AIMS Mathematics

AIMS Mathematics, 8(8): 18809–18832.
DOI: 10.3934/math.2023957
Received: 23 February 2023
Revised: 07 May 2023
Accepted: 18 May 2023
Published: 02 June 2023

http://www.aimspress.com/journal/Math

Research article

Industry 4.0 project prioritization by using q-spherical fuzzy rough analytic hierarchy process

Ahmad Bin Azim[1,*], Ahmad ALoqaily[2,3], Asad Ali[1,*], Sumbal Ali[1] and Nabil Mlaiki[2]

[1] Department of Mathematics & Statistics, Hazara University, Mansehra, 21120 Pakistan
[2] Department of Mathematics and Sciences, Prince Sultan University. Riyadh, 11586 Saudi Arabia
[3] School of Computer, Data, and Mathematical Sciences, Western Sydney University, Australia-Sydney-2150

* **Correspondence:** Email: ahmadbinazim@gmail.com, asad_maths@hu.edu.pk;
Tel: +923138273403, +923339508166.

Abstract: The Fourth Industrial Revolution, also known as Industry 4.0, is attracting a significant amount of attention because it has the potential to revolutionize a variety of industries by developing a production system that is fully automated and digitally integrated. The implementation of this transformation, however, calls for a significant investment of resources and may present difficulties in the process of adapting existing technology to new endeavors. Researchers have proposed integrating the Analytic Hierarchy Process (AHP) with extensions of fuzzy rough sets, such as the three-dimensional q-spherical fuzzy rough set (q-SFRS), which is effective in handling uncertainty and quantifying expert judgments, to prioritize projects related to Industry 4.0. This would allow the projects to be ranked in order of importance. In this article, a novel framework is presented that combines AHP with q-SFRS. To calculate aggregated values, the new framework uses a new formula called the q-spherical fuzzy rough arithmetic mean, when applied to a problem involving the selection of a project with five criteria for evaluation and four possible alternatives, the suggested framework produces results that are robust and competitive in comparison to those produced by other multi-criteria decision-making approaches.

Keywords: Industry 4.0; project prioritization; q-spherical fuzzy rough sets; q-spherical fuzzy rough analytic hierarchy process
Mathematics Subject Classification: 60L70, 68N17

---

1. Introduction

In 2017, Lu [31] presented the fourth industrial revolution, which includes remote-control components, ERP, industrial internet, adaptive production, and collaborative product development. These technologies led to "smart manufacturing". Adaptive manufacturing improves production performance for integrated infrastructure and appropriate methodologies. Industry 4.0 includes digitization, optimization, and customization of production; automation and adaptation; human-machine interaction (HMI); value-added services and businesses; and automatic data exchange and communication, according to Posada et al. [32]. In addition to an autonomous structure, smart manufacturing's ICT-connected technologies offer Industry 4.0 opportunities and benefits. Smart manufacturing triggers robust tailored production, real-time integrated system tracking, and value chain optimization. Integrated manufacturing services and business models reduce operating costs. Technology selection criteria are crucial to Industry 4.0 technologies. Gossen et al. [33] recommend analyzing the interaction between selection criteria and alternative technologies. Smart manufacturing requires IT-focused approaches and digitalization, so all possible scenarios should be researched using a multi-criteria decision and selection method to find the best Industry 4.0 technology. The novel q-spherical fuzzy rough set has fuzzy membership functions with membership, non-membership, and hesitation parameters and lower and upper set approximations. To keep their q sum below 1, these parameters can be individually specified between 0 and 1. q-spherical fuzzy rough sets accurately and adaptably describe linguistic terms due to their structure. This research uses the q-spherical fuzzy rough AHP approach to prioritize Industry 4.0 projects in an organization. This enhances the standard AHP technique to handle linguistic fuzziness and ambiguity in decision-making problems. The fuzzy set theory develops from the more traditional crisp set theory. To add to that, Zadeh was the pioneer in bringing up the concept of a fuzzy set, which considers only those grades that were awarded a positive grade. The correlation between the problems of Industry 4.0 project prioritization and fuzzy set theory lies in the fact that many of the criteria used to evaluate Industry 4.0 projects are inherently fuzzy. For example, the benefits of implementing new technology may not be immediately clear, or the risks associated with a project may be difficult to quantify. In such cases, fuzzy set theory can be used to represent these criteria more flexibly and intuitively. By using fuzzy set theory, decision-makers can assign degrees of membership to different project criteria, which allows for a more nuanced and realistic evaluation of the projects. This can help organizations prioritize their Industry 4.0 projects more effectively, leading to better outcomes and competitive advantage. In conclusion, the problems of Industry 4.0 project prioritization and fuzzy set theory are closely connected. The fuzzy set theory [1] develops from the more traditional crisp set theory. To add to that, Zadeh was the pioneer in bringing up the concept of a fuzzy set, which considers only those grades that were awarded a positive grade. Fuzzy set theory can provide decision-makers with a more flexible and intuitive way to evaluate Industry 4.0 projects, helping them to make better decisions and achieve their strategic goals. As a development of the fuzzy set, the intuitionistic fuzzy set (IFS) was proposed by Atanassov [2]. It includes both positive and negative grades, but only if their sum is less than or equal to 1. In other words, both good and bad grades are considered by the IFS. IFS has become more important since it was first introduced, and it has been used a lot to solve problems with making decisions. In 2014, Cuong [3] added to the ideas of fuzzy sets and IFS by coming up with a new idea he called a picture fuzzy (PF) set. In this idea, he talks about grades that are positive, neutral, and negative. Using intuitionistic fuzzy frank power aggregation operations, Zhang et al. [4] developed multiple attribute group decision-making procedures in 2015. Seikh and Mandal [5] have decided to make fuzzy Dombi aggregation operators that are easy to understand and have used them to solve MCDM problems. Also, Zeng et al. [6] made the MCDM by combining social network analysis with an intuitionistic fuzzy hybrid contingent aggregation operator. It is important to remember that in IFS, the restriction sum $(\mu^{+}, \mu^{-}) \in [0, 1]$ limit the chance of getting both a good grade "+" and a bad grade "-" for the same work. To get around this problem, Yager [7] built what is now called the Pythagorean fuzzy set as an extension of the IFS (PyFS). For this generalization, we will use the condition that must be met, which can be written as $0 \leq \mu^{+^2} + \mu^{-^2} \leq 1$. PyFS is a better mathematical tool, and it gives decision-makers (DMs) in fuzzy set theory more freedom to use the information they have at hand. After PyFS came out, other academics tried to make "forks" of it by adding more "aggregation operators" (AOs). Because of this, Akram et al. [8] showed some fuzzy Pythagorean Dombi AOs. Also, Garg [9] makes different PyF aggregation operations based on the level of confidence and uses them in DM situations. Also, Wang and Garg [10] came up with an algorithm for MADM that works based on the PyFS principle and uses the interactive Archimedean norm operations. Wu et al. [11] showed that the MAGDM is flexible by using the information fusion approach and uncertain Pythagorean fuzzy sets. PyFS is a limited idea that has been brought to the attention of researchers because when DMs give 0.8 as a positive grade (PG) and 0.7 as a negative grade (NG), then sum $(0.8^2 + 0.7^2) \leq \notin [0, 1]$, PyFS cannot be used, so PyFS cannot solve this problem in the current situation. Yager [12] came up with the idea of a q-rung orthopair fuzzy set (q-ROFS) as a generalization of IFS and PyFS to make this task easier. In a recent work [13] written by Xing et al., the authors presented various point-weighted AOs for q-ROFSs and discussed how these AOs may be used for MCDM issues. Liu and Wang [14] have also made several q-ROFS-based AOs for MADM problems. AOs have been made for these. It is important to note that PG and NG are the only grades that are mentioned in any of the concepts. The abstinence grade (AG) is completely ignored, even though it is important to talk about the AG in many different types of real-life situations. Cuong and Kreinovich [15] brought this problem to light, and they came up with a new idea that is called a picture fuzzy set (PFS) in published works. PFS is a better tool for dealing with the ambiguity and uncertainty that come with MCDM issues. Note that the PFS uses the rule that the sum of $(\mu^{+}, \mu', \mu^{-}) \in [0, 1]$, must be between 0 and 1. This makes the PFS different from all other theories and makes it more general. Wei [16], which is based on PFS, also talks about a few PF Hamacher AOs. If all the membership grades end up being 0.5 (positive), 0.4 (negative), or 0.2 (negative), then PFS is not a valid formation because its main requirement is not met. This is because the formula $0.5 + 0.2 + 0.4 \notin [0,1]$ doesn't work in this case. Mahmood et al. [17] think that a spherical fuzzy set (SFS) could be used to control this problem. Since SFS has been around, researchers have been paying more attention to it. Also, Deli and Cagman [18] have come up with some theories about fuzzy spherical numbers and a method called the MCDM method. Raq et al. [19] have also shown some cosine similarity measures based on SFS, as well as how they can be used to solve DM problems. Ashraf et al. [20] also come up with several symmetric-based AOs for the SF data. To address the problem of uncertainty, Kahraman [21] developed the q-spherical fuzzy set. He used the idea to help him figure out what to do. It is thought to be a bigger version of what is called a "spherical fuzzy set". In q-SFS, the element is described by one of three degrees: positive (μ^{+}), neutral (μ'), or negative (μ^{-}). However $0 \leq (\mu^{+})^{q} + (\mu')^{q} + (\mu^{-})^{q} \leq 1$, must be true. Opinions in this q-spherical fuzzy set include not only "yes" and "no", but also "don't know" and "refuse". It is required that the sum of the q powers of (μ^{+}), (μ'), and (μ^{-}), respectively, be less than or equal to one. So, q-SFSs (q-Spherical Fuzzy Sets) give decision makers a larger number of preferences to choose from so they can make their own decisions about membership, non-membership, and hesitant degrees.

Rough sets (RS) are a way to deal with the ambiguity that was first suggested by Pawlak [23,24]. This setup is like ambiguity and uncertainty from a mathematical point of view. Rough set theory (RST), which is an expansion of classical set theory, uses the relation, which is a way to show how information systems work, as its main tool. People have noticed that the Pawlak relational semantics theory's equivalence relation is limited in many real-world situations. Considering this, numerous authors have supplemented Pawlak's rough set theory by introducing non-equivalence relations, and similarity relations are illustrated in [25,26].

Even though intuitionistic fuzzy sets, Pythagorean fuzzy sets, and q-rung orthopair fuzzy sets all have their uses, they can only look at binary options like yes or no, and people's opinions are never just yes or no. Take voting as an example. There are four possible outcomes: voting yes, voting no, not voting at all, or not voting at all. No theory that is currently accepted can explain this event. Remember that the picture fuzzy set and the spherical fuzzy set can solve these kinds of problems, even though they have some limits. The q-spherical fuzzy set, on the other hand, has proven to be the best way to deal with this kind of problem. Also, keep in mind that the current theories must overcome problems in their structures or conditions, as shown below.

(1) IFRS [27], PyFRS [28], and q-ROFRS [29,30] are all well-known theories that fail horribly when all three possible grades are included in the data (positive, neutral, and negative).

(2) It is feasible to consider voting with the help of the concept of a picture fuzzy rough set (PFSRS), but this is constrained by the presence of lower and upper approximations like $0 \leq \underline{\mu}^{+} + \underline{\mu}' + \underline{\mu}^{-} \leq 1$ and $0 \leq \overline{\mu}\wedge + +\overline{\mu}' + \overline{\mu}^{-} \leq 1$. On the other hand, when information is given to decision-makers as q-spherical fuzzy rough sets (q-SFRS), which include lower and upper approximations like {(0.9,0.8,0.9), (0.9,0.7,0.9)}, etc., be aware that the sum of the lower and upper approximation values is outside the range [0, 1]. This means that $(0 \nleq 0.9^2 + 0.8^2 + 0.9^2 \nleq 1)$ and $(0 \nleq 0.9^2 + 0.7^2 + 0.9^2 \nleq 1)$ that can never be handled by SFRS and that limit the SFRS concept.

