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Research article

A novel method for calculating the contribution rates of economic growth factors

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Abstract: The common production functions include the Leontief production function, the Cobb-Douglas (C-D) production function, the constant elasticity of substitution (CES) production function, the variable elasticity of substitution (VES) production function and so on. With different elasticity of substitution of factor, the production functions have different ranges of applications. In the production functions, the C-D production function is used the most widely because of its simple form, while the CES production function and the VES production function have limitations in applications due to their complicated forms. However, the C-D production function has the elasticity of substitution of factors of 1, and the CES production function has the elasticity of substitution of factors which is not 1 but a constant, so the two production functions both have limitations in applications. The VES production function with the variable elasticity of substitution is more practical in some application cases. This paper studies the applications of the VES production function model and gives a method of calculating the contribution rates of economic growth factors scientifically. As for the parameter estimation of the model, this paper gives an improved Sine Cosine Algorithm (SCA) to enhance the convergence rate and precision. Finally, the paper makes an empirical analysis on the contribution rates of economic growth factors of Shanghai City, China, using the method proposed.

Keywords: production function; economic growth; contribution rates of factors; SCA
Mathematics Subject Classification: 65K10, 91B02, 93B40

1. Introduction

Before the introduction of the production function, researchers have performed some studies with the functional relationship in the economic field. In 1934, the American economist Douglas formally put forward the term of production function, which was built on the theoretical basis of marginal productivity and reflected the relationship between the production factor input and the product output. Research on the production function developed further after the appearance of the Cobb-Douglas (C-D) production function. In August 1936, American economist Leontief whose-ancestral home was in the Soviet Union-proposed the Leontief production function with a fixed factor rate. In 1957, American scholar Solow measured the technological progress of America from 1909 to 1949 using the Q=A(t)f(K, L) production function, and since then this production function has been used in the studies on technological progress, improving its role and status in quantitative analyses. In 1961, Solow, Chenery, and Minhas et al. proposed the generalized production function, i.e. the constant elasticity of substitution (CES) production function. In 1968, Sato and Hoffman proposed the variable elasticity of substitution (VES) production function and developed the C-D production function. Then, the modified C-D production functions appeared gradually.

In the Western economics, the development of economic growth theory and models went through three stages [1–6]: The Harrod-Domer model, the Solow model and the new growth model, or endogenous growth model. The first-stage model represented by the Harrod-Domer model introduced the growth problem into the research field of economics using the Leontief production function and constructed the Keynes-style growth model with the characteristic that capital and the labor are not substitutable. Theorists generally believe that the Harrod-Domer model makes the mistake of choosing a static production function as the analysis tool to study the dynamic economic growth problem and thus draws a conclusion failing to conform to the reality. However, when an economic entity has a lot of idle capital goods for various causes or a lot of unemployed workforce, and the problem of idle capital goods or labor redundance exists for a long time, the economic entity presents strong rigidity and then the production function has the static characteristic, in which case the Leontief production function shall be a production function with practical applicability. Solow realized the stable growth of economies in a simple model using the neoclassical production function. It is widely believed to have solved the key problem of Harrod-Domer model. In the Solow model, the technological progress is exogenous. The C-D production function used has the homogeneity of the first degree. The production technology represented by the function has constant returns to scale. Further, the capital and the labor have diminishing marginal returns. The function meets the three properties of the neoclassical production function and thus is a production function of good quality. People often use the function for the empirical tests of neoclassical growth theories. The research on new growth theory led by Lucas and Romer made the problems like technological progress endogenous and formed a new wave of economic growth study since the 1980s.

In the Leontief production function, the Cobb-Douglas (C-D) production function, the constant elasticity of substitution (CES) production function and the variable elasticity of substitution (VES) production function used commonly, their elasticities of substitution of factors are different [7–10]. The Leontief production function has the elasticity of substitution of factors of 0 and the C-D production function has the elasticity of substitution of factors of 1, so the two production functions both have many defects. The CES production function considers that the elasticity of substitution of factors in the production function varies in different sections and enterprises. However, in the CES production function, the elasticity of substitution can be any constant, but the constant is invariable for a specific production function, while, in fact, the elasticity of substitution may be variable rather than constant in different sample points. To reflect the characteristics of elasticity of substitution, the VES production function's elasticity of substitution is no longer constant but varies as the difficulty of substitution of factors or technological level changes. Therefore, the VES production function's coefficient of variable elasticity of substitution conforms to the actual economic situation better relative to the CES production function's coefficient of invariable elasticity of substitution.

