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Research article

Model-free control approach to uncertain Euler-Lagrange equations with a Lyapunov-based L_∞-gain analysis

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Abstract: This paper considers a model-free control approach to Euler-Lagrange equations and proposes a new quantitative performance measure with its Lyapunov-based computation method. More precisely, this paper aims to solve a trajectory tracking problem for uncertain Euler-Lagrange equations by using a model-free controller with a proportional-integral-derivative (PID) control form. The L_∞-gain is evaluated for the closed-loop systems obtained through the feedback connection between the Euler-Lagrange equation and the model-free controller. To this end, the input-to-state stability (ISS) for the closed-loop systems is first established by deriving an appropriate Lyapunov function. The study further extends these arguments to develop a computational approach to determine the L_∞-gain. Finally, the theoretical validity and effectiveness of the proposed quantitative performance measure are demonstrated through a simulation of a 2-degree-of-freedom (2-DOF) robot manipulator, which is one of the most representative examples of Euler-Lagrange equations.

Keywords: Euler-Lagrange equations; input-to-state stability (ISS); model-free control; optimal tracking control; L_∞-gain; Lyapunov function
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1. Introduction

Motion control is an essential aspect of industrial systems, wherein achieving accurate trajectory tracking represents a primary objective. In order to achieve high accuracy in motion control, a number of methods have been introduced in the context of industrial systems [1] by noting that the dynamics of physical systems are generally described by Euler-Lagrange equations. The control approaches discussed in that study can be interpreted as providing various model-based methods based on the position control mode, in which position, velocity and torque controllers are connected in series so that torque set-point and velocity set-point is generated by the velocity and position controllers, respectively. Motivated by the success in achieving high tracking performances via the control methods in [1], a number of model-based approaches have been developed subsequently to tackle various control problems in Euler-Lagrange equations, e.g., adaptive control [2, 3], disturbance observer [4], sliding mode control [5, 6], neural network control [7] and so on. More interestingly, the adaptive control approach is also significantly extended in [8] by taking a leakage-type adaptive law tailored to solving input saturation problems in Euler-Lagrange systems.

Even though some important properties in control systems such as stability and tracking performance could be often achieved by using the aforementioned model-based control approaches, it should be required for taking these approaches to derive accurate mathematical models of the control systems. However, obtaining exact dynamic information of real control systems becomes more difficult as the size and complexity of the control systems become larger. For such large-scale systems, there should exist inescapable errors between the nominal model and the real dynamics, and they often lead to instability and/or performance deterioration for the feedback systems between the real system and a controller obtained through a model-based approach. To solve this difficulty in taking model-based control approaches, there have been various studies on robust control [9, 10]. However, the relevant model uncertainties should be characterized for employing these studies in terms of norm-boundness, structurality and so on. Hence, this requirement makes it difficult to directly apply the arguments in robust control [9, 10] to real systems.

As an intrinsic method for alleviating the difficulty occurring from the above model-based control approaches, one could take the model-free control approaches as in [11, 12]. Some dynamic characteristics for real systems, such as a positive definiteness of the inertia matrix and a skew-symmetric property relevant to the inertia matrix and Coriolis matrix in Euler-Lagrange equations, are taken in those studies for ensuring the convergence of the associated state trajectory and/or the stability and robustness of the corresponding closed-loop systems, although the overall model information is not employed in the model-free control approaches. More interestingly, the control inputs in the model-free control architectures are usually generated in a heuristic fashion, based on the online or offline input/output data from the real systems. However, it is also a non-trivial task to establish an immediately applicable control law as a model-free structure from such input/output data due to a number of processes of trial and error.

In connection with this, one of the most representative and readily applicable model-free control methods is the proportional-integral-derivative (PID) control, which is also often applied to a number of control systems, irrespective of the associated system models. For instance, two-layer structure optimal setting control systems with exogenous disturbances are considered with PID control law for a bottom loop in [13], the nonlocal boundary condition for parabolic reaction-diffusion equations with PID controllers is tackled in [14], a PID control law for affine-nonlinear uncertain dynamical systems is considered in [15], an observer-based adaptive PID controller subject to cyberattacks is discussed in [16] and an event-triggered PID controller with respect to quadrotors is introduced in [17].

To clarify some theoretical effectiveness of applying the PID controller to Euler-Lagrange equations, the global asymptotic stability (GAS) for Euler-Lagrange equations equipped with a PID controller is discussed [18–21], in which there is no consideration of external disturbances. A concept of input-to-state stability (ISS) with the consideration of external disturbances is introduced in [22] for the first time, and this issue is further theoretically tackled in [23]. More precisely, the notion of ISS means that the states of the systems are bounded even if unknown bounded disturbances, such as parameter perturbations and model uncertainties, exist. Those studies [22, 23] further shed new light on the arguments relevant to the disturbance ISS for nonlinear systems [24–27]. Roughly speaking, the stability of nonlinear systems could be established by constructing adequate Lyapunov functions in the employment of the ISS or disturbance ISS, in which the arguments on Lyapunov stability [28, 29] play significant roles in deriving the relevant assertions.

Furthermore, the Lyapunov arguments with respect to the stability for Euler-Lagrange equations could be more sophisticated by using the arguments on passivity [28, 29], in which it is intrinsically assumed that the signals considered in systems have finite energy, i.e., those of finite L_2 norms. In this sense, the applications of passivity-based arguments to Euler-Lagrange equations are confined to themselves to the treatment of decaying signals in the time-domain. However, it is quite practically meaningful to take the corresponding performance specifications on the time-domain bound for the relevant error signals in Euler-Lagrange equations, i.e., the L_∞ norm, rather than taking the L_2 norm of those signals. More importantly, there is no discussion on the L_∞-gain with respect to Euler-Lagrange equations in aforementioned studies because it is quite difficult to lead to an analytic form for the L_∞-gain with respect to nonlinear systems, while the L_∞-induced norm for linear equations is deeply studied in [30–33].

With this in mind, this paper aims at deriving sophisticated arguments on the L_∞-gain of the closed-loop systems obtained by connecting the Euler-Lagrange equation and a PID control law. To do this, we first consider a trajectory tracking problem of Euler-Lagrange equations and develop a sort of PID control law for achieving a desired tracking performance. By noting the dynamical properties of the Euler-Lagrange equation, we also deal with the disturbance ISS associated with the provided PID controller by constructing a relevant Lyapunov function. At the same line, we propose a new quantitative performance measure for Euler-Lagrange equations by re-interpreting the corresponding Lyapunov function through the L_∞ norm of signals. To put it concisely, the L_∞-gain is taken as the quantitative performance measure for dealing with the corresponding trajectory tracking errors caused by unknown elements in the time-domain bounds. More importantly, the quantitative performance measure (i.e., the L_∞-gain) of the closed-loop systems could be obtained by computing the minimum eigenvalue of an adequately constructed positive definite matrix. A computational approach to taking the corresponding control parameters is also derived based on the quantitative performance measure.

To summarize, the contributions of this paper on a model-free control approach to Euler-Lagrange equations are as follows.

- **Synthesis of an effective PID control law**: A readily applicable model-free controller for the Euler-Lagrange equations is introduced.
- **Disturbance ISS**: The disturbance ISS for the closed-loop systems obtained through the feedback connection between the Euler-Lagrange equations and the PID controller is ensured by deriving an adequate Lyapunov function.
- **Quantitative performance measure**: With respect to the characteristics of desired control objectives in Euler-Lagrange equations, the L_∞-gain for the aforementioned closed-loop systems is defined.
- **Performance analysis**: The explicit method for computing the above L_∞ gain is established.
- **Computational approach to taking parameters**: We introduce a computational approach to

taking the corresponding control parameters based on the quantitative performance measure together with its computation method.

Finally, it should be remarked that some earlier results of this paper were partially presented at the conference [34], but they are limited to the PD control law and no computational approach to taking the relevant parameters was provided in that study. In this sense, the arguments derived in this paper can be regarded as generalized and extended versions of those in [34].