The concept q-spherical fuzzy rough set was first introduced by Azim et al. [22] in 2023. This kind of fuzzy set has good things about both the rough set and the q-spherical fuzzy set. Because of this, the most important thing that this work adds to what we already know is a practical way to make decisions that work for the q-spherical fuzzy rough set. In q-SFRS, there are a total of three independent parameters with lower and upper set approximations. For the sake of future study, we want to create brand new aggregation operators in addition to defuzzification processes. As a result, considering everything, the q-spherical fuzzy rough set seems to be a promising new idea, opening the door to a wide variety of opportunities for study in the future. Reviewing the relevant literature shows that in the years after the first industrial revolution, several revolutionary changes caused the manufacturing sector to think deeply about its ability to be both effective and efficient. On the other hand, Lu et al. [31] say that corporations are likely to investigate the root cause of problems that come up when making their products. To reach this goal, complex technologies and structures that were once separate must be brought together to make important advances. In 2011, the German government was the first to use the term "Industry 4.0" at the Hannover Fair. This was done by integrating information systems better and making sure that different industrial processes worked together better. Interoperability and constant data exchange are two of the most important things to think about when making Industry 4.0-style distributed manufacturing systems. A real-time tracking system is also needed to predict how a system will act so that production can be controlled automatically. When enabling technologies are used a lot, these effects influence the whole manufacturing process as well as the standardization of production. The main things that drive these creative innovations are the ability to meet specific consumer needs, the allocation and use of resources, and the shortening of the average product's lifespan. Qin et al. [35] say that this is why applications of Industry 4.0 are leading to big breakthroughs, and these applications have the potential to help the economy grow. For instance, the associated sector is responsible for about 17% of the gross domestic product and almost 32 million jobs across the European Union. In 2015, Posada et al. [32] showed that visual computing is a key technology that makes Industry 4.0 and the industrial internet possible. Gossen et al. [33] introduced the idea of using multiple criteria to choose track and trace technologies as part of a plan to stop counterfeiting. Dogan and Oztaysi [35] in 2018 talk about how fuzzy decision-making could be used to choose in-store behavioral analytics technology. Lee et al. [36] introduced a cyber-physical systems architecture for manufacturing systems that are based on Industry 4.0 projects. In the end, Lasi et al. [37] talked about the improvements in automation, digitization, and networking that are needed for the sector to work well and are part of the technologies that support it. To start, it is important to coordinate the many basic needs, such as mobile technologies, RFIP/RTLS technologies, and sensors, to make the advances that Industry 4.0 promises and create value in the manufacturing process. Also, for Industry 4.0 to be used, protocols and procedures for protecting distributed systems and cyber security systems need to be made the same. Shen et al. [38] proposed in 2010 that if you want to use or buy new technology, you should choose it based on the principle of choosing the best option out of several competing options. To keep up with modern technological changes, the company needs to get digital skills and spread them throughout the organization. As an example of how technology selection issues can be used, several studies have come up with different ways to judge how well people make decisions based on different criteria. As part of the technology selection problem, this has been done. Saatay [39] came up with the AHP in 1987 to solve complicated problems with multiple criteria. It was then made into a fuzzy form so that it could be used to choose technologies based on imperfect information. Oztaysi [40] in 2014 suggested that when choosing IT, an AHP with a fuzzy TOPSIS-Grey technique built-in should be used. Oztaysi also said that choosing the right technology means spending a lot of money to build the right infrastructure and running the risk of having to change the way things are done. Because of this, there needs to be a more scientific and methodical way to make decisions about which technologies to use. This will increase the number of successful Industry 4.0 adaptations for both the current system and the new system. So, the choice of technology could be seen as an example of a group decision-making (GDM) issue, which involves analyzing and interpreting possible solutions while considering the qualities of several different decision criteria. Oztaysi et al. [41] in 2019 suggested a hesitant fuzzy AHP to choose gamification features that can be used in the field of demand-side energy management. Solar technology is being looked at. Vermat et al. [42] in 2021 used an interval-valued intuitionistic fuzzy-analytic hierarchy process to measure the impact of security attributes in the fog-based internet of things paradigm. This helped them choose alternative fuel vehicles. In addition to fuzzy AHP, fuzzy TOPSIS, and fuzzy ANP are also ways to deal with problems involving making decisions based on more than one factor. Rajak and Shah [43] in 2019 came up with an AHP-integrated fuzzy TOPSIS-based way to find mobile application health systems (mHealth). Zhao et al. [44] in 2019 presented a fuzzy TOPSIS technique for energy-aware fuzzy job-shop scheduling for engine remanufacturing at the level of multiple machines. In their 2019 study, Fetanat et al. [45] introduced the idea of an energy justice-based decision-making framework for waste-to-energy technologies selection in sustainable waste management: A case of Iran. The concept of covering-based multi-granulation (I, T)-fuzzy rough set models and applications in multi-attribute group decision-making was introduced by Zhan [46] in 2019. The idea of a novel type of soft rough covering and its application to multicriteria group decision-making is presented by Zhan and Alcantud [47] in 2019. The concept of fuzzy β-covering based (I, T)-fuzzy rough set models and applications to multi-attribute decision-making is presented by Zhang et al. [48] in 2019. The concept of an N -soft set approach to rough sets is given by Alcantud et al. [49]

in 2019. Rao and Parnichkun [50] introduced the concept of flexible manufacturing system selection using a combinatorial mathematics-based decision-making method in 2009. Maniya and Bhatt [51] in 2011 published an article on the selection of a flexible manufacturing system using the preference selection index method. Karande and Chakraborty [52] in 2013 presented the evaluation and selection of flexible manufacturing systems using MACBETH method. Mathew and Thomas [53] in 2019 introduced the idea of interval-valued multi-criteria decision-making methods for the selection of flexible manufacturing systems. Zhang et al. [54] in 2021 introduced the concept of multi-source information fusion based on rough set theory. Yuan et al. [55] in 2021 presented the idea about the attribute reduction methods in fuzzy rough set theory: An overview, comparative experiments, and new directions. Che et al. [56] in 2023 presented the concept of learning instance-level label correlation distribution for multi-label classification with fuzzy rough sets. Ali et al. [57] in 2023 introduced a novel idea of averaging aggregation operators under the environment of q-rung orthopair picture fuzzy soft sets and their applications in MADM problems.

The remaining parts of the work are organized as follows: Section 2 provides an extensive overview of fundamental concepts such as fuzzy sets (FS), picture fuzzy sets (PFS), spherical fuzzy sets (SFS), q-spherical fuzzy sets (q-SFS), rough sets (RS), and q-spherical fuzzy rough sets (q-SFRS). The aim is to establish the necessary background knowledge for the subsequent sections. In Section 3, we discuss the operational laws and aggregation operators specifically designed for q-spherical fuzzy rough numbers (q-SFRNs). These laws and operators serve as essential tools for performing computations and aggregations within the q-spherical fuzzy rough AHP framework. Section 4 delves into the q-spherical fuzzy rough AHP technique, providing a detailed breakdown of its steps. This section offers a comprehensive understanding of how the q-spherical fuzzy rough AHP method is applied and implemented. Section 5 applies the q-spherical fuzzy AHP concept to evaluate Industry 4.0 project prioritization. We discuss the specific application of the q-spherical fuzzy rough AHP technique in this context, demonstrating how it can be effectively used to assess and compare different Industry 4.0 project prioritization. Section 6 focuses on the managerial implications of the q-spherical fuzzy rough AHP approach. We emphasize the importance of conducting comparative analysis to compare rankings obtained from different MCDM techniques and performing sensitivity analysis to evaluate the method's robustness. These analyses contribute to informed and reliable decision-making processes. The final Section 7 presents the conclusion, summarizing the key findings and contribution.

2. Preliminaries

This section investigates the concepts of the fuzzy set (FS), picture fuzzy set (PFS), spherical fuzzy set (SFS), q-spherical fuzzy set (q-SFS), and rough set (RS), in addition to operational laws and aggregation operators for q-spherical fuzzy numbers (q-SFSs), q-spherical fuzzy relation, and q-spherical fuzzy rough approximation space.

Definition 2.1. Zadeh [1] came up with the idea of a fuzzy set in 1965. It is a generalization of the crisp set. Mathematically, it's defined as:

$$A = \{\langle x, \mu_A(x)\rangle : x \in X\} \tag{1}$$

with the condition,

$$0 \leq \mu_A(x) \leq 1.$$

Where $\mu_A(x): X \to [0,1]$ represents the membership function.

Definition 2.2. Picture fuzzy set is the generalization of fuzzy set and intuitionistic fuzzy set, which was proposed by Cuong et al. [3] in 2014. Mathematically it is defined as:

$$A = \{\langle x, {\mu^{+}}_{A}(x), {\mu'}_{A}(x), {\mu^{-}}_{A}(x)\rangle : x \in X\} \tag{2}$$

with the condition,

$$0 \leq {\mu^{+}}_{A}(x) + {\mu'}_{A}(x) + {\mu^{-}}_{A}(x) \leq 1.$$

Where ${\mu^{+}}_{A}(x): X \to [0,1], {\mu'}_{A}(x): X \to [0,1]$ and ${\mu^{-}}_{A}(x): X \to [0,1]$ represents the positive, neutral, and negative membership functions respectively.

Definition 2.3. The spherical fuzzy set is the generalization of the picture fuzzy set proposed by Mehmood et al. [17] in 2019. Mathematically it is defined as:

$$A = \{\langle x, {\mu^{+}}_{A}(x), {\mu'}_{A}(x), {\mu^{-}}_{A}(x)\rangle : x \in X\} \tag{3}$$

with the condition,

$$0 \leq \left({\mu^{+}}_{A}(x)\right)^{2} + \left({\mu'}_{A}(x)\right)^{2} + \left({\mu^{-}}_{A}(x)\right)^{2} \leq 1.$$

Where ${\mu^{+}}_{A}(x): X \to [0,1], {\mu'}_{A}(x): X \to [0,1]$ and ${\mu^{-}}_{A}(x): X \to [0,1]$ represents the positive, neutral, and negative membership functions respectively.

Definition 2.4. A q-Spherical fuzzy set is the generalization of the spherical fuzzy set proposed by Kahraman et al. [21] in 2020. Mathematically it is defined as:

Let X be a non-empty set. A q-spherical fuzzy set A_{q-SFS} is of the form

$$A_{q-SFS} = \left\{\langle x, {\mu^{+}}_{A_{q-SFS}}(x), {\mu'}_{A_{q-SFS}}(x),\ {\mu^{-}}_{A_{q-SFS}}(x)\rangle : x \in X\right\} \tag{4}$$

with the condition that

$$\left(0 \leq \left({\mu^{+}}_{A_{q-SFS}}(x)\right)^{q} + \left({\mu'}_{A_{q-SFS}}(x)\right)^{q} + \left(\ {\mu^{-}}_{A_{q-SFS}}(x)\right)^{q} \leq 1, \mathrm{q} \geq 1\right).$$

Where ${\mu^{+}}_{A_{q-SFS}}(x): X \to [0,1]$, ${\mu'}_{A_{q-SFS}}(x): X \to [0,1]$ and ${\mu^{-}}_{A_{q-SFS}}(x): X \to [0,1]$ is the positive, neutral, and negative membership functions respectively.

Definition 2.5. Pawlak [23] was the first to introduce the concept of a rough set in 1982, which is defined as:

Let R be an arbitrary binary relation on $U_1 \times U_2$, then the triplet (U_1, U_2, R) is called approximation space. For any, $X \subseteq U_1$ and $A \subseteq U_2$ the lower approximation $\underline{R}(A)$ and the upper approximation $\overline{R}(A)$ are defined as

$$\begin{pmatrix} \underline{R}(A) = \{x \in U_1 : [x]_A \subseteq X\} \\ \overline{R}(A) = \{x \in U_1 : [x]_A \cap X \neq \phi\} \end{pmatrix}. \tag{5}$$

Where $[x]_A$ denote indiscernibility.

Then the pair $\left(\underline{R}(A), \overline{R}(A)\right)$ is called a rough set.