In current economic studies, the C-D production function and the CES production function are applied more widely than the VES production function, for which an important reason is that the VES production function has complicated forms and more difficult parameter estimations. Currently, many scholars have made theoretical and applied studies on the C-D production function [11–15] and the CES production function [16–21], and a few researchers have explored the VES production function [22–25]. The studies mainly focus on the fields of agriculture, industry, environment, education, science & technology, firefighting and so on. Study content includes the influencing factor analysis, the total factor productivity analysis, the production elasticity analysis, the production elasticity analysis, the potential productivity analysis, the efficiency measure calculation and so on, in which the calculation of contribution rates of economic growth influencing factors is an important subject, and only a few scholars have made related studies. On the basis of the entropy weight method, Zhu and Yang [26] measured the contribution rate of vocational education to village vitalization with the C-D production function by constructing the village vitalization and development index. To determine the contribution rate of inland water transportation to the economy in Hubei province, Wang et al. [27] analyzed and tested the long-run equilibrium relationship between inland water transportation and economy in Hubei province using co-integration analysis, basing the analysis on Hubei province's inland water transportation development, and measured and calculated the contribution rate of inland water transportation to the economy in Hubei province using the extended C-D production function model. Lv et al. [28] introduced the technology input factor on the basis of the C-D production function to construct a new production function model, and used the data of Shandong province, China in 2003-2014 to build a production function of wheat yield of Shandong province and then determined the contribution rate of technology input to wheat yield. Cheng et al. [29] proposed a new method calculating the contribution rates of energy and other influencing factors to economic growth using the modified CES production function model. Wei and Gang [30] analyzed the contribution rate of technological progress of the construction industry in Sichuan province, China, from 2006-2015 using the C-D production function. Starting from the C-D production function model, Zhang [31] selected related data of Quanzhou City, China, from 2003-2014 to analyze the contributions of three production factor inputs of labor, capital and technology to economic growth. Most of the scholars explored the contribution rates of factors using the C-D production function which had its defects and thus was not applicable in some cases. This paper uses the VES production function to calculate the contribution rates of economic growth factors.

The VES production function has many forms, in which a common one is

$$Y = AK^{\mu\frac{1}{(1+c)}}(L + \frac{b}{1+c}K)^{\mu\frac{c}{(1+c)}}. \tag{1.1}$$

This paper modifies it as:

$$\begin{aligned} Y &= A(t)K^{\mu\frac{1}{(1+c)}}(L + \tfrac{b}{1+c}K)^{\mu\frac{c}{(1+c)}} \\ &= Ae^{\lambda t}K^{\mu\frac{1}{(1+c)}}(L + \tfrac{b}{1+c}K)^{\mu\frac{c}{(1+c)}}, \end{aligned} \tag{1.2}$$

where $A(t) = A_0e^{\lambda t}$ is the technological progress level, Y is the output, K is the capital input, L is the labor input, μ is the return on scale, c is the factor's output elasticity parameter, λ is the technological progress rate, and A, λ, b, c and μ are the parameters to be estimated.

The VES production function model is complicated, in which case the conventional method has poor precision when estimating parameters, so modern intelligent algorithm is a good choice [32–34]. The paper uses a modern intelligent algorithm, i.e. the Sine Cosine Algorithm (SCA) [35–39], which has a few parameters, a simple structure and can be realized easily. The conventional SCA has a slow rate of convergence and poor precision, so the paper gives an improved SCA, i.e. the ISCA [40–42]. The ISCA has a significantly improved rate of convergence and precision.

The VES production function has many good characteristics. It meets the law of diminishing marginal returns and the law of diminishing marginal rate of technological substitution. Its isoquant curve has strict convexity. Its elasticity of substitution varies as K/L varies. It satisfies Euler's theorem. However, the VES production function has a complicated form and thus is limited in applications. This paper researches the applications of the VES production function.

As to the applications of the production function model, a few scholars have researched the contribution rates of influencing factors of economic growth using the C-D production function. However, there are two problems: first, the C-D production function has its defects and is not suitable in some cases; second, the scholars all calculated the contribution rates of factors using the growth rate equation derived with the C-D production function, but the equation substituted differential expression with difference expression in calculations and thus caused a deviation. In this case, this paper gives a method of calculating the contribution rates of economic growth factors scientifically using the VES production function. This method has great theoretical guidance significance for the accurate calculation of contribution rates and important practical significance for the scientific decision making of the management.

The research in this paper includes the following three aspects of content: (1) the method to build the VES production function, i.e. giving a parameter estimation method for the modified VES production function model and then getting a real model; (2) the method to calculate the contribution rates of economic growth factors, i.e. deriving calculation formulas with the calculus factor analysis; (3) an empirical analysis, i.e. calculating the contribution rates of economic growth factors of Shanghai City, China.

2. The VES production function model's parameter estimation method

This paper uses the SCA to estimate the model's parameters.