This paper is organized as follows. We first introduce the underlying mathematical notations used in this paper in Section 2. We next formulate the problem definition relevant to a trajectory tracking problem of Euler-Lagrange equations in Section 3. A new quantitative performance measure (i.e., the L_∞-gain) for Euler-Lagrange equations together with its computation method is given in Section 4. We then verify in Section 5 the effectiveness and validity of the main results through a simulation result associated with a trajectory tracking problem of a 2-degree-of-freedom (2-DOF) robot manipulator, whose dynamics are one of the most representative examples of Euler-Lagrange equations. Finally, conclusion remarks are given in Section 6.

2. Mathematical notations

This section provides the underlying mathematical notations used for establishing the main results in this paper.

We denote the sets of ν-dimensional real numbers, ν-dimensional nonnegative real numbers and ν-dimensional symmetric matrices by $\mathbb{R}^\nu$, $\mathbb{R}_+^\nu$ and $\mathbb{S}^\nu$, respectively. The notation $\succ$ ($\succeq$) is used for $M_1, M_2 \in \mathbb{S}^\nu$ to mean the binary relation such that

$$\begin{aligned}&M_1 \succ M_2\ (M_1 \succeq M_2)\\ &\Leftrightarrow x^T(M_1 - M_2)x > 0\ (x^T(M_1 - M_2)x \geq 0), \quad \forall x \in \mathbb{R}^\nu (x \neq 0).\end{aligned}$$

In other words, $M_1 \succ (\succeq) M_2$ implies that $M_1 - M_2$ is a positive (semi-)definite matrix. Furthermore, we denote the minimum eigenvalue of a matrix $(\cdot)$ and the ν-dimensional identity matrix by $\lambda_{\min}(\cdot)$ and I_ν, respectively.

We denote the Euclidean-norm of a vector $f(t) \in \mathbb{R}^\nu$ by $|f(t)|_2$, i.e.,

$$|f(t)|_2 := \left(\sum_{i=0}^{\nu} f_i^2(t)\right)^{1/2} = \left(f^T(t)f(t)\right)^{1/2}$$

while the ∞-norm of a vector $f(t) \in \mathbb{R}^\nu$ is denoted by $|\cdot|_\infty$, i.e.,

$$|f(t)|_\infty := \max_{1\leq i\leq \nu} |f_i(t)|$$

where $f_i(t)$ implies the ith element of the vector $f(t)$. With respect to this, we use the notation $\|\cdot\|_\infty$ for the L_∞ norm of a real-valued vector function, i.e.,

$$\|f\|_\infty := \operatorname*{ess\,sup}_{0\leq t<\infty} \max_{1\leq i\leq \nu} |f_i(t)| = \operatorname*{ess\,sup}_{0\leq t<\infty} |f(t)|_\infty.$$

We next introduce the classes of functions considered in this paper as follows [28, 29]. A function $\beta(\cdot) : \mathbb{R}_+ \to \mathbb{R}_+$ is said to be of class $\mathcal{K}$, if it is continuous, strictly increasing and $\beta(0) = 0$. A function

$\beta(\cdot) : \mathbb{R}_+ \to \mathbb{R}_+$ is said to be of class $\mathcal{K}_\infty$, if it satisfies all the conditions of class $\mathcal{K}$ as well as the condition that $\beta(t) \to \infty$ as $t \to \infty$. A function $\xi(\cdot, \cdot) : \mathbb{R}_+ \times \mathbb{R}_+ \to \mathbb{R}_+$ is said to be of class $\mathcal{KL}$, if $\xi(\cdot, t)$ is of class $\mathcal{K}$ for each fixed t, $\xi(s, \cdot)$ is monotonically decreasing and converges to 0 as $t \to \infty$ for each fixed s, i.e., $\xi(s, t) \to 0$ $(t \to \infty)$.

By using these notations, we also introduce some fundamental properties of the disturbance input-to-stability (ISS) as follows [22–25].

Definition 1. *Let us consider the system*

$$\dot{x}(t) = A(x, t)x(t) + B_1(x, t)w(t) + B_2(x, t)u(t) \tag{2.1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $w(t) \in \mathbb{R}^{n_w}$ is the exogenous input vector and $u(t) \in \mathbb{R}^{n_u}$ is the control input vector. The disturbance ISS is defined for this system when there exist a class $\mathcal{K}$ function β and a class $\mathcal{KL}$ function ξ such that the solution for (2.1) exists for all $t \geq 0$ and satisfies

$$|x(t)|_2 \leq \beta\left(\sup_{0 \leq s \leq t} |w(s)|_2\right) + \xi(|x(0)|_2, t) \tag{2.2}$$

for an arbitrary $x(0) \in \mathbb{R}^n$ and for a bounded exogenous input function $w(\cdot)$.

Remark 1. *The disturbance ISS can be also regarded as a generalized version of the global asymptotic stability (GAS) since the inequality (2.2) is essentially equivalent to the GAS for the case of $w \equiv 0$.*

Because a direct application of the arguments in Definition 1 in establishing the disturbance ISS for Euler-Lagrange equations is quite difficult, we introduce the following necessary and sufficient condition, by which the difficulty in the employment of disturbance ISS would be alleviated.

Lemma 1. *The system described by (2.1) is said to be disturbance input-to-state stable, if and only if there exist a positive definite and radially unbounded function $V(x, t)$, a class $\mathcal{K}$ function β_1, and a class $\mathcal{K}_\infty$ function β_2 such that the following inequality holds:*

$$\frac{dV(x, t)}{dt} \leq \beta_1(|w(t)|_2) - \beta_2(|x(t)|_2). \tag{2.3}$$

Furthermore, this radially unbounded function $V(x, t)$ is called also an ISS-Lyapunov function for the system given by (2.1).

Remark 2. *The implication of taking (2.3) could be also interpreted as ensuring the fact that all trajectories with respect to nonlinear systems satisfying the disturbance ISS should converge to a specific ball centered at the origin $x = 0$, in which the radius is determined depending on the L_∞ norm of w. In other words, such a ball establishes also the attractive property [29] for systems with the disturbance ISS property.*

Based on Lemma 1, the disturbance ISS for Euler-Lagrange equations equipped with a PID control law would be dealt with in the subsequent sections.

3. Problem definition

The problem definition to be tackled in this paper is formulated in this section. Let us first take the n-dimensional Euler-Lagrange equation given by

$$M(q(t))\ddot{q}(t) + C(q(t), \dot{q}(t))\dot{q}(t) + g(q(t)) = \tau(t) + d(t) \tag{3.1}$$

where $M(q(t)) \in \mathbb{R}^{n\times n}$ is the Inertia matrix, $C(q(t), \dot{q}(t)) \in \mathbb{R}^{n\times n}$ is the Coriolis and centrifugal torques matrix, $g(q(t)) \in \mathbb{R}^n$ is the gravitational torque vector, $q(t) \in \mathbb{R}^n$ is the n-dimensional generalized coordinates, $\tau(t) \in \mathbb{R}^n$ is the control input torque vector and $d(t) \in \mathbb{R}^n$ is the unknown disturbance vector.

For this Euler-Lagrange equation (3.1), we aim to achieve the trajectory tracking performance described by

$$q(t) \to q_d(t), \quad (t \to \infty) \tag{3.2}$$

where $q_d(t)$ is the reference trajectory determined depending on the desired control objectives. With respect to quantitative interpretations of the above trajectory tracking performance, we also define the tracking error as $e(t) := q_d(t) - q(t)$. Then, we say the Euler-Lagrange equation of (3.1) satisfies the tracking performance of (3.2) if the L_∞ norm $||e||_\infty$ is bounded. Subsequently, the tracking performance of the Euler-Lagrange equation (3.1) is said to be improved when the maximum of the L_∞ norm $||e||_\infty$ becomes smaller.

