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Research article

A new scheme of dispersion charts based on neoteric ranked set sampling

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Abstract: There are certain areas of science and technology, such as agriculture, ecology, and environmental studies, that emphasize designing competent sampling strategies. The ranked set schemes, particularly the neoteric ranked set sampling (NRSS), are one method that meets such objectives. The NRSS provides plans that incorporates expert knowledge while choosing samples, which is beneficial. This study proposes a novel scheme for creating dispersion charts based on NRSS. The proposed scheme aims to improve the accuracy of dispersion charts by reducing the impact of outliers and non-normality in data sets. As a highly effective method in estimating population parameters, NRSS is used to select samples from the data set. The proposed dispersion charts are assessed based on individual performance measure criteria at shifts of different magnitudes. The dispersion charts created using this new scheme are compared with traditional dispersion charts, and the results demonstrate that the proposed scheme produces charts with higher accuracy and robustness. The study highlights the potential benefits of using NRSS-based dispersion charts in various fields, including quality control, environmental monitoring, and process control. An actual data application from a non-isothermal continuous stirred tank chemical reactor model further validates the simulation results.

Keywords: average runs length; control charts; inter quartile range; mean absolute deviation;

neoteric ranked set sampling
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1. Introduction

Statistical process control (SPC) employs a well-established tool known as a control chart to achieve the desired level of performance. Product quality is greatly affected by the variation observed in the production process over time. Control charts are used to properly control this variation and detect any assignable causes in a timely manner. These charts graphically display the monitoring process of a parameter related to a quality characteristic. Control charts are helpful in industrial statistics, as they help identify any assignable causes that may impact product quality during manufacturing. The main objective of a control chart is to promptly detect any assignable causes that lead to defective products. Control charts guarantee that the manufacturing process is in an in control (IC) state when there are no assignable causes and in an out-of-control (OC) state when there are, by monitoring any process discrepancies caused by assignable factors.

Control charting techniques are powerful tools used for monitoring process means and dispersion within the framework of a statistical process control (SPC) [1]. The basic Shewhart charts are effective tools for fault detection in the process and have undergone various modifications over time to cater to changing demands and requirements. The most notable developments in control charts are the cumulative sum (CUSUM) chart (see [2]) and the exponentially weighted moving average (EWMA) chart (see [3]), both of which are memory-based control charts that incorporate current and past process data. In any manufacturing process, monitoring and stabilizing process variability are crucial before tracking the process location [4]. Different dispersion estimators are used to design variability control charts, depending on the nature and requirements of the process [5]. Abbas et al. [6] proposed efficient scheme charts to identify faults while monitoring process dispersion, which was later enhanced by CUSUM charts to detect changes in the dispersion parameter of industrial processes [7].

Selecting suitable sampling plans plays a vital role in achieving pre-specified goals, such as stabilizing the variation in the process. There are certain areas of science, such as agriculture, ecology, and environmental studies, where the focus is designing efficient sampling designs. Ranked set sampling (RSS) is one method that fulfills such objectives. Initially, simple random sampling (SRS) was good choice [8], but SRS assures that each selected sample adequately represents the whole population [9]. The cases where the population needed to be more efficiently represented the alternative sampling strategies such as RSS are used [10]. The SPC community designed control charts for RSS that shows an efficient performance for SRS based charts. Later, the extensions of RSS, such as the median ranked set sampling (MRSS) and the extreme ranked set sampling (ERSS), are introduced under different sampling environments [11]. The improved control charts for monitoring process mean and variation are designed after incorporating RSS, MRSS, or ERSS schemes to enhance the quality of a product [12]. Abujiya and Muttlak [13] uncovered extensions of RSS and MRSS, such as double RSS (DRSS) and double MRSS (DMRSS) schemes, to increase the competence of the control chart while monitoring the process mean. Al-Nasser and Al-Rawwash [14] extended the idea of RSS and used a robust ranked set sampling scheme to design a location control chart. Al-Omari and Haq [15] offered a sampling plan named double L ranked set sampling for the estimation of the population mean. The new process dispersion chart based on RSS designed showed superiority over the existing chart [16].

Recently, Zamanzade, and Al-Omari [17] introduced a new RSS scheme named neoteric ranked set sampling (NRSS) that effectively estimates the population's mean and variance. The technical difference between NRSS and RSS is the structure and preparation for a single k^2 sample component. Once these arrangements are finalized, the k units are selected concerning their ranks to compose the final sample. The basic mathematical structure reveals that NRSS performs better than the RSS and SRS while estimating the mean and variance at different sample sizes, the association among main and auxiliary variables, and the probability distribution of different setups. Nawaz et al. [18] designed charts such as Shewhart, CUSUM, and EWMA using the NRSS to monitor process means. Koyuncu et al. [19] developed NRSS-based charts for monitoring means under bivariate unbalanced skewed distributions. Koyuncu and Karagoz [20] introduced a new structure of robust $\bar{X}$-chart and R-chart using NRSS. Another charting structure based on NRSS, CSCUSUM, and CSEWMA charts for the monitoring process efficiently outperformed the competing charts [21]. Recently, NRSS has been getting attention from the statistical process control community [22–25].

RSS method is a preferred sampling design where efficient sampling design are required. The RSS methods showed certain advantage over SRS method. The neoteric ranked set sampling (NRSS) is a recently developed method that comes up with a certain advantage over existing RSS methods, particularly to address the issue of variability in the process. The variation in any process is undeniable, where slight variations are acceptable, but significant variations are unacceptable in the process [26]. This work is significant because it proposes a new scheme for designing dispersion charts that are more accurate and informative compared to traditional charts and enable control structures to capture defects in the process parameter more accurately. The NRSS scheme allows researchers to collect more representative and precise data by selecting samples from subsets of a population that are more homogeneous than the population. The available work in the literature on NRSS is for monitoring process mean [18–25], while this study is more focused on the monitoring of process dispersion. This study considered different dispersion estimators (mean absolute deviation (*MAD)*, interquartile range *(IQR)*, range *(R)*, and standard deviation (*S)*) and designed corresponding dispersion control charts established on *(SRS*, *RSS*, *MRSS*, *ERSS*, and *NRSS)* sampling schemes. The novelty of this work lies in its innovative approach to data analysis using NRSS and its potential to improve the accuracy and precision of dispersion charts. The designs of dispersion charts are assessed using individual performance measures of an average run length. The run length standard deviation is used for the run length distribution's scatteredness. The designed setting of dispersion charts is evaluated at different sample sizes $(n=5$ and $7)$ and under perfect correlation $(\rho=1.0)$ and imperfect correlation $(\rho=0.25$, 0.5, and $0.75)$ ranking scenarios.

The remaining article is arranged in the subsequent lines: Section 2 provides the basic description of ranked set schemes. Section 3 presents the definition and formulas for dispersion estimators based on NRSS, and Section 4 contains the general dispersion control charting structures under NRSS and the description of performance measures. Section 5 includes the efficiency assessment of proposed charts under the standard scenario. Section 6 discusses the actual data application, and finally, Section 7 provides conclusions and recommendations.

2. Sampling schemes

This section defines in detail the ranked set sampling schemes considered in this study for monitoring process dispersion. The SRS scheme remains a simple choice for constructing and assessing control charts [27]. The sampling scheme of SRS is the most straightforward, assuming each sample contains an equal probability of occurrence. The precision of the estimates from SRS depends on the standard error of estimates, while the standard error is directly affected by the sample size. A new scheme of control charts designed for ranked set structures (i.e., RSS, MRSS, ERSS, and NRSS) shows superiority over the competing charts for SRS. The detailed description of ranked set schemes considered in this study is as follows.

2.1. Ranked set sampling

The idea of sampling design-based RSS, familiarized by McIntyre [10], is more efficient and effective than SRS regarding the standard error of estimates. The RSS schemes are of value when sampling designs needs careful consideration for sample selection, specifically, for the sciences such as agriculture, ecology, and environmental sciences where the focus is designing efficient sampling designs. For such a situation, the investigator needs to think beyond SRS and design more effective sampling schemes, such as the RSS schemes. The RSS sampling method is suitable for situations where units can be easily ranked based on their importance variable but are difficult and expensive to measure, as noted by Abujiya [28]. Under this scheme, sets are randomly selected, and the ranker should not be aware of the specific unit selected for a complete measurement. The RSS scheme considers the following pattern:

- Select n^2 elements randomly from under a study population built on the n sets of the n random samples.
- The observations selected for each set are arranged to weigh based on either the researcher's expertise or auxiliary variable.
- After the ranking process is finished, the observation with the lowest rank is selected from the first set. Then, the value with the second-lowest rank is chosen from the second set, and this process is repeated until the observation with the highest rank is selected from the last set, resulting in a total of n elements selected from n sets.
- The process continues m times to obtain $m*n$ observations through m cycles.

