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Research article

An integrated group decision-making technique under interval-valued probabilistic linguistic T-spherical fuzzy information and its application to the selection of cloud storage provider

Shahid Hussain Gurmani[1], Zhao Zhang[1,*] and Rana Muhammad Zulqarnain[2]

[1] School of Mathematical Sciences, Zhejiang Normal University, Jinhua, Zhejiang 321004, China
[2] Department of Mathematics, University of Management and Technology, Sialkot Campus, 51310, Pakistan

* **Correspondence:** Email: zhaozhang@zjnu.cn; Tel: +8615888980790.

Abstract: Cloud storage is crucial in today's digital era due to its accessibility, scalability, cost savings, collaboration and enhanced security features. The selection of a reliable cloud storage provider is a significant multi-attribute group decision-making (MAGDM) problem that involves intrinsic relationships among the various alternatives, attributes and decision DMs. Due to the uncertain and incomplete nature of the evaluation data for cloud storage providers, i.e., quality of service and user feedback, the identification of appropriate cloud storage providers with accurate service ranking remains an open research challenge. To address the above-mentioned challenge, this work proposes the concept of interval-valued probabilistic linguistic T-spherical fuzzy set (IVPLt-SFS). Then, some basic operations and a score function are defined to compare two or more IVPLt-SF numbers (IVPLt-SFNs). For information fusion, two aggregation operators for IVPLt-SFN are also developed. Next, an extended TOPSIS method-based group decision-making technique under interval-valued probabilistic linguistic T-spherical fuzzy information is established to solve the MAGDM problem. Finally, a numerical example is given to illustrate the practicability and usefulness of the designed approach and its suitability as a decision-making tool for selecting a cloud storage provider. Comparative and sensitivity analysis confirmed that this paper enriches the theory and methodology of the selection problem of cloud storage provider and MAGDM analysis.

Keywords: T-spherical fuzzy set; linguistic T-spherical fuzzy set; multi-attribute group decision-making; aggregation operator; TOPSIS method
Mathematics Subject Classification: 94D05, 03B52

1. Introduction

Cloud storage is the practice of storing digital data on remote servers accessed via the internet instead of local physical devices. In comparison to conventional storage techniques, it has many benefits. It allows users to store, access, and manage their data from any location with an internet connection [1]. The selection of a cloud storage provider is crucial. It guarantees the safety, dependability and accessibility of data that has been saved. Strong data security measures are provided by a reliable source, lowering the possibility of breaches or illegal access [2]. When several decision-makers (DMs) or stakeholders with varied preferences and opinions are participating in the selection process, the cloud storage provider selection problem changes into a multi-attribute group decision-making (MAGDM) problem. Multiple attributes or criteria must be taken into account in these circumstances in order to reach a wise and unified choice [3].

Multiple decision-makers (DMs) evaluate and rank alternatives using a variety of attributes or criteria in a decision-making process known as MAGDM. The attributes or criteria might be qualitative or quantitative, such as cost, quality, efficiency or subjective aspects. MAGDM aims to incorporate the preferences of several DMs into a collective decision or rating while taking into account each one's unique priorities, opinions and areas of skills. It entails using strategies and tactics to compile and evaluate the DMs' ideas, sometimes using mathematical or statistical methods, in order to facilitate group decision-making and find common ground or a compromise between opposing points of view [4–10]. MAGDM is frequently utilized in numerous fields, including business [11], management [12–14], engineering [15,16] and social sciences [17], where decisions often require input from multiple stakeholders with different perspectives. For the MAGDM process to be successful, the DMs must communicate, work together and coordinate effectively. However, it can also be challenging due to potential conflicts, differing opinions and the need for effective decision-making processes to account for varying perspectives. Properly executed, MAGDM can lead to better decision-making outcomes that reflect a more comprehensive consideration of various attributes and perspectives.

1.1. Relevant literature, research gap and motivations

MAGDM problems often involve uncertain or ambiguous information, such as subjective preferences or incomplete data. Due to its capacity to handle uncertainty, linguistic modeling and assist group decision-making, DMs use fuzzy set (FS) theory proposed by Zadeeh [18] in the context of MAGDM. Classical FSs allow for the representation of uncertainty using membership degree (MD) and are capable of handling vague or ambiguous information. However, it does not explicitly capture the hesitation or uncertainty in making decisions or assigning non-membership degree (NMD) to elements. The development of an intuitionistic fuzzy set (IFS) [19] has been motivated by the need for a more comprehensive and nuanced approach to decision-making, especially in situations where DMs may have varying levels of confidence or hesitation in their judgments. It uses separate MD and NMD to represent uncertainty more precisely, with the restriction that the sum of MD and NMD must lie between zero and one. Yager [20,21] introduced Pythagorean fuzzy set (PyFS) by expanding the condition that the square sum of the MD and NMD must be within [0,1]. Later, he generalized this idea by presenting a parameter 'q' such that the sum of the qth power of MD and qth power of NMD must lie between zero and one [22]. This kind of set was named as q-rung orthopair fuzzy set (q-ROFs). q-ROF is a more general and flexible approach to modeling uncertainty and decision-making problems.

Although, many developments have been made in providing freedom to the DMs by introducing various extended and hybrid fuzzy sets. However, there are instances in which the decision-making process becomes complicated by the involvement of abstinence, and then the DMs have no tool to tackle this type of issues. For example, in the voting system, some individuals refuse to give their opinions. Cuong and Kreinovich developed the concept of the picture fuzzy set (PFS) to address this kind of situation [23]. The authors introduced abstinence degree (AD) together with MD and NMD and defined the requirement that MD, AD and NMD added together must be less than or equal to one. PFS-based regret theory was proposed by He and Wang [24] for the evaluation of new energy vehicles from an online review. Frank t-norm and t-conform-based new operations for PFS were offered by Seikh et al. in [25]. PFS can describe AD information that IFS, P_yFS and q-ROFS cannot, but it has the limitation of failing if the sum of 3 degrees exceeds one. Mahmood et al. [26] expanded the idea of the spherical fuzzy set (SFS) in this regard. Then they promoted it to the generalized form, i.e., T-spherical fuzzy set (t-SFS), in order to free DMs from the constraints of MD, AD,and ND allocation with a larger decision space and allow them to express their preferences and opinions more freely. The extended fuzzy sets discussed above are all special instances of t-SFS, which has received much attention from academics due to its generalized t-SFS without any restrictions. t-SFS have been used in several research areas, such as decision-making, pattern recognition, image processing and more. For example, in [27] the proposed method considers both the economic and environmental aspects of supplier selection. In [28], the authors proposed Archimedean aggregation operators using t-SF information and developed an MAGM technique to use it for the selection of surgical instruments. Interval-valued T-SFS-based decision-making framework was utilized by Pirbalouti et al. [29] to manage hydrogen refueling station leakage.

However, DMs frequently deal with difficult issues including ambiguous and inaccurate information. This type of information is frequently expressed in natural language, but linguistic expressions are difficult for traditional mathematical models to represent and handle. By emergence of the concept of a linguistic term set (LTS) [30–32] in fuzzy set theory, DMs have a tool that allows them to represent and manipulate linguistic variables in a flexible and accurate way. This implies that professionals can use linguistic terms to deal with ambiguous and imprecise information in a more natural and intuitive way and to make better decisions based on the available data. For instance, in a medical diagnosis system, the expert might want to use linguistic terms like "mild", "moderate", and "severe" to indicate the severity of a patient's symptoms. The expert can use fuzzy set operations to combine the various symptoms and make a diagnosis by associating each term with a fuzzy set, modeling the uncertainty in each term's meaning. Various fuzzy sets have been combined with LTSs. For example, Yazdi [33] explained system safety and reliability analysis using different linguistic methods under a fuzzy environment. Chen et al. [34] suggested the linguistic IFS (LIFS), which combines the IFS and LTS. Extensions of the LIFS have been suggested for some cases where the LIFS fails, such as the linguistic PYFS (LPYFS) [35], linguistic PFS [36], linguistic SFSs [37], linguistic t-SFS and linguistic interval valued t-SFSs [38,39].

Fuzzy set theory is a mathematical framework that allows for the representation of uncertainty and imprecision in data. It does this by allowing for partial membership in a set, meaning that an element can belong to a set to a degree or with a certain level of certainty, rather than simply being a member or not. Probability theory, on the other hand, is a mathematical framework for dealing with uncertainty and randomness. It allows for the quantification of the likelihood of a particular event occurring, based on available evidence or prior knowledge. By combining probability and fuzzy set theory, researchers have created more powerful models for dealing with uncertainty in complex systems. For example, in [40], the authors have assigned probability to the occurrence of each event in t-SFSs. Luo and Liu applied the probabilistic interval-valued hesitant P_yFS for the selection process of project private partners [41]. Liu and Huang [42] introduced the idea of probabilistic linguistic q-rung orthopair fuzzy set and used it to construct a consensus reaching process. Xu et al. [43] extended the TOPSIS method under interval-valued probabilistic Lq-ROF information. In [44], Pang et al. introduced the concept of probabilistic linguistic term set (PLTS).

Furthermore, MAGDM techniques have been thoroughly studied using aggregation operators, similarity and distance measures, as well as some other well-known traditional decision-making methods, such as the TOPSIS method [45], VIKOR method [46], AHP method [47], multi-attributive border approximation area comparison (MABAC) method [48] and TODIM model [49,50]. These approaches indicated above have been extended by numerous researchers employing various types of fuzzy information. For example, Gurmani et al [51] extended TOPSIS method under q-rung orthopair hypersoft set. In [52], Tahir et al. designed an aggregation operator based MADM methodology. Wątróbski et al. [53] prolonged the DARIA-TOPSIS method and used it for the assessment of sustainable cities and communities. The TOPSIS technique is being extended in our study to the IVPLt-SF scenario. The TOPSIS methodology is a quick and efficient method for making decisions that aims to find the best option that is the closest to the positive ideal solution (PIS) and farthest from the negative ideal solution (NIS). Numerous academics have proposed TOPSIS adaptations for various fuzzy environments during the last few decades [54,55].

The selection of a cloud storage provider is a highly intricate matter characterized by considerable uncertainty. It should be noted that different companies offer distinct facilities based on a variety of terms and conditions. As an authority in the field, decision-making relies heavily on personal experience and skill. Given the presence of numerous uncertain factors, the evaluator faces multiple values when assessing a company's attributes, each having a different level of significance. To capture these differences in decision-making information provided by DMs, the introduction of probability serves the purpose of quantifying the significance levels. In this context, the data is presented using the IVPLt-SFS format, which incorporates interval-valued probability values that account for membership, abstinence and non-membership degrees. Even though the studies mentioned above have been proven to be useful and adaptable in a variety of situations, some DMs could choose to use interval values to convey their uncertainty and irresolution in complicated systems [56–59]. In this paper, we introduce interval values into the probability and propose the idea of interval-valued probabilistic linguistic T-spherical fuzzy set (IVPLt-SFS). As an extension of Lt-SFS, the IVPLt-SFS significantly broadens the field of information description. The IVPLt-SFS has a strong capacity to manage heterogeneous information concurrently, which makes it more versatile, adaptive and accurate. DMs have more freedom to communicate their assessment values and clearly articulate their perspectives regarding potential candidate alternatives when using IVPLt-SFSs.

1.2. Major contributions of the study

The primary contributions of this study are as follows:

(1) The interval-valued probabilistic linguistic T-spherical fuzzy set (IVPLt-SFS) notion is one we put out.

(2) For IVPLt-SF information, several fundamental operations, comparison rules, Euclidean distance and two aggregation operators are defined. Additionally, some of their characteristics are also investigated.

(3) An extended IVPt-SF TOPSIS method-based MAGDM technique is designed for the selection of cloud storage provider.

(4) Finally, the comparison and sensitivity analysis is carried out, which shows the robustness and effectiveness of the proposed model.

The organization of this paper is as follow: Section 2 presents the fundamental concepts related to this paper. Section 3 is devoted to the inception of IVPLt-SFS. In Section 4, two aggregation operators for IVPLt-SFNs are presented. Section 5 is about the development of TOPSIS method-based MAGDM technique under IVPLt-SF environment. Practical example and comparison analysis is provided in Section 6. Finally, Section 7 provides a brief conclusion of the study.

2. Preliminaries

In this section, some basic concept related to t-SFSs have been reviewed.