As a preliminary step to establishing methods for tackling the above performance tracking problem, we introduce the following properties of Euler-Lagrange equations (3.1) [35]:

(i) $q(\cdot), \dot{q}(\cdot), \ddot{q}(\cdot), q_d(\cdot), \dot{q}_d(\cdot), \ddot{q}_d(\cdot), d(\cdot)$ are assumed to be continuous and bounded functions of t with the zero initial condition.

(ii) $0 \prec M$ and there exist constants $m_*, m^* \in \mathbb{R}_+$ such that

$$m_* I_n \preceq M(q(t)) \preceq m^* I_n, \ \forall q(t) \in \mathbb{R}^n. \tag{3.3}$$

(iii) There exist constants $c^*, g^* \in \mathbb{R}_+$ such that

$$|C(q(t), x)y|_2 \leq c^* |x|_2 |y|_2, \forall q(t), x, y \in \mathbb{R}^n, \tag{3.4}$$

$$|g(q(t))|_2 \leq g^*, \ \forall q(t) \in \mathbb{R}^n. \tag{3.5}$$

(iv) $\dot{M} = C + C^T$ and all the initial conditions $C(q(0), \dot{q}(0)), g(q(0))$ are zeros.

4. Main results

This section provides the main results of this paper, i.e., a PID control law, which is readily applicable to Euler-Lagrange equations, together with the relevant arguments on the disturbance ISS and the L_∞-gain analysis. These results could be further extended to deriving a computational approach to determining the corresponding control parameters. To this end, this section is divided into two subsections, the arguments associated with the PID control law and the computational approach to determining control parameters, respectively.

4.1. Synthesis of a PID control law with its theoretical extensions

In this subsection, we are in a position to consider the PID control law given by

$$\dot{\epsilon}(t) = e(t), \tag{4.1}$$

$$\tau(t) = \alpha\left(K_I\epsilon(t) + K_P e(t) + \dot{e}(t)\right) =: \alpha h(t) \tag{4.2}$$

where $\alpha > 0$ is a scalar parameter, K_I and K_P are diagonal positive definite matrices such that $K_P^2 > 2K_I$. Because the PID control law given by (4.2) is used in this paper, it is natural to consider the behavior of the signals $\epsilon(t)$, $e(t)$ and $\dot{e}(t)$ when we proceed to construct the corresponding mathematical proofs.

As a preliminary step to prove the disturbance ISS associated with the PID control law described by (4.2), we define the exogenous input vector $w(t) \in \mathbb{R}^n$ as

$$w(t) := M(q(t))\dot{s}(t) + C(q(t), \dot{q}(t))s(t) + g(q(t)) - d(t) \tag{4.3}$$

where

$$s(t) := \dot{q}_d(t) + K_P e(t) + K_I\epsilon(t). \tag{4.4}$$

Then, defining the state vector $x(t) \in \mathbb{R}^{3n}$ as

$$x(t) := \begin{bmatrix}\epsilon^T(t) & e^T(t) & \dot{e}^T(t)\end{bmatrix}^T \tag{4.5}$$

leads to the state-space equation described by

$$\dot{x}(t) = A(x,t)x(t) + B(x,t)w(t) - B(x,t)\tau(t) \tag{4.6}$$

where

$$A(x,t) := \begin{bmatrix} 0 & I & 0 \\ 0 & 0 & I \\ -M^{-1}CK_I & -M^{-1}CK_P - K_I & -M^{-1}C - K_P \end{bmatrix}, \tag{4.7}$$

$$B(x,t) := \begin{bmatrix} 0 \\ 0 \\ M^{-1} \end{bmatrix}. \tag{4.8}$$

One of the main contributions in this paper is to show that the closed-form equation obtained by connecting the state-space equation (4.6) with the PID control law (4.2) satisfies the disturbance ISS. With respect to this, let us consider the Lyapunov function $V(x,t)$ defined as

$$V(x,t) = \frac{1}{2}x^T P x \tag{4.9}$$

where

$$P := \begin{bmatrix} K_I M K_I + \alpha K_I K_P & K_I M K_P + \alpha K_I & K_I M \\ K_P M K_I + \alpha K_I & K_P M K_P + \alpha K_P & K_P M \\ M K_I & M K_P & M \end{bmatrix}.$$

Here, $V(x,t)$ is a obviously positive function for all $x \in \mathbb{R}^{3n}$, because it can also be represented by

$$V(x,t) = \frac{1}{2}h^T M h + \frac{1}{2}\begin{bmatrix}\epsilon \\ e\end{bmatrix}^T \begin{bmatrix}\alpha K_I K_P & \alpha K_I \\ \alpha K_I & \alpha K_P\end{bmatrix}\begin{bmatrix}\epsilon \\ e\end{bmatrix}. \tag{4.10}$$

Based on this representation, we give the following theorem relevant to the disturbance ISS.

Theorem 1. *The closed-form equation obtained by connecting the state-space equation (4.6) and the PID control law (4.2) satisfies disturbance ISS.*

Proof. Let us first note from (4.2) and (4.4) that

$$s(t) = h(t) + \dot{q}(t). \tag{4.11}$$

Substituting this into (4.3) derives that

$$\begin{aligned} w(t) &= M(q(t))(\dot{h}(t) + \ddot{q}(t)) + C(q(t),\dot{q}(t))(h(t) + \dot{q}(t)) + g(q(t)) - d(t) \\ &= M(q(t))\ddot{q}(t) + C(q(t),\dot{q}(t))\dot{q}(t) + g(q(t)) - d(t) + M(q(t))\dot{h}(t) + C(q(t),\dot{q}(t))h(t). \end{aligned} \tag{4.12}$$

This admits from (3.1) the representation

$$M(q(t))\dot{h}(t) + C(q(t),\dot{q}(t))h(t) = w(t) - \tau(t). \tag{4.13}$$

Combining (4.13) with (4.2) also leads to

$$M(q(t))\dot{h}(t) + C(q(t),\dot{q}(t))h(t) = w(t) - \alpha h(t). \tag{4.14}$$

On the other hand, we can obtain from (4.10) that

$$\begin{aligned} \dot{V}(x,t) =& \frac{1}{2}\left(\dot{h}^T M h + h^T M \dot{h} + h^T \dot{M} h + \begin{bmatrix}e \\ \dot{e}\end{bmatrix}^T \begin{bmatrix}\alpha K_I K_P & \alpha K_I \\ \alpha K_I & \alpha K_P\end{bmatrix}\begin{bmatrix}\epsilon \\ e\end{bmatrix} + \begin{bmatrix}\epsilon \\ e\end{bmatrix}^T \begin{bmatrix}\alpha K_I K_P & \alpha K_I \\ \alpha K_I & \alpha K_P\end{bmatrix}\begin{bmatrix}e \\ \dot{e}\end{bmatrix}\right) \\ =& h^T M \dot{h} + \frac{1}{2}h^T \dot{M} h + \begin{bmatrix}\epsilon \\ e\end{bmatrix}^T \begin{bmatrix}\alpha K_I K_P & \alpha K_I \\ \alpha K_I & \alpha K_P\end{bmatrix}\begin{bmatrix}e \\ \dot{e}\end{bmatrix}, \end{aligned} \tag{4.15}$$

since K_I and K_P are diagonal matrices. This further admits from the assertion (iv) that

$$\begin{aligned} \dot{V}(x,t) =& h^T M \dot{h} + \frac{1}{2}h^T \dot{M} h + \begin{bmatrix}\epsilon \\ e\end{bmatrix}^T \begin{bmatrix}\alpha K_I K_P & \alpha K_I \\ \alpha K_I & \alpha K_P\end{bmatrix}\begin{bmatrix}e \\ \dot{e}\end{bmatrix} \\ =& h^T M \dot{h} + \frac{1}{2}h^T (C + C^T) h + \begin{bmatrix}\epsilon \\ e\end{bmatrix}^T \begin{bmatrix}\alpha K_I K_P & \alpha K_I \\ \alpha K_I & \alpha K_P\end{bmatrix}\begin{bmatrix}e \\ \dot{e}\end{bmatrix} \\ =& h^T (M\dot{h} + Ch) + \begin{bmatrix}\epsilon \\ e\end{bmatrix}^T \begin{bmatrix}\alpha K_I K_P & \alpha K_I \\ \alpha K_I & \alpha K_P\end{bmatrix}\begin{bmatrix}e \\ \dot{e}\end{bmatrix}. \end{aligned} \tag{4.16}$$