Let $Z_{(i:n)j}$ $(i=1,2,\ldots,n; j=1,2,\ldots,m)$ define as the i^{th} sample having the size of n observations from the i^{th} order statistic in the j^{th} cycle. Takahashi and Wakimoto [29] proposed an unbiased estimator for the mean and the variance under a perfect ranking scenario:

$$\bar{Z}_{[RSS]j} = \frac{1}{n}\sum_{i=1}^{n} Z_{(i:n)j} \text{ and } \sigma^2{}_{\bar{Z}_{[RSS]j}} = \frac{1}{n^2}\sum_{i=1}^{n} \sigma^2{}_{Z_{(i:n)}}, \tag{1}$$

where $\sigma^2{}_{Z_{(i:n)}} = E\{Z_{(i:n)} - E(Z_{(i:n)})\}^2$ is the population variance under the i^{th} order statistic having perfect ranking scenario. Dell and Clutter [29] estimated the mean and variance under the imperfect ranking scenario of RSS. Suppose (Z, Y) is the bivariate random sample, and the regression between Z and Y is linear. Let $Y_{(i:n)j}$ define as the auxiliary variable in the position of *the* i^{th} unit of the i^{th} ranked value in the j^{th} cycle and $Z_{(i:n)j}$ as the variable of interest. Then, the relation between Z and Y is defined as:

$$Z_{[i:n]j} = \mu_z + \rho\frac{\sigma_Z}{\sigma_Y}\left(Y_{[i:n]j} - \mu_Y\right) + \epsilon_{ij},\ i=1, 2, \ldots, n; j=1, 2, \ldots, m. \tag{2}$$

where μ_Y, μ_Z, σ_Y, and σ_Z are the means and standard deviations of Y and Z, while $\in_{ij} \sim N\left(0, {\sigma^2}_Z(1-\rho^2)\right)$, ρ is the correlation among auxiliary variable *(Y)* and primary variable *(Z)*. Now, the unbiased estimators for the mean and variance for the leading variable Z that is ranked using the auxiliary variable Y are given as:

$$\bar{Z}_{[IRSS]j} = \frac{1}{n}\sum_{i=1}^{n} Z_{(i:n)j} \text{ and } {\sigma^2}_{\bar{Z}_{[IRSS]j}} = \frac{{\sigma_Z}^2}{n}\left\{(1-\rho^2) + \frac{\rho^2}{n{\sigma_Y}^2}\sum_{i=1}^{n} {\sigma^2}_{Y(i:n)j}\right\}, \tag{3}$$

where ${\sigma^2}_{Y_{(i:n)}} = E\left\{Y_{(i:n)} - E\left(Y_{(i:n)}\right)\right\}^2$ is the population variance under an imperfect ranking scenario for the i^{th} order statistic [30].

2.2. Median ranked set sampling

Mostly ranking the units without error is more complicated than using an alternative sampling scheme of MRSS [31]. The ranked set scheme is an extension of RSS, and the procedure is as follows:
- First n^2 elements are randomly designated from the marked population and arranged like RSS.
- Second the selected observations in each set use expert opinion or the auxiliary variable.
- Now, for the case of odd observations, select the median observation element from each set.
- For the case of even observations, select *the* $n/2^{th}$ element with the smallest ranked from the first $n/2$ sets, and from the unique n/2 groups, select $(n + 2)/2^{th}$ ranked component that results in n units. The process carried till m times to obtain $m*n$ observations through m cycles.
- In the case of an even number of observations, choose the element ranked as $n/2^{th}$ with the smallest rank from the first $n/2^{th}$ sets, and from the remaining $n/2^{th}$ sets, select the component ranked as $(n+2)/2^{th}$, resulting in a total of n units. This process is repeated m times to obtain $m*n$ observations over m cycles.

2.3. Extreme ranked set sampling

Another alternative scheme of RSS is the ERSS [32]. The layout to select samples using ERSS is simple and convenient in extreme units located quickly. In ERSS, select the lowermost ranked elements from the lower fraction of the n sets and the uppermost rated parts from an upper fraction of n sets. ERSS exhibits superior properties in terms of efficiency for estimating the mean when compared to both RSS and MRSS. The procedure to select samples using ERSS as follows:
- From the population under study, select n^2 elements and arrange them like RSS and then rank concerning auxiliary variable or personal judgment.
- For the case of even samples, select the most minor ranked units from the first $n/2$ sets, and select the remaining $n/2$ sets largest ranked units.
- In the case of an odd sample, the first $(n-1)/2^{th}$ smallest units are selected from the first $(n-1)/2^{th}$ elements, and the remaining $(n-1)/2^{th}$ largest units are selected from the remaining elements. The median of the sample is then computed from one of the sets. This process is repeated m times to obtain a total of $n*m$ units.

2.4. Neoteric ranked set sampling

Zamanzade and Al-Omari [17] proposed an alternative to RSS, known as NRSS, in which all collected elements are ranked within a single set instead of being split into n collections of n^2 elements. The procedure of NRSS can be as follows:

- Select n^2 sampling units randomly from under study population.
- Using visual assessment or any other method, rank the n^2 sampling elements in increasing order of magnitude.
- For odd sample size, the units selected as: $[\frac{n+1}{2}+(j-1)n]$, $i=1, 2, \ldots, n$ and $j = 1, 2, \ldots, m$.
- For the case of even samples, select $[q+(j-1)n]^{th}$ units, where $q=\frac{n}{2}$, if j is even and $q=\frac{n+2}{2}$, if j is odd, $i=1, 2, \ldots, n$ and $j = 1, 2, \ldots, m$.
- Finally, repeat these steps m times to obtain the $n*m$ samples.

Let $us\ select\ Z_{\left[\frac{n+1}{2}\right]}$, $Z_{\left[\frac{3n+1}{2}\right]}$, $Z_{\left[\frac{5n+1}{2}\right]}$, …, $Z_{\left[\frac{2n^2-n+1}{2}\right]}$ tangible measurement for odd sample size n, and select the case of even sample size $Z_{\left[\frac{n+2}{2}\right]}$, $Z_{\left[\frac{3n}{2}\right]}$, $Z_{\left[\frac{5n+2}{2}\right]}$, …, $Z_{\left[\frac{2n^2+2}{2}\right]}$ units. The estimators of population mean and variance as:

$$\bar{Z}_{NRSS} = \frac{1}{nm}\sum_i^n \sum_j^m Z_{[q+(i-1)n]j} \tag{4}$$

for $i = 1, 2,\ldots,n$ and $j = 1,2,\ldots,m$.

$$Var(\bar{Z}_{NRSS}) = \frac{1}{nm^2}\sum_{i=1}^n Var\left(Z_{[q+(i-1)n]}\right) + \frac{2}{nm^2}\sum_{i<k}^n Cov\left(Z_{[q+(i-1)n]}, Z_{[q+(k-1)n]}\right). \tag{5}$$

The SPC research community is actively designing charts constructed on ranked set scenarios due to its superiority over SRS [17,18,33–35].

3. Dispersion statistics

This section provides a detailed description of different dispersion estimators to measure process variation. It is essential to evaluate the degree of variation in the process to know the scatteredness of estimated statistics from the center. There are numerous dispersion estimators used to design charting structures to evaluate process variability range (R), standard deviation (S), interquartile range (IQR), and median absolute deviation (MAD) [36]. The detail of dispersion statistics under the neoteric ranked set scheme is as below:

3.1. Range(R)

The range is the most straightforward dispersion statistic, computed by subtracting the minimum value from the maximum value. This type of dispersion statistic only relies on two extreme values. The mathematical expression for the j^{th} observation is as follows:

$$R_{NRSSj} = Z_{(n:n)j} - Z_{(1:n)j}, \quad i = 1,2,\ldots,n;\ j = 1,2,\ldots,m \tag{6}$$

where $Z_{(n:n)j}$ and $Z_{(1:n)j}$ represent the n^{th} order statistic for the subgroup of the size n in the j^{th} cycle.

3.2. Standard deviation (SD)

The commonly used and widely acknowledged measure of dispersion is the standard deviation.

This dispersion estimator has wide applications compared to the range, as it considers all the values of the data set rather than the extreme ones. The mathematical form of the estimator is as under:

$$S_{NRSSj} = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(Z_{(i:n)j} - \bar{Z}_j)^2}\ ,\ i = 1,2,\dots,n;\ j = 1,2,\dots,m, \tag{7}$$

where $Z_{(i:n)j}$ represents the i^{th} order statistic and $\bar{Z}_j$ is the mean of a sample of size *n* in the j^{th} cycle.

3.3. Interquartile range (IQR)

The IQR dispersion estimator deals with the extent of the middle 50% of the values in the data set. The IQR is computed by subtracting the lower quartile from the upper quartile of the desired data set. The mathematical form of the estimator is

$$IQR_{NRSSj} = \frac{Z_{(0.75:n)j} - Z_{(0.25:n)j}}{1.34898},\ i = 1,2,\dots,n; j = 1,2,\dots,m \tag{8}$$

where $Z_{(0.75:n)j}$ and $Z_{(0.25:n)j}$ are the upper and lower quartiles of the i^{th} order statistics in the j^{th} cycle.

3.4. Median absolute deviation (MAD)

MAD is a type of robust estimator to measure process dispersion. The mathematical formation is under:

$$MAD_{NRSSj} = 1.4826\, med\left|Z_{(1;n)j} - \tilde{Z}_j\right|,\ i = 1,2,\dots,n; j = 1,2,\dots,m \tag{9}$$

where $Z_{(1;n)j}$ represents the i^{th} order statistic and $\tilde{Z}_j$ as the median of the subgroup of size *n* in the j^{th} cycle, the term "*med*" stands for the median.

The dispersion estimators described above are frequently used in literature to design dispersion control charts [37].

4. Proposed dispersion control charts

This segment describes the structure of dispersion charts under NRSS. This study's control charts for other ranked set schemes are designed on similar lines. Let samples from bivariate normal such as pair of observations (Z_i, Y_i), are obtained from an in-control process where Z_i is the primary variable and Y_i is the auxiliary variable. Dispersion estimators are computed using sample observations under NRSS. Suppose $\widehat{\mathrm{H}}_{NRSS}$ is a dispersion statistic obtained using NRSS for Eqs (6)–(9), and then a pivotal quantity is defined as $\mathrm{T}_{NRSS} = \frac{\widehat{\mathrm{H}}_{NRSS}}{\sigma}$, where σ is the process standard deviation. The mean value of T_{NRSS} under NRSS as

$$E(\mathrm{T}_{NRSS}) = \frac{1}{\sigma}E\left(\widehat{\mathrm{H}}_{NRSS}\right) \tag{10}$$

Suppose $E(\mathrm{T}_{NRSS}) = \upsilon_2$ is the pre-specified value of a pre-specified sampling distribution; υ_2 entirely depends upon sample size *n*. Then, the unbiased estimator for process standard deviation is defined as $\hat{\sigma} = \frac{E\left(\widehat{\mathrm{H}}_{NRSS}\right)}{\upsilon_2}$. Then, the corresponding probability limits under NRSS are defined as

$$LCL_{NRSS} = T_{NRSS(\alpha/2)} \frac{E(\widehat{\text{H}}_{NRSS})}{\upsilon_2} \text{ with } \Pr\left(T_{NRSS} \leq T_{NRSS(\alpha/2)}\right) = \alpha/2 \tag{11}$$

$$UCL_{NRSS} = T_{NRSS[1-\alpha/2]} \frac{E(\widehat{\text{H}}_{NRSS})}{\upsilon_2} \text{ with } \Pr\left(T_{NRSS} \leq T_{NRSS[1-\alpha/2]}\right) = 1 - \alpha/2 \tag{12}$$

The quantile points $\left(\frac{\alpha}{2}\right) th$ and $\left(1 - \frac{\alpha}{2}\right) th$ depend upon the subgroup sample size for every calculated value of $\widehat{\text{H}}_{NRSS}$ [38].

