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Research article

Study of nonlinear generalized Fisher equation under fractional fuzzy concept

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Abstract: Fractional calculus can provide an accurate model of many dynamical systems, which leads to a set of partial differential equations (PDE). Fisher's equation is one of these PDEs. This article focuses on a new method that is used for the analytical solution of Fuzzy nonlinear time fractional generalized Fisher's equation (FNLTFGFE) with a source term. While the uncertainty is considered in the initial condition, the proposed technique supports the process of the solution commencing from the parametric form (double parametric form) of a fuzzy number. Next, a joint mechanism of natural transform (NT) coupled with Adomian decomposition method (ADM) is utilized, and the nonlinear term is calculated through ADM. The obtained solution of the unknown function is written in infinite series form. It has been observed that the solution obtained is rapid and accurate. The result proved that this method is more efficient and less time-consuming in comparison with all other methods. Three examples are presented to show the efficiency of the proposed techniques. The result shows that uncertainty plays an important role in analytical sense. i.e., as the uncertainty decreases, the solution approaches a classical solution. Hence, this method makes a very useful contribution towards the solution of the fuzzy nonlinear time fractional generalized Fisher's equation. Moreover, the matlab (2015) software has been used to draw the graphs.

Keywords: double parametric form; nonlinear fractional partial differential equation; natural transform; Adomian decomposition method
Mathematics Subject Classification: 34A08, 34A12, 47H10

1. Introduction

It is obvious that fractional calculus (FC) is the generalization of classical calculus that deals with the operations of differentiation and integration of non-integer (fractional) order. As the idea of fractional operators has been presented or introduced almost with the development of the classical ones (classical calculus). Moreover, in this context, fractional dynamics geometry (fractional geometry) and fractional differential equations (FDE) are only a few of the practical areas for which the fractional calculus (theory) has been rapidly developed [1]. Fractional calculus has several uses and has been applied to a variety of fields, including control engineering [2, 3], signal processing [4], biosciences [5], engineering and mathematics and so on [6–12]. It is crucial to develop analytical and numerical methods for the solutions of FDEs because most FDEs are difficult to solve precisely. In this regard, many proposals have been made to propose analytical methods and formulas for the exact solution of FDEs. Like as, the Adomian decomposition method [13], variational iteration method [14], the Homotopy perturbation method [15], and additive operating splitting (AOS) method [16], except these some other numerical methods have also been presented for the solutions of FDEs [17–24].

Moreover, with the passage of time, it has been observed that partial differential equations have some drawbacks, and one of these limitations or drawbacks is the initial value that appears in a problem. Because developing accurate (fractional differential equations) in the context of mathematical modelling is not an easy task, as it requires a comprehension of real physical phenomena, it is a fact that uncertainty exists in most real-world problems, and determining the initial value is a difficult task due to the presence of uncertainty. This uncertainty is a moment of concern for scientists and researchers. Therefore, several researchers have tried and succeeded in proposing some new concepts to deal with such uncertainty. The fuzzy sets theory [25] is one of the most famous and well-known concepts among these, which have the ability to handle such things as PDEs having uncertainties. In this regard, the first contribution to dealing with such PDEs (with uncertainty) can be seen in [26]. Further contributions are available in [27–33]. The work done by Agarwal et al. [26] has inspired many researchers in the field of mathematics to set up some new methods, such as explicit solution [30], solution of FFDEs through fuzzy laplace transform [34, 35], solution of FFDEs via LADM [36], solution of FFDEs via natural transform and homotopy perturbation method [37] and recently the solution of FFDEs by natural Adomian decomposition method (NADM) [38]. In addition, FFDEs are further studied in terms of the Riemann-Liouville H-derivative [39–41]. In general, it is observed that fuzzy fractional partial differential equations (FFPDEs) are studied due to their wide applications in many research areas, and in this connection, Ronald Fisher [42] developed a specific type of partial differential equation, the so-called Fisher equation.

It is our deep interest to solve the time-fractional Fisher's equation by implementing a new technique, called natural Adomian decomposition method shortly (NADM). In contrast, the Adomian decomposition method is a powerful method for solving both linear and nonlinear, homogeneous and nonhomogeneous partial differential equations, as well as integro-differential equations of integer and non-integer order, which gives us convergent series with exact/better approximate solutions [29,43,44]. The general response expression contains parameters that describe the order of the fractional derivative and uncertainties that can be varied to obtain various responses.

The general form of such PDE's is [45] as

$$\frac{\partial v(y,t)}{\partial t} = \frac{\partial^2 v(y,t)}{\partial y^2} - v(y,t)(v(y,t) - 1). \tag{1.1}$$

In Eq (1.1), the function $v(y,t)$ shows the "population density". Whereas, the equation is used to examine/investigate the spread of faulty gene in a specific area/population. Now according to [45], mutant gene (means frequency of the mutant gene) for the aforesaid equation (Eq (1.1)) is shown by $v(y,t)$, for the provided point y and time t. Due to its importance Fisher equations (FEs) has been used widely in many fields, like as, ecology, Neolithic-transition, heat and mass-transfer, branching Brownian motion, epidemic, bacteria, and many others [46,47]. Moreover, the specific solution for the aforesaid PDEs was first presented by Ablowitz et al. [48]. In addition, for further updated results, we refer [49–51].