For the known observation value $(K_t, L_t, Y_t), (t = 1, 2, \cdots, n)$, there is

$$Y_t = Ae^{\lambda t} K_t^{\mu \frac{1}{(1+c)}} (L_t + \frac{b}{1+c} K_t)^{\mu \frac{c}{(1+c)}} + \varepsilon_t. \tag{2.1}$$

This paper writes

$$G(\eta) = \sum_{t=1}^{n} \varepsilon_t^2 = \sum_{t=1}^{n} \left\{ Y_t - Ae^{\lambda t} K^{\mu \frac{1}{(1+c)}} (L + \frac{b}{1+c} K)^{\mu \frac{c}{(1+c)}} \right\}^2, \tag{2.2}$$

and makes it have the minimum. Then, we can get parameter estimate $\eta = (A, \lambda, b, c, \mu)$. It is essentially a nonlinear optimization problem and the paper gives an SCA.

The fitness function is

$$G = \sum_{t=1}^{n} \varepsilon_t^2 = \sum_{t=1}^{n} \left\{ Y_t - Ae^{\lambda t} K^{\mu \frac{1}{(1+c)}} (L + \frac{b}{1+c} K)^{\mu \frac{c}{(1+c)}} \right\}^2. \tag{2.3}$$

2.1. The standard SCA

SCA was first proposed by Australian scholar Mirjalili in 2016 as a novel swarm intelligent optimization algorithm. SCA requires a few parameters to be set, can be realized easily and can achieve the optimization objective through the sine and cosine functions' property iterations. It has been proved that SCA is superior to PSO (Particle Swarm Optimization) and GA (Genetic Algorithm) in both convergence precision and convergence rate.

In SCA, we suppose the swarm size is N, the search space is d-dimensional, and individual i's position in the D^{th} dimension space can be represented as $X_i = \left(x_i^1, x_i^2, \cdots, x_i^D\right), i = 1, 2, \cdots, N$. The algorithm first generates the positions of N individuals, calculates the fitness value of each individual in the swarm, and then sorts and records the optimal individual currently and its position. In each iteration, the individuals in the swarm update positions according to the following formula (2.4)

$$X(t+1) = \begin{cases} X(t) + r_1 \times \sin r_2 \times |r_3 \times X^* - X(t)|, & r_4 < 0.5, \\ X(t) + r_1 \times \cos r_2 \times |r_3 \times X^* - X(t)|, & r_4 \geq 0.5. \end{cases} \tag{2.4}$$

where t is current number of iterations, X^* is the position of the current optimal individual, $r_2 \in [0, 2\pi]$, $r_3 \in [0, 2]$ and $r_4 \in [0, 1]$ are three random parameters, and r_1 is called the control parameter decreasing linearly from a to 0 as the number of iterations increases in the standard SCA, i.e.

$$r_1 = a - a\frac{t}{T_{\max}}, \tag{2.5}$$

where a is a constant, and $T_{\max}$ is the maximum number of iterations.

Finally, the loop iteration ends when the SCA meets the end conditions.

In basic SCA, the swarm update formula (2.4) uses the sine and cosine functions, so Mirjalili called the algorithm the Sine Cosine Algorithm. In the SCA, there are four parameters r_1, r_2, r_3 and r_4, in which the key parameter is r_1 controlling the conversion of the algorithm from the global search to the local development. When $r_1 > 1$, the values of functions $r_1 \sin(r_2)$ and $r_1 \cos(r_2)$ may be more than 1 or less than -1; when $r_1 \leq 1$, the values of functions $r_1 \sin(r_2)$ and $r_1 \cos(r_2)$ must be in the range of -1-1. According to the design principle of SCA, the algorithm first performs a global search and then a local development, i.e. when $|r_1 \sin(r_2)| > 1$ or $|r_1 \cos(r_2)| > 1$, the algorithm performs a global search, and when $|r_1 \sin(r_2)| \leq 1$ or $|r_1 \cos(r_2)| \leq 1$, the algorithm performs a local development. Therefore, when the value of r_1 is big, the algorithm tends to perform a global search, and when the value of r_1 is small, the algorithm tends to ferform a local development.

2.2. ISCA

2.2.1. Dynamic inertia weight

We introduce an inertia weight w, and get the improved individual position update formula as follows:

$$X(t+1) = \begin{cases} w(t)X(t) + r_1 \times \sin r_2 \times |r_3 \times X^* - X(t)|, & r_4 < 0.5, \\ w(t)X(t) + r_1 \times \cos r_2 \times |r_3 \times X^* - X(t)|, & r_4 \geq 0.5. \end{cases} \tag{2.6}$$

where $w(t) = \sin[\frac{\pi}{2}(1 - h\frac{t}{T_{\max}})^{\alpha}]$, t is the current number of iterations, $T_{\max}$ is the maximum number of iterations, $\alpha > 0$, $0 \le h \le 1$. Inertia weight w decreases as the number of iterations increases. In the early evolution, a large value of w is good for the global search, while in the later evolution, a small value of w is good for the accurate local search, thus improving the algorithm's convergence precision and rate.

