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*Research article*

# Positive ground state solutions for a class of fractional coupled Choquard systems

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**Abstract:** In this paper, we combine the critical point theory and variational method to investigate the following a class of coupled fractional systems of Choquard type

$$\begin{cases} (-\Delta)^s u + \lambda_1 u = (I_\alpha * |u|^p)|u|^{p-2}u + \beta v & \text{in } \mathbb{R}^N, \\ (-\Delta)^s v + \lambda_2 v = (I_\alpha * |v|^p)|v|^{p-2}v + \beta u & \text{in } \mathbb{R}^N, \end{cases}$$

with $s \in (0, 1),\ N \geq 3,\ \alpha \in (0, N),\ p > 1,\ \lambda_i > 0$ are constants for $i = 1,\ 2, \beta > 0$ is a parameter, and $I_\alpha(x)$ is the Riesz Potential. We prove the existence and asymptotic behaviour of positive ground state solutions of the systems by using constrained minimization method and Hardy-Littlewood-Sobolev inequality. Moreover, nonexistence of nontrivial solutions is also obtained.

**Keywords:** fractional Choquard systems; ground state solution; asymptotic behaviour
**Mathematics Subject Classification:** 35J65, 47J05, 47J30

## 1. Introduction

In this paper, we are interested in establishing the existence and nonexistence results of nontrivial solutions for the coupled fractional Schrödinger systems of Choquard type

$$\begin{cases} (-\Delta)^s u + \lambda_1 u = (I_\alpha * |u|^p)|u|^{p-2}u + \beta v & \text{in } \mathbb{R}^N, \\ (-\Delta)^s v + \lambda_2 v = (I_\alpha * |v|^p)|v|^{p-2}v + \beta u & \text{in } \mathbb{R}^N, \end{cases} \tag{1.1}$$

where $s \in (0, 1),\ N \geq 3,\ \alpha \in (0, N),\ p > 1,\ \lambda_i > 0$ are constants for $i = 1,\ 2, \beta > 0$ is a parameter, and $I_\alpha(x)$ is the Riesz Potential defined as

$$I_\alpha(x) = \frac{\Gamma(\frac{N-\alpha}{2})}{\Gamma(\frac{\alpha}{2})\pi^{\frac{N}{2}}2^\alpha|x|^{N-\alpha}}, \quad x \in \mathbb{R}^N \setminus \{0\},$$

where $\Gamma$ is the Gamma function.

Here, the nonlocal Laplacian operator $(-\Delta)^s$ with $s \in (0,1)$ of a function $u : \mathbb{R}^N \to R$ is expressed by the formula

$$(-\Delta)^s u(x) = C(N,s)\text{P.V.} \int_{\mathbb{R}^N} \frac{u(x)-u(z)}{|x-z|^{N+2s}} dz,$$

where P.V. stand for the Cauchy principal value on the integral, and $C(N,s)$ is some positive normalization constant (see [1] for details).

It can also be defined as a pseudo-differential operator

$$\mathcal{F}((-\Delta)^s f)(\xi) = |\xi|^{2s}\mathcal{F}(f)(\xi) = |\xi|^{2s}\hat{f}(\xi),$$

where $\mathcal{F}$ is the Fourier transform.

The problem (1.1) presents nonlocal characteristics in the nonlinearity as well as in the (fractional) diffusion because of the appearance of the terms $(I_\alpha * |u|^p)|u|^{p-2}u$ and $(I_\alpha * |v|^p)|v|^{p-2}v$. This phenomenon raises some mathematical puzzles that make the study of such problems particularly interesting. We point out that when $s = 1$, $\lambda_1 = 1$, $p = 2$, $N = 3$, $\alpha = 2$ and $\beta = 0$, (1.1) reduces to the Choquard-Pekar equation

$$-\Delta u + u = (I_2 * |u|^2)u, \quad \text{in } \mathbb{R}^3, \tag{1.2}$$

which appeared in 1954 by Pekar [2] describing a polaron at rest in the quantum theory. In 1976, Choquard [3] used this equation to model an electron trapped in its own hole and considered it as an approximation to Hartree-Fock theory of one-component plasma. Subsequently, in 1996 Penrose [4] investigated it as a model for the self-gravitating collapse of a quantum mechanical wave function; see also [5]. The first investigations for existence and uniqueness of ground state solutions of (1.2) go back to the work of Lieb [6]. Lions [7] generalized the result in [6] and proved the existence and multiplicity of positive solutions of (1.2). In addition, the existence and qualitative results of solutions of power type nonlinearities $|u|^{p-2}u$ and for more generic values of $\alpha \in (0,N)$ are discussed by variational method, where $N \geq 3$, see [8–12]. Under almost necessary conditions on the nonlinearity $F$ in the spirit of H. Berestycki and P. L. Lions [13], Moroz and Schaftingen [14] considered the existence of a ground state solution $u \in H^1(\mathbb{R}^N)$ to the nonlinear Choquard equation

$$-\Delta u + u = (I_\alpha * F(u))F'(u), \quad \text{in } \mathbb{R}^N.$$

When $s \in (0,1)$, Laskin [15] introduced the fractional power of the Laplace operator in (1.1) as an extension of the classical local Laplace operator in the study of nonlinear Schrödinger equations, replacing the path integral over Brownian motions with Lévy flights [16]. This operator has concrete applications in a wide range of fields, see [1, 17] and the references therein. Equations involving the fractional Laplacian together with local nonlinearities and the system of weakly coupled equations has been investigated extensively in recent years, and some research results can be found in [18–21].

When $\beta = 0$, the system (1.1) can be reduced to two single Choquard equations

$$(-\Delta)^s u + \lambda_1 u = (I_\alpha * |u|^p)|u|^{p-2}u \quad \text{in } \mathbb{R}^N \tag{1.3}$$

and

$$(-\Delta)^s v + \lambda_2 v = (I_\alpha * |v|^p)|v|^{p-2}v \quad \text{in } \mathbb{R}^N. \tag{1.4}$$

Equations (1.3) and (1.4) arise from the search for standing wave solutions of the following time-dependent fractional Choquard equation:

$$i\frac{\partial\Psi}{\partial t} = (-\Delta)^s\Psi + \lambda\Psi - (I_\alpha * |\Psi|^p)|\Psi|^{p-2}\Psi, \quad (t, x) \in \mathbb{R}_+ \times \mathbb{R}^N,$$

where $i$ denotes the imaginary unit.

In [22], by minimizing

$$S(u) = \frac{\|(-\Delta)^{\frac{s}{2}}u\|_2^2 + \lambda_1\|u\|_2^2}{(\int_{\mathbb{R}^N}(I_\alpha * |u|^p)|u|^p)^{\frac{1}{p}}}$$

on $H^s(\mathbb{R}^N)\backslash\{0\}$, the authors obtained the existence of ground state solution of (1.3) with $p \in (1+\frac{\alpha}{N}, \frac{N+\alpha}{N-2s})$ (see [22, Theorem 4.2]).

Of course, scalar problems can be extended to systems. It is easy to see that the system (1.1) can be regarded as a counterpart of the following systems with standard Laplace operator

$$\begin{cases} -\Delta u + u = (I_\alpha * |u|^p)|u|^{p-2}u + \lambda v & \text{in } \mathbb{R}^N, \\ -\Delta v + v = (I_\alpha * |v|^p)|v|^{p-2}v + \lambda u & \text{in } \mathbb{R}^N. \end{cases}$$

In [23], Chen and Liu studied the systems of Choquard type, when $p \in (1 + \frac{\alpha}{N}, \frac{N+\alpha}{N-2})$, they obtained the existence of ground state solutions of the systems. Yang et al. [24] considered the corresponding critical case.