Then, substituting (4.14) into (4.16) leads to

$$\dot{V}(x,t) = h^T (M\dot{h} + Ch) + \begin{bmatrix}\epsilon \\ e\end{bmatrix}^T \begin{bmatrix}\alpha K_I K_P & \alpha K_I \\ \alpha K_I & \alpha K_P\end{bmatrix}\begin{bmatrix}e \\ \dot{e}\end{bmatrix}$$

$$= h^T w - \alpha h^T h + \begin{bmatrix} \epsilon \\ e \end{bmatrix}^T \begin{bmatrix} \alpha K_I K_P & \alpha K_I \\ \alpha K_I & \alpha K_P \end{bmatrix} \begin{bmatrix} e \\ \dot{e} \end{bmatrix}$$

$$= h^T w - \alpha h^T h + \frac{\alpha}{2} x^T \begin{bmatrix} 0 & K_I K_P & K_I \\ K_I K_P & 2K_I & K_P \\ K_I & K_P & 0 \end{bmatrix} x$$

$$= h^T w - \frac{\alpha}{2} h^T h - \frac{\alpha}{2} h^T h + \frac{\alpha}{2} x^T \begin{bmatrix} 0 & K_I K_P & K_I \\ K_I K_P & 2K_I & K_P \\ K_I & K_P & 0 \end{bmatrix} x$$

$$= h^T w - \frac{\alpha}{2} h^T h - \frac{\alpha}{2} x^T \begin{bmatrix} K_I & K_P & I_n \end{bmatrix}^T \begin{bmatrix} K_I & K_P & I_n \end{bmatrix} x + \frac{\alpha}{2} x^T \begin{bmatrix} 0 & K_I K_P & K_I \\ K_I K_P & 2K_I & K_P \\ K_I & K_P & 0 \end{bmatrix} x$$

$$= h^T w - \frac{\alpha}{2} h^T h - \frac{\alpha}{2} x^T \begin{bmatrix} K_I^2 & 0 & 0 \\ 0 & (K_P^2 - 2K_I) & 0 \\ 0 & 0 & I_n \end{bmatrix} x. \tag{4.17}$$

Furthermore, if we note from the AM–GM inequality that

$$h^T w \le \frac{\alpha}{2} h^T h + \frac{1}{2\alpha} w^T w, \tag{4.18}$$

then (4.17) can be further represented by

$$\begin{aligned} \dot{V}(x,t) &= h^T w - \frac{\alpha}{2} h^T h - \frac{\alpha}{2} x^T \begin{bmatrix} K_I^2 & 0 & 0 \\ 0 & (K_P^2 - 2K_I) & 0 \\ 0 & 0 & I_n \end{bmatrix} x \\ &\le \frac{1}{2\alpha} w^T w - \frac{\alpha}{2} x^T \begin{bmatrix} K_I^2 & 0 & 0 \\ 0 & (K_P^2 - 2K_I) & 0 \\ 0 & 0 & I_n \end{bmatrix} x \\ &\le \frac{1}{2\alpha} |w(t)|_2^2 - \frac{\alpha}{2} \eta \cdot |x(t)|_2^2 \end{aligned} \tag{4.19}$$

where

$$\eta := \min\{\lambda_{\min}(K_I^2), \lambda_{\min}((K_P^2 - 2K_I)), \lambda_{\min}(I_n)\}. \tag{4.20}$$

Because $\frac{1}{2\alpha}|w(t)|_2^2$ and $\frac{\alpha}{2}\eta|x(t)|_2^2$ are class $\mathcal{K}_\infty$ functions of $|w(t)|_2$ and $|x(t)|_2$, respectively, it is obvious from (4.19) and Lemma 1 that the closed-form equation obtained by (4.6) and (4.2) satisfies the disturbance ISS. This completes the proof. □

The disturbance ISS discussed in Theorem 1 clearly implies that the state vector signal $x(\cdot)$ is bounded if the exogenous vector signal $w(\cdot)$ is bounded, i.e.,

$$||w||_\infty < \infty \Rightarrow ||x||_\infty < \infty. \tag{4.21}$$

Next, we are concerned with computing the L_∞-gain of the closed-form equation. With this regard, the performance index function $\gamma(t)$ is first defined as the ∞-norm of the state vector $x(t)$ when $\dot{V}(x,t) \ge 0$. Otherwise, $\gamma(t)$ is defined as 0 (i.e., when $\dot{V}(x,t) < 0$). Based on the performance index function, we introduce the following theorem relevant to the L_∞-gain of the closed-form equation.

Theorem 2. *Denote a lower bound of α by $\alpha_0 (> 0)$ and decompose α as $\alpha = \alpha_0 + \alpha_1$ with $\alpha_1 > 0$. We also define the matrix T as*

$$T := \begin{bmatrix} 2K_I^2 & K_I K_P & K_I \\ K_P K_I & 2(K_P^2 - K_I) & K_P \\ K_I & K_P & 2I_n \end{bmatrix}.$$

Then, the following inequality holds:

$$||\gamma||_\infty \leq \sqrt{\frac{n}{\alpha_0}} \cdot \sqrt{\frac{1}{\alpha_1 \lambda_{\min}(T)}} ||w||_\infty. \tag{4.22}$$

Proof. Similarly for the proof of Theorem 1, let us first note that

$$h^T w \leq \frac{\alpha_0}{2} h^T h + \frac{1}{2\alpha_0} w^T w. \tag{4.23}$$

Then, we could see from (4.17) that

$$\begin{aligned}
\dot{V}(x,t) &= h^T w - \frac{\alpha}{2} h^T h - \frac{\alpha}{2} x^T \begin{bmatrix} K_I^2 & 0 & 0 \\ 0 & (K_P^2 - 2K_I) & 0 \\ 0 & 0 & I_n \end{bmatrix} x \\
&\leq \frac{1}{2\alpha_0} w^T w - \frac{\alpha_1}{2} h^T h - \frac{\alpha_1}{2} x^T \begin{bmatrix} K_I^2 & 0 & 0 \\ 0 & (K_P^2 - 2K_I) & 0 \\ 0 & 0 & I_n \end{bmatrix} x \\
&= \frac{1}{2\alpha_0} w^T w - \frac{\alpha_1}{2} x^T \left(\begin{bmatrix} K_I & K_P & I_n \end{bmatrix}^T \begin{bmatrix} K_I & K_P & I_n \end{bmatrix} + \begin{bmatrix} K_I^2 & 0 & 0 \\ 0 & (K_P^2 - 2K_I) & 0 \\ 0 & 0 & I_n \end{bmatrix} \right) x \\
&= \frac{1}{2\alpha_0} w^T w - \frac{\alpha_1}{2} x^T T x \\
&\leq \frac{1}{2\alpha_0} w^T w - \frac{\alpha_1}{2} \lambda_{\min}(T) x^T x.
\end{aligned} \tag{4.24}$$

From the fact that $|w(t)|_2^2 \leq n|w(t)|_\infty^2$ and $-|x(t)|_2^2 \leq -|x(t)|_\infty^2$, (4.24) can be further simplified to

$$\begin{aligned}
\dot{V}(x,t) &\leq \frac{n}{2\alpha_0} |w(t)|_\infty^2 - \frac{\alpha_1}{2} \lambda_{\min}(T) |x(t)|_\infty^2 \\
&\leq \frac{n}{2\alpha_0} ||w||_\infty^2 - \frac{\alpha_1}{2} \lambda_{\min}(T) |x(t)|_\infty^2.
\end{aligned} \tag{4.25}$$

Since (4.25) corresponds to deriving an upper bound of $\dot{V}(x,t)$, the performance index vector $\gamma(t)$ does not exceed the ∞-norm $|x(t)|_\infty$, which makes the right-hand side (RHS) of (4.25) zero. This completes the proof. □

This paper is in a position to take the RHS of (4.22) as the Lyapunov-based quantitative performance measure associated with suppressing the maximum amplitude of the trajectory tracking error x, i.e., $||x||_\infty$. Furthermore, it can be concluded from Theorem 2 that the L_∞ norm $||\gamma||_\infty$ is decreasing within a convergence order, which is not slower than $1/\sqrt{\alpha_1 \lambda_{\min}(T)}$. This implication is expected to establish that $||x||_\infty$ is also decreasing in a convergence order no smaller than $1/\sqrt{\alpha_1 \lambda_{\min}(T)}$. Such an expectation together with the validity of the Lyapunov-based quantitative performance measure for reducing $||x||_\infty$ will be demonstrated through simulation results in Section 5.