5. Performance measures

The evaluation of proposed dispersion charts is based on evaluation measures named the average run length *(ARL)* and for scatteredness of run length, the standard deviation of run length *(SDRL)*, such as [39–42]. The ARL values represent the average number of units that must be observed before an out-of-control unit is detected. Two types of ARL values exist: the in-control ARL (ARL_0) and the out-of-control ARL (ARL_1). ARL_0 is the average number of samples plotted before a control chart signals an out-of-control condition under normal operating conditions. For example, if $ARL_0 = 1/\alpha$, where α is the false alarm rate (FAR), and α = 0.0027, then $ARL_0 = 1/0.0027 = 370$. This means that an out-of-control signal is detected after 370 samples, even if the investigation process is operating under normal conditions. The ARL_1 value is the mean number of samples before out-of-control state detection in a process after shift. Let $ARL_1 = \frac{1}{1-\beta}$ when $1\text{-}\beta$ is the probability to signal an out-of-control state, and if $1\text{-}\beta = 0.25$, then ARL_1 will be $1/0.25 = 4.0$; this reflects that the designed chart required 4.0 units before the identification of shift in the process. The *SDRL* is the measure of scatteredness of the run length and is computed for $SDRL_0$ (in-control situation) and $SDRL_1$ for the unstable process mean, for example, [43–45].

6. Simulative setting

A comprehensive simulation study is performed to evaluate the proposed charts under different sampling schemes. This study consists of four dispersion *(*i.e., *MAD, IQR, R, S)* charts designed and assessed based on altered selections of dispersion statistics described in Section 3. The different dispersion charts for this study for various sampling schemes as: for *SRS* as *MAD_SRS, IQR_SRS, R_SRS, S_SRS* charts; for *RSS* as *MAD_RSS, IQR_RSS, R_RSS, S_RSS* charts; for *MRSS* as *MAD_MRSS, IQR_MRSS, R_MRSS, S_MRSS* charts; for *ERSS* as *MAD_ERSS, IQR_ERSS, R_ERSS, S_ERSS* charts and for *NRSS* as *MAD_NRSS, IQR_NRSS, R_NRSS, S_NRSS* charts. For the case of a normal process, the corresponding coefficients and quantile points are calculated for the structure of control limits of different control charts. The dispersion statistics and corresponding control limits are computed independently after simulating the random samples (100,000) of size $n=5$ *and* seven from a bivariate normal distribution *(BVN)*, i.e., $BVN\left[\mu = \begin{pmatrix}\mu_X \\ \mu_Y\end{pmatrix}, \Sigma = \begin{pmatrix}1 & \rho \\ \rho & 1\end{pmatrix}\right]$. The false alarm rate *(α)* is selected as *0.005* that, which results in an ARL_0 of *200*. The shifts of different magnitude range from *1.1–3.0* (i.e., λ=*1.1*, *1.2*, *1.25*, *1.3*, *1.4*, *1.5*, *1.75*, *2.0*, *2.5*, *3.0*), and ARL_1 values are computed at each shift. The manufacturing process is affirmed as the unstable process for the case when process standard deviation *(σ)* diverts from the in-control state, such as σ_I to σ_C, i.e., $\sigma_C = \lambda\sigma_I$, while λ be the shift value arises in process standard deviation. For the in-control state $\lambda=1$, that means no shift in σ_I, indicating the stable state of the process dispersion, and whenever $\lambda>1$ represents an increase in the process dispersion (σ_I). The recital of the under-study charts is evaluated at various shift *(λ)* values for the case of perfect correlation ($\rho=1$) and the possibility of the imperfect correlation ($\rho\neq1$) for *RSS*, *MRSS*, *ERSS* and *NRSS*. We have selected different correlation coefficient values, as: $\rho=0.0, 0.25, 0.5, 0.75, 1$, along with various samples of size $n=5$ and 7. For the case when $\rho=0.0$, the ranked set samplings are essentially the same as *SRS*. The simulation results are obtained for every combination of n, ρ, and λ.

7. Performance comparison

We have performed extensive simulations to compare the performance of proposed and competing dispersion charts using different sampling schemes. Tables 1–4 present the ARL_1 and $SDRL_1$ values using different sampling strategies at varying levels of n, ρ, and λ. The graphical representation for selected *ARL* values is also provided in Figures *1–5*. The *ARL* values are plotted against λ for an improved pictorial evaluation. The main findings of the simulation study are as follows:

(1) Let's assume ARL_0=200, the ARL_1 values decrease with the increase of shift (λ) values for sampling schemes used in this study. As the sample size increases, the ARL_1 values decrease such as: for $n=5$ and $\lambda=1.1$, $ARL_1=126.8$, while for $n=7$ and $\lambda=1.1$, $ARL_1=120.7$ for *MAD_SRS,* for detail see Table 1.

(2) As the values of correlation coefficient (ρ) increase, the ARL_1 values decrease as: for *MAD_RSS* at $\rho=0.25$, $\lambda=1.1$ and $n=5$, the ARL_1 values is 116.2 while at same λ and n, the ARL_1 value at $\rho=0.5$ is 110.3, for detail see Table 1. The pattern is the same for other sampling schemes *(MAD_MRSS, MAD_ERSS,* and *MAD_NRSS)*. This pattern is the same for the perfect ranking scenario (i.e., $\rho=1.0$). The simulation results also shows that for the case of $\rho=1.0$, *MAD_RSS, MAD_MRSS, MAD_ERSS,* and *MAD_NRSS*) charts show superior performance compared to imperfect ranking scenarios ($\rho=0.25, 0.5, 0.75$) i.e., for *MAD_NRSS* the $ARL_1=79.5$ when $\rho=1.0$ and ($ARL_1=110.9$, $\rho=0.25$); ($ARL_1=103.0$, $\rho=0.5$); ($ARL_1=95.0$, $\rho=0.75$) (for detail see Table 1).

(3) *MAD_SRS* chart shows inferior results compared to other charts (*MAD_RSS, MAD_MRSS, MAD_ERSS,* and *MAD_NRSS*) charts such as: *MAD_SRS* gives $ARL_1=71.8$, while *MAD_RSS* gives $ARL_1=66.5$, *MAD_MRSS* gives $ARL_1=70.2$, *MAD_ERSS* gives $ARL_1=62.3$, *MAD_NRSS* gives $ARL_1=58.4$ at $\lambda=1.1$, $\rho=0.25$ and $n=5$) (for detail see Table 1).

(4) The increase in n improves the recital of dispersion charts, such as: when $n=5$ and $\lambda=1.1$ [(*MAD_SRS*; $ARL_1=126.8$); (*MAD_RSS*; $ARL_1=116.2$); (*MAD_MRSS;* $ARL_1=123.5$); (*MAD_ERSS;* $ARL_1=114.0$); (*MAD_NRSS*; $ARL_1=110.9$)] while at $n=7$ and $\lambda=1.1$ [(*MAD_SRS*; $ARL_1=120.7$); (*MAD_RSS*; $ARL_1=107.5$); (*MAD_MRSS*; $ARL_1=118.7$); (*MAD_ERSS*; $ARL_1=105.2$); (*MAD_NRSS*; $ARL_1=102.8$)] (for detail see Table 1).

(5) The performance of IQR-Charts is slightly better than MAD-Charts for various ρ and n under *SRS, RSS, MRSS, ERSS,* and *NRSS*, such as at $n=5$, $\rho=0.5$ and $\lambda=1.1$ the ARL_1 values for *MAD_NRSS* is *103.0* and the ARL_1 value for *IQR_NRSS* is *97.0*, which show the slight advantage of *IQR_NRSS* over *MAD_NRSS* (for detail see Tables 1 and 2).

(6) The *IQR_NRSS* chart shows significant improvement as n increases, such as: for the perfect ranking scenario under *NRSS* when $\lambda=1.1$ and $n=5$, $ARL_1=68.0$, while for $n=7$, $ARL_1=59.0$, indicating a *13.24%* decrease in ARL_1 value (for detail see Table 1).

(7) The R-Charts and S-Charts have superior performance over MAD-Charts and IQR-Charts under *SRS, RSS, MRSS, ERSS, and NRSS* at different values of ρ and n (for detail see Tables 1–4).

(8) The magnitude of difference in ARL_1 values for S-Charts under *SRS, RSS, MRSS, ERSS,* and *NRSS* is superior to MAD-Charts, IQR-Charts, and R-Charts with the increase in sample size. For example, *NRSS* at $\rho=1.0$ and $\lambda=1.1$, there are 12.96%, 13.24%, 24.46%, and 43.43% decreases in ARL_1 values for *MAD_NRSS, IQR_NRSS, R-NRSS,* and *S_NRSS* charts respectively when sample size *(n)* increased from *5* to *7* (see Tables 1–4).