Definition 2.6. Azim et al. [22] 2023 introduce the concept of a q-spherical fuzzy rough set, which is defined as

Let A$\subseteq U_2$, then the lower and upper approximation of A for (U_1, U_2, R) is defined by

$$A_{q-SFRS} = (\underline{A}_{q-SFRS}, \bar{A}_{q-SFRS} =$$
$$\left\{s, \langle \mu^{+}{}_{\underline{A}_{q-SFRS}}(s), \mu'{}_{\underline{A}_{q-SFRS}}(s),\ \mu^{-}{}_{\underline{A}_{q-SFRS}}(s), \mu^{+}{}_{\bar{A}_{q-SFRS}}(s), \mu'{}_{\bar{A}_{q-SFRS}}(s),\ \mu^{-}{}_{\bar{A}_{q-SFRS}}(s)\rangle : s \in U_1\right\}, \tag{6}$$

where,

$$\mu^{+}{}_{\underline{A}_{q-SFRS}}(s) = \wedge_{s\in U_2}\{\mu^{+}{}_{R}(r,s)\wedge\mu^{+}{}_{\boldsymbol{A}}(s)\}$$
$$\mu'{}_{\underline{A}_{q-SFRS}}(s) = \vee_{s\in U_2}\{\mu'{}_{R}(r,s)\vee\mu'{}_{\boldsymbol{A}}(s)\}$$
$$\mu^{-}{}_{\underline{A}_{q-SFRS}}(s) = \vee_{s\in U_2}\{\ \mu^{-}{}_{R}(r,s)\vee\ \mu^{-}{}_{\boldsymbol{A}}(s)\}$$
$$\mu^{+}{}_{\bar{A}_{q-SFRS}}(s) = \vee_{s\in U_2}\{\mu^{+}{}_{R}(r,s)\vee\mu^{+}{}_{\boldsymbol{A}}(s)\}$$
$$\mu'{}_{\bar{A}_{q-SFRS}}(s) = \wedge_{s\in U_2}\{\mu'{}_{R}(r,s)\wedge\mu'{}_{\boldsymbol{A}}(s)\}$$
$$\mu^{-}{}_{\bar{A}_{q-SFRS}}(s) = \wedge_{s\in U_2}\{\ \mu^{-}{}_{R}(r,s)\wedge\ \mu^{-}{}_{\boldsymbol{A}}(s)\},$$

with the condition that

$$\left\{\begin{matrix}\left(0 \le \left(\mu^{+}{}_{\underline{A}_{q-SFRS}}(s)\right)^{q} + \left(\mu'{}_{\underline{A}_{q-SFRS}}(s)\right)^{q} + \left(\ \mu^{-}{}_{\underline{A}_{q-SFRS}}(s)\right)^{q} \le 1\right), \\ \left(0 \le \left(\mu^{+}{}_{\bar{A}_{q-SFRS}}(s)\right)^{q} + \left(\mu'{}_{\bar{A}_{q-SFRS}}(s)\right)^{q} + \left(\ \mu^{-}{}_{\bar{A}_{q-SFRS}}(s)\right)^{q} \le 1\right)\end{matrix}\right\}.$$

The pair of q-spherical fuzzy sets is then said to represent a q-spherical fuzzy rough set (q-SFRS) if $\underline{A}_{q-SFRS} \neq \bar{A}_{q-SFRS}$. For simplicity, we write $A = (\underline{A}_{q-SFRS}, \bar{A}_{q-SFRS})$ and the expression $A = (\underline{A}_{q-SFRS}, \bar{A}_{q-SFRS})$ is called a q-spherical fuzzy rough number. $A_{q-SFRSi}$ denotes the collection of all q-SFRNs.

Example 2.1. Let's say that a person in charge of making decisions, Z, purchases a home, as shown in the set $U_1 = \{\ h_1, h_2, h_3, h_4, h_5\}$ that are now being considered.

Let U_2 = {beautiful (a_1), size (a_2), expensive (a_3) *and* location (a_4)} is the set of alternatives. A person who makes decisions Z would want to buy a home from among those that are currently on the market and which, to the greatest degree possible, satisfy the requirements that have been outlined.

Consider the fact that decision-maker Z displays the attractiveness of the homes in the form of a q-SFR relation, which can be found in Table 1 below.

Table 1. q-Spherical fuzzy rough relation.

R	a_1	a_2	a_3	a_4
h_1	(0.7,0.2,0.4)	(0.5,0.2,0.7)	(0.6,0.5,0.3)	(0.7,0.2,0.5)
h_2	(0.6,0.3,0.1)	(0.3,0.2,0.5)	(0.3,0.2,0.4)	(0.5,0.2,0.5)
h_3	(0.3,0.1,0.2)	(0.7,0.4,0.3)	(0.2,0.3,0.7)	(0.3,0.4,0.1)
h_4	(0.4,0.5,0.2)	(0.8,0.4,0.5)	(0.5,0.3,0.4)	(0.4,0.3,0.2)
h_5	(0.6,0.1,0.5)	(0.4,0.3,0.5)	(0.9,0.3,0.5)	(0.3,0.2,0.1)

Consider a decision-maker Z that provides the optimal normal decision object A, which is a q-SFS subset over the attribute set U_2; that is to say, A= {(a1/ (0.8,0.1,0.3)), (a2/ (0.8,0.4,0.3)), (a3/ (0.5,0.1,0.4)), (a4/ (0.5,0.4,0.6))}.

In this case, we use Definition 3.4 to determine the lower and upper approximations of A approximations regarding (U1, U2, R, A).

$\underline{A}_{q-SFRS}$={(h1/(0.5,0.5,0.7)),(h2/(0.3,0.4,0.6)),(h3/(0.2,0.4,0.7)),(h4/(0.4,0.5,0.6)),(h5/(0.3,0.4,0.6))}.

$\bar{A}_{q-SFRS}$={(h1/(0.8,0.1,0.3)),(h2/(0.8,0.1,0.1)),(h3/(0.8,0.1,0.1)),(h4/(0.8,0.1,0.2)),(h5/(0.9,0.1,0.1))}.

Hence A_{q-SFRS} = {(h1/(0.5,0.5,0.7),(0.8,0.1,0.3)),(h2/(0.3,0.4,0.6),(0.8,0.1,0.1)),(h3/(0.2,0.4,0.7),(0.8,0.1,0.1)),(h4/(0.4,0.5,0.6),(0.8,0.1,0.2)),(h5/(0.3,0.4,0.6),(0.9,0.1,0.1))}.

3. Operations of q-spherical fuzzy rough sets

This section examines the basic operational laws and aggregation operators of q-spherical fuzzy rough numbers.

Definition 3.1. Basic operators

(i) Intersection: Let A_{q-SFRS} and B_{q-SFRS} are two q-SFRNs in (U1, U2, R), then

$A_{q-SFRS} \cap B_{q-SFRS} = \underline{A}_{q-SFRS} \cap \underline{B}_{q-SFRS}, \bar{A}_{q-SFRS} \cap \bar{B}_{q-SFRS}$ where,

$$\underline{A}_{q-SFRS} \cap \underline{B}_{q-SFRS} = \left\langle \min\left\{\mu^{+}_{\underline{A}_{q-SFSR}}, \mu^{+}_{\underline{B}_{q-SFRS}}\right\}, \max\left\{\mu'_{\underline{A}_{q-SFRS}}, \mu'_{\underline{B}_{q-SFRS}}\right\}, \min\left\{1 - \left(\left(\min\left\{\mu^{+}_{\underline{A}_{q-SFSR}}, \mu^{+}_{\underline{B}_{q-SFRS}}\right\}\right)^{q} + \left(\max\left\{\mu'_{\underline{A}_{q-SFSR}}, \mu'_{\underline{B}_{q-SFRS}}\right\}\right)^{q}, \min\left\{\mu^{-}_{\underline{A}_{q-SFSR}}, \mu^{-}_{\underline{B}_{q-SFRS}}\right\}\right)\right\}\right\rangle.$$

$$\bar{A}_{q-SFRS} \cap \bar{B}_{q-SFRS} = \left\langle \min\left\{\mu^{+}_{\bar{A}_{q-SFRS}}, \mu^{+}_{\bar{B}_{q-SFRS}}\right\}, \max\left\{\mu'_{\bar{A}_{q-SFRS}}, \mu'_{\bar{B}_{q-SFRS}}\right\}, \min\left\{1 - \left(\left(\min\left\{\mu^{+}_{\bar{A}_{q-SFRS}}, \mu^{+}_{\bar{B}_{q-SFRS}}\right\}\right)^{q} + \left(\max\left\{\mu'_{\bar{A}_{q-SFRS}}, \mu'_{\bar{B}_{q-SFRS}}\right\}\right)^{q}, \min\left\{\mu^{-}_{\bar{A}_{q-SFRS}}, \mu^{-}_{\bar{B}_{q-SFRS}}\right\}\right)\right\}\right\rangle.$$

(ii) Union: Let A_{q-SFRS} and B_{q-SFRS} are two q-SFRNs in (U1, U2, R), then

$A_{q-SFRS} \cup B_{q-SFRS} = \underline{A}_{q-SFRS} \cup \underline{B}_{q-SFRS}, \bar{A}_{q-SFRS} \cup \bar{B}_{q-SFRS}$ where,

$$\underline{A}_{q-SFRS} \cup \underline{B}_{q-SFSR} = \left\langle \max\left\{\mu^{+}_{\underline{A}_{q-SFRS}}, \mu^{+}_{\underline{B}_{q-SFRS}}\right\}, \min\left\{\mu'_{\underline{A}_{q-SFRS}}, \mu'_{\underline{B}_{q-SFRS}}\right\}, \max\left\{1 - \left(\left(\max\left\{\mu^{+}_{\underline{A}_{q-SFSR}}, \mu^{+}_{\underline{B}_{q-SFRS}}\right\}\right)^{q} + \left(\min\left\{\mu'_{\underline{A}_{q-SFSR}}, \mu'_{\underline{B}_{q-SFRS}}\right\}\right)^{q}, \max\left\{\mu^{-}_{\underline{A}_{q-SFSR}}, \mu^{-}_{\underline{B}_{q-SFRS}}\right\}\right)\right\}\right\rangle.$$

$$\bar{A}_{q-SFRS} \cup \bar{B}_{q-SFRS} = \left\langle \max\left\{\mu^{+}_{\bar{A}_{q-SFRS}}, \mu^{+}_{\bar{B}_{q-SFRS}}\right\}, \min\left\{\mu'_{\bar{A}_{q-SFRS}}, \mu'_{\bar{B}_{q-SFRS}}\right\}, \max\left\{1 - \left(\left(\max\left\{\mu^{+}_{\bar{A}_{q-SFRS}}, \mu^{+}_{\bar{B}_{q-SFRS}}\right\}\right)^{q} + \left(\min\left\{\mu'_{\bar{A}_{q-SFRS}}, \mu'_{\bar{B}_{q-SFRS}}\right\}\right)^{q}, \max\left\{\mu^{-}_{\bar{A}_{q-SFRS}}, \mu^{-}_{\bar{B}_{q-SFRS}}\right\}\right)\right\}\right\rangle.$$