4.2. Computational approach to taking control parameters

This subsection is concerned with introducing a computational approach to determining the corresponding control parameters in (4.2). We can see from (4.2) that the parameters K_P and K_I could be interpreted as weighting constants of e and ϵ, respectively, while the scalar parameter $\alpha > 0$ plays the role of a common weighting constant of $K_P e, K_I \epsilon$ and $\dot{e}$. Here, it should be remarked from the proof of Theorem 1 that it is sufficient to take the scalar parameter α larger than 0 as well as the positive diagonal matrices K_P and K_I when we are only interested in the disturbance ISS.

With respect to more constructive methods for determining the control parameters rather than the above arguments, the parameters α, K_P and K_I could be determined as follows. Based on the fact that the scalar parameter α is decomposed as $\alpha = \alpha_0 + \alpha_1$ where α_0 denotes an lower bound of α, a relatively small $\alpha_0 > 0$ is first selected to guarantee the disturbance ISS for the corresponding closed-loop system. Once α_0 is fixed, the next step is to select a relatively small initial value of α_1, for instance, $\alpha_1 = 1$ or $\alpha_1 = 2$. If the initial value of α_1 is also selected, it is required to take K_P and K_I by which $K_I||\epsilon||_\infty, K_P||e||_\infty$ and $||\dot{e}||_\infty$ have a similar order of accuracy. After K_P and K_I are also fixed, the value of α_1 would be modulated by using the Lyapunov-based performance measure (4.22) such that $\max\{||\epsilon||_\infty, ||e||_\infty, ||\dot{e}||_\infty\} = ||x||_\infty$ becomes small corresponding to a control specification desired by the users. This might be achieved by taking larger α_1 for fixed α_0, K_P and K_I.

5. Simulation results

This section verifies the validity and effectiveness of the main results in this paper, i.e., the arguments of Theorems 1 and 2, through a simulation result for the 2-degree-of-freedom (2-DOF) planar robot manipulator Σ as shown in Figure 1, whose dynamics is one of the most representative examples of Euler-Lagrange equations.

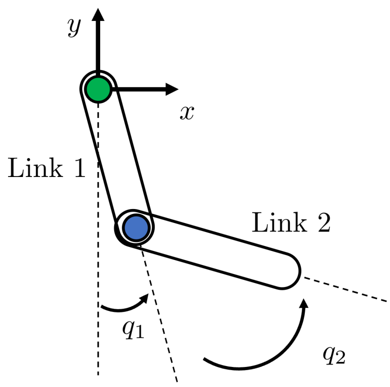


Figure 1. 2-DOF planar robot manipulator Σ.

Let us assume that the dynamics of Σ is described by

$$M(q(t))\ddot{q}(t) + C(q(t), \dot{q}(t))\dot{q}(t) + g(q(t)) = \tau(t) + d(t)$$

with

$$M(q(t)) = \begin{bmatrix} 7.96 + 2.4\cos(q_2(t)) & 0.96 + 1.2\cos(q_2(t)) \\ 0.96 + 1.2\cos(q_2(t)) & 5.96 \end{bmatrix},$$

$$C(q(t),\dot{q}(t)) = \begin{bmatrix} -1.2\sin(q_2(t))\dot{q}_2(t) & -1.2\sin(q_2(t))(\dot{q}_1(t)+\dot{q}_2(t)) \\ 1.2\sin(q_2(t))\dot{q}_1(t) & 0 \end{bmatrix},$$
$$g(q(t)) = \begin{bmatrix} 11.77\sin(q_1(t)+q_2(t)) + 19.62\sin(q_1(t)) \\ 11.77\sin(q_1(t)+q_2(t)) \end{bmatrix}$$

where $q_i(t)$ $(i = 1, 2)$ is the ith joint position and the values of model parameters are obtained from [8, 36]. We also assume that the disturbance vector as shown in Figure 2 is applied to this 2-DOF planar robot manipulator and the reference trajectories are described by the the periodic functions

$$q_{d1}(t) = \begin{cases} -\frac{1}{\pi^2}\sin(\frac{\pi}{2}t) + \frac{1}{2\pi}t & 0 \le t \le 4, \\ \frac{1}{\pi^2}\sin(\frac{\pi}{2}t) - \frac{1}{2\pi}(t-8) & 4 \le t \le 8, \end{cases},$$
$$q_{d2}(t) = \begin{cases} -\frac{1}{\pi^2}\sin(\pi t) + \frac{1}{\pi}t & 0 \le t \le 2, \\ \frac{1}{\pi^2}\sin(\pi t) - \frac{1}{\pi}(t-4) & 2 \le t \le 4. \end{cases}$$

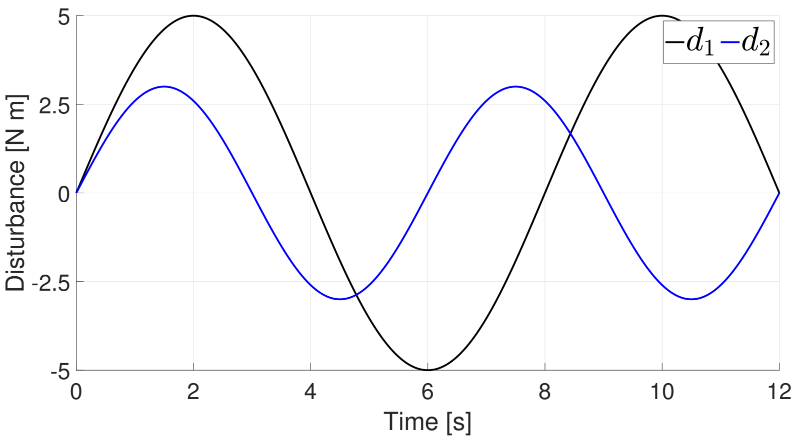


Figure 2. The external disturbance.

To take into account a practical situation relevant to uncertain elements, the velocity measurements are also assumed to be obtained from tachometers in each link with noise, i.e.,

$$y(t) = \begin{bmatrix} q(t) \\ \dot{q}(t) \end{bmatrix} + \begin{bmatrix} 0 \\ n(t) \end{bmatrix}$$

with the normal distribution given by $n \sim N(0, 5 \cdot 10^{-3})$.

