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Research article

Robust optimal reinsurance strategy with correlated claims and competition

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Abstract: This paper investigates the robust optimal reinsurance strategy, which simultaneously takes into account the ambiguity aversion, the correlated claims and the joint interests of an insurer and a reinsurer. The correlated claims mean that future claims are correlated with historical claims, which are measured by an extrapolative bias. The joint interests of the insurer and the reinsurer are reflected by the competition between them. To better reflect competition, we assume that the insurer and the reinsurer are engaged in related insurance business. The insurer is allowed to purchase proportional reinsurance or acquire a new business. Under ambiguity aversion and the criterion of maximizing the expected utility of terminal wealth, we obtain explicit solutions for the robust optimal reinsurance strategy and the corresponding value function by using the stochastic dynamic programming approach. Furthermore, we obtain the optimal reinsurance strategy under four typical cases. A series of numerical experiments were carried out to illustrate how the robust optimal reinsurance strategy varies with model parameters, and the result analyses reveal some interesting phenomena and provide useful guidance for reinsurance in reality.

Keywords: correlated claims; competition; robust reinsurance strategy; ambiguity aversion; stochastic control
Mathematics Subject Classification: 62P05, 91B28, 93E20

1. Introduction

Nowadays, with the rapid development of economy and society, more and more individuals and institutions are beginning to buy insurance. This leads to a rapid increase in the premium income of the insurer; meanwhile, the claim risks faced by the insurer also increase. However, it is usually difficult to undertake claim risks for an insurer only depending upon his premium. Therefore, the insurer is likely to purchase reinsurance in order to transfer claim risks. The technique of stochastic control theory and the stochastic dynamic programming approach are widely used to cope with the optimal reinsurance problem. Different optimization criteria have been proposed in the literature, e.g., minimizing the probability of ruin criterion (see [1–3]), maximizing the expected utility criterion (see [4–6]) and maximizing or minimizing mean-variance (MV) criterion (see [7–9]).

The above studies only considered the interest of the insurer, and did not pay attention to the reinsurer's interest. In fact, the reinsurance problem involves the interests of both the insurer and the reinsurer. An insurer pays a reinsurance premium to a reinsurer in order to obtain compensation from the reinsurer in case a claim occurs, while a reinsurer is willing to bear the insurer's claims in exchange for the reinsurance premium. The authors of [10] considered the joint interests of an insurer and a reinsurer by maximizing the weighted sum of the insurer's and the reinsurer's MV utilities. The authors of [11] studied the optimal reinsurance problem that minimizes the convex combination of the value-at-risk measures of the insurer's loss and the reinsurer's loss under two types of constraints. The authors of [12] considered the joint interests of an insurer and a reinsurer by maximizing the product of the insurer's and the reinsurer's utilities. The authors of [13] and [14] considered the joint interests of an insurer and a reinsurer by maximizing the expected utility of the weighted sum of the insurer's wealth and the reinsurer's wealth. And the difference is that [13] considered the exponential utility, while [14] considered the hyberbolic absolute risk aversion utility. However, none of these studies considered competition.

With the development of economy and society, competition exists widely among investors, managers, insurers and reinsurers. However, it is a difficult issue to quantify competition. The authors of [15] demonstrated that relative performance can be used to quantify competition, which has recently been used to describe the competition between two insurers. The authors of [16] studied the competition between two insurers, where all of the surplus process parameters and asset price parameters were modeled by a common random factor and Markov chain. The authors of [17] investigated the competition between two insurers in the longevity risk transfer market. The authors of [18] studied the competition between two MV insurers, where the stock's price process is the constant elasticity of variance model. The authors of [19] considered the competition between two insurers under different classes of premium principles, and the price of the risky asset follows an Ornstein-Uhlenbeck process. In practice, there are often many, say $n > 2$, insurers whose reinsurance strategies are related. It is more reasonable to study the reinsurance problem among n insurers. The authors of [20] first considered the competition among n insurers.

At present, few scholars study the competition between an insurer and a reinsurer. The authors of [21] recently studied the competition between an insurer and a reinsurer. Generally speaking, the insurer competes with the reinsurer in terms of insurance business or reinsurance business. As what was explained in [21], a Swiss reinsurance company engaged the property insurance business in China, and Ping An insurance company of China engaged the reinsurance business. Nevertheless, the authors of [21] did not consider this form of competition. This paper intends to extend the competition form of the results in [21].

In practice, insurers usually renew the terms of insurance contract based on policy-holders' past claims. The authors of [22–24] have shown that future claims are correlated with historical claims. Regarding discrete time models, the authors of [25] empirically studied the effect of firms' past loss on premium, and they demonstrated that the premium is paid according to the formula $(1 + \varpi)(\theta \mathrm{A}L + (1 - \theta)\mathrm{E}L)$. Here $\theta \in (0, 1)$ is the weight factor, $\varpi > 0$ is the safety factor and $\mathrm{A}L$ and $\mathrm{E}L$ respectively represent the firm's past yearly loss and the yearly expected loss. However, it is difficult to quantify the correlation between future claims and historical claims for a continuous time model. The authors of [26–28] established the correlation between future claims and historical claims by using an extrapolative bias model, which was pioneered in [29]. Furthermore, the discrete premium in [25] above-mentioned is extended to the continuous case. However, these studies did not consider the interest of the reinsurer. In practice, people expect to know the insurer's reinsurance decision when taking into account the joint interests of the insurer and the reinsurer.

Insurers often face an uncertain environment when making reinsurance decisions. It is well accepted that insurers are generally ambiguity-averse. This is because they must have enough funds to pay the claim of the policy-holders. The authors of [30] first proposed a robust control approach under the continuous-time framework to study ambiguity aversion. Nowadays, there are many studies on robust reinsurance and/or investment under ambiguity aversion. The authors of [31] investigated the robust optimal excess-of-loss reinsurance and investment problem with jumps. The authors of [32] considered the robust optimal dynamic reinsurance problem under the mean-RVaR premium principle. The authors of [33] investigated the robust reinsurance under a learning framework. The authors of [34] studied the robust optimal reinsurance problem for an ambiguity-averse insurer (AAI), where the insurer cannot obtain the perfect information of the claim. To the best of our knowledge, only the authors of [28] studied the robust optimal reinsurance problem under correlated claims; however, they did not consider the interest of the reinsurer.

In view of the aforementioned state-of-the-art work, we will study the robust optimal reinsurance problem and simultaneously take into account the ambiguity aversion, the joint interests of an insurer and a reinsurer and the correlated claims. The insurer and the reinsurer can also engage in insurance business, and their insurance businesses are correlated. The insurer can reduce his claim risk by purchasing proportional reinsurance or acquiring a new business, which includes playing the role of a reinsurer. The joint interests of the insurer and the reinsurer are reflected by their competition. We construct the competition model through a relative wealth process. The interests of the insurer and the reinsurer are reflected through their respective wealth in the relative wealth process. That is, the relative wealth process includes the joint interests of the insurer and the reinsurer. Furthermore, the insurer determines the optimal reinsurance strategy under the relative wealth process. Considering the uncertainty of model, we assume that the insurer is ambiguity-averse. By maximizing the expected exponential utility of terminal wealth, we obtain explicit solutions for the robust optimal reinsurance strategy and the corresponding value function under the worst-case market scenario. To analyze the effects of key model features on the robust optimal reinsurance strategy, we consider four special cases, i.e., the no-competition case, the uncorrelated insurance businesses case, the no-ambiguity-aversion case and the uncorrelated claims case. Finally, we conduct numerical experiments to examine the effects of key model features on the robust optimal reinsurance strategy.

Compared to the existing literature, the main contributions of this paper are as follows:

- We simultaneously take into account the ambiguity aversion, the joint interests of an insurer and a reinsurer, and the correlated claims.
- The insurer can reduce his claim risk by purchasing proportional reinsurance or acquiring a new business, and the reinsurer can also engage in insurance business; furthermore, we quantify the competition between the insurer and the reinsurer.
- We obtain the explicit robust optimal reinsurance strategy, which can guide the actual reinsurance activities more effectively.
- We systematically analyze the influences of time, the correlated claims, the dependence of

insurance business, the competition degree and the ambiguity aversion coefficient on the robust optimal reinsurance strategy through numerical experiments. Furthermore, the result analyses reveal some interesting phenomena, extending the result of [28], and provide useful guidance for reinsurance in reality.

The rest of the paper is organized as follows. The model settings and assumptions are given in Section 2. In Section 3, we introduce the robust reinsurance problem with correlated claims and competition. In Section 4, we obtain explicit solutions for the robust optimal reinsurance strategy and the corresponding value function, and we also provide the verification theorem. Section 5 illustrates our theoretical results through numerical experiments. The final section summarizes the paper.

2. Model setting

Throughout this paper, we assume that all random variables and processes are defined on a filtered probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\in[0,T]}, \mathbb{P})$ satisfying the usual conditions, i.e., $\mathcal{F}_t$ is right continuous and $\mathbb{P}$-complete; $\mathcal{F}_t$ stands for the information available until time t; T is the terminal time of reinsurance; $\mathbb{P}$ is the reference measure.

