AIMS Mathematics, 8(8): 19172–19209.
DOI: 10.3934/math.2023979
Received: 19 November 2022
Revised: 29 May 2023
Accepted: 30 May 2023
Published: 07 June 2023

http://www.aimspress.com/journal/Math

Research article

A bi-level humanitarian response plan design model considering equity and efficiency—the example of Yemen

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Abstract: Yemen has suffered from a civil war since 2015, which caused the largest famine in the world at this time. People came in need of urgent humanitarian relief in all sectors. In this situation, the donor countries are offering funds to non-profit humanitarian organizations to help Yemen in critical sectors, such as food, health, water, education and other sectors. We propose a new bi-level optimization distribution model for large-scale emergency logistics in Yemen. The upper-level model aims to minimize the unmet demand. The lower-level model seeks to maximize the funds sent to affected areas that fulfill the needs of the affected people by appealing to the donor countries to increase the funds. This model ensures a satisfying rate of equity and efficiency distribution among aid recipients of all governorates of Yemen based on their needs. We consider in this work the top ten donor countries, the nine sectors of the sustainable development goals, the five top humanitarian organizations and twenty-two disastrous regions. The model is applied and validated using actual data collected from Yemen in 2021. The results indicate the necessity of redistributing funds to all governorates of Yemen according to their needs and the priority of the supporting sectors. This proposed model is essential to humanitarian relief decision-makers in general and workers in Yemen in particular as it ensures the continuous flow of aid from donors to beneficiaries and is equitable and effectively distributed. It also gives a glimpse of the importance of continuing to appeal for fundraising from the donors to increase funds and their importance to cover the most significant percentage of those affected.

Keywords: humanitarian response plan; bi-level optimization; equity distribution; efficacy; unmet demand
Mathematics Subject Classification: 90B06, 90C29

1. Introduction

War is among the deadliest disasters for humans at all levels. It accounts for about a third of all hazards globally and affects more people than other natural disasters. Wars also lead to massive loss of life and the destruction of infrastructure and resources. Diseases and famine in war zones spread quickly due to a lack of services. It is a fertile ground for the emergence of terrorist groups and armed militias and human rights are violated in these areas and people are deprived of their most basic right, which is to live in peace free of fear, hunger and disease. Therefore, humans' preservation, development and prosperity are the basis for forming sustainable development goals. Hence the goal of international organizations is to develop humanitarian plans to confront disasters, the latest of which is the COVID-19 pandemic. The need for humanitarian aid increases in places where there is more than one disaster, such as areas of conflict, famine and disease. Various emergency supplies are needed and the quantity is usually high. One common obstacle is the lack of supplies in the first phase of the rescue. Moreover, the duration of the rescue from the war is prolonged. The demand for the type and quantity of emergency supplies depends on the stage of the rescue danger.

On the other hand, Yemen, one of the poorest countries in the world, has been suffering from a complex civil war since 2015, in which many people have been killed and Yemen was recently classified as going through the worst humanitarian disaster in half a century [1].

The ACLED dataset indicates that from Jan 2015 to December 2021, the armed conflicts caused 155,123 fatalities in Yemen; where the number of fatalities were caused by battles (93,493 fatalities), by explosions/remote violence (58,893 fatalities), by violence against civilians (2,274 fatalities), by strategic developments (341 fatalities), by riots (77 fatalities) and by protests (45 fatalities).

According to the report issued by OCHA [2] on Yemen, the estimated population of Yemen is 30.8 M, where the estimated number of people in need is 20.7 M (67%). Moreover, the estimated number of people in acute need is 12.1 M (39%). The humanitarian situation in Yemen can be summarized in the following two tables:

Table 1 reports the needy people by sector, as the most significant number of people need health, then food and after that they need protection, wash, sanitation and hygiene. Most of the neediest groups are boys and girls, then women and men, respectively. Figure 1 illustrates the spread and clusters of the needy people at the governorate level.

Table 1. People in need by sector.

Cluster/Sector	People in need (M.)	People in acute need (M.)	Men (M.)	Women (M.)	Boys (M.)	Girls (M.)
Food Security and Agriculture	16.14	5.1	3.94	3.88	4.25	4.07
Nutrition	7.56	4.71	-	2.46	2.60	2.50
Health	20.07	11.55	4.96	4.89	5.22	5.00
WASH, Sanitation and Hygiene	15.37	8.66	3.47	3.47	4.30	4.13
Education	5.54	2.93	0.27	0.21	2.64	2.42
Protection	15.77	8.04	3.62	3.54	4.39	4.22
Shelter and NFI	7.32	2.91	1.70	1.65	2.03	1.94
Camp Coordination & Camp Management	1.19	1.17	0.27	0.27	0.33	0.32
Refugees and Migrants Multisector	0.27	0.28	0.13	0.08	0.03	0.03

Table 2 summarizes the data related to the humanitarian situation in Yemen (2016–2022). It can be noted that the peak of the deterioration of the situation in Yemen was 2019, but on the other hand, the funding coverage was 87% of the needs. The situation has slightly improved in 2020 and 2021 compared to 2019, but the situation is still deteriorating. The funding coverage decreased to a large extent so that the year 2021 had the least the funding coverage (57%) compared to the previous five years.

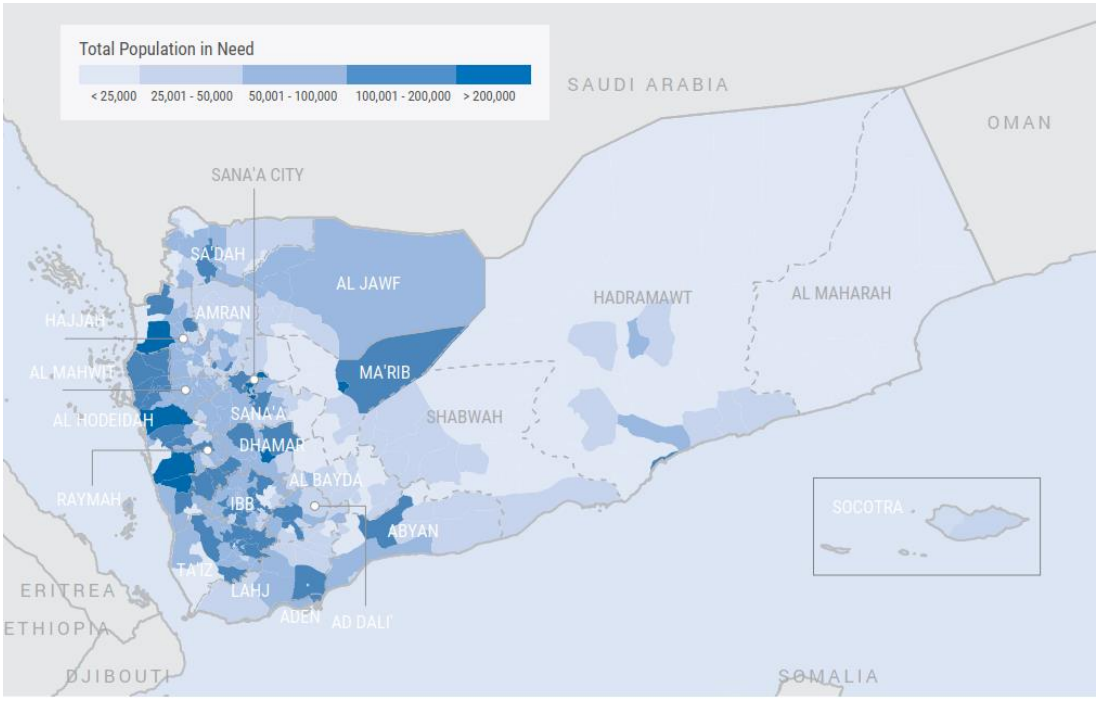

Figure 1. People in need by governorate.

Table 2. Trends of the humanitarian situation in Yemen (2016–2022). Source: FTS, GHO.

Year	People in need (M.)	People targeted (M.)	Requirements (B. US$)	Funding coverage
2022	20.7	16.0	3.9	
2021	20.7	16.0	3.9	57%
2020	24.0	15.6	3.2	59%
2019	24.1	24.1	4.2	87%
2018	22.2	13.1	3.6	81%
2017	18.8	10.3	3.1	75%
2016	21.2	13.6	2.3	63%

This work is inspired by some of the difficulties challenged by donor countries and humanitarian organizations in the distribution of aids and relief to affected areas in Yemen. The main concerns of donors are to ensure that relief is distributed equitably, impartially and transparently to all affected areas. Another concern is to meet all needs of people by urging donor countries to increase funds and fulfill their pledges. From this point of view, the contributions of this work are as follows:

- ❖ We propose a new bi-level optimization model that aims to maximize the funds sent by donors to affected people and minimize the unmet demand.

- ❖ We consider four dimensions in the proposed mathematical model: donors, sectors, intermediary humanitarian organizations and beneficiaries.
- ❖ We consider the desired level of equity and efficiency of distribution among the affected areas or at the level of sectors to ensure effective and equitable distribution among all beneficiaries.
- ❖ We derivate the single-level model from the proposed model using the Karush-Kuhn-Tucker (KKT) conditions.
- ❖ We apply the proposed model to discuss a real case study on the Yemen Humanitarian Response Plan 2021.

The following is how the rest of the article is structured: A brief review of related literature is in Section 2. The proposed mathematical model is presented in Section 3. A description of the data is provided in Section 4. The results are reported and discussed in Section 5. Finally, Section 6 presents the conclusion with the limitations and directions for future research.

2. Related background and literature review

a. Bi-level programming

A bi-level program is a mathematical program in which decision-making takes place at a hierarchical level and with the interaction of two decision-makers. So, the decision-maker at the upper level (the leader) seeks to find the optimal solution for the objectives under a set of constraints while considering the optimal solution for the decision-maker (the follower) at the lower-level model. The basic form of the bi-level programming is proposed by Bracken J [3], as follows;

$$\begin{aligned} &\min_{x\in X} \ F(x,y) \\ &\text{s.t} \ \ G(x,y) \le 0 \\ &y \in P(x) \\ &\text{where} \\ &P(x) = \underset{y\in Y}{\operatorname{argmin}} f(x,y) \\ &\text{s.t} \ \ g(x,y) \le 0 \end{aligned} \tag{1}$$

where $F, f: R^m \times R^n \to R$, $G: R^m \times R^n \to R^p$ and $g: R^m \times R^n \to R^q$ are continuous and twice differentiable functions.

b. Literature review

Humanitarian logistics is defined as the activities of planning, implementing and controlling the storage and flow of goods and information between the origin and consumption point to satisfy the needs of beneficiaries. Logistics and humanitarian relief distributions in disasters and armed conflicts face some difficulties, such as security risks, unmet needs and distribution bias. So, met demand and equity distribution are two essential goals of relief distribution during large-scale disasters. Hence, decision-makers must work to overcome these difficulties to meet the needs of the beneficiaries in all affected areas in an urgent, fair and satisfactory fashion. Moreover, particularly in conflict zones, humanitarian relief operations face obstacles, including restrictions on imports, visas and movement permits and aid delivery to many communities that need it most.