Motivated by the above mentioned works, in this paper, we aim to study the existence of positive ground state solutions of the systems (1.1). This class of systems has two new characteristics: One is the presence of the fractional Laplace and the Choquard type functions which are nonlocal, the other is its lack of compactness inherent to problems defined on unbounded domains. In order to overcome such difficulties, next we introduce a special space where we are able to recover some compactness.

First we use $\|\cdot\|_p$ denote the norm of $L^p(\mathbb{R}^N)$ for any $1 \le p < \infty$. The Hilbert space $H^s(\mathbb{R}^N)$ is defined by

$$H^s(\mathbb{R}^N) := \left\{u \in L^2(\mathbb{R}^N) : \int_{\mathbb{R}^N}\int_{\mathbb{R}^N}\frac{|u(x) - u(z)|^2}{|x - z|^{N+2s}}dxdz < +\infty\right\}$$

with the scalar product and norm given by

$$\langle u, v\rangle := \int_{\mathbb{R}^N}(-\Delta)^{\frac{s}{2}}u(-\Delta)^{\frac{s}{2}}vdx + \int_{\mathbb{R}^N}uvdx,$$
$$\|u\| := (\|(-\Delta)^{\frac{s}{2}}u\|_2^2 + \|u\|_2^2)^{\frac{1}{2}},$$

where

$$\|(-\Delta)^{\frac{s}{2}}u\|_2^2 := \frac{C(N, s)}{2}\int_{\mathbb{R}^N}\int_{\mathbb{R}^N}\frac{|u(x) - u(z)|^2}{|x - z|^{N+2s}}dxdz.$$

The radial space $H_r^s(\mathbb{R}^N)$ of $H^s(\mathbb{R}^N)$ is defined as

$$H_r^s(\mathbb{R}^N) := \{u \in H^s(\mathbb{R}^N)|u(x) = u(|x|)\}$$

with the $H^s(\mathbb{R}^N)$ norm.

Let

$$\|u\|_{\lambda_i}^2 := \|(-\Delta)^{\frac{s}{2}}u\|_2^2 + \lambda_i\|u\|_2^2, \quad i = 1, 2$$

for convenience. It is easy to obtain that $\|\cdot\|_{\lambda_i}$ and $\|\cdot\|$ are equivalent norms in $H^s(\mathbb{R}^N)$. Denote $H := H^s(\mathbb{R}^N) \times H^s(\mathbb{R}^N)$ and $H_r := H_r^s(\mathbb{R}^N) \times H_r^s(\mathbb{R}^N)$. The norm of $H$ is given by

$$\|(u, v)\|_H^2 = \|u\|_{\lambda_1}^2 + \|v\|_{\lambda_2}^2, \quad \text{for all } (u, v) \in H.$$

The energy functional $E_\beta$ associated to (1.1) is

$$\begin{aligned} E_\beta(u, v) =& \frac{1}{2}\int_{\mathbb{R}^N}[|(-\Delta)^{\frac{s}{2}}u|^2 + |(-\Delta)^{\frac{s}{2}}v|^2 + \lambda_1|u|^2 + \lambda_2|v|^2]dx - \frac{1}{2p}\int_{\mathbb{R}^N}(I_\alpha * |u|^p)|u|^p dx \\ & - \frac{1}{2p}\int_{\mathbb{R}^N}(I_\alpha * |v|^p)|v|^p dx - \beta\int_{\mathbb{R}^N} uv dx, \quad \text{for all } (u, v) \in H. \end{aligned} \tag{1.5}$$

It is easy to obtain that $E_\beta \in C^1(H, \mathbb{R})$ and

$$\begin{aligned} \langle E_\beta'(u, v), (\varphi, \psi)\rangle =& \int_{\mathbb{R}^N}[(-\Delta)^{\frac{s}{2}}u(-\Delta)^{\frac{s}{2}}\varphi + (-\Delta)^{\frac{s}{2}}v(-\Delta)^{\frac{s}{2}}\psi + \lambda_1 u\varphi + \lambda_2 v\psi]dx \\ & - \int_{\mathbb{R}^N}(I_\alpha * |u|^p)|u|^{p-2}u\varphi dx - \int_{\mathbb{R}^N}(I_\alpha * |v|^p)|v|^{p-2}v\psi dx \\ & - \beta\int_{\mathbb{R}^N}(v\varphi + u\psi)dx \end{aligned} \tag{1.6}$$

for all $(\varphi, \psi) \in H$.

$(u, v)$ is called a nontrivial solution of (1.1) if $u_\beta \not\equiv 0$, $v_\beta \not\equiv 0$ and $(u, v) \in H$ solves (1.1). A positive ground state solution $(u, v)$ of (1.1) is a nontrivial solution of (1.1) such that $u > 0$, $v > 0$ which has minimal energy among all nontrivial solutions. In order to find positive ground state solutions of (1.1), we need to investigate the existence of the minimum value of $E_\beta$, defined in (1.5) under the Nehari manifold constraint

$$\mathcal{N}_\beta = \{(u, v) \in H \setminus \{(0, 0)\} : \langle E_\beta'(u, v), (u, v)\rangle = 0\}. \tag{1.7}$$

Define

$$m_\beta = \inf\{E_\beta(u, v) : (u, v) \in \mathcal{N}_\beta\}.$$

Furthermore, define $E_{0,i} : H^s(\mathbb{R}^N) \to \mathbb{R}$ by

$$E_{0,i}(u) = \frac{1}{2}\int_{\mathbb{R}^N}|(-\Delta)^{\frac{s}{2}}u|^2 dx + \frac{\lambda_i}{2}\int_{\mathbb{R}^N}u^2 dx - \frac{1}{2p}\int_{\mathbb{R}^N}(I_\alpha * |u|^p)|u|^p dx, \quad i = 1, 2. \tag{1.8}$$

We introduce the Nehari manifolds

$$\mathcal{N}_{0,i} := \left\{u \in H^s(\mathbb{R}^N) \setminus \{0\} : \|(-\Delta)^{\frac{s}{2}}u\|_2^2 + \lambda_i\|u\|_2^2 - \int_{\mathbb{R}^N}(I_\alpha * |u|^p)|u|^p dx = 0\right\}, \quad i = 1, 2. \tag{1.9}$$

A ground state solution of (1.3) (or (1.4)) is a solution with minimal energy $E_{0,1}$ (or $E_{0,2}$ ) and can be characterized as

$$\min_{u\in\mathcal{N}_{0,1}} E_{0,1}(u) \ (or \min_{u\in\mathcal{N}_{0,2}} E_{0,2}(u)).$$

The main results of our paper are the following.

**Theorem 1.1.** *Suppose* $s \in (0,1)$, $N \geq 3$, $\alpha \in (0,N)$ *and* $p \in (1+\frac{\alpha}{N}, \frac{\alpha+N}{N-2s})$, *then the system (1.1) possesses a positive radial ground state solution* $(u_\beta, v_\beta) \in \mathcal{N}_\beta$ *with* $E_\beta(u_\beta, v_\beta) = m_\beta > 0$ *for any* $0 < \beta < \sqrt{\lambda_1\lambda_2}$. *Moreover,* $(u_\beta, v_\beta) \to (u_0, v_0)$ *in* $H$ *as* $\beta \to 0^+$, *where* $(u_0, v_0)$ *is a positive radial ground state solution for the system (1.1) with* $\beta = 0$, *namely,* $u_0$ *and* $v_0$ *are positive radial ground state solutions to problems (1.3) and (1.4), respectively.*

**Remark 1.1.** *In comparison with [19], this paper has several new features. Firstly, the system (1.1) contains the Choquard type terms which are more difficult to deal with. Secondly, Lemma 3.11 in [19] shows that* $(u_\beta, v_\beta) \to (u_0, v_0)$ *in* $H$ *as* $\beta \to 0^+$, *where either* $v_0 \equiv 0$ *and* $u_0$ *is a ground state solution to one single equation, or* $u_0 \equiv 0$ *and* $v_0$ *is a ground state solution to the other single equation. While we prove that* $(u_0, v_0)$ *is a positive radial ground state solution for the system (1.1) with* $\beta = 0$. *Finally, the difference in asymptotic behavior is that it is obtained in this paper that* $u_0 > 0$ *and* $v_0 > 0$ *are positive radial ground state solutions to problems (1.3) and (1.4), respectively (see Theorem 1.3 in [19]).*

Finally, by using the Pohožaev identity (4.1) of the system (1.1), we have the following non-existence result.