Definition 2.1. [38] Let X be a universe of discourse and $S=\{s_t \mid t=0,1,...,\tau\}$ be a continuous linguistic term set; then a Lt-SFS is defined as,

$$T=\left\{\left\langle x, s_{\mu}(x), s_{\eta}(x), s_{\nu}(x)\right\rangle \middle| x\in X\right\}$$

where $s_{\mu}(x), s_{\eta}(x), s_{\nu}(x)\in S$ are knows as linguistic membership degree, linguistic abstinence degree and linguistic non-membership degrees of the element $x\in X$ to the set T, respectively, satisfying the constraint $0\le \mu^q(x)+\eta^q(x)+\nu^q(x)\le t^q$ for a positive number $q\ge 1$ and $\pi_T = s_{\sqrt[q]{t^q-\left(\mu^q(x)+\eta^q(x)+\nu^q(x)\right)}}$ is known as linguistic refusal degree of x in T. For simplicity, the triplet $T=\left(s_{\mu}, s_{\eta}, s_{\nu}\right)$ is known as an Lt-SF number (Lt-SFN).

Definition 2.2. [38] Let $T=\left(s_{\mu}, s_{\eta}, s_{\nu}\right)$, $T_1=\left(s_{\mu_1}, s_{\eta_1}, s_{\nu_1}\right)$ and $T_2=\left(s_{\mu_2}, s_{\eta_2}, s_{\nu_2}\right)$ be three Lt-SFNs and $\lambda>0$. Then, following are the Dombi operations for Lt-SFNs:

(i) $$T_1\oplus T_2=\left(s_{\sqrt[q]{t^q-\frac{t^q}{1+\left\{\left(\frac{\mu_1^q}{t^q-\mu_1^q}\right)^{\xi}+\left(\frac{\mu_2^q}{t^q-\mu_2^q}\right)^{\xi}\right\}^{1/\xi}}}}, s_{\sqrt[q]{\frac{t^q}{1+\left\{\left(\frac{t^q-\eta_1^q}{\eta_1^q}\right)^{\xi}+\left(\frac{t^q-\eta_2^q}{\eta_2^q}\right)^{\xi}\right\}^{1/\xi}}}}, s_{\sqrt[q]{\frac{t^q}{1+\left\{\left(\frac{t^q-\nu_1^q}{\nu_1^q}\right)^{\xi}+\left(\frac{t^q-\nu_2^q}{\nu_2^q}\right)^{\xi}\right\}^{1/\xi}}}} \right)$$

(ii) $$T_1 \otimes T_2 = \left(s_{\sqrt[q]{\frac{t^q}{1+\left\{\left(\frac{t^q-\mu_1^q}{\mu_1^q}\right)^{\xi}+\left(\frac{t^q-\mu_2^q}{\mu_2^q}\right)^{\xi}\right\}^{1/\xi}}}}, s_{\sqrt[q]{t^q-\frac{t^q}{1+\left\{\left(\frac{\eta_1^q}{t^q-\eta_1^q}\right)^{\xi}+\left(\frac{\eta_2^q}{t^q-\eta_2^q}\right)^{\xi}\right\}^{1/\xi}}}}, s_{\sqrt[q]{t^q-\frac{t^q}{1+\left\{\left(\frac{\nu_1^q}{t^q-\nu_1^q}\right)^{\xi}+\left(\frac{\nu_2^q}{t^q-\nu_2^q}\right)^{\xi}\right\}^{1/\xi}}}} \right)$$

(iii) $$\lambda.T = \left(s_{\sqrt[q]{t^q-\frac{t^q}{1+\left\{\lambda\left(\frac{\mu^q}{t^q-\mu^q}\right)^{\xi}\right\}^{1/\xi}}}}, s_{\sqrt[q]{\frac{t^q}{1+\left\{\lambda\left(\frac{t^q-\eta^q}{\eta^q}\right)^{\xi}\right\}^{1/\xi}}}}, s_{\sqrt[q]{\frac{t^q}{1+\left\{\lambda\left(\frac{t^q-\nu^q}{\nu^q}\right)^{\xi}\right\}^{1/\xi}}}} \right)$$

(iv) $$T^{\lambda} = \left(s_{\sqrt[q]{\frac{t^q}{1+\left\{\lambda\left(\frac{t^q-\mu^q}{\mu^q}\right)^{\xi}\right\}^{1/\xi}}}}, s_{\sqrt[q]{t^q-\frac{t^q}{1+\left\{\lambda\left(\frac{\eta^q}{t^q-\eta^q}\right)^{\xi}\right\}^{1/\xi}}}}, s_{\sqrt[q]{t^q-\frac{t^q}{1+\left\{\lambda\left(\frac{\nu^q}{t^q-\nu^q}\right)^{\xi}\right\}^{1/\xi}}}} \right)$$

3. Interval-valued probabilistic linguistic T-spherical fuzzy set

This section presents the concept of IVPLt-SFS, beginning with its definition. It also includes algebraic operations, a comparison rule for IVLPLt-SF numbers (IVLPLt-SF) and a distance measure between them.

Definition 3.1. Let X be a universal set and $S=\left\{s_{\alpha}\left|s_0 \le s_{\alpha} \le s_t, \alpha \in [0,t]\right.\right\}$ continuous linguistic term set. An interval-valued probabilistic linguistic T-SFS (IVPLt-SFS) $\tilde{A}$ defined on X is expressed as

$$\tilde{A}=\left\{\left\langle x, \mu_{\tilde{A}}(x) \middle| p_{\tilde{A}}(x), \eta_{\tilde{A}}(x) \middle| h_{\tilde{A}}(x), v_{\tilde{A}}(x) \middle| d_{\tilde{A}}(x), \right\rangle \middle| x \in X\right\} \quad (3.1)$$

where $\mu_{\tilde{A}}(x)$, $\eta_{\tilde{A}}(x)$, $v_{\tilde{A}}(x) \in S$ are three sets of some linguistic terms, which are called as LMD, LAD and LNMD of the element $x \in X$ to $\tilde{A}$ respectively. $p_{\tilde{A}}(x)=\left[p_{\tilde{A}}^L(x), p_{\tilde{A}}^U(x)\right]$, $p_{\tilde{A}}^L(x)=\inf p_{\tilde{A}}(x)$ and $p_{\tilde{A}}^U(x)=\sup p_{\tilde{A}}(x)$; $h_{\tilde{A}}(x)=\left[h_{\tilde{A}}^L(x), h_{\tilde{A}}^U(x)\right]$, $h_{\tilde{A}}^L(x)=\inf h_{\tilde{A}}(x)$ and $h_{\tilde{A}}^U(x)=\sup h_{\tilde{A}}(x)$; $d_{\tilde{A}}(x)=\left[d_{\tilde{A}}^L(x), d_{\tilde{A}}^U(x)\right]$, $d_{\tilde{A}}^L(x)=\inf d_{\tilde{A}}(x)$ and $d_{\tilde{A}}^U(x)=\sup d_{\tilde{A}}(x)$ are three interval values, denoting the interval-valued probabilistic information of $p_{\tilde{A}}(x)$, $h_{\tilde{A}}(x)$ and $d_{\tilde{A}}(x)$ respectively.

Additionally, $s_{\varphi}, s_{\phi}, s_{\gamma} \in S$, $\left(\varphi^{+}\right)^q+\left(\phi^{+}\right)^q+\left(\gamma^{+}\right)^q \le t^q$ for a positive integer $(q \ge 1)$, where $s_{\varphi} \in \mu_{\tilde{A}}(x), s_{\phi} \in \eta_{\tilde{A}}(x), s_{\gamma} \in v_{\tilde{A}}(x)$ and $s_{\varphi^{+}}=\bigcup_{s_{\varphi} \in \mu_{\tilde{A}}(x)} \max\left\{s_{\varphi}\right\}$, $s_{\phi^{+}}=\bigcup_{s_{\phi} \in \mu_{\tilde{A}}(x)} \max\left\{s_{\phi}\right\}$ and $s_{\gamma^{+}}=\bigcup_{s_{\gamma} \in \mu_{\tilde{A}}(x)} \max\left\{s_{\gamma}\right\}$, $\#\mu, \#\eta, \#v$ denote the number of values in μ, η and v respectively. Here,

$p_{\tilde{A}}(x), h_{\tilde{A}}(x), d_{\tilde{A}}(x) \subseteq [0,1]$ such that $\sum_{i=1}^{\#\mu} p_{\tilde{A}}^{U}(x)_i \leq 1$, $\sum_{j=1}^{\#\eta} h_{\tilde{A}}^{U}(x)_j \leq 1$ and $\sum_{k=1}^{\#\nu} d_{\tilde{A}}^{U}(x)_k \leq 1$. We call the triplet $\tilde{A} = \left\{ \mu_{\tilde{A}}(x) \middle| p_{\tilde{A}}(x), \eta_{\tilde{A}}(x) \middle| h_{\tilde{A}}(x), \nu_{\tilde{A}}(x) \middle| d_{\tilde{A}}(x), \right\}$ an IVPLt-SF number (IVPLt-SFN) and is simply denoted as $\tilde{a} = \left\{ \mu \middle| p_{\mu}, \eta \middle| h_{\eta}, \nu \middle| d_{\nu} \right\}$.

Example 1. Let $S = \left\{ s_{\alpha} \middle| s_0 \leq s_{\alpha} \leq s_7, \alpha \in [0,7] \right\}$ be LTS. Suppose that two DMs are asked to evaluate whether a healthcare provider is a good investment in terms of its potential value, they will need to conduct a comprehensive assessment of the provider's financial and operational performance. To do this, they might review financial statements, analyze operational metrics, evaluate the competitive landscape, consider future growth potential and estimate the provider's overall value based on projected cash flows. Ultimately, the DMs will draw on their knowledge and expertise to determine whether the healthcare provider is a wise investment choice. For that, the first expert considers 30–50% for a positive outcome, 10–20% for abstinence, and 20–40% for a negative outcome. To express his opinion, the decision maker uses linguistic variables, specifically "s_5" to indicate a positive opinion, "s_2" to indicate abstinence, and "s_3" to indicate a negative opinion. In other words, the expert has some degree of uncertainty about the investment opportunity but feels confident enough to express an opinion. Using linguistic variables may reflect the decision maker's preference for qualitative or subjective assessments rather than strictly numerical or quantitative approaches. The second expert has more nuanced opinions about the investment. He considers 20–50% for a positive outcome and uses the linguistic expression "s_4" to express his opinion. He also believes that there is a 30–50% chance that the investment is worth it, and he uses the linguistic expression "s_6" to express this opinion. To express the abstinence degree, he chooses 10–15% by using the linguistic expression "s_3" However, he still has some doubts about the investment, as they believe there is a 30–45% chance that it is not worth it, and use the linguistic expression "s_2" to express this opinion. The evaluation given by the two individuals can then be described as

$\tilde{a}_1 = \left\{ \left\{ s_5 \middle| [0.3, 0.5] \right\}, \left\{ s_2 \middle| [0.1, 0.2] \right\}, \left\{ s_3 \middle| [0.2, 0.4] \right\} \right\}$, and

$\tilde{a}_1 = \left\{ \left\{ s_4 \middle| [0.2, 0.5], s_6 \middle| [0.3, 0.5] \right\}, \left\{ s_3 \middle| [0.1, 0.15] \right\}, \left\{ s_4 \middle| [0.3, 0.45] \right\} \right\}$.

We can see that the value of linguistic variable in this example is '7' and if we consider $q > 3$ then it is easy to spot that $5^3 + 2^3 + 3^3 = 160 < t^3 = 343$ and $6^3 + 3^3 + 4^3 = 307 < t^3 = 343$. Which means that the constraint holds and meets the definition of IVPLt-SFS.

Remark 1. From Definition 3.2, it can be seen that

(1) When $q = 1$, then $\tilde{A}$ is reduced to interval-valued probabilistic linguistic picture fuzzy set.

(2) When $q = 2$, then $\tilde{A}$ is reduced to interval-valued probabilistic linguistic spherical fuzzy set.

(3) When $q = 1$, $\eta_{\tilde{A}}(x) = 0$ then $\tilde{A}$ is reduced to interval-valued probabilistic linguistic intuitionistic fuzzy set.

(4) When $q = 2$, $\eta_{\tilde{A}}(x) = 0$ then $\tilde{A}$ is reduced to interval-valued probabilistic linguistic Pythagorean fuzzy set.