(iii) Addition: Let A_{q-SFS} and B_{q-SFS} are two q-SFRNs in (U1, U2, R), then

$A_{q-SFRS} \oplus B_{q-SFRS} = \underline{A}_{q-SFRS} \oplus \underline{B}_{q-SFRS}, \bar{A}_{q-SFRS} \oplus \bar{B}_{q-SFRS}$ where,

$$\underline{A}_{q-SFRS} \oplus \underline{B}_{q-SFRS} = \Bigg\langle \left(\left(\mu^{+}_{\underline{A}_{q-SFRS}}\right)^{q} + \left(\mu^{+}_{\underline{B}_{q-SFRS}}\right)^{q} - \left(\mu^{+}_{\underline{A}_{q-SFRS}}\right)^{q}\left(\mu^{+}_{\underline{B}_{q-SFRS}}\right)^{q}\right)^{\frac{1}{q}}, \left(\mu'_{\underline{A}_{q-SFRS}}\right)^{q}\left(\mu'_{\underline{B}_{q-SFRS}}\right)^{q}, \Bigg(\Bigg(\left(1 - \left(\mu^{+}_{\underline{B}_{q-SFRS}}\right)^{q}\right)\left(\mu^{-}_{\underline{A}_{q-SFRS}}\right)^{q} +$$

$$\left(1-\left(\mu^{+}{}_{\underline{A}_{q-SFRS}}\right)^{q}\right)\left(\mu^{-}{}_{\underline{B}_{q-SFRS}}\right)^{q}\right)-\left(\mu^{-}{}_{\underline{A}_{q-SFRS}}\right)^{q}\left(\mu^{-}{}_{\underline{B}_{q-SFRS}}\right)^{q}\right)^{\frac{1}{q}}\Big\rangle.$$

$$\bar{A}_{q-SFRS}\oplus\bar{B}_{q-SFRS}=\Big\langle\left(\left(\mu^{+}{}_{\bar{A}_{q-SFRS}}\right)^{q}+\left(\mu^{+}{}_{\bar{B}_{q-SFRS}}\right)^{q}-\left(\mu^{+}{}_{\bar{A}_{q-SFRS}}\right)^{q}\left(\mu^{+}{}_{\bar{B}_{q-SFRS}}\right)^{q}\right)^{\frac{1}{q}},\left(\mu'{}_{\bar{A}_{q-SFRS}}\right)^{q}\left(\mu'{}_{\bar{B}_{q-SFRS}}\right)^{q},\left(\left(\left(1-\left(\mu^{+}{}_{\bar{B}_{q-SFRS}}\right)^{q}\right)\left(\mu^{-}{}_{\bar{A}_{q-SFRS}}\right)^{q}+\left(1-\left(\mu^{+}{}_{\bar{A}_{q-SFRS}}\right)^{q}\right)\left(\mu^{-}{}_{\bar{B}_{q-SFRS}}\right)^{q}\right)-\left(\mu^{-}{}_{\bar{A}_{q-SFRS}}\right)^{q}\left(\mu^{-}{}_{\bar{B}_{q-SFRS}}\right)^{q}\right)^{\frac{1}{q}}\Big\rangle.$$

(iv) Multiplication: Let A_{q-SFRS} and B_{q-SFRS} are two q-SFRNs in (U1, U2, R), then $A_{q-SFRS}\otimes B_{q-SFRS}=\underline{A}_{q-SFRS}\otimes\underline{B}_{q-SFRS},\bar{A}_{q-SFRS}\otimes\bar{B}_{q-SFRS}$, where,

$$\underline{A}_{q-SFRS}\otimes\underline{B}_{q-SFRS}=\Big\langle\left(\mu^{+}{}_{\underline{A}_{q-SFRS}}\right)^{q}\left(\mu^{+}{}_{\underline{B}_{q-SFRS}}\right)^{q},\left(\left(\mu'{}_{\underline{A}_{q-SFRS}}\right)^{q}+\left(\mu'{}_{\underline{B}_{q-SFRS}}\right)^{q}-\left(\mu'{}_{\underline{A}_{q-SFRS}}\right)^{q}\left(\mu'{}_{\underline{B}_{q-SFRS}}\right)^{q}\right)^{\frac{1}{q}},\left(\left(\left(1-\left(\mu'{}_{\underline{B}_{q-SFRS}}\right)^{q}\right)\left(\mu^{-}{}_{\underline{A}_{q-SFRS}}\right)^{q}+\left(1-\left(\mu'{}_{\underline{A}_{q-SFRS}}\right)^{q}\right)\left(\mu^{-}{}_{\underline{B}_{q-SFRS}}\right)^{q}\right)-\left(\mu^{-}{}_{\underline{A}_{q-SFRS}}\right)^{q}\left(\mu^{-}{}_{\underline{B}_{q-SFRS}}\right)^{q}\right)^{\frac{1}{q}}\Big\rangle.$$

(v) Multiplication by a scaler: Let A_{q-SFRS} be a q-SFRNs in (U1, U2, R) and λ >0 then $\lambda A_{q-SFRS}=\lambda\underline{A}_{q-SFRS},\lambda\bar{A}_{q-SFRS}$ where,

$$\lambda\underline{A}_{q-SFRS}=\Big\langle\left(1-\left(1-\left(\mu^{+}{}_{\underline{A}_{q-SFRS}}\right)^{q}\right)^{\lambda}\right)^{\frac{1}{q}},\left(\mu'{}_{\underline{A}_{q-SFRS}}\right)^{\lambda},\left[\left(1-\left(\mu^{+}{}_{\underline{A}_{q-SFRS}}\right)^{q}\right)^{\lambda}-\left(1-\left(\mu^{+}{}_{\underline{A}_{q-SFRS}}\right)^{q}-\left(\mu^{-}{}_{\underline{A}_{q-SFRS}}\right)^{q}\right)^{\lambda}\right]^{\frac{1}{q}}\Big\rangle.$$

$$\lambda\bar{A}_{q-SFRS}=\Big\langle\left(1-\left(1-\left(\mu^{+}{}_{\bar{A}_{q-SFRS}}\right)^{q}\right)^{\lambda}\right)^{\frac{1}{q}},\left(\mu'{}_{\bar{A}_{q-SFRS}}\right)^{\lambda},\left[\left(1-\left(\mu^{+}{}_{\bar{A}_{q-SFRS}}\right)^{q}\right)^{\lambda}-\left(1-\left(\mu^{+}{}_{\bar{A}_{q-SFRS}}\right)^{q}-\left(\mu^{-}{}_{\bar{A}_{q-SFRS}}\right)^{q}\right)^{\lambda}\right]^{\frac{1}{q}}\Big\rangle.$$

(vi) λ Power of $\bar{A}_{q-SFS}$: Let A_{q-SFRS} be a q-SFRNs in (U1, U2, R) and λ >0 then

$$A_{q-SFRS}{}^{\lambda}=\underline{A}_{q-SFRS}{}^{\lambda},\bar{A}_{q-SFRS}{}^{\lambda}$$ where,

$$\underline{A}_{q-SFRS}{}^{\lambda}=\left\langle\left(\mu^{+}{}_{\underline{A}_{q-SFRS}}\right)^{\lambda},\left(1-\left(1-\left(\mu'{}_{\underline{A}_{q-SFRS}}\right)^{q}\right)^{\lambda}\right)^{\frac{1}{q}},\left[\left(1-\left(\mu'{}_{\underline{A}_{q-SFRS}}\right)^{q}\right)^{\lambda}-\left(1-\left(\mu'{}_{\underline{A}_{q-SFRS}}\right)^{q}-\left(\mu^{-}{}_{\underline{A}_{q-SFRS}}\right)^{q}\right)^{\lambda}\right]^{\frac{1}{q}}\right\rangle.$$

$$\bar{A}_{q-SFRS}{}^{\lambda}=\left\langle\left(\mu^{+}{}_{\bar{A}_{q-SFRS}}\right)^{\lambda},\left(1-\left(1-\left(\mu'{}_{\bar{A}_{q-SFRS}}\right)^{q}\right)^{\lambda}\right)^{\frac{1}{q}},\left[\left(1-\left(\mu'{}_{\bar{A}_{q-SFRS}}\right)^{q}\right)^{\lambda}-\left(1-\left(\mu'{}_{\bar{A}_{q-SFRS}}\right)^{q}-\left(\mu^{-}{}_{\bar{A}_{q-SFRS}}\right)^{q}\right)^{\lambda}\right]^{\frac{1}{q}}\right\rangle.$$

Definition 3.2. For two q-SFRNs $A_{q-SFRS}=\langle\mu^{+}{}_{\underline{A}_{q-SFRS}},\mu'{}_{\underline{A}_{q-SFRS}},\ \mu^{-}{}_{\underline{A}_{q-SFRS}},\mu^{+}{}_{\bar{A}_{q-SFRS}},\mu'{}_{\bar{A}_{q-SFRS}},\ \mu^{-}{}_{\bar{A}_{q-SFRS}}\rangle$ and $B_{q-SFRS}=\langle\mu^{+}{}_{\underline{B}_{q-SFRS}},\mu'{}_{\underline{B}_{q-SFRS}},\ \mu^{-}{}_{\underline{B}_{q-SFRS}},\mu^{+}{}_{\bar{B}_{q-SFRS}},\mu'{}_{\bar{B}_{q-SFRS}},\ \mu^{-}{}_{\bar{B}_{q-SFRS}}\rangle$, then the following are valid under the condition $\lambda,\lambda_1,\lambda_2>0$.

$(i)\ A_{q-SFRS}\oplus B_{q-SFRS}=B_{q-SFRS}\oplus\ A_{q-SFRS}$

$(ii)\ A_{q-SFRS}\otimes B_{q-SFRS}=B_{q-SFRS}\otimes A_{q-SFRS}$

$(iii)\ \lambda(A_{q-SFRS}\oplus B_{q-SFRS})=\lambda A_{q-SFRS}\oplus\ \lambda B_{q-SFRS}$

$(iv)\ \lambda_1(A_{q-SFRS})\oplus\lambda_2(A_{q-SFRS})=(\lambda_1\oplus\lambda_2)A_{q-SFRS}$

$(v)\ (A_{q-SFRS}\otimes B_{q-SFRS})^{\lambda}=A_{q-SFRS}{}^{\lambda}\otimes B_{q-SFRS}{}^{\lambda}$

$(vi)\ A_{q-SFRS}{}^{\lambda_1}\otimes A_{q-SFRS}{}^{\lambda_2}=A_{q-SFRS}{}^{\lambda_1+\lambda_2}$.

Definition 3.3. q-Spherical Fuzzy Rough Arithmetic Mean (q-SFRAM) concerning, $w=(w_1,w_2,w_3,\dots,w_n)$; $w_i\in[0,1]$; $\sum_{i=1}^{n}w_i=1$, q-SFRAM is defined as

$$\text{q}-\text{SFRAM}_{w}\left(A_{q-SFRS1},A_{q-SFRS2},A_{q-SFRS3},\dots,A_{q-SFRSn}\right)$$

$$=w_1A_{q-SFRS1}\oplus w_2A_{q-SFRS2}\oplus w_3A_{q-SFRS3}\oplus,\dots,\oplus\ w_nA_{q-SFRSn}$$

$$=\left\langle\left[1-\prod_{i=1}^{n}\left(1-\left(\mu^{+}{}_{\underline{A}_{q-SFRSi}}\right)^{q}\right)^{w_i}\right]^{\frac{1}{q}},\prod_{i=1}^{n}\left(\mu'{}_{\underline{A}_{q-SFRSi}}\right)^{w_i},\left[\prod_{i=1}^{n}\left(1-\left(\mu^{+}{}_{\underline{A}_{q-SFRSi}}\right)^{q}\right)^{w_i}-\prod_{i=1}^{n}\left(1-\left(\mu^{+}{}_{\underline{A}_{q-SFRSi}}\right)^{q}-\left(\mu^{-}{}_{\underline{A}_{q-SFRSi}}\right)^{q}\right)^{w_i}\right]^{\frac{1}{q}},\left[1-\prod_{i=1}^{n}\left(1-\left(\mu^{+}{}_{\bar{A}_{q-SFRSi}}\right)^{q}\right)^{w_i}\right]^{\frac{1}{q}},\prod_{i=1}^{n}\left(\mu'{}_{\bar{A}_{q-SFRSi}}\right)^{w_i},\left[\prod_{i=1}^{n}\left(1-\left(\mu^{+}{}_{\bar{A}_{q-SFRSi}}\right)^{q}\right)^{w_i}-\prod_{i=1}^{n}\left(1-\left(\mu^{+}{}_{\bar{A}_{q-SFRSi}}\right)^{q}-\left(\mu^{-}{}_{\bar{A}_{q-SFRSi}}\right)^{q}\right)^{w_i}\right]^{\frac{1}{q}}\right\rangle.$$