**Theorem 1.2.** *Suppose* $p \geq \frac{\alpha+N}{N-2s}$ *or* $p \leq 1+\frac{\alpha}{N}$, *then the system (1.1) does not admit non-trivial solutions.*

**Remark 1.2.** *According to Theorem 1.2, we can know that the range of* $p \in (1+\frac{\alpha}{N}, \frac{\alpha+N}{N-2s})$ *is optimal for the existence of nontrivial solutions to the system (1.1).*

The rest of this paper is as following. In Section 2, we introduce some preliminary results and notions. In Section 3, we obtain the existence of ground state solutions of the system (1.1) and we also investigate their asymptotic behaviour. In Section 4, we get the nonexistence result.

Throughout this paper, we use " $\to$ " and " $\rightharpoonup$ " to denote the strong convergence and weak convergence in the correlation function space, respectively. $o_n(1)$ denotes a sequence which converges to 0 as $n \to \infty$. $C$ will always denote a positive constants, which may vary from line to line.

## 2. Preliminaries

It is well known that the following properties which follow from the fractional Sobolev embedding

$$H^s(\mathbb{R}^N) \hookrightarrow L^q(\mathbb{R}^N), \quad q \in [2, 2_s^*], \text{ where } 2_s^* := \frac{2N}{N-2s}.$$

If $1+\frac{\alpha}{N} < p < \frac{\alpha+N}{N-2s}$, we have that $2 < \frac{2Np}{N+\alpha} < 2_s^*$, the space $H_r^s(\mathbb{R}^N)$ compactly embedded into $L^{\frac{2Np}{N+\alpha}}(\mathbb{R}^N)$.

First of all, let us recall the Hardy-Littlewood-Sobolev inequality.

**Lemma 2.1.** *(Hardy-Littlewood-Sobolev inequality [23]) Let* $0 < \alpha < N$, $r,\ q > 1$ *and* $1 \leq s < t < \infty$ *be such that*

$$\frac{1}{r} + \frac{1}{q} = 1 + \frac{\alpha}{N}, \quad \frac{1}{s} - \frac{1}{t} = \frac{\alpha}{N}.$$

*(i) For any* $u \in L^r(\mathbb{R}^N)$ *and* $v \in L^q(\mathbb{R}^N)$, *we have*

$$\left| \int_{\mathbb{R}^N} (I_\alpha * u)v \right| \leq C(N, \alpha, q) \|u\|_r \|v\|_q. \tag{2.1}$$

*If* $p \in (1+\frac{\alpha}{N}, \frac{\alpha+N}{N-2s})$ *and* $r = q = \frac{2N}{N+\alpha}$*, then*

$$\left|\int_{\mathbb{R}^N} (I_\alpha * |u|^p)|u|^p\right| \le C(N,\alpha,p)\|u\|^{2p}_{\frac{2Np}{N+\alpha}}, \tag{2.2}$$

*where the sharp constant* $C(N,\alpha,p)$ *is*

$$C(N,\alpha,p) = C_\alpha(N) = \pi^{\frac{N-\alpha}{2}} \frac{\Gamma(\frac{\alpha}{2})}{\Gamma(\frac{N+\alpha}{2})} \left\{\frac{\Gamma(\frac{N}{2})}{\Gamma(N)}\right\}^{-\frac{\alpha}{N}}.$$

*(ii) For any* $u \in L^s(\mathbb{R}^N)$*, we have*

$$\|I_\alpha * u\|_t \le C(N,\alpha,s)\|u\|_s. \tag{2.3}$$

*Here,* $C(N,\alpha,s)$ *is a positive constant which depends only on* $N$*,* $\alpha$ *and* $s$*, and satisfies*

$$\limsup_{\alpha\to 0} \alpha C(N,\alpha,s) \le \frac{2}{s(s-1)}\omega_{N-1},$$

*where* $\omega_{N-1}$ *denotes the surface area of the* $N-1$ *dimensional unit sphere.*

Next, the following result is crucial in the proof of the Theorem 1.1.

**Lemma 2.2.** *Assumption* $N \in \mathbb{N}$*,* $0 < \alpha < N$ *and* $p \in (1+\frac{\alpha}{N}, \frac{\alpha+N}{N-2s})$*. Let* $\{u_n\} \subset H^s(\mathbb{R}^N)$ *be a sequence satisfying that* $u_n \rightharpoonup u$ *weakly in* $H^s(\mathbb{R}^N)$ *as* $n \to \infty$*, then*

$$\lim_{n\to\infty} \int_{\mathbb{R}^N} (I_\alpha * |u_n|^p)|u_n|^p - \int_{\mathbb{R}^N} (I_\alpha * |u_n - u|^p)|u_n - u|^p = \int_{\mathbb{R}^N} (I_\alpha * |u|^p)|u|^p. \tag{2.4}$$

To show Lemma 2.2, we state the classical Brezis-Lieb lemma [25].

**Lemma 2.3.** *Let* $\Omega \subseteq \mathbb{R}^N$ *be an open subset and* $1 \le r < \infty$*. If*
*(i)* $\{u_n\}_{n\in\mathbb{N}}$ *is bounded in* $L^r(\Omega)$*.*
*(ii)* $u_n \to u$ *almost everywhere on* $\Omega$ *as* $n \to \infty$*, then for every* $q \in [1, r]$*,*

$$\lim_{n\to\infty} \int_\Omega \left||u_n|^q - |u_n - u|^q - |u|^q\right|^{\frac{r}{q}} = 0. \tag{2.5}$$

Here we also need to mention sufficient conditions for weak convergence (see for example [25, Proposition 4.7.12]).

**Lemma 2.4.** *Assume* $\Omega$ *be an open subset of* $\mathbb{R}^N$*,* $1 < q < \infty$ *and the sequence* $\{u_n\}_{n\in\mathbb{N}}$ *is bounded in* $L^q(\Omega)$*. If* $u_n \to u$ *almost everywhere on* $\Omega$ *as* $n \to \infty$*, we have that* $u_n \rightharpoonup u$ *weakly in* $L^q(\Omega)$*.*

In view of Lemmas 2.3 and 2.4 we have the following proof.