(5) When $p_{\tilde{A}}^{L}(x) = p_{\tilde{A}}^{U}(x)$, $h_{\tilde{A}}^{L}(x) = h_{\tilde{A}}^{U}(x)$, and $d_{\tilde{A}}^{L}(x) = d_{\tilde{A}}^{U}(x)$, then $\tilde{A}$ is reduced to probabilistic linguistic t-SFS.

Definition 3.2. Let $\tilde{a}=\left\{\mu\left|p_{\mu},\eta\right|h_{\eta},v\left|d_{v}\right.\right\}$, $\tilde{a}_1=\left\{\mu_1\left|p_{\mu_1},\eta_1\right|h_{\eta_1},v_1\left|d_{v_1}\right.\right\}$ and $\tilde{a}_2=\left\{\mu_2\left|p_{\mu_2},\eta_2\right|h_{\eta_2},v_2\left|d_{v_2}\right.\right\}$ be any three IVPLt-SFNs and $\lambda>0$, then

$$\text{(1)}\quad \tilde{a}_1\oplus\tilde{a}_2=\bigcup_{\substack{s_{\varphi_1}\in\mu_1,s_{\phi_1}\in\eta_1,s_{\gamma_1}\in v_1\\ s_{\varphi_2}\in\mu_2,s_{\phi_2}\in\eta_2,s_{\gamma_2}\in v_2}}\left\{\begin{array}{l}\left\{s_{\left(\varphi_1^q+\varphi_2^q+\varphi_1^q\varphi_2^q/t^q\right)^{1/q}}\left|\left[p_{\varphi_1}^L p_{\varphi_2}^L,p_{\varphi_1}^U p_{\varphi_2}^U\right]\right.\right\},\left\{s_{(\phi_1\phi_2/t)}\left|\left[h_{\phi_1}^L h_{\phi_2}^L,h_{\phi_1}^U h_{\phi_2}^U\right]\right.\right\},\\ \left(s_{\left(\gamma_1\gamma_2/t\right)}\left|\left[d_{\gamma_1}^L d_{\gamma_2}^L,d_{\gamma_1}^U d_{\gamma_2}^U\right]\right.\right)\end{array}\right\} \tag{3.2}$$

$$\text{(2)}\quad \tilde{a}_1\otimes\tilde{a}_2=\bigcup_{\substack{s_{\varphi_1}\in\mu_1,s_{\phi_1}\in\eta_1,s_{\gamma_1}\in v_1\\ s_{\varphi_2}\in\mu_2,s_{\phi_2}\in\eta_2,s_{\gamma_2}\in v_2}}\left\{\begin{array}{l}\left\{s_{(\varphi_1\varphi_2/t)}\left|\left[p_{\varphi_2}^L p_{\varphi_2}^L,p_{\varphi_1}^U p_{\varphi_2}^U\right]\right.\right\},\left(s_{\left(\phi_1\phi_2/t\right)}\left|\left[d_{\phi_1}^L d_{\phi_2}^L,d_{\phi_1}^U d_{\phi_2}^U\right]\right.\right),\\ \left\{s_{\left(\gamma_1^q+\gamma_2^q+\gamma_1^q\gamma_2^q/t^q\right)^{1/q}}\left|\left[d_{\gamma_1}^L d_{\gamma 2}^L,d_{\gamma_1}^U d_{\gamma 2}^U\right]\right.\right\}\end{array}\right\} \tag{3.3}$$

$$\text{(3)}\quad \lambda\tilde{a}=\bigcup_{s_\varphi\in\mu,s_\phi\in\eta,s_\gamma\in v}\left\{\left\{s_{t\left(1-\left(1-\varphi^q/t^q\right)^{\lambda}\right)^{1/q}}\left|\left[p_\varphi^L,p_\varphi^U\right]\right.\right\},\left\{s_{t(\phi/t)^{\lambda}}\left|\left[h_\phi^L,h_\phi^U\right]\right.\right\},\left\{s_{t(\gamma/t)^{\lambda}}\left|\left[d_\gamma^L,d_\gamma^U\right]\right.\right\}\right\} \tag{3.4}$$

$$\text{(4)}\quad \left(\tilde{a}\right)^{\lambda}=\bigcup_{s_\varphi\in\mu,s_\phi\in\eta,s_\gamma\in v}\left\{\begin{array}{l}\left\{s_{t(\varphi/t)^{\lambda}}\left|\left[p_\varphi^L,p_\varphi^U\right]\right.\right\},\left\{s_{t\left(1-\left(1-\phi^q/t^q\right)^{\lambda}\right)^{1/q}}\left|\left[h_\phi^L,h_\phi^U\right]\right.\right\},\\ \left\{s_{t\left(1-\left(1-\gamma^q/t^q\right)^{\lambda}\right)^{1/q}}\left|\left[d_\gamma^L,d_\gamma^U\right]\right.\right\}\end{array}\right\} \tag{3.5}$$

Theorem 3.1. Let $\tilde{a}=\left\{\mu\left|p_{\mu},\eta\right|h_{\eta},v\left|d_{v}\right.\right\}$, $\tilde{a}_1=\left\{\mu_1\left|p_{\mu_1},\eta_1\right|h_{\eta_1},v_1\left|d_{v_1}\right.\right\}$ and $\tilde{a}_2=\left\{\mu_2\left|p_{\mu_2},\eta_2\right|h_{\eta_2},v_2\left|d_{v_2}\right.\right\}$ be any three IVPLt-SFNs and $\lambda,\lambda_1,\lambda_2>0$, then we have

1) $\tilde{a}_1\oplus\tilde{a}_2=\tilde{a}_2\oplus\tilde{a}_1$

2) $\tilde{a}_1\otimes\tilde{a}_2=\tilde{a}_2\otimes\tilde{a}_1$

3) $\lambda\tilde{a}_2\oplus\lambda\tilde{a}_1=\lambda\left(\tilde{a}_1\oplus\tilde{a}_2\right)$

4) $\lambda_1\tilde{a}\oplus\lambda_2\tilde{a}=\left(\lambda_1\oplus\lambda_2\right)\tilde{a}$

5) $\left(\tilde{a}_1\otimes\tilde{a}_2\right)^{\lambda}=\tilde{a}_1^{\lambda}\otimes\tilde{a}_2^{\lambda}$

6) $\left(\tilde{a}_1\right)^c\oplus\left(\tilde{a}_2\right)^c=\left(\tilde{a}_1\oplus\tilde{a}_2\right)^c$.

Proof. The proof of (3) and (5) will be discussed in this section and the other proofs are similar.
(3) By Eq (3.4), we have

$$\lambda \tilde{a}_1 = \bigcup_{s_{\varphi_1}\in\mu_1, s_{\phi_1}\in\eta_1, s_{\gamma_1}\in\nu_1} \left\{ \left\{ s_{t\left(1-\left(1-\varphi_1^q/t^q\right)^{\lambda}\right)^{1/q}} \left[\left[p_{\varphi_1}^L, p_{\varphi_1}^U \right]\right] \right\}, \left\{ s_{t(\phi_1/t)^{\lambda}} \left[\left[h_{\phi_1}^L, h_{\phi_1}^U \right]\right] \right\}, \left\{ s_{t(\gamma_1/t)^{\lambda}} \left[\left[d_{\gamma_1}^L, d_{\gamma_1}^U \right]\right] \right\} \right\} \tag{3.6}$$

$$\lambda \tilde{a}_2 = \bigcup_{s_{\varphi_2}\in\mu_2, s_{\phi 2}\in\eta_2, s_{\gamma_2}\in\nu_2} \left\{ \left\{ s_{t\left(1-\left(1-\varphi_2^q/t^q\right)^{\lambda}\right)^{1/q}} \left[\left[p_{\varphi_2}^L, p_{\varphi_2}^U \right]\right] \right\}, \left\{ s_{t(\phi_2/t)^{\lambda}} \left[\left[h_{\phi_2}^L, h_{\phi_2}^U \right]\right] \right\}, \left\{ s_{t(\gamma_2/t)^{\lambda}} \left[\left[d_{\gamma_2}^L, d_{\gamma_2}^U \right]\right] \right\} \right\} \tag{3.7}$$

$$\lambda \tilde{a}_1 \oplus \lambda \tilde{a}_2 = \bigcup_{s_{\varphi_1}\in\mu_1, s_{\phi_1}\in\eta_1, s_{\gamma_1}\in\nu_1} \left\{ \left\{ s_{t\left(1-\left(1-\varphi_1^q/t^q\right)^{\lambda}\right)^{1/q}} \left[\left[p_{\varphi_1}^L, p_{\varphi_1}^U \right]\right] \right\}, \left\{ s_{t(\phi_1/t)^{\lambda}} \left[\left[h_{\phi_1}^L, h_{\phi_1}^U \right]\right] \right\}, \left\{ s_{t(\gamma_1/t)^{\lambda}} \left[\left[d_{\gamma_1}^L, d_{\gamma_1}^U \right]\right] \right\} \right\} \oplus$$
$$\bigcup_{s_{\varphi_2}\in\mu_2, s_{\phi 2}\in\eta_2, s_{\gamma_2}\in\nu_2} \left\{ \left\{ s_{t\left(1-\left(1-\varphi_2^q/t^q\right)^{\lambda}\right)^{1/q}} \left[\left[p_{\varphi_2}^L, p_{\varphi_2}^U \right]\right] \right\}, \left\{ s_{t(\phi_2/t)^{\lambda}} \left[\left[h_{\phi_2}^L, h_{\phi_2}^U \right]\right] \right\}, \left\{ s_{t(\gamma_2/t)^{\lambda}} \left[\left[d_{\gamma_2}^L, d_{\gamma_2}^U \right]\right] \right\} \right\}$$

Then, by Eq (3.2), we obtain

$$\lambda \tilde{a}_1 \oplus \lambda \tilde{a}_2 = \bigcup_{s_{\varphi_i}\in\mu_i, s_{\phi_i}\in\eta_i, s_{\gamma_i}\in\nu_i;(i=1,2)} \left\{ \begin{array}{l} \left\{ s_{t\left(1-\prod_{i=1}^{2}\left(1-\varphi_i^q/t^q\right)^{\lambda}\right)^{1/q}} \left[\left[\prod_{i=1}^{2} p_{\varphi_i}^L, \prod_{i=1}^{2} p_{\varphi_i}^U \right]\right] \right\}, \\ \left\{ s_{t\prod_{i=1}^{n}(\phi_i/t)^{\lambda}} \left[\left[\prod_{i=1}^{2} h_{\phi_i}^L, \prod_{i=1}^{2} h_{\phi_i}^U \right]\right] \right\}, \left\{ \left\{ s_{t\prod_{i=1}^{n}(\gamma_i/t)^{\lambda}} \left[\left[\prod_{i=1}^{2} d_{\gamma_i}^L, \prod_{i=1}^{2} d_{\gamma_i}^U \right]\right] \right\} \right\} \end{array} \right\}$$

(5) For any IVPLt-SFN $\lambda_1\tilde{a} \oplus \lambda_2\tilde{a} = (\lambda_1 \oplus \lambda_2)\tilde{a}$, by Eq (3.4) we have

$$\lambda_i \tilde{a} = \bigcup_{s_{\varphi}\in\mu, s_{\phi}\in\eta, s_{\gamma}\in\nu} \left\{ \left\{ s_{t\left(1-\left(1-\varphi^q/t^q\right)^{\lambda_i}\right)^{1/q}} \left[\left[p_{\varphi}^L, p_{\varphi}^U \right]\right] \right\}, \left\{ s_{t(\phi/t)^{\lambda_i}} \left[\left[h_{\phi}^L, h_{\phi}^U \right]\right] \right\}, \left\{ s_{t(\gamma/t)^{\lambda_i}} \left[\left[d_{\gamma}^L, d_{\gamma}^U \right]\right] \right\} \right\}$$

where $i = 1, 2$.