Table 1. ARLs and SDRLs values for MAD- chart under different ranked set sampling schemes at ARL0=200.

n=5																		
Sampling Schemes	SRS		RSS								MRSS							
ρ	0		0.25		0.5		0.75		1		0.25		0.5		0.75		1	
λ	ARL	SDRL	ARL	SDRL	ARL	SDRL	ARL	SDRL	ARL	ADRL	ARL	SDRL	ARL	SDRL	ARL	SDRL	ARL	ADRL
1.1	126.8	123.2	116.2	113	110.3	107.7	102.8	106.8	95.3	93.9	123.5	122.1	119.8	116.1	115.8	113.4	108.8	104
1.2	71.8	70.7	66.5	64.1	60.7	56.8	57.8	55.3	51.9	49.4	70.2	69.8	68.2	65.6	66.1	64.8	62.4	60.7
1.25	57	56.6	50.8	49	46.1	44.7	44.3	42.7	40.3	38.3	54.9	52.3	52.4	50.6	50.1	49	47.8	46.2
1.3	43.9	42.7	40.5	38.5	37.2	35.7	35.9	34	31.2	30.4	42.3	41	41.8	40.6	40.3	39.6	37.1	36
1.4	28.2	27.8	26.4	25.7	24.5	23.4	21.7	20.8	19.2	18.1	27.2	26.8	26.6	25	25.2	24.4	22.2	21.3
1.5	20.2	19.8	18.7	17.6	16.7	15.8	14.7	13.8	12.5	11.6	20.6	19.1	19.5	18.7	18	17.2	16.9	15.3
1.75	10.2	9.5	9	8.3	8.4	8.1	7.2	7.9	6.5	5.5	10	9.3	9	8.2	8.1	8	7.1	6.4
2	6.4	5.8	5.6	5.2	5.6	5	5.3	4.7	4.6	4.1	6	5.6	6	5.2	6.1	5.6	5.4	4.9
2.5	3.5	3	3.1	2.8	3	2.5	3.1	2.5	2.6	2	3.5	3	3.3	2.8	3.3	2.8	3.2	2.7
3	2.5	2	2.2	2.1	2.2	1.5	2.2	1.6	2	1.4	2.4	1.8	2.4	1.8	2.4	1.8	2.4	1.9
n=7																		
λ	ARL	SDRL	ARL	SDRL	ARL	SDRL	ARL	SDRL	ARL	ADRL	ARL	SDRL	ARL	SDRL	ARL	SDRL	ARL	ADRL
1.1	120.7	116.9	107.5	103.4	103.7	101	95.5	94.4	90.5	88	118.7	117.5	113.1	111.4	103.9	100.2	95.4	92.3
1.2	66.9	64.1	60.9	58.1	57	55.8	51.6	49.9	40	39.7	65.3	64.4	62.4	60.3	58.9	56.2	50.8	48.3
1.25	50.6	48.4	43.2	42.4	41	39.4	38.7	36.8	30.5	29.7	46.4	45.4	44.8	42.6	42.9	40.7	36.8	35.9
1.3	39.8	38.5	31.3	30.5	30.2	29.6	28.1	26.4	23.3	21.1	34.5	33.1	33.8	31.8	32.9	31.8	29.3	27.8
1.4	25.8	24.1	20	18.5	18.5	17.3	16.6	15.9	14.8	13	22.4	20.7	21.7	19.3	20.5	19.4	18.9	17
1.5	17.9	16.4	14.4	13.5	13	11.9	12.3	11.7	8.8	8.2	15.3	14.2	14.7	13.7	13.6	12.9	12.7	11.4
1.75	9.1	8.6	6.4	6	5.6	5	5.1	4.1	4.1	3.4	7.2	6.5	6.6	5.9	5.7	5.1	5.2	4.6
2	5.6	4.2	4	3.4	3.6	2.9	3.7	3.1	2.7	2.2	4.5	4	4.1	3.6	4	3.8	4	3.4
2.5	3.1	1.9	2.4	1.9	2.3	2	2.3	1.7	1.7	1.2	2.5	1.8	2.4	1.8	2.4	1.9	2.4	1.8
3	1.8	1.2	1.7	1.1	1.6	1.1	1.7	1	1.4	0.7	1.9	1.2	1.8	1.2	1.9	1.2	1.8	1.2

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n=5																
Sampling Schemes	ERSS								NRSS							
ρ	0.25		0.5		0.75		1		0.25		0.5		0.75		1	
λ	ARL	SDRL	ARL	SDRL	ARL	SDRL	ARL	ADRL	ARL	SDRL	ARL	SDRL	ARL	SDRL	ARL	ADRL
1.1	114	112.6	106.4	105.8	98.7	95.5	91.8	89.2	110.9	112.8	103	101.8	95	92.8	79.5	74.8
1.2	62.3	60.6	59.4	58.1	49.6	47.2	42	41.4	58.4	57.8	56.1	54.9	52.6	51.3	35.3	34.2
1.25	45	43.8	43	41.5	36.3	35.7	30.2	29.2	43	42.1	42.9	41.2	40.4	39.4	24.4	24.6
1.3	37.2	36.5	35.7	34.2	28.9	27.4	24.5	21.2	36.2	35.7	35.9	33.9	33.6	32.9	17.2	18.7
1.4	23.7	22.9	22	21.2	19.3	18.1	15.5	12.4	22.1	21.6	20.8	19.2	19.4	18	9.3	8.6
1.5	16.6	15.1	15.1	13.8	13.2	12.1	10.1	8.9	15.7	14	13.8	11.9	12.8	11.9	6.1	5.2
1.75	8.7	7.2	7.5	6.1	6.6	5.6	4.6	4	8.1	7.2	7.2	6.8	6.5	5.1	3.1	2.5
2	6.3	5.1	5.6	4.1	4.3	3.7	2.9	2.3	5.6	4.8	5.4	5	4.9	4.4	1.9	1.3
2.5	3.5	3	3	2.5	2.6	2	1.8	1.2	3.2	2.6	3.2	2.6	2.9	2.4	1.3	0.6
3	2.6	2.1	2.4	1.8	2	1.4	1.5	0.8	2.3	1.7	2.4	1.9	2.1	1.6	1.1	0.3
n=7																
λ	ARL	SDRL	ARL	SDRL	ARL	SDRL	ARL	ADRL	ARL	SDRL	ARL	SDRL	ARL	SDRL	ARL	ADRL
1.1	105.2	104	98.9	95.5	92.5	88.9	82.6	81.4	102.8	104.9	99.5	95.1	91.3	90.7	69.2	67.8
1.2	55.8	54	55.8	53.7	45.9	43	33.4	32.7	53.6	52.4	53.5	52.6	48.2	46.5	20.3	19.2
1.25	42.3	41.7	39.7	37.2	33.8	31.8	24.4	22.8	41.8	40	39.1	38	36	35.5	12.6	11
1.3	30.9	29.2	28.2	26.9	20.9	19.7	18.3	17.8	31.1	29.9	29.6	29.1	27.3	27.7	8.4	7.9
1.4	19.4	18.1	16.9	14.4	14.7	12	11.2	9.7	19.9	18.8	18	17	16.3	15	4.8	4
1.5	13.4	12.4	11.8	9	10.9	8.5	7.1	5.5	12.7	11.8	12.7	11.1	11.9	10.8	3.5	3
1.75	5.5	4.2	4.9	3.5	4.2	3.6	3.1	2.4	6.7	6.4	6.5	5.7	5.7	5	1.4	1.8
2	3.5	2.8	3.4	2.4	2.8	2.3	2	1.4	4.2	3.6	4.2	3.7	3.6	2.9	1.1	0.7
2.5	2.1	1.9	2.1	1.9	1.7	1.1	1.4	0.9	2.5	1.9	2.4	1.8	2.2	1.6	1	0.2
3	1.6	1.1	1.5	1.2	1.4	0.8	1.2	0.6	1.8	1.1	1.8	1.2	1.6	1	1	0.1

Table 2. ARLs and SDRLs values for IQR- chart under different ranked set sampling schemes at ARL0=200.

n=5																		
Sampling Schemes	SRS		RSS								MRSS							
ρ	0		0.25		0.5		0.75		1		0.25		0.5		0.75		1	
λ	ARL	SDRL	ARL	SDRL	ARL	SDRL	ARL	SDRL	ARL	SDRL	ARL	SDRL	ARL	SDRL	ARL	SDRL	ARL	SDRL
1.1	120.5	119.1	115	112.5	106.1	105.3	100.2	98	91	89	120.4	120.6	116.5	114.2	109.1	106.6	100	103.1
1.2	64	62.7	62.5	60.8	57.7	55.4	51.7	50.9	43.6	41.2	62.3	60.8	60.9	60.7	54.4	53.9	50.1	48.3
1.25	48.6	47.9	47.4	46.9	44.1	43.8	38.5	36.7	33.6	32.2	47.8	46.8	46.8	50.7	46	44.1	39.5	38.8
1.3	38.6	37.2	36.4	34.8	34.9	33.6	27.1	26.2	25.6	24.8	38.1	37.6	37.6	38.5	34.1	33.6	30.2	29.9
1.4	24	22.7	23	21.7	22.3	21.2	18.3	17.1	16	15.3	24.3	23.1	23.7	27.1	22.4	20.9	19.2	18.3
1.5	16.9	15.7	16	15.5	15.8	15.8	13.3	12.5	11.4	10.8	17.6	15.2	17.8	16.8	16.1	15.8	14.2	13.1
1.75	8.7	8	8.7	8.1	8.7	7.7	7.4	7.1	5.6	5	9.3	7.8	9.2	8.9	8.3	7.5	7.8	7.1
2	5.6	4.9	5.3	4.8	5	4.5	4.4	3.8	3.6	2.9	5.9	5	5.5	5.2	5.4	4.9	5.8	5.4
2.5	3.1	2.6	3.2	2.5	3.1	2.6	2.7	2.2	2.3	1.7	3.4	2.5	3.1	2.5	3.1	2.6	3.2	2.5
3	2.2	1.7	2.3	1.6	2.3	1.7	2.1	1.5	1.7	1	2.3	1.6	2.4	1.8	2.3	1.8	2.3	1.7
n=7																		
λ	ARL	SDRL	ARL	SDRL	ARL	SDRL	ARL	SDRL	ARL	SDRL	ARL	SDRL	ARL	SDRL	ARL	SDRL	ARL	SDRL
1.1	115.9	113.8	109.5	107	101.6	98	98.3	91.1	84.4	82.9	112	110.4	110.9	107.2	103.1	113.3	93.1	109.4
1.2	61.3	59.5	58.5	56.5	55.1	53.4	48.6	49.2	37.5	36	61.7	59	58.5	57.7	50.8	48.4	46.1	45.2
1.25	45.2	42.8	43.2	41.1	37.6	35.4	32.5	33.1	25.6	23.5	44.6	41.1	41.7	40.3	38.9	40.2	37.2	35.8
1.3	35.6	34.1	33.5	32.8	29.9	26.2	25	23.8	20.6	18.9	34.6	31.4	33.4	30.7	30.2	29.6	27.4	26.5
1.4	22.6	20.9	20.4	19.9	17.8	15.5	15.6	15	12.9	10.2	20.5	17.3	19.8	17.8	17.9	17	17.8	16.7
1.5	14.4	13.8	13.5	12.1	11.8	9.9	10	10.8	7.1	5.6	14	11.5	12.8	11.4	12.1	11.8	11.7	10.2
1.75	7.5	6.9	6.1	5.8	5	4.3	4.6	4	2.8	2.3	6.8	5.1	6.5	5.1	5.6	5	5.6	5.1
2	4.4	3.9	3.4	2.9	3.2	2.6	3	2.6	1.8	1.2	3.5	2.9	4.3	2.9	3.4	2.9	3.3	2.9
2.5	2.9	1.6	2.1	1.5	2	1.4	1.8	1.3	1.3	0.6	2	1.4	2.2	1.4	2	1.4	2	1.4
3	1.8	0.9	1.6	0.9	1.4	0.8	1.4	0.7	1.2	0.4	1.5	0.9	1.5	0.9	1.6	1	1.5	0.8