5.1. Effectiveness and validity of Theorems 1 and 2

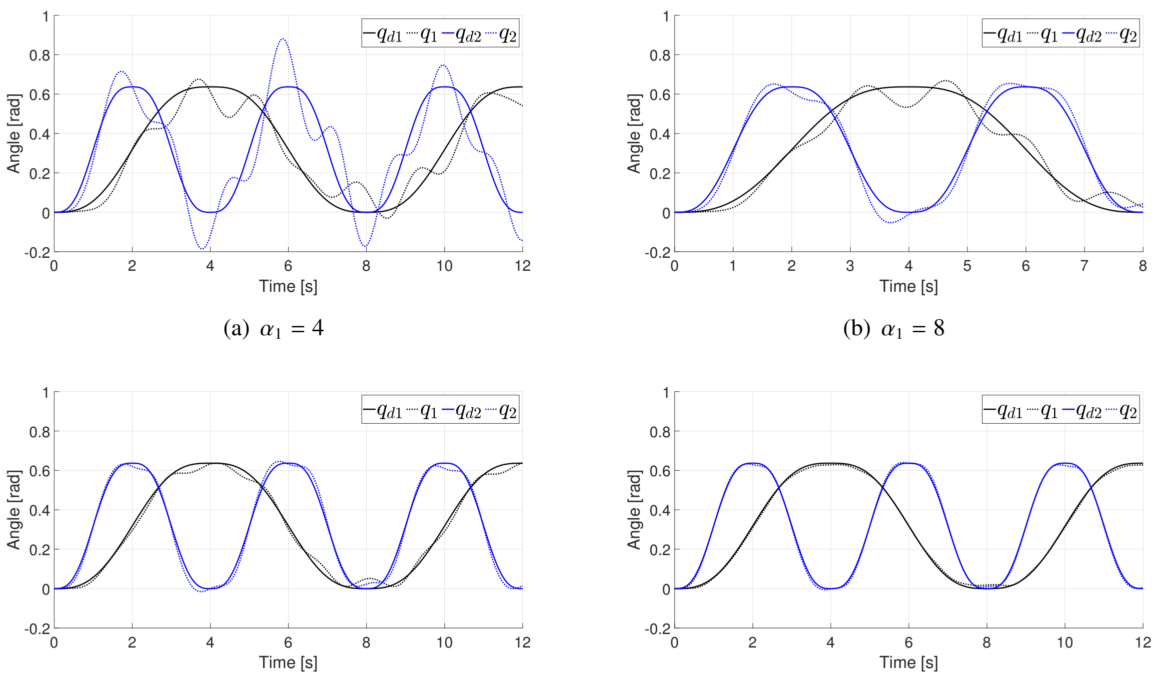
This section provides some simulation results to examine the effectiveness of Theorems 1 and 2. More precisely, the arguments with respect to the disturbance ISS and the arguments with respect to the convergence rate for $\|x\|_\infty$ are verified through the simulation results.

Based on the computational approach introduced in Subsection 4.2, we are concerned with the PID control law given by

$$\tau = (\alpha_0 + \alpha_1)(\dot{e} + K_P e + K_I \epsilon) \tag{5.1}$$

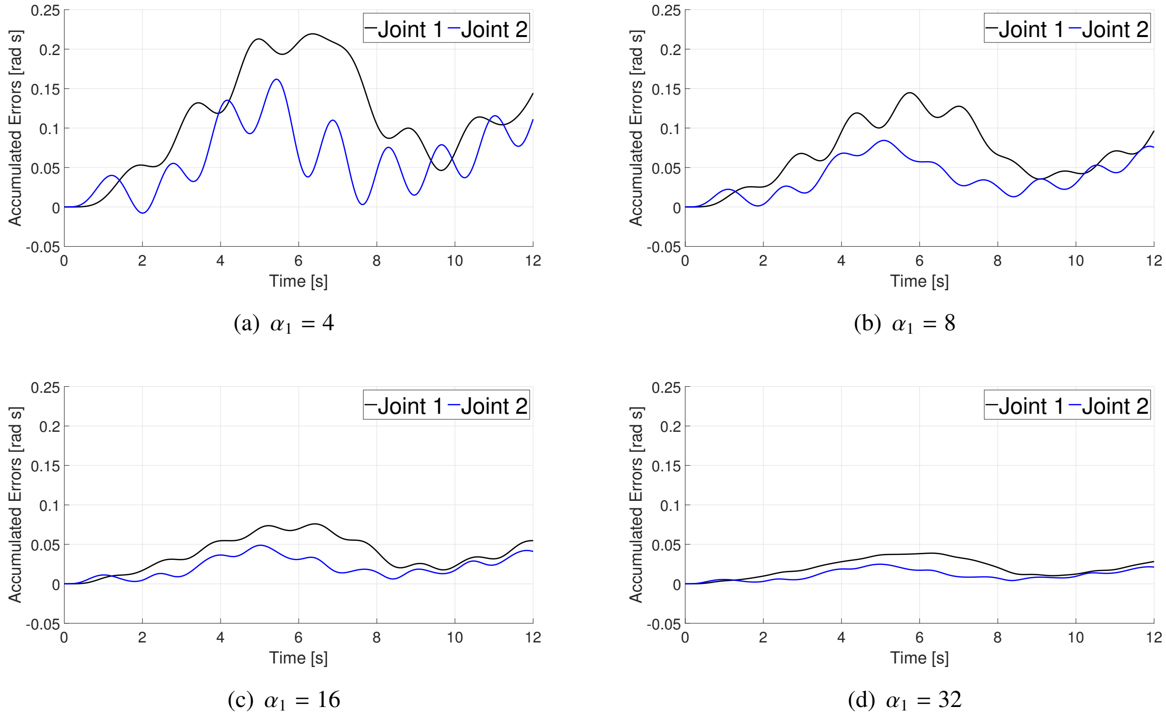
where the corresponding control parameters are determined to be $\alpha_0 = 2$, $K_P = 15I_2$ and $K_I = 15I_2$ with the initial value of α_1 as 4. With these control parameter values, the feedback system obtained by connecting (4.6) and (4.2) is expected to satisfy the disturbance ISS by the arguments in Theorem 1. With respect to the arguments in Theorem 2, the L_∞ norm $||x||_\infty$ is further expected to be smaller as α_1 increases because taking α_1 larger obviously makes $\alpha_1 \lambda_{\min}(T)$ in (4.22) become larger. In this respect, we also take $\alpha_1 = 8$, $\alpha_1 = 16$ and $\alpha_1 = 32$ to examine the validity of the convergence rate for $||x||_\infty$ corresponding to taking larger α_1 mentioned above.

The simulation results for the real position vector $q(t) \in \mathbb{R}^2$ and the reference position vector $q_d(t) \in \mathbb{R}^2$ are shown in Figure 3, while the results for the accumulated position error vector $\epsilon(t) \in \mathbb{R}^2$, the position error vector $e(t) \in \mathbb{R}^2$, the velocity error vector $\dot{e}(t) \in \mathbb{R}^2$ and the control input torque vector $\tau(t) \in \mathbb{R}^2$ are shown in Figures 4–7, respectively. Furthermore, the simulation results for the L_∞ norm $||x||_\infty$ and $\sqrt{\frac{1}{\alpha_1 \lambda_{\min}(T)}}$ are shown in Table 1.