Then, according to Eqs (3.2), (3.6) and (3.7)

$$\lambda_1 \tilde{a} \oplus \lambda_2 \tilde{a} = \bigcup_{s_{\varphi}\in\mu, s_{\phi}\in\eta, s_{\gamma}\in\nu} \left\{ \begin{array}{l} \left\{ s_{t\left(1-\prod_{i=1}^{2}\left(1-\varphi_i^q/t^q\right)^{\lambda_i}\right)^{1/q}} \left[\left[\prod_{i=1}^{2} p_{\varphi_i}^L, \prod_{i=1}^{2} p_{\varphi_i}^U \right]\right] \right\}, \\ \left\{ s_{t\prod_{i=1}^{n}(\phi_i/t)^{\lambda_i}} \left[\left[\prod_{i=1}^{2} h_{\phi_i}^L, \prod_{i=1}^{2} h_{\phi_i}^U \right]\right] \right\}, \left\{ \left\{ s_{t\prod_{i=1}^{n}(\gamma_i/t)^{\lambda_i}} \left[\left[\prod_{i=1}^{2} d_{\gamma_i}^L, \prod_{i=1}^{2} d_{\gamma_i}^U \right]\right] \right\} \right\} \end{array} \right\}$$

$$= \bigcup_{s_{\varphi}\in\mu, s_{\phi}\in\eta, s_{\gamma}\in\nu} \left\{ \left\{ s_{t\left(1-\left(1-\varphi^q/t^q\right)^{\lambda_1+\lambda_2}\right)^{1/q}} \left[\left[p_{\varphi}^L, p_{\varphi}^U \right]\right] \right\}, \left\{ s_{t(\phi/t)^{\lambda_1+\lambda_2}} \left[\left[h_{\phi}^L, h_{\phi}^U \right]\right] \right\}, \left\{ s_{t(\gamma/t)^{\lambda_1+\lambda_2}} \left[\left[d_{\gamma}^L, d_{\gamma}^U \right]\right] \right\} \right\}$$

$$= (\lambda_1 \oplus \lambda_2)\tilde{a}.$$

Thus, we have shown that $\lambda_1\tilde{a}\oplus\lambda_2\tilde{a}=(\lambda_1\oplus\lambda_2)\tilde{a}$ holds.

Definition 3.3. Let $\tilde{a}=\{\mu|p_\mu,\eta|h_\eta,v|d_v\}$ be any IVPLt-SFN defined on LTS $S=\{s_\alpha|s_0\le s_\alpha\le s_t,\alpha\in[0,t]\}$, where $s_\varphi\in\mu_{\tilde{A}}(x),s_\phi\in\eta_{\tilde{A}}(x),s_\gamma\in v_{\tilde{A}}(x)$. Then the definitions of score function $SF(\tilde{a})$ and $AF(\tilde{a})$ of $\tilde{a}$ are given as follows:

$$SF(\tilde{a})=\left(\frac{1}{\#\mu}\sum_{i=1,\varphi_i\in\mu}^{\#\mu}\left(\varphi_i p_i^L\right)^q+\frac{1}{\#\mu}\sum_{i=1,\varphi_i\in\mu}^{\#\mu}\left(\varphi_i p_i^U\right)^q\right)-\left(\frac{1}{\#v}\sum_{i=1,\gamma_i\in v}^{\#v}\left(\gamma_i d_i^L\right)^q+\frac{1}{\#v}\sum_{i=1,\gamma_i\in v}^{\#v}\left(\gamma_i d_i^U\right)^q\right) \tag{3.8}$$

and

$$AF(\tilde{a})=\left(\frac{1}{\#\mu}\sum_{i=1,\varphi_i\in\mu}^{\#\mu}\left(\varphi_i p_i^L\right)^q+\frac{1}{\#\mu}\sum_{i=1,\varphi_i\in\mu}^{\#\mu}\left(\varphi_i p_i^U\right)^q\right)+\left(\frac{1}{\#v}\sum_{i=1,\gamma_i\in v}^{\#v}\left(\gamma_i d_i^L\right)^q+\frac{1}{\#v}\sum_{i=1,\gamma_i\in v}^{\#v}\left(\gamma_i d_i^U\right)^q\right). \tag{3.9}$$

Definition 3.4. Let $\tilde{a}_1=\{\mu_1|p_{\mu_1},\eta_1|h_{\eta_1},v_1|d_{v_1}\}$ and $\tilde{a}_2=\{\mu_2|p_{\mu_2},\eta_2|h_{\eta_2},v_2|d_{v_2}\}$ be any two IVPLt-SFNs, then

(1) If $SF(\tilde{a}_1)>SF(\tilde{a}_2)$, then $\tilde{a}_1>\tilde{a}_2$;

(2) If $SF(\tilde{a}_1)<SF(\tilde{a}_2)$, then $\tilde{a}_1<\tilde{a}_2$;

(3) If $SF(\tilde{a}_1)=SF(\tilde{a}_2)$, then there are two possible situations:

(i) If $AF(\tilde{a}_1)>AF(\tilde{a}_2)$, then $\tilde{a}_1>\tilde{a}_2$;

(ii) If $AF(\tilde{a}_1)=AF(\tilde{a}_2)$, then $\tilde{a}_1=\tilde{a}_2$.

Definition 3.5. Let $\tilde{a}_1=\{\mu_1|p_{\mu_1},\eta_1|h_{\eta_1},v_1|d_{v_1}\}$ and $\tilde{a}_2=\{\mu_2|p_{\mu_2},\eta_2|h_{\eta_2},v_2|d_{v_2}\}$ be any two IVPLt-SFNs, then the Euclidean distance measure for two IVPLt-SFNs is defined as follows:

$$dis(\tilde{a}_1,\tilde{a}_2)=\frac{1}{4t}\left(\begin{array}{l}\sum_{i=1}^{\#\mu_1}\sum_{j=1}^{\#\mu_2}\left|\frac{\varphi_i p_i^L-\varphi_j p_j^L}{\#\mu_1\#\mu_2}\right|^q+\sum_{i=1}^{\#\mu_1}\sum_{j=1}^{\#\mu_2}\left|\frac{\varphi_i p_i^U-\varphi_j p_j^U}{\#\mu_1\#\mu_2}\right|^q\\+\sum_{i=1}^{\#\gamma_1}\sum_{j=1}^{\#\gamma_2}\left|\frac{\gamma_i d_i^L-\gamma_j d_j^L}{\#\gamma_1\#\gamma_2}\right|^q+\sum_{i=1}^{\#\gamma_1}\sum_{j=1}^{\#\gamma_2}\left|\frac{\gamma_i d_i^U-\gamma_j d_j^U}{\#\gamma_1\#\gamma_2}\right|^q\end{array}\right)^{1/q}. \tag{3.10}$$

4. Aggregation operators for IVPLt-SFNs

Aggregation operators refer to mathematical functions that are utilized to summarize or combine multiple values into a single value. They are commonly used in data analysis, decision-making and database management to compute summary statistics like counts, averages, minimum and maximum values, sums and more. These operators are useful for analyzing large datasets and can provide valuable insights into the data. Therefore, in this section, two aggregation operators have been developed, namely; IVPLt-SF weighted averaging (IVPLt-SFWA) operator and IVPLt-SF weighted geometric (IVPLt-SFGA) operator to fuse the DMs assessment information.

Definition 4.1. Let $\tilde{a}_1 = \left\{ \mu_i \left| p_{\mu_i}, \eta_i \right| h_{\eta_i}, v_i \left| d_{v_i} \right. \right\} (i = 1, 2, ..., n)$ be a collection of IVPLt-SFNs and $w = (w_1, w_2, ..., w_n)^T$ be the corresponding weight vector, such that $\sum_{i=1}^{n} w_i = 1$ and $0 \le w_i \le 1$. Then, the IVPLt-SFWA operator of dimension is a mapping $IVPLt - SFWA : \Omega^n \to \Omega$ such that

$$IVPLt - SFWA(\tilde{a}_1, \tilde{a}_2, ..., \tilde{a}_n) = \bigoplus_{i=1}^{n} (w_i \alpha_i). \tag{4.1}$$

Theorem 4.1. Let $\tilde{a}_1 = \left\{ \mu_i \left| p_{\mu_i}, \eta_i \right| h_{\eta_i}, v_i \left| d_{v_i} \right. \right\} (i = 1, 2, ..., n)$ be a collection of IVPLt-SFNs. Then, the aggregation results obtained by IVPLt-SFWA operator is still an IVPLt-SFNs and is expressed as,

$$IVPLt - SFWA(\tilde{a}_1, \tilde{a}_2, ..., \tilde{a}_n) = \bigcup_{s_{\varphi_i} \in \mu_i, s_{\phi_i} \in \eta_i, s_{\gamma_i} \in v_i} \left\{ \begin{array}{l} \left\{ s_{t\left(1 - \prod_{i=1}^{n} \left(1 - \varphi_i^q / t^q\right)^{w_i}\right)^{1/q}} \left[\left[\prod_{i=1}^{n} p_{\varphi_i}^L, \prod_{i=1}^{n} p_{\varphi_i}^U \right] \right] \right\}, \\ \left\{ s_{t \prod_{i=1}^{n} (\phi_i / t)^{w_i}} \left[\left[\prod_{i=1}^{n} h_{\phi_i}^L, \prod_{i=1}^{n} h_{\phi_i}^U \right] \right] \right\}, \left\{ \left\{ s_{t \prod_{i=1}^{n} (\gamma_i / t)^{w_i}} \left[\left[\prod_{i=1}^{n} d_{\gamma_i}^L, \prod_{i=1}^{n} d_{\gamma_i}^U \right] \right] \right\} \right\} \end{array} \right\}. \tag{4.2}$$

Proof. According to Eq (3.4), we can get

$$w_i \alpha_i = \bigcup_{s_{\varphi_i} \in \mu_i, s_{\phi_i} \in \eta_i, s_{\gamma_i} \in v_i} \left\{ \left\{ s_{t\left(1 - \left(1 - \varphi_i^q / t^q\right)^{w_i}\right)^{1/q}} \left[\left[p_{\varphi_i}^L, p_{\varphi_i}^U \right] \right] \right\}, \left\{ s_{t(\phi_i / t)^{w_i}} \left[\left[h_{\phi_i}^L, h_{\phi_i}^U \right] \right] \right\}, \left\{ s_{t(\gamma_i / t)^{w_i}} \left[\left[d_{\gamma_i}^L, d_{\gamma_i}^U \right] \right] \right\} \right\}$$

then we can obtain,

$$\bigoplus_{i=1}^{n} (w_i \alpha_i) = \bigcup_{s_{\varphi_i} \in \mu_i, s_{\phi_i} \in \eta_i, s_{\gamma_i} \in v_i} \left\{ \begin{array}{l} \left\{ s_{t\left(1 - \prod_{i=1}^{n} \left(1 - \varphi_i^q / t^q\right)^{w_i}\right)^{1/q}} \left[\left[\prod_{i=1}^{n} p_{\varphi_i}^L, \prod_{i=1}^{n} p_{\varphi_i}^U \right] \right] \right\}, \\ \left\{ s_{t \prod_{i=1}^{n} (\phi_i / t)^{w_i}} \left[\left[\prod_{i=1}^{n} h_{\phi_i}^L, \prod_{i=1}^{n} h_{\phi_i}^U \right] \right] \right\}, \left\{ \left\{ s_{t \prod_{i=1}^{n} (\gamma_i / t)^{w_i}} \left[\left[\prod_{i=1}^{n} d_{\gamma_i}^L, \prod_{i=1}^{n} d_{\gamma_i}^U \right] \right] \right\} \right\} \end{array} \right\}$$

which completes the proof of Theorem 4.1.

Definition 4.2. Let $\tilde{a}_1 = \left\{ \mu_i \left| p_{\mu_i}, \eta_i \right| h_{\eta_i}, v_i \left| d_{v_i} \right. \right\} (i = 1, 2, ..., n)$ be a collection of IVPLt-SFNs and $w = (w_1, w_2, ..., w_n)^T$ be the corresponding weight vector, such that $\sum_{i=1}^{n} w_i = 1$ and $0 \le w_i \le 1$. Then, the IVPLt-SFWG operator of dimension is a mapping $IVPLt - SFWG : \Omega^n \to \Omega$ such that

$$IVPLt - SFWG(\tilde{a}_1, \tilde{a}_2, ..., \tilde{a}_n) = \bigotimes_{i=1}^{n} (\alpha_i)^{w_i} \tag{4.3}$$

Theorem 4.2. Let $\tilde{a}_1 = \left\{ \mu_i \left| p_{\mu_i}, \eta_i \right| h_{\eta_i}, v_i \left| d_{v_i} \right. \right\} \left(i = 1, 2, ..., n \right)$ be a collection of IVPLt-SFNs. Then, the aggregation results obtained by the IVPLt-SFWG operator is still an IVPLt-SFNs and is expressed as,

$$IVPLt-SFWG\left(\tilde{a}_1, \tilde{a}_2, ..., \tilde{a}_n\right) = \bigcup_{s_{\varphi_i} \in \mu_i, s_{\phi_i} \in \eta_i, s_{\gamma_i} \in v_i} \left\{ \begin{array}{l} \left\{ s_{t \prod_{i=1}^{n} (\varphi_i / t)^{w_i}} \left| \left[\prod_{i=1}^{n} p_{\varphi_i}^{L}, \prod_{i=1}^{n} p_{\varphi_i}^{U} \right] \right. \right\}, \left\{ s_{t \left(1 - \prod_{i=1}^{n} \left(1 - \phi_i^q / t^q \right)^{w_i} \right)^{1/q}} \left| \left[\prod_{i=1}^{n} h_{\phi_i}^{L}, \prod_{i=1}^{n} h_{\phi_i}^{U} \right] \right. \right\}, \\ \left\{ s_{t \left(1 - \prod_{i=1}^{n} \left(1 - \gamma_i^q / t^q \right)^{w_i} \right)^{1/q}} \left| \left[\prod_{i=1}^{n} d_{\gamma_i}^{L}, \prod_{i=1}^{n} d_{\gamma_i}^{U} \right] \right. \right\} \end{array} \right\} \tag{4.4}$$

Proof of Theorem 4.1 is similar to the Theorem 4.1, which is omitted here.