2.2.2. Dynamic control parameter

To further enhance the local search ability in later iterations, we change parameter r_1's linear decrease into the nonlinear trigonometric functional decrease, i.e.

$$r_1 = a\sin[\frac{\pi}{2}(1 - g\frac{t}{T_{\max}})^{\beta}], \tag{2.7}$$

where $1 < a \le 2$, $0 \le g \le 1$ and $\beta > 0$.

2.2.3. Steps of the algorithm

In the algorithm, each individual corresponds to a solution. The optimization process is as follows:

Step 1: The algorithm initializes the solution set $x_i^j, i = 1, 2, \cdots, N, j = 1, 2, \cdots D$ in which N is the swarm size, and D is the number of dimensions of the problem to be optimized;

Step 2: The algorithm calculates each individual's fitness and finds the global optimal;

Step 3: The algorithm updates the position of individual according to formula (2.6);

Step 4: The algorithm judges whether the current individual's fitness is superior to the global optimal solution of the swarm before. If it is superior, the current individual's fitness value replaces the previous global optimal solution. If it is not superior, the algorithm returns to Step 3.

Step 5: If $t < T_{\max}$, the algorithm returns to Step 3; if not, the algorithm ends and outputs the result.

3. The calculation method for contribution rates of economic growth factors

We write $B(t) = Ae^{\lambda t}$.

In this case, the VES production function model is

$$Y(t) = B(t)K^{\mu\frac{1}{(1+c)}}(L + \frac{b}{1+c}K)^{\mu\frac{c}{(1+c)}}. \tag{3.1}$$

We differentiate the formula above and get:

$$dY = \frac{\partial Y}{\partial B}dB + \frac{\partial Y}{\partial K}dK + \frac{\partial Y}{\partial L}dL, \tag{3.2}$$

where

$$\frac{\partial Y}{\partial B} = K^{\mu\frac{1}{(1+c)}}(L + \frac{b}{1+c}K)^{\mu\frac{c}{(1+c)}} = \frac{Y}{Ae^{\lambda t}}, \tag{3.3}$$

$$\begin{aligned} \frac{\partial Y}{\partial K} &= Ae^{\lambda t}\mu\frac{1}{1+c}K^{\mu\frac{1}{1+c}-1}(L + \frac{b}{1+c}K)^{\frac{\mu c}{1+c}} \\ &+Ae^{\lambda t}K^{\mu\frac{1}{1+c}}\frac{\mu c}{1+c}(L + \frac{b}{1+c}K)^{\frac{\mu c}{1+c}-1}\frac{b}{1+c} \\ &= Ae^{\lambda t}\mu\frac{1}{1+c}\frac{1}{K}\frac{Y}{Ae^{\lambda t}} \\ &+Ae^{\lambda t}\frac{\mu c}{1+c}\frac{b}{1+c}(\frac{Ae^{\lambda t}K^{\frac{\mu}{1+c}}}{Y})^{\frac{1+c}{\mu c}}\frac{Y}{Ae^{\lambda t}} \\ &= \frac{\mu}{1+c}YK^{-1} + \frac{\mu cb}{(1+c)^2}A^{\frac{1+c}{\mu c}}Y^{1-\frac{1+c}{\mu c}}K^{\frac{1}{c}}e^{\frac{1+c}{\mu c}\lambda t}, \end{aligned} \tag{3.4}$$

$$\begin{aligned}
&\frac{\partial Y}{\partial L} = Ae^{\lambda t}K^{\mu\frac{1}{1+c}}\frac{\mu c}{1+c}(L+\frac{b}{1+c}K)^{\frac{\mu c}{1+c}-1}\\
&= Ae^{\lambda t}K^{\mu\frac{1}{1+c}}\frac{\mu c}{1+c}(\frac{Y}{Ae^{\lambda t}K^{\mu\frac{1}{1+c}}})^{\frac{1+c}{\mu c}(\frac{\mu c}{1+c}-1)}\\
&= A^{\frac{1+c}{\mu c}}\frac{\mu c}{1+c}Y^{1-\frac{1+c}{\mu c}}K^{\frac{1}{c}}e^{\frac{1+c}{\mu c}\lambda t}.
\end{aligned} \tag{3.5}$$

We suppose the economic vector (Y, K, L) varies in the form of the $L(t)$ curve from period 1 to period n. The paper chooses $L(t)$ as an exponential curve, i.e.

$$\begin{cases} Y = Y_0 e^{at}, \\ K = K_0 e^{h_1 t}, \quad (1 \le t \le n). \\ L = L_0 e^{h_2 t}. \end{cases} \tag{3.6}$$

In this case, the technological progress factor's value of influence on economic growth is

$$\begin{aligned}
\Delta Y_A &= \int_{L(t)} \frac{\partial Y}{\partial B} dB\\
&= \int_{L(t)} \frac{Y}{Ae^{\lambda t}} d(Ae^{\lambda t})\\
&= \int_1^n \frac{Y_0 e^{at}}{Ae^{\lambda t}} Ae^{\lambda t}\lambda dt\\
&= \frac{Y_0\lambda}{a}(e^{an} - e^{a}).
\end{aligned} \tag{3.7}$$