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n=5																
Sampling Schemes	ERSS								NRSS							
ρ	0.25		0.5		0.75		1		0.25		0.5		0.75		1	
λ	ARL	SDRL	ARL	SDRL	ARL	SDRL	ARL	SDRL	ARL	SDRL	ARL	SDRL	ARL	SDRL	ARL	SDRL
1.1	113.5	112.5	103.7	101.1	92.6	90.8	83.7	80.2	101.2	101.1	97	96.4	90.6	89.4	68	66.7
1.2	60.4	58.9	55.1	53.9	43.4	41.5	35.1	33.2	47.2	47.6	46.1	45.7	37.9	36.6	20.8	18.2
1.25	41.4	40.6	42.9	41.3	35.7	34.9	28.9	27.5	33.2	32	32.2	31.5	26	25.2	14.1	13.5
1.3	33.6	32.4	33	32	26.7	25	22.5	21.3	25.1	25	24.4	23.2	18.1	17.6	9	7.6
1.4	21.8	20.5	21.6	20.6	18.3	17.1	14.1	13.5	14.6	14	13.9	12.5	10.7	10.2	5.4	3.8
1.5	15	13.8	14.8	13.1	11.9	10.7	9	8.3	9.3	8.7	9	8.5	6.9	6.4	3.5	2.6
1.75	8.4	7.9	8	7.5	6.7	6.1	4.2	3.5	4.5	4	4.3	3.8	3.3	2.7	1.9	1
2	5.4	5.1	5.1	4.6	4	3.4	2.8	2.3	2.8	2.2	2.7	2.2	2.2	1.6	1.2	0.4
2.5	3.2	2.7	2.9	2.4	2.5	1.8	1.7	1.1	1.7	1.1	1.7	1	1.4	0.8	1	0.1
3	2.2	1.8	2.3	1.7	1.9	1.3	1.3	0.7	1.4	0.7	1.3	0.7	1.2	0.5	1	0.1
n=7																
λ	ARL	SDRL	ARL	SDRL	ARL	SDRL	ARL	SDRL	ARL	SDRL	ARL	SDRL	ARL	SDRL	ARL	SDRL
1.1	105.1	103.8	94.4	92.1	90.3	89.4	76.6	74.6	100.4	100.8	90.2	89.2	83.2	82.4	59	57.9
1.2	53.3	51.5	41.7	39.1	41.1	40	30.3	28.9	46.2	45.4	39.9	38.8	34	32.5	16.4	14.1
1.25	40.7	38.4	29.4	29.2	26.4	25.3	21.2	19.2	32.2	31.2	28.8	27	24.6	23.7	11.3	9.7
1.3	29.9	28.7	23.4	21.4	19.6	18.5	16.8	14.1	24.2	26	22.6	21.9	18	17.7	7	5.5
1.4	17.8	16.3	13.6	13.2	11	10.1	9.6	8.9	14.3	14.8	12.6	11.3	10.5	9.1	4.7	3.1
1.5	12.2	12.1	9.6	8.6	7.6	7.4	5.4	4.8	10.2	9.8	8.7	8.1	7.6	7	2.8	1.6
1.75	5.7	5.3	4.6	4	3.7	3.1	1.8	1.2	4.7	4.2	4	4	3.7	3.1	1.2	0.8
2	3.6	2.9	3.1	2.5	2.4	1.8	1.4	0.7	3.1	2.5	2.9	2.3	2.4	1.8	1	0.3
2.5	2	1.4	1.7	1.1	1.5	0.9	1.1	0.3	1.8	1.2	1.8	1.2	1.6	0.9	1	0.1
3	1.5	0.8	1.5	0.8	1.2	0.6	1	0.2	1.4	0.8	1.4	0.8	1.3	0.6	1	0.1

Table 3. ARLs and SDRLs values for R- chart under different ranked set sampling schemes at ARL0=200.

n=5																		
Sampling Schemes	SRS		RSS								MRSS							
ρ	0		0.25		0.5		0.75		1		0.25		0.5		0.75		1	
λ	ARL	SDRL	ARL	SDRL	ARL	SDRL	ARL	SDRL	ARL	SDRL	ARL	SDRL	ARL	SDRL	ARL	SDRL	ARL	SDRL
1.1	112.4	109.8	106	103.8	97	95.4	93.6	92.7	80.1	78.1	109	105.8	103.3	100.3	101.7	101.1	90.6	93.6
1.2	51.7	53.6	48.6	49.1	44	43.2	42.6	42.8	35	41.3	50	48.8	48.7	47.6	46.7	44.2	42.1	40.4
1.25	40.2	38.6	37.5	34.8	32.4	35.4	30.5	30.2	26.7	28.4	39	38.9	36.3	32.3	35.2	34.8	30.9	29.8
1.3	30.5	29.9	29.5	30.2	24.6	23.2	24	22.8	21.9	22.8	29.1	27.9	28	25.1	25.8	23.5	23.5	22.6
1.4	19.5	19.2	17.3	16.5	17.2	15.9	17	17.1	15.5	16.9	18.6	16.9	18.5	16	18.5	17.7	17.8	15.4
1.5	14	13.3	12.5	12.1	11.5	10.5	11.1	10.9	10.5	10.8	12.9	12.2	12.7	11.7	12.8	11.8	11.1	10.9
1.75	6.8	6.3	6.3	5.8	5.8	5.5	5.7	5.5	5.6	5.2	6.4	6	7.2	6.4	6.8	6.4	5.9	5.1
2	4.3	3.7	4	3.4	3.8	3.2	4	3.5	3.6	3.5	4	3.4	4.3	3.8	4.1	3.6	3.6	2.1
2.5	2.5	1.9	2.4	1.7	2.2	1.6	2.3	1.8	2.3	1.6	2.3	1.7	2.5	1.9	2.5	1.9	2.1	1.9
3	1.9	1.3	1.8	1.2	1.8	1.2	1.8	1.2	1.7	1	1.8	1.2	1.9	1.3	1.9	1.3	1	0.5
n=7																		
λ	ARL	SDRL	ARL	SDRL	ARL	SDRL	ARL	SDRL	ARL	SDRL	ARL	SDRL	ARL	SDRL	ARL	SDRL	ARL	SDRL
1.1	105.2	109	96.7	96.1	90.1	86.6	85.8	84.8	78.5	68.9	104.4	101.4	96.8	94.1	91.6	81.8	85.1	82.4
1.2	48.7	47.1	45.3	45.7	42.2	40.2	39.7	37.6	33.9	31.5	48	47.5	45.9	43.9	43.4	39	38	37.9
1.25	37.9	35.2	36.7	36.8	32	30	29.6	28.3	20.4	22.7	37.9	35.7	35.7	34.7	33.6	29.3	24.9	23.9
1.3	28.1	26.3	26.9	26.1	24	23.1	21.5	20.8	14.8	17.4	27.1	25.2	27.4	26.9	23.9	21	19	18.9
1.4	17.5	16.1	16.4	16.2	15.6	15.1	14.6	14.2	9.8	11.4	17.1	15.8	17.2	15.8	16.2	13.7	15.7	14.1
1.5	12.3	11.7	11.7	11.2	10.7	10.1	9.8	8.6	8.4	7.3	12.3	10.5	11.5	10.8	10.4	9.1	10.7	10.2
1.75	5.5	5.5	5.4	4.5	5.5	4.9	5.2	4.8	4.4	3.8	6.4	5.3	5.9	5.6	5.9	4.3	5.9	5.3
2	3.6	3.1	3.4	2.8	3.4	2.7	3.4	2.8	2.9	2.4	3.9	2.8	3.7	3.1	3.9	2.6	3.5	3
2.5	2.2	1.6	2.1	1.4	2	1.4	1.9	1.4	1.8	1.1	2.4	1.5	2.1	1.5	2.1	1.3	2.1	1.5
3	1.7	1.1	1.6	0.9	1.5	0.8	1.5	0.8	1.4	0.7	1.8	1	1.6	1	1.5	0.9	1.6	1