Now considering the time fractional Fisher's equations (TFFEs) as

$$\frac{\partial^\gamma v(y,t)}{\partial t^\gamma} = \frac{\partial^2 v(y,t)}{\partial y^2} - v(y,t)(v(y,t) - 1), \quad t > 0, \quad 0 < \gamma \leq 1, \tag{1.2}$$

with the available initial-condition(s) $v(y,0) = \psi(y)$, which is non-linear equation, and by putting $\gamma = 1$, we get the standard Fisher's equation (1.1).

In fuzzy sense the above Eq (1.2) can be presented as follows:

$$\frac{\partial^\gamma \bar{v}(y,t)}{\partial t^\gamma} = \frac{\partial^2 \bar{v}(y,t)}{\partial y^2} - \bar{v}(y,t)(\bar{v}(y,t) - 1), \quad t > 0, \quad 0 < \gamma \leq 1, \tag{1.3}$$

provided fuzzy condition (initial condition) $\bar{v}(0,y) = \bar{\psi}(y)$, where $0 < \gamma \leq 1$ shows the non integer order (fractional order) of the function (fuzzy function) $\bar{v}(y,t)$.

Since generally, the determination of initial values is very difficult. It always involves uncertainty quantities in fractional order differential equations dealing with real physical phenomena. To handle uncertainty quantities, many researchers used several new concepts. The one that stands out among the concepts is fuzzy set theory. Hence, with the established analysis, we become able to deal with differential equations of fractional order possessing uncertainties at initial values. On the other hand using natural decomposition technique is an updated method because natural transform contains Laplace and Sumudu transform are special cases. Also, using such technique does not need any kind of discretization or collocation. In addition, the used method is easy to implement and also simple in compilation. In addition, as local fractional derivative is a generalization of differentiation and integration of the functions defined on fractal sets. Being local in nature these derivatives have proven useful in studying fractional differentiability properties of highly irregular and nowhere differentiable functions. As fractional derivative has numerous definitions including Caputo, Reimann-Liuoville, etc. These definitions, however, are non-local in nature, which makes them unsuitable for investigating properties related to local scaling or fractional differentiability (see details [52]).

The paper is organized as follows: In Section 1, the introduction is discussed. In Section 2, we present some basic definitions and formulas that are employed in this paper. In Section 3, a general algorithm is developed. Section 4 contains the examples, and in Section 5, a brief conclusion is presented.

2. Preliminaries

Here, we'll talk about several fundamental findings that were employed in this research.

Definition 2.1. *[40, 53–56] A fuzzy set "ν" is said to be a fuzzy number, if "ν" is (both normalized and convex, piece-wise continuous, and the closure is compact).*

In level wise form the fuzzy number(ν) can be written as

$$[\nu]_{\jmath} = \begin{cases} \{y \in X : \nu(y) \geq (\jmath)\}, & if\ 1 > \jmath > 0, \\ cl\{y \in X : \nu(y) > 0\}, & if\ \jmath = 0. \end{cases}$$

Definition 2.2. *[55] A fuzzy number(ν) is represented with three points as $\nu = (y_1, y_2, y_3)$ is known as triangular fuzzy number (TFN).*

Mathematically we can write as

$$\nu = \begin{cases} 0, & a \leq y_1, \\ \frac{a-y_1}{y_2-y_1}, & y_1 \leq a \leq y_2, \\ \frac{y_3-a}{y_3-y_2}, & y_2 \leq a \leq y_3, \\ 0, & a \geq y_3. \end{cases} \tag{2.1}$$

Moreover, in sense of r-cut, the triangular fuzzy number in interval form can be written as

$$\nu = [y_1, y_2, y_3] = ((y_2 - y_1)\beta + y_1, y_3 - (y_3 - y_2)\beta), \quad 1 \geq \beta \geq 0.$$

Definition 2.3. *[45] A fuzzy number "ν" in double parametric form can be written as*

$$\mathcal{A} = \hbar(\bar{\kappa} - \kappa) + \kappa,$$

where interval form(r-cut) of κ is, k =[$\underline{\kappa}$,$\bar{\kappa}$], $\underline{\kappa}$ and $\bar{\kappa}$ shows the lower bounds and upper bounds and $\hbar \in [0, 1]$.

Definition 2.4. *[40] (Operations on fuzzy numbers) Let $\nu_1 = ({}_l\nu_1, {}_u\nu_1)$ and $\nu_2 = ({}_l\nu_2, {}_u\nu_2)$ be any two different numbers (fuzzy numbers), and $s \in R$ be any scalar (arbitrary scalar), then the various operations are*

(1) Addition: $[{}_l\nu_1, {}_u\nu_1] + [{}_l\nu_2, {}_u\nu_2] = [{}_l\nu_1 +_l \nu_2, {}_u\nu_1 +_u \nu_2]$.
(2) Subtraction: $[{}_l\nu_1, {}_u\nu_1] - [{}_l\nu_2, {}_u\nu_2] = [{}_l\nu_1 -_u \nu_1, {}_u\nu_1 - {}_l\nu_2]$.
(3) Scalar multiplication:

$$s.\nu = \begin{cases} (s.{}_l\nu_1, s.{}_u\nu_1), & s \geq 0, \\ (s.{}_u\nu_1, s.{}_l\nu_1), & s < 0. \end{cases}$$