Definition 3.4. q-Spherical Fuzzy Rough Geometric Mean (q-SFRGM) for $w = (w_1, w_2, w_3, \dots, w_n)$; $w_i \in [0,1]$; $\sum_{i=1}^{n} w_i = 1$, q-SFRGM is defined as

$$\mathrm{q-SFRGM}\left(A_{q-SFRS1}, A_{q-SFRS2}, A_{q-SFRS3}, \dots, A_{q-SFRSn}\right)$$
$$= w_1 A_{q-SFRS1} \otimes w_2 A_{q-SFRS2} \otimes w_3 A_{q-SFRS3} \otimes, \dots, \otimes w_n A_{q-SFRSn}$$
$$= \left\langle \prod_{i=1}^{n} \left(\mu^{+}{}_{\underline{A}_{q-SFRSi}}\right)^{w_i}, \left[1 - \prod_{i=1}^{n} \left(1 - \left(\mu'{}_{\underline{A}_{q-SFRSi}}\right)^{q}\right)^{w_i}\right]^{\frac{1}{q}}, \left[\prod_{i=1}^{n} \left(1 - \left(\mu'{}_{\underline{A}_{q-SFRSi}}\right)^{q}\right)^{w_i} - \prod_{i=1}^{n} \left(1 - \left(\mu'{}_{\underline{A}_{q-SFRSi}}\right)^{q} - \left(\mu^{-}{}_{\underline{A}_{q-SFRSi}}\right)^{q}\right)^{w_i}\right]^{\frac{1}{q}}, \prod_{l=1}^{n} \left(\mu^{+}{}_{\bar{A}_{q-SFRSi}}\right)^{w_i}, \left[1 - \prod_{i=1}^{n} \left(1 - \left(\mu'{}_{\bar{A}_{q-SFRSi}}\right)^{q}\right)^{w_i}\right]^{\frac{1}{q}}, \left[\prod_{i=1}^{n} \left(1 - \left(\mu'{}_{\bar{A}_{q-SFRSi}}\right)^{q}\right)^{w_i} - \prod_{i=1}^{n} \left(1 - \left(\mu'{}_{\bar{A}_{q-SFRSi}}\right)^{q} - \left(\mu^{-}{}_{\bar{A}_{q-SFRSi}}\right)^{q}\right)^{w_i}\right]^{\frac{1}{q}} \right\rangle.$$

Definition 3.5. The score function of sorting q-SFRNs is defined by

$$Score\left(A_{q-SFRS}\right) = \frac{1}{3}\left(2 + \left(\mu^{+}{}_{\underline{A}_{q-SFRS}}\right)^{q} + \left(\mu^{+}{}_{\bar{A}_{q-SFRS}}\right)^{q} - \left(\mu'{}_{\underline{A}_{q-SFRS}}\right)^{q} - \left(\mu'{}_{\bar{A}_{q-SFRS}}\right)^{q} - \left(\mu^{-}{}_{\underline{A}_{q-SFRS}}\right)^{q} - \left(\mu^{-}{}_{\bar{A}_{q-SFRS}}\right)^{q}\right), Score\left(A_{q-SFRS}\right) \in [-1,1], q \geq 1.$$

Note that if $A_{q-SFRS} < B_{q-SFRS}$ if and only if

$$Score\left(A_{q-SFRS}\right) < Score\left(B_{q-SFRS}\right) \text{ or}$$

$$Score\left(A_{q-SFRS}\right) = Score\left(B_{q-SFRS}\right) \text{ and } Accuracy\left(A_{q-SFRS}\right) < Accuracy\left(B_{q-SFRS}\right).$$

4. q-Spherical fuzzy analytic hierarch process

Step 1: Constructing the hierarchical structure is the first step. At this stage of the process, a hierarchical structure with at least 3 stages will be constructed.
Step 2: Construct pairwise comparisons by making use of q-spherical fuzzy rough judgments matrices and basing them on the linguistic terms presented in Table 2.

Table 2. Linguistic terms and their corresponding q-spherical fuzzy rough numbers.

	$(\underline{\mu}, \underline{\nu}, \underline{\pi})$	$(\overline{\mu}, \overline{\nu}, \overline{\pi})$	Score Index (SI)
Absolutely more Importance (AMI)	(0.95, 0.55, 0.55)	(0.91, 0.51, 0.51)	9
Very High Importance (VHI)	(0.90, 0.60, 0.60)	(0.86, 0.56, 0.56)	7
High Importance (HI)	(0.85, 0.65, 0.65)	(0.81, 0.61, 0.61)	5
Slightly More Importance (SMI)	(0.80, 0.70, 0.70)	(0.76, 0.66, 0.66)	3
Equally Importance (EI)	(0.75, 0.75, 0.75)	(0.71, 0.71, 0.71)	1
Slightly Low Importance (SLI)	(0.70, 0.80, 0.70)	(0.66, 0.76, 0.66)	1/3
Low Importance (LI)	(0.65, 0.85, 0.65)	(0.61, 0.81, 0.61)	1/5
Very Low Importance (VLI)	(0.60, 0.90, 0.60)	(0.56, 0.86, 0.56)	1/7
Absolutely Low Importance	(0.55, 0.95, 0.55)	(0.51, 0.91, 0.51)	1/9

Step 3: Do a check to ensure that each pairwise comparison matrix (J) is consistent. Converting the linguistic terms in the pairwise comparison matrix into their respective score indices is the first step in accomplishing this goal. After that, apply the classical consistency check.

Step 4: Determine the q-spherical fuzzy rough local weights of the alternatives and the criteria. Find out how much each of the possibilities weighs. Calculate the weight of each alternative regarding each criterion by using the q-spherical fuzzy rough averaging mean operator. For calculating the q-spherical fuzzy rough weights, the weighted arithmetic mean is the preferred approach.

$$\mathrm{q-SFRAM}_w\left(A_{q-SFRS1}, A_{q-SFRS2}, A_{q-SFRS3}, \dots, A_{q-SFRSn}\right) = w_1 A_{q-SFRS1} \oplus w_2 A_{q-SFRS2} \oplus w_3 A_{q-SFRS3} \oplus, \dots, \oplus w_n A_{q-SFRSn}$$

$$= \left\langle \left[1 - \prod_{i=1}^{n}\left(1 - \left(\mu^{+}{}_{\underline{A}_{q-SFRSi}}\right)^{q}\right)^{w_i}\right]^{\frac{1}{q}}, \prod_{i=1}^{n}\left(\mu'{}_{\underline{A}_{q-SFRSi}}\right)^{w_i}, \left[\prod_{i=1}^{n}\left(1 - \left(\mu^{+}{}_{\underline{A}_{q-SFRSi}}\right)^{q}\right)^{w_i} - \prod_{i=1}^{n}\left(1 - \left(\mu^{+}{}_{\underline{A}_{q-SFRSi}}\right)^{q} - \left(\mu^{-}{}_{\underline{A}_{q-SFRSi}}\right)^{q}\right)^{w_i}\right]^{\frac{1}{q}}, \left[1 - \prod_{i=1}^{n}\left(1 - \left(\mu^{+}{}_{\bar{A}_{q-SFRSi}}\right)^{q}\right)^{w_i}\right]^{\frac{1}{q}}, \prod_{i=1}^{n}\left(\mu'{}_{\bar{A}_{q-SFRSi}}\right)^{w_i}, \left[\prod_{i=1}^{n}\left(1 - \left(\mu^{+}{}_{\bar{A}_{q-SFRSi}}\right)^{q}\right)^{w_i} - \prod_{i=1}^{n}\left(1 - \left(\mu^{+}{}_{\bar{A}_{q-SFRSi}}\right)^{q} - \left(\mu^{-}{}_{\bar{A}_{q-SFRSi}}\right)^{q}\right)^{w_i}\right]^{\frac{1}{q}} \right\rangle.$$

Where $w = \frac{1}{n}$.

Step 5: To obtain the global weights, first establish the hierarchy layer sequencing. While trying to estimate the final ranking orders for the alternatives, it is necessary to integrate the spherical fuzzy weights at each level. This calculation is performed in reverse order, beginning with the lowest level (alternatives) and working its way up to the top level (goal).

At this point, 2 separate action options could be taken. The first one is to defuzzify the criterion weights by making use of the score function $S(w_j)$ given in Eq (7) and normalize them by making use of Eq (8), and then use q-spherical fuzzy rough multiplication in Eqs (9) and (10).

$$S(w_j) = \frac{1}{3}\left(2 + \underline{\mu}^q + \overline{\mu}^q - \underline{\nu}^q - \overline{\nu}^q - \underline{\pi}^q - \overline{\pi}^q\right), q \geq 1. \tag{7}$$

Normalize the criteria weights by using Eq (8)

$$\widetilde{\boldsymbol{w}}_{\boldsymbol{j}} = \frac{S(w_j)}{\sum_{j=1}^{n} S(w_j)} \tag{8}$$

$$\underline{A}_{S_{ij}} = \widetilde{\boldsymbol{w}}_{\boldsymbol{j}} \underline{A}_{S_i} = \left\langle \left(1 - \left(1 - \left(\mu^{+}{}_{\underline{A}_{q-SFRS}}\right)^{q}\right)^{\widetilde{\boldsymbol{w}}_{\boldsymbol{j}}}\right)^{\frac{1}{q}}, \left(\mu'{}_{\underline{A}_{q-SFRS}}\right)^{\widetilde{\boldsymbol{w}}_{\boldsymbol{j}}}, \left[\left(1 - \left(\mu^{+}{}_{\underline{A}_{q-SFRS}}\right)^{q}\right)^{\widetilde{\boldsymbol{w}}_{\boldsymbol{j}}} - \left(1 - \left(\mu^{+}{}_{\underline{A}_{q-SFRS}}\right)^{q} - \left(\mu^{-}{}_{\underline{A}_{q-SFRS}}\right)^{q}\right)^{\widetilde{\boldsymbol{w}}_{\boldsymbol{j}}}\right]^{\frac{1}{q}} \right\rangle. \tag{9}$$

$$\bar{A}_{S_{ij}} = \widetilde{\boldsymbol{w}}_{\boldsymbol{j}}\bar{A}_{S_i} = \left\langle \left(1-\left(1-\left({\mu^+}_{\bar{A}_{q-SFRS}}\right)^q\right)^{\widetilde{w}_j}\right)^{\frac{1}{q}}, \left({\mu'}_{\bar{A}_{q-SFRS}}\right)^{\widetilde{w}_j}, \left[\left(1-\left({\mu^+}_{\bar{A}_{q-SFRS}}\right)^q\right)^{\widetilde{w}_j} - \left(1-\left({\mu^+}_{\bar{A}_{q-SFRS}}\right)^q - \left({\mu^-}_{\bar{A}_{q-SFRS}}\right)^q\right)^{\widetilde{w}_j}\right]^{\frac{1}{q}} \right\rangle. \tag{10}$$

The final q-spherical fuzzy rough AHP score (F) for each alternative A_i can be derived by performing the q-spherical fuzzy arithmetic addition over each global preference weight as outlined in Eqs (11) and (12).