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n=5																
Sampling Schemes	ERSS								NRSS							
ρ	0.25		0.5		0.75		1		0.25		0.5		0.75		1	
λ	ARL	SDRL	ARL	SDRL	ARL	SDRL	ARL	SDRL	ARL	SDRL	ARL	SDRL	ARL	SDRL	ARL	SDRL
1.1	100.7	98.9	94	101.4	90.7	87.7	77.3	75.6	99.3	97.7	91.7	92.1	87.4	86	60.5	60.2
1.2	44.7	42.5	41.6	51.4	41	40.9	33.2	31.4	42.4	41.9	39.4	38.3	36.3	36.1	17.3	16.6
1.25	36.3	35.1	30.1	39.2	29.9	28	23.5	22	34.6	32.7	29.1	28.4	25.4	24.9	10.3	9.8
1.3	27.9	26.5	22.7	28.3	23.2	21.7	18.7	15.6	25	23.5	21.5	20.2	19.8	17.5	7	6.5
1.4	17.1	16.7	16.5	18.7	16.3	16	13.7	12.2	16.9	15.3	13.7	13.2	13.6	10.3	3.7	3.2
1.5	11.7	11.5	10.7	13	9.9	9.7	8.3	7.7	10	9.3	8.8	8.2	6.9	6.3	2.4	1.8
1.75	6.2	5.5	5.5	5.8	5.5	5.2	4.4	4.1	5.3	3.8	4.1	3.7	3.8	2.7	1.4	0.7
2	4.1	3.7	3.7	3.5	3.5	3.1	2.8	2.2	3.7	2.1	2.6	2	2.2	1.6	1.1	0.3
2.5	2.4	1.8	2	1.8	2.1	1.5	1.7	1.1	1.9	1.1	1.6	1	1.4	0.8	1	0.1
3	1.9	1.3	1.7	1.3	1.6	0.9	1.3	0.7	1.4	0.7	1.3	0.7	1.2	0.5	1	0.1
n=7																
λ	ARL	SDRL	ARL	SDRL	ARL	SDRL	ARL	SDRL	ARL	SDRL	ARL	SDRL	ARL	SDRL	ARL	SDRL
1.1	93.5	92.2	86	84.6	80.4	78.5	70.2	68.6	88.1	78.7	78.4	73.5	74.1	72.2	45.7	40.1
1.2	42.1	40.2	39.1	38.1	36.3	34.1	29.6	27.2	37.5	31.1	31.6	34.9	29.7	28	11.8	8.2
1.25	33.9	31.5	29.2	27.6	27.1	25.6	18.1	17.6	27.6	26.3	25.9	24.3	21.9	19.5	7.1	4.6
1.3	24.2	23.3	21.1	19.1	19.3	18.4	12.1	11.7	19.6	18.1	18.2	17.6	16.9	14.2	5.4	2.8
1.4	15.7	15	14.5	13.4	12.4	11.9	8	8.1	11	9.6	10.4	9.9	9.9	7.2	2.9	1.3
1.5	10.7	9.4	9.6	9.1	8	7.5	6.5	5.9	7.1	6.7	6.6	6.2	6	4.5	2	0.7
1.75	5.4	5	5	4.5	3.8	3.2	3	2.5	3.3	2.8	3.1	2.6	2.5	1.9	1.1	0.2
2	3.4	2.7	3	2.6	2.5	2	1.8	1.1	2.1	1.6	2	1.4	1.7	1	1	0.1
2.5	2	1.4	1.8	1.2	1.5	0.8	1.3	0.6	1.4	0.7	1.3	0.7	1.1	0.5	1	0
3	1.5	0.9	1.4	0.8	1.3	0.6	1.1	0.3	1.2	0.4	1.1	0.4	1.1	0.3	1	0.1

Table 4. ARLs and SDRLs values for S- chart under different ranked set sampling schemes at ARL0=200.

n=5																		
Sampling Schemes	SRS		RSS								MRSS							
ρ	0		0.25		0.5		0.75		1		0.25		0.5		0.75		1	
λ	ARL	SDRL	ARL	SDRL	ARL	SDRL	ARL	SDRL	ARL	SDRL	ARL	SDRL	ARL	SDRL	ARL	SDRL	ARL	SDRL
1.1	109.5	106.6	101.8	100	99.1	99	95.6	91.1	83.3	79.4	112	104.2	106.8	104.9	98.4	96.5	93.4	84.5
1.2	43.3	42.4	42.8	43.7	42.1	40.7	39.5	40.3	33.5	32.1	44	43.5	45	43.2	41.5	45.9	38.3	40.9
1.25	30	28.7	29.4	28.8	30.1	29.3	28.5	27.8	24.1	24.4	30.8	30	32.1	33	29.7	30	27.4	26.8
1.3	22.8	21.9	21.7	20.7	22.1	21	20.7	20	16.8	15.2	23.1	23.8	23.8	23	21.1	22.1	20.6	19.6
1.4	12.1	11.6	12.2	11.6	12.1	12.9	10.9	10.4	8.6	7.5	13.1	12	12.7	12.2	12.7	11.4	12.7	12.7
1.5	9	8.6	8.5	7.8	8.3	7.7	7.4	7.5	5.7	5.2	9.1	8.9	8.5	8.8	8.3	7.5	8.2	7.7
1.75	4.2	3.2	4	3.6	3.7	3.1	3.1	3	2.4	2.1	3.9	3.5	3.7	3.6	4	3.6	4.1	3.5
2	3	2.1	2.6	2	2.5	2	2	1.8	1.8	1.4	2.6	2.1	2.7	2.1	2.4	1.8	2.5	1.9
2.5	1.7	1	1.6	1	1.6	1	1.3	0.9	1.2	0.6	1.6	1	1.7	1.1	1.6	0.9	1.6	1
3	1.4	0.6	1.3	0.6	1.3	0.6	1.1	0.6	1.1	0.4	1.3	0.6	1.3	0.6	1.3	0.6	1.3	0.6
n=7																		
λ	ARL	SDRL	ARL	SDRL	ARL	SDRL	ARL	SDRL	ARL	SDRL	ARL	SDRL	ARL	SDRL	ARL	SDRL	ARL	SDRL
1.1	98.6	97.5	91.2	90.5	87.2	81.6	82.4	93.7	75.5	75.2	95.8	95.5	90.7	88.5	88.4	90.2	83.5	89.7
1.2	37.2	35.4	34	32.8	32.9	32.9	30.7	31.5	25.2	24	37.5	38.4	34.5	33.2	34.7	34.1	31.2	37.1
1.25	26.4	25.8	22.7	22.8	22.1	21.1	20.1	19.4	17.5	15.9	25	24.1	23	23.3	23.7	23.5	22.1	22.9
1.3	18.8	17.2	16.6	16	15.6	14.6	14	13.8	11.8	10.2	17.8	18.2	15.8	14.5	17	16	16.3	15.7
1.4	10.9	8.7	9.5	8.2	8.8	8	7.8	7.5	6.1	5.5	9.3	9	9.1	7.7	9.1	8.7	9	9
1.5	7.2	6.7	5.8	5	5.3	4.9	4.9	4.5	3.8	3.3	6	5.5	5.5	4.8	5.9	5.5	6.1	5.9
1.75	3.6	3	2.7	2.2	2.7	2.1	2.4	1.9	1.9	1.2	2.8	2.3	2.7	2.1	2.9	2.2	2.9	2.3
2	2.8	1.4	1.8	1.2	1.8	1.1	1.7	1	1.3	0.6	1.9	1.2	1.8	1.1	1.9	1.3	1.8	1.3
2.5	1.3	0.6	1.3	0.6	1.3	0.5	1.2	0.4	1.1	0.3	1.3	0.7	1.3	0.6	1.3	0.6	1.3	0.6
3	1.1	0.3	1.1	0.3	1.1	0.4	1.1	0.3	1	0.1	1.1	0.4	1.1	0.4	1.1	0.4	1.1	0.3

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n=5																
Sampling Schemes	ERSS								NRSS							
ρ	0.25		0.5		0.75		1		0.25		0.5		0.75		1	
λ	ARL	SDRL	ARL	SDRL	ARL	SDRL	ARL	SDRL	ARL	SDRL	ARL	SDRL	ARL	SDRL	ARL	SDRL
1.1	98.9	97.6	99.3	98.4	85.6	84.4	71.4	68.2	96.9	96.3	90.4	90.3	80.7	80.8	57.1	56.2
1.2	40.5	45	40.1	40.3	34.5	35.8	27	25.6	39.3	38.4	36.4	35.4	31.7	30.1	15.4	14.8
1.25	28	31.6	28.4	30	24.3	23.3	18.3	17.6	29.7	28.7	27.3	26.7	23	22.4	9.4	8.9
1.3	20.4	22.8	20.4	20	17.5	16.7	13.8	12.9	20	18.5	19.5	19	16.4	15.9	6.2	5.7
1.4	11.6	12.9	12	11.3	9.1	8.9	6.4	6.1	11.2	11.8	11	10.5	9	8.6	3.3	2.8
1.5	8.2	7.7	7.9	7.7	6.3	5.8	4.2	4	8	7.8	7.7	7.3	6	5.5	2.2	1.6
1.75	4.1	3.8	3.8	3.1	3.7	2	2.3	1.7	4.1	3.6	3.7	3.2	3	2.4	1.3	0.6
2	2.6	1.8	2.5	2	2.4	1.3	1.5	0.9	2.6	2	2.4	1.8	2	1.4	1.1	0.3
2.5	1.6	0.9	1.5	0.9	1.6	0.7	1.2	0.4	1.6	1	1.6	0.9	1.4	0.7	1	0.1
3	1.3	0.7	1.3	0.6	1.1	0.4	1.1	0.2	1.3	0.6	1.3	0.6	1.2	0.4	1	0
n=7																
λ	ARL	SDRL	ARL	SDRL	ARL	SDRL	ARL	SDRL	ARL	SDRL	ARL	SDRL	ARL	SDRL	ARL	SDRL
1.1	88.1	85.4	80.4	78.1	70.5	69.3	51.7	50.6	82	80	75.5	74.7	64.3	64.5	32.3	31.6
1.2	32.8	30.2	30.7	30.3	24.9	22.8	14.4	13.7	30.9	30.5	28.4	27	23	22.6	6.7	6
1.25	20.8	18.4	19.4	19.8	15.8	14.7	9.4	8.4	19.9	18.1	19.8	18.3	15	14.6	3.9	3.4
1.3	13.7	12.9	13.9	12.3	11.8	9.3	6.2	5.8	12.9	11.2	13.4	12.9	10.6	10.3	2.7	2.1
1.4	7.6	7.3	7.7	7.2	6.5	4.9	3.4	2.7	7.4	7.2	7.1	6.5	5.9	5.3	1.6	1
1.5	5.2	4.6	4.7	4.3	4	3	2.2	1.7	5	5.2	5.2	4.7	3.8	3.3	1.2	0.5
1.75	2.7	2.1	2.4	1.9	2.4	1.3	1.3	0.6	2.7	2.2	2.6	2.1	2	1.5	1	0.1
2	1.8	1.1	1.7	1.1	1.4	0.7	1.1	0.3	1.9	1.3	1.8	1.2	1.5	0.8	1	0
2.5	1.3	0.6	1.2	0.5	1.1	0.3	1	0.1	1.3	0.6	1.7	0.6	1.1	0.4	1	0
3	1.1	0.4	1.1	0.3	1	0.2	1	0	1.1	0.4	1.1	0.3	1	0.2	1	0