The capital factor's value of influence on economic growth is

$$\begin{aligned}
\Delta Y_K &= \int_{L(t)} \frac{\partial Y}{\partial K} dK\\
&= \int_{L(t)} [\frac{\mu}{1+c} Y K^{-1} + \frac{\mu c b}{(1+c)^2} A^{\frac{1+c}{\mu c}} Y^{1-\frac{1+c}{\mu c}} K^{\frac{1}{c}} e^{\frac{1+c}{\mu c}\lambda t}] dK,
\end{aligned} \tag{3.8}$$

i.e.

$$\begin{aligned}
\Delta Y_K &= \int_{L(t)} \frac{\partial Y}{\partial K} dK\\
&= \int_1^n [\frac{\mu}{1+c} Y K^{-1} + \frac{\mu c b}{(1+c)^2} A^{\frac{1+c}{\mu c}} Y^{1-\frac{1+c}{\mu c}} K^{\frac{1}{c}} e^{\frac{1+c}{\mu c}\lambda t}] dK\\
&= \int_1^n [\frac{\mu}{1+c} Y_0 e^{at} (K_0 e^{h_1 t})^{-1} + \frac{\mu c b}{(1+c)^2} A^{\frac{1+c}{\mu c}} (Y_0 e^{at})^{1-\frac{1+c}{\mu c}} (K_0 e^{h_1 t})^{\frac{1}{c}} e^{\frac{1+c}{\mu c}\lambda t}] d(K_0 e^{h_1 t}),
\end{aligned} \tag{3.9}$$

then we get the integral

$$\begin{aligned}
\Delta Y_K &= \frac{\mu h_1 Y_0}{a(1+c)}[e^{an} - e^{a}]\\
&+ \frac{\frac{\mu c b h_1}{(1+c)^2} A^{\frac{1+c}{\mu c}} Y_0^{\frac{\mu c-1-c}{\mu c}} K_0^{1+\frac{1}{c}}}{\frac{1+c}{\mu c}\lambda + \frac{1+c}{c}h_1 + \frac{a(\mu c-1-c)}{\mu c}} [e^{(\frac{1+c}{\mu c}\lambda + \frac{1+c}{c}h_1 + \frac{a(\mu c-1-c)}{\mu c})n} - e^{(\frac{1+c}{\mu c}\lambda + \frac{1+c}{c}h_1 + \frac{a(\mu c-1-c)}{\mu c})}].
\end{aligned} \tag{3.10}$$

The labor factor's value of influence on economic growth is

$$\begin{aligned}
\Delta Y_L &= \int_{L(t)} \frac{\partial Y}{\partial L} dL\\
&= \int_{L(t)} A^{\frac{1+c}{\mu c}} \frac{\mu c}{1+c} Y^{1-\frac{1+c}{\mu c}} K^{\frac{1}{c}} e^{\frac{1+c}{\mu c}\lambda t} dL,
\end{aligned} \tag{3.11}$$

i.e.

$$\Delta Y_L = \int_1^n A^{\frac{1+c}{\mu c}} \frac{\mu c}{1+c} (Y_0 e^{at})^{1-\frac{1+c}{\mu c}} (K_0 e^{h_1 t})^{\frac{1}{c}} e^{\frac{1+c}{\mu c}\lambda t} d(L_0 e^{h_2 t}), \tag{3.12}$$

then we get the integral

$$\Delta Y_L = \frac{\frac{\mu c}{1+c} L_0 h_2 A^{\frac{1+c}{\mu c}} Y_0^{\frac{\mu c-1-c}{\mu c}} K_0^{\frac{1}{c}}}{\frac{a(\mu c-1-c)}{\mu c}+\frac{h_1}{c}+\frac{1+c}{\mu c}\lambda+h_2} \times[e^{(\frac{a(\mu c-1-c)}{\mu c}+\frac{h_1}{c}+\frac{1+c}{\mu c}\lambda+h_2)n} - e^{(\frac{a(\mu c-1-c)}{\mu c}+\frac{h_1}{c}+\frac{1+c}{\mu c}\lambda+h_2)}]. \tag{3.13}$$

Therefore, the technological progress's rate of contribution to economic growth from period 1 to period n is

$$\frac{\Delta Y_A}{\Delta Y_A + \Delta Y_K + \Delta Y_L}. \tag{3.14}$$

Capital's rate of contribution to economic growth from period 1 to period n is

$$\frac{\Delta Y_K}{\Delta Y_A + \Delta Y_K + \Delta Y_L}. \tag{3.15}$$

Labor's rate of contribution to economic growth from period 1 to period n is

$$\frac{\Delta Y_L}{\Delta Y_A + \Delta Y_K + \Delta Y_L}. \tag{3.16}$$

4. Empirical analysis on the contribution rates of economic growth influencing factors of Shanghai City, China

To research the economic growth situation in Shanghai City, China and explore the growth method, we calculate the contribution rates of input factors to economic growth. The paper selects the nominal gross regional product Y (RMB 0.1 billion) as an overall representative index, and considers fixed-asset investment K (RMB 0.1 billion) and the number of employees L (10,000 people) as economic influencing factors for the analysis. See Table 1 for the data.