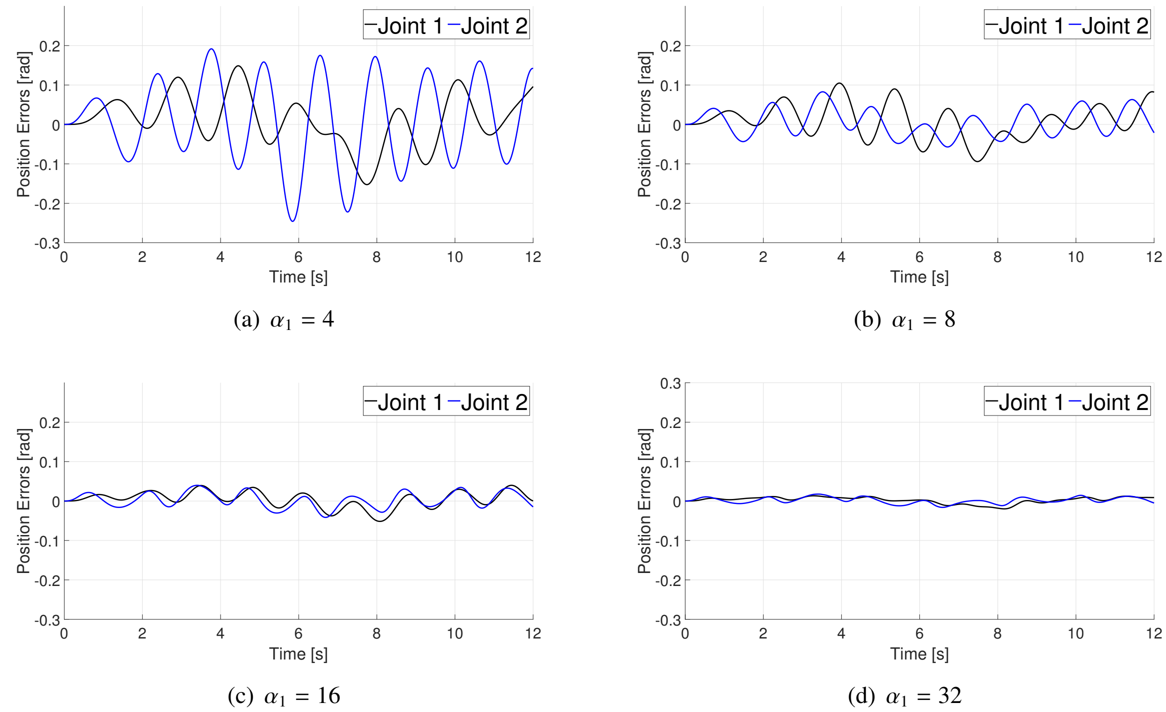
(a) $\alpha_1 = 4$ (b) $\alpha_1 = 8$ (c) $\alpha_1 = 16$ (d) $\alpha_1 = 32$

Figure 3. Results for the real position vector $q(t)$ and the reference position vector $q_d(t)$ with the proposed control law.



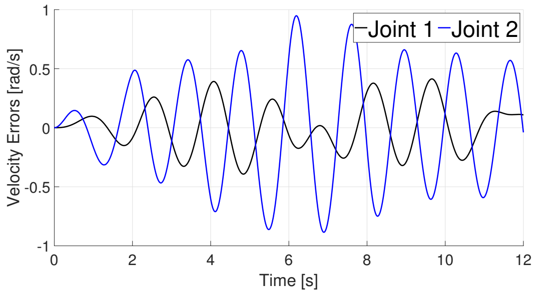
(a) $\alpha_1 = 4$ (b) $\alpha_1 = 8$ (c) $\alpha_1 = 16$ (d) $\alpha_1 = 32$

Figure 4. Results for the accumulated position error $\epsilon(t)$ with the proposed control law.

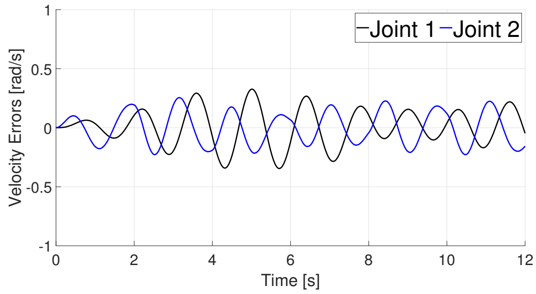


(a) $\alpha_1 = 4$ (b) $\alpha_1 = 8$ (c) $\alpha_1 = 16$ (d) $\alpha_1 = 32$

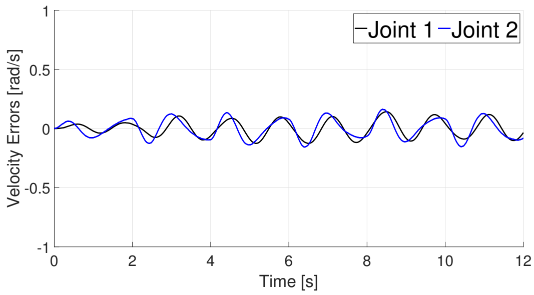
Figure 5. Results for the position error $e(t)$ with the proposed control law.



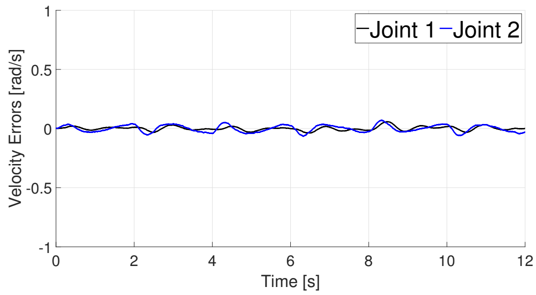
(a) $\alpha_1 = 4$



(b) $\alpha_1 = 8$

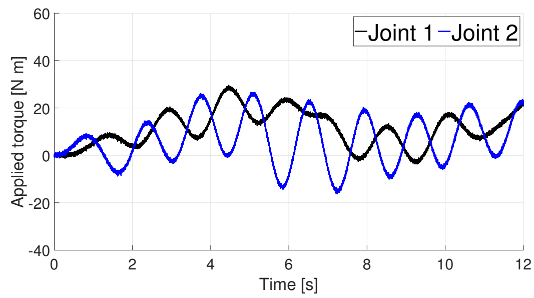


(c) $\alpha_1 = 16$

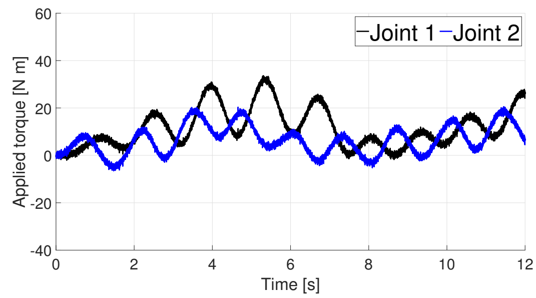


(d) $\alpha_1 = 32$

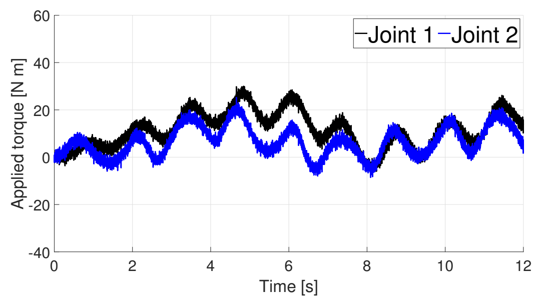
Figure 6. Results for the velocity error $\dot{e}(t)$ with the proposed control law.



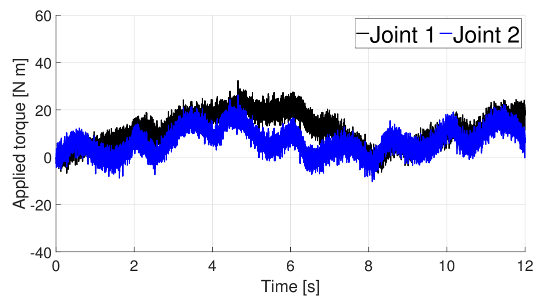
(a) $\alpha_1 = 4$



(b) $\alpha_1 = 8$



(c) $\alpha_1 = 16$



(d) $\alpha_1 = 32$

Figure 7. Results for the control input torque $\tau(t)$ with the proposed control law.

Table 1. Results for $||x||_\infty$ and $\sqrt{\frac{1}{\alpha_1 \lambda_{\min}(T)}}$ with the proposed control law.

α_1 ($\alpha_0 = 2$)	4	8	16	32
$\|\|x\|\|_\infty$	0.9483	0.3451	0.1622	0.0702
$\sqrt{\frac{1}{\alpha_1 \lambda_{\min}(T)}}$	0.4359	0.3082	0.2180	0.1541

We can observe from Figure 3 that the arguments in Theorem 1 with respect to the disturbance ISS are verified for all the taken control parameter values since the corresponding tracking errors are bounded. More importantly, we can also see from Figure 3 that making α_1 larger leads to an improved accuracy for the considered trajectory tracking problem. Furthermore, it can be observed from Figures 4–6 that all the peak magnitudes of the accumulated position error $\epsilon(t)$, the position error $e(t)$ and the velocity error $\dot{e}(t)$ are reduced by making α_1 larger. The aforementioned intuitive implication of the results in Theorem 2 with respect to the convergence rate for $||x||_\infty$ might be verified through these observations. Regarding a more sophisticated interpretation of the simulation results in a theoretical side, we note from the results in Table 1 that the L_∞ norm $||x||_\infty$ decreases within a order no slower than $1/\sqrt{\alpha_1 \lambda_{\min}(T)}$. This clearly demonstrates the effectiveness of the arguments in Theorem 2.