Definition 2.5. *[52] In "Caputo" sense the fractional order derivatives (FOD's) of order "$\beta > 0$" for the f-function "$\nu(\zeta, t)$" over the interval $[\emptyset, \emptyset_0]$ is as*

$${}_{\emptyset}^{C}D_{\emptyset_0}^{\beta}\nu(y, t) = \frac{1}{\Gamma(\zeta - \beta)} \int_{\emptyset}^{\emptyset_0} (y - \zeta)^{n-\beta-1} \nu^n(\zeta, t) d\zeta, \ \ \emptyset_0 > \emptyset.$$

Definition 2.6. *[52] Mittage-Leffler function $E_\beta(t)$ is defined as follows:*

$$E_\beta(t) = \sum_{n=0}^{\infty} \frac{t^n}{\Gamma(n\beta + 1)}.$$

Definition 2.7. *[54] Natural transform for a fuzzy function $v(y, t)$ is defined as*

$$\mathcal{R}(u, s) = \mathcal{N}[v(y, t)] = \int_0^{\infty} (e)^{-st} \odot v(y, ut)dt, t > 0.$$

Definition 2.8. *[55] Natural transform of the "γ" order derivative of a function is given by as*

$$N[\mathcal{D}_t^\gamma v(y, t)] = \frac{s^\gamma}{u^\gamma} N(v(y, t)) - \sum_{i=0}^{\gamma-1} \frac{s^{\gamma-i-1}}{u^{\gamma-i}} v^i(y, 0).$$

Definition 2.9. *[40] The Adomian decomposition method (ADM) for the fuzzy function (FF) "$v(y, t)$" can be shown in infinite series as*

$$v(y, t) = \sum_{i=0}^{\infty} v_i(y, t). \tag{2.2}$$

In above Eq (2.2) the components of the function (fuzzy function) "$v_i, i \geq 0$" can be calculated in a recursive and a fashionable way. Whereas, the nonlinear term of the functions (fuzzy functions) $\mathcal{F}(u)$ of the aforesaid PDEs can be expressed by an infinite series of the ADM, which is known as Adomian polynomial A_n given in the form

$$\mathcal{F}(u) = \sum_{n=0}^{\infty} A_n(u_o, u_1, u_2, ...u_n),$$

where the Adomian polynomial A_n for the nonlinear term can be calculated as

$$A_n = \frac{1}{n!} \frac{d^n}{d\leftthreetimes^n} \left[F\left(\sum_{i=0}^{n} \leftthreetimes^i u_i \right) \right]_{\leftthreetimes=0}, \quad n = 0, 1, 2, 3, ...$$

3. Mathematical formulation of the model

In this portion, we will establish a mathematical technique in order to get a better knowledge about Natural transformation coupled with Adomian decomposition method (NADM). Hence, a nonlinear time fractional PDEs of order γ is considered, i.e.,

$$\mathcal{D}_t^\gamma v(y, t) + \mathcal{R}v(y, t) + \mathcal{F}v(y, t) = \mathcal{H}(y, t), \tag{3.1}$$

having initial condition as $v(y, 0) = \psi(y)$, where $\mathcal{D}_t^\gamma v(y, t)$, $\mathcal{R}v(y, t)$, $\mathcal{F}v(y, t)$ and $\mathcal{H}(y, t)$ represents the Caputo fractional derivative, linear differential operator, non linear differential operator and the source term respectively.

Now according to Definition 2.2, the above PDE can be written as

$$[\mathcal{D}_t^\gamma v(y, \beta, t), \mathcal{D}_t^\gamma \bar{v}(y, \beta, t)] + [\mathcal{R}v(y, \beta, t), \mathcal{R}\bar{v}(y, \beta, t)] + [\mathcal{F}v(y, \beta, t), \mathcal{F}\bar{v}(y, \beta, t)] = [\mathcal{H}(y, \beta, t), \bar{\mathcal{H}}(y, \beta, t)].$$

Using Definition 2.3 we can write as

$$\begin{aligned}&\Big[\hbar(\mathcal{D}_t^\gamma \bar{v}(y,\beta,t)-\mathcal{D}_t^\gamma v(y,\beta,t))+\mathcal{D}_t^\gamma v(y,\beta,t)\Big]+\Big[\hbar(\mathcal{R}\bar{v}(y,\beta,t)-\mathcal{R}v(y,\beta,t))+\mathcal{R}v(y,\beta,t)\Big]\\&+\Big[\hbar(\mathcal{F}\bar{v}(y,\beta,t)+\mathcal{F}v(y,\beta,t))-\mathcal{F}v(y,\beta,t)\Big]\\=\;&\Big[\hbar(\bar{\mathcal{H}}(y,\beta,t)+\mathcal{H}(y,\beta,t))-\mathcal{H}(y,\beta,t)\Big].\end{aligned}$$