**Proof of Lemma 2.2.** For every $n \in \mathbb{N}$. We have that

$$\begin{aligned}
&\int_{\mathbb{R}^N} (I_\alpha * |u_n|^p)|u_n|^p - \int_{\mathbb{R}^N} (I_\alpha * |u_n - u|^p)|u_n - u|^p \\
&= \int_{\mathbb{R}^N} (I_\alpha * (|u_n|^p - |u_n - u|^p))(|u_n|^p - |u_n - u|^p) \\
&\quad + 2\int_{\mathbb{R}^N} (I_\alpha * (|u_n|^p - |u_n - u|^p))|u_n - u|^p.
\end{aligned}$$

Since $1 + \frac{\alpha}{N} < p < \frac{\alpha+N}{N-2s}$, we have that $2 < \frac{2Np}{N+\alpha} < 2_s^*$, then the space $H^s(\mathbb{R}^N)$ is embedded continuously in $L^{\frac{2Np}{N+\alpha}}(\mathbb{R}^N)$. Moreover, $u_n \rightharpoonup u$ weakly in $H^s(\mathbb{R}^N)$ as $n \to \infty$. Thus, the sequence $\{u_n\}_{n\in\mathbb{N}}$ is bounded in $L^{\frac{2Np}{N+\alpha}}(\mathbb{R}^N)$. By (2.5) with $q = p$ and $r = \frac{2Np}{N+\alpha}$, we have that

$$|u_n|^p - |u_n - u|^p \to |u|^p$$

strongly in $L^{\frac{2N}{N+\alpha}}(\mathbb{R}^N)$ as $n \to \infty$. By (2.3), we have that $I_\alpha$ defines a linear continuous map from $L^{\frac{2N}{N+\alpha}}(\mathbb{R}^N)$ to $L^{\frac{2N}{N-\alpha}}(\mathbb{R}^N)$, then

$$I_\alpha * (|u_n|^p - |u_n - u|^p) \to I_\alpha * |u|^p$$

in $L^{\frac{2N}{N-\alpha}}(\mathbb{R}^N)$ as $n \to \infty$. By (2.2), we have

$$\int_{\mathbb{R}^N} (I_\alpha * (|u_n|^p - |u_n - u|^p))(|u_n|^p - |u_n - u|^p) = \int_{\mathbb{R}^N} (I_\alpha * |u|^p)|u|^p + o_n(1).$$

In view of Lemma 2.4, we get $|u_n - u|^p \rightharpoonup 0$ weakly in $L^{\frac{2N}{N+\alpha}}(\mathbb{R}^N)$ as $n \to \infty$. Thus,

$$\int_{\mathbb{R}^N} (I_\alpha * (|u_n|^p - |u_n - u|^p))|u_n - u|^p = o_n(1).$$

The proof is thereby complete. □

**Lemma 2.5.** *Let $0 < \alpha < N$, $p \in (1 + \frac{\alpha}{N}, \frac{\alpha+N}{N-2s})$ and the sequence $\{u_n\}_{n\in\mathbb{N}} \subset H^s(\mathbb{R}^N)$ be such that $u_n \rightharpoonup u \in H^s(\mathbb{R}^N)$ weakly in $H^s(\mathbb{R}^N)$ as $n \to \infty$. Let $\phi \in H^s(\mathbb{R}^N)$, we have*

$$\lim_{n\to\infty} \int_{\mathbb{R}^N} (I_\alpha * |u_n|^p)|u_n|^{p-2}u_n\phi = \int_{\mathbb{R}^N} (I_\alpha * |u|^p)|u|^{p-2}u\phi. \tag{2.6}$$

*Proof.* Since $u_n \rightharpoonup u$ weakly in $H^s(\mathbb{R}^N)$ as $n \to \infty$, then $u_n \to u$ a.e. in $\mathbb{R}^N$. By the fractional Sobolev embedding $H^s(\mathbb{R}^N) \hookrightarrow L^q(\mathbb{R}^N)$ with $q \in [2, 2_s^*]$, we see that $\{u_n\}_{n\in\mathbb{N}}$ is bounded in $L^2(\mathbb{R}^N) \cap L^{2_s^*}(\mathbb{R}^N)$. Since $2 < \frac{2Np}{N+\alpha} < 2_s^*$, then $\{|u_n|^p\}$ and $\{|u_n|^{q-2}u_n\}$ are bounded in $L^{\frac{2N}{N+\alpha}}(\mathbb{R}^N)$ and $L^{\frac{q}{q-1}}(\mathbb{R}^N)$ with $q \in [2, 2_s^*]$, respectively, up to a subsequence, we get

$$|u_n|^{q-2}u_n \rightharpoonup |u|^{q-2}u \text{ weakly in } L^{\frac{q}{q-1}}(\mathbb{R}^N),$$

$$|u_n|^p \rightharpoonup |u|^p \text{ weakly in } L^{\frac{2N}{N+\alpha}}(\mathbb{R}^N). \tag{2.7}$$

In view of the Rellich theorem, $u_n \to u$ in $L^t_{loc}(\mathbb{R}^N)$ for $t \in [1, 2_s^*)$ and $|u_n|^{p-2}u_n \to |u|^{p-2}u$ in $L_{loc}^{\frac{2Np}{(p-1)(N+\alpha)}}(\mathbb{R}^N)$ (see [26, Theorem A.2]), then we have that $|u_n|^{p-2}u_n\phi \to |u|^{p-2}u\phi$ in $L^{\frac{2N}{N+\alpha}}(\mathbb{R}^N)$ for any $\phi \in C_0^\infty(\mathbb{R}^N)$, where $C_0^\infty(\mathbb{R}^N)$ denotes the space of the functions infinitely differentiable with compact support in $\mathbb{R}^N$. By (2.3), we get

$$I_\alpha * (|u_n|^{p-2}u_n\phi) \to I_\alpha * (|u|^{p-2}u\phi) \tag{2.8}$$

in $L^{\frac{2N}{N-\alpha}}(\mathbb{R}^N)$. Therefore, by (2.7) and (2.8) we get

$$\begin{aligned}
&\int_{\mathbb{R}^N}(I_\alpha * |u_n|^p)|u_n|^{p-2}u_n\phi - \int_{\mathbb{R}^N}(I_\alpha * |u|^p)|u|^{p-2}u\phi \\
=& \int_{\mathbb{R}^N}(I_\alpha * (|u_n|^{p-2}u_n\phi))|u_n|^p - \int_{\mathbb{R}^N}(I_\alpha * (|u|^{p-2}u\phi))|u|^p \\
=& \int_{\mathbb{R}^N}\left[I_\alpha * (|u_n|^{p-2}u_n\phi) - I_\alpha * (|u|^{p-2}u\phi)\right]|u_n|^p \\
&+ \int_{\mathbb{R}^N}(I_\alpha * (|u|^{p-2}u\phi))(|u_n|^p - |u|^p) \\
\to& \, 0
\end{aligned}$$

as $n \to \infty$. Since $C_0^\infty(\mathbb{R}^N)$ is dense in $H^s(\mathbb{R}^N)$, we reach the conclusion. □

**Lemma 2.6.** *(see [27, Theorem 3.7]) Let $f$, $g$ and $h$ be three non-negative Lebesgue measurable functions on $\mathbb{R}^N$. Let*

$$W(f, g, h) := \int_{\mathbb{R}^N}\int_{\mathbb{R}^N} f(x)g(y)h(x-y)dxdy,$$

*we get*

$$W(f^*, g^*, h^*) \geq W(f, g, h),$$

*where $f^*$, $g^*$ and $h^*$ denote the symmetric radial decreasing rearrangement of $f$, $g$ and $h$.*

**Lemma 2.7.** *(see [22, Theorem 1.1]) Under the assumptions of Theorem 1.1, there exists a ground state solution $u \in H^s(\mathbb{R}^N)$ ( $v \in H^s(\mathbb{R}^N)$) to problem (1.3) ((1.4)) which is positive, radially symmetric. Moreover, the minima of the energy functional $E_{0,1}$ ($E_{0,2}$) on the Nehari manifold $\mathcal{N}_{0,1}$ ($\mathcal{N}_{0,2}$) defined in (1.9) satisfies $\min_{u\in\mathcal{N}_{0,1}} E_{0,1}(u) > 0$ ($\min_{u\in\mathcal{N}_{0,2}} E_{0,2}(u) > 0$).*

## 3. Proof of Theorem 1.1

For any $(u, v) \in \mathcal{N}_\beta$, we have

$$\begin{aligned}
E_\beta(u, v) &= \left(\frac{1}{2} - \frac{1}{2p}\right)\left(\|(u, v)\|_H^2 - 2\beta\int_{\mathbb{R}^N} uvdx\right) \\
&= \left(\frac{1}{2} - \frac{1}{2p}\right)\left(\int_{\mathbb{R}^N}(I_\alpha * |u|^p)|u|^p dx + \int_{\mathbb{R}^N}(I_\alpha * |v|^p)|v|^p dx\right).
\end{aligned}$$

This shows that $E_\beta$ is coercive on $\mathcal{N}_\beta$. Next we show, through a series of lemmas, that $m_\beta$ is attained by some $(u, v) \in \mathcal{N}_\beta$ which is a critical point of $E_\beta$ considered on the whole space $H$, and therefore a ground state solution to (1.1).

