AIMS Mathematics

AIMS Mathematics, 8(7): 15670–15688.
DOI: 10.3934/math.2023800
Received: 24 February 2023
Revised: 3 April 2023
Accepted: 11 April 2023
Published: 27 April 2023

http://www.aimspress.com/journal/Math

Research article

Analytical solutions to a class of fractional coupled nonlinear Schrödinger equations via Laplace-HPM technique

Baojian Hong[1,*], Jinghan Wang[2] and Chen Li[3]

[1] Faculty of Mathematical Physics, Nanjing Institute of Technology, Nanjing 211167, China
[2] Faculty of Electric Power Engineering, Nanjing Institute of Technology, Nanjing 211167, China
[3] Faculty of Architectural Engineering, Nanjing Institute of Technology, Nanjing 211167, China

* **Correspondence:** Email: hbj@njit.edu.cn.

Abstract: In this article, a class of fractional coupled nonlinear Schrödinger equations (FCNLS) is suggested to describe the traveling waves in a fractal medium arising in ocean engineering, plasma physics and nonlinear optics. First, the modified Kudryashov method is adopted to solve exactly for solitary wave solutions. Second, an efficient and promising method is proposed for the FCNLS by coupling the Laplace transform and the Adomian polynomials with the homotopy perturbation method, and the convergence is proved. Finally, the Laplace-HPM technique is proved to be effective and reliable. Some 3D plots, 2D plots and contour plots of these exact and approximate solutions are simulated to uncover the critically important mechanism of the fractal solitary traveling waves, which shows that the efficient methods are much powerful for seeking explicit solutions of the nonlinear partial differential models arising in mathematical physics.

Keywords: fractional coupled nonlinear Schrödinger equation; modified Kudryashov method; Laplace transform; Homotopy perturbation method; exact solutions; approximate solutions
Mathematics Subject Classification: 65Mxx, 34A08

1. Introduction

In recent years, with the wide applications of fractional calculus, more and more nonlinear phenomena came down to fractional models: for examples, various discontinuous phenomena in mechanics [1], chaotic oscillations [2], ecological and economic systems [3], two-scale thermal science [