Equity distribution is one of the difficulties that are difficult to achieve in conflict areas, in which equity refers to fairness in the distribution of aids among recipients [4]. Equity is measured by the

maximal ratio between the proportions of the satisfied demand of each pair of demand points. An equitable solution is achieved by a set of constraints, achieving the minimal percentages by certain parameters. Several studies addressed the fairness distribution in mathematical models. Some studies included fairness in the objective functions, while others included it in the constraints. Shehadeh and Snyder [5] reviewed the different measures of equity and then studied the static and mobile healthcare facility locations under uncertainty and fairness restrictions. A similar study in [6] reviewed literature that dealt with equity and analyzed some mathematical formulas for how to introduce equity into the objective functions of models, comparing and evaluating them. The authors in [7] measured the equity by comparing the fulfillment rates, arrival times and deprivation times. Afterward, they balanced between equity and efficiency. Then, the model was applied in the Haiti earthquake case. In [8], they proposed a mixed-integer model for minimizing the total cost of distributing food donations and wastage cost while maintaining maximum equitability, efficiency and effectiveness in the distribution. The authors in [1] designed a framework of humanitarian supply chains in conflict zones subject to the inherent risks. They applied the proposed framework in the Yemen case. In [9], the authors proposed a stochastic model that addressed uncertain demand and disturbances during transportation. The goals of this model were to maximize efficiency and equity. Also, genetic algorithm (GA) was used to solve the proposed model on real data obtained from the Kartal district of Istanbul. Xiaoping Li et al. in [10] proposed a mathematical model for distributing gasoline fairly and efficiently during natural disasters. Hurricane Sandy in New Jersey was used to test the model. Equity is achieved through defining a constraint to maximize the minimum ratio between the total quantity of outputs to the region's needs. Noham and Tzur [11] proposed a mathematical model that hybridizes between the design of a relief network and the study of the effect of incentives for improving humanitarian relief operations in line with the humanitarian behavioral aspects working in the network. Also, a vision was presented for how to ensure balance, equity and efficiency. The authors in [12] and [13] proposed mathematical models to maximize the amount of donated food from the food bank, a distribution that ensures equality and effectiveness among all beneficiaries. They also provided a management vision for capacity investment in collaboration with local agencies to improve the food bank's ability to achieve these equity and efficiency goals. Mohammad Firouz et al. [14] developed a flexible, robust model that considers efficiency and equity to achieve equity. The proposed model was tested in a food bank, which gives an administrative vision for charitable works, helping the stakeholders make optimal decisions. A similar study in [15] introduced a flexible, robust model considering three axes of efficiency, efficacy and equity. Then, they applied the proposed model to the home healthcare problem. In [16], they suggested a mathematical model for minimizing the total unmet demand for those affected by a disaster. The proposed model was formulated as a weighted total of the unsaturated demand for all affected, for all relief items and overall time periods. Constraints have been proposed to impose a minimum level of service and through which equity among disaster victims is achieved over all time periods. Enayati and Özaltın [17] proposed a mathematical model for an optimal distribution of influenza vaccines that ensures the quality and fairness of the distribution. Through this model, the number of doses distributed is minimized and, in turn, the outbreak of the disease is eliminated in its early stages. Mathematical models have been proposed in [18,19] for distributing vaccines in developing countries. The proposed models can achieve equitable distribution of vaccines. Moreover, it can select manufacturers, plan capacity, allocate orders and manage waiting time. Z. Liu et al. in [20] proposed a two-stage fuzzy random mixed integer optimization model using a hybrid intelligent algorithm to solve facility location problems under an uncertain environment. M.M. Miah et al. in [21] solved the uncertain multi-

objective transportation problems. While S. Kousar et al. in [22] proposed a neutrosophic fuzzy multi-objective optimization and they applied the model to solve a crop production problem. S. Shiripoura and N.M. Amirib in [23] formulated an integer nonlinear programming (INLP) model to solve a location-allocation-routing problem for the distribution of the injured in disaster response scenarios. The authors in [24] developed a robust stochastic model that considers the locations of the facility and inventory and the equitable distribution. The proposed model includes two stages, the stage of determining the optimal location and capacity and the stage of scheduling the distribution that aims for fairness and for minimizing the costs of logistics services. Mollah et al. [25] addressed the humanitarian logistics and relief distributions during floods. The main objective is the total cost which is the sum of the cost for transporting population and relief-kits and penalty cost associated with the un-evacuated in-need population. Two methodologies are developed for the problem based on mixed-integer programming techniques and genetic algorithms. Both of the algorithms are run on the hypothetically developed data as well as real-life data and the results are compared. GA achieves a much better result. Chen et al. [26] addressed the relief material allocation problem based on bi-level programming including two objectives. The first objective is to minimize the weighted distribution time to deliver all relief materials, which represents the upper level. The second objective is to maximize the minimum fulfillment rate of all affected sites required for every kind of relief material, which represents the lower level. An improved differential evolution (IDE) algorithm is used to solve this model. The numerical results are compared with several conventional differential evolution algorithms. Safaei et al. [27] developed a robust bi-level optimization model for a supply–distribution relief network under uncertainty in demand and supply parameters. In the upper-level of the hierarchy, the number and location of transfer depots and the amount of victims' demand for relief commodities are determined with the aim of minimizing logistics costs and maximizing the satisfaction at demand points. Whereas the lower-level of the hierarchy identifies convenient suppliers with the lower risk and determines the optimal order. It aims to minimize the supply risk and satisfy demand under disaster scenarios. Saranwong S and Likasiri [28] developed a robust bi-level optimization model for an integrated model of distribution and production processes. Optimizing the distribution centers (DC) locations and allocating supplies to minimize the total cost represents the upper-level model, while minimizing the total transportation cost for all customers represents the lower-level model. Five hybrid (meta) heuristic methods are proposed to solve each level of the problem. Safaei et al. [29] reformulated the bi-level programming as a single-level linear problem and used the goal programming for solving the model. The upper level aims to minimize total operational cost and total unsatisfied demand considering the effect of distribution locations of relief supplies, while the lower-level aims to minimize the total supply risk. Camacho-Vallejo et al. [30] proposed a bilevel model to minimize both the total response time and the total cost. Moreover, they reduced the model into a nonlinear single-level mathematical model to solve it. Shokr et al. [31] proposed a robust bi-level model to minimize both relief chain costs and unmet demand. Also, they solved the model using the developed Benders decomposition algorithm and applied the model using a real-world example. Cao et al. [32] proposed a fuzzy bi-level model for multi-period post-disaster relief distribution. Three functions were minimized in the upper-level model, namely the unmet demand rate, potential environmental risks and emergency costs. Survivors' perceived satisfaction was maximized on the lower level. Xuehong Gao in [33] proposed a bi-level stochastic mixed-integer nonlinear model where the aim of the upper level is to minimize the total dissatisfaction level, while the aim of the lower level is to minimize transportation time. Xueping Li et al. [34] proposed a mathematical model to maximize

the size of relief items in disaster areas subject to the cost constraints and distribution facilities in order to cover the needs of the most affected people. The authors in [35,36] proposed bi-objective stochastic optimization models considering multi-commodity to minimize the total transportation time and maximize the fairness by minimizing the unmet demand.

An interesting study in [37] proposed a bi-level multi-objective scenario-based model that takes into account public donations, efficiency, supply risks, optimal selection of suppliers, coverage of the demand and the optimal facility locations. Hezam in [38] proposed an optimization model to maximize the funds and minimize the unmet demand in COVID-19 global humanitarian response plan with equity constraints.

c. Research gaps

Reviewing the literature, it appears that the bi-level optimization model that aims to minimize the unmet demand by maximizing the funds sent from donors and considering multiple sectors fairly and effectively has not yet been studied. Herein, we propose a new bi-level model considering the amount of funds sent from donor countries and the extent to which they meet the demand for each affected region from several sectors. Four dimensions were taken in this study: donors, intermediary humanitarian organizations, sectors, as well as a number of affected areas. It is also desired that the proposed model distributes funds fairly and effectively at the level of regions and sectors.

3. Proposed model

During wars, infrastructure is destroyed, the economy is disrupted and sources of income are cut off, which directly causes great harm to all people in these areas. This situation calls for the intervention of humanitarian organizations to provide rapid relief assistance to the affected people.

The humanitarian response plan consists of four phases. In the first phase: in order for humanitarian organizations to carry out their whole duty, they must know the extent of the disaster through field surveys and identify the needs of each region from each sector. In the second phase: humanitarian organizations launch appeals for fundraising from donor countries. Furthermore, the donor countries, in turn, pledge to support organizations with the funds. However, sometimes there are some difficulties as:

- There is a lack of sufficient funds (Due to such a scenario, we will try to minimize the unmet demand).
- There is a failure to fulfill pledges (Motivated by such a circumstance, we will try to maximize the funds sent from donor countries and humanitarian organizations and appeal to them to increase the funds).
- Only certain regions are supplied for regional, political, or other considerations (As a result of such scenarios, we will set a percentage to ensure fairness and effectiveness of distribution among all governorates).

In the third phase: the humanitarian organizations, after receiving the funds, send them to the affected areas through local agents. In the final phase, the beneficiaries receive their needs, which alleviate their suffering and achieve sustainable development goals.

In this model, we will assume that we have a number of donor countries indexed by $i \in I$, and the funds will be sent to a number of humanitarian organizations indexed by $j \in J$ to cover a number of

sectors indexed by $l \in L$ for covering the needs of a number of regions indexed by $k \in K$. Let P_i be the maximum funds that can be sent by the donor country i, and let H_{il} is the maximum funds that can be sent by the donor country i for covering the sector l (We need this because some donor countries only support some sectors and not others, such as the health sector and confronting COVID-19). We assume that QP_l is the maximum funds that can be sent for the sector l. We denote to the requirements of the humanitarian organization j from the sector l by Q_{jl}. Let D_{kl} denote to the requirements of the affected area k from the sector l, let D_k denote to the requirements of the affected area k from all sectors and let D_l denote to the requirements of the sector l for all affected areas. We define x_{ijl} as the nonnegative decision variable which represents the amount funds sent by donor country i to humanitarian organization j for covering the sector l. Also, we define y_{jkl} is the nonnegative decision variable which represents the amount funds received by affected area k from the humanitarian organization j for covering the sector l. The corresponding network of the humanitarian supply chain is illustrated in Figure 2.

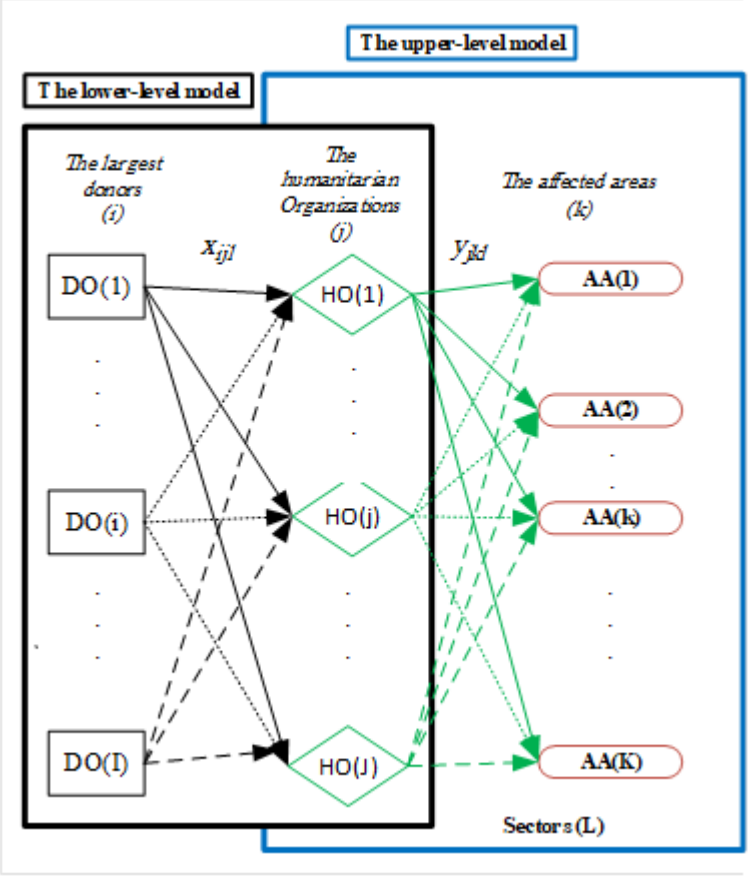

Figure 2. Schematic view of humanitarian supply chain.

The list of all nomenclatures is defined below:

Nomenclature.

Sets:

I	Set of all the donor countries, indexed as $i \in I$;
J	Set of all the humanitarian organization, indexed as $j \in J$;
K	Set of all the governorates of Yemen, indexed as $k \in K$.