Without losing generality, suppose that there are two kinds of insurance businesses in the insurance market, which are named as insurance businesses 1 and 2. The insurance business i's, $i = 1, 2$, cumulative claims in the time interval $[0, t]$ is denoted by

$$L_i(t) = \sum_{k=1}^{G_i(t)} Y_k^i.$$

Here Y_k^i is the size of the insurance business i's kth claim, and $\{Y_k^i, k = 1, 2, \ldots\}$ forms a sequence of independent and nonnegative random variables following a common distribution. The common random variable of $\{Y_k^i, k = 1, 2, \ldots\}$ is denoted as Y^i. Y^i has a finite mean $\mu_{i1} = \mathrm{E}(Y^i)$ and second-order moment $\mu_{i2} = \mathrm{E}[(Y^i)^2]$. $N_i(t)$ and $N(t)$ are mutually independent Poisson processes with intensities of $\lambda_i \geq 0$ and $\lambda \geq 0$, respectively. $N(t)$ is the number of claims caused by the common shock of the two insurance businesses, which reflects the interdependence of the two insurance businesses. $N_i(t)$ is the number of claims independent of the common shock. $G_i(t) = N(t) + N_i(t)$ is the counting process, representing the number of claims of business i up to time t. According to the additivity of the Poisson process, $G_i(t)$ is a Poisson process with an intensity of $\lambda_i + \lambda$.

It is known from [35] that the aggregate claims process $\sum_{k=1}^{G_i(t)} Y_k^i$ can be approximated by the following Brownian motion with drift

$$\sum_{k=1}^{G_i(t)} Y_k^i \approx (\lambda + \lambda_i)\mu_{i1} t - \sqrt{(\lambda + \lambda_i)\mu_{i2}}\, W_i(t), \tag{2.1}$$

where $\{W_i(t), t \in [0, T]\}$ is the standard Brownian motion. The two Brownian motions $W_1(t)$ and $W_2(t)$ are correlated with the correlation coefficient of

$$\rho := \frac{\lambda \mu_{11} \mu_{21}}{\sqrt{(\lambda + \lambda_1)(\lambda + \lambda_2)\mu_{12}\mu_{22}}}.$$

For presentation convenience, we define notations $p_i = (\lambda + \lambda_i)\mu_{i1}$ and $q_i = \sqrt{(\lambda + \lambda_i)\mu_{i2}}$, $i = 1, 2$.

In reality, future claims are usually correlated with historical claims. Therefore, the insurers always take the occurred claims, especially the recent claims into consideration when renewing the insurance contracts. Considering this and inspired by [29], especially by [26] and [28], we define the exponential weighted average of historical losses as follows

$$v(t) = \int_0^t \delta e^{-\delta(t-s)} dL_1(s - ds),\ 0 < \delta < 1, \tag{2.2}$$

which is an exponential weighted average of historical claims. $v(0) = v_0 = 0$, δ is the extrapolation intensity and $dL_1(s-ds) = L_1(s) - L_1(s-ds)$ represents the claim amount occurred from time $s-ds$ to time s. Equation (2.2) means that the insurer uses the exponential weighted average of historical claims to forecast the expected future claim. By using model (2.2), we quantify the correlation between future claims and historical claims.

To ensure that the weighted average of historical claims is finite, we introduce the following assumption.

Assumption 2.1. We assume that

$$|v(t)| \leq M,\ \text{for } \forall t \in [0, T], \mathbb{P}\text{-a.s.},$$

where M is a positive constant.

Substituting (2.1) into (2.2) and according to the derivative of the definite integral with a variable upper limit, we derive the differential form of $v(t)$ as

$$dv(t) = \delta p_1 e^{-\delta t} - \delta q_1 dW_1(t). \tag{2.3}$$

Suppose that there exists one insurer in the insurance market who is engaged in insurance business 1. His premium rate c_1 can be determined according to the exponential weighted average of historical losses and the expected future claims. By the exponential weighted average of historical losses, we can easily obtain the total weight of the past claims is $\int_0^t \delta e^{-\delta(t-s)} ds = 1 - e^{-\delta t}$, which is always less than one. Hence, a weight $e^{-\delta t}$ is naturally given to the expectation of the future claims. Then, the premium rate c_1 is given by $c_1 = (1 + \eta_1)(v(t) + p_1 e^{-\delta t})$, where $\eta_1 > 0$ is the safety loading of the insurer.

To transfer claim risks, the insurer adopts reinsurance. For each $t \in [0, T]$, the reinsurance level is associated with the parameter $a(t)$, where $a(t) \in [0, +\infty)$ is the retention level of the insurer. $0 \leq a(t) \leq 1$ corresponds to a proportional reinsurance. $a(t) > 1$ corresponds to a new business requirement; for example, the insurer plays the role of a reinsurer. By acquiring a new business, the insurer will increase his income to hedge the claim risks. For more information about this reinsurance pattern, please see [36]. For this reinsurance pattern, the insurer only needs to pay $a(t)$ of each claim, and the reinsurer pays the rest $(1 - a(t))$. Let $(1 + \eta_2)(1 - a(t))(v(t) + p_1 e^{-\delta t})$ be the reinsurance premium rate, where $\eta_2 > 0$ is the safety loading of the reinsurer. To exclude the insurer's arbitrage behavior, we require $\eta_2 > \eta_1$. The insurer can invest his surplus in a risk-free money account with a constant interest rate $r > 0$. With the reinsurance and money account being incorporated, the wealth process $X_1(t)$ of the insurer satisfies the following stochastic differential equation

$$\begin{aligned} dX_1(t) &= [rX_1(t) + c_1 - (1 + \eta_2)(1 - a(t))(v(t) + p_1 e^{-\delta t})]dt \\ &\quad - a(t)[(v(t) + p_1 e^{-\delta t})dt - q_1 dW_1(t)] \\ &= [rX_1(t) + c_1 - (1 + \eta_2)(v(t) + p_1 e^{-\delta t}) + a(t)\eta_2(v(t) + p_1 e^{-\delta t})]dt \\ &\quad + a(t) q_1 dW_1(t). \end{aligned} \tag{2.4}$$

Considering the insurance business in practice, we assume that the reinsurer not only accepts the reinsurance business of the insurer, but also engages in insurance business 2. The premium rate received by the reinsurer from insurance business 2 is c_2. Similar to that of the insurer, the reinsurer can also invest his surplus in the risk-free money account with the constant interest rate $r > 0$. With the reinsurance and money account being incorporated, the wealth process $X_2(t)$ of the reinsurer satisfies the following stochastic differential equation

$$\begin{aligned} dX_2(t) &= [rX_2(t) + c_2 + (1 + \eta_2)(1 - a(t))(v(t) + p_1 e^{-\delta t})]dt \\ &\quad - (1 - a(t))[(v(t) + p_1 e^{-\delta t})dt - q_1 dW_1(t)] - [p_2 dt - q_2 dW_2(t)] \\ &= [rX_2(t) + c_2 - p_2 + (1 - a(t))\eta_2(v(t) + p_1 e^{-\delta t})]dt \\ &\quad + (1 - a(t))q_1 dW_1(t) + q_2 dW_2(t). \end{aligned} \tag{2.5}$$

From the above discussion, it is obvious that the interdependence of the insurer's insurance business and the reinsurer's insurance business is due to the common Poisson process $N(t)$. When signing a reinsurance contract, it is clear that both the insurer and reinsurer are concerned about the reinsurance premium $(1+\eta_2)(1-a(t))(v(t)+p_1 e^{-\delta t})$. The insurer wants $(1+\eta_2)(1-a(t))(v(t)+p_1 e^{-\delta t})$ to be as small as possible, while the reinsurer wants $(1+\eta_2)(1-a(t))(v(t)+p_1 e^{-\delta t})$ to be as large as possible. In addition, the insurer and the reinsurer are engaged in dependent insurance business. Therefore, competition between the insurer and the reinsurer exists both in insurance business and in the formulation of the reinsurance premium. In insurance practice, after signing a reinsurance contract, the insurer not only cares about his own wealth $X_1(t)$, but he also cares about the wealth $X_2(t)$ obtained by the reinsurer from the reinsurance contract. In practice, since the reinsurer may also sign reinsurance contracts with other insurers, the wealth of the reinsurer is generally greater than $X_2(t)$. However, for this insurer, he may most care about the wealth $X_2(t)$ of the reinsurer. This is because the decision of this insurer will only affect $X_2(t)$, and will not affect other wealth assets of the reinsurer. If $X_1(t) > X_2(t)$, the insurer will continue to perform the reinsurance contract. If $X_1(t) < X_2(t)$, the insurer may consider amending the reinsurance contract, i.e., increasing the proportion of claims paid by himself, thus reducing the reinsurer's wealth obtained from the reinsurance contract. Therefore, after signing the reinsurance contract, the insurer will always simultaneously pay attention to $X_1(t)$ and $X_2(t)$. To this end, similar to [21], we introduce the following relative performance model

$$(1 - \tau)X_1(t) + \tau(X_1(t) - X_2(t)).$$

Here $\tau \in [0, 1]$ captures the competition intensity between the insurer and the reinsurer, and measures the insurer's sensitivities to the performance of the other party. A large τ means that the insurer puts more weight on the performance of the reinsurer and cares more about his relative wealth increase. When $\tau = 0$, the insurer only cares about his own wealth.