Table 1. Relevant data of economic growth in Shanghai, China.

Time	Y	K	L
2000	4812.15	1869.67	745.24
2001	5257.66	1994.73	752.26
2002	5795.02	2187.06	792.04
2003	6804.04	2452.11	813.05
2004	8101.55	3084.66	836.87
2005	9197.13	3542.55	863.32
2006	10598.86	3925.09	885.51
2007	12878.68	4458.61	909.08
2008	14536.9	4829.45	1053.24
2009	15742.44	5273.33	1064.42
2010	17915.41	5317.67	1090.76
2011	20009.68	5067.09	1104.33
2012	21305.59	5254.38	1115.5
2013	23204.12	5647.79	1137.35
2014	25269.75	6016.43	1197.31
2015	26887.02	6352.7	1361.51
2016	29887.02	6755.88	1365.24
2017	32925.01	7246.6	1372.65
2018	36011.82	7623.42	1375.66
2019	38155.32	8012.22	1376.2
2020	38700.58	8837.48	1418.25

We suppose the VES production function is

$$Y = Ae^{\lambda t}K^{\mu\frac{1}{(1+c)}}(L + \frac{b}{1+c}K)^{\mu\frac{c}{(1+c)}}. \tag{4.1}$$

We use the ISCA for the model's parameter estimation. The values of parameters are as follows: swarm size $N = 30$, the number of dimensions $D = 5$, $h = 0.90$, $g = 0.90$, $\alpha = 0.5, \beta = 0.5$ and the maximum number of iterations $T_{\max} = 500$. The SCA algorithm runs for 20 times. Figure 1 gives the optimal fitness value convergence curves of the SCA algorithm and the ISCA algorithm running for 20 times. The optimal fitness obtained with the SCA is 2.3886e+07 and the number of iterations was 406. The optimal fitness obtained with the ISCA algorithm is 1.5974e+07 and the number of iterations was 220. We can see that the ISCA algorithm convergence rate and precision is both superior to that of the conventional SCA algorithm.

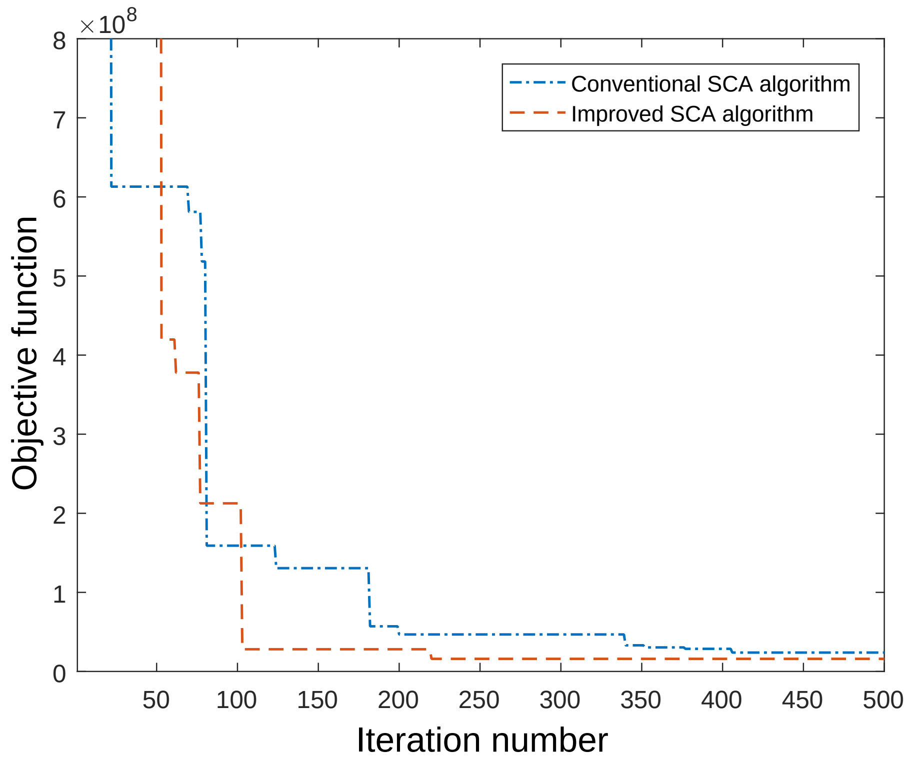

Figure 1. The chart of two algorithms' fitness variation curves with the changes of the number of iteration.