We begin with some basic properties of $E_\beta$ and $\mathcal{N}_\beta$.

**Lemma 3.1.** *For every $(u, v) \in H \setminus \{(0, 0)\}$, there exists some $t > 0$ such that $(tu, tv) \in \mathcal{N}_\beta$.*

*Proof.* Indeed, $(tu, tv) \in \mathcal{N}_\beta$ is equivalent to

$$\|(tu, tv)\|_H^2 = \int_{\mathbb{R}^N}(I_\alpha * |tu|^p)|tu|^p + \int_{\mathbb{R}^N}(I_\alpha * |tv|^p)|tv|^p + 2\beta t^2\int_{\mathbb{R}^N} uv,$$

which is solved by

$$t = \left( \frac{\|(u,v)\|_H^2 - 2\beta \int_{\mathbb{R}^N} uv}{\int_{\mathbb{R}^N} (I_\alpha * |u|^p)|u|^p + \int_{\mathbb{R}^N} (I_\alpha * |v|^p)|v|^p} \right)^{\frac{1}{2p-2}}. \tag{3.1}$$

By inequality

$$\begin{aligned} 2\beta \int_{\mathbb{R}^N} uv < 2\sqrt{\lambda_1\lambda_2} \int_{\mathbb{R}^N} uv &\le \int_{\mathbb{R}^N} \lambda_1 u^2 + \lambda_2 v^2 \\ &\le \|u\|_{\lambda_1}^2 + \|v\|_{\lambda_1}^2 = \|(u,v)\|_H^2, \end{aligned}$$

we have that

$$\|(u,v)\|_H^2 - 2\beta \int_{\mathbb{R}^N} uv > \|(u,v)\|_H^2 - \|(u,v)\|_H^2 = 0.$$

Therefore we get $t > 0$. □

**Lemma 3.2.** *The following assertions hold:*
*(i) There exists $c > 0$ such that $\|(u,v)\|_H \ge c$ for any $(u,v) \in \mathcal{N}_\beta$.*
*(ii) $m_\beta = \inf_{(u,v)\in\mathcal{N}_\beta} E_\beta(u,v) > 0$ for all fixed $0 < \beta < \sqrt{\lambda_1\lambda_2}$.*
*(iii) Let $u_1$, $v_1$ are positive solutions of (1.3) and (1.4) respectively, and let $t > 0$ be such that $(tu_1, tv_1) \in \mathcal{N}_\beta$, then $0 < t < 1$.*

*Proof.* (i) In view of the definition of $\mathcal{N}_\beta$, by the Hardy-Littlewood-Sobolev inequality (2.2), for any $(u,v) \in \mathcal{N}_\beta$, we have

$$\begin{aligned} \|u\|_{\lambda_1}^2 + \|v\|_{\lambda_2}^2 = &\int_{\mathbb{R}^N} (I_\alpha * |u|^p)|u|^p + \int_{\mathbb{R}^N} (I_\alpha * |v|^p)|v|^p + 2\beta \int_{\mathbb{R}^N} uv \\ &\le C(N,\alpha,p)(\|u\|_{\frac{2Np}{N+\alpha}}^{2p} + \|v\|_{\frac{2Np}{N+\alpha}}^{2p}) + \frac{\beta}{\sqrt{\lambda_1\lambda_2}} \left( 2\sqrt{\lambda_1\lambda_2} \int_{\mathbb{R}^N} uv \right) \\ &\le C_1 C(N,\alpha,p)(\|u\|_{\lambda_1}^{2p} + \|v\|_{\lambda_2}^{2p}) + \frac{\beta}{\sqrt{\lambda_1\lambda_2}} \left( \int_{\mathbb{R}^N} \lambda_1 u^2 + \lambda_2 v^2 \right) \\ &\le C_1 C(N,\alpha,p)(\|u\|_{\lambda_1}^2 + \|v\|_{\lambda_2}^2)^p + \frac{\beta}{\sqrt{\lambda_1\lambda_2}} (\|u\|_{\lambda_1}^2 + \|v\|_{\lambda_2}^2), \end{aligned}$$

where $C_1 > 0$ denotes the fractional Sobolev embedding constant and $C_1$ does not depend on $u$ and $v$. This means that

$$\left(1 - \frac{\beta}{\sqrt{\lambda_1\lambda_2}}\right) \|(u,v)\|_H^2 \le C_1 C(N,\alpha,p) \|(u,v)\|_H^{2p}.$$

Since $0 < \beta < \sqrt{\lambda_1\lambda_2}$, we have $\|(u,v)\|_H \ge c$, where

$$c = \left( \frac{\sqrt{\lambda_1\lambda_2} - \beta}{C_1 C(N,\alpha,p)\sqrt{\lambda_1\lambda_2}} \right)^{\frac{1}{2p-2}} > 0. \tag{3.2}$$

(ii) For any $(u, v) \in \mathcal{N}_\beta$, we have

$$\begin{aligned} E_\beta(u,v) &= \left(\frac{1}{2}-\frac{1}{2p}\right)\left(\|(u,v)\|_H^2 - 2\beta\int_{\mathbb{R}^N} uv\right) \\ &\geq \left(\frac{1}{2}-\frac{1}{2p}\right)\left(\|(u,v)\|_H^2 - \frac{\beta}{\sqrt{\lambda_1\lambda_2}}(\|u\|_{\lambda_1}^2 + \|v\|_{\lambda_2}^2)\right) \\ &\geq \left(\frac{1}{2}-\frac{1}{2p}\right)\left(1-\frac{\beta}{\sqrt{\lambda_1\lambda_2}}\right)\|(u,v)\|_H^2. \end{aligned} \tag{3.3}$$

Since $p > 1$, we obtain $m_\beta \geq (\frac{1}{2} - \frac{1}{2p})(1 - \frac{\beta}{\sqrt{\lambda_1\lambda_2}})c^2 > 0$.