L	Set of all the sectors type, indexed as $l \in L$.
Decision variables:	
x_{ijl}	The mount funds for sector l from the donor country i to the humanitarian organization j;
y_{jkl}	The mount funds for sector l from the humanitarian organization j to the governorate k;
Parameters:	
w_{ijl}	The weight of priority to send fund for sector l from the donor country i to the humanitarian organization j;
w'_{jkl}	The weight of priority to send fund for sector l from the humanitarian organization j to the governorate k;
D_{kl}	The demand of the governorate k for each sector l where each governorate's demand is proportional to its population;
D_l	The total demand of the sector l for all governorates;
D_k	The total demand of the governorate k for all sectors;
P_i	The maximum funds from the donor country i;
F_l	The minimum funded for the sector l;
QP_l	The maximum funds for the sector l;
DF_j	The Total funding for the humanitarian organization j;
β	Parameter of deviation from the rate of total demand;
θ	Parameter of deviation from the needs of each other governorates;
ω_{min}	The minimum level of governorate k satisfaction;
σ_{min}	The minimum level of sector l satisfaction;
τ_{min}	The minimum level of governorate k satisfaction of the sector l;
π_{min}	The minimum level of the utilization rate, where $\pi_{min} = 1$ at the perfect efficiency
μ	Lagrange multipliers
$\bar{G}$	The percentage between satisfied demand at the total demand for all governorates
G_k	The proportion between met demand at the total demand for each governorate k.

The proposed model:

Upper-Level Model:

$$z_1 = min\left[\sum_{k=1}^{K}\sum_{l=1}^{L} D_{kl} - \sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{l=1}^{L} y_{jkl}\right]. \tag{2}$$

$$\sum_{k=1}^{K}\sum_{l=1}^{L} y_{jkl} \le DF_j, \quad \forall j \in J. \tag{3}$$

$$|G_k - \bar{G}| \le \beta, \qquad \forall k \in K. \tag{4}$$

$$|G_k - G_{k'}| \le \theta \ \forall k, k' \in K, k \ne k'.$$

$$G_k = \frac{\sum_{j=1}^{J}\sum_{l=1}^{L} y_{jkl}}{\sum_{l=1}^{L} D_{kl}}.$$

$$G_{k'} = \frac{\sum_{j=1}^{J}\sum_{l=1}^{L} y_{jk'l}}{\sum_{l=1}^{L} D_{k'l}}. \tag{5}$$

$$\bar{G} = \frac{\sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{l=1}^{L} y_{jkl}}{\sum_{k=1}^{K}\sum_{l=1}^{L} D_{kl}}.$$

$$\frac{\sum_{j=1}^{J}\sum_{l=1}^{L} y_{jkl}}{\sum_{l=1}^{L} D_{kl}} \ge \omega_{min}, \ \forall k \in K. \tag{6}$$

$$\frac{\sum_{j=1}^{J}\sum_{k=1}^{K} y_{jkl}}{\sum_{k=1}^{K} D_{kl}} \ge \sigma_{min}, \ \forall l \in L. \tag{7}$$

$$\frac{\sum_{j=1}^{J} y_{jkl}}{D_{kl}} \geq \tau_{min},\ \forall k \in K,\ l \in L. \tag{8}$$

Lower-Level Model:

$$z_2 = \max\left[\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{l=1}^{L} w_{ijl}x_{ijl} + \sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{l=1}^{L} w'_{jkl}y_{jkl}\right]. \tag{9}$$

$$\sum_{i=1}^{I} x_{ijl} = \sum_{k=1}^{K} y_{jkl},\ \ \forall j \in J, l \in L. \tag{10}$$

$$\sum_{i=1}^{I}\sum_{j=1}^{J} x_{ijl} \geq F_l,\ \forall\, l \in L. \tag{11}$$

$$\sum_{i=1}^{I}\sum_{j=1}^{J} x_{ijl} \leq QP_l,\ \forall l \in L. \tag{12}$$

$$\sum_{j=1}^{J}\sum_{l=1}^{L} x_{ijl} \leq P_i,\ \forall i \in I. \tag{13}$$

$$\frac{\sum_{j=1}^{J}\sum_{l=1}^{L} x_{ijl}}{P_i} \geq \pi_{min}\ \ \forall i \in I. \tag{14}$$

$$\sum_{i=1}^{I}\sum_{l=1}^{L} x_{ijl} \geq DF_j,\ \ \forall j \in J. \tag{15}$$

$$x_{ijl} \in \mathbb{Z}^+ \cup \{0\}, \forall i \in L,\ j \in J,\ l \in L. \tag{16}$$

$$y_{jkl} \in \mathbb{Z}^+ \cup \{0\}, \forall j \in J,\ k \in K,\ l \in L. \tag{17}$$

The bi-level problem is defined by Constraints (2)–(17). In (2), the objective function of the upper level appears and it shows the leader wanting to minimize the unmet demand. Constraint (3) ensures that humanitarian organizations j can not send more than the funding obtained from donor countries.

Constraint (4) indicates that the difference between the met demand rate for each governorate k and the total met demand rate does not exceed β, while constraint (5) specifies that the absolute difference in the ratio of demand fulfilled between any two governorates. That means the difference between the met demand rate of the governorate k and the met demand rate of governorate $k', k \neq k'$does not exceed the θ.

To simplify, we can rewrite the constraints (4), (5) as:

$$-\beta \leq \frac{\sum_{j=1}^{J}\sum_{l=1}^{L} y_{jkl}}{\sum_{l=1}^{L} D_{kl}} - \frac{\sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{l=1}^{L} y_{jkl}}{\sum_{k=1}^{K}\sum_{l=1}^{L} D_{kl}} \leq \beta, \qquad \forall k \in K. \tag{18}$$

$$-\theta \leq \frac{\sum_{j=1}^{J}\sum_{l=1}^{L} y_{jkl}}{\sum_{l=1}^{L} D_{kl}} - \frac{\sum_{j=1}^{J}\sum_{l=1}^{L} y_{jk'l}}{\sum_{l=1}^{L} D_{k'l}} \leq \theta\ \forall k, k' \in K, k \neq k'. \tag{19}$$

which can also be simplified more as:

$$\frac{\sum_{j=1}^{J}\sum_{l=1}^{L} y_{jkl}}{\sum_{l=1}^{L} D_{kl}} - \frac{\sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{l=1}^{L} y_{jkl}}{\sum_{k=1}^{K}\sum_{l=1}^{L} D_{kl}} \leq \beta, \qquad \forall k \in K. \tag{20}$$

$$\frac{\sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{l=1}^{L} y_{jkl}}{\sum_{k=1}^{K}\sum_{l=1}^{L} D_{kl}} - \frac{\sum_{j=1}^{J}\sum_{l=1}^{L} y_{jkl}}{\sum_{l=1}^{L} D_{kl}} \leq \beta, \qquad \forall k \in K. \tag{21}$$

$$\frac{\sum_{j=1}^{J}\sum_{l=1}^{L} y_{jkl}}{\sum_{l=1}^{L} D_{kl}} - \frac{\sum_{j=1}^{J}\sum_{l=1}^{L} y_{jk'l}}{\sum_{l=1}^{L} D_{k'l}} \leq \theta \ \forall k, k' \in K, k \neq k'. \tag{22}$$

$$\frac{\sum_{j=1}^{J}\sum_{l=1}^{L} y_{jk'l}}{\sum_{l=1}^{L} D_{k'l}} - \frac{\sum_{j=1}^{J}\sum_{l=1}^{L} y_{jkl}}{\sum_{l=1}^{L} D_{kl}} \leq \theta \ \forall k, k' \in K, k \neq k'. \tag{23}$$

At perfect equity these constraints become

$$\frac{\sum_{j=1}^{J}\sum_{l=1}^{L} y_{jkl}}{\sum_{l=1}^{L} D_{kl}} = \frac{\sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{l=1}^{L} y_{jkl}}{\sum_{k=1}^{K}\sum_{l=1}^{L} D_{kl}}. \tag{24}$$

$$\frac{\sum_{j=1}^{J}\sum_{l=1}^{L} y_{jkl}}{\sum_{l=1}^{L} D_{kl}} = \frac{\sum_{j=1}^{J}\sum_{l=1}^{L} y_{jk'l}}{\sum_{l=1}^{L} D_{k'l}}. \tag{25}$$

This means that each governorate must receive the same fraction of its needs at the perfect equity point.

Constraint (6) imposes a minimum percentage (ω_{min}) for the total funds sent to the governorate k, while constraint (7) sets a minimum percentage (at least $\sigma_{min}\%$) covering sector l. Also, constraint (7) specifies a minimum percentage (at least $\tau_{min}\%$) of needs governorate k from the sector l. Constraints (4)–(8) present the equity constraints. The equity was specified as the maximum of the minimum ratio of total met demand over the total demand.

Expressions (9)–(17) makes this problem a bi-level programming model; hence, Expression (9) is called the objective function of the lower level that indicates the desire to maximize the funds send by the donors to the humanitarian organizations and then to governorates of Yemen. Constraint (10) states that the total funds received by humanitarian organizations j from the sector l are same as the total funds sent to affected areas. Constraint (11) reflects the minimum funding of the sector l, while constraint (12) reflects the maximum requirements of the sector l. Constraint (13) guarantees a donor i cannot send more than the available funding. Constraint (14) is related to the efficiency level, where the perfect efficiency rate equals one. Constraint (15) ensures that the donor countries must send to the humanitarian organizations j more than the receive in funding. Finally, constraints (16,17) indicate the non-negativity for the decision variables related to the fundings. On the other hand, and due to the bi-level mathematical model being an NP-hard problem and classified as a complex model, the computational complexity of bilevel programming problems is exceptionally high especially when solving large-scale and high-dimensional practical applications, such as with the humanitarian relief distribution. The authors in [39,40] showed that the natural complexity of the bilevel problem is $\sum_{k}^{P} hard$, where k is the k^{th} level of the polynomial hierarchy.

Derivation the single-level model

The Lagrangian function associated with the lower model (9)-(17) can be defined as:

$$\begin{gathered} L(x,y,\mu,\lambda) = -\left[\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{l=1}^{L} w_{ijl}x_{ijl} + \sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{l=1}^{L} w'_{jkl}y_{jkl}\right] + \sum_{j=1}^{J}\sum_{l=1}^{L}\lambda_{jl}\left[\sum_{i=1}^{I} x_{ijl} - \right. \\ \left.\sum_{k=1}^{K} y_{jkl}\right] + \sum_{l=1}^{L}\mu_l\left[F_l - \sum_{i=1}^{I}\sum_{j=1}^{J} x_{ijl}\right] + \sum_{l=1}^{L}\mu'_l\left[\sum_{i=1}^{I}\sum_{j=1}^{J} x_{ijl} - QP_l\right] + \\ \sum_{i=1}^{I}\mu_i\left[\sum_{j=1}^{J}\sum_{l=1}^{L} x_{ijl} - P_i\right] + \sum_{i=1}^{I}\mu'_i\left[\pi_{min}\, P_i - \sum_{j=1}^{J}\sum_{l=1}^{L} x_{ijl}\right] + \sum_{j=1}^{J}\mu_j\left[DF_j - \sum_{i=1}^{I}\sum_{l=1}^{L} x_{ijl}\right]. \end{gathered} \tag{26}$$

Both necessary and sufficient KKT conditions for the optimality in the lower model can be used to convert the bi-level model to its single-level model, which is easy to solve. Hence, the following four KKT conditions are replaced by the lower-level model.

Stationarity constraints: this kind of constraint is directly derived from the Lagrangian function (26). Here, the gradient of the Lagrangian function concerning the lower-level decision variables must be

equal to zero.

$$\nabla\Big[-\big[\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{l=1}^{L}w_{ijl}x_{ijl}^{*}+\sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{l=1}^{L}w'_{jkl}y_{jkl}^{*}\big]+\sum_{l=1}^{L}\mu_{l}\big[F_{l}-\sum_{i=1}^{I}\sum_{j=1}^{J}x_{ijl}^{*}\big]+\sum_{l=1}^{L}\mu'_{l}\big[\sum_{i=1}^{I}\sum_{j=1}^{J}x_{ijl}^{*}-QP_{l}\big]+\sum_{i=1}^{I}\mu_{i}\big[\sum_{j=1}^{J}\sum_{l=1}^{L}x_{ijl}^{*}-P_{i}\big]+\sum_{i=1}^{I}\mu'_{i}\big[\pi_{min}\,P_{i}-\sum_{j=1}^{J}\sum_{l=1}^{L}x_{ijl}^{*}\big]+\sum_{j=1}^{J}\mu_{j}\big[DF_{j}-\sum_{i=1}^{I}\sum_{l=1}^{L}x_{ijl}^{*}\big]+\sum_{j=1}^{J}\sum_{l=1}^{L}\lambda_{jl}\big[\sum_{i=1}^{I}x_{ijl}^{*}-\sum_{k=1}^{K}y_{jkl}^{*}\big]\Big]=0. \tag{27}$$

Primal feasibility constraints: The KKT primal feasibility conditions imply that the lower-level constraints should be satisfied with the optimal value of the variables. These consist of constraints (10)–(17).