It may be noted that β and $\hbar$ are parameters and $\beta,\hbar\in[0,1]$ is the parametric (double parametric) form of the partial differential equation means fuzzy partial differential equations. Hence, we can write as

$$\begin{aligned}&\Big[\hbar(\mathcal{D}_t^\gamma \bar{v}(y,\beta,t)-\mathcal{D}_t^\gamma v(y,\beta,t))+\mathcal{D}_t^\gamma v(y,\beta,t)\Big]=\mathcal{D}_t^\gamma \hat{v}(y,\hbar,\beta,t),\\&\Big[\hbar(\mathcal{R}\bar{v}(y,\beta,t)-\mathcal{R}v(y,\beta,t))+\mathcal{R}v(y,\beta,t)\Big]=\mathcal{R}\hat{v}(y,\hbar,\beta,t),\\&\Big[\hbar(\mathcal{F}\bar{v}(y,\beta,t)+\mathcal{F}v(y,\beta,t))-\mathcal{F}v(y,\beta,t)\Big]=\mathcal{F}\hat{v}(y,\hbar,\beta,t),\\&\Big[\hbar(\bar{\mathcal{H}}(y,\beta,t)+\mathcal{H}(y,\beta,t))-\mathcal{H}(y,\beta,t)\Big]=\hat{\mathcal{H}}(y,\hbar,\beta,t).\end{aligned}$$

Equation (3.1) will take the form as

$$\mathcal{D}_t^\gamma \hat{v}(y,\hbar,\beta,t)+\mathcal{R}\hat{v}(y,\hbar,\beta,t)+\mathcal{F}\hat{v}(y,\hbar,\beta,t)=\hat{\mathcal{H}}(y,\hbar,\beta,t), \tag{3.2}$$

with associated fuzzy initial condition $\bar{v}(y,0)=\bar{\psi}(y)$.

Now applying natural transform to Eq (3.2), we have

$$N[\mathcal{D}_t^\gamma \hat{v}(y,\hbar,\beta,t)+\mathcal{R}\hat{v}(y,\hbar,\beta,t)+\mathcal{F}\hat{v}(y,\hbar,\beta,t)]=N[\hat{\mathcal{H}}(y,\hbar,\beta,t)]. \tag{3.3}$$

$$\frac{s^\gamma}{u^\gamma}N(\hat{v}(y,\hbar,\beta,t))-\sum_{i=0}^{\gamma-1}\frac{s^{\gamma-i-1}}{u^{\gamma-i}}\hat{v}^i(y,\hbar,\beta,0)+N[\mathcal{R}\hat{v}(y,\hbar,\beta,t)]+N[\mathcal{F}\hat{v}(y,\hbar,\beta,t)]=N[\hat{\mathcal{H}}(y,\hbar,\beta,t)],$$

$$N(\hat{v}(y,\hbar,\beta,t))=\frac{1}{s}\bar{\psi}(y,\hbar,\beta)+\frac{u^\gamma}{s^\gamma}N[-\mathcal{R}\hat{v}(y,\hbar,\beta,t)-\mathcal{F}\hat{v}(y,\hbar,\beta,t)]+\frac{u^\gamma}{s^\gamma}N[\hat{\mathcal{H}}(y,\hbar,\beta,t)],$$

$$N(\hat{v}(y,\hbar,\beta,t))=\frac{1}{s}\bar{\psi}(y,\hbar,\beta)+\frac{u^\gamma}{s^\gamma}N[-\mathcal{R}\hat{v}(y,\hbar,\beta,t)-\mathcal{F}\hat{v}(y,\hbar,\beta,t)]+\frac{u^\gamma}{s^{\gamma+1}}\hat{\mathcal{H}}(y,\hbar,\beta,t).$$

Now taking the inverse natural transform we get

$$\hat{v}(y,\hbar,\beta,t)=\bar{\psi}(y,\beta,\hbar)-N^{-1}[\frac{u^\gamma}{s^\gamma}N[\mathcal{R}\hat{v}(y,\hbar,\beta,t)+\mathcal{F}\hat{v}(y,\hbar,\beta,t)]]+\frac{t^\gamma}{\Gamma(\gamma+1)}\hat{\mathcal{H}}(y,\hbar,\beta,t).$$

Now applying the Adomian decomposition method we have

$$\sum_{i=0}^{\infty}\hat{v}_i(y,\hbar,\beta,t)=\bar{\psi}(y,\hbar,\beta)-N^{-1}\Big[\frac{u^\gamma}{s^\gamma}N[\sum_{i=0}^{\infty}\hat{v}_i(y,\hbar,\beta,t)+\sum_{n=0}^{\infty}A_n]\Big]+\frac{t^\gamma}{\Gamma(\gamma+1)}\hat{\mathcal{H}}(y,\hbar,\beta,t),$$

where

$$A_n=\frac{1}{n!}\frac{d^n}{d\leftthreetimes^n}\Big[F\Big(\sum_{i=0}^{n}\leftthreetimes^i u_i\Big)\Big]_{\leftthreetimes=0},\quad n=0,1,2,3,...$$