The simulation results described above indicate improvement in the performance of different dispersion *(MAD, IQR, R,* and *S)* charts under *NRSS* compared to charts designed using existing sampling schemes, as seen in detail in Tables 1–4. The outcomes of performance measures reflect that the MAD-Charts are inferior compared to other charts, while S-Charts showed superiority over other charts under different sampling schemes. The performance of IQR-Charts is slightly better than MAD-Charts, while S-Charts have an advantage over R-Charts. The rise in sample size *(n)* impacts the recital of different dispersion charts and significantly decreases the ARL_1 values significantly, for detail see Tables 1–4. The performance of varying dispersion (*MAD*, *IQR*, *R*, *S*) charts is on the lower side for *SRS*, while their performance is efficient for *NRSS*. The ranked set scheme of *RSS* has an advantage over *MRSS*, while *NRSS* showed better performance than *ERSS*. Among the ranked set schemes, *MRSS* gives inferior results for different dispersion charts, while *ERSS* performs better than *RSS*. In this study, among all sampling schemes, *NRSS* is revealed to be the best for the monitoring of process variation.

The dispersion charts under a perfect ranking scenario ($\rho=1$) provide efficient results compared to imperfect ranking scenarios ($\rho=0.25$, 0.5, 0.75). The changes in sample size *(n)* also have an impact on the performance of different dispersion charts under different ranking scenarios ($\rho=0.25,0.5,0.75,1.0$). The simulation results conclude that when n=7, S_NRSS charts have superiority over all the other charts for perfect and imperfect ranking scenarios.

We have also presented the ARL_1 values graphically for different dispersion (*MAD, IQR, R,* and *S*) charts using *SRS, RSS, MRSS, ERSS,* and *NRSS* at a fixed $ARL_0=200$, for detail see Figures 1–5. The ARL_1 values are plotted on the *y*-axis against different shifts *(λ)* plotted on *the x*-axis. Figure 1 presents the ARL_1 comparison of the MAD-Chart for different sampling structures considered in this study under a perfect ranking scenario *(ρ=1.0)*. The curve under *NRSS* (the green line) is on the lower side, indicating the superiority of MAD_NRSS over MAD_SRS, MAD_RSS, MAD_MRSS, and MAD_ERSS charts. The magnitude of the difference is small when $n=5$ (ss Figure 1(a)), while this difference is high when $n=7$ (see Figure 1(b)), particularly when $\lambda=1.2$, 1.25, and 1.3. Figure 2 presents the ARL_1 values for IQR-Charts using different sampling schemes under a perfect ranking scenario. The pattern and magnitude of the difference are almost the same with MAD-Charts with a slight variation that indicates the superiority of IQR_NRSS chart over IQR_SRS, IQR_RSS, IQR_MRSS, and IQR_ERSS charts (see Figures 2).

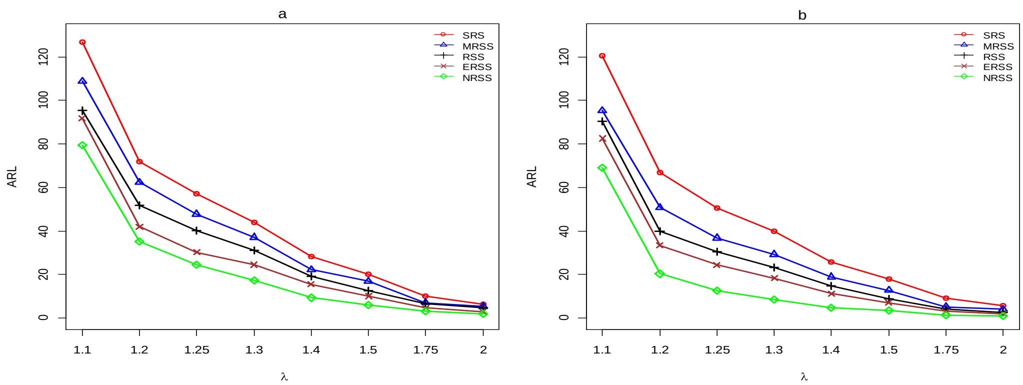

Figure 1. ARL curves MAD-chart under different sampling schemes for *(a)* $n=5$; *(b)* $n=7$ at $ARL_0=200$ and $\rho=1.0$.

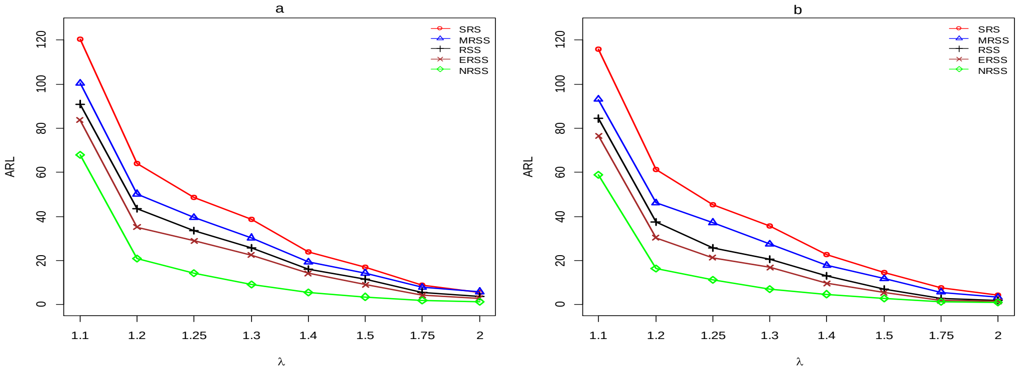

Figure 2. ARL curves IQR-Chart under different sampling schemes for *(a)* $n=5$; *(b)* $n=7$ at $ARL_0=200$ and $\rho=1.0$.

Figure 3 displays the ARL_1 values for R-Charts using different sampling schemes under a perfect ranking scenario; similarly, the performance of the R_NRSS chart is best compared to R_SRS, R_RSS, R_MRSS, R_ERSS, and R_NRSS charts. Figure 4 comes up with the same line pattern reflecting the best performance of the S_NRSS chart under a perfect ranking scenario. Figure 5 presents the ARL_1 values of different dispersion (MAD_NRSS, IQR_NRSS, R_NRSS and S_NRSS) charts using NRSS when $(\rho=1.0)$ and $n=5$ and 7. Figure 5(a) reflects that the IQR_NRSS has a slight advantage over the MAD_NRSS chart, while the R_NRSS and S_NRSS charts show almost equivalent performance. Figure 5(b) demonstrates the dominance of the S_NRSS chart above the rest of the charts here with a significant magnitude of difference. Figure 6 present the ARL_1 comparison of different dispersion control charts under RSS at different level of correlation coefficients ((a) $\rho=0.25$; (b) $\rho=0.50$; (c) $\rho=0.75$; (d) $\rho=1.0$). The curves on the lower side indicate the improved detection ability of the control chart, such as the green curve representing S_RSS chart is on the lower side among all indicating the most efficient chart. The graphical display concludes with the recommendation of the S_NRSS chart under a perfect ranking scenario.

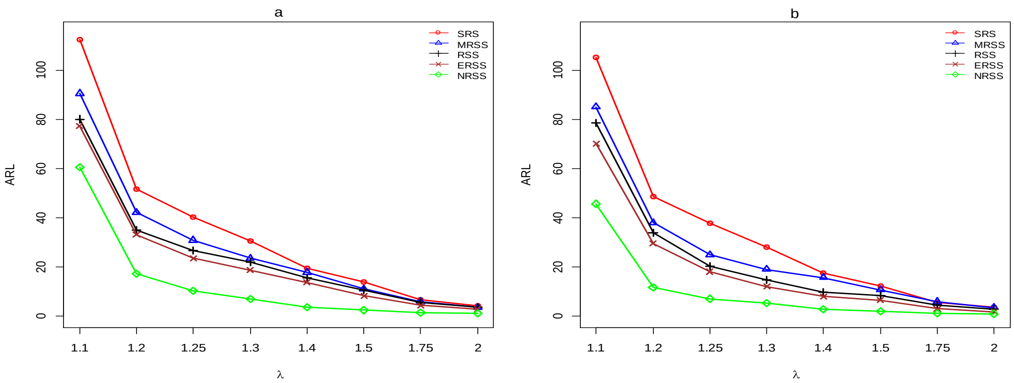

Figure 3. ARL curves R-chart under different sampling schemes for *(a)* $n=5$; *(b)* $n=7$ at $ARL_0=200$ and $\rho=1.0$.

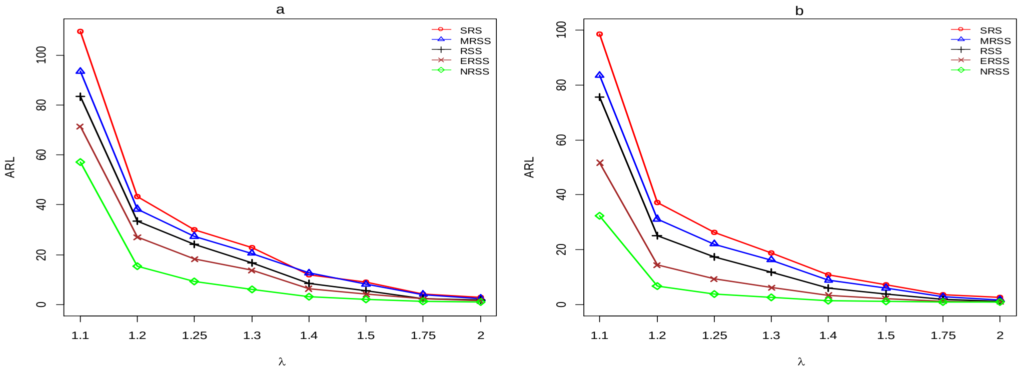

Figure 4. ARL curves S-chart under different sampling schemes for *(a) n=5*; *(b) n=7* at ARL_0*=200* and ρ*=1.0*.