(iii) Since $u_1, v_1$ are positive solutions of (1.3) and (1.4) respectively, and $(tu_1, tv_1) \in \mathcal{N}_\beta$, we have

$$\|u_1\|_{\lambda_1}^2 + \|v_1\|_{\lambda_2}^2 = \int_{\mathbb{R}^N}(I_\alpha * |u_1|^p)|u_1|^p + \int_{\mathbb{R}^N}(I_\alpha * |v_1|^p)|v_1|^p \tag{3.4}$$

and

$$t^2\left(\|u_1\|_{\lambda_1}^2 + \|v_1\|_{\lambda_2}^2 - 2\beta\int_{\mathbb{R}^N} u_1v_1\right) = t^{2p}\left(\int_{\mathbb{R}^N}(I_\alpha * |u_1|^p)|u_1|^p + \int_{\mathbb{R}^N}(I_\alpha * |v_1|^p)|v_1|^p\right). \tag{3.5}$$

Combining (3.4) and (3.5), we have

$$t^{2p-2} = \frac{\|u_1\|_{\lambda_1}^2 + \|v_1\|_{\lambda_2}^2 - 2\beta\int_{\mathbb{R}^N} u_1v_1}{\|u_1\|_{\lambda_1}^2 + \|v_1\|_{\lambda_2}^2} < 1.$$

The proof is complete. □

**Proof of Theorem 1.1.** Let $(u_n, v_n) \in \mathcal{N}_\beta$ be a minimizing sequence for $E_\beta$, namely such that $E_\beta(u_n, v_n) \to m_\beta$. By (3.3), we know that $\{(u_n, v_n)\}_{n\in\mathbb{N}}$ is bounded in $H$. In view of Lemma 3.1, there exists $t_n > 0$ such that $(t_n|u_n|, t_n|v_n|) \in \mathcal{N}_\beta$. Then

$$\begin{aligned} t_n^{2p-2} &= \frac{\|(|u_n|,|v_n|)\|_H^2 - 2\beta\int_{\mathbb{R}^N}|u_n||v_n|}{\int_{\mathbb{R}^N}(I_\alpha * |u_n|^p)|u_n|^p + \int_{\mathbb{R}^N}(I_\alpha * |v_n|^p)|v_n|^p} \\ &\leq \frac{\|(u_n,v_n)\|_H^2 - 2\beta\int_{\mathbb{R}^N}u_nv_n}{\int_{\mathbb{R}^N}(I_\alpha * |u_n|^p)|u_n|^p + \int_{\mathbb{R}^N}(I_\alpha * |v_n|^p)|v_n|^p} = 1. \end{aligned}$$

Hence, we have that $0 < t_n \leq 1$. Since

$$\begin{aligned} E_\beta(t_n|u_n|, t_n|v_n|) &= \left(\frac{1}{2}-\frac{1}{2p}\right)t_n^{2p}\left(\int_{\mathbb{R}^N}(I_\alpha * |u_n|^p)|u_n|^p + \int_{\mathbb{R}^N}(I_\alpha * |v_n|^p)|v_n|^p\right) \\ &\leq \left(\frac{1}{2}-\frac{1}{2p}\right)\left(\int_{\mathbb{R}^N}(I_\alpha * |u_n|^p)|u_n|^p + \int_{\mathbb{R}^N}(I_\alpha * |v_n|^p)|v_n|^p\right) \\ &= E_\beta(u_n, v_n). \end{aligned}$$

For this reason we can assume that $u_n \geq 0$ and $v_n \geq 0$. Let $u_n^*$ and $v_n^*$ denote the symmetric decreasing rearrangement of $u_n$, respectively $v_n$. By Lemma 2.6 with $f(x) = |u_n(x)|^p$, $g(y) = |u_n(y)|^p$, $h(x-y) = |x-y|^{\alpha-N}$, we have

$$\int_{\mathbb{R}^N}(I_\alpha * |u_n^*|^p)|u_n^*|^p \geq \int_{\mathbb{R}^N}(I_\alpha * |u_n|^p)|u_n|^p. \tag{3.6}$$

In addition, it is well known that

$$\int_{\mathbb{R}^N} |(-\Delta)^{\frac{s}{2}} u_n^*|^2 \le \int_{\mathbb{R}^N} |(-\Delta)^{\frac{s}{2}} u_n|^2 \quad \text{and} \quad \int_{\mathbb{R}^N} |u_n^*|^2 = \int_{\mathbb{R}^N} |u_n|^2 \tag{3.7}$$

(see [28, Theorem 3]). By Hardy-Littlewood inequality and Riesz rearrangement inequality (see [28]),

$$\int_{\mathbb{R}^N} u_n^* v_n^* \ge \int_{\mathbb{R}^N} u_n v_n. \tag{3.8}$$

By (3.6)–(3.8) we have

$$\begin{aligned} E_\beta(u_n^*, v_n^*) &= \frac{1}{2}(\|u_n^*\|_{\lambda_1}^2 + \|v_n^*\|_{\lambda_2}^2) - \frac{1}{2p}\int_{\mathbb{R}^N} (I_\alpha * |u_n^*|^p)|u_n^*|^p - \frac{1}{2p}\int_{\mathbb{R}^N} (I_\alpha * |v_n^*|^p)|v_n^*|^p - \beta \int_{\mathbb{R}^N} u_n^* v_n^* \\ &\le \frac{1}{2}(\|u_n\|_{\lambda_1}^2 + \|v_n\|_{\lambda_2}^2) - \frac{1}{2p}\int_{\mathbb{R}^N} (I_\alpha * |u_n|^p)|u_n|^p - \frac{1}{2p}\int_{\mathbb{R}^N} (I_\alpha * |v_n|^p)|v_n|^p - \beta \int_{\mathbb{R}^N} u_n v_n \\ &= E_\beta(u_n, v_n). \end{aligned}$$

Therefore, we can further assume that $(u_n, v_n) \in H_r$. By (3.3), we have that $\{(u_n, v_n)\}$ is bounded in $H$, there exists $(u_\beta, v_\beta) \in H$ and $u_\beta \ge 0$, $v_\beta \ge 0$ such that up to subsequences, $(u_n, v_n) \rightharpoonup (u_\beta, v_\beta)$ weakly in $H$. Moreover, we also can assume that $u_n \to u_\beta$, $v_n \to v_\beta$ a.e. in $\mathbb{R}^N$ and $(u_\beta, v_\beta) \in H_r$. Since $\{(u_n, v_n)\}_{n\in\mathbb{N}} \subset \mathcal{N}_\beta$, we have

$$\begin{aligned} \int_{\mathbb{R}^N} (I_\alpha * |u_n|^p)|u_n|^p + \int_{\mathbb{R}^N} (I_\alpha * |v_n|^p)|v_n|^p &= \|u_n\|_{\lambda_1}^2 + \|v_n\|_{\lambda_2}^2 - 2\beta \int_{\mathbb{R}^N} u_n v_n \\ &\ge \left(1 - \frac{\beta}{\sqrt{\lambda_1\lambda_2}}\right)\|(u_n, v_n)\|_H^2 \ge \left(1 - \frac{\beta}{\sqrt{\lambda_1\lambda_2}}\right)c^2. \end{aligned}$$

By (2.4), we obtain

$$\int_{\mathbb{R}^N} (I_\alpha * |u_\beta|^p)|u_\beta|^p + \int_{\mathbb{R}^N} (I_\alpha * |v_\beta|^p)|v_\beta|^p \ge \left(1 - \frac{\beta}{\sqrt{\lambda_1\lambda_2}}\right)c^2 > 0,$$

which means $u_\beta \not\equiv 0$ or $v_\beta \not\equiv 0$.