After some necessary calculation, the final result can be written in the form as

$$v(y,\hbar,r,t)=\sum_{i=0}^{\infty}\hat{v}_i(y,\hbar,r,t).$$

4. Results and discussions

Example 4.1. [45] Let us imagine the nonlinear homogenous fuzzy time fractional Fisher's equation as

$$\mathcal{D}_t^{\gamma}\hat{v}(y,\hbar,\beta,t) = \frac{\partial^2\hat{v}(y,\hbar,\beta,t)}{\partial y^2} + 6\hat{v}(y,\hbar,\beta,t)[1-\hat{v}(y,\hbar,\beta,t)], \quad \gamma\in(0,1). \tag{4.1}$$

Having IC's as $\hat{v}(y,\hbar,\beta,0) = \bar{\kappa}\frac{1}{(1+e^y)^2}$, where $\kappa = [-1,0,1]$, shows the triangular fuzzy number, moreover, in $r-cut$ form κ will be as $[\kappa,\bar{\kappa}] = [\beta-1, 1-\beta]$, in sense of double parametric form we have

$$\kappa = \hbar[\kappa-\bar{\kappa}] + \kappa = \hbar(2-\beta)+1-\beta, \quad \hbar, r\in[0,1].$$

Equation (4.1) can be written as

$$\mathcal{D}_t^{\gamma}\hat{v}(y,\hbar,\beta,t) = \frac{\partial^2\hat{v}(y,\hbar,\beta,t)}{\partial y^2} + 6\hat{v}(y,\hbar,\beta,t) - 6\hat{v}^2(y,\hbar,\beta,t). \tag{4.2}$$

Taking natural transform of (4.2), we have

$$N[\mathcal{D}_t^{\gamma}\hat{v}(y,\hbar,\beta,t)] = N\Big[\frac{\partial^2\hat{v}(y,\hbar,\beta,t)}{\partial y^2} + 6\hat{v}(y,\hbar,\beta,t) - 6\hat{v}^2(y,\hbar,\beta,t)\Big],$$

$$\frac{s^{\gamma}}{u^{\gamma}}(\hat{v}(y,s,\hbar,\beta,t)) - \sum_{i=0}^{\gamma-1}\frac{s^{\gamma-i-1}}{u^{\gamma-i}}\hat{v}^i(y,s,\hbar,\beta,0) = N\Big[\frac{\partial^2\hat{v}(y,\hbar,\beta,t)}{\partial y^2} + 6\hat{v}(y,\hbar,\beta,t) - 6\hat{v}^2(y,\hbar,\beta,t)\Big],$$

$$\hat{v}(y,s,\beta,\hbar,t) = \frac{1}{s}\frac{\kappa}{(1+e^y)^2} + \frac{u^{\gamma}}{s^{\gamma}}N\Big[\frac{\partial^2\hat{v}(y,\hbar,\beta,t)}{\partial y^2} + 6\hat{v}(y,\hbar,\beta,t) - 6\hat{v}^2(y,\hbar,\beta,t)\Big].$$

Applying the inverse natural transform, we have

$$\hat{v}(y,\hbar,\beta,t) = \frac{\kappa}{(1+e^y)^2} + N^{-1}\Big[\frac{u^{\gamma}}{s^{\gamma}}N[\frac{\partial^2\hat{v}(y,\hbar,\beta,t)}{\partial y^2} + 6\hat{v}(y,\hbar,\beta,t) - 6\hat{v}^2(y,\hbar,\beta,t)]\Big].$$

Using Definition 2.9, we have

$$\sum_{i=0}^{\infty}\hat{v}_i(y,\hbar,\beta,t) = \frac{\kappa}{(1+e^y)^2} + N^{-1}\Big[\frac{u^{\gamma}}{s^{\gamma}}N[\frac{\partial^2}{\partial y^2}\sum_{i=0}^{\infty}\hat{v}_i(y,\hbar,\beta,t) + 6\sum_{i=0}^{\infty}\hat{v}_i(y,\hbar,\beta,t) - 6\sum_{n=0}^{\infty}A_n]\Big],$$

where

$$A_n = \frac{1}{n!}\frac{d^n}{d\leftthreetimes^n}\Big[F\Big(\sum_{i=0}^{n}\leftthreetimes^i u_i\Big)\Big]_{\leftthreetimes=0}, \quad n = 0,1,2,3,...$$

After some calculation and comparing the terms, we have

$$\hat{v}_0 = \frac{\kappa}{(1+e^y)^2}.$$

$$\hat{v}_1 = \Big(\frac{6\kappa}{(1+e^y)^2} + \frac{6\kappa e^{2y}}{(1+e^y)^4} - \frac{2\kappa e^y}{(1+e^y)^3} - \frac{6\kappa^2}{(1+e^y)^4}\Big)\frac{t^{\gamma}}{\Gamma(\gamma+1)}.$$

$$\hat{v}_2 = \Big(\frac{36\kappa}{(1+e^y)^2} - \frac{108\kappa^2}{(1+e^y)^4} + \frac{72\kappa^3}{(1+e^y)^5} + \frac{24\kappa^2 e^y}{(1+e^y)^4} + \frac{114\kappa e^{2y}}{(1+e^y)^4} - \frac{26\kappa e^y}{(1+e^y)^3} - \frac{144\kappa e^{3y}}{(1+e^y)^5} + \frac{24\kappa^2 e^y}{(1+e^y)^5} - \frac{72\kappa^2 e^{2y}}{(1+e^y)^5} + \frac{120\kappa e^{4y}}{(1+e^y)^6} - \frac{120\kappa^2 e^{2y}}{(1+e^y)^6}\Big)\frac{t^{2\gamma}}{\Gamma(2\gamma+1)}.$$

