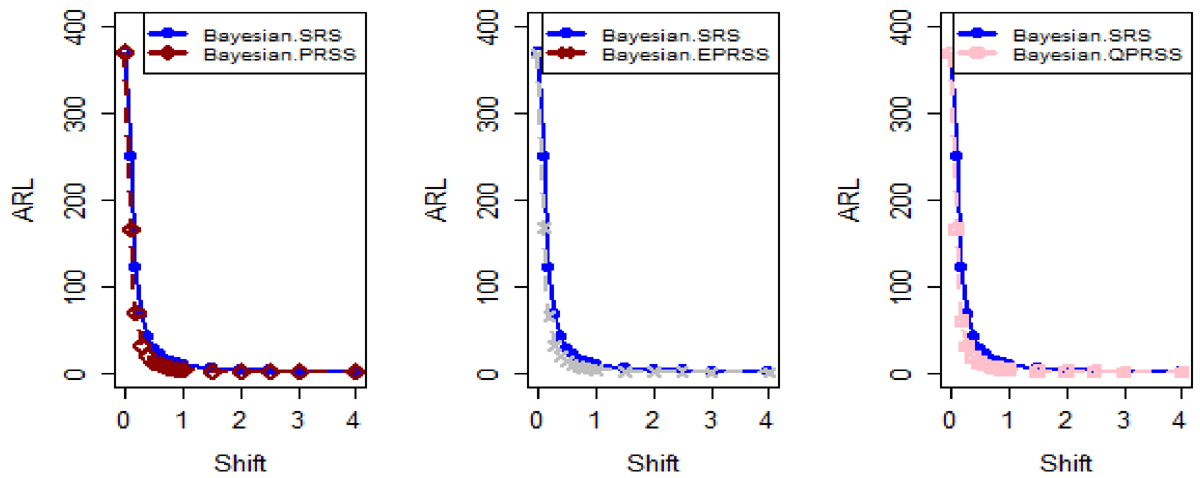

Figure 3. Results of using posterior predictive distribution under the conditions of distinct PRSS schemes.

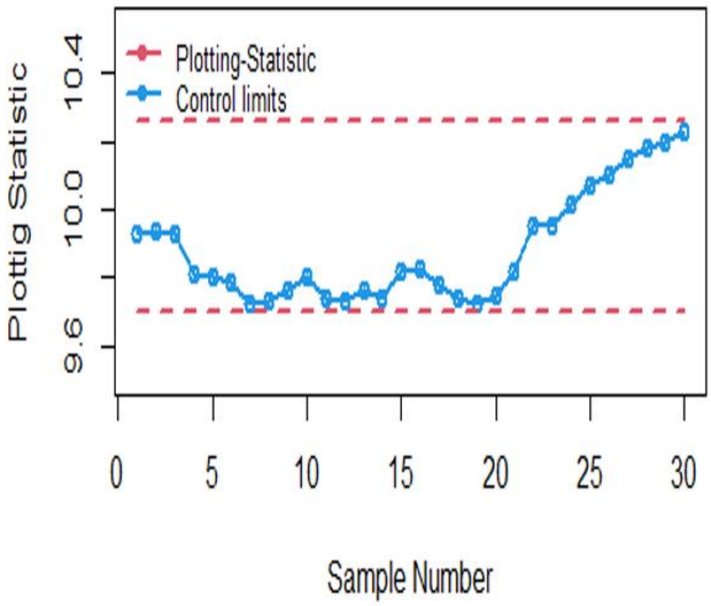

Figure 4. EWMA control chart obtained under the conditions of posterior and posterior predictive distirbutions, SRS and SELF applications.