$$F = (\underline{F}, \overline{F}) = \left(\sum_{j=1}^{n} \underline{A}_{S_{ij}}, \sum_{j=1}^{n} \bar{A}_{S_{ij}}\right) = \left(\underline{A}_{S_{i1}} \oplus \underline{A}_{S_{i2}} \oplus \ldots \oplus \underline{A}_{S_{in}}, \bar{A}_{S_{i1}} \oplus \bar{A}_{S_{i2}} \oplus \ldots \oplus \bar{A}_{S_{in}} \forall\, i\right)$$

that is

$$\underline{A}_{S_{i1}} \oplus \underline{A}_{S_{i2}} = \left\langle \left(\left({\mu'}_{\underline{A}_{S_{i1}}}\right)^q + \left({\mu'}_{\underline{A}_{S_{i2}}}\right)^q - \left({\mu'}_{\underline{A}_{S_{i1}}}\right)^q \left({\mu'}_{\underline{A}_{S_{i2}}}\right)^q\right)^{\frac{1}{q}}, \left({\mu^+}_{\underline{A}_{S_{i1}}}\right)^q \left({\mu^+}_{\underline{A}_{S_{i2}}}\right)^q, \left(\left(\left(1-\left({\mu'}_{\underline{A}_{S_{i2}}}\right)^q\right)\left({\mu^-}_{\underline{A}_{S_{i1}}}\right)^q + \left(1-\left({\mu'}_{\underline{A}_{S_{i1}}}\right)^q\right)\left({\mu^-}_{\underline{A}_{S_{i2}}}\right)^q\right) - \left({\mu^-}_{\underline{A}_{S_{i1}}}\right)^q \left({\mu^-}_{\underline{A}_{S_{i2}}}\right)^q\right)^{\frac{1}{q}} \right\rangle. \tag{11}$$

$$\bar{A}_{S_{i1}} \oplus \bar{A}_{S_{i2}} = \left\langle \left(\left({\mu'}_{\bar{A}_{S_{i1}}}\right)^q + \left({\mu'}_{\bar{A}_{S_{i2}}}\right)^q - \left({\mu'}_{\bar{A}_{S_{i1}}}\right)^q \left({\mu'}_{\bar{A}_{S_{i2}}}\right)^q\right)^{\frac{1}{q}}, \left({\mu^+}_{\bar{A}_{S_{i1}}}\right)^q \left({\mu^+}_{\bar{A}_{S_{i2}}}\right)^q, \left(\left(\left(1-\left({\mu'}_{\bar{A}_{S_{i2}}}\right)^q\right)\left({\mu^-}_{\bar{A}_{S_{i1}}}\right)^q + \left(1-\left({\mu'}_{\bar{A}_{S_{i1}}}\right)^q\right)\left({\mu^-}_{\bar{A}_{S_{i2}}}\right)^q\right) - \left({\mu^-}_{\bar{A}_{S_{i1}}}\right)^q \left({\mu^-}_{\bar{A}_{S_{i2}}}\right)^q\right)^{\frac{1}{q}} \right\rangle. \tag{12}$$

The second method to proceed is to carry on without utilizing any form of defuzzification. In this case, the q-spherical fuzzy rough global preference weights are generated with the help of the Eqs (13) and (14).

$$\underline{A}_{S_{i1}} \otimes \underline{A}_{S_{i2}} = \left\langle \left({\mu^+}_{\underline{A}_{S_{i1}}}\right)^q \left({\mu^+}_{\underline{A}_{S_{i2}}}\right)^q, \left(\left({\mu'}_{\underline{A}_{S_{i1}}}\right)^q + \left({\mu'}_{\underline{A}_{S_{i2}}}\right)^q - \left({\mu'}_{\underline{A}_{S_{i1}}}\right)^q \left({\mu'}_{\underline{A}_{S_{i2}}}\right)^q\right)^{\frac{1}{q}}, \left(\left(\left(1-\left({\mu'}_{\underline{A}_{S_{i2}}}\right)^q\right)\left({\mu^-}_{\underline{A}_{S_{i1}}}\right)^q + \left(1-\left({\mu'}_{\underline{A}_{S_{i1}}}\right)^q\right)\left({\mu^-}_{\underline{A}_{S_{i2}}}\right)^q\right) - \left({\mu^-}_{\underline{A}_{S_{i1}}}\right)^q \left({\mu^-}_{\underline{A}_{S_{i2}}}\right)^q\right)^{\frac{1}{q}} \right\rangle. \tag{13}$$

$$\bar{A}_{S_{i1}} \otimes \bar{A}_{S_{i2}} = \left\langle \left({\mu^+}_{\bar{A}_{S_{i1}}}\right)^q \left({\mu^+}_{\bar{A}_{S_{i2}}}\right)^q, \left(\left({\mu'}_{\bar{A}_{S_{i1}}}\right)^q + \left({\mu'}_{\bar{A}_{S_{i2}}}\right)^q - \left({\mu'}_{\bar{A}_{S_{i1}}}\right)^q \left({\mu'}_{\bar{A}_{S_{i2}}}\right)^q\right)^{\frac{1}{q}}, \left(\left(\left(1-\left({\mu'}_{\bar{A}_{S_{i2}}}\right)^q\right)\left({\mu^-}_{\bar{A}_{S_{i1}}}\right)^q + \left(1-\left({\mu'}_{\bar{A}_{S_{i1}}}\right)^q\right)\left({\mu^-}_{\bar{A}_{S_{i2}}}\right)^q\right) - \left({\mu^-}_{\bar{A}_{S_{i1}}}\right)^q \left({\mu^-}_{\bar{A}_{S_{i2}}}\right)^q\right)^{\frac{1}{q}} \right\rangle. \tag{14}$$

The final score (F) is calculated by using Eqs (11) and (12).

Step 6: Defuzzify the final score for each workable alternative by utilizing the scoring function that is

provided in Eq (7).
Step 7: Rank the alternative in order according to the scores once they have been defuzzified. The alternative with the highest value is the best one.

5. Project prioritization using q spherical fuzzy rough AHP

The textile manufacturing company serves as the primary subject of the real-world case study. The business is working on a variety of initiatives that fall under the umbrella of Industry 4.0, and it must establish a hierarchy for these projects. The projects are described in the following way. A cloud-based (ERP) system gives users access to software programmers as well as shared computing resources including processor power, memory, and disc storage space. These resources can be accessed via the Internet. These computer resources are held in reserve in off-site data centers and can be used by a wide variety of applications running on a variety of platforms. It is possible to work from anywhere, at any time, with customers, to collaborate data, and to generate reports in real-time and without interruption if you use a cloud-based ERP system. With real-time manufacturing tracking, in addition to being linked to the identity and position of an object, information about the surrounding environment is also included. Manufacturing information needs to be gathered and analyzed, and choices need to be taken as a direct result of the analysis for the company to improve its productivity and the effectiveness of its machines. For businesses to make quick and reliable decisions, they want immediate access to production data. The collection of real-time data from the shop floor allows for the tracking of jobs and processes, which is essential for meeting delivery deadlines. The ability to adapt to varied working situations is a feature shared by robotic warehouse systems, advanced sensor structures, and virtual vision capabilities. At the storage facility, automated guided vehicles, which are a crucial component of these systems, can provide motion and position control using powerful algorithms with a high level of precision. After conducting interviews with various experts and doing a review of the relevant literature, a set of criteria is developed to prioritize the project.

The following are its criteria: Cost (C_1): It provides an estimate of the overall financial commitment required to set up the system. The level of risk (C_2): This indicates that connected factories may be susceptible to shutdowns or other forms of attack. (C_3): Even apart from the fact that the implementation of the Industry 4.0 paradigm in manufacturing has remarkable potential. Location (C_4): General area where the project work is performed. Impact on the benefit to the customer (C_5): As a result of the increased connection and use of analytical capabilities in all processes, the constructed business model will change towards being more data-centric, customer-focused, and practically applicable.

6. Real-world application

A company has it in its plans to invest in initiatives related to Industry 4.0. After identifying the four most important projects, the organization now concentrates on determining the order in which the projects will be completed. To accomplish this goal, the preceding section specified the four primary criteria that must be met. After completing the preceding steps of the q-spherical fuzzy rough AHP, the first step is to calculate the pairwise comparison matrix, which is given in Table 3.

Table 3. The linguistic evaluation of the criteria.

⧖	C_1	C_2	C_3	C_4	C_5
C_1	*SMI*	*HI*	*LI*	*SMI*	*EI*
C_2	*EI*	*HI*	*HI*	*HI*	*EI*
C_3	*HI*	*SMI*	*SMI*	*SMI*	*LI*
C_4	*SMI*	*VHI*	*LI*	*VHI*	*LI*
C_5	*VHI*	*VHI*	*HI*	*LI*	*SMI*

At later stages, the linguistic terms are converted into q-spherical fuzzy rough numbers, which are shown in Table 4.

Table 4. q-Spherical Fuzzy Rough Evaluations.

	C_1	C_2	C_3	C_4	C_5
C_1	(0.80, 0.70, 0.70), (0.76, 0.66, 0.66)	(0.85, 0.65, 0.65), (0.81, 0.61, 0.61)	(0.65, 0.85, 0.65), (0.61, 0.81, 0.61)	(0.80, 0.70, 0.70), (0.76, 0.66, 0.66)	(0.75, 0.75, 0.75), (0.71, 0.71, 0.71)
C_2	(0.75, 0.75, 0.75), (0.71, 0.71, 0.71)	(0.85, 0.65, 0.65), (0.81, 0.61, 0.61)	(0.85, 0.65, 0.65), (0.81, 0.61, 0.61)	(0.85, 0.65, 0.65), (0.81, 0.61, 0.61)	(0.75, 0.75, 0.75), (0.71, 0.71, 0.71)
C_3	(0.85, 0.65, 0.65), (0.81, 0.61, 0.61)	(0.80, 0.70, 0.70), (0.76, 0.66, 0.66)	(0.80, 0.70, 0.70), (0.76, 0.66, 0.66)	(0.80, 0.70, 0.70), (0.76, 0.66, 0.66)	(0.65, 0.85, 0.65), (0.61, 0.81, 0.61)
C_4	(0.80, 0.70, 0.70), (0.76, 0.66, 0.66)	(0.90, 0.60, 0.60), (0.86, 0.56, 0.56)	(0.65, 0.85, 0.65), (0.61, 0.81, 0.61)	(0.90, 0.60, 0.60), (0.86, 0.56, 0.56)	(0.65, 0.85, 0.65), (0.61, 0.81, 0.61)
C_5	(0.90, 0.60, 0.60), (0.86, 0.56, 0.56)	(0.90, 0.60, 0.60), (0.86, 0.56, 0.56)	(0.85, 0.65, 0.65), (0.81, 0.61, 0.61)	(0.65, 0.85, 0.65), (0.61, 0.81, 0.61)	(0.80, 0.70, 0.70), (0.76, 0.66, 0.66)

The q-spherical fuzzy rough arithmetic mean operator will be used in the next phase to do the calculation of the aggregated values. After that, the values that have been defuzzified are computed, and then, after that, they are normalized, and finally, the weights of the criterion are obtained, which is given in Table 5.

Table 5. Aggregated values, defuzzified values, and normalized weights.

q-SFRAM	Defuzzified Values	Normalized Weights
(0.7822,0.7270,0.7022), (0.7412,0.6868,0.6574)	0.5157	0.1743
(0.8175,0.6883,0.6884), (0.7767,0.6482,0.6493)	0.6055	0.2047
(0.7916,0.7170,0.6921), (0.7507,0.6768,0.6468)	0.5461	0.1846
(0.8183,0.7113,0.6673), (0.7740,0.6708,0.6089)	0.6090	0.2051
(0.8442,0.6741,0.6552) (0.8014,0.6338,0.6050)	0.6842	0.2313

When this stage of the process has been performed to pairwise comparison matrices of the alternatives regarding each criterion, normalized weights of the alternatives will have been obtained. In the end, the weight of each alternative is computed in conjunction with the criteria that are linked with it to determine the global weight of each alternative about each criterion. In the last step, the total global weights of an alternative are added together to get the alternative's overall weight (see Table 6).

Table 6. Global weights of the alternatives.

·	C_1	C_2	C_3	C_4	C_5	*Total*
A_1	0.1380	0.2370	0.3640	0.3470	0.2910	1.3770
A_2	0.7690	0.8740	0.9640	0.1340	0.2230	2.9640
A_3	0.6040	0.5130	0.4920	0.3370	0.2130	2.1590
A_4	0.3760	0.4620	0.6940	0.7240	0.9850	3.2410

The graphical representation of the global weights of the alternatives is shown in Figure 1.

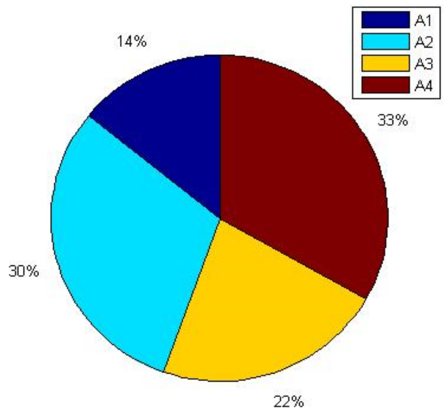

Figure 1. Graphical representation of the global weights of the alternatives.

According to the findings, the project A_4 is the most beneficial, followed by projects A_2, A_3 and A_1.