Let

$$X^a(t) := (1 - \tau)X_1(t) + \tau(X_1(t) - X_2(t))$$

be the relative wealth process of the insurer. Then $X^a(t)$ can be described as

$$\begin{aligned} dX^a(t) &= [rX^a(t) + c_1 - \tau c_2 + \tau p_2 - (1 + \eta_2)(v(t) + p_1 e^{-\delta t}) \\ &\quad + \eta_2(v(t) + p_1 e^{-\delta t})((1 + \tau)a(t) - \tau)]dt \\ &\quad + [(1 + \tau)a(t) - \tau]q_1 dW_1(t) - \tau q_2 dW_2(t). \end{aligned} \tag{2.6}$$

We quantify the competition between the insurer and the reinsurer through the relative wealth. In the relative wealth process (2.6), $X_1(t)$ embodies the interest of the insurer and $X_2(t)$ embodies the interest of the reinsurer. In what follows, the insurer will determine the optimal reinsurance strategy based on the relative wealth process (2.6). That is, the insurer determines the optimal reinsurance strategy by simultaneously takeing into account the joint interests of the insurer and the reinsurer.

The above-mentioned model corresponds to a traditional framework. It is assumed that the insurer is ambiguity-neutral, and that he has complete confidence in the model (2.6) provided by the probability distribution $\mathbb{P}$. However, in many situations, the insurer cannot know exactly the true $\mathbb{P}$; thus, any particular probability distribution used to describe the model would lead to potential model misspecification. For this reason, we shall incorporate the probability distribution uncertainty into the reinsurance problem for an AAI. We assume that the AAI uses the model (2.6) as his reference model for the wealth process. Nevertheless, he is sceptical about this reference model and takes alternative models into consideration to determine a robust optimal strategy. Parallel to [30], the alternative models are defined by a class of probability distributions which are equivalent to $\mathbb{P}$:

$$\mathcal{Q} := \{\mathbb{Q} | \mathbb{Q} \sim \mathbb{P}\}.$$

In what follows, we transform model (2.6) from probability distribution $\mathbb{P}$ to probability distribution $\mathbb{Q}$. First, we introduce a process $\{\theta(t) = (\theta_1(t), \theta_2(t)) | t \in [0, T]\}$ satisfying the following:

- $\theta_1(t)$ and $\theta_2(t)$ are $\{\mathcal{F}_t\}_{t\geq 0}$-measurable, for each $t \in [0, T]$;
- $\theta_1(t)$ and $\theta_2(t)$ satisfy the following Novikov's condition

$$\mathrm{E}\left[\exp\left\{\frac{1}{2}\int_0^T [(\theta_1(t))^2 + (\theta_2(t))^2]dt\right\}\right] < +\infty.$$

We denote the space of all such processes $\theta(t)$ by Θ.

To change the probability distribution from $\mathbb{P}$ to $\mathbb{Q}$, we define for each $\theta(t) \in \Theta$ a real-valued process $\{\Lambda^\theta(t) | t \in [0, T]\}$ on $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, \mathbb{P})$ by

$$\Lambda^\theta(t) = \exp\left\{-\int_0^t \theta_1(s)dW_1(s) - \int_0^t (\theta_1(s))^2 ds - \int_0^t \theta_2(s)dW_2(s) - \int_0^t (\theta_2(s))^2 ds\right\}.$$

Under the Novikov's condition, similar to that in [37], we can prove that $\Lambda^\theta(t)$ is a $\mathbb{P}$-martingale. Hence, $\mathrm{E}[\Lambda^\theta(t)] = 1$. For each $\theta(t) \in \Theta$, we apply the Radon-Nikodym derivative to define a new probability distribution $\mathbb{Q}^\theta$ that is absolutely continuous with respect to $\mathbb{P}$ on $\mathcal{F}_T$ as

$$\frac{d\mathbb{Q}^\theta}{d\mathbb{P}}\Big|_{\mathcal{F}_T} := \Lambda^\theta(T),$$

and the set of all $\mathbb{Q}^\theta$ is denoted by $\mathcal{Q}$.

According to Girsanov's theorem, under the alternative measure $\mathbb{Q}^\theta$, the stochastic processes $W_1^\theta(t)$ and $W_2^\theta(t)$ are standard Brownian motions, where

$$dW_1^\theta(t) = dW_1(t) + \theta_1(t)dt, \ dW_2^\theta(t) = dW_2(t) + \theta_2(t)dt.$$

Since Brownian motions $W_1(t)$ and $W_2(t)$ are correlated with the correlation coefficient of ρ, Brownian motions $W_1^\theta(t)$ and $W_2^\theta(t)$ are correlated with the correlation coefficient of ρ. Furthermore, the dynamics of the wealth process $X^a(t)$ in (2.6) under $\mathbb{Q}^\theta$ becomes

$$\begin{aligned} dX^a(t) = &\left[rX^a(t) + c_1 - \tau c_2 + \tau p_2 - (1+\eta_2)(v(t) + p_1 e^{-\delta t})\right. \\ &+ \eta_2(v(t) + p_1 e^{-\delta t})((1+\tau)a(t) - \tau) \\ &- \theta_1((1+\tau)a(t) - \tau)q_1 + \theta_2 \tau q_2]dt \\ &+ [(1+\tau)a(t) - \tau]q_1 dW_1^\theta(t) - \tau q_2 dW_2^\theta(t), \end{aligned} \tag{2.7}$$

with $X^a(0) = x_0$. Correspondingly, the measure of extrapolative bias $v(t)$ in (2.3) becomes

$$dv(t) = \left[\delta p_1 e^{-\delta t} + \theta_1(t)\delta q_1\right] ds - \delta q_1 dW_1^\theta(t). \tag{2.8}$$

In order to introduce the robust reinsurance problem, we define the admissible strategy under the probability distribution $\mathbb{Q}^\theta$ as follows.

Definition 2.1. (Admissible Strategy). For any fixed $t \in T$, a reinsurance strategy $a(t)$ is said to be admissible if it satisfies the following conditions:

(i) $a(t)$ is progressively measurable with respect to $\{\mathcal{F}_t\}_{t\geq 0}$ for each $t \in [0, T]$;

(ii) $\forall\ t \in [0, T]$, $a(t) \in [0, +\infty)$;

(iii) $\mathrm{E}_{t,x,v}\{\int_0^T a^2(t)dt\} < +\infty$, where $\mathrm{E}_{t,x,v}[\cdot] = \mathrm{E}[\cdot|X^a(t) = x, v(t) = v]$.

The set of all admissible reinsurance strategies is denoted by $\mathcal{A}$.

In this section, we have proposed a new reinsurance model with correlated claims and competition. Compared with [28], our model has the following characteristics.

- The reinsurer can engage in insurance business.
- We quantify the competition between the insurer and the reinsurer.
- We simultaneously take into account the ambiguity aversion, the joint interests of the insurer and the reinsurer, and the correlated claims.

Therefore, we extend the model of [28].

3. Robust reinsurance problem with correlated claims and competition

In this section, we construct the robust optimal reinsurance problem. We first introduce the following penalty function:

$$\Phi(t, X^a(t), v(t), \theta(t)) = \frac{(\theta_1(t))^2}{2\phi_1(t, X^a(t), v(t))} + \frac{(\theta_2(t))^2}{2\phi_2(t, X^a(t), v(t))}, \tag{3.1}$$

where $\phi_1(t, X^a(t), v(t))$ and $\phi_2(t, X^a(t), v(t))$ are nonnegative and capture the AAI's ambiguity aversions.

This penalty function can effectively reflect the penalty due to the deviation from the reference model, i.e., the model under the probability distribution $\mathbb{P}$. The first and second terms describe the penalty caused by $W_1(t)$ and $W_2(t)$, respectively. The larger the $\phi_1(t, X^a(t), v(t))$ and $\phi_2(t, X^a(t), v(t))$, the less the insurer will be penalized, the less faith in the reference model had by the insurer, i.e., the insurer is more likely to consider the alternative model, which is the model under the probability distribution $\mathbb{Q}^\theta$.

On the basis of the penalty function $\Phi(t, X^a(t), \nu(t), \theta(t))$, we formulate a robust optimal reinsurance problem which chooses the optimal strategy in the worst-case scenario as follows:

$$\sup_{a\in\mathcal{A}}\inf_{\theta\in\Theta} \mathrm{E}_{t,x,\nu}\left\{\int_t^T \Phi(s, X^a(s), \nu(s), \theta(s))ds + U(X^a(T))\right\}, \tag{3.2}$$

where the expectation is with respect to the alternative probability distribution $\mathbb{Q}^\theta$, and $U(\cdot)$ is the utility function which is assumed to be continuous, increasing and strictly concave.