Using the ISCA, we get

$$(A, \lambda, b, c, \mu) = (5.7404, 0.0619, -0.1891, 0.3587, 0.9338), \tag{4.2}$$

and then have

$$\begin{aligned} Y &= Ae^{\lambda t}K^{\mu\frac{1}{(1+c)}}(L + \tfrac{b}{1+c}K)^{\mu\frac{c}{(1+c)}} \\ &= 5.7404e^{0.0619t}K^{0.6873}(L-0.1392K)^{0.2465}. \end{aligned} \tag{4.3}$$

We calculate and get the model's coefficient of determination $R^2 = 1 - \frac{\sum(Y_t - \hat{Y}_t)^2}{\sum(Y_t - \bar{Y})^2} = 0.9937$.

We can see that the model's coefficient of determination is close to 1, so the model's fitness precision is high.

We suppose (Y, K, L) all change on an exponential curve path, then get the path $L(t)$

$$\begin{cases} Y = 4.9789\mathrm{e}+03 e^{0.1061t}, \\ K = 1.4290\mathrm{e}+03 e^{0.0725t}, \quad (1 \le t \le n). \\ L = 327.8705 e^{0.0353t}. \end{cases} \tag{4.4}$$

i.e.

$$\begin{aligned} &(Y_0, K_0, L_0, a, h_1, h_2) \\ &= (4.9789\mathrm{e}+03, 1.4290\mathrm{e}+03, 327.8705, 0.1061, 0.0725, 0.0353), \end{aligned} \tag{4.5}$$

so

$$\begin{aligned} \Delta Y_A &= \tfrac{Y_0 \lambda}{a}(e^{an} - e^{a}) \\ &= 2.3719\mathrm{e}+04, \end{aligned} \tag{4.6}$$

$$\begin{aligned} \Delta Y_K &= \tfrac{\mu h_1 Y_0}{a(1+c)}[e^{an} - e^{a}] \\ &+ \frac{\frac{\mu c b h_1}{(1+c)^2} A^{\frac{1+c}{\mu c}} Y_0^{\frac{\mu c-1-c}{\mu c}} K_0^{1+\frac{1}{c}}}{\frac{1+c}{\mu c}\lambda + \frac{1+c}{c}h_1 + \frac{a(\mu c-1-c)}{\mu c}} [e^{(\frac{1+c}{\mu c}\lambda + \frac{1+c}{c}h_1 + \frac{a(\mu c-1-c)}{\mu c})n} - e^{(\frac{1+c}{\mu c}\lambda + \frac{1+c}{c}h_1 + \frac{a(\mu c-1-c)}{\mu c})}] \\ &= 1.4617\mathrm{e}+04, \end{aligned} \tag{4.7}$$

$$\begin{aligned} \Delta Y_L &= \frac{\frac{\mu c}{1+c} L_0 h_2 A^{\frac{1+c}{\mu c}} Y_0^{\frac{\mu c-1-c}{\mu c}} K_0^{\frac{1}{c}}}{\frac{a(\mu c-1-c)}{\mu c} + \frac{h_1}{c} + \frac{1+c}{\mu c}\lambda + h_2} \\ &\times [e^{(\frac{a(\mu c-1-c)}{\mu c} + \frac{h_1}{c} + \frac{1+c}{\mu c}\lambda + h_2)n} - e^{(\frac{a(\mu c-1-c)}{\mu c} + \frac{h_1}{c} + \frac{1+c}{\mu c}\lambda + h_2)}] \\ &= 1.9768\mathrm{e}+03. \end{aligned} \tag{4.8}$$

Then, we calculate and get that the technological progress' rate of contribution to economic growth from year 2000 to year 2020 is

$$\frac{\Delta Y_A}{\Delta Y_A + \Delta Y_K + \Delta Y_L} = 0.5884 = 58.84\%. \tag{4.9}$$

Capital's rate of contribution to economic growth from year 2000 to year 2020 is

$$\frac{\Delta Y_K}{\Delta Y_A + \Delta Y_K + \Delta Y_L} = 0.3626 = 36.26\%. \tag{4.10}$$

Labor's rate of contribution to economic growth from year 2000 to year 2020 is

$$\frac{\Delta Y_L}{\Delta Y_A + \Delta Y_K + \Delta Y_L} = 0.0490 = 4.90\%. \tag{4.11}$$

We can see that the factors' rates of contributions to economic growth from year 2000 to year 2020 are as follows: technological progress' contribution rate is 58.84%; capital's contribution rate is 36.26%; labor's contribution rate is 4.90%. It shows that Shanghai's economic growth mainly depends on technological progress, next on capital input and least on labor. The results are consistent with reality in Shanghai City, China.

5. Discussion

The VES production function has many characteristics. It meets the law of diminishing marginal returns and the law of diminishing marginal rate of technological substitution. Its isoquant curve has strict convexity. Its elasticity of substitution varies as K/L varies. It satisfies the Euler's theorem. Further, it shows good statistical properties in the case of empirical analysis.