By (2.4) and Fatou's lemma, we have

$$\|u_\beta\|_{\lambda_1}^2 + \|v_\beta\|_{\lambda_2}^2 - 2\beta \int_{\mathbb{R}^N} u_\beta v_\beta \le \int_{\mathbb{R}^N} (I_\alpha * |u_\beta|^p)|u_\beta|^p + \int_{\mathbb{R}^N} (I_\alpha * |v_\beta|^p)|v_\beta|^p.$$

Let $t > 0$ such that $(tu_\beta, tv_\beta) \in \mathcal{N}_\beta$, we have

$$t = \left(\frac{\|(u_\beta, v_\beta)\|_H^2 - 2\beta \int_{\mathbb{R}^N} u_\beta v_\beta}{\int_{\mathbb{R}^N} (I_\alpha * |u_\beta|^p)|u_\beta|^p + \int_{\mathbb{R}^N} (I_\alpha * |v_\beta|^p)|v_\beta|^p}\right)^{\frac{1}{2p-2}} \le 1.$$

Hence,

$$\begin{aligned} m_\beta \le E_\beta(tu_\beta, tv_\beta) &= \left(\frac{1}{2} - \frac{1}{2p}\right) t^{2p} \left(\int_{\mathbb{R}^N} (I_\alpha * |u|^p)|u_\beta|^p + \int_{\mathbb{R}^N} (I_\alpha * |v_\beta|^p)|v_\beta|^p\right) \\ &\le \left(\frac{1}{2} - \frac{1}{2p}\right)\left(\int_{\mathbb{R}^N} (I_\alpha * |u_\beta|^p)|u_\beta|^p + \int_{\mathbb{R}^N} (I_\alpha * |v_\beta|^p)|v_\beta|^p\right) \\ &= \lim_{n\to\infty} E_\beta(u_n, v_n) = m_\beta. \end{aligned}$$

Thus, we can deduce that $t = 1$ and $m_\beta$ is achieved by $(u_\beta, v_\beta) \in \mathcal{N}_\beta$ with $u_\beta \geq 0$, $v_\beta \geq 0$. Now we know that $(u_\beta, v_\beta)$ be non-negative and radial ground state solution of (1.1). Since (1.1) has no semitrivial solution, namely $(u_\beta, 0)$ and $(0, v_\beta)$ are no solutions of (1.1), we infer that $u_\beta \not\equiv 0$ and $v_\beta \not\equiv 0$. By the strong maximum principle, we get $u_\beta > 0$ and $v_\beta > 0$, then $(u_\beta, v_\beta)$ be positive and radial ground state solution of (1.1).

Next we consider the asymptotic behavior of the ground state solution.

Suppose $\{\beta_n\}$ be a sequence which satisfies $\beta_n \in (0, \min\{\frac{1}{2}, \sqrt{\lambda_1\lambda_2}\})$ and $\beta_n \to 0$ as $n \to \infty$. Let $(u_{\beta_n}, v_{\beta_n})$ be the positive radial ground state solution of (1.1) obtained above, we claim $\{(u_{\beta_n}, v_{\beta_n})\}$ is bounded in $H$. Indeed, let $\phi$, $\psi$ are the positive solutions of (1.3) and (1.4) respectively. By (iii) of Lemma 3.2, we have that $(t_n\phi, t_n\psi) \in \mathcal{N}_{\beta_n}$, where $0 < t_n < 1$. Hence, by (1.5) and (1.6), we have

$$
\begin{aligned}
E_{\beta_n}(u_{\beta_n}, v_{\beta_n}) \leq E_{\beta_n}(t_n\phi, t_n\psi) &= E_{\beta_n}(t_n\phi, t_n\psi) - \frac{1}{2p}\langle E'_{\beta_n}(t_n\phi, t_n\psi), (t_n\phi, t_n\psi)\rangle \\
&= \left(\frac{1}{2} - \frac{1}{2p}\right)\left(\|(t_n\phi, t_n\psi)\|_H^2 - 2\beta_n t_n^2 \int_{\mathbb{R}^N} \phi\psi\right) \\
&< \left(\frac{1}{2} - \frac{1}{2p}\right)\|(\phi, \psi)\|_H^2 := D.
\end{aligned}
$$

Therefore, let $c_0 = \min\{\frac{1}{2}, \sqrt{\lambda_1\lambda_2}\}$, for $n$ large enough, we have

$$
\begin{aligned}
D > E_{\beta_n}(u_{\beta_n}, v_{\beta_n}) &= E_{\beta_n}(u_{\beta_n}, v_{\beta_n}) - \frac{1}{2p}\langle E'_{\beta_n}(u_{\beta_n}, v_{\beta_n}), (u_{\beta_n}, v_{\beta_n})\rangle \\
&\geq \left(\frac{1}{2} - \frac{1}{2p}\right)(1 - \beta_n)\|(u_{\beta_n}, v_{\beta_n})\|_H^2 > c_0\left(\frac{1}{2} - \frac{1}{2p}\right)\|(u_{\beta_n}, v_{\beta_n})\|_H^2,
\end{aligned}
$$

from which we deduce that $\{(u_{\beta_n}, v_{\beta_n})\}$ is bounded in $H$. Thus, there exists $(u_0, v_0) \in H$ such that, up to a subsequences, $(u_{\beta_n}, v_{\beta_n}) \rightharpoonup (u_0, v_0)$ in $H$ as $n \to \infty$ and $u_0 \geq 0, v_0 \geq 0$. Moreover by (3.2) we have that

$$
c_n = \left(\frac{\sqrt{\lambda_1\lambda_2} - \beta_n}{C_1 C(N, \alpha, p)\sqrt{\lambda_1\lambda_2}}\right)^{\frac{1}{2p-2}}
$$

is an increasing sequence and $\|(u_{\beta_n}, v_{\beta_n})\|_H^2 > c_1 > 0$, hence we have that $u_0 \not\equiv 0$ or $v_0 \not\equiv 0$. It is easy to observe that $E'_0(u_0, v_0) = 0$, thus $u_0$, $v_0$ are the solutions of (1.3) and (1.4), respectively. Since

$$
\begin{aligned}
&\|(u_{\beta_n}, v_{\beta_n}) - (u_0, v_0)\|_H^2 \\
&= \langle E'_{\beta_n}(u_{\beta_n}, v_{\beta_n}) - E'_0(u_0, v_0), (u_{\beta_n}, v_{\beta_n}) - (u_0, v_0)\rangle + \int_{\mathbb{R}^N}(I_\alpha * |u_{\beta_n}|^p)|u_{\beta_n}|^p \\
&\quad + \int_{\mathbb{R}^N}(I_\alpha * |v_{\beta_n}|^p)|v_{\beta_n}|^p - \int_{\mathbb{R}^N}(I_\alpha * |u_{\beta_n}|^p)|u_{\beta_n}|^{p-2}u_{\beta_n}u_0 - \int_{\mathbb{R}^N}(I_\alpha * |v_{\beta_n}|^p)|v_{\beta_n}|^{p-2}v_{\beta_n}v_0 \\
&\quad + \int_{\mathbb{R}^N}(I_\alpha * |u_0|^p)(|u_0|^p - |u_0|^{p-2}u_0u_{\beta_n}) + \int_{\mathbb{R}^N}(I_\alpha * |v_0|^p)(|v_0|^p - |v_0|^{p-2}v_0v_{\beta_n}) \\
&\quad + \beta_n\int_{\mathbb{R}^N}(2u_{\beta_n}v_{\beta_n} - u_{\beta_n}v_0 - v_{\beta_n}u_0),
\end{aligned}
\tag{3.9}
$$

by Lemmas 2.1, 2.2, 2.5 and above equality (3.9), we can conclude that $(u_{\beta_n}, v_{\beta_n}) \to (u_0, v_0)$ in $H$ as $n \to \infty$.