5. Multi-attribute group decision-making with IVPLt-SF information

Multi-attribute group decision-making with interval valued probabilistic linguistic T-spherical fuzzy information is a complex process used to evaluate and rank multiple options based on various criteria. With this approach, DMs communicate their ideas and preferences by providing IVPLt-SF information. Decision maker's information is a representation of uncertainty that takes into consideration their level of confidence in their evaluation of the available options. Compared to conventional fuzzy sets, the IVPLt-SFS is a kind of fuzzy set that may represent information in a more adaptable and sophisticated manner. The procedure entails establishing the standards and potential solutions, gathering and combining the IVPLt-SF data and rating the potential solutions according to their overall effectiveness. To address this issue, a variety of tools and procedures are available, such as the TOPSIS, VIKOR and ELECTRE methods.

The TOPSIS (Technique for Order of Preference by Similarity to Ideal Solution) method is a decision-making approach used to rank options based on their similarity to the ideal solution and dissimilarity to the negative ideal solution. This method is commonly used for multi-criteria decision-making and can be adapted to handle the complexity of MAGDM with interval valued probabilistic linguistic T-spherical fuzzy information. In this paper, we will extend the TOPSIS method using IVPLt-SF information.

Extended TOPSIS method based methodology for MAGDM under IVPLt-SFSs

In this section, we aim to provide an MAGDM methodology based on extend TOPSIS model where the evaluation information of DMs take the form of IVPLt-SFNs.

Suppose that there are 'm' alternatives of the form $X_i \left(i = 1, 2, ..., m \right)$ which are to be evaluated based on 'n' attributes $G_j \left(j = 1, 2, ..., n \right)$, whose weighting vector is given as $\omega = \left(\omega_1, \omega_2, ..., \omega_n \right)^T$, satisfying the constraint that $\sum_{j=1}^{n} \omega_j = 1$ and $0 \le \omega_j \le 1$. Consider that there are 'K' DMs $\left\{ E^{(1)}, E^{(2)}, ..., E^{(k)} \right\}$ with weighting vector $\left(\rho_1, \rho_2, ..., \rho_k \right)$. Assume that the '$K$' DMs have evaluated the alternatives X_i under the attributes G_j and give their evaluation information in the form IVPLt-SFNs $Y^{(\lambda)} = \left(y_{ij}^{(\lambda)} \right)$, $\lambda = 1, 2, ..., k$ where $y_{ij}^{(\lambda)} = \left\{ \mu_{ij}^k \left| p_{\mu_{ij}^k}, \eta_{ij}^k \right| h_{\eta_{ij}^k}, v_{ij}^k \left| d_{v_{ij}^k} \right. \right\}$. The TOPSIS technique is expanded to the IVPLt-SF scenario in order to determine the best option in the decision-making issue

(Figure 1). With all of the given information, the computation procedure can be shown in the following.

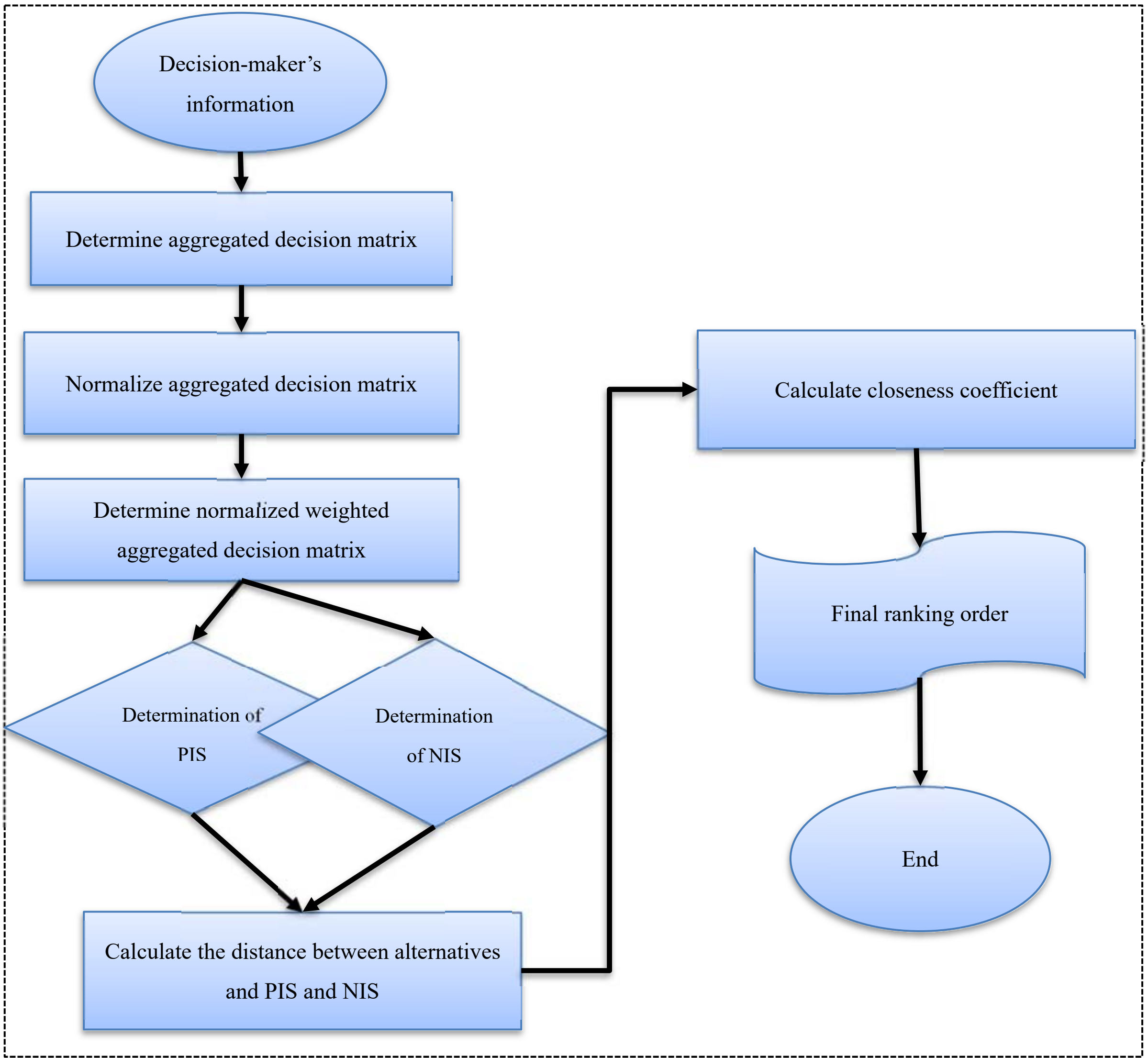

Figure 1. Flowchart of the proposed technique.

Step 1. Get the individual assessments of the DMs in the form of IVPLt-SF setting.

$$Y^{(\lambda)} = \left(y_{ij}^{(\lambda)}\right), \quad \lambda = 1, 2, \ldots, k \tag{5.1}$$

where

$$y_{ij}^{(\lambda)} = \left\{ \mu_{ij}^{k} \middle| p_{\mu_{ij}^{k}}, \eta_{ij}^{k} \middle| h_{\eta_{ij}^{k}}, v_{ij}^{k} \middle| d_{v_{ij}^{k}} \right\}.$$

Step 2. The second step involves aggregating all the information the DMs provided into a collective one using an IVPLt-SFWA operator. According to Eq (4.2):

$$y_{ij} = IVPLt - SFWA\left(y_{ij}^{(1)}, y_{ij}^{(2)}, \ldots, y_{ij}^{(k)}\right) = \bigoplus_{\lambda=1}^{k} \rho_{\lambda} y_{ij}^{(\lambda)}. \tag{5.2}$$

Step 3. Normalize the group IVPLt-SF decision matrix. If all the criteri are of the same type, then do nothing; if the criterion is cost type, then the cost type criterion should be converted into a benefit type. The normalized group decision matrix is shown as follow:

$$Y = y_{ij} = \left\{ \mu_{ij}^k \left| p_{\mu_{ij}^k}, \eta_{ij}^k \right| h_{\eta_{ij}^k}, v_{ij}^k \left| d_{v_{ij}^k} \right. \right\} = \begin{cases} \left\{ \mu_{ij}^k \left| p_{\mu_{ij}^k}, \eta_{ij}^k \right| h_{\eta_{ij}^k}, v_{ij}^k \left| d_{v_{ij}^k} \right. \right\}, & \text{for benefit type } G_j \\ \left\{ v_{ij}^k \left| d_{v_{ij}^k}, \eta_{ij}^k \right| h_{\eta_{ij}^k}, \mu_{ij}^k \left| p_{\mu_{ij}^k} \right. \right\}, & \text{for cost type } G_j \end{cases}. \tag{5.3}$$

Step 4. The weighted collective decision matrix should be computed in the third stage utilizing the attribute weights. According to Eq (3.7):

$$\overline{Y} = \left(\overline{y_{ij}} \right)_{m \times n} = \left(\omega_j y_{ij} \right)_{m \times n} \tag{5.4}$$

where $(i = 1, 2, ..., m; j = 1, 2, ..., n)$.

Step 5. In the third step, we must use the following equations to get the IVPLt-SF positive ideal solution (IVPLt-SF-PIS) y_j^+ and IVPLt-SF negative ideal solution (IVPLt-SF-NIS) y_j^- by using the following equations:

$$\begin{aligned} y_j^+ &= \left\{ \tilde{a}_1^+, \tilde{a}_2^+, ... \tilde{a}_n^+ \right\} \\ &= \left\{ \left\{ \max SF\left(\tilde{a}_{i1}\right) \right\}, \left\{ \max SF\left(\tilde{a}_{i2}\right) \right\}, ..., \left\{ \max SF\left(\tilde{a}_{in}\right) \right\} \right\} \end{aligned} \tag{5.5}$$

$$\begin{aligned} y_j^- &= \left\{ \tilde{a}_1^-, \tilde{a}_2^-, ... \tilde{a}_n^- \right\} \\ &= \left\{ \left\{ \min SF\left(\tilde{a}_{i1}\right) \right\}, \left\{ \min SF\left(\tilde{a}_{i2}\right) \right\}, ..., \left\{ \min SF\left(\tilde{a}_{in}\right) \right\} \right\} \end{aligned} \tag{5.6}$$

where $(i = 1, 2, ..., m; j = 1, 2, ..., n)$.

Step 6. The distances between each alternative with PIS and NIS are calculated based on Euclidean distance measure by using Eq (3.10):

$$\beta_i^+ = \sum_{j=1}^{n} d\left(\overline{y_{ij}}, y_j^+ \right); \tag{5.7}$$

$$\beta_i^- = \sum_{j=1}^{n} d\left(\overline{y_{ij}}, y_j^- \right) \tag{5.8}$$

where $(i = 1, 2, ...m)$ and $(j = 1, 2, ...n)$

Step 7. Now, we will calculate the closeness ratio $\Xi_i \, (i = 1, 2, ...m)$ of the alternatives by using the following equation:

$$\Xi_i = \frac{\beta_i^-}{\beta_i^+ + \beta_i^-}. \tag{5.9}$$

Step 8. Rank all the alternatives in descending order. The best alternative will be the one with greater values.