7. Managerial implications

The managerial implication of the developed novel q-spherical fuzzy rough AHP model is that it can provide support to managers and decision-makers for making strategic decisions and bring out robust and reliable results. This framework can be used by managers of different industries, in different applications like supplier selection, maintenance strategy selection, evaluation of robots under industrial conditions, material handling equipment selection, and many other decision-making processes. However, the decision-making processes involved in this framework will depend on experts' preferences and the individuals involved in the decision-making process. To check the relevance and robustness of the result, a comparative analysis and sensitivity analysis is also performed.

7.1 Comparative analysis

The comparison of rank obtained from q-spherical fuzzy rough AHP and different MCDM techniques is shown in Table 7. Four methods are used for comparative analysis. The combinatorial mathematics-based decision-making method applied by Rao and Parnichkun [50] finds the relative importance between criteria and calculates the permanent function to obtain the final score of alternatives. The preference selection index method applied by Maniya and Bhatt [51] calculates the preference value between each alternative and its variation is used to calculate the final preference score of the alternative. MACBETH, implemented by Karande and Chakraborty [52] is based on an additive value model. The interval-valued MCDM method encompasses interval-valued TOPSIS, interval-valued EDAS, and interval-valued CODAS, which was proposed by Mathew and Thomas [53]. In summary, the comparative analysis has helped in understanding the variation in ranks obtained from different MCDM methods. In the present example, the ranks obtained from different MCDM methods are different from the rank obtained from spherical fuzzy rough AHP, so it is competitive with other conventional MCDM methods. The methods applied by previous researchers are less computationally expensive (as a single numeric value is assigned to linguistic terms) but cannot capture uncertainty in decision-making. Although an 11-point scale is used for more accurate results in the preference selection index method [51] but assigning a crisp numeric value to the linguistic terms leads to a less reliable result. Other methodologies such as the combinatorial mathematics-based decision-making method [50] and MACBETH [52] have also used crisp numeric values to solve the problem. Interval-valued MCDM methods [53] assign a range of values rather than a single numeric value but do not consider the vagueness and uncertainty aspect of decision-makers while solving the problem.

Table 7. Comparison of ranking using different methods.

	Combinatorial mathematics-based decision-making method [50]	Preference selection index method [51]	MACBETH [52]	Interval-valued MCDM method [53]	q-spherical fuzzy rough AHP [This paper]
A_1	2	2	2	2	4
A_2	1	1	1	1	2
A_3	3	3	4	4	3
A_4	4	4	3	3	1

The graphical representation of ranking using different methods is shown in Figure 2.

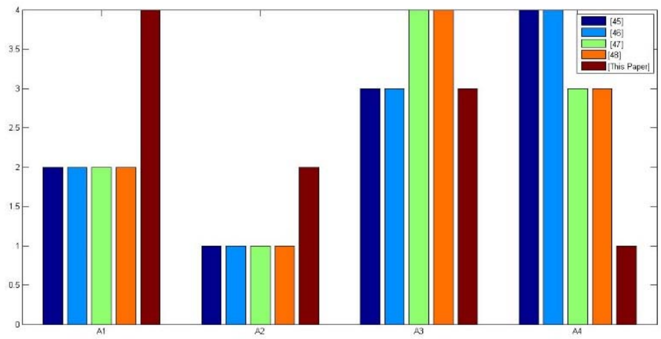

Figure 2. Graphical representation of ranking using different methods.

7.2 Sensitivity analysis

The robustness of the solutions and ranking of alternatives is checked through a sensitivity analysis that is performed by changing the weights of criteria to check the variation in the ranking of alternatives. The sets of weights used for sensitivity analysis are shown in Table 8. On defuzzifying the criteria weights, it is found that $[(0.8442,0.6741,0.6552),(0.8014,0.6338,0.6050)]$ has the highest weight value. Also, it can be seen in Table 8 that the value $[(0.8442,0.6741,0.6552),(0.8014,0.6338,0.6050)]$ is allotted to criteria C_1, C_2, C_3, C_4 and C_5 in Set 1, Set 2, Set 3, Set 4, and Set 5 respectively (i.e., arranged in a diagonal array). Set 1 is the criteria weights obtained from the q-spherical fuzzy rough AHP. The result of the sensitivity analysis is represented in Figure 3. It can be observed in Figure 3 that A_4 is the best alternative for all five different sets of weights, proving that the novel q-spherical fuzzy rough AHP gives a robust result.

Table 8. Weights for sensitivity analysis.

	C_1	C_2	C_3	C_4	C_5
Set 1	(0.7822,0.7270,0.7022), (0.7412,0.6868,0.6574)	(0.8175,0.6883,0.6884), (0.7767,0.6482,0.6493)	(0.7916,0.7170,0.6921), (0.7507,0.6768,0.6468)	(0.8183,0.7113,0.6673), (0.7740,0.6708,0.6089)	(0.8442,0.6741,0.6552) (0.8014,0.6338,0.6050)
Set 2	(0.8175,0.6883,0.6884), (0.7767,0.6482,0.6493)	(0.7822,0.7270,0.7022), (0.7412,0.6868,0.6574)	(0.8175,0.6883,0.6884), (0.7767,0.6482,0.6493)	(0.8175,0.6883,0.6884), (0.7767,0.6482,0.6493)	(0.7822,0.7270,0.7022), (0.7412,0.6868,0.6574)
Set 3	(0.8175,0.6883,0.6884), (0.7767,0.6482,0.6493)	(0.7916,0.7170,0.6921), (0.7507,0.6768,0.6468)	(0.7822,0.7270,0.7022), (0.7412,0.6868,0.6574)	(0.8175,0.6883,0.6884), (0.7767,0.6482,0.6493)	(0.7822,0.7270,0.7022), (0.7412,0.6868,0.6574)
Set 4	(0.8175,0.6883,0.6884), (0.7767,0.6482,0.6493)	(0.8175,0.6883,0.6884), (0.7767,0.6482,0.6493)	(0.8175,0.6883,0.6884), (0.7767,0.6482,0.6493)	(0.7822,0.7270,0.7022), (0.7412,0.6868,0.6574)	(0.7822,0.7270,0.7022), (0.7412,0.6868,0.6574)
Set 5	(0.8175,0.6883,0.6884), (0.7767,0.6482,0.6493)	(0.7916,0.7170,0.6921), (0.7507,0.6768,0.6468)	(0.8175,0.6883,0.6884), (0.7767,0.6482,0.6493)	(0.8183,0.7113,0.6673), (0.7740,0.6708,0.6089)	(0.7822,0.7270,0.7022), (0.7412,0.6868,0.6574)

The graphical representation of the results of the sensitivity analysis of the alternatives is exhibited in Figure 3.

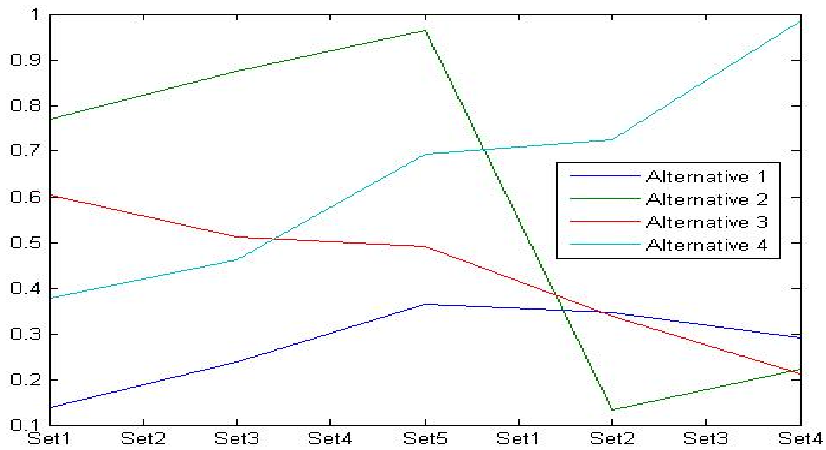

Figure 3. Result of sensitivity analysis.

8. Conclusions and recommendations for further work

This article illustrates a novel q-spherical fuzzy rough AHP which has been proven to be successful in handling imprecision in the decision-making process. The newly developed three-dimensional extension of fuzzy rough sets (i.e., q-spherical fuzzy rough sets are incorporated into different MCDM techniques). In the paper, q-spherical fuzzy rough AHP is used to calculate the weights of criteria. A new q-spherical fuzzy rough arithmetic mean formula is proposed for calculating the aggregated values. For quantification of judgment, the q-spherical fuzzy rough AHP approach was used to rank the importance of several Industry 4.0 projects. The framework is applied in Industries 4.0 projects prioritization selection problem and is found that ranks obtained from q-spherical fuzzy AHP are different when the results are compared with other multi-attribute decision-making approaches. In this paper vagueness and uncertainty of decision-makers are taken into consideration and the methodology also describes the procedure to be followed in the case of multiple experts, which was not covered by previous researchers. A sensitivity analysis (by changing weights) was performed to check the robustness of the proposed method and it is found that the best alternative remains the same for variations in the set of weights. Future research can combine different MCDM techniques with a q-spherical fuzzy rough set to obtain new hybrid MCDM techniques. The novel q-spherical fuzzy rough AHP can also be applied in a range of engineering applications. In the future, researchers can also bring out two arithmetic operations i.e., subtraction and division for the q-spherical fuzzy rough sets.

Acknowledgments

The authors A. ALoqaily and N. Mlaiki would like to thank Prince Sultan University for paying the APC and for the support through the TAS research lab.

Conflict of interest

The authors disclose no conflict of interest in publishing this paper.

References

1. L. A. Zadeh, Fuzzy sets, *Inf. Control*, **8** (1965), 338–353. https://doi.org/10.1016/S0019-9958(65)90241-X
2. K. T. Atanassov, Intuitionistic fuzzy sets, *Fuzzy Set. Syst.*, **20** (1986), 87–96. https://doi.org/10.1016/S0165-0114(86)80034-3
3. B. C. Cuong, V. Kreinovich, Picture fuzzy sets, *J. Comput. Sci. Cyb.*, **30** (2014), 409–420.
4. X. Zhang, P. Liu, Y. Wang, Multiple attribute group decision-making methods based on intuitionistic fuzzy frank power aggregation operators, *J. Intell. Fuzzy Syst.*, **29** (2015), 2235–2246. https://doi.org/10.3233/IFS-151699
5. M. R. Seikh, U. Mandal, Intuitionistic fuzzy Dombi aggregation operators and their application to multiple attribute decision-making, *Granular Comput.*, **6** (2021), 473–488. https://doi.org/10.1007/s41066-019-00209-y
6. S. Zeng, N. Zhang, C. Zhang, W. Su, L. A. Carlos, Social network multiple-criteria decision-making approach for evaluating unmanned ground delivery vehicles under the Pythagorean fuzzy environment, *Technol. Forecast. Soc.*, **175** (2022), 121–414. https://doi.org/10.1016/j.techfore.2021.121414
7. R. R. Yager, *Pythagorean fuzzy subsets*, In: 2013 joint IFSA world congress and NAFIPS annual meeting (IFSA/NAFIPS), IEEE, 2013, 57–61. https://doi.org/10.1109/IFSA-NAFIPS.2013.6608375
8. M. Akram, W. A. Dudek, J. M. Dar, Pythagorean Dombi fuzzy aggregation operators with application in multicriteria decision-making, *Int. J. Intell. Syst.*, **34** (2019), 3000–3019. https://doi.org/10.1002/int.22183
9. H. Garg, Confidence levels-based Pythagorean fuzzy aggregation operators and its application to the decision-making process, *Comput. Math. Organ. Th.*, **23** (2017), 546–571. https://doi.org/10.1007/s10588-017-9242-8
10. L. Wang, H. Garg, Algorithm for multiple attribute decision-making with interactive Archimedean norm operations under Pythagorean fuzzy uncertainty, *Int. J. Comput. Intell. Syst.*, **14** (2021), 503–927. https://doi.org/10.2991/ijcis.d.201215.002
11. Q. Wu, W. Lin, L. Zhou, Y. Chen, H. Chen, Enhancing multiple attribute group decision-making flexibility based on information fusion technique and hesitant Pythagorean fuzzy sets, *Comput. Ind. Eng.*, **127** (2019), 954–970. https://doi.org/10.1016/j.cie.2018.11.029
12. R. R. Yager, Generalized orthopair fuzzy sets, *IEEE T. Fuzzy Syst.*, **25** (2016), 1222–1230. https://doi.org/10.1109/TFUZZ.2016.2604005
13. Y. Xing, R. Zhang, Z. Zhou, J. Wang, Some q-rung orthopair fuzzy point weighted aggregation operators for multi-attribute decision making, *Soft Comput.*, **23** (2019), 11627–11649. https://doi.org/10.1007/s00500-018-03712-7
14. P. Liu, P. Wang, Some q-rung orthopair fuzzy aggregation operators and their applications to multiple-attribute decision making, *Int. J. Intell. Syst.*, **33** (2018), 259–280. https://doi.org/10.1002/int.21927
15. B. C. Cuong, V. Kreinovich, *Picture fuzzy sets-a new concept for computational intelligence problems*, In: 2013 third world congress on information and communication technologies (WICT 2013), IEEE, 2013, 1–6. https://doi.org/10.1109/WICT.2013.7113099
16. G. Wei, Picture fuzzy Hamacher aggregation operators and their application to multiple attribute decision making, *Fund. Inform.*, **157** (2018), 271–320. https://doi.org/10.3233/FI-2018-1628
17. T. Mahmood, K. Ullah, Q. Khan, N. Jan, An approach toward decision-making and medical diagnosis problems using the concept of spherical fuzzy sets, *Neural Comput. Appl.*, **31** (2019), 7041–7053. https://doi.org/10.1007/s00521-018-3521-2