In view of Lemma 2.7, we can assume that $u_1$, $v_1$ are positive ground state solutions to (1.3) and (1.4) respectively, and let $t_n > 0$ such that $(t_n u_1, t_n v_1) \in \mathcal{N}_{\beta_n}$. In view of (iii) of Lemma 3.2, we know that $0 < t_n < 1$. Furthermore, by (3.1) we have that

$$t_n = \left( \frac{\|(u, v)\|_H^2 - 2\beta_n \int_{\mathbb{R}^N} uv}{\int_{\mathbb{R}^N} (I_\alpha * |u|^p)|u|^p + \int_{\mathbb{R}^N} (I_\alpha * |v|^p)|v|^p} \right)^{\frac{1}{2p-2}}$$

is an increasing sequence and $t_n > t_1 > 0$, then we know that $t_n \to 1$. Consequently, we have

$$E_0(u_1, v_1) \le E_0(u_0, v_0) = \lim_{n\to\infty} E_{\beta_n}(u_{\beta_n}, v_{\beta_n}) \le \lim_{n\to\infty} E_{\beta_n}(t_n u_1, t_n v_1) = E_0(u_1, v_1). \tag{3.10}$$

Obviously $E_0(u_0, v_0)$ is the sum of the energy of $u_0$ and $v_0$ for the single equation (1.3) and (1.4) respectively, namely

$$E_0(u_0, v_0) = E_{0,1}(u_0) + E_{0,2}(v_0),$$

where $E_{0,2} : H^s(\mathbb{R}^N) \to \mathbb{R}$ is the energy functional of (1.4), which is defined similarly to $E_{0,1}$, and $E_0(u_1, v_1)$ is the sum of the energy of $u_1$ and $v_1$ for the single equation (1.3) and (1.4), respectively, namely

$$E_0(u_1, v_1) = E_{0,1}(u_1) + E_{0,2}(v_1).$$

Since $u_1$, $v_1$ are positive ground state solutions to (1.3) and (1.4) respectively, we have

$$E_{0,1}(u_0) \ge E_{0,1}(u_1) \quad \text{and} \quad E_{0,2}(v_0) \ge E_{0,2}(v_1).$$

By (3.10), we get $E_{0,1}(u_0) = E_{0,1}(u_1)$ and $E_{0,2}(v_0) = E_{0,2}(v_1)$. By Lemma 2.7, we know that $u_0$, $v_0$ are positive ground state solutions of (1.3) and (1.4) respectively.

Let $u_0^*$ and $v_0^*$ denote the symmetric decreasing rearrangement of $u_0$ and $v_0$ respectively. By Lemma 2.6 with $f(x) = |u_0(x)|^p$, $g(y) = |u_0(y)|^p$, $h(x - y) = |x - y|^{\alpha-N}$, we have

$$\int_{\mathbb{R}^N} (I_\alpha * |u_0^*|^p)|u_0^*|^p \ge \int_{\mathbb{R}^N} (I_\alpha * |u_0|^p)|u_0|^p. \tag{3.11}$$

In addition, we know that

$$\int_{\mathbb{R}^N} |(-\Delta)^{\frac{s}{2}} u_0^*|^2 \le \int_{\mathbb{R}^N} |(-\Delta)^{\frac{s}{2}} u_0|^2 \quad \text{and} \quad \int_{\mathbb{R}^N} |u_0^*|^2 = \int_{\mathbb{R}^N} |u_0|^2 \tag{3.12}$$

(see [28, Theorem 3]). By (3.11) and (3.12) we have

$$\begin{aligned} E_0(u_0^*, v_0^*) &= \frac{1}{2}(\|u_0^*\|_{\lambda_1}^2 + \|v_0^*\|_{\lambda_2}^2) - \frac{1}{2p} \int_{\mathbb{R}^N} (I_\alpha * |u_0^*|^p)|u_0^*|^p - \frac{1}{2p} \int_{\mathbb{R}^N} (I_\alpha * |v_0^*|^p)|v_0^*|^p \\ &\le \frac{1}{2}(\|u_0\|_{\lambda_1}^2 + \|v_0\|_{\lambda_2}^2) - \frac{1}{2p} \int_{\mathbb{R}^N} (I_\alpha * |u_0|^p)|u_0|^p - \frac{1}{2p} \int_{\mathbb{R}^N} (I_\alpha * |v_0|^p)|v_0|^p \\ &= E_0(u_0, v_0). \end{aligned}$$

Therefore, we can further assume that $(u_0, v_0) \in H_r$. This completes the proof of Theorem 1.1. □

## 4. Nonexistence

In this section, in order to prove the nonexistence of nontrivial solutions, we need to use the following Pohožaev identity type:

**Lemma 4.1.** *Let $N \geq 3$ and $(u, v) \in H$ be any solution of (1.1). Then, $(u, v)$ satisfies the Pohožaev identity*

$$\begin{aligned} &\frac{N-2s}{2}\int [|(-\Delta)^{\frac{s}{2}}u|^2 + |(-\Delta)^{\frac{s}{2}}v|^2]dx + \frac{N}{2}\int (\lambda_1|u|^2 + \lambda_2|v|^2)dx \\ &\qquad = \frac{N+\alpha}{2p}\left(\int (I_\alpha * |u|^p)|u|^p dx + \int (I_\alpha * |v|^p)|v|^p dx\right) + N\beta\int uvdx. \end{aligned} \tag{4.1}$$

*Proof.* The proof is similar to the argument of Theorem 1.13 in [22]. □

**Proof of Theorem 1.2.** Let $\langle E'_\beta(u, v), (u, v)\rangle = 0$, by (1.6), we have

$$\begin{aligned} \int [|(-\Delta)^{\frac{s}{2}}u|^2 + |(-\Delta)^{\frac{s}{2}}v|^2 + \lambda_1|u|^2 + \lambda_2|v|^2]dx = &\int (I_\alpha * |u|^p)|u|^p dx \\ &+ \int (I_\alpha * |v|^p)|v|^p dx + 2\beta\int uvdx \end{aligned} \tag{4.2}$$

for all $(u, v) \in H$.

Combining the Pohožaev identity (4.1) and (4.2), we can see that

$$\begin{aligned} 0 = &\left(N - 2s - \frac{N+\alpha}{p}\right)\int [|(-\Delta)^{\frac{s}{2}}u|^2 + |(-\Delta)^{\frac{s}{2}}v|^2]dx \\ &+ \left(N - \frac{N+\alpha}{p}\right)\int (\lambda_1|u|^2 + \lambda_2|v|^2)dx + \left(\frac{N+\alpha}{p} - N\right)\int 2\beta uvdx. \\ = &\left(N - 2s - \frac{N+\alpha}{p}\right)\int [|(-\Delta)^{\frac{s}{2}}u|^2 + |(-\Delta)^{\frac{s}{2}}v|^2]dx \\ &+ \left(N - \frac{N+\alpha}{p}\right)\int (\lambda_1|u|^2 + \lambda_2|v|^2 - 2\beta uv)dx. \end{aligned} \tag{4.3}$$

Since $\lambda_1 > 0$, $\lambda_2 > 0$ and $0 < \beta < \sqrt{\lambda_1\lambda_2}$, we have

$$\lambda_1|u|^2 + \lambda_2|v|^2 \geq 2\sqrt{\lambda_1\lambda_2}uv > 2\beta uv.$$

Thus, if both the coefficients are non-positive, that is

$$N - 2s - \frac{N+\alpha}{p} \leq 0 \quad \text{and} \quad N - \frac{N+\alpha}{p} \leq 0,$$

then we get $p \leq 1 + \frac{\alpha}{N}$, which jointly with (4.3) leads us to a contradiction. Therefore, the solution of (1.1) is the trivial one. Similarly, if they are nonnegative, that is $p \geq \frac{N+\alpha}{N-2s}$, we get that nontrivial solutions of (1.1) cannot exist. Therefore, the range of $1 + \frac{\alpha}{N} < p < \frac{N+\alpha}{N-2s}$ is optimal for the existence of nontrival solutions of the problem (1.1). This completes the proof. □

## 5. Conclusions

In this present paper, we combine the critical point theory and variational method to investigate a class of coupled fractional systems of Choquard type. By using constrained minimization method and Hardy-Littlewood-Sobolev inequality, we establish the existence and asymptotic behaviour of positive ground state solutions of the systems. Furthermore, nonexistence of nontrivial solutions is also obtained. In the next work, we will focus on the research of normalized solutions to fractional couple Choquard systems.

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## Conflicts of interest

The authors declare that they have no conflicts of interest.

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