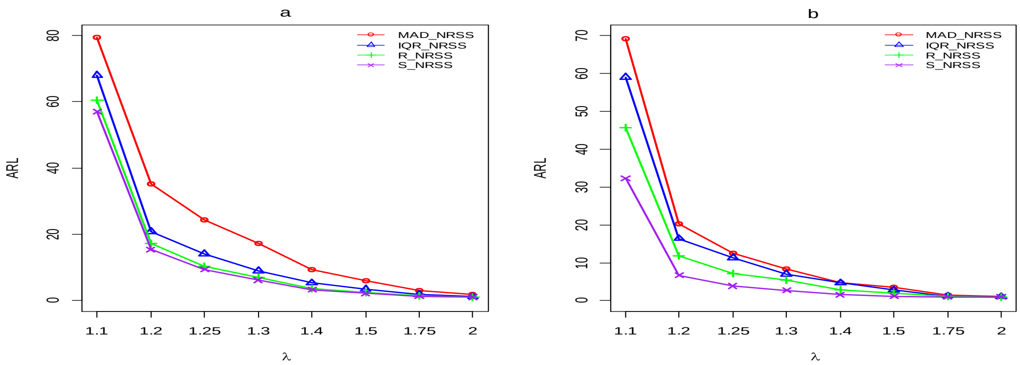

Figure 5. ARL curves of different dispersion charts under NRSS for *(a) n=5*; *(b) n=7* at ARL_0*=200* and ρ*=1.0*.

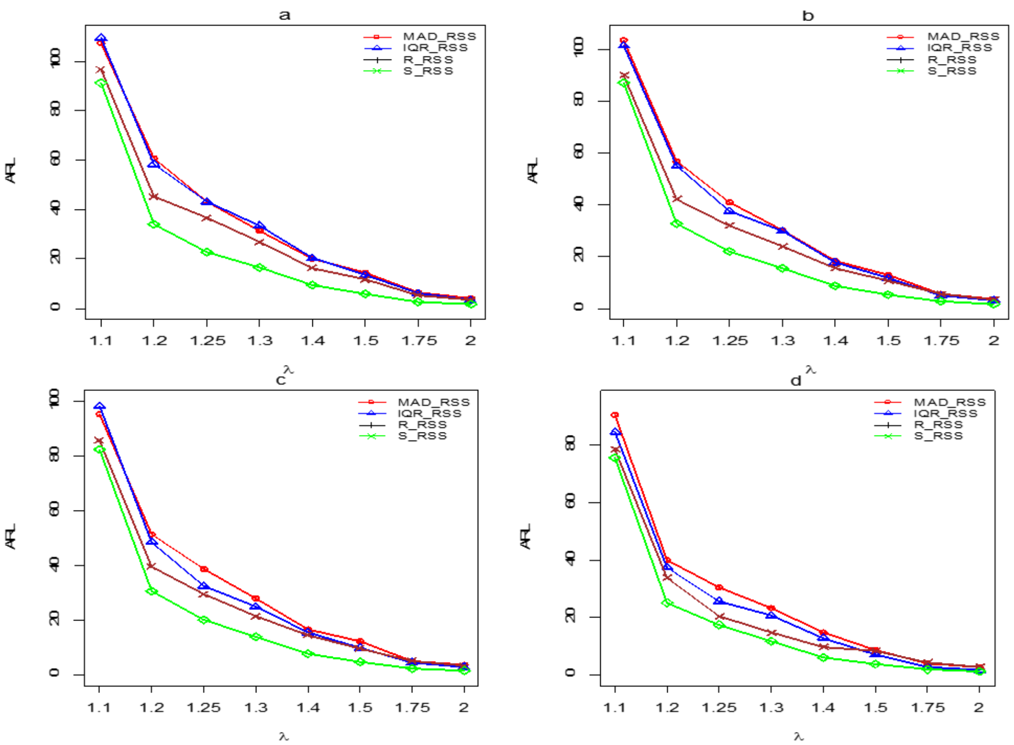

Figure 6. ARL curves of different dispersion charts under RSS for *(a)* ρ*=0.25*; *(b)* ρ*=0.50*; *(c)* ρ*=0.75* and *(d)* ρ*=1.0*.

8. Case study

To validate the dispersion charts for ranked set schemes [46] dataset of a non-isothermal continuous stirred tank chemical reactor (CSTR) process was used. The dataset was comprised of 1024 observations collected over a period of 30 seconds and contained nine variables, with details available in [47]. The CSTR process data is widely employed as a benchmark for fault detection and diagnosis. This study focused on Z as the study variable (CAS, inlet concentration of solvent flow in *mole/m3*) and Y as the auxiliary variable (FS, solvent flow rate in *m3/min*). However, the dataset was insufficient to conduct ranked set schemes, so the parameters (mean, standard deviations, and correlation) of the two variables were normalized and utilized to generate a similar bivariate dataset, as shown in Table 5. First, we generate 100 subsamples of size $n^2=25$ from a bivariate normal distribution using the descriptive measures presented in Table 5, when the process is under in-control state.

The control limits are computed when the process is under an in-control state for the most efficient (S_ERSS and S_NRSS) charts as declared in simulation study. The control limits for these charts under ERSS and NRSS are shown in Figure 7. Now for the case of a stable process, control limits of S_ERSS and S_NRSS charts are computed. These control limits are further used for the next 50 subsamples of size $n^2=25$ when a shift of 1.3 is introduced in process standard deviation (i.e., $\sigma_C = \lambda\sigma_I$; λ=1.3). The dispersion statistics are computed for 150 subsamples (first 100 in-control and last 50 out-of-control) under ERSS and NRSS schemes (see Figure 7). The plotted dispersion statistics in Figure 7 indicate that S_NRSS chart signals more out-of-control points (i.e., 7 spotted with red dots) than that of S_ERSS chart (i.e., 3 spotted with red dots). These findings are the same as we have in simulation study defined in Section 5. The simulative and real data case study results revealed that the dispersion charts under NRSS are beneficial for the detection of the faults in manufacturing process compared to dispersion charts under SRS or other RSS schemes in this study.

Table 5. Descriptive statistics of CAS (inlet concentration of solvent flow) and FS (flow rate of the solvent).

CAS		FS	
Mean	0.9027	Mean	0.1013
Standard Error	0.0035	Standard	0.0015
Median	0.9032	Median	0.1042
Standard deviation	0.1120	Standard	0.0482
Kurtosis	–0.1783	Kurtosis	–0.0785
Skewness	–0.0875	Skewness	–0.0621
Range	0.6291	Range	0.2768
Minimum	0.5710	Minimum	–0.0356
Maximum	1.2002	Maximum	0.2412
Q1	0.8312	Q1	0.0699
Q3	0.9745	Q3	0.1336
W statistic (Shapiro-Wilk normality test)	0.9985	W statistic (Shapiro-Wilk normality test)	0.9975
p-value	0.1658	p-value	0.1260

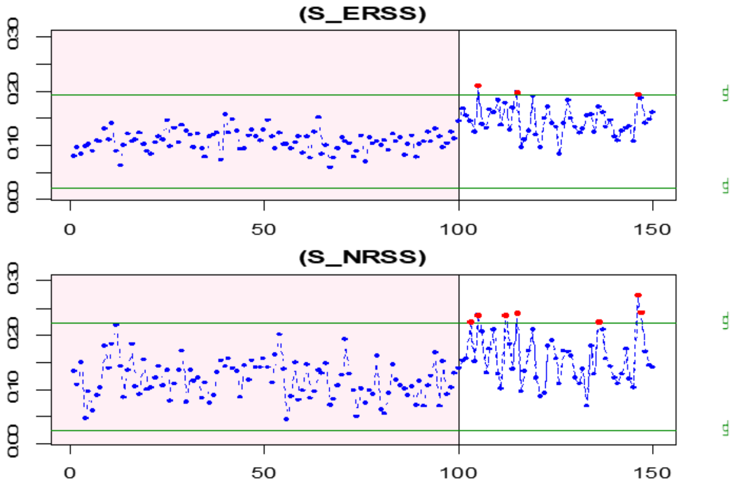

Figure 7. Dispersion control charts under NRSS using CSTR data.

9. Conclusions and recommendations

This study presents different dispersion (*MAD*, *IQR*, *R*, and *S*) charts using *SRS*, *RSS*, *MRSS*, *ERSS*, and *NRSS* under perfect $(\rho=1.0)$ and imperfect $(\rho=0.25$, 0.5 and $0.75)$ ranking. After these settings, sample sizes $n=5$ *and* 7 are generated from the bivariate normal process. The performance of the proposed dispersion charts is evaluated at different shifts $(\lambda=1.1–3.0)$ using various sampling schemes at a fixed ARL_0 of 200. The ARL_1 values are obtained at each shift (λ) value under different sampling schemes taking $\rho=0.0$, 0.25, 0.5, 0.75 and 1.0, and the results are presented in tables and graphs.

The ARL_1 values decrease with the rise in shift (λ), sample size (n), and correlation coefficient (ρ) values for all dispersion charts. The dispersion charts designed for ranked set sampling performed efficiently under perfect ranking scenarios compared to imperfect ranking scenarios. Among the ranked set sampling schemes in this study, the *NRSS* is more efficient for both perfect and imperfect ranking scenarios. The simulative and case study findings conclude that under NRSS dispersion charts performed efficiently at different ρ.

In contrast, the S_NRSS chart under a perfect ranking scenario $(\rho=1.0)$ outperforms other dispersion charts in this study. The designed dispersion estimators under NRSS can be extended for the case of different memory-type dispersion charts. The research idea in this study can also be prolonged with multivariate charts.

Use of AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

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Conflict of interest

The authors declare no conflict of interest.

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