Next, we analyze the elasticity of substitution of the VES production function built. The elasticity of substitution refers to the ratio of the percentage variation of factor rate to the percentage variation of the technological substitution rate caused by the percentage variation of the technological substitution rate in the case of fixed output. The elasticity of substitution of the VES production function built is

$$\rho = 1 + b\frac{K}{L} = 1 - 0.1891\frac{K}{L}. \tag{5.1}$$

Because $\frac{K}{L}$'s variation rises per year as time goes on, ρ presents a trend of decrease per year. We get the average elasticity of substitution of $\bar{\rho} = 0.1512$. From the elasticity of substitution, we can see it is largely deviates from 0, so it is not suitable to build a Leontief production function model; it also largely deviates from 1, so it is not suitable to build a C-D production function model either. The elasticity of substitution varies by year, so the VES production function model built in this paper is suitable. Considering that the elasticity of substitution is a linear function of time t , we get $\rho(t) = 0.4664 - 0.02865t$. The smaller ρ is, the smaller the probability of substituting labor with capital is. From the value of ρ calculated, we can see that the elasticity of substitution decreases by 0.02865 yearly on average, and the capital intensity of production increases as time goes.

Next, we perform an analysis on the contribution rates calculated according to the growth rate equation. With the VES production function model built, we calculate and get that the technological progress' growth rate is 6.19%, the capital's output elasticity is 1.9320 and the labor's output elasticity is 0.7687. It can be seen that 6.19% of the average annual growth rate of GDP, which is 10.99%, is generated by the technological progress, so the technological progress makes a great contribution to economic growth with a contribution rate of 58.84%. Capital's elasticity coefficient and average annual growth rate are relatively large (1.9320 and 8.08%, respectively), while the labor's elasticity coefficient and average annual growth rate are relatively small (0.7687 and 3.27%, respectively). Further, the contribution rate is influenced by the factor's average annual growth rate and elasticity coefficient together. In this case, in economic growth, the capital has less contribution with a contribution rate of 36.26% and the labor has the least contribution with a contribution rate of 4.90%.

6. Conclusions

This paper explores the VES production function, and for the model's parameter estimation, gives an intelligent optimization solving method, i.e. ISCA. With this algorithm, the model's parameter estimation has a faster rate of convergence and higher precision. As for practical applications, the paper gives a method for calculating the contribution rates of economic growth influencing factors scientifically and thus avoids large calculation errors brought by conventional methods.

This paper calculates the contribution rates of economic growth influencing factors of Shanghai City, China. Calculation results show that in the period from 2000 to 2020, technological progress had a contribution rate of 58.84%, capital had a contribution rate of 36.26%, and labor had a contribution rate of 4.90% to the economic growth in Shanghai City, China. Technological progress has become a main factor in promoting economic growth, followed by capital and labor. From the VES production function model built, we can see that, in Shanghai City, the elasticity of substitution of production factors has been declining, indicating that the capital intensity of production increases as time goes on.

The method proposed has great significance for the in-depth research on production function models, and promotes the practical applications of VES production function models in particular.

It should be noted that the VES production function built is a macro aggregate production function without being divided into the low-end or high-end production sectors. In fact, China's productivity has an unbalanced structure, so it is more proper to build a structural production function. In addition, this paper only considers the influences of technological progress, capital and labor on economic growth, but in fact there are many economic growth influencing factors such as energy consumption, system efficiency and so on. We can build the Superimposed production function model with a multiple structure and the extended production function model with more input factors in the future.

7. Recommendations

(1)We should make the element of innovation play a promoting role to achieve catch-up development. After the development of two periods, China's innovation urgently needs to move from an introduction and simulation stage to an overtaking stage to promote technological progress.

(2)We should redouble the efforts on market opening to make more products accessible to the international market. The key factor for success in reform and opening-up policy is opening up to the world, participating in international trade and introducing international capital to develop domestic economy.

(3)We should build a green China and attach great importance to the supply of ecological elements. In economic growth, the restrictions of resources and environment have become increasingly significant.

(4)We should deepen the market-oriented reform and improve the efficiency of system. From the perspective of systemic efficiency, we should deepen the market-oriented reform, continuously improve the socialist market economy system and promote systemic efficiency to provide a solid system guarantee for high-quality development.

(5)We should ensure the system can provide fair supplies for free to the broad masses of laborers. When entering into the high-quality development stage, from the perspective of the supply end, we need to improve the quantity and quality of laborers.

(6)We should stick to making a structural reform to help to bring about new economic growth points. Currently, we should lead the surplus low-end and high-end industry sectors to transfer to strategically new economic growth points and the factors of surplus industry sectors to become the points of new economic growth points, and finally make the economic structure reasonable and stable and realize high-quality economic development.

Use of AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

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Conflict of interest

The authors declare that there is no conflict of interest regarding the publication of this paper.

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