To solve problem (3.2), we define the optimal value function as

$$V(t, x, \nu) = \sup_{a\in\mathcal{A}}\inf_{\theta\in\Theta} \mathrm{E}_{t,x,\nu}\left\{\int_t^T \Phi(s, X^a(s), \nu(s), \theta(s))ds + U(X^a(T))\right\}. \tag{3.3}$$

Let $C^{1,2,2}([0,T]\times R\times R^+)$ denote the space of $\varphi(t, x, \nu)$ such that $\varphi(t, x, \nu)$ and its derivatives $\varphi_t(t, x, \nu)$, $\varphi_x(t, x, \nu)$, $\varphi_{xx}(t, x, \nu)$, $\varphi_\nu(t, x, \nu)$ and $\varphi_{\nu\nu}(t, x, \nu)$ are continuous on $[0, T] \times R \times R^+$. For any function $\varphi(t, x, \nu) \in C^{1,2,2}([0, T] \times R \times R^+)$ and any fixed $a(t) \in \mathcal{A}$, the usual infinitesimal generator $\mathcal{B}^{\theta,a}$ for the wealth process $X^a(t)$ in (2.7) is defined as

$$\begin{aligned}
\mathcal{B}^{\theta,a}\varphi(t, x, \nu) = {} & \varphi_t(t, x, \nu) + \Big[rx + c_1 - \tau c_2 + \tau p_2 - (1 + \eta_2)(\nu + p_1 e^{-\delta t}) \\
& + \eta_2(\nu(t) + p_1 e^{-\delta t})((1 + \tau)a(t) - \tau) - \theta_1((1 + \tau)a(t) - \tau)q_1 + \theta_2\tau q_2\Big]\varphi_x(t, x, \nu) \\
& + \frac{1}{2}\Big[((1 + \tau)a(t) - \tau)^2 q_1^2 + \tau^2 q_2^2 - 2((1 + \tau)a(t) - \tau)\tau\rho q_1 q_2\Big]\varphi_{xx}(t, x, \nu) \\
& + \Big[\delta p_1 e^{-\delta t} + \theta_1(t)\delta q_1\Big]\varphi_\nu(t, x, \nu) + \frac{1}{2}\delta^2 q_1^2 \varphi_{\nu\nu}(t, x, \nu) \\
& + \big[-\delta q_1((1 + \tau)a(t) - \tau)q_1 + 2\rho\delta q_1 q_2 \tau\big]\varphi_{x\nu}(t, x, \nu).
\end{aligned} \tag{3.4}$$

According to the optimality principle of dynamic programming, we can derive the Hamilton-Jacobi-Bellman-Isaacs equation for problem (3.3) as

$$\sup_{a\in\mathcal{A}}\inf_{\theta\in\Theta}\Big\{\mathcal{B}^{\theta,a}V(t, x, \nu) + \Phi(t, x, \nu, \theta)\Big\} = 0, \tag{3.5}$$

with the boundary condition being $V(T, x, \nu) = U(x)$.

The following proposition is essential in solving the associated stochastic control problem.

Proposition 3.1. Suppose that there exist a function $H(t, x, \nu) \in C^{1,2,2}([0, T] \times R \times R^+)$ and a control strategy $(\theta^*, a^*) \in (\Theta, \mathcal{A})$ such that

(1) $\mathcal{B}^{\theta,a^*}H(t, x, \nu) + \Phi(t, x, \nu, \theta) \geq 0$ for all $\theta \in \Theta$;

(2) $\mathcal{B}^{\theta^*,a}H(t, x, \nu) + \Phi(t, x, \nu, \theta^*) \leq 0$ for all $a \in \mathcal{A}$;

(3) $\mathcal{B}^{\theta^*,a^*}H(t, x, \nu) + \Phi(t, x, \nu, \theta^*) = 0$;

(4) for all $(\theta, a) \in (\Theta, \mathcal{A})$, $\lim_{t\to T^-} H(t, X^a(t), \nu(t)) = U(X^a(T))$;

(5) $\{H(\epsilon, X^a(\epsilon), \nu(\epsilon))\}_{\epsilon\in\Xi}$ and $\{\Phi(\epsilon, X^a(\epsilon), \nu(\epsilon), \theta(\epsilon))\}_{\epsilon\in\Xi}$ are uniformly integrable, where Ξ denotes the set of stopping times $\epsilon \leq \Xi$.

Then $H(t, x, \nu) = V(t, x, \nu)$ and (θ^*, a^*) is an optimal control strategy.

The proof of this proposition is similar to that of Theorem 3.2 in [38], so we omit it here.

In the next section, we shall apply Proposition 3.1 to verify that the candidate reinsurance strategy and the corresponding value function are indeed optimal.

4. Robust optimal reinsurance strategy and verification theorem

In this section, the aim is to derive the robust optimal reinsurance strategy for problem (3.3). We assume that the AAI has an exponential utility function, i.e.,

$$U(x) = -\frac{1}{m}e^{-mx}, \tag{4.1}$$

where $m > 0$ is the absolute risk aversion parameter. Inspired by [39], we define the ambiguity preference functions as

$$\phi_i(t, x, v) = -\frac{\beta_i}{mV(t, x, v)},\ i = 1, 2. \tag{4.2}$$

Here $\beta_i, i = 1, 2$, are positive constants and stand for the ambiguity aversion parameters.

The following theorem provides the candidate robust optimal reinsurance strategy and the corresponding value function for problem (3.3) .

Theorem 4.1. For problem (3.3) with the exponential utility function (4.1) and the ambiguity aversion coefficients specified in (4.2), the candidate robust optimal reinsurance strategy is given by

$$a^*(t) = \frac{1}{1+\tau}\left[\tau + \frac{\eta_2(v + p_1e^{-\delta t}) + (m+\beta_1)\delta q_1^2(2A(t) + B(t)) + mq_1q_2\tau\rho e^{r(T-t)}}{(m+\beta_1)q_1^2e^{r(T-t)}}\right] \vee 0, \tag{4.3}$$

and the candidate worst-case measure is determined by

$$\begin{cases} \theta_1^*(t) = -\beta_1 q_1\Big[\delta(2A(t) + B(t)) - ((1+\tau)a^*(t) - \tau)e^{r(T-t)}\Big], \\ \theta_2^*(t) = -\beta_2\tau q_2 e^{r(T-t)}, \end{cases} \tag{4.4}$$

where $A(t)$ and $B(t)$ are defined by (4.7) and (4.8), respectively. For $a^*(t) > 0$, the candidate optimal value function is given by

$$H(t, x, v) = -\frac{1}{m}e^{-m\left[xe^{r(T-t)} + f(t,v)\right]}, \tag{4.5}$$

where

$$f(t, v) = A(t)v^2 + B(t)v + D(t), \tag{4.6}$$

and

$$A(t) = \frac{\eta_2}{4\lambda_1\mu_{12}\delta(m+\beta_1)}\left[e^{2\delta\eta_2(T-t)} - 1\right], \tag{4.7}$$

$$\begin{aligned} B(t) = e^{\eta_2\delta(T-t)}\int_t^T \Bigg\{&2A(s)\Big[\delta p_1e^{-\delta s} - 2\rho\delta q_1q_2\tau me^{r(T-s)} + \delta mq_1q_2\tau\rho e^{r(T-s)} \\ &+ \eta_2\delta p_1e^{-\delta s}\Big] - (1+\eta_2)e^{r(T-s)} + \frac{\eta_2p_1e^{-\delta s}}{(m+\beta_1)q_1^2} + \frac{\eta_2mq_2\tau\rho e^{r(T-s)}}{(m+\beta_1)q_1}\Bigg\}e^{-\eta_2\delta(T-s)}ds, \end{aligned} \tag{4.8}$$

$$\begin{aligned} D(t) = \int_t^T \Bigg\{&\Big[\delta p_1e^{-\delta s} - 2m\rho\delta q_1q_2\tau e^{r(T-s)} + \eta_2p_1\delta e^{-\delta s} + \delta mq_1q_2\tau\rho e^{r(T-s)}\Big]B(s) \\ &+ \Big[c_1 - \tau c_2 - (1+\eta_2)p_1e^{-\delta s} + \tau p_2\Big]e^{r(T-s)} - \frac{m+\beta_2}{2}\tau^2q_2^2e^{2r(T-s)} + \frac{1}{2q_1^2(m+\beta_1)} \\ &\times\Big(\eta_2^2p_1^2e^{-2\delta s} + m^2q_1^2q_2^2\tau^2\rho^2e^{2r(T-s)} + 2\eta_2mq_1q_2\tau\rho p_1e^{-\delta s}e^{r(T-s)}\Big)\Bigg\}ds. \end{aligned} \tag{4.9}$$

Proof. According to boundary condition $V(T,x,v) = U(x)$, we conjecture that the candidate value function has the following structure

$$H(t,x,v) = -\frac{1}{m}e^{-m\left[xe^{r(T-t)}+f(t,v)\right]}, \tag{4.10}$$

where $f(t,v)$ is a function to be determined. It satisfies the boundary condition $f(T,v) = 0$. The partial derivatives of $H(t,x,v)$ are then given by

$$\begin{cases} H_t(t,x,v) = \left[rxe^{r(T-t)} - f_t(t,v)\right]mH(t,x,v), \\ H_x(t,x,v) = -me^{r(T-t)}H(t,x,v), H_v(t,x,v) = -mf_vH(t,x,v), \\ H_{xx}(t,x,v) = m^2e^{2r(T-t)}H(t,x,v), \\ H_{xv}(t,x,v) = m^2f_ve^{r(T-t)}H(t,x,v), \\ H_{vv}(t,x,v) = m^2(f_v)^2H(t,x,v) - mf_{vv}H(t,x,v). \end{cases} \tag{4.11}$$

Substituting (4.11) into (3.5), we obtain after some simplifications that

$$\begin{aligned} \sup_{a\in\mathcal{A}}\inf_{\theta\in\Theta}\Bigg\{ & \frac{\theta_1^2(t)}{2\beta_1} + \frac{\theta_2^2(t)}{2\beta_2} + f_t + \Big[rx + c_1 - \tau c_2 + \tau p_2 - (1+\eta_2)(v + p_1e^{-\delta t}) \\ & + \eta_2(v(t) + p_1e^{-\delta t})((1+\tau)a(t) - \tau) - \theta_1((1+\tau)a(t) - \tau)q_1 + \theta_2\tau q_2\Big]e^{r(T-t)} \\ & - \frac{m}{2}\left[((1+\tau)a(t)-\tau)^2q_1^2 + \tau^2q_2^2 - 2((1+\tau)a(t)-\tau)\tau\rho q_1q_2\right]e^{2r(T-t)} \\ & + \left[\delta p_1e^{-\delta t} + \theta_1(t)\delta q_1\right]f_v + \frac{1}{2}\delta^2q_1^2(-mf_v^2 + f_{vv}) \\ & - [-\delta q_1((1+\tau)a(t)-\tau)q_1 + 2\rho\delta q_1q_2\tau]mf_ve^{r(T-t)}\Bigg\} = 0, \end{aligned} \tag{4.12}$$

where f is a short notation for $f(t,v)$.