Complementary slackness conditions: these conditions define the general relationship between primal constraints and their associated Lagrange multipliers, in which the multiplication of the slack variables in the primal constraints and the respective multipliers are equal to zero. We formulated the primal constraints (11)–(15) as constraints (28) to (32).

$$\mu_{l}\sum_{l=1}^{L}\big[F_{l}-\sum_{i=1}^{I}\sum_{j=1}^{J}x_{ijl}^{*}\big]=0,\ \forall\, l\in L. \tag{28}$$

$$\mu'_{l}\sum_{l=1}^{L}\big[\sum_{i=1}^{I}\sum_{j=1}^{J}x_{ijl}^{*}-QP_{l}\big]=0,\ \forall\, l\in L. \tag{29}$$

$$\mu_{i}\sum_{i=1}^{I}\big[\sum_{j=1}^{J}\sum_{l=1}^{L}x_{ijl}^{*}-P_{i}\big]=0,\ \forall\, i\in I. \tag{30}$$

$$\mu'_{i}\sum_{i=1}^{I}\big[\pi_{min}\,P_{i}-\sum_{j=1}^{J}\sum_{l=1}^{L}x_{ijl}^{*}\big]=0,\ \forall\, i\in I. \tag{31}$$

$$\mu_{j}\sum_{j=1}^{J}\big[DF_{j}-\sum_{i=1}^{I}\sum_{l=1}^{L}x_{ijl}^{*}\big]=0,\ \forall\, j\in J. \tag{32}$$

Dual feasibility constraints: The KKT dual feasibility conditions ensure the feasibility of the optimal solution to the dual problem. Hence, the Lagrange multipliers associated with greater than or equal to zero constraints must be defined as in (33), while the Lagrange multipliers associated with other constraints are unrestricted in sign.

$$\mu_{l},\mu'_{l},\mu_{i},\mu'_{i},\mu_{j}\geq 0,\forall l,i,j. \tag{33}$$

Therefore, the single-level formulation is obtained by:

The upper-level objective function (Equation (2)).

Subject to:

- *The upper-level constraints (Equations (3) to (8))*
- *The primal feasibility constraints (Equations (10) to (17))*
- *The stationarity constraints (Equation (27))*
- *The complementary slackness constraints (Equations (28) to (32))*
- *The dual feasibility constraints (Equation (33))*

4. Case study

Before the investigation of the case study, we will implement the proposed model on a simple test example to ensure the validity and effectiveness of the proposed model. In this example, there are no unmet demands. Therefore, it is expected that all affected areas will receive their full requirements, which makes the satisfying rates of equity and efficiency distribution among aid recipients equal to one.

4.1. Simple example

In this subsection, our goal is to test a simple example for illustrating the validity and performance of the proposed method. In this example, we assumed two donors, two humanitarian organizations, two sectors and three affected areas.

Set $P_1 = 200, P_2 = 100$, $DF_1 = 150, DF_2 = 150$, $QP_1 = 200, QP_2 = 150$, $F_1 = 180, F_2 = 120$. Further, we assumed the other related parameter values as given in Table 3.

Table 3. Sample demand.

Affected area	Sector (1)	Sector (2)	**Total D_k**
Affected area (1)	80	20	100
Affected area (2)	50	50	100
Affected area (3)	50	50	100
Sum D_l	180	120	300

The results of the optimal distribution plan for this example are shown in Tables 4 and 5.

Table 4. The results of the test example (Donors, Humanitarian Organizations, Sectors).

	Sector (1)	Sector (2)	
Donor (1)	101.82	36.70	Humanitarian Organizations (1)
Donor (2)	5.74	5.74	150
Donor (1)	60.74	0.74	Humanitarian Organizations (2)
Donor (2)	11.7	76.82	150
Sum	180	120	

Table 5. The results of the test example (Humanitarian Organizations, Affected Areas, Sectors).

	Sector (1)	Sector (2)	
Humanitarian Organizations (1)	79.63	0.00	Affected Area (1)
Humanitarian Organizations (2)	0.37	20.00	100
Humanitarian Organizations (1)	0.00	0.00	Affected Area (2)
Humanitarian Organizations (2)	50.00	50.00	100
Humanitarian Organizations (1)	27.93	42.44	Affected Area (3)
Humanitarian Organizations (2)	22.07	7.56	100
Sum	180	120	

We can see clearly from the obtained results that equitably and effective distribution was achieved because the unmet demand in this example equals zero and the donors' funds covered all requirements of the affected areas. Hence, the results of this simple test example confirm the robustness and performance of the proposed model.

4.2. Data description of the case study

Although the proposed model is applicable to various humanitarian response plans, with some minor adaptations, here we apply it to the humanitarian response plans in Yemen 2021. We now provide a case study on Yemen to validate the model. Yemen is a country that is situated at the southern end of the Arabian Peninsula in Western Asia. It has a total area of 527,970 sq. km. and an estimated population of 30,041,712. Yemen consists of twenty-two governorates are Abyan, Aden, Al Bayda, Al Dhalee, Al Hudaydah, Al Jawf, Al Maharah, Al Mahwit, Amanat Al Asimah, Amran, Dhamar, Hadramaut, Hajjah, Ibb, Lahj, Marib, Raymah, Sa'ada, Sana'a, Shabwah, Socotra and Taizz. Data are analyzed that were gathered in 2021 from a financial tracing service (https://fts.unocha.org/appeals/1024/summary) FTS.

We consider the largest ten sources of the response plan, namely: United States of America, Saudi Arabia, Germany, United Arab Emirates, European Commission, World Bank, United Kingdom, Japan, Central Emergency Response Fund and Canada, and we collect the funds from other donors in one named as "other donors".

Table 6 presents the largest donors and their funding.

Table 6. Top ten donor countries with their financial pledges.

Donor countries	Funding for response plan/appeal (US$) ($\boldsymbol{P_i}$)
United States of America, Government of	588,044,047
Saudi Arabia (Kingdom of), Government of	348,391,212
Germany, Government of	234,030,552
United Arab Emirates, Government of	230,000,000
European Commission	165,129,153
World Bank	120,829,200
United Kingdom, Government of	92,308,168
Japan, Government of	56,956,203
Central Emergency Response Fund	54,663,733
Canada, Government of	54,286,633
Other Donors	274,443,218
Sum	2,219,082,119

Moreover, Table 7 summarizes the overall funding, funded, Required fund, and the unmet requirement for Yemen 2021.

Table 7. Yemen funding overview 2021.

Appeal	Overall funding (US$)	Funded (US$)	Required (US$)	Unmet requirements (US$)
Yemen 2021	$2,845,190,184	$2,210,973,015	$3,853,456,397	$1,642,483,382

The donor countries send the funds to non-profit international or national organizations; more than 250 such organizations are working in Yemen. In this work, we considered the five top organizations and the rest of the organizations are listed under the name "others". Table 8 lists the details of the humanitarian organizations.

Table 8. The five largest humanitarian organizations with funding.

The humanitarian organizations	Funding US$ $(\boldsymbol{DF_j})$
World Food Programme	$1,177,048,828
United Nations Children's Fund	$202,937,171
United Nations High Commissioner for Refugees	$156,123,420
UN agencies and NGOs (details not yet provided)	$76,396,195
United Nations Population Fund	$62,322,877
Other humanitarian organizations	$567,932,128

We also consider nine categories of sectors: Food Security and Agriculture; Nutrition; Health; WASH, Sanitation and Hygiene; Education; Protection; Shelter and NFI; Camp Coordination & Camp Management; Refugees and Migrants Multisector; and other clusters/sectors (shared). Table 9 reports the requirements for each sector and the existing funding with its percentage coverage.

Table 9. Top ten sectors to be financed.

Cluster/Sector	Required (US$) $(\boldsymbol{QP_l})$	Funded (US$) $(\boldsymbol{F_l})$	Coverage (%)
Food Security and Agriculture	$1,707,979,939	$1,040,707,983	60.90%
Nutrition	$442,926,563	$238,041,723	53.70%
Health	$438,800,000	$90,139,435	20.50%
WASH, Sanitation and Hygiene	$330,703,801	$45,797,520	13.80%
Education	$257,750,026	$90,062,591	34.90%
Protection	$218,000,000	$89,129,815	40.90%
Shelter and NFI	$207,600,000	$37,992,092	18.30%
Camp Coordination & Camp Management	$61,340,000	$5,207,237	8.50%
Refugees and Migrants Multisector	$58,738,565	$2,759,978	4.70%
Other clusters/sectors (shared)	$129,617,503	$71,335,941	55%
Sum	$3,853,456,397	$1,711,174,315	44%

Since the actual demand in each governorate is challenging to identify due to many factors affecting demand, the reasonable assumption is to consider the needs in each governorate to be correlated with the estimated population of that governorate. So, the governorate's demand was calculated in proportion to its population from the total need for this sector. Table 10 lists the estimated demand of each governorate from each sector.

Table 10. The needs of each governorate from each sector (D_{kl}).

Governorate	Population	Percent of total demand	Food Security and Agriculture	Health	WASH, Sanitation and Hygiene	Nutrition	Education	Protection	Shelter and NFI	Camp Coordination & Camp Management	Refugees and Migrants Multisector	Other clusters/sectors (shared)	**Total D_k**
A. Al Asimah	3406643	11.34%	193679971	49758647.19	37500851.77	$50,226,587.46	$29,228,105.30	$24,720,567.66	23541238	6955778	6660783	$14,698,248.86	436970778
Abyan	615154	2.05%	34973729	8985159.54	6771710.15	$9,069,657.78	$5,277,860.31	$4,463,912.44	4250955	1256038	1202770	$2,654,133.87	7890592
Aden	997308	3.32%	56700565	14567037.67	10978520.34	$14,704,029.01	$8,556,641.61	$7,237,042.42	6891789	2036331	1949970	$4,302,969.57	127924896
Al Bayda	775404	2.58%	44084521	11325828.41	8535766.87	$11,432,338.76	$6,652,763.37	$5,626,778.93	5358345	1583241	1516096	$3,345,546.03	9946122
Al Dhale'e	779656	2.60%	44326262	11387934.64	8582573.55	$11,495,029.06	$6,689,244.42	$5,657,633.89	5387728	1591923	1524410	$3,363,891.64	100006631
Al Hudaydah	2985122	9.94%	169714978	43601760.57	32860683.57	$44,011,800.25	$25,611,565.45	$21,661,768.01	20628363	6095105	5836611	$12,879,560.92	382902195
Al Jawf	603816	2.01%	34329123	8819552.65	6646899.69	$8,902,493.49	$5,180,583.24	$4,381,637.37	4172605	1232888	1180601	$2,605,215.12	7745159
Al Maharah	169327	0.56%	9626852	2473250.78	1863977.74	$2,496,509.72	$1,452,781.34	$1,228,734.43	1170116	345737	331074	$730,575.64	2171960
Al Mahwit	774511	2.58%	44033750	11312784.93	8525936.59	$11,419,172.62	$6,645,101.66	$5,620,298.80	5352174	1581418	1514350	$3,341,693.11	9934668
Amran	1205960	4.01%	68563186	17614683.48	13275393.75	$17,780,335.49	$10,346,821.16	$8,751,141.75	8333656	2462363	2357934	$5,203,216.25	154688730
Dhamar	2176229	7.24%	123726487	31786779.83	23956264.61	$32,085,709.07	$18,671,475.23	$15,791,973.57	15038595	4443485	4255036	$9,389,523.77	279145328
Hadramut	1510895	5.03%	85899843	22068673.25	16632165.28	$22,276,211.47	$12,963,083.65	$10,963,926.09	10440876	3084987	2954153	$6,518,884.05	193802803