5.2. Comparison to conventional method

This section provides some simulation results to examine the superiority of the proposed method over the conventional method obtained by combining the PID control and the computed-torque control (CTC) [37]. This conventional method is called the PID-CDC throughout the paper, and it is described by

$$\dot{\epsilon} = e(t), \tag{5.2}$$

$$\tau = \hat{M}(q)(\ddot{q}_d + K_D^{[c]}\dot{e} + K_P^{[c]}e + K_I^{[c]}\epsilon) + \hat{C}(q(t), \dot{q}(t))\dot{q} + \hat{g}(q(t)) \tag{5.3}$$

where $\hat{M}, \hat{C}$ and $\hat{g}$ are the estimated (i.e., nominal) values of M, C and g, respectively, and the control parameters $K_d^{[c]}, K_p^{[c]}$ and $K_I^{[c]}$ are diagonal positive definite matrices. Assuming that the accurate dynamic model cannot be derived due to some model uncertainties, we take the estimated values of M, C and g respectively as

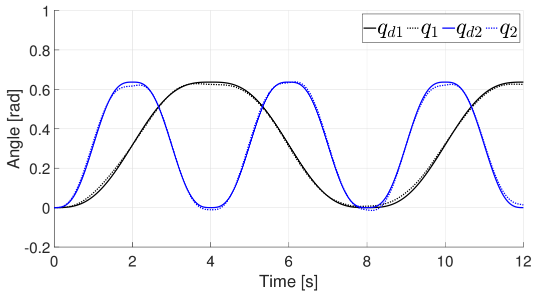
$$\hat{M}(q(t)) = \begin{bmatrix} 9.52 + 2.88\cos(q_2(t)) & 1.15 + 1.44\cos(q_2(t)) \\ 1.15 + 1.44\cos(q_2(t)) & 7.15 \end{bmatrix},$$

$$\hat{C}(q(t), \dot{q}(t)) = \begin{bmatrix} -1.44\sin(q_2(t))\dot{q}_2(t) & -1.44\sin(q_2(t))(\dot{q}_1(t) + \dot{q}_2(t)) \\ 1.44\sin(q_2(t))\dot{q}_1(t) & 0 \end{bmatrix},$$

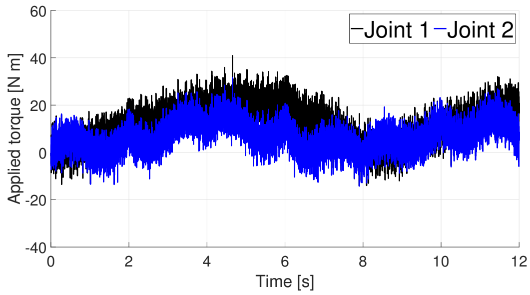
$$\hat{g}(q(t)) = \begin{bmatrix} 14.12\sin(q_1(t) + q_2(t)) + 23.54\sin(q_1(t)) \\ 14.12\sin(q_1(t) + q_2(t)) \end{bmatrix}.$$

Finally, the control parameters are taken by $K_d^{[c]} = 2\omega_c I_2$, $K_p^{[c]} = \omega_c^2 I_2$ and $K_I^{[c]} = 2\omega_c^3 I_2$ with the cut-off frequency ω_c.

We first take the same parameter values for the proposed controller as in the last case of the previous subsection, i.e., $\alpha_0 = 2, \alpha_1 = 32, K_P = 15I_2$ and $K_I = 15I_2$. To make a comparison between the proposed and conventional controllers fair, we then determine the control parameters for the latter, by which both the tracking performances relevant to the two controllers become similar to each other. In other words, we take the cut-off frequency ω_c in $K_d^{[c]}$, $K_p^{[c]}$ and $K_I^{[c]}$ to make the L_∞ norm of the state x to be close to that of the case of $\alpha_1 = 32$ in Table 1, i.e., $||x||_\infty = 0.07$. The simulation results for the real position vector $q(t) \in \mathbb{R}^2$ and the reference position vector $q_d(t) \in \mathbb{R}^2$ are shown in Figure 8(a), while the control input torque vector $\tau(t) \in \mathbb{R}^2$ are shown in Figure 8(b). In a quantitative comparison between the proposed and conventional controllers, the simulation results for $||x||_\infty$, $||\Delta\tau||_\infty$ and $||\tau||_\infty$ are shown in Table 2, where $\Delta\tau$ is the input difference in a unit time defined as $\Delta\tau := \tau(t + \delta) - \tau(t)$ with the operating period δ in the controllers.



(a) Reference and real position vector.



(b) Control input torque.

Figure 8. Results for the conventional control law (PID-CTC).

Table 2. Comparison between the proposed and conventional control laws.

	$\|\|x\|\|_\infty$	$\|\|\Delta\tau\|\|_\infty$	$\|\|\tau\|\|_\infty$
Proposed control law	0.0702	14.00	32.38
Conventional control law	0.0701	26.42	40.52

It could be observed from Figure 7(d) and Figure 8(b) that the torque input from the conventional controller is more turbulent than that from the proposed controller, although both controllers achieve similar tracking performances as observed from Figure 3(d) and Figure 8(a) as well as Table 2.

This tendency can be more obviously observed from Table 2 by showing that the peak values of the input difference and the torque input in the conventional controller are quite larger than those of the proposed controller; the peak values of the proposed controller are about 53% and 80% of those of the conventional controller, respectively.

We can conclude from these observations that the proposed controller would lead to more smooth control actions even for similar tracking performances to the conventional controller. More importantly, obtaining the exact values of M, C and g becomes more difficult as the dimension of Euler-Lagrange equations increases, and thus the proposed controller can be expected to be used more effectively for such large-scale Euler-Lagrange equations than the conventional controller. This superiority of the proposed controller over the conventional controller in the effectiveness of torque input might be arising from the fact that the parameter $\hat{M}(q)K_D^{[c]}$ relevant to the derivative term in the latter is usually much larger than the similar parameter α in the former as seen from (5.3) and (4.2).

6. Conclusions

This paper was concerned with introducing a new Lyapunov-based quantitative performance measure associated with suppressing the peak magnitude of trajectory tracking errors for an Euler-Lagrange equation. We proposed a PID control law readily applicable to the Euler-Lagrange equation and evaluated the effect of disturbances on the system states by showing that the feedback system obtained through the connection between the Euler-Lagrange equation and the proposed PID control law satisfies the disturbance ISS. The main idea was to introduce an adequate Lyapunov function with respect to the disturbance ISS, and the relevant arguments could be further effectively employed for the L_∞-gain analysis with respect to the corresponding trajectory tracking problem. More precisely, the relation between the associated control parameters and the L_∞-gain was shown to be described in terms of the minimum eigenvalue of a suitably constructed positive definite matrix. The theoretical validity and the practical effectiveness of the overall arguments developed in this paper were demonstrated through some simulation results for a 2-DOF robot manipulator.

On the other hand, we would like to note that the tracking accuracy might be improved through an additional implementation of a friction compensator. Furthermore, an auto-tuning technique for the proposed PID controller undoubtedly contributes to wider applications of this paper to practical Euler-Lagrange equations. In this sense, it is quite meaningful to extend the arguments for deriving (4.22) tailored to a friction compensator and/or an auto-tuning algorithm for the proposed PID controller, but this issue seems to be quite difficult and is left as an interesting future work.

Use of AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

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Conflict of interest

All authors declare no conflicts of interest in this paper.

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