6. Application

In the following part, a numerical example and a comparison with existing techniques are provided to further elucidate the applicability of IVPLt-SFSs:

6.1. Numerical example of an IVPLt-SFN MAGDM problem

Cloud storage providers offer a third-party service that allows companies to store their digital data, such as files, documents, images and videos on remote servers that are hosted by the provider. This type of storage enables companies to store and access their data over the internet from anywhere, using various devices such as computers, smartphones and tablets. Using cloud storage can provide several benefits to companies, including scalability, accessibility, cost savings, security and collaboration features. Cloud storage providers offer scalable storage capacity, which means that companies can adjust their storage needs based on their business requirements. They also offer easy access to data from anywhere and anytime, which is convenient for remote teams or employees. Using cloud storage can also save companies money, as they do not have to invest in expensive hardware or maintenance. Cloud storage providers offer several security features to protect data from unauthorized access or loss, including data encryption, access controls and disaster recovery mechanisms. Finally, cloud storage providers offer collaboration features that enable teams to work together more effectively, regardless of their location.

When choosing a cloud storage provider, there are several criteria that companies should consider. Let us consider that a company invited three DMs $D=\left\{D^{(1)},D^{(2)},D^{(3)}\right\}$ with different knowledge and experiences having weighting vector $\rho=\left(0.36,0.24,0.4\right)$ to review some world-renown cloud storage providers. These DMs short-listed three alternatives:

$X_1-Amazon\ Web\ Services$; $X_2-Alibaba\ Cloud$; and $X_3-Tencent\ Cloud$

The four effective attributes they settled on and used to the company's selection procedure are explained as follows:

Storage capacity and scalability $\left(Z_1\right)$**:** The quantity of storage space, the business needs, and the provider's ability to grow storage capacity are two key factors to consider when choosing a cloud storage provider. This implies that companies should evaluate their current and future demands for data storage and select a supplier that can satisfy those needs. It is crucial to choose a supplier that will allow the organization to increase its storage capacity without moving providers or buying new gear. By only paying for the storage space they actually use, scalability may help businesses save money. It is also vital to consider the provider's pricing strategy because some use pay-as-you-go or tiered pricing. Ultimately, choosing a cloud storage provider with the appropriate scalability and storage capacity helps ensure that the business has the space to retain its data and can quickly adjust its storage requirements as it expands.

Accessibility $\left(Z_2\right)$**:** The second aspect to consider when selecting a cloud storage provider is accessibility. A provider who offers easy accessibility may boost productivity by enabling customers to access data from any place and on any device. Businesses should look for service providers who offer durable connections, nimble online and mobile applications and reliable data transfer methods. Consideration must be given to the provider's ability to integrate applications while maintaining ease of access.

Security $\left(Z_3\right)$**:** The third aspect to consider when selecting a cloud storage provider is security.

Businesses should seek service providers with robust security processes, such as encryption, access controls, data backups and disaster recovery plans. Additionally, service providers must adhere to rules particular to their business and have a transparent data privacy policy. The security, integrity and accessibility of data stored on the cloud platform must all be guaranteed.

Vendor Stability (Z_4)**:** Vendor stability is a term used to describe a cloud storage provider's financial stability, standing and durability. When choosing a cloud storage service, it's crucial to consider stability. This entails evaluating their financial standing, performance history and standing within their sector. A recognized, financially secure supplier is more likely to provide dependable services and has the funds to make changes. The likelihood of unexpected disruptions or the requirement for data migration is decreased by longevity in the market, which also shows their dedication. Assessing vendor stability aids in ensuring your firm receives dependable and trustworthy cloud storage services.

The decision hierarchy of the cloud storage provider selection problem is further explained in Figure 2.

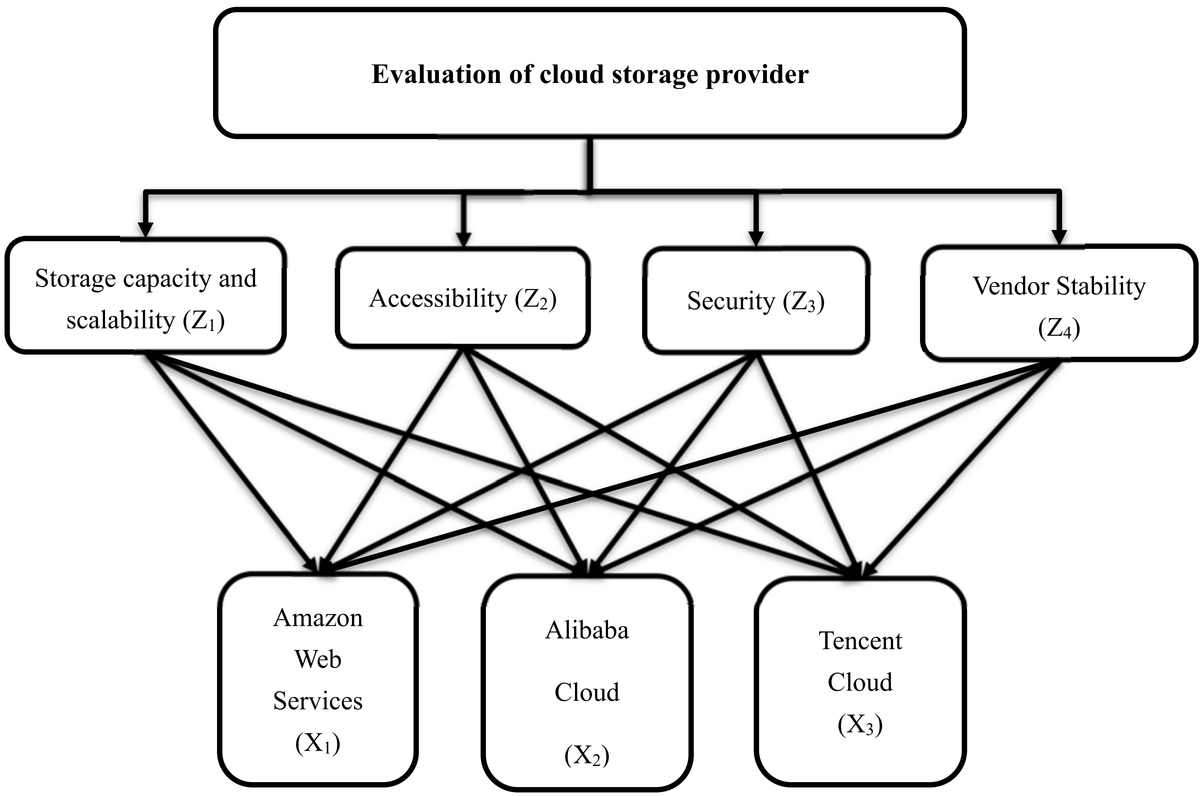

Figure 2. Decision hierarchy of cloud storage provider selection.

These four attributes explained above have weighting vector $w=(0.10, 0.30, 0.15, 0.45)$, and the evaluation matrices provided by DMs are $R^{(1)}, R^{(2)}$ and $R^{(3)}$, which are expressed in the form of IVPLt-SFS with $t=7$ and $q=3$, are given in Tables 1–3. In Section 5, the MAGDM technique is proposed as a way to determine the optimal alternative by aggregating the opinions of multiple DMs. The technique involves various computational processes, and the findings and outcomes are outlined and discussed below.

Step 1. The three DMs evaluate the short-listed alternatives $\{X_1, X_2, X_3\}$ and gave their assessment in the form of IVPLt-SFN, shown in Tables 1–3.
Step 2. By using the weights of DMs and Eqs (4.2) and (5.2), the collective decision matrix is given in Table 4.
Step 3. All the attributes are of the same type. Therefore, there is no need for normalization. We will proceed to the next step.

Table 1. Assessment matrix provided by decision-maker $D^{(1)}$.

	Z_1
X_1	$\{\{s_4\Vert[.3,.5]\},\{s_5\Vert[.1,.2]\},\{s_2\Vert[.2,.4]\}\}$
X_2	$\{\{s_3\Vert[.1,.5],s_6\Vert[.4,.5]\},\{s_1\Vert[.1,.3]\},\{s_4\Vert[.2,.6]\}\}$
X_3	$\{\{s_4\Vert[.1,.8]\},\{s_3\Vert[.3,.4]\},\{s_5\Vert[.2,.8]\}\}$
	Z_2
X_1	$\{\{s_2\Vert[.2,.6]\},\{s_3\Vert[.5,.7]\},\{s_1\Vert[.1,.5]\}\}$
X_2	$\{\{s_4\Vert[.5,.7]\},\{s_1\Vert[.2,.4]\},\{s_3\Vert[.5,.7]\}\}$
X_3	$\{\{s_2\Vert[.4,.7]\},\{s_1\Vert[.2,.4],s_6\Vert[.3,.6]\},\{s_4\Vert[.1,.3]\}\}$
	Z_3
X_1	$\{\{s_6\Vert[.3,.4]\},\{s_4\Vert[.2,.5]\},\{s_1\Vert[.1,.9]\}\}$
X_2	$\{\{s_3\Vert[.2,.5]\},\{s_4\Vert[.4,.6]\},\{s_4\Vert[.3,.4],s_5\Vert[.3,.6]\}\}$
X_3	$\{\{s_2\Vert[.6,.7]\},\{s_1\Vert[.3,.6]\},\{s_6\Vert[.2,.4]\}\}$
	Z_4
X_1	$\{\{s_3\Vert[.2,.5]\},\{s_4\Vert[.1,.6],s_6\Vert[.2,.4]\},\{s_1\Vert[.1,.3],s_2\Vert[.1,.4]\}\}$
X_2	$\{\{s_4\Vert[.4,.6]\},\{s_2\Vert[.4,.7],\},\{s_3\Vert[.3,.8]\}\}$
X_3	$\{\{s_4\Vert[.4,.7]\},\{s_3\Vert[.1,.4]\},\{s_5\Vert[.2,.6]\}\}$

Table 2. Assessment matrix provided by decision-maker $D^{(2)}$.

	Z_1
X_1	$\{\{s_5 \| [.3,.5]\}, \{s_4 \| [.2,.3]\}, \{s_2 \| [.4,.5]\}\}$
X_2	$\{\{s_4 \| [.4,.5]\}, \{s_2 \| [.1,.3]\}, \{s_5 \| [.5,.6]\}\}$
X_3	$\{\{s_4 \| [.2,.7]\}, \{s_3 \| [.5,.8]\}, \{s_2 \| [.4,.5]\}\}$
	Z_2
X_1	$\{\{s_3 \| [.2,.6], s_4 \| [.2,.6]\}, \{s_3 \| [.2,.4]\}, \{s_4 \| [.3,.5]\}\}$
X_2	$\{\{s_4 \| [.2,.45]\}, \{s_2 \| [.2,.25]\}, \{s_5 \| [.1,.2]\}\}$
X_3	$\{\{s_6 \| [.3,.6]\}, \{s_2 \| [.2,.4]\}, \{s_4 \| [.3,.6]\}\}$
	Z_3
X_1	$\{\{s_6 \| [.3,.4]\}, \{s_4 \| [.2,.5]\}, \{s_1 \| [.1,.9]\}\}$
X_2	$\{\{s_5 \| [.3,.4]\}, \{s_1 \| [.1,.3]\}, \{s_4 \| [.3,.4]\}\}$
X_3	$\{\{s_5 \| [.3,.5]\}, \{s_3 \| [.4,.5]\}, \{s_3 \| [.1,.6]\}\}$
	Z_4
X_1	$\{\{s_4 \| [.4,.7]\}, \{s_3 \| [.1,.4]\}, \{s_5 \| [.2,.6]\}\}$
X_2	$\{\{s_1 \| [0.4,0.5], s_4 \| [0.2,0.5]\}, \{s_2 \| [0.1,0.3]\}, \{s_5 \| [0.5,0.6]\}\}$
X_3	$\{\{s_2 \| [.1,.5]\}, \{s_3 \| [.2,.4]\}, \{s_2 \| [.3,.8]\}\}$

Table 3. Assessment matrix provided by decision-maker $D^{(3)}$.