According to the first-order optimality conditions, the processes $\theta_1(t)$ and $\theta_2(t)$ solving the inner infimum problem in (4.12) are given by

$$\begin{cases} \theta_1^*(t) = -\beta_1q_1\left[\delta f_v - ((1+\tau)a(t)-\tau)e^{r(T-t)}\right], \\ \theta_2^*(t) = -\beta_2\tau q_2e^{r(T-t)}. \end{cases} \tag{4.13}$$

Inserting (4.13) into (4.12), we obtain

$$\begin{aligned} \sup_{a\in\mathcal{A}}\Bigg\{ & -\frac{q_1^2\beta_1}{2}\left[\delta f_v - ((1+\tau)a(t)-\tau)e^{r(T-t)}\right]^2 - \frac{\beta_2}{2}\tau^2q_2^2e^{2r(T-t)} + f_t + \Big[c_1 - \tau c_2 \\ & + \tau p_2 - (1+\eta_2)(v + p_1e^{-\delta t}) + \eta_2(v(t) + p_1e^{-\delta t})((1+\tau)a(t)-\tau)\Big]e^{r(T-t)} \\ & - \frac{m}{2}\left[((1+\tau)a(t)-\tau)^2q_1^2 + \tau^2q_2^2 - 2((1+\tau)a(t)-\tau)\tau\rho q_1q_2\right]e^{2r(T-t)} + \delta p_1e^{-\delta t}f_v \\ & + \frac{1}{2}\delta^2q_1^2(-mf_v^2 + f_{vv}) - [-\delta q_1((1+\tau)a(t)-\tau)q_1 + 2\rho\delta q_1q_2\tau]mf_ve^{r(T-t)}\Bigg\} = 0. \end{aligned} \tag{4.14}$$

According to the first-order optimality condition for $a(t)$ with respect to the superemum problem in (4.14), we have

$$\hat{a}(t) = \frac{1}{1+\tau}\left[\tau + \frac{\eta_2(\nu + p_1 e^{-\delta t}) + (m+\beta_1)\delta q_1^2 f_\nu + m q_1 q_2 \tau\rho e^{r(T-t)}}{(m+\beta_1) q_1^2 e^{r(T-t)}}\right]. \tag{4.15}$$

Substituting (4.15) into (4.14), we have

$$\begin{aligned}
& f_t + \left[\delta p_1 e^{-\delta t} - 2\rho\delta q_1 q_2 \tau m e^{r(T-t)} + \eta_2\delta(\nu + p_1 e^{-\delta t}) + \delta m q_1 q_2 \tau\rho e^{r(T-t)}\right] f_\nu \\
& + \frac{1}{2}\delta^2 q_1^2 f_{\nu\nu} + \nu^2 \frac{\eta_2^2}{2(m+\beta_1) q_1^2} + \nu\left[\frac{\eta_2 p_1 e^{-\delta t}}{(m+\beta) q_1^2} + \frac{\eta_2 m q_2 \tau\rho e^{r(T-t)}}{(m+\beta) q_1} - (1+\eta_2) e^{r(T-t)}\right] \\
& + \left[c_1 - \tau c_2 - (1+\eta_2) p_1 e^{-\delta t} + \tau p_2\right] e^{r(T-t)} - \frac{m+\beta_2}{2}\tau^2 q_2^2 e^{2r(T-t)} + \frac{1}{2 q_1^2 (m+\beta_1)} \\
& \times \left(\eta_2^2 p_1^2 e^{-2\delta t} + m^2 q_1^2 q_2^2 \tau^2 \rho^2 e^{2r(T-t)} + 2\eta_2 m q_1 q_2 \tau\rho p_1 e^{-\delta t} e^{r(T-t)}\right) = 0, f(T,\nu) = 0.
\end{aligned} \tag{4.16}$$

We assume that the solution of (4.16) is

$$f(t,\nu) = A(t)\nu^2 + B(t)\nu + D(t).$$

Then, we obtain

$$f_t = A'(t)\nu^2 + B'(t)\nu + D'(t),\ f_\nu = 2A(t)\nu + B(t),\ f_{\nu\nu} = 2A(t). \tag{4.17}$$

Substituting (4.17) into (4.16), we have after some simplifications that

$$\begin{aligned}
& \nu^2\left\{A'(t) + 2\eta_2\delta A(t) + \frac{\eta_2^2}{2(m+\beta_1) q_1^2}\right\} \\
& + \nu\left\{B'(t) + \eta_2\delta B(t) + 2A(s)\left[\delta p_1 e^{-\delta t} - 2\rho\delta q_1 q_2 \tau m e^{r(T-t)} + \delta m q_1 q_2 \tau\rho e^{r(T-t)}\right.\right. \\
& \left.\left. + \eta_2\delta p_1 e^{-\delta t}\right] - (1+\eta_2) e^{r(T-t)} + \frac{\eta_2 p_1 e^{-\delta t}}{(m+\beta_1) q_1^2} + \frac{\eta_2 m q_2 \tau\rho e^{r(T-t)}}{(m+\beta_1) q_1}\right\} \\
& + \left\{D'(t) + \left[\delta p_1 e^{-\delta t} - 2m\rho\delta q_1 q_2 \tau e^{r(T-t)} + \eta_2 p_1 \delta e^{-\delta t} + \delta m q_1 q_2 \tau\rho e^{r(T-t)}\right] B(t)\right. \\
& + \left[c_1 - \tau c_2 - (1+\eta_2) p_1 e^{-\delta t} + \tau p_2\right] e^{r(T-t)} - \frac{m+\beta_2}{2}\tau^2 q_2^2 e^{2r(T-t)} + \frac{1}{2 q_1^2 (m+\beta_1)} \\
& \left. \times \left(\eta_2^2 p_1^2 e^{-2\delta t} + m^2 q_1^2 q_2^2 \tau^2 \rho^2 e^{2r(T-t)} + 2\eta_2 m q_1 q_2 \tau\rho p_1 e^{-\delta t} e^{r(T-t)}\right)\right\}.
\end{aligned} \tag{4.18}$$

To solve (4.18), we separate the terms with ν^2, ν and the constant term, respectively. It can then be seen that the following system of ordinary differential equations should hold

$$A'(t) + 2\eta_2\delta A(t) + \frac{\eta_2^2}{2(m+\beta_1) q_1^2},\ A(T) = 0, \tag{4.19}$$

$$B'(t) + \eta_2\delta B(t) + 2A(s)\Big[\delta p_1 e^{-\delta t} - 2\rho\delta q_1 q_2 \tau m e^{r(T-t)} + \delta m q_1 q_2 \tau\rho e^{r(T-t)} + \eta_2\delta p_1 e^{-\delta t}\Big]$$
$$-(1+\eta_2)e^{r(T-t)} + \frac{\eta_2 p_1 e^{-\delta t}}{(m+\beta_1)q_1^2} + \frac{\eta_2 m q_2 \tau\rho e^{r(T-t)}}{(m+\beta_1)q_1},\ B(T) = 0, \tag{4.20}$$

$$D'(t) + \Big[\delta p_1 e^{-\delta t} - 2m\rho\delta q_1 q_2 \tau e^{r(T-t)} + \eta_2 p_1 \delta e^{-\delta t} + \delta m q_1 q_2 \tau\rho e^{r(T-t)}\Big]B(t)$$
$$+\Big[c_1 - \tau c_2 - (1+\eta_2)p_1 e^{-\delta t} + \tau p_2\Big]e^{r(T-t)} - \frac{m+\beta_2}{2}\tau^2 q_2^2 e^{2r(T-t)} + \frac{1}{2q_1^2(m+\beta_1)} \tag{4.21}$$
$$\times\Big(\eta_2^2 p_1^2 e^{-2\delta t} + m^2 q_1^2 q_2^2 \tau^2\rho^2 e^{2r(T-t)} + 2\eta_2 m q_1 q_2 \tau\rho p_1 e^{-\delta t} e^{r(T-t)}\Big) = 0,\ D(T) = 0.$$

Solving these ordinary differential equations gives us $A(t)$, $B(t)$ and $D(t)$ as those shown in (4.7), (4.8) and (4.9).

Combining (4.6) and (4.10), we can obtain the candidate optimal value function in (4.5).