$$\hat{v}_3 = \Big(\frac{-254\kappa e^y}{(1+e^y)^2} + \frac{1590\kappa e^{2y}}{(1+e^y)^4} + \frac{2484\kappa^2 e^y}{(1+e^y)^5} - \frac{15612\kappa^2 e^{2y}}{(1+e^y)^6} - \frac{2376\kappa^3 e^y}{(1+e^y)^6} + \frac{8304\kappa^3 e^{2y}}{(1+e^y)^7} + \frac{168\kappa^2 e^y}{(1+e^y)^4} - \frac{48\kappa^2 e^{2y}}{(1+e^y)^5} + \frac{120\kappa^2 e^{3y}}{(1+e^y)^6} - \frac{4752\kappa e^{3y}}{(1+e^y)^5} + \frac{7440\kappa e^{4y}}{(1+e^y)^6} - \frac{4320\kappa e^{4y}}{(1+e^y)^6} - \frac{720\kappa^2 e^{4y}}{(1+e^y)^7} + \frac{216\kappa}{(1+e^y)^2} - \frac{8856\kappa^2}{(1+e^y)^4} + \frac{432\kappa^3}{(1+e^y)^5} + \frac{18744\kappa^2 e^{3y}}{(1+e^y)^7} + \frac{9072\kappa^3}{(1+e^y)^6} - \frac{6048\kappa^4}{(1+e^y)^7} - \frac{5904\kappa^3 e^y}{(1+e^y)^7} - \frac{25365\kappa^2 e^{4y}}{(1+e^y)^8} + \frac{8856\kappa^3 e^{2y}}{(1+e^y)^8} - \frac{6998\kappa^3}{(1+e^y)^8} - \frac{31104\kappa^6}{(1+e^y)^{10}} + \frac{39142\kappa^4 e^{2y}}{(1+e^y)^8} - \frac{151776\kappa^2 e^{6y}}{(1+e^y)^{10}} - \frac{60480\kappa^4 e^{2y}}{(1+e^y)^{10}} - \frac{86400\kappa^2 e^{8y}}{(1+e^y)^{12}} - \frac{1728\kappa^3 e^y}{(1+e^y)^7} + \frac{15552\kappa^5}{(1+e^y)^9} - \frac{42816\kappa^3 e^{3y}}{(1+e^y)^9} - \frac{12384\kappa^4 e^{2y}}{(1+e^y)^9} - \frac{15552\kappa^4 e^y}{(1+e^y)^9} + \frac{60192\kappa^3 e^{4y}}{(1+e^y)^{10}} - \frac{3456\kappa^5 e^y}{(1+e^y)^9} + \frac{29952\kappa^4 e^{3y}}{(1+e^y)^{10}} + \frac{6912\kappa^5 e^y}{(1+e^y)^{10}} - \frac{34560\kappa^4 e^{4y}}{(1+e^y)^{11}} + \frac{17280\kappa^5 e^y}{(1+e^y)^{11}} - \frac{9216\kappa^3 e^{3y}}{(1+e^y)^8} + \frac{23328\kappa^3 e^{4y}}{(1+e^y)^9} + \frac{3556\kappa^4 e^{3y}}{(1+e^y)^9} - \frac{5760\kappa^3 e^{5y}}{(1+e^y)^9} + \frac{39072\kappa^2 e^{5y}}{(1+e^y)^9} - \frac{20736\kappa^3 e^{5y}}{(1+e^y)^{10}} + \frac{34560\kappa^2 e^{7y}}{(1+e^y)^{11}} - \frac{40320\kappa^3 e^{5y}}{(1+e^y)^{11}} + \frac{5760\kappa^4 e^{3y}}{(1+e^y)^{11}} + \frac{17280\kappa^3 e^{6y}}{(1+e^y)^{11}} + \frac{2880\kappa^3 e^{6y}}{(1+e^y)^{12}}\Big)\frac{t^{3\gamma}}{\Gamma(3\gamma+1)}.$$

In the same way, the other terms can be found, and hence the n^{th} term can be written as

$$\hat{v}_i(y,\hbar,\beta,t) = \hat{v}_0 + \hat{v}_1\frac{t^{\gamma}}{\Gamma(\gamma+1)} + \hat{v}_2\frac{t^{2\gamma}}{\Gamma(2\gamma+1)} + \hat{v}_3\frac{t^{3\gamma}}{\Gamma(3\gamma+1)} + \hat{v}_4\frac{t^{4\gamma}}{\Gamma(4\gamma+1)} + \ldots + \hat{v}_n\frac{t^{n\gamma}}{\Gamma(n\gamma+1)},$$

$$\hat{v}_i(y,\hbar,\beta,t) = \sum_{n=0}^{\infty}\hat{v}_n\frac{(t^{\gamma})^n}{\Gamma(n\gamma+1)}.$$

Here, we present 3D profile of fuzzy solution of Example 4.1 in Figure 1.