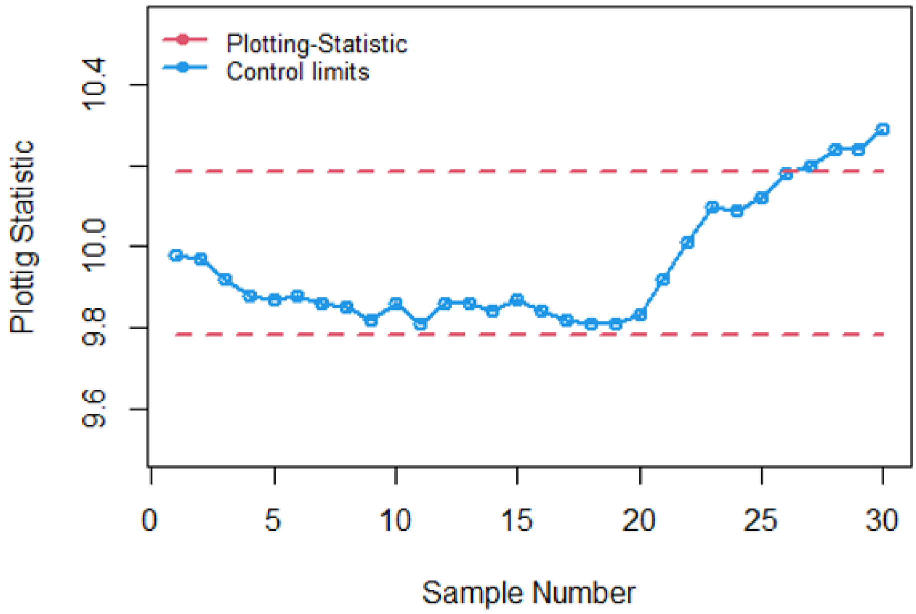

Figure 5. EWMA control chart obtianed under the conditions of posterior and posterior predictive distributions, PRSS and SELF application

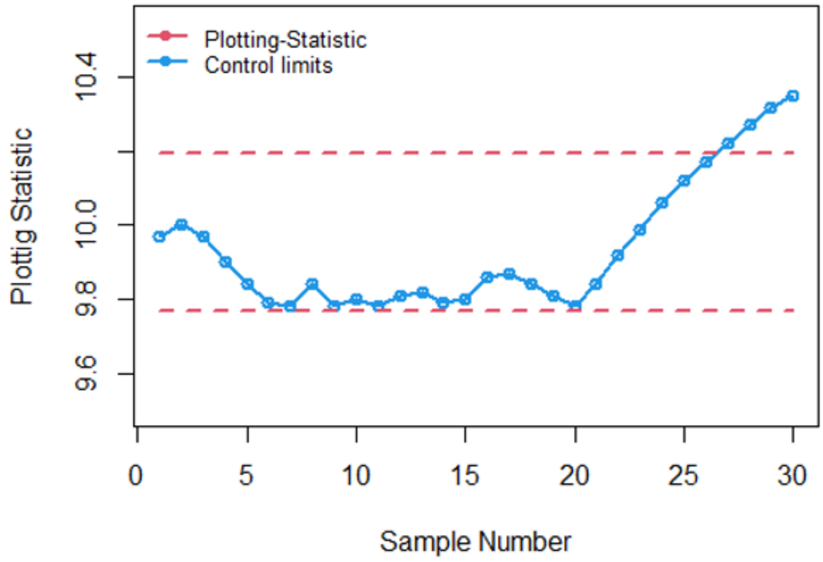

Figure 6. Bayesian-EWMA control chart obtained under the conditions of QPRSS and SELF application.

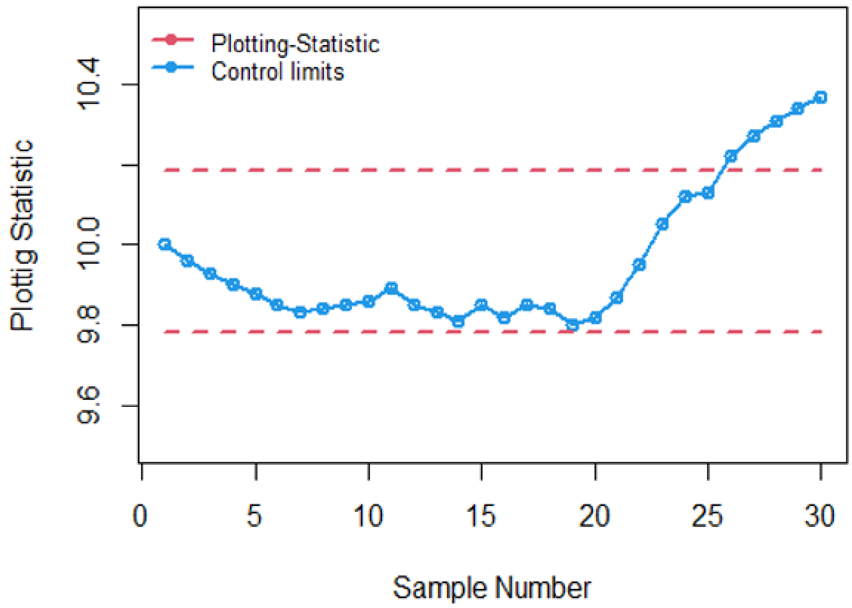

Figure 7. EWMA control chart obtained under the conditions of posterior and posterior predictive distributions, EPRSS and SELF application

The Bayesian EWMA for SRS with the SELF for posterior and posterior predictive distributions is shown in Figure 4, which depicts that all computed points are within the control limits. The proposed Bayesian EWMA for PRSS, EPRSS, and QPRSS indicates a shift at the 26^{th}, 25^{th} and 24^{th} sample points respectively and showed that the process is out of control. The real-life illustration proves that the proposed Bayesian EWMA control charts for different PRSS schemes based on posterior and predictive posterior distribution are efficient in detecting out-of-control signals in comparison with the counterpart.