Inserting (4.6) into (4.15), we have

$$\hat{a}(t) = \frac{1}{1+\tau}\left[\tau + \frac{\eta_2(\nu + p_1 e^{-\delta t}) + (m+\beta_1)\delta q_1^2(2A(t)\nu + B(t)) + m q_1 q_2 \tau\rho e^{r(T-t)}}{(m+\beta_1)q_1^2 e^{r(T-t)}}\right]. \tag{4.22}$$

If the reinsurance strategy $\hat{a}(t) > 0$, then the candidate robust optimal reinsurance strategy is $a^*(t) = \hat{a}(t)$, and the corresponding optimal value function is given by (4.5). If $\hat{a}(t) \le 0$, the $\sup_a\{\cdots\}$ in (4.14) is achieved at the point 0 since the function in the interior of $\sup_a\{\cdots\}$ is a decreasing function with respect to $a(t)$ in the interval $[0, +\infty)$. This implies that the candidate robust optimal reinsurance strategy must be $a^*(t) = 0$. Then, inserting $a^*(t) = 0$ into (4.14), we can obtain the optimal value function as we did for the case $a^*(t) = \hat{a}(t)$. This case means that the insurer transfers all of the claims to the reinsurer, which hardly occurs in practice. Therefore, we do not calculate the value function in this case. □

In what follows, we shall apply Proposition 3.1 to verify that the candidate robust optimal reinsurance strategy and the corresponding candidate value function given in Theorem 4.1 are indeed optimal.

Theorem 4.2. (Verification theorem). The candidate robust optimal strategies $a^*(t)$ and $\theta^*(t) = (\theta_1^*(t), \theta_2^*(t))$ given by (4.3) and (4.4) respectively are indeed robust optimal strategies, and the candidate value function $H(t,x,\nu)$ given by (4.5) is the optimal value function $V(t,x,\nu)$ defined by (3.3).

Proof. From Theorem 4.1, it is easy to see that the conditions (2)–(4) in Proposition 3.1 hold for $H(t,x,\nu)$. Because $H(t,x,\nu)$ is a smooth function, it is generally assumed in the literature (for example, [39] and [37]) that the optimization problem

$$\inf_{\theta\in\Theta}\sup_{u\in\mathcal{U}} \mathrm{E}_{t,x,\nu}\left\{\int_t^T \Phi(s, X_s^u, \nu(s), \theta(s))ds + U(X_T^u)\right\}, \tag{4.23}$$

is equivalent to problem (3.5). Hence, the condition (1) in Proposition 3.1 holds. To verify that the condition (5) in Proposition 3.1 holds, the candidate strategy $(a^*(t), \theta^*(t))$ and the candidate value function $H(t,x,\nu)$ under the probability distribution $\mathbb{Q}^{\theta^*}$ need to satisfy the following three properties:

(i) $\mathrm{E}\left(\sup_{t\in[0,T]}|H(t, X^{a^*}(t), \nu(t))|^4\right) < \infty$;

(ii) $a^*(t)$ is an admissible strategy;

(iii) $\mathrm{E}\left(\sup_{t\in[0,T]}\left|\frac{(\theta_1^*(t))^2}{2\phi_1(t,X^{a^*}(t),\nu(t))} + \frac{(\theta_2^*(t))^2}{2\phi_2(t,X^{a^*}(t),\nu(t))}\right|^2\right) < \infty$.

Similar to inequality (91) in [28], we can find that property (i) holds. From the solution process of the Hamilton-Jacobi-Bellman-Isaacs equation (3.5), we know that conditions (i) and (ii) in Definition 2.1 are satisfied, and that the strategy $a^*(t)$ is deterministic and state-independent; thus, condition (iii) in Definition 2.1 holds. Thus, $a^*(t)$ is an admissible strategy. To prove property (iii), let

$$\bar{\Gamma}(t) = \frac{m(\theta_1^*(t))^2}{2\beta_1} + \frac{m(\theta_2^*(t))^2}{2\beta_2}.$$

Obviously, $\bar{\Gamma}(t)$ is bounded. According to (4.2) with $H(t,x,v)$ instead of $V(t,x,v)$, we obtain

$$\begin{aligned}
&\mathrm{E}\left(\sup_{t\in[0,T]}\left|\frac{(\theta_1^*(t))^2}{2\phi_1(t,X^{a^*}(t),v(t))} + \frac{(\theta_2^*(t))^2}{2\phi_2(t,X^{a^*}(t),v(t))}\right|^2\right)\\
=&\mathrm{E}\left(\sup_{t\in[0,T]}\left|\bar{\Gamma}(t)\right|^2\left|H(t,X^{a^*}(t),v(t))\right|^2\right)\\
\le&\mathrm{E}\left(\sup_{t\in[0,T]}\left|\bar{\Gamma}(t)\right|^4\right)^{\frac{1}{2}}\left(\sup_{t\in[0,T]}\left|H(t,X^{a^*}(t),v(t))\right|^4\right)^{\frac{1}{2}} < \infty.
\end{aligned}$$

The first inequality follows from the Cauchy-Schwarz inequality, and the second inequality follows from property (i). Thus, Theorem 4.2 holds. □

To more intuitively analyze the effects of key model features on the robust optimal reinsurance strategy, we consider four special cases: the no-competition case, the uncorrelated insurance businesses case, the no-ambiguity-aversion case and the uncorrelated claims case. The resulting robust optimal reinsurance strategies for these four special cases can be directly derived from Theorem 4.1. The corresponding value functions can also be similarly obtained. However, their expressions are rather complicated and are thus omitted here.

Corollary 4.1. If we do not consider the competition between the insurer and reinsurer (that is, only consider the interest of the insurer), i.e., $\tau = 0$, the robust optimal reinsurance strategy is given by

$$a^*(t) = \frac{\eta_2(v + p_1e^{-\delta t}) + (m+\beta_1)\delta q_1^2(2A(t) + \tilde{B}(t))}{(m+\beta_1)q_1^2e^{r(T-t)}} \vee 0, \tag{4.24}$$

where

$$\begin{aligned}
\tilde{B}(t) = e^{\eta_2\delta(T-t)}\int_t^T &\left\{2A(s)\left[\delta p_1e^{-\delta s} + \eta_2\delta p_1e^{-\delta s}\right] - (1+\eta_2)e^{r(T-s)}\right.\\
&\left.+\frac{\eta_2p_1e^{-\delta s}}{(m+\beta_1)q_1^2}\right\}e^{-\eta_2\delta(T-s)}ds,
\end{aligned}\tag{4.25}$$

and $A(t)$ is given by (4.7).

The robust optimal reinsurance strategy (4.24) is similar to (28) in [28]. In other words, our model extends the robust optimal reinsurance strategy in [28] to the competition case, i.e., the case of simultaneously considering the joint interests of the insurer and the reinsurer.

Corollary 4.2. If we do not consider the interdependence between insurance businesses, i.e., $\rho = 0$, the robust optimal reinsurance strategy is given by

$$a^*(t) = \frac{1}{1+\tau}\left[\tau + \frac{\eta_2(v + p_1e^{-\delta t}) + (m+\beta_1)\delta q_1^2(2A(t) + \tilde{B}(t))}{(m+\beta_1)q_1^2e^{r(T-t)}}\right] \vee 0, \tag{4.26}$$

where $A(t)$ and $\tilde{B}(t)$ are given by (4.7) and (4.25), respectively. Furthermore, if we do not consider correlated claims, i.e., $\delta = \nu(t) = 0$, then (4.26) reduces to

$$a^*(t) = \frac{1}{1+\tau}\left[\tau + \frac{\eta_2 p_1}{(m+\beta_1)q_1^2 e^{r(T-t)}}\right]. \tag{4.27}$$

We can see that the robust optimal reinsurance strategy (4.27) is similar to (3.5) in [21]. This shows that our model extends the robust optimal reinsurance strategy in [21] to a more general case.

Corollary 4.3. If we do not consider ambiguity aversion, i.e., $\beta_1 = \beta_2 = 0$, the robust optimal reinsurance strategy is given by

$$a^*(t) = \frac{1}{1+\tau}\left[\tau + \frac{\eta_2(\nu + p_1 e^{-\delta t}) + m\delta q_1^2(2\hat{A}(t) + \hat{B}(t)) + mq_1q_2\tau\rho e^{r(T-t)}}{mq_1^2 e^{r(T-t)}}\right] \vee 0, \tag{4.28}$$

where

$$\hat{A}(t) = \frac{\eta_2}{4\lambda_1\mu_{12}\delta m}\left[e^{2\delta\eta_2(T-t)} - 1\right], \tag{4.29}$$

$$\begin{aligned}\hat{B}(t) = e^{\eta_2\delta(T-t)}\int_t^T &\Big\{2A(s)\Big[\delta p_1 e^{-\delta s} - 2\rho\delta q_1q_2\tau m e^{r(T-s)} + \delta m q_1 q_2\tau\rho e^{r(T-s)} \\ &+ \eta_2\delta p_1 e^{-\delta s}\Big] - (1+\eta_2)e^{r(T-s)} + \frac{\eta_2 p_1 e^{-\delta s}}{mq_1^2} + \frac{\eta_2 m q_2\tau\rho e^{r(T-s)}}{mq_1}\Big\}e^{-\eta_2\delta(T-s)}ds.\end{aligned} \tag{4.30}$$

We find that (4.3) and (4.28) have very similar forms. However, we cannot arbitrarily say that ambiguity aversion has no significant influence on the robust optimal reinsurance strategy. In the next section, we will concretely analyze the influence of ambiguity aversion on the robust optimal reinsurance strategy through numerical experiments.

Corollary 4.4. If we do not consider correlation among claims, i.e., $\nu(t) = \delta = 0$, the robust optimal reinsurance strategy is given by

$$a^*(t) = \frac{1}{1+\tau}\left[\tau + \frac{\eta_2 p_1 + mq_1q_2\tau\rho e^{r(T-t)}}{(m+\beta_1)q_1^2 e^{r(T-t)}}\right]. \tag{4.31}$$

Comparing (4.3) and (4.31), we find that (4.31) becomes simpler. This shows that the correlated claims have a significant impact on the robust optimal reinsurance strategy. However, the specific forms of influence will be further discussed in the next section.