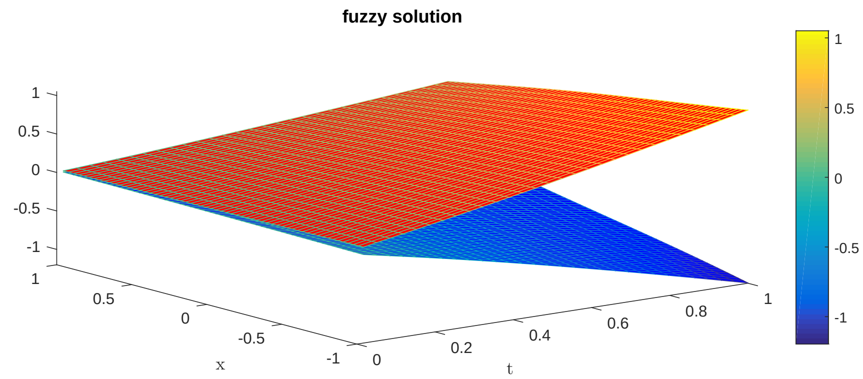

Figure 1. 3D profile of fuzzy solution of Example 4.1.

Example 4.2. [45] Let us consider the non-linear homogenous fuzzy time fractional Fisher's (Burgers Fisher's) equation as

$$\mathcal{D}_t^{\gamma}\hat{v}(y,\hbar,\beta,t) = \frac{\partial^2\hat{v}(y,\hbar,\beta,t)}{\partial y^2} - \hat{v}(y,\hbar,\beta,t)\frac{\partial\hat{v}(y,\hbar,\beta,t)}{\partial y} - \hat{v}(y,\hbar,\beta,t)[\hat{v}(y,\hbar,\beta,t)-1], \quad \gamma\in(0,1), \tag{4.3}$$

with IC's as $\hat{v}(y,\hbar,\beta,0) = \bar{\kappa}\Big(\frac{1}{2} + \frac{1}{2}tanh(\frac{y}{4})\Big)$, where $\kappa = [-1,0,1]$, shows the triangular fuzzy number, moreover, in $r-cut$ form κ can be written as $[\underline{\kappa},\bar{\kappa}]$= $[\beta-1, 1-\beta]$, in sense of double parametric form this can be written as κ= $\hbar[\underline{\kappa}-\bar{\kappa}]+\underline{\kappa}$= $\hbar(2-\beta)+1-\beta$, $\hbar,\beta\in[0,1]$.

Equation (4.3) can be written as

$$\mathcal{D}_t^{\gamma}\hat{v}(y,\hbar,\beta,t) = \frac{\partial^2\hat{v}(y,\hbar,\beta,t)}{\partial y^2} - \hat{v}(y,\hbar,\beta,t)\frac{\partial\hat{v}(y,\hbar,\beta,t)}{\partial y} - \hat{v}^2(y,\hbar,\beta,t) - \hat{v}(y,\hbar,\beta,t).$$

Following the same procedure, we get

$$\begin{aligned}
\hat{v}_0 &= \bar{\kappa}\Big(\frac{1}{2}+\frac{1}{2}\tanh(\frac{y}{4})\Big).\\
\hat{v}_1 &= \Big(\tanh\frac{y}{4}\sec h^2\frac{y}{4}(\frac{\kappa}{16}-\frac{\kappa^2}{16}) - \frac{\kappa^2}{16}\sec h^2\frac{y}{4} - \frac{\kappa}{2}\tanh\frac{y}{4} - \frac{\kappa^2}{4}\tanh^2\frac{y}{4}\\
&\quad -\frac{\kappa^2}{2}\tanh\frac{y}{4} - \frac{\kappa^2}{4} - \frac{\kappa}{2}\Big)\frac{t^{\gamma}}{\Gamma(\gamma+1)}.\\
\hat{v}_2 &= \Big(\frac{\kappa}{2}+\frac{3}{4}\kappa+\frac{\kappa^3}{4}+(\frac{\kappa}{2}+\frac{3}{2}\kappa^2+\frac{3}{4}\kappa^3)\tanh(\frac{y}{4}) + (\frac{3\kappa^2+3\kappa^3}{4})\tanh^2(\frac{y}{4}) + \frac{\kappa^3}{4}\tanh^3(\frac{y}{4})\\
&\quad +(\frac{\kappa^2-4\kappa^3-4\kappa^4}{128})\sec h^2(\frac{y}{4}) + (\frac{-40\kappa^2+4\kappa-17\kappa^4}{1024})\sec h^4(\frac{y}{4})\\
&\quad +\frac{\kappa^3}{1024}\sec h^6(\frac{y}{4}) + (\frac{-16\kappa-8\kappa^2-10\kappa^3-13\kappa^4}{128})\tanh(\frac{y}{4})\sec h^2(\frac{y}{4})\\
&\quad +(\frac{16\kappa-8\kappa^2-12\kappa^3-\kappa^4}{512})\tanh(\frac{y}{4})\sec h^4(\frac{y}{4}) + (\frac{-\kappa^4}{512})\tanh(\frac{y}{4})\sec h^6(\frac{y}{4})\\
&\quad +(\frac{-16\kappa^2-7\kappa^3-16\kappa^4}{128})\tanh^2(\frac{y}{4})\sec h^2(\frac{y}{4}) + (\frac{8\kappa^3-7\kappa^4}{512})\tanh^2(\frac{y}{4})\sec h^4(\frac{y}{4})\\
&\quad +(\frac{2\kappa-7\kappa^4}{128})\tanh^3(\frac{y}{4})\sec h^2(\frac{y}{4}) + (\frac{-\kappa^2-2\kappa^3-\kappa^4}{512})\tanh^3(\frac{y}{4})\sec h^4(\frac{y}{4})\\
&\quad +\frac{\kappa^3}{128}\tanh^4(\frac{y}{4})\sec h^4(\frac{y}{4})\Big)\frac{t^{2\gamma}}{\Gamma(2\gamma+1)}.
\end{aligned}$$