Data Set II

We evaluated the effectiveness of the proposed Bayesian EWMA control chart for PRSS designs by applying it to data from the hard-bake process in semiconductor manufacturing, as studied by Montgomery [1]. Semiconductor manufacturing is closely related to industrial engineering, and the hard-bake process is critical in this field. The field of industrial engineering often involves designing and optimizing processes used in semiconductor manufacturing to ensure that they are efficient, effective, and reliable. This article examines a data set comprising 45 samples, each consisting of five wafers and totaling 225 observations. Flow width is measured in microns, and the time interval between samples is one hour. The first 30 samples, containing 150 observations, are considered to be from the in-control process (referred to as the phase-I data set). The remaining 15 samples, comprising 75 observations, are categorized as the out-of-control process (referred to as the phase-II data set).

The use of Bayesian theory in conjunction with the EWMA and AEWMA control charts is shown in Figures 8 and 9 for SRS using posterior and posterior predictive distributions and under the condition of applying the standard error of the mean (SELF). As demonstrated in these figures, all calculated points fell within the control limits for the EWMA chart, while Figure 9 highlights an out-of-control signal on the 40th sample. The suggested Bayesian-EWMA control chart using PRSS, EPRS, and QPRSS detected a shift at the 36th, 37th, and 38th sample points, as shown in Figures 10, 11, and 12. These figures reveals that the process was out of control. Based on the outcomes displayed in Figures 1–12, it can be inferred that the suggested Bayesian EWMA control chart exhibited superior performance in comparison to the traditional EWMA and AEWMA control charts based on Bayesian theory. The Bayesian EWMA control chart consistently provided tighter control limits and better detection of out-of-control signals. This was particularly evident in the case of small sample sizes, where the Bayesian EWMA control chart was able to detect shifts earlier than the other control charts. Overall, these findings suggest that the Bayesian EWMA control chart is a promising tool for monitoring process performance for PRSS designs.

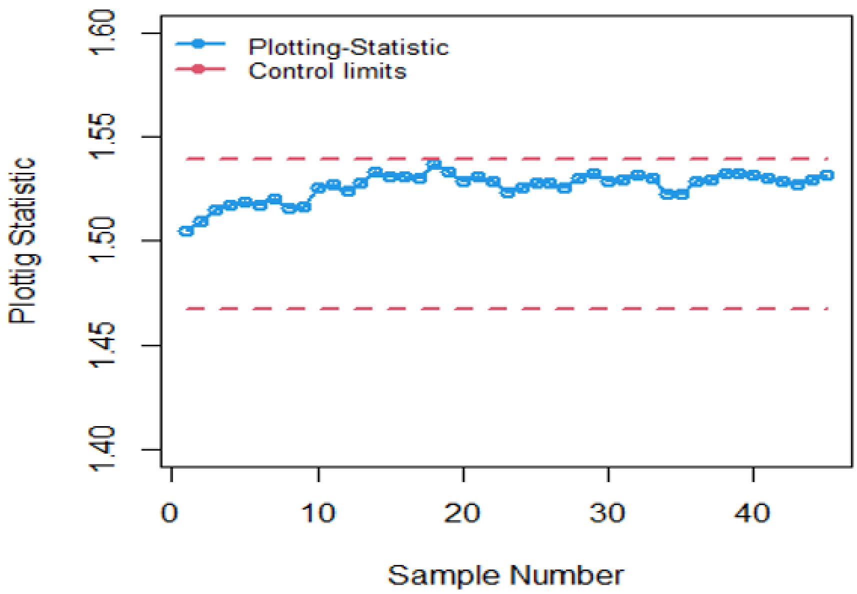

Figure 8. Bayesian EWMA control chart obtained under the conditions of posterior and posterior predictive distributions, SRS and SELF application.

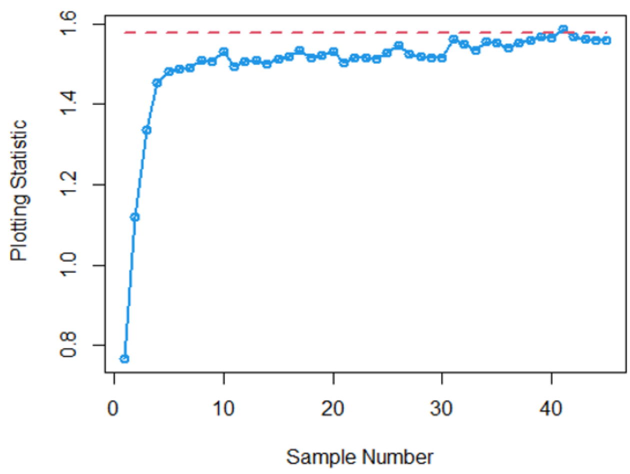

Figure 9. The Bayesian AEWMA control chart obtained under the conditions of SRS and SELF application.