5. Numerical experiment

In this section, we will conduct a series of numerical experiments to illustrate the effects of time, the correlated claims, the dependence of insurance business, the competition degree and the ambiguity-aversion coefficient on the robust optimal reinsurance strategy. We also analyze the similarities and differences between the robust optimal reinsurance strategy obtained when the correlated claims, the dependence of insurance business and the competition are considered and are not considered, respectively.

By referring to the relevant studies such as [6, 21, 27, 28], we set the basic parameters as those in Table 1.

Table 1. Values of model parameters.

t	T	λ_1	λ_2	λ	μ_{11}	μ_{12}	μ_{21}	μ_{22}	m	β_1	β_2	r	δ	ν	η_1	η_2	τ
0	10	0.8	0.5	0.04	2.2	2.7	1.6	2.3	0.2	0.2	0.4	0.02	0.25	0.4	0.15	0.5	0.5

5.1. The effects of model parameters and dependence on the robust optimal reinsurance strategy

In this subsection, we examine the effects of time, the correlated claims, the competition degree and the ambiguity aversion coefficient on the robust optimal reinsurance strategy when the dependence between the insurer's insurance business and the reinsurer's insurance business is considered and is not considered, respectively.

Figure 1 illustrates the effect of time t on $a^*(t)$ when the dependence of the insurance business is considered ($\lambda > 0$) and is not considered ($\lambda = 0$). From Figure 1, we can see that $a^*(t)$ is increasing with respect to t, regardless of whether the dependence of the insurance business is considered or not. Since we consider the correlated claims, the insurer can obtain more claim information as time goes on. Therefore, as time passes, the insurer would reduce his reinsurance willingness.

For given time $t = 0$, Figure 2 shows the influence of the extrapolation intensity δ on the robust optimal reinsurance strategy $a^*(0)$ when the dependence of the insurance business is considered ($\lambda > 0$) and is not considered ($\lambda = 0$). The larger the δ, the larger the weighted average $v(t)$. The weighted average $v(t)$ is negatively correlated with the risk of the wealth process; hence, $v(t)$ can offset more risk of the wealth process as δ increases. And then, the insurer decreases his reinsurance demand as δ increases. Therefore, in Figure 2, we see that $a^*(0)$ is an increasing function of δ, regardless of whether the dependence of insurance businesses is considered or not.

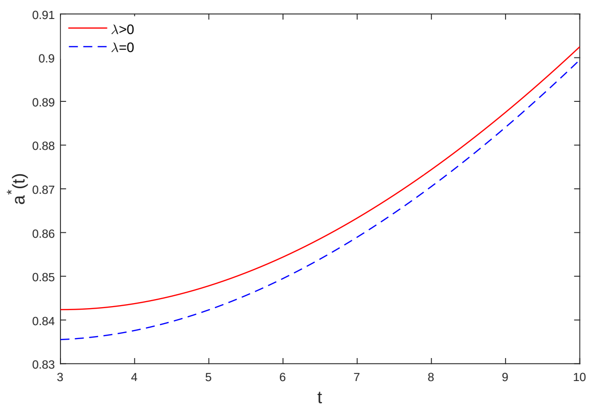

Figure 1. Effect of t on $a^*(t)$.

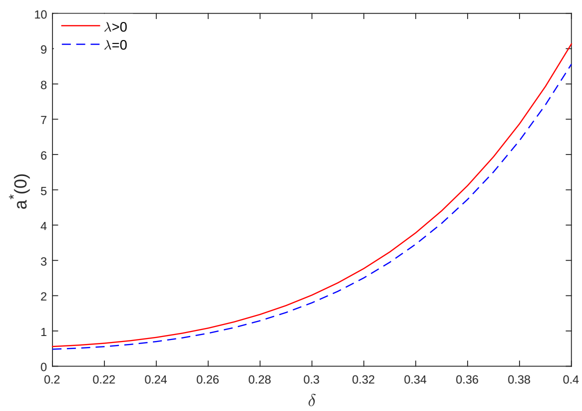

Figure 2. Effect of δ on $a^*(0)$.

For given time $t = 0$, Figure 3 illustrates the effect of τ on the robust optimal reinsurance strategy $a^*(0)$ when the dependence of the insurance business is considered ($\lambda > 0$) and is not considered ($\lambda = 0$). From Figure 3, we can see that $a^*(0)$ is increasing with respect to τ, regardless of whether the dependence of the insurance business is considered or not. A larger τ means that the insurer becomes more concerned about his relative wealth. This makes the insurer become more eager to surpass the reinsurer's wealth. Hence, the insurer increases the value of $a^*(0)$ in order to reduce the reinsurance premium.

For given time $t = 0$, Figure 4 discloses the effect of β_1 on the robust optimal reinsurance strategy $a^*(0)$ when the dependence of the insurance business is considered ($\lambda > 0$) and is not considered ($\lambda = 0$). From Figure 4, we can see that $a^*(0)$ decreases with respect to β_1, regardless of whether the dependence of the insurance business is considered or not. This is intuitive, as β_1 stands for the ambiguity-aversion coefficient. With the increase of β_1, the insurer would purchase more reinsurance to disperse the underlying distribution uncertainty risk.

From Figures 1–4, we can also see that when considering the dependence of an insurance business, the insurer keeps a larger $a^*(t)$ and $a^*(0)$. This is because, when considering the dependence of an insurance business, the insurer can simultaneously grasp the claims of multiple insurance businesses. Therefore, the insurer can more efficiently grasp the potential claims' risk. This directly leads to the increase of his retention level and the decrease of his reinsurance premium expenditure.

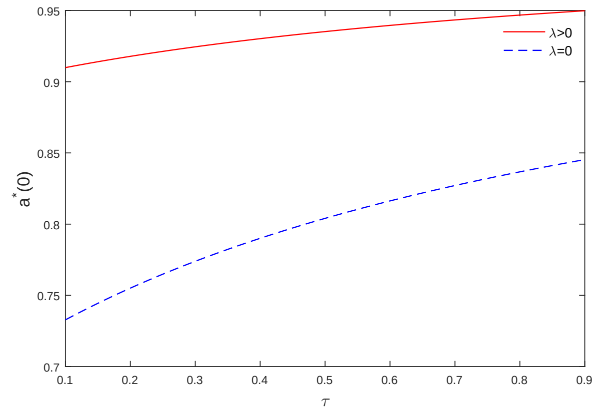

Figure 3. Effect of τ on $a^*(0)$.

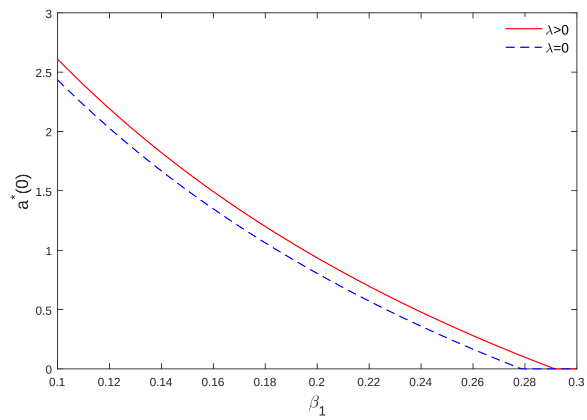

Figure 4. Effect of β_1 on $a^*(0)$.

5.2. The effects of model parameters and correlated claims on the robust optimal reinsurance strategy

In this subsection, we examine the effect of time, the competition degree and the ambiguity aversion coefficient on the robust optimal reinsurance strategy when the correlated claims are considered and not considered, respectively.

Figure 5 shows the effect of t on $a^*(t)$ when the correlated claims are considered ($\delta > 0, \nu(t) > 0$) and not considered ($\delta = \nu(t) = 0$), where we set $\eta_2 = 0.3$. For given time $t = 0$, Figures 6 and 7 respectively show the effects of τ and β_1 on $a^*(0)$ when the correlated claims are considered ($\delta > 0, \nu(t) > 0$) and not considered ($\delta = \nu(t) = 0$). Similar to Figure 1, we see in Figure 5 that $a^*(t)$ is increasing with respect to t, regardless of whether the correlated claims are considered or not. Similar to Figures 3 and 4, we see in Figures 6 and 7 that $a^*(0)$ is increasing and decreasing with respect to τ and β_1, respectively, regardless of whether the correlated claims are considered or not. More importantly, we find from Figures 5–7, that, when the correlated claims are considered, the insurer keeps a larger $a^*(t)$ and $a^*(0)$. This is because, when the correlated claims are considered, the insurer can infer future claim information based on historical claim information. Therefore, the insurer can better grasp the claim risk. Hence, at this time, the insurer reduces his reinsurance willingness. Thus, this directly causes him to keep a large $a^*(t)$ and $a^*(0)$ to reduce the expenditure of the reinsurance premium.

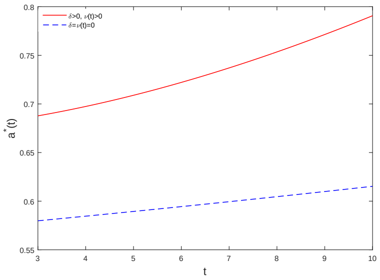

Figure 5. Effect of t on $a^*(t)$.