In the same way, the other terms can be found, and hence the n^{th} term can be written as

$$\begin{aligned}
\hat{v}_i(y,\hbar,\beta,t) &= \hat{v}_0 + \hat{v}_1\frac{t^{\gamma}}{\Gamma(\gamma+1)} + \hat{v}_2\frac{t^{2\gamma}}{\Gamma(2\gamma+1)} + \hat{v}_3\frac{t^{3\gamma}}{\Gamma(3\gamma+1)} + \hat{v}_4\frac{t^{4\gamma}}{\Gamma(4\gamma+1)} + ... + \hat{v}_n\frac{t^{n\gamma}}{\Gamma(n\gamma+1)}.\\
\hat{v}_i(y,\hbar,\beta,t) &= \sum_{n=0}^{\infty}\hat{v}_n\frac{(t^{\gamma})^n}{\Gamma(n\gamma+1)}.
\end{aligned}$$

We have presented the 3D profile of fuzzy solutions in Figures 2 and 3 respectively.

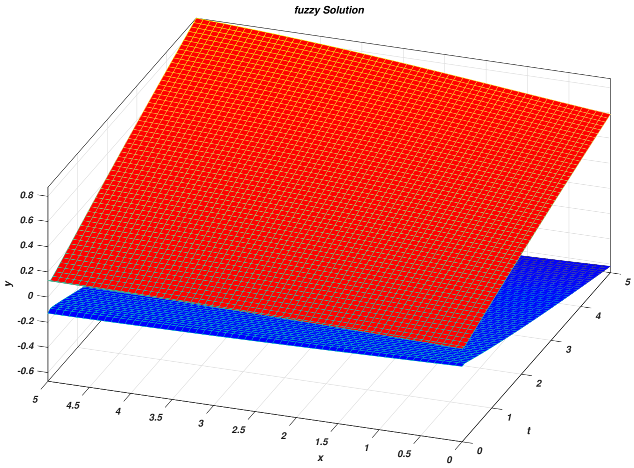

Figure 2. 3D profile of fuzzy solution of Example 4.2 .

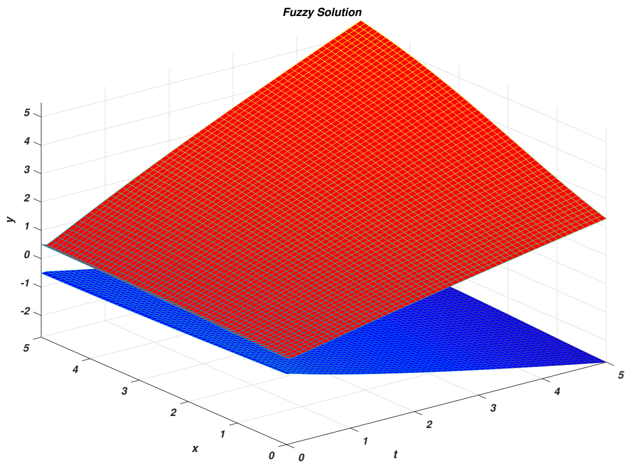

Figure 3. 3D profile of fuzzy solution of Example 4.2.

5. Conclusions

A useful technique of the integral transform (natural transform) coupled with the Adomian decomposition method (ADM) has been presented for the analytical solution of fuzzy nonlinear time fractional generalized Fisher's equation using the parametric (double parametric) form of fuzzy numbers, whereas the analytical results were given in series form. Furthermore, the efficiency of the proposed technique was checked by assigning different values to the fuzzy parameters "κ" and "$\hbar$". During analysis, it has been observed that as the uncertainty increases, the distance between the upper and lower solutions increases, i.e., the distance between the solutions is directly proportional to the increasing of uncertainty, which means that as the uncertainty decreases, the fuzzy solution converts (approaches) to classical solutions. In an analytical sense, the current techniques are efficient, reliable, and less time-consuming as compared to the other techniques. And hence the result shows that the techniques (proposed techniques) are computationally efficient. Moreover, it is suggested that the formulation presented is simple and can be extended to other linear and nonlinear fractional order partial differential equations. It is hoped that the simplicity of this formulation will initiate a new interest for multiple purposes, like de noising, image processing, image segmentation, etc.

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Conflict of interest

The authors declare that they have no conflicts of interest.

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