Continued on next page

Governorate	Population	Percent of total demand	Food Security and Agriculture	Health	WASH, Sanitation and Hygiene	Nutrition	Education	Protection	Shelter and NFI	Camp Coordination & Camp Management	Refugees and Migrants Multisector	Other clusters/sectors (shared)	**Total D_k**
Hajjah	2510327	8.36%	142721166	36666734.83	27634066.95	$37,011,556.14	$21,537,948.62	$18,216,381.48	17347343	5125655	4908276	$10,831,017.80	322000146
Ibb	3080130	10.25%	175116526	44989481.43	33906546.29	$45,412,571.51	$26,426,709.22	$22,351,200.89	21284905	6289095	6022374	$13,289,480.96	395088890
Lahj	1058219	3.52%	60163576	15456725.54	11649038.03	$15,602,083.68	$9,079,242.05	$7,679,047.79	7312708	2160701	2069065	$4,565,775.23	135737962
Marib	495634	1.65%	28178585	7239407.63	5456015.55	$7,307,488.47	$4,252,410.00	$3,596,606.35	3425025	1011999	969080	$2,138,454.74	63575072
Raymah	646854	2.15%	36775989	9448181.09	7120668.64	$9,537,033.67	$5,549,838.02	$4,693,945.94	4470015	1320764	1264751	$2,790,906.20	82972092
Sa’ada	981401	3.27%	55796195	14334694.33	10803413.63	$14,469,500.67	$8,420,163.71	$7,121,612.04	6781865	2003852	1918868	$4,234,337.48	125884502
Sana’a	1469960	4.89%	83572540	21470761.99	16181546.49	$21,672,677.33	$12,611,872.06	$10,666,878.11	10158000	3001405	2874115	$6,342,266.54	188552063
Shabwah	665881	2.22%	37857742	9726096.26	7330120.79	$9,817,562.42	$5,713,084.70	$4,832,016.83	4601499	1359614	1301953	$2,872,999.80	85412689
Socotra	68247	0.23%	3880089	996840.11	751273.51	$1,006,214.60	$585,541.40	$495,239.62	471614	139349	133439	$294,457.44	8754056
Taizz	3065034	10.20%	174258264	44768983.84	33740367.19	$45,190,000.33	$26,297,189.49	$22,241,655.60	21180586	6258271	5992857	$13,224,347.99	393152523
Sum D_l	30041712	1707979939	438800000	33070380 1	442926563	257750026	218000000	207600000	61340000	58738565	129617503	1707979939	3853456397

5. Data analysis and results

We assume that all weights $w_{ijl} = w'_{jkl} = 1$ and the minimum levels of σ_{min}, τ_{min} and ω_{min} are equal to 40%; we found by experimentation that this value is the best percentage. The mathematical model was implemented by LINGO 18 software. We can see clearly from Table 11 that each governorate received at least 40% of its needs. It can be noted that the governorates with the smallest needs obtained the highest rates, such as the governorates of Al Maharah, Raymah and Socotra, which got a rate of 73%. Meanwhile, the large governorates with the highest needs got low percentages. For example, Hajjah got only 40% of its needs, Amanat Al-Asimah and Aden got 45% of its needs and the rest of the governorates got different rates, between 40% and 73%. Moreover, we can see that all governorates received 58% of the total needs.

Table 11. Comparison between the estimated demand and obtained funds (y_{kjl}) using the proposed model for each governorate.

Governorate	Demand (D_k)	Results (y_{kjl})	Ratio
A. Al Asimah	436,970,777	196,636,850	45%
Abyan	78,905,926	39,452,963	50%
Aden	127,924,896	57,566,203	45%
Al Bayda	99,461,225	68,122,795	68%
Al Dhale'e	100,006,630	70,418,078	70%
Al Hudaydah	382,902,195	230,212,004	60%
Al Jawf	77,451,598	38,725,799	50%
Al Maharah	21,719,608	15,765,567	73%
Al Mahwit	99,346,680	67,970,842	68%
Amran	154,688,730	107,309,223	69%
Dhamar	279,145,328	167,288,434	60%
Hadramut	193,802,803	134,704,600	70%
Hajjah	322,000,145	128,800,058	40%
Ibb	395,088,890	221,805,958	56%
Lahj	135737962	91,172,004	67%
Marib	63,575,072	45,355,812	71%
Raymah	82,972,091	60,226,783	73%
Sa'ada	125,884,502	79,550,142	63%
Sana'a	188,552,062	124199357	66%
Shabwah	85,412,688	42706344	50%
Socotra	8,754,056	6354289	73%
Taizz	393,152,523	224,738,003	57%
Total	3,853,456,397	2,219,082,119	**58%**

Figure 3 depicts the obtained results for the Yemen case. Further, Tables 15–25 (in Appendix A) show the funds sent by each donor to humanitarian organizations for all sectors, and we notice that the total amount sent by each donor is equal to the funds granted by the donor, and this means that they sent all the funds they donated, which indicates that the efficacy rate is 100%.

Tables 26–47 (in Appendix B) report the funds received by each governorate from humanitarian

organizations to cover each sector. Due to the lack of funding, some sectors were not covered in some governorates, so we can redistribute the governorate's share to include the most necessary sectors. This allows flexibility for the decision-maker to redistribute the funds in the governorate to spend in the necessary sectors because, in some governorates, sectors are more important than others, unlike other governorates. For example, coverage of the displaced sector in Ma'rib governorate is very important to cover due to the large number of displaced people in this governorate.

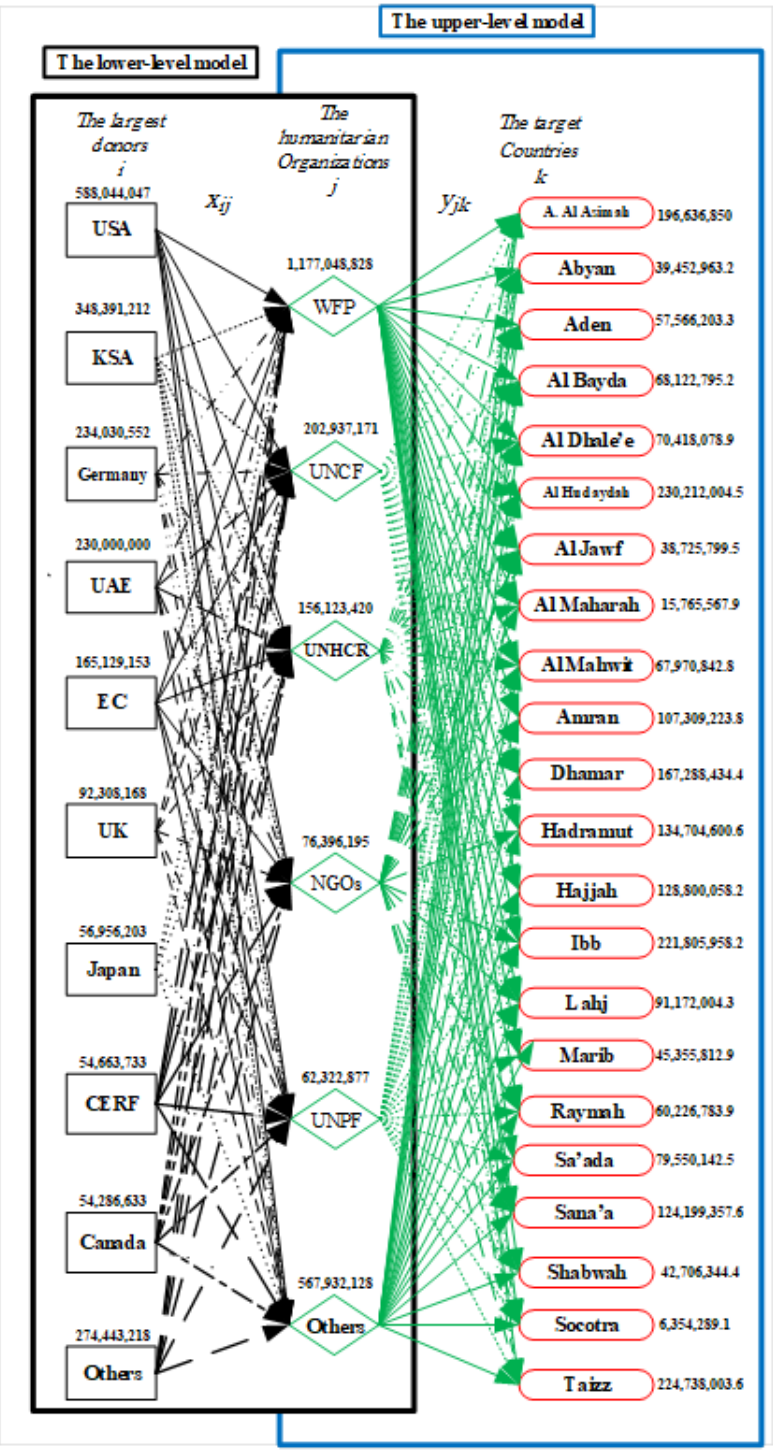

Figure 3. The obtained results of the case study.

Table 12 shows the relationship between the results, the requirements and the funding for each sector.

We can clearly see that the model's results covered 58% of the requirements as a total, and in return, the funds sent, according to the model's results, increased by 30% compared with the actual funding. All sectors increased by up to 50% relative to the actual funding, except for the food security and agriculture sector, which increased by 17%. On the other hand, there is a disparity between sectors compared to their requirements due to that available funds only covered 58%.

Table 12. The numerical results of the sectors and their ratios to required and funded.

Cluster/Sector	Results	Required (US$)	Results/ Required %	Funded (US$)	Results/Funded%
Food Security and Agriculture	1,213,382,621	$1,707,979,939	71%	$1,040,707,983	117%
Health	135,209,153	$438,800,000	31%	$90,139,435	150%
WASH, Sanitation and Hygiene	68,696,280	$330,703,801	21%	$45,797,520	150%
Nutrition	357,062,585	$442,926,563	81%	$238,041,723	150%
Education	135,093,887	$257,750,026	52%	$90,062,591	150%
Protection	133,694,723	$218,000,000	61%	$89,129,815	150%
Shelter and NFI	56,988,138	$207,600,000	27%	$37,992,092	150%
Camp Coordination & Camp Management	7,810,856	$61,340,000	13%	$5,207,237	150%
Refugees and Migrants Multisector	4,139,967	$58,738,565	7%	$2,759,978	150%
Other clusters/sectors (shared)	107,003,912	$129,617,503	83%	$71,335,941	150%
Sum	2,219,082,119	$3,853,456,397	58%	$1,711,174,315	130%

Sensitivity analysis

To analyze how the minimum levels of σ_{min}, τ_{min} and ω_{min} values affect the distribution ratio in each affected area, six values of σ_{min}, τ_{min} and ω_{min} were tested and Table 13 shows the best minimum level values of σ_{min}, τ_{min} and ω_{min} are 40%, where all affected areas received at least 40% and the total received for all affected was the highest among other ratios.

Moreover, if we raise the percentage to 60% or 70%, or reduce it to 25% or 30%, the distribution will be uneven between the governorates, as some of them will get higher percentages, especially the governorates with the slightest need. This uneven distribution is due to the lack of funds sent by donor countries. Hence, we found that the best percentage is 40% to reduce disparities and achieve a balance between governorates.

Overall, we can conclude the following main points:

- Equity can be achieved more as we increase the values of equity parameters.
- It requires sending more funds from donor countries the more equity parameters we increase.
- Unmet demand decreases as donors increase funds.
- The proposed model ensures that 40% of each governorate's needs are met. The minimum rate of equity is dynamic so that the decision-maker can change it according to the availability of funds.
- The weights can be adjusted based on other factors such as the interests and competencies of

any humanitarian organization, such as the WHO, which will have a higher weight in the field of health than others, as well as the WFP will contribute to meeting the food sector more and therefore requires receiving more aid for this sector. The weights of each governorate depend on the security level and humanitarian situation, the displaced, the stability of the local government and other factors.