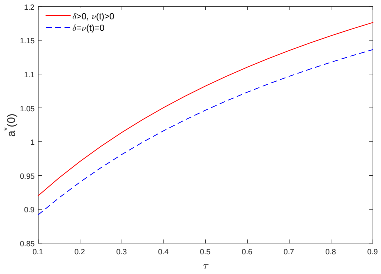

Figure 6. Effect of τ on $a^*(0)$.

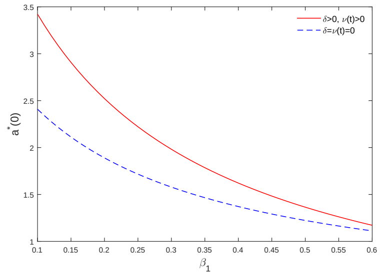

Figure 7. Effect of β_1 on $a^*(0)$.

5.3. The effects of model parameters and competition on the robust optimal reinsurance strategy

In this subsection, we examine the effects of time, the correlated claims and the ambiguity aversion coefficient on the robust optimal reinsurance strategy when the competition between the insurer and the reinsurer is considered and is not considered, respectively.

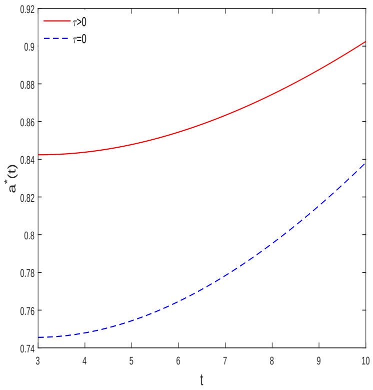

Figure 8. Effect of t on $a^*(t)$.

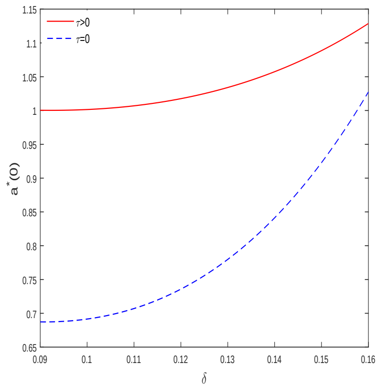

Figure 9. Effect of δ on $a^*(0)$.

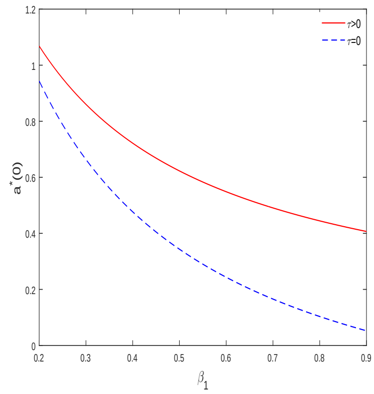

Figure 10. Effect of β_1 on $a^*(0)$.

Figure 8 shows the effect of t on $a^*(t)$ when the competition between the insurer and the reinsurer is considered ($\tau > 0$) and is not considered ($\tau = 0$). For given time $t = 0$, Figures 9 and 10 respectively illustrate the effects of δ and β_1 on $a^*(0)$ when the competition between the insurer and the reinsurer is considered ($\tau > 0$) and is not considered ($\tau = 0$). Similar to Figure 1, we see in Figure 8 that $a^*(t)$ is increasing with respect to t, regardless of whether the competition is considered or not. Similar to Figures 2 and 4, we see in Figures 9 and 10 that $a^*(0)$ is increasing and decreasing with respect to δ and β_1, respectively, regardless of whether the competition is considered or not. More importantly, we find from Figures 8–10, that, when the competition is considered, the insurer keeps a larger $a^*(t)$ and $a^*(0)$. The reason is similar to that for Figure 3. That is, when considering competition, the insurer hopes to surpass the reinsurer's wealth. While purchasing reinsurance can reduce his risk, it is nonetheless costly because the insurer needs to pay the reinsurance premium to the reinsurer for the reinsurance protection. This would decrease his wealth value relative to that of his competitor. Hence, the insurer keeps a larger $a^*(t)$ and $a^*(0)$ when the competition is considered.

5.4. Joint effects of model parameters on the robust optimal reinsurance strategy

Above, we have analyzed the effects of time, the correlated claims, the dependence of the insurance business, the competition degree and the ambiguity aversion coefficient on the robust optimal reinsurance strategy. To further analyze which of these model parameters has a more significant influence on the robust optimal reinsurance strategy, for given time $t = 0$, we examine some joint effects of these model parameters on $a^*(0)$ under the general situation, i.e., all the of above-mentioned model features are considered. The results are shown in Figures 11–14.

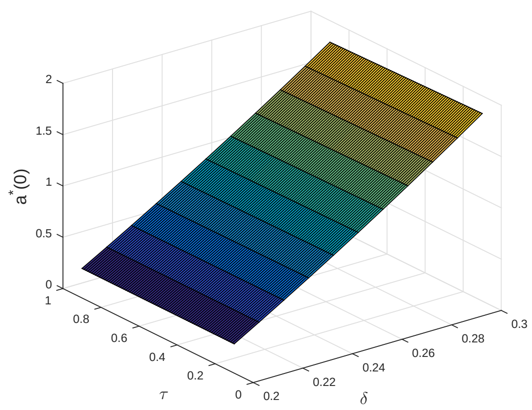

Figure 11. Joint effects of δ and τ on $a^*(0)$.

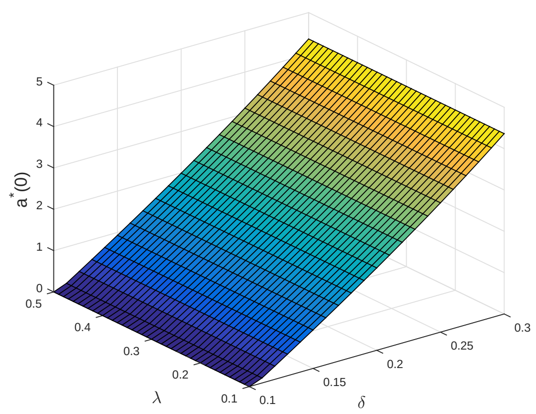

Figure 12. Joint effects of δ and λ on $a^*(0)$.

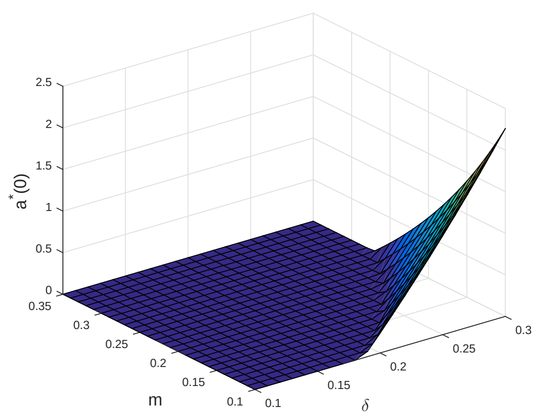

Figure 13. Joint effects of β_1 and m on $a^*(0)$.

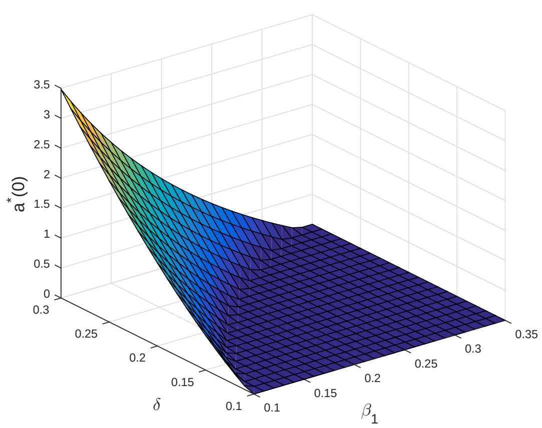

Figure 14. Joint effects of δ and β_1 on $a^*(0)$.

From Figures 11–14, we can see that the influence of the extrapolation intensity δ has the most significant influence on $a^*(0)$. This means that the correlated claims have the most significant influence on the insurer's reinsurance decision. This may be because the insurer can better grasp the possible claim risk when considering the correlated claims. This shows the necessity and importance of considering the correlated claims.

6. Conclusions

A robust optimal reinsurance strategy selection problem was studied in this paper. To reduce claim risk, the insurer can purchase proportional reinsurance or acquire a new business; to increase wealth, the reinsurer can engage in insurance business. The interdependence between the insurance businesses of both the insurer and the reinsurer is considered. Meanwhile, the correlation between the insurer's future claims and historical claims is also considered. Furthermore, we have quantified the competition between the insurer and the reinsurer through relative wealth. These model settings extend the model in [28]. Under ambiguity aversion and the criterion of maximizing the expected utility of terminal wealth, we have obtained explicit solutions for the robust optimal reinsurance strategy and the corresponding value function by using the stochastic dynamic programming approach. Finally, a series of numerical experiments were carried out to examine the influences of the model parameters on the robust optimal reinsurance strategy.

There are still some issues worthy of investigation in the future. First, we considered the joint interests of the insurer and the reinsurer through competition, but we did not consider the decision of the reinsurer. It is interesting to consider the decision of the reinsurer. Second, it is also interesting to consider other reinsurance patterns, e.g., the excess-of-loss reinsurance and per-loss reinsurance. Third, it is worthwhile to consider other forms of competition between the insurer and the reinsurer.

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Conflict of interest

The author declares that he has no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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