	Z_1
X_1	$\{\{s_5\|[.4,.5], s_6\|[.3,.5]\}, \{s_2\|[.3,.9]\}, \{s_6\|[.3,.6]\}\}$
X_2	$\{\{s_4\|[.2,.45]\}, \{s_4\|[.2,.25]\}, \{s_2\|[.1,0.7], s_5\|[.2,0.3]\}\}$
X_3	$\{\{s_5\|[.1,.2]\}, \{s_3\|[.4,.5]\}, \{s_6\|[.6,.9]\}\}$
	Z_2
X_1	$\{\{s_3\|[.2,.45], s_5\|[.4,.55]\}, \{s_3\|[.2,.6]\}, \{s_6\|[.7,.9]\}\}$
X_2	$\{\{s_2\|[.3,.6]\}, \{s_6\|[.4,.5]\}, \{s_3\|[.3,.9]\}\}$
X_3	$\{\{s_5\|[.2,.4]\}, \{s_3\|[.3,.2], s_4\|[.1,.8]\}, \{s_6\|[.3,.6]\}\}$
	Z_3
X_1	$\{\{s_3\|[.3,.6], s_5\|[.2,.4]\}, \{s_2\|[.2,.5]\}, \{s_1\|[.4,.7]\}\}$
X_2	$\{\{s_3\|[.3,.5]\}, \{s_3\|[.1,.4]\}, \{s_4\|[.4,.6]\}\}$
X_3	$\{\{s_6\|[.3,.6]\}, \{s_2\|[.2,.4]\}, \{s_4\|[.3,.6]\}\}$
	Z_4
X_1	$\{\{s_3\|[.5,.6]\}, \{s_1\|[.3,.4]\}, \{s_4\|[.4,.6]\}\}$
X_2	$\{\{s_5\|[.3,.5], s_6\|[.3,.4]\}, \{s_3\|[.1,.4]\}, \{s_4\|[.3,.5]\}\}$
X_3	$\{\{s_3\|[.4,.5]\}, \{s_5\|[.2,.5]\}, \{s_4\|[.5,.6]\}\}$

Table 4. Aggregated group decision matrix.

	Z_1
X_1	$\{\{s_{4.714}\|[0.036,0.125], s_{5.3258}\|[0.027,0.125]\}, \{s_{3.285}\|[0.006,0.054]\}, \{s_{3.1037}\|[0.024,0.12]\}\}$
X_2	$\left\{\begin{matrix}\{s_{3.7135}\|[0.008,0.1125], s_{5.1024}\|[0.032,0.1125]\}, \{s_{3.9193}\|[0.001,0.063]\}, \\ \{s_{3.1982}\|[0.01,0.252], s_{4.6141}\|[0.02,0.108]\}\end{matrix}\right\}$
X_3	$\{\{s_{4.485}\|[0.002,0.112]\}, \{s_3\|[0.06,0.16]\}, \{s_{4.3166}\|[0.048,0.36]\}\}$
	Z_2
X_1	$\left\{\begin{matrix}\{s_{2.7274}\|[0.008,0.162], s_{3.1}\|[0.008,0.162], s_{4.017}\|[0.016,0.198], s_{4.1796}\|[0.016,0.198]\} \\ \{s_3\|[0.02,0.168]\}, \{s_{2.856}\|[0.021,0.225]\}\end{matrix}\right\}$
X_2	$\{\{s_{3.4985}\|[0.03,0.189]\}, \{s_{2.4183}\|[0.016,0.05]\}, \{s_{3.3913}\|[0.015,0.126]\}\}$
X_3	$\left\{\{s_{4.9253}\|[0.024,0.168]\}, \left\{\begin{matrix}s_{1.8327}\|[0.012,0.032], s_{3.4933}\|[0.018,0.048], \\ s_{2.0562}\|[0.004,0.128], s_{3.9193}\|[0.006,0.192]\end{matrix}\right\}, \{s_{4.7043}\|[0.009,0.108]\}\right\}$
	Z_3
X_1	$\{\{s_{5.4302}\|[0.027,0.096], s_{5.7017}\|[0.018,0.064]\}, \{s_{3.0314}\|[0.008,0.125]\}, \{s_1\|[0.004,0.567]\}\}$
X_2	$\{\{s_{3.7783}\|[0.018,0.1]\}, \{s_{2.5562}\|[0.004,0.072]\}, \{s_4\|[0.036,0.096], s_{4.3346}\|[0.036,0.144]\}\}$
X_3	$\{\{s_{5.1676}\|[0.054,0.21]\}, \{s_{1.7176}\|[0.024,0.12]\}, \{s_{4.3198}\|[0.006,0.144]\}\}$
	Z_4
X_1	$\left\{\begin{matrix}\{s_{3.3112}\|[0.04,0.21]\}, \{s_{2.1441}\|[0.003,0.096], s_{2.4811}\|[0.006,0.064]\}, \\ \{s_{2.562}\|[0.008,0.108], s_{3.2881}\|[0.008,0.144]\}\end{matrix}\right\}$
X_2	$\left\{\begin{matrix}\{s_{4.2646}\|[0.048,0.15], s_{4.485}\|[0.024,0.15], s_{5.0548}\|[0.048,0.12], s_{5.1844}\|[0.024,0.12]\}, \\ \{s_{2.3522}\|[0.004,0.084]\}, \{s_{3.8049}\|[0.045,0.24]\}\end{matrix}\right\}$
X_3	$\{\{s_{3.3199}\|[0.016,0.175]\}, \{s_{3.6801}\|[0.004,0.08]\}, \{s_{3.6703}\|[0.03,0.288]\}\}$

Step 3. By using attributes weight and Eqs (3.7) and (5.4), we compute a weighted aggregated decision matrix which is given in Table 5.

Table 5. Weighted aggregated group decision matrix.

	Z_1
X_1	$\{\{s_{2.3068}\mid[0.036,0.125], s_{2.6845}\mid[0.027,0.125]\}, \{s_{6.49}\mid[0.006,0.054]\}, \{s_{6.4532}\mid[0.024,0.12]\}\}$
X_2	$\{\{s_{1.7653}\mid[0.008,0.1125], s_{2.5405}\mid[0.032,0.1125]\}, \{s_{6.6055}\mid[0.001,0.063]\}, \{s_{6.4726}\mid[0.01,0.252], s_{6.7142}\mid[0.02,0.108]\}\}$
X_3	$\{\{s_{2.1765}\mid[0.002,0.112]\}, \{s_{6.4313}\mid[0.06,0.16]\}, \{s_{6.6696}\mid[0.048,0.36]\}\}$
	Z_2
X_1	$\{\{s_{1.8387}\mid[0.008,0.162], s_{2.0971}\mid[0.008,0.162], s_{2.7541}\mid[0.016,0.198], s_{2.8751}\mid[0.016,0.198]\}, \{s_{5.4288}\mid[0.02,0.168]\}, \{s_{5.3493}\mid[0.021,0.225]\}\}$
X_2	$\{\{s_{2.3782}\mid[0.03,0.189]\}, \{s_{5.0889}\mid[0.016,0.05]\}, \{s_{5.6322}\mid[0.015,0.126]\}\}$
X_3	$\{\{s_{3.4581}\mid[0.024,0.168]\}, \{s_{4.6827}\mid[0.012,0.032], s_{5.6825}\mid[0.018,0.048], s_{4.8472}\mid[0.004,0.128], s_{5.8821}\mid[0.006,0.192]\}, \{s_{6.2132}\mid[0.009,0.108]\}\}$
	Z_3
X_1	$\{\{s_{3.1373}\mid[0.027,0.096], s_{3.3547}\mid[0.018,0.064]\}, \{s_{6.1742}\mid[0.008,0.125]\}, \{s_{5.228}\mid[0.004,0.567]\}\}$
X_2	$\{\{s_{2.056}\mid[0.018,0.1]\}, \{s_{6.0183}\mid[0.004,0.072]\}, \{s_{6.4364}\mid[0.036,0.096], s_{6.5144}\mid[0.036,0.144]\}\}$
X_3	$\{\{s_{2.9428}\mid[0.054,0.21]\}, \{s_{5.6698}\mid[0.024,0.12]\}, \{s_{6.5111}\mid[0.006,0.144]\}\}$
	Z_4
X_1	$\{\{s_{2.5632}\mid[0.04,0.21]\}, \{s_{4.1102}\mid[0.003,0.096], s_{4.3893}\mid[0.006,0.064]\}, \{s_{4.4531}\mid[0.008,0.108], s_{4.9823}\mid[0.008,0.144]\}\}$
X_2	$\{\{s_{3.3433}\mid[0.048,0.15], s_{3.5308}\mid[0.024,0.15], s_{4.035}\mid[0.048,0.12], s_{4.1549}\mid[0.024,0.12]\}, \{s_{4.2851}\mid[0.004,0.084]\}, \{s_{5.3206}\mid[0.045,0.24]\}\}$
X_3	$\{\{s_{2.5701}\mid[0.016,0.175]\}, \{s_{5.2413}\mid[0.004,0.08]\}, \{s_{5.235}\mid[0.03,0.288]\}\}$

Step 4. According to Table 5 and Eqs (5.5) and (5.6), we can compute the IVPLt-SF-PIS $y_j^+ \left(j = 1,2,3,4 \right)$ and IVPLt-SF-NIS $y_j^- \left(j = 1,2,3,4 \right)$, which are given below.

$$y_1^+ = \left\{ \left\{ s_{2.3068} \left| \left[0.036, 0.125\right], s_{2.6845} \right| \left[0.027, 0.125\right] \right\}, \left\{ s_{6.49} \left| \left[0.006, 0.054\right] \right\}, \left\{ s_{6.4532} \left| \left[0.024, 0.12\right] \right\} \right\}$$

$$y_2^+ = \left\{ \left\{ s_{3.4581} \left| \left[0.024, 0.168\right] \right\}, \left\{ \begin{matrix} s_{4.6827} \left| \left[0.012, 0.032\right], s_{5.6825} \right| \left[0.018, 0.048\right], \\ s_{4.8472} \left| \left[0.004, 0.128\right], s_{5.8821} \right| \left[0.006, 0.192\right] \end{matrix} \right\}, \left\{ s_{6.2132} \left| \left[0.009, 0.108\right] \right\} \right\}$$

$$y_3^+ = \left\{ \left\{ s_{2.056} \left| \left[0.018, 0.1\right] \right\}, \left\{ s_{6.0183} \left| \left[0.004, 0.072\right] \right\}, \left\{ s_{6.4364} \left| \left[0.036, 0.096\right], s_{6.5144} \right| \left[0.036, 0.144\right] \right\} \right\}$$

$$y_4^+ = \left\{ \begin{matrix} \left\{ s_{2.5632} \left| \left[0.04, 0.21\right] \right\}, \left\{ s_{4.1102} \left| \left[0.003, 0.096\right], s_{4.3893} \right| \left[0.006, 0.064\right] \right\}, \\ \left\{ s_{4.4531} \left| \left[0.008, 0.108\right], s_{4.9823} \right| \left[0.008, 0.144\right] \right\} \end{matrix} \right\}$$

$$y_1^- = \left\{ \left\{ s_{2.1765} \left| \left[0.002, 0.112\right] \right\}, \left\{ s_{6.4313} \left| \left[0.06, 0.16\right] \right\}, \left\{ s_{6.6696} \left| \left[0.048, 0.36\right] \right\} \right\}$$

$$y_2^- = \left\{ \begin{matrix} \left\{ s_{1.8387} \left| \left[0.008, 0.162\right], s_{2.0971} \right| \left[0.008, 0.162\right], s_{2.7541} \left| \left[0.016, 0.198\right], s_{2.8751} \right| \left[0.016, 0.198\right] \right\} \\ \left\{ s_{5.4288} \left| \left[0.02, 0.168\right] \right\}, \left\{ s_{5.3493} \left| \left[0.021, 0.225\right] \right\} \end{matrix} \right\}$$

$$y_3^- = \left\{ \left\{ s_{3.1373} \left| \left[0.027, 0.096\right], s_{3.3547} \right| \left[0.018, 0.064\right] \right\}, \left\{ s_{6.1742} \left| \left[0.008, 0.125\right] \right\}, \left\{ s_{5.228} \left| \left[0.004, 0.567\right] \right\} \right\}$$

$$y_4^- = \left\{ \left\{ s_{2.5701} \left| \left[0.016, 0.175\right] \right\}, \left\{ s_{5.2413} \left| \left[0.004, 0.08\right] \right\}, \left\{ s_{5.235} \left| \left[0.03, 0.288\right] \right\} \right\}$$

Step 5. The distances between each alternative with PIS and NIS are computed by using Eqs (3.8), (5.7) and (5.8), which are given below:

$$\beta_1^+ = 0.0410, \ \beta_2^+ = 0.0405, \ \beta_3^+ = 0.0735$$

$$\beta_1^- = 0.0539, \ \beta_2^- = 0.0640, \ \beta_3^- = 0.0466$$

Step 6. The closeness ratio $\Xi_i \left(i = 1, 2, 3\right)$ of the alternatives is computed by using Eq (5.9).

$$\Xi_1 = 0.5679, \ \Xi_2 = 0.6124, \ \Xi_3 = 0.3880$$

Step 7. The value of the closeness ratio indicates the overall ranking order of the alternatives which is $X_2 > X_1 > X_3$. We conclude that '$X_2 - Alibaba\ Cloud$' is the best suitable cloud storage provider among all.