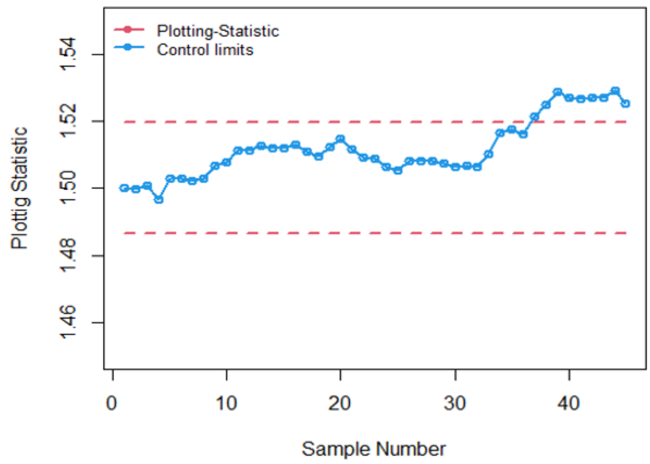

Figure 10. EWMA control chart obtained under the conditions of posterior and posterior predictive distributions, PRSS and SELF.

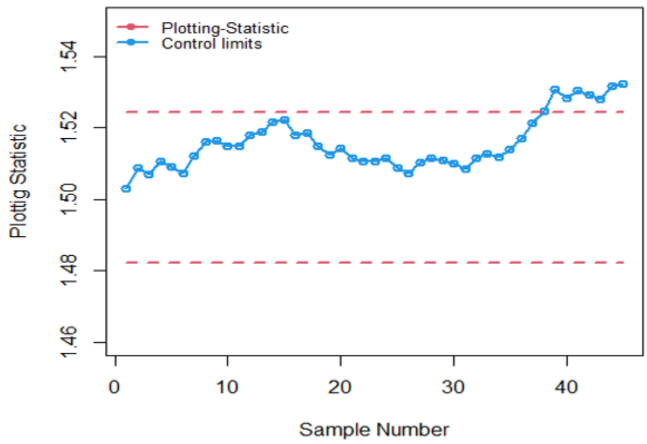

Figure 11. Bayesian-EWMA control chart obtained under the conditions of QPRSS and SELF application.

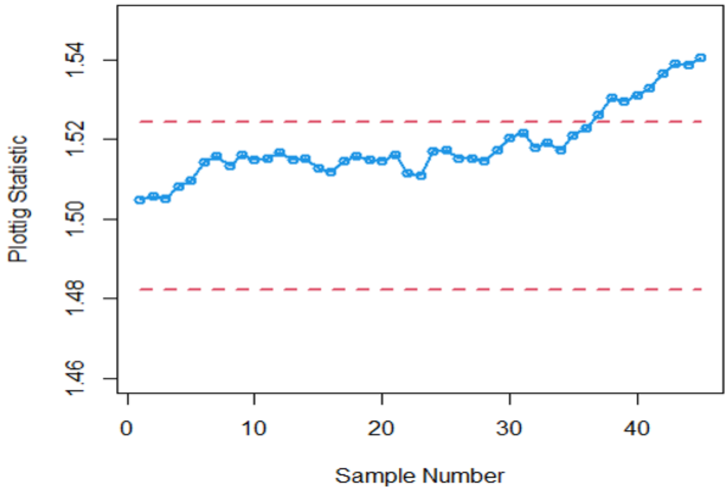

Figure 12. EWMA control chart using obtained under the conditions of posterior and posterior predictive distributions, EPRSS and SELF application.

8. Conclusions

The EWMA statistic gives more weightage to the current sample information than the previous samples. This study, the Bayesian EWMA control chart was constructed by applying the Bayesian-EWMA for PRSS designs. The suggested design was found to be better than the Bayesian EWMA under the SRS scheme. The performance of the proposed control chart was evaluated by computing the ARL and SDRL values through extensive Monte Carlo simulation runs. The computational results have been provided in tabular format. The simulation study findings have been graphically presented to highlight on the efficacy of the proposed Bayesian EWMA control chart foring different PRSS sampling designs. The current study has the potential to be extended to different sampling schemes, particularly successive sampling, as well as to non-normal distributions.

Use of AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

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Conflict of interest

The authors declare no conflict of interest.

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