- ❖ The model results indicate the importance of reducing unmet demand and increasing funding from donor countries.
- ❖ Despite the importance of available sufficient funding to cover all needs, we also point out that the distributive fairness is fundamental, especially in the most deserving regions.

Table 13. Analysis of the minimum levels of σ_{min}, τ_{min} and ω_{min} values.

Governorate	σ_{min}, τ_{min} and ω_{min}					
	0.25	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**
A. Al Asimah	**20%**	45%	45%	44%	71%	51%
Abyan	56%	68%	50%	78%	71%	60%
Aden	56%	45%	45%	78%	71%	60%
Al Bayda	56%	68%	68%	78%	71%	60%
Al Dhale'e	56%	67%	70%	78%	71%	60%
Al Hudaydah	45%	45%	60%	56%	45%	**16%**
Al Jawf	56%	67%	50%	78%	71%	60%
Al Maharah	56%	69%	73%	78%	71%	60%
Al Mahwit	56%	67%	68%	78%	71%	60%
Amran	56%	69%	69%	56%	71%	54%
Dhamar	55%	68%	60%	52%	71%	**21%**
Hadramut	56%	47%	70%	59%	71%	54%
Hajjah	55%	60%	40%	50%	71%	47%
Ibb	**18%**	**31%**	56%	50%	25%	**25%**
Lahj	56%	69%	67%	57%	71%	53%
Marib	56%	69%	71%	78%	71%	60%
Raymah	56%	69%	73%	78%	71%	60%
Sa'ada	56%	67%	63%	78%	71%	60%
Sana'a	45%	62%	66%	78%	61%	60%
Shabwah	56%	69%	50%	78%	71%	60%
Socotra	56%	69%	73%	78%	71%	60%
Taizz	**7%**	42%	57%	**33%**	**1%**	48%
Total	41%	54%	58%	58%	56%	45%

6. Conclusions

Our study proposed a novel bi-objective optimization model for examining the Yemen Humanitarian Response Plan 2021, where the actual data have been collected from FTS. Both level models aim to achieve fairness and effective distribution among the Yemeni governorates, minimize the unmet demand and maximize the funds granted by donor countries and intermediary UN organizations. Our results provide minimum distributional fairness of 40% to satisfy the governorates' demand based on the purely humanitarian aspect, away from political or regional tendencies.

Furthermore, we have noted some limitations of our study that may help shape future research directions. Finally, we hope that our study provides insight into the importance of equitable and efficient distribution, meeting unmet demand and understanding the humanitarian response plan, which will better reflect on the effectiveness of humanitarian relief efforts.

6.1. Limitations and further research directions

The humanitarian response plan, in general, is complex and depends on many overlapping factors, the most prominent of which is an appeal to donor countries to increase grants and fulfill their pledges. The plan also faces security, humanitarian and political risks, especially in war zones, which calls for a rapid response to such risks and more cooperative efforts between humanitarian organizations.

Therefore, reality cannot be simulated with a mathematical model that can be easily solved. Nevertheless, the proposed model presents a vision and broad lines for equitable distribution among the governorates. Hence, we recommend that donors and international and local organizations take advantage of the proposed mathematical models to achieve the minimum level of fairness, effectiveness and flexibility based on feedback for funding the urgent sectors.

Some main Limitations are considered as directions of future studies as:

- Although the proposed model is general and can be applied to most similar cases with some minor modifications, we used data specific to Yemen, and for this reason, the model can be applied to similar cases, especially since data are available for most countries to achieve a certain level of fair and effective distribution and minimize the unmet demand.
- We considered only ten donors, six non-profit humanitarian organizations, ten sectors and twenty-two governates. So, more than these nodes of the network model can consider in the future works.
- We considered the demand of each governate according to its populations only. We do not take considering the security factor and other factors. Recently, Yemen has been ruled by many conflicting governments in different areas of Yemen, so the regions differ from each other in terms of security and economics, the availability of job and salary opportunities, the difficulty of moving between them and the presence of refugees and displaced persons in some areas. Therefore, these factors are essential to be considered in future studies in determining the actual needs for each area. However, the study was relied upon by [1]. Also, there are difficulties in measuring and identifying these factors easily, and the ratio of the estimate to the actual estimate will be inaccurate.
- We considered certain data, and some parameters can be considered as uncertain data. As we referred to before, the difficulties in estimating each sector's actual demand for each governorate. So, the building mathematical model considering the uncertainty (robust, fuzziness, rough, etc.) is important to future works.
- The present model minimized unmet demand as the objective function in the upper-level model and maximized the sent funds in the lower-level model. Hence, adding more objective functions into both level models as minimizing the delivery times and minimizing the emergency risks will be promising topics in future works.
- The proposed model had been solved by LINGO 18 software "Hyper version". So, introducing an efficient solution approach as a metaheuristic approach for larger instances will be interesting work in future work.

Table 14. Abbreviations and Acronyms.

CERF	Central Emergency Response Fund
EC	European Commission's Humanitarian Aid and Civil Protection Department
FAO	Food and Agriculture Organization of the United Nations
FTS	Financial Tracking Service
GHO	Global Humanitarian Overview
GHRP	Global Humanitarian Response Plan
OCHA	United Nations Office for the Coordination of Humanitarian Affairs
OD	Other humanitarian organizations
UK	United Kingdom
UNFPA	United Nations Population Fund
UNHCR	United Nations High Commissioner for Refugees
UNICEF	United Nations Children's Fund
UN-NGOs	UN agencies and NGOs (details not yet provided)
USA	United States of America
WB	World Bank
WFP	World Food Programme
WHO	World Health Organization

Use of AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

Data Availability Statement

The data presented in this study are available on https://fts.unocha.org/appeals/1024/summary).

Acknowledgements

The authors extend their appreciation to the Deputyship for Research & Innovation, Ministry of Education in Saudi Arabia for funding this research work through the project no. (IFKSUOR3–037–1).

We should like to thank the Editors of the journal as well as the anonymous reviewers for their valuable suggestions that make the paper stronger and more consistent.

Conflicts of Interest

The authors declare no conflict of interest. The funders had no role in the design of the study; in the collection, analyses, or interpretation of data; in the writing of the manuscript, or in the decision to publish the results.

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Appendix A

Table 15. The funds sent by United States of America (x_{1jl}).

	FS	Health	WSH	Nutrition	Education	Protection	Shelter and NFI	CCCM	RMM	OS	**Total**
WFP	27120.13	0	0	51875715	113000000	55280414	41784241	0	4711.944	67867732	329460641.7
UNCF	0	24780531	36526262	23474122	13393882	13770104	7606216	930957	133426.5	9609099	130224599.2
UNHCR	0	24536294	0	0	9079231	36051815	4172593	332649.2	650964.1	12043645	86867191.96
NGOs	0	0	0	0	0	614357.3	0	0	590276	0	1204633.318
UNPF	0	8819543	0	0	0	22351187	3425014	0	0	5011442	39607186.04
OO	0	0	0	0	0	0	0	679794.8	0	0	679794.7928
Total	27120.13	58136368	36526262	75349837	135000000	128000000	56988064	1943401	1379379	94531919	**588044047**

Table 16. The funds sent by Saudi Arabia (x_{2jl}).

6	FS	Health	WSH	Nutrition	Education	Protection	Shelter and NFI	CCCM	RMM	OS	**Total**
WFP	123561275.4	1073895	1.234568	1264303	1.234568	1.234568	1.234568	67556.1	73154.77	1.234568	126040190.3
UNCF	0.627727525	1.234568	1.234568	1.234568	1.234568	1.234568	1.234568	1.234568	1.234568	1.234568	11.73883845
UNHCR	26245600.65	1.234568	1.234568	0	1.234568	1.234568	1.234568	1.234568	1.234568	1.234568	26245610.53
NGOs	0	1.234568	1.234568	1.234568	1.234568	1.234568	1.234568	0	1.234568	0	8.641975164
UNPF	1.234567881	1.234568	1.234568	0	1.234568	1.234568	1.234568	0	0	1.234568	8.641975164
OO	9818034.538	0	0	183000000	1.234568	3212478	1.234568	1.234568	1.234568	2068.368	196105382.2
Total	159624912.4	1073900	6.172839	184000000	7.407407	3212484	7.407407	67559.8	73159.71	2073.306	**348391212**

Table 17. The funds sent by Germany (x_{3jl}).

	FS	Health	WSH	Nutrition	Education	Protection	Shelter and NFI	CCCM	RMM	OS	**Total**
WFP	142747301.7	72029977	465404	874492	1.234568	1.234568	1.234568	5534408	0	1.234568	221651587.9
UNCF	0.62772717	1.234568	1.234568	1.234568	1.234568	1.234568	1.234568	1.234568	1.234568	1.234568	11.7388381
UNHCR	565598.5019	1.234568	1.234568	0	1.234568	1.234568	1.234568	1.234568	1.234568	1.234568	565608.3785
NGOs	1.234567881	1.234568	1.234568	1.234568	1.234568	1.234568	1.234568	1.234568	1.234568	0	11.11111093
UNPF	0	1.234568	1.234568	0	1.234568	1.234568	1.234568	0	0	1.234568	7.407407284
OO	9818034.538	0	1429436	97466.03	1.234568	466317.2	1.234568	1.234568	1.234568	2066.879	11813325.5
Total	153130936.6	72029982	1894845	971960.5	7.407407	466323.4	7.407407	5534413	4.938272	2071.817	**234030552**

Table 18. The funds sent by United Arab Emirates (x_{4jl}).

	FS	Health	WSH	Nutrition	Education	Protection	Shelter and NFI	CCCM	RMM	OS	**Total**
WFP	0	0	0	0	1.234568	1.234568	1.234568	0	0	1.234568	4.938271523
UNCF	0	1.234568	1.234568	1.234568	1.234568	1.234568	1.234568	1.234568	1.234568	1.234568	11.11111093
UNHCR	0	1.234568	1.234568	0	1.234568	1.234568	1.234568	1.234568	1.234568	1.234568	9.876543045
NGOs	1.234567881	1.234568	0	1.234568	1.234568	1.234568	1.234568	1.234568	1.234568	0	9.876543045
UNPF	1.234567881	1.234568	0	0	0	1.234568	1.234568	0	0	1.234568	6.172839403
OO	229999953.1	0	0	0	1.234568	0	1.234568	1.234568	1.234568	0	229999958
Total	229999955.6	4.938272	2.469136	2.469136	6.172839	6.172839	7.407407	4.938272	4.938272	4.938272	**230000000**

Table 19. The funds sent by European Commission (x_{5jl}).

	FS	Health	WSH	Nutrition	Education	Protection	Shelter and NFI	CCCM	RMM	OS	**Total**
WFP	81093567.71	0	0	865786	1.234568	1.234568	1.234568	0	71595.58	1.234568	82030954.23
UNCF	72712458.14	1.234568	1.234568	1.234568	1.234568	1.234568	1.234568	1.234568	1.234568	1.234568	72712469.25
UNHCR	565598.5019	1.234568	0	0	1.234568	1.234568	1.234568	1.234568	1.234568	1.234568	565607.1439
NGOs	1.234567881	1.234568	0	1.234568	1.234568	1.234568	1.234568	1.234568	1.234568	0	9.876543045
UNPF	1.234567881	1.234568	0	0	0	1.234568	1.234568	0	0	1.234568	6.172839403
OO	9818034.538	0	0	0	1.234568	0	1.234568	1.234568	1.234568	2066.845	9820106.322
Total	164189661.4	4.938272	1.234568	865788.5	6.172839	6.172839	7.407407	4.938272	71600.52	2071.784	**165129153**

Table 20. The funds sent by world bank (x_{6jl}).