Sensitivity analysis

This part examines the effect of the parameter 'q'. Here, we take $3 \le q \le 10$ and the outcomes are given in Table 6 and illustrated graphically in Figure 3 as well. According to the closeness ratio obtained from using different values of parameter '$3 \le q \le 10$' and again using each stage of the suggested process, it was discovered that the alternatives' ranking did not change, which is '$X_2 > X_1 > X_3$, with the best alternative being X_2 and worst alternative X_3.

Table 6. Influence of parameters 'q' on the ranking of alternatives.

Parameter	Closeness ratio	Ranking order	Best alternative
$q=3$	$\Xi_1 = 0.5679$, $\Xi_2 = 0.6124$, $\Xi_3 = 0.3880$	$X_2 > X_1 > X_3$	X_2
$q=4$	$\Xi_1 = 0.5526$, $\Xi_2 = 0.6080$, $\Xi_3 = 0.3804$	$X_2 > X_1 > X_3$	X_2
$q=5$	$\Xi_1 = 0.5314$, $\Xi_2 = 0.6027$, $\Xi_3 = 0.3740$	$X_2 > X_1 > X_3$	X_2
$q=6$	$\Xi_1 = 0.5213$, $\Xi_2 = 0.5880$, $\Xi_3 = 0.3710$	$X_2 > X_1 > X_3$	X_2
$q=7$	$\Xi_1 = 0.5160$, $\Xi_2 = 0.5769$, $\Xi_3 = 0.3660$	$X_2 > X_1 > X_3$	X_2
$q=8$	$\Xi_1 = 0.5100$, $\Xi_2 = 0.5700$, $\Xi_3 = 0.3590$	$X_2 > X_1 > X_3$	X_2
$q=9$	$\Xi_1 = 0.5055$, $\Xi_2 = 0.5650$, $\Xi_3 = 0.3509$	$X_2 > X_1 > X_3$	X_2
$q=10$	$\Xi_1 = 0.4970$, $\Xi_2 = 0.5560$, $\Xi_3 = 0.3459$	$X_2 > X_1 > X_3$	X_2

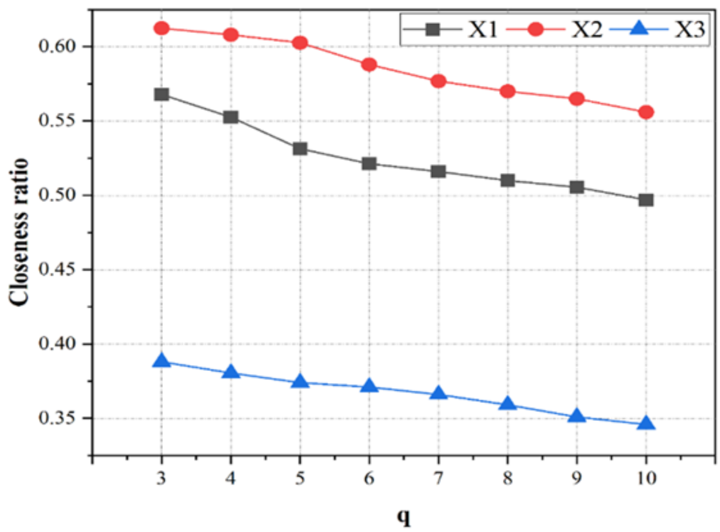

Figure 3. Impact of parameter 'q' on final ranking of alternatives.

One more thing is also worth noting: The closeness ratios decrease as the parameter value increases. The value of 'q' in this study represents the complexity of the decision environment and conditions. The complexity is higher when the value of 'q' means is higher. The final ranking order and best alternative obtained by using the different values of parameter 'q' on our proposed method are the same. However, when the data provided by DMs is more complicated and ambiguous, the results can differ. In real-world decision problems, the DMs may change the value of 'q' depending on the situation to achieve appropriate decision results.

6.2. Comparative analysis

A fuzzy set [18] can only handle situations when the MD of the elements is present and cannot handle circumstances where the NMD of the elements is also present. The MD and NMD scenarios can be handled by IFSs [19]. There are numerous applications of fuzzy set theory that can cope with MD, AD and NMD, and were made possible by the development of PFSs [23]. Later, Mahmood et al. [26] developed t-SFSs by improving the restrictions of PFS, which cannot handle the circumstances in which the sum of MD, AD and NMD exceeds one. Further, Lt-SFS [38] was offered by providing more freedom to the DMs so that they could give their assessments in a qualitative manner. Lt-SFSs thus enable professionals the freedom to present their analysis with the fewest limitations. Due to this, we employed IVPLt-SFSs to gather evaluation data from professionals for this study. As we discussed in the introduction, there is no literature available about the presentation of probability in Lt-SFS. However, probabilistic linguistic q-rung orthopair fuzzy weighted averaging (PLq-ROFWA) [42] and IVPLq-ROFS [43] are the special cases of the proposed approach. For example, when the linguistic abstinence degree is considered to be zero or the internal-valued probability is a single-valued probability in the above given practical example, the final results obtained by using the techniques proposed in [42] and [43] are compared with our suggested method which is shown in Table 7. We can see from Table 7 that the final ranking obtained is slightly different than our proposed approach, but the best option is still the same, which is X_2. Further, we have discussed the qualitative analysis of our designed methodology with some existing approaches. From Table 8 and Figure 4, It has come to light that the method suggested in this study is more comprehensive than the ones already in use.

Table 7. Comparison analysis of the proposed model to the existing techniques using considered data.

Methods	Closeness ratio/Score values	Ranking of alternatives	Best option
PLq-ROFWA [42]	$SC(X_1)=0.4671$, $SC(X_2)=0.5890$, $SC(X_3)=0.3910$	$X_2 > X_1 > X_3$	X_2
IVPLq-ROFS [43]	$SC(X_1)=0.4279$, $SC(X_2)=0.6511$, $SC(X_3)=0.6345$	$X_2 > X_3 > X_1$	X_2

Continued on next page

Methods	Closeness ratio/Score values	Ranking of alternatives	Best option
Proposed method	$\Xi_1 = 0.5679$, $\Xi_2 = 0.6124$, $\Xi_3 = 0.3880$	$X_2 > X_1 > X_3$	X_2

Table 8. Qualitative comparison analysis of the proposed model with the existing techniques.

Set	Parameter	Involvement of abstinence degree	Involvement of linguistic term	Involvement of probability	Involvement of interval-valued probability	Computational complexity
Lt-SFS [38]	Yes	Yes	Yes	No	No	Relatively high
PLTS [60]	No	No	Yes	Yes	No	Relatively high
IVPLTS [61]	No	No	Yes	Yes	Yes	Relatively high
PLT [62]	No	No	Yes	Yes	No	Relatively high
TOPSIS-based PLTS [63]	No	No	Yes	Yes	No	Relatively high
Pt-SHFS [40]	Yes	Yes	No	Yes	No	Relatively high
PLq-ROFWA) [42]	Yes	No	Yes	Yes	No	Relatively high
IVPLq-ROFS [43]	Yes	No	Yes	Yes	Yes	Relatively high
Proposed method	Yes	Yes	Yes	Yes	Yes	Low

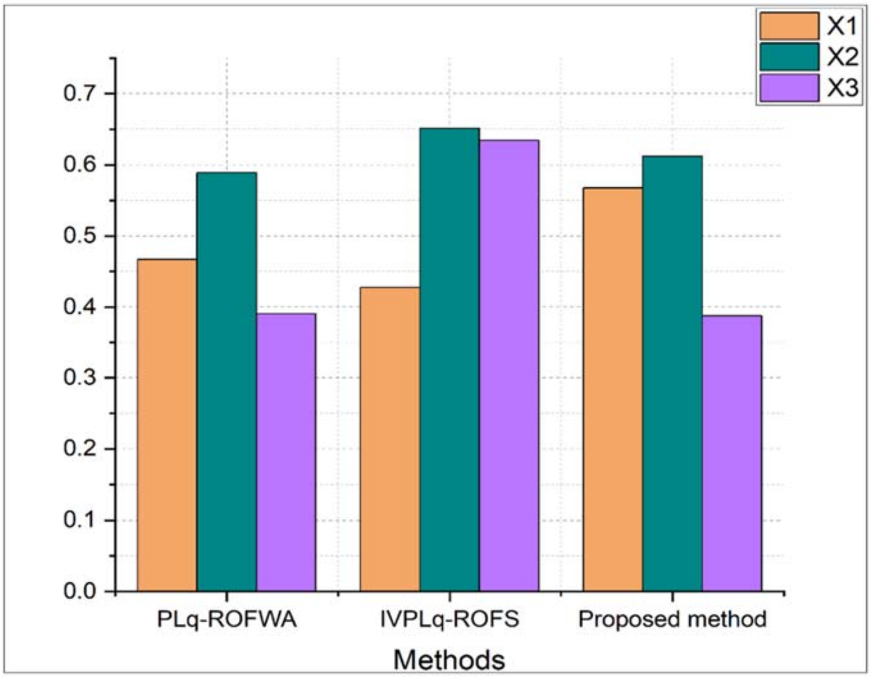

Figure 4. Comparison of existing methods with the proposed approach.

6.3. Advantages of the proposed method

Through this investigation and comparison analysis, we have come to the conclusion that our suggested strategy provides the following advantages over other existing techniques. (1) Since they can handle different types of assessment information by employing the IVPLPFSs, the evaluation process is more flexible; (2) Information fusion is more accurate compared with IVPLt-SFWA and IVPLt-SFWG operators; (3) More freedom for DMs when offering their opinions because of the relaxed restrictions of the novel information expression method; (4) The calculation complexity is significantly reduced due to the innovative distance measurement method we have introduced. This means that evaluations can be directly utilized without the need for normalization. As a result, our proposed methods have a broader range of applications and provide more accurate calculation results compared to other existing methods.

7. Conclusions

This paper introduces a new way of information expression, based on which a novel MAGDM technique is established. We first gave a brief introduction to a few current and inspirational perspectives. Then, as a foundation for solving a real-world MAGDM problem, the definition of IVPLt-SFS and a few fundamental principles were provided, such as the basic operational laws, the comparison and ranking rules and the distance measurement method. In addition, the extension of the IVPLt-SFWA and IVPLt-SFWG operators and their mathematical properties were offered. These operators were employed in the following section to aggregate assessments from various levels. After that, we presented a comprehensive framework to define a typical MAGDM problem and thoroughly propose and exhibit the application procedures of the suggested approach. The IVPLt-SFWA-TOPSIS method's specific calculation steps were provided throughout the problem of choosing a cloud storage provider. In the end, the impact of the parameter on final decision results and comparison analysis revealed that our technique is more flexible, robust and efficient than the other existing methods in describing fuzzy information.

Limitations and future directions

However, the idea of large-scale group decision-making (LSGDM) has drawn much attention from scholars in today's information-driven cultivation. LSGDM is the term used to describe a decision-making procedure including a significant number of participants and requiring consideration of several opinions, preferences and criteria which inevitably enhances the intricacy involved in reaching a conclusive decision [64]. Furthermore, in real-world LSGDM situations, consensus checking and improving, clustering, a more comprehensive information expression approach and an automated consensus-reaching mechanism are typically necessary [65].

This paper allocates limited attention to the issues mentioned above, and future research endeavors could place greater emphasis on addressing these aspects. In the future, it may be investigated whether the suggested approach may be used to solve LSGDM issues. It should also be noted that the formula for calculating the closeness ratio in the TOPSIS approach used in our work is outdated. Future work may also be concentrated using the updated closeness ratio formula. In this paper, the proposed method is employed for the selection of cloud storage provider. In the future, our goal is to apply this developed method to other real-life decision problems, such as medical diagnosis problems [26], bio-medical waste management [66], risk assessment [29] and electric-vehicle site selection problems [67].

Use of AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

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Conflict of interest

The authors have no conflict of interest.

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