18. I. Deli, N. Çağman, *Spherical fuzzy numbers and multi-criteria decision-making*, In: Decision Making with Spherical Fuzzy Sets, Springer, **392** (2021), 53–84. https://doi.org/10.1007/978-3-030-45461-6_3
19. M. Rafiq, S. Ashraf, S. Abdullah, T. Mahmood, S. Muhammad, The cosine similarity measures of spherical fuzzy sets and their applications in decision making, *J. Intell. Fuzzy Syst.*, **36** (2019), 6059–6073. https://doi.org/10.3233/JIFS-181922
20. S. Ashraf, S. Abdullah, M. Aslam, Symmetric sum-based aggregation operators for spherical fuzzy information: Application in multi-attribute group decision making problem, *J. Intell. Fuzzy Syst.*, **38** (2020), 5241–5255. https://doi.org/10.3233/JIFS-191819
21. C. Kahraman, B. Oztaysi, S. C. Onar, I. Otay, *q-Spherical fuzzy sets and their usage in multi-attribute decision making*, In: Developments of Artificial Intelligence Technologies in Computation and Robotics, World Scientific, **12** (2020), 217–225. https://doi.org/10.1142/9789811223334_0027
22. A. B. Azim, A. Aloqaily, A. Ali, S. Ali, N. Mlaiki, F. Hussain, q-Spherical fuzzy rough sets and their usage in multi-attribute decision-making problems, *AIMS Math.*, **8** (2023), 8210–8248. https://doi.org/10.3934/math.2023415
23. Z. Pawlak, Rough sets, *Int. J. Comput. Inform. Sci.*, **11** (1982), 341–356. https://doi.org/10.1007/BF01001956
24. Z. Pawlak, Rough set theory and its applications to data analysis, *Cybern. Syst.*, **29** (1998), 661–688. https://doi.org/10.1080/019697298125470
25. Y. Yao, Constructive and algebraic methods of the theory of rough sets, *Inform. Sciences*, **109** (1998), 21–47. https://doi.org/10.1016/S0020-0255(98)00012-7
26. J. Dai, S. Gao, G. Zheng, Generalized rough set models determined by multiple neighborhoods generated from a similarity relation, *Soft Comput.*, **22** (2018), 2081–2094. https://doi.org/10.1007/s00500-017-2672-x
27. J. Zhan, B. Sun, Covering-based intuitionistic fuzzy rough sets and applications in multiattribute decision-making, *Artif. Intell. Rev.*, **53** (2020), 671–701. https://doi.org/10.1007/s10462-018-9674-7
28. B. Sun, S. Tong, W. Ma, T. Wang, C. Jiang, An approach to MCGDM based on multi granulation Pythagorean fuzzy rough set over two universes and its application to the medical decision problem, *Artif. Intell. Rev.*, **55** (2022), 1887–1913. https://doi.org/10.1007/s10462-021-10048-6
29. H. Garg, M. Atef, Cq-ROFRS: Covering q-rung orthopair fuzzy rough sets and its application to the multi-attribute decision-making process, *Complex Intell. Syst.*, **8** (2022), 2349–2370. https://doi.org/10.1007/s40747-021-00622-4
30. S. Ashraf, N. Rehman, H. AlSalman, A. H. Gumaei, A decision-making framework using q-rung orthopair probabilistic hesitant fuzzy rough aggregation information for the drug selection to treat COVID-19, *Complexity*, **2022** (2022). https://doi.org/10.1155/2022/5556309
31. Y. Lu, Industry 4.0: A survey on technologies, applications, and open research issues, *J. Ind. Inf. Integ.*, **6** (2017), 1–10. https://doi.org/10.1016/j.jii.2017.04.005
32. J. Posada, C. Toro, I. Barandiaran, D. Oyarzun, D. Stricker, R. De Amicis, et al., Visual computing as a key enabling technology for industry 4.0 and industrial internet, *IEEE Comput. Graph.*, **35** (2015), 26–40. https://doi.org/10.1109/MCG.2015.45
33. E. Gossen, E. Abele, M. Rauscher, Multi-criterial selection of track and trace technologies for an anti-counterfeiting strategy, *Procedia CIRP*, **57** (2016), 73–78. https://doi.org/10.1016/j.procir.2016.11.014

34. J. Qin, Y. Liu, R. Grosvenor, A categorical framework of manufacturing for industry 4.0 and beyond, *Procedia CIRP*, **52** (2016), 173–178. https://doi.org/10.1016/j.procir.2016.08.005
35. O. Dogan, B. Öztaysi, In-store behavioral analytics technology selection using fuzzy decision making, *J. Enterp. Inf. Manag.*, **31** (2018), 612–630. https://doi.org/10.1108/JEIM-02-2018-0035
36. J. Lee, B. Bagheri, H. A. Kao, A cyber-physical systems architecture for industry 4.0-based manufacturing systems, *Manuf. Lett.*, **3** (2015), 18–23. https://doi.org/10.1016/j.mfglet.2014.12.001
37. H. Lasi, P. Fettke, H. G. Kemper, T. Feld, M. Hoffmann, Industry 4.0, *Bus. Inform. Syst. Eng.*, **6** (2014), 239–242. https://doi.org/10.1007/s12599-014-0334-4
38. Y. C. Shen, S. H. Chang, G. T. Lin, H. C. Yu, A hybrid selection model for emerging technology, *Techno. Forecast. Soc.*, **77** (2010), 151–166. https://doi.org/10.1016/j.techfore.2009.05.001
39. R. W. Saaty, The analytic hierarchy process—what it is and how it is used, *Math. Model.*, **9** (1987), 161–176. https://doi.org/10.1016/0270-0255(87)90473-8
40. B. Oztaysi, A decision model for information technology selection using AHP integrated TOPSIS-Grey: The case of content management systems, *Knowl.-Based Syst.*, **70** (2014), 44–54. https://doi.org/10.1016/j.knosys.2014.02.010
41. B. Oztaysi, O. Dogan, H. Gul, *Selection of gamification elements for demand-side energy management: An application using hesitant fuzzy AHP*, In: R&D Manag. Knowl. Era: Challenges Emerg. Technol., 2019, 299–322. https://doi.org/10.1007/978-3-030-15409-7_11
42. R. Verma, S. Chandra, Interval-valued intuitionistic fuzzy-analytic hierarchy process for evaluating the impact of security attributes in fog-based internet of things paradigm, *Comput. Commun.*, **175** (2021), 35–46. https://doi.org/10.1016/j.comcom.2021.04.019
43. M. Rajak, K. Shaw, Evaluation, and selection of mobile health (mHealth) applications using AHP and fuzzy TOPSIS, *Technol. Soc.*, **59** (2019), 101186. https://doi.org/10.1016/j.techsoc.2019.101186
44. J. Zhao, S. Peng, T. Li, S. Lv, M. Li, H. Zhang, Energy-aware fuzzy job-shop scheduling for engine remanufacturing at the multi-machine level, *Front. Mech. Eng.*, **14** (2019), 474–488. https://doi.org/10.1007/s11465-019-0560-z
45. A. Fetanat, H. Mofid, M. Mehrannia, G. Shafipour, Informing energy justice based decision-making framework for waste-to-energy technologies selection in sustainable waste management: A case of Iran, *J. Clean. Prod.*, **228** (2019), 1377–1390. https://doi.org/10.1016/j.jclepro.2019.04.215
46. J. Zhan, B. Sun, J. C. R. Alcantud, Covering based multi granulation (I, T)-fuzzy rough set models and applications in multi-attribute group decision-making, *Inform. Sciences*, **476** (2019), 290–318.
47. J. Zhan, J. C. R. Alcantud, A novel type of soft rough covering and its application to multicriteria group decision making, *Artif. Intell. Rev.*, **52** (2019), 2381–2410. https://doi.org/10.1016/j.jclepro.2019.04.215
48. K. Zhang, J. Zhan, W. Wu, J. C. R. Alcantud, Fuzzy β-covering based (I, T)-fuzzy rough set models and applications to multi-attribute decision-making, *Comput. Indust. Eng.*, **128** (2019), 605–621. https://doi.org/10.1016/j.cie.2019.01.004
49. J. C. R. Alcantud, F. Feng, R. R. Yager, An N-soft set approach to rough sets, *IEEE T. Fuzzy Syst.*, **28** (2019), 2996–3007. https://doi.org/10.1109/TFUZZ.2019.2946526
50. R. V. Rao, M. Parnichkun, Flexible manufacturing system selection using a combinatorial mathematics-based decision-making method, *Int. J. Prod. Res.*, **47** (2009), 6981–6998. https://doi.org/10.1080/00207540802389227
51. K. Maniya, M. Bhatt, The selection of flexible manufacturing system using preference selection index method, *Int. J. Ind. Syst. Eng.*, **9** (2011), 330–349. https://doi.org/10.1504/IJISE.2011.043142

52. P. Karande, S. Chakraborty, Evaluation and selection of flexible manufacturing systems using MACBETH method, *Int. J. Serv. Oper. Manag.*, **16** (2013), 123–144. https://doi.org/10.1504/IJSOM.2013.055576
53. M. Mathew, J. Thomas, Interval-valued multi-criteria decision-making methods for the selection of flexible manufacturing system, *Int. J. Data Network Sci.*, **3** (2019), 349–358. https://doi.org/10.5267/j.ijdns.2019.4.001
54. P. Zhang, T. Li, G. Wang, C. Luo, H. Chen, J. Zhang, et al., Multi-source information fusion based on rough set theory: A review, *Inform. Fusion*, **68** (2021), 85–117. https://doi.org/10.1016/j.inffus.2020.11.004
55. Z. Yuan, H. Chen, P. Xie, P. Zhang, J. Liu, T. Li, Attribute reduction methods in fuzzy rough set theory: An overview, comparative experiments, and new directions, *Appl. Soft Comput.*, **107** (2021), 107353. https://doi.org/10.1016/j.asoc.2021.107353
56. X. Che, D. Chen, J. Mi, Learning instance-level label correlation distribution for multi-label classification with fuzzy rough sets, *IEEE T. Fuzzy Syst.*, 2023, 1–13. https://doi.org/10.1109/TFUZZ.2023.3248060
57. S. Ali, A. Ali, A. B. Azim, A. Aloqaily, N. Mlaiki, Averaging aggregation operators under the environment of q-rung orthopair picture fuzzy soft sets and their applications in MADM problems, *AIMS Math.*, **8** (2023), 9027–9053. https://doi.org/10.3934/math.2023452