	FS	Health	WSH	Nutrition	Education	Protection	Shelter and NFI	CCCM	RMM	OS	**Total**
WFP	71990871.48	968884.8	464859.9	865786	1.234568	1.234568	1.234568	65996.91	2544624	1.234568	76901027.74
UNCF	0	1.234568	1.234568	1.234568	1.234568	1.234568	1.234568	1.234568	1.234568	1.234568	11.11111093
UNHCR	565598.5019	1.234568	0	0	1.234568	1.234568	1.234568	1.234568	1.234568	1.234568	565607.1439
NGOs	0	1.234568	1.234568	1.234568	1.234568	1.234568	1.234568	1.234568	1.234568	0	9.876543045
UNPF	27962869.79	1.234568	1.234568	0	0	1.234568	1.234568	0	0	1.234568	27962875.97
OO	0	0	14836024	97432.02	1.234568	464140.7	1.234568	1.234568	1.234568	2066.845	15399668.16
Total	100519339.8	968889.8	15300887	963220.5	6.172839	464146.9	7.407407	66001.84	2544629	2071.784	**120829200**

Table 21. The funds sent by United Kingdom (x_{7jl}).

	FS	Health	WSH	Nutrition	Education	Protection	Shelter and NFI	CCCM	RMM	OS	**Total**
WFP	72029312.46	0	1.234568	0	1.234568	1.234568	1.234568	0	0	1.234568	72029318.63
UNCF	0	1.234568	1.234568	1.234568	1.234568	1.234568	1.234568	1.234568	1.234568	1.234568	11.11111093
UNHCR	565598.468	1.234568	0	0	1.234568	1.234568	1.234568	1.234568	1.234568	1.234568	565607.1099
NGOs	0	1.234568	1.234568	1.234568	1.234568	1.234568	1.234568	0	1.234568	0	8.641975164
UNPF	1.234567881	1.234568	0	0	0	1.234568	1.234568	0	0	1.234568	6.172839403
OO	7255738.963	0	0	0	1.234568	0	1.234568	1.234568	1.234568	12457472	19713216.33
Total	79850651.13	4.938272	3.703704	2.469136	6.172839	6.172839	7.407407	3.703704	4.938272	12457477	**92308168**

Table 22. The funds sent by Japan (x_{8jl}).

	FS	Health	WSH	Nutrition	Education	Protection	Shelter and NFI	CCCM	RMM	OS	**Total**
WFP	43002170.45	991552.8	470526.9	956457.7	1.234568	1.234568	1.234568	66351.09	0	1.234568	45487063.84
UNCF	0.627727244	1.234568	1.234568	1.234568	1.234568	1.234568	1.234568	1.234568	1.234568	1.234568	11.73883817
UNHCR	565598.502	1.234568	1.234568	0	1.234568	1.234568	1.234568	1.234568	1.234568	1.234568	565608.3785
NGOs	0	1.234568	1.234568	1.234568	1.234568	1.234568	1.234568	1.234568	1.234568	0	9.876543045
UNPF	498798.9911	1.234568	1.234568	0	1.234568	1.234568	1.234568	0	0	1.234568	498806.3985
OO	9818034.538	1.234568	0	97786.21	1.234568	486808.7	1.234568	1.234568	1.234568	2067.191	10404702.77
Total	53884603.11	991558.9	470531.8	1054246	7.407407	486814.8	7.407407	66356.03	4.938272	2072.13	**56956203**

Table 23. The funds sent by Central Emergency Response Fund (x_{9jl}).

	FS	Health	WSH	Nutrition	Education	Protection	Shelter and NFI	CCCM	RMM	OS	Total
WFP	0	0	0	0	1.234568	1.234568	1.234568	0	0	1.234568	4.938271523
UNCF	0	1.234568	1.234568	1.234568	1.234568	1.234568	1.234568	1.234568	1.234568	1.234568	11.11111093
UNHCR	54663686.09	1.234568	1.234568	0	1.234568	1.234568	1.234568	1.234568	1.234568	1.234568	54663695.96
NGOs	0	1.234568	0	1.234568	1.234568	1.234568	1.234568	1.234568	1.234568	0	8.641975164
UNPF	0	1.234568	1.234568	0	1.234568	1.234568	1.234568	0	0	1.234568	7.407407284
OO	0	0	0	0	1.234568	0	1.234568	1.234568	1.234568	0	4.938271523
Total	54663686.09	4.938272	3.703704	2.469136	7.407407	6.172839	7.407407	4.938272	4.938272	4.938272	**54663733**

Table 24. The funds sent by Canada (x_{10jl}).

	FS	Health	WSH	Nutrition	Education	Protection	Shelter and NFI	CCCM	RMM	OS	**Total**
WFP	27033082.8	941329.6	14014349	755565.3	1.234568	1.234568	1.234568	65566.36	71165.03	1.234568	42881062.56
UNCF	0	1.234568	1.234568	1.234568	1.234568	1.234568	1.234568	1.234568	1.234568	1.234568	11.11111093
UNHCR	565598.5019	1.234568	1.234568	0	1.234568	1.234568	1.234568	1.234568	1.234568	1.234568	565608.3784
NGOs	0	1.234568	1.234568	1.234568	1.234568	1.234568	1.234568	1.234568	1.234568	0	9.876543045
UNPF	486243.2123	1.234568	0	0	0	1.234568	1.234568	0	0	1.234568	486248.1506
OO	9818034.538	0	0	97001.47	1.234568	436585.5	1.234568	1.234568	1.234568	2066.425	10353692.92
Total	37902959.05	941334.6	14014352	852569.2	6.172839	436591.7	7.407407	65571.29	71169.97	2071.363	**54286633**

Table 25. The funds sent by other donors (x_{11jl}).

	FS	Health	WSH	Nutrition	Education	Protection	Shelter and NFI	CCCM	RMM	OS	**Total**
WFP	169205159.6	1067093	489381.9	9737802	1.234568	1.234568	1.234568	67529.53	0	1.234568	180566971.3
UNCF	0.627727519	1.234568	1.234568	1.234568	1.234568	1.234568	1.234568	1.234568	1.234568	1.234568	11.73883844
UNHCR	565598.502	1.234568	0	0	1.234568	1.234568	1.234568	1.234568	1.234568	1.234568	565607.144
NGOs	1.234567881	1.234568	1.234568	82831085	1.234568	1.234568	1.234568	1.234568	1.234568	0	82831094.76
UNPF	1.234567881	1.234568	0	0	0	1.234568	1.234568	0	0	1.234568	6.172839403
OO	9818034.538	1.234568	0	98964.64	1.234568	560453.2	1.234568	1.234568	1.234568	2068.342	10479526.88
Total	179588795.7	1067100	489384.3	92667853	6.172839	560459.4	7.407407	67534.47	4.938272	2073.28	**274443218**

Appendix B

Table 26. The funds received by A. Al Asimah (y_{1jl}).

	FS	Health	WSH	Nutrition	Education	Protection	Shelter and NFI	CCCM	RMM	OS	**Total**
WFP	146390996.4	0	0	0	0	0	0	0	0	0	146390996.4
UNCF	0	0	0	0	0	0	0	0	0	0	0
UNHCR	0	0	0	0	0	0	0	0	0	0	0
NGOs	0	0	0	0	0	0	0	0	0	0	0
UNPF	0	0	0	0	0	0	0	0	0	0	0
OO	0	0	0	50226587	0	0	0	0	0	19266.16	50245853.62
Total	146390996.4	0	0	50226587	0	0	0	0	0	19266.16	**196636850**

Table 27. The funds received by Abyan (y_{2jl}).

	FS	Health	WSH	Nutrition	Education	Protection	Shelter and NFI	CCCM	RMM	OS	**Total**
WFP	0	0	3353096	0	5277860	0	0	628019.2	601384.9	0	9860360.262
UNCF	0	4492580	0	0	0	0	0	0	0	0	4492579.77
UNHCR	8879559.825	0	0	0	0	4463912	0	0	0	2654134	15997606.14
NGOs	0	0	0	0	0	0	0	0	0	0	0
UNPF	0	0	0	0	0	0	0	0	0	0	0
OO	0	0	32759.25	9069658	0	0	0	0	0	0	9102417.033
Total	8879559.825	4492580	3385855	9069658	5277860	4463912	0	628019.2	601384.9	2654134	**39452963.21**

Table 28. The funds received by Aden (y_{3jl}).

	FS	Health	WSH	Nutrition	Education	Protection	Shelter and NFI	CCCM	RMM	OS	**Total**
WFP	0	0	0	0	0	7237042	71843.74	1018166	863712.1	4302970	13493733.34
UNCF	0	14567038	0	0	0	0	293519	0	0	0	14860556.69
UNHCR	14507860.84	0	0	0	0	0	0	0	0	0	14507860.84
NGOs	0	0	0	14704029	0	0	12.34568	0	0	0	14704041.35
UNPF	0	0	0	0	0	0	0	0	0	0	0
OO	0	0	0	0	0	0	11.11111	0	0	0	11.11111093
Total	14507860.84	14567038	0	14704029	0	7237042	365386.2	1018166	863712.1	4302970	**57566203.34**

Table 29. The funds received by Al Bayda (y_{4jl}).

	FS	Health	WSH	Nutrition	Education	Protection	Shelter and NFI	CCCM	RMM	OS	**Total**
WFP	0	11325828	0	0	6652763	0	0	0	0	0	17978590.91
UNCF	0.435729849	0	0	0	0	0	0	791620.7	0	3345546	4137167.152
UNHCR	0.435729849	0	0	0	0	0	0	0	0	0	0.435729849
NGOs	0	0	0	0	0	0	0	0	0	0	0
UNPF	28947918.17	0	0	0	0	0	0	0	0	0	28947918.17
OO	0	0	0	11432339	0.863732	5626779	0	0	0	0	17059118.56
Total	28947919.04	11325828	0	11432339	6652763	5626779	0	791620.7	0	3345546	**68122795.23**

Table 30. The funds received by Al Dhale'e (y_{5jl}).

	FS	Health	WSH	Nutrition	Education	Protection	Shelter and NFI	CCCM	RMM	OS	**Total**
WFP	31028383.64	11387935	0	0	6689236	5657634	0	795961.6	0	0	55559149.68
UNCF	0	0	0	0	0	0	0	0	0	3363892	3363891.642
UNHCR	0	0	0	0	0	0	0	0	0	0	0
NGOs	0	0	0	0	0	0	0	0	0	0	0
UNPF	0	0	0	0	0	0	0	0	0	0	0
OO	0	0	0	11495029	8.519924	0	0	0	0	0	11495037.58
Total	31028383.64	11387935	0	11495029	6689244	5657634	0	795961.6	0	3363892	**70418078.9**

Table 31. The funds received by Al Hudaydah (y_{6jl}).

	FS	Health	WSH	Nutrition	Education	Protection	Shelter and NFI	CCCM	RMM	OS	**Total**
WFP	169714978	0	0	22005900	25611565	0	0	0	0	12879561	230212004.5
UNCF	0	0	0	0	0	0	0	0	0	0	0
UNHCR	0	0	0	0	0	0	0	0	0	0	0
NGOs	0	0	0	0	0	0	0	0	0	0	0
UNPF	0	0	0	0	0	0	0	0	0	0	0
OO	0	0	0	0	0	0	0	0	0	0	0
Total	169714978	0	0	22005900	25611565	0	0	0	0	12879561	**230212004.5**

Table 32. The funds received by Al Jawf (y_{7jl}).

	FS	Health	WSH	Nutrition	Education	Protection	Shelter and NFI	CCCM	RMM	OS	**Total**
WFP	0	0	0	0	5180583	4381637	0	616444.1	0	0	10178664.73
UNCF	0	0	0	0	0	0	0	0	0	2605215	2605215.115
UNHCR	7908214.4	0	0	0	0	0	4172605	0	0	0	12080819.53
NGOs	0	0	0	4451247	0	0	0	0	590288.3	0	5041535.069
UNPF	0	8819553	0	0	0	0	0	0	0	0	8819552.654
OO	0	0	0	0	0	0	0	0	12.34568	0	12.34567881
Total	7908214.4	8819553	0	4451247	5180583	4381637	4172605	616444.1	590300.7	2605215	**38725799.45**

Table 33. The funds received by Al Maharah (y_{8jl}).

	FS	Health	WSH	Nutrition	Education	Protection	Shelter and NFI	CCCM	RMM	OS	**Total**
WFP	1500616.445	0	1831218	0	1452781	0	585058	345736.6	331073.8	0	6046484.632
UNCF	1871004.187	0	0	0	0	0	0	0	0	0	1871004.187
UNHCR	0	2473251	0	0	0	0	0	0	0	0	2473250.779
NGOs	0	0	0	2496510	0	614367.2	0	0	0	0	3110876.941
UNPF	0	0	0	0	0	0	0	0	0	0	0
OO	1500616.445	0	32759.25	0	0	0	0	0	0	730575.6	2263951.338
Total	4872237.077	2473251	1863978	2496510	1452781	614367.2	585058	345736.6	331073.8	730575.6	**15765567.88**

Table 34. The funds received by Al Mahwit (y_{9jl}).

	FS	Health	WSH	Nutrition	Education	Protection	Shelter and NFI	CCCM	RMM	OS	**Total**
WFP	0	11312785	0	0	6645102	0	0	790709	0	3341693	22090288.71
UNCF	0	0	0	11419173	0	5620299	0	0	0	0	17039471.43
UNHCR	28841082.68	0	0	0	0	0	0	0	0	0	28841082.68
NGOs	0	0	0	0	0	0	0	0	0	0	0
UNPF	0	0	0	0	0	0	0	0	0	0	0
OO	0	0	0	0	0	0	0	0	0	0	0
Total	28841082.68	11312785	0	11419173	6645102	5620299	0	790709	0	3341693	**67970842.82**

Table 35. The funds received by Amran (y_{10jl}).

	FS	Health	WSH	Nutrition	Education	Protection	Shelter and NFI	CCCM	RMM	OS	**Total**
WFP	0	0	0	0	10346821	4375571	8333656	0	0	0	23056048.12
UNCF	0	0	0	0	0	0	0	0	0	0	0
UNHCR	0	0	7.407407	0	0	0	0	0	0	0	7.407407284
NGOs	0	0	8.641975	1.685607	0	0	0	0	0	0	10.32758195
UNPF	0	0	0	0	0	0	0	0	0	0	0
OO	47994230.19	0	13275378	17780334	0	0	0	0	0	5203216	84253157.94
Total	47994230.19	0	13275394	17780335	10346821	4375571	8333656	0	0	5203216	**107309223.8**

Table 36. The funds received by Dhamar (y_{11jl}).

	FS	Health	WSH	Nutrition	Education	Protection	Shelter and NFI	CCCM	RMM	OS	**Total**
WFP	458120.1101	15893380	0	0	0	0	7519298	0	0	0	23870797.66
UNCF	0	0	0	0	0	0	0	0	0	0	0
UNHCR	0	0	0	0	0	15791974	0	0	0	9389524	25181497.34
NGOs	0	9.876543	0	32085709	0	0	0	0	0	0	32085718.95
UNPF	0	0	0	0	0	0	0	0	0	0	0
OO	86150420.45	0	0	0	0	0	0	0	0	0	86150420.45
Total	86608540.56	15893390	0	32085709	0	15791974	7519298	0	0	9389524	**167288434.4**

Table 37. The funds received by Hadramut (y_{12jl}).

	FS	Health	WSH	Nutrition	Education	Protection	Shelter and NFI	CCCM	RMM	OS	**Total**
WFP	60129890.23	1.664025	0	0	0	0	5220438	0	0	0	65350330.1
UNCF	0	0	16599406	1.636603	12963084	0	0	0	0	0	29562491.31
UNHCR	0	0	0	0	0	10963926	0	0	0	0	10963926.09
NGOs	0	0	0	1.683304	0	0	0	0	0	0	1.683303764
UNPF	0	0	0	0	0	0	0	0	0	0	0
OO	0	0	32759.25	22276208	0	0	0	0	0	6518884	28827851.45
Total	60129890.23	1.664025	16632165	22276211	12963084	10963926	5220438	0	0	6518884	**134704600.6**

Table 38. The funds received by Hajjah (y_{13jl}).

	FS	Health	WSH	Nutrition	Education	Protection	Shelter and NFI	CCCM	RMM	OS	**Total**
WFP	0	18333366	0	0	21537949	18216381	17347343	0	0	10831018	86266057.17
UNCF	0	0	13784274	0	0	0	0	0	0	0	13784274.22
UNHCR	10211188.27	0	0	0	0	0	0	0	0	0	10211188.27
NGOs	0	0	0	18505778	0	0	0	0	0	0	18505778.07
UNPF	0	0	0	0	0	0	0	0	0	0	0
OO	0	1.234568	32759.25	0	0	0	0	0	0	0	32760.48857
Total	10211188.27	18333367	13817033	18505778	21537949	18216381	17347343	0	0	10831018	**128800058.2**

Table 39. The funds received by Ibb (y_{14jl}).

	FS	Health	WSH	Nutrition	Education	Protection	Shelter and NFI	CCCM	RMM	OS	**Total**
WFP	175116526.3	0	0	6.902607	0	1.981394	0	0	0	13289481	188406016.1
UNCF	0	0	0	11048743	0	0	0	0	0	0	11048743.16
UNHCR	0	0	0	0	0	0	0	0	0	0	0
NGOs	0	0	0	0	0	0	0	0	0	0	0
UNPF	0	0	0	0	0	22351197	0	0	0	0	22351196.93
OO	0	0	0	0	0	1.980924	0	0	0	0	1.980923846
Total	175116526.3	0	0	11048750	0	22351201	0	0	0	13289481	**221805958.2**

Table 40. The funds received by Lahj (y_{15jl}).

	FS	Health	WSH	Nutrition	Education	Protection	Shelter and NFI	CCCM	RMM	OS	**Total**
WFP	6492866.114	0	0	0	0	0	0	0	0	4565775	11058641.34
UNCF	26887399.71	0	5791755	0	0	7679048	7312708	0	0	0	47670910.21
UNHCR	0	7728363	0	0	9079242	0	0	0	0	0	16807604.82
NGOs	0	0	0	0	0	0	0	0	0	0	0
UNPF	0	0	4.938272	0	0	0	0	0	0	0	4.938271523
OO	0	0	32759.25	15602084	0	0	0	0	0	0	15634842.94
Total	33380265.83	7728363	5824519	15602084	9079242	7679048	7312708	0	0	4565775	**91172004.25**

Table 41. The funds received by Marib (y_{16jl}).

	FS	Health	WSH	Nutrition	Education	Protection	Shelter and NFI	CCCM	RMM	OS	**Total**
WFP	4876581.48	3956388	0	0	2126198	3596606	0	1011989	969080.3	0	16536842.67
UNCF	4876581.48	0	0	0	0	0	0	0	0	0	4876581.48
UNHCR	0	0	0	0	0	0	0	0	0	0	0
NGOs	4.938271523	0	0	5679038	2.716053	0	0	9.876543	0	0	5679055.097
UNPF	0	0	0	0	3.703704	0	3425025	0	0	2138455	5563483.571
OO	9971841.461	0	2728008	0	0.863731	0	0	0	0	0	12699850.1
Total	19725009.36	3956388	2728008	5679038	2126205	3596606	3425025	1011999	969080.3	2138455	**45355812.92**

Table 42. The funds received by Raymah (y_{17jl}).

	FS	Health	WSH	Nutrition	Education	Protection	Shelter and NFI	CCCM	RMM	OS	**Total**
WFP	25679993.3	0	7087908	0	2774905	4693942	2235004	660382.2	0	2790905	45923039.26
UNCF	9837.130751	4724086	0	0	0	0	1.234568	0	0	0	4733923.972
UNHCR	0	1.234568	0	0	1.234568	0	0	0	0	0	2.469135761
NGOs	0	1.234568	0	0	9.629626	1.234568	0	0	0	0	12.09876154
UNPF	0	1.234568	1.234568	0	1.234568	1.234568	1.234568	0	0	0	6.172839403
OO	0	1.234568	32759.25	9537034	2.098293	1.234568	1.234568	0	0	1.234568	9569799.965
Total	25689830.43	4724091	7120669	9537034	2774919	4693946	2235007	660382.2	0	2790906	**60226783.93**

Table 43. The funds received by Sa'ada (y_{18jl}).

	FS	Health	WSH	Nutrition	Education	Protection	Shelter and NFI	CCCM	RMM	OS	**Total**
WFP	0	0	0	0	0	7121610	0	0	0	4234337	11355947.05
UNCF	39057336.48	0	0	1.234568	0	0	0	0	0	0	39057337.72
UNHCR	0	14334692	0	0	0	0	0	332661.5	0	0	14667353.38
NGOs	0	1.234568	0	0	0	0	0	0	0	0	1.234567881
UNPF	0	1.234568	0	0	0	1.234568	0	0	0	0	2.469135761
OO	0	0	0	14469499	0	1.234568	0	0	0	0	14469500.67
Total	39057336.48	14334694	0	14469501	0	7121612	0	332661.5	0	4234337	**79550142.52**

Table 44. The funds received by Sana'a (y_{19jl}).

	FS	Health	WSH	Nutrition	Education	Protection	Shelter and NFI	CCCM	RMM	OS	**Total**
WFP	83572540.44	1.234568	0	0	12611872	0	0	0	0	6342267	102526680.3
UNCF	0	0	0	1.234568	0	0	0	0	0	0	1.234567881
UNHCR	0	0	0	0	0	0	0	0	0	0	0
NGOs	0	0	0	1.234568	0	0	0	0	0	0	1.234567881
UNPF	0	0	0	0	0	0	0	0	0	0	0
OO	0	0	0	21672675	0	0	0	0	0	0	21672674.86
Total	83572540.44	1.234568	0	21672677	12611872	0	0	0	0	6342267	**124199357.6**

Table 45. The funds received by Shabwah (y_{20jl}).

	FS	Health	WSH	Nutrition	Education	Protection	Shelter and NFI	CCCM	RMM	OS	**Total**
WFP	0	4863048	3632301	0	5713085	0	0	0	0	0	14208433.97
UNCF	0	0	0	0	0	0	0	0	0	0	0
UNHCR	14520569.76	0	0	0	0	4832016	0	0	650976.5	0	20003561.81
NGOs	0	0	0	4908781	0	1.234568	0	0	0	0	4908782.443
UNPF	0	0	0	0	0	0	0	0	0	2873000	2872999.798
OO	0	0	32759.25	0	0	0	0	679807.1	0	0	712566.3925
Total	14520569.76	4863048	3665060	4908781	5713085	4832017	0	679807.1	650976.5	2873000	**42706344.42**

Table 46. The funds received by Socotra (y_{21jl}).

	FS	Health	WSH	Nutrition	Education	Protection	Shelter and NFI	CCCM	RMM	OS	**Total**
WFP	10301.22257	0	0	0	0	0	471613.5	0	0	0	481914.7311
UNCF	10301.22257	996840.1	350839.6	1006215	430810.5	470769.6	0	139348.6	133438.8	294457.4	3833020.471
UNHCR	0	0	0	0	0	0	0	0	0	0	0
NGOs	0	0	0	0	0	0	0	0	0	0	0
UNPF	0	0	0	0	0	0	0	0	0	0	0
OO	2006594.646	0	32759.25	0	0	0	0	0	0	0	2039353.9
Total	2027197.091	996840.1	383598.8	1006215	430810.5	470769.6	471613.5	139348.6	133438.8	294457.4	**6354289.103**

Table 47. The funds received by Taizz (y_{22jl}).

	FS	Health	WSH	Nutrition	Education	Protection	Shelter and NFI	CCCM	RMM	OS	**Total**
WFP	25718068.04	0	0	45190000	0	0	0	0	0	5289738	76197806.33
UNCF	0	0	0	0	0	0	0	0	0	1.234568	1.234567881
UNHCR	0	0	0	0	0	0	0	0	0	0	0
NGOs	0	0	0	0	0	0	0	0	0	0	0
UNPF	0	0	0	0	0	0	0	0	0	0	0
OO	148540196.1	0	0	0	0	0	0	0	0	0	148540196.1
Total	174258264.1	0	0	45190000	0	0	0	0	0	5289739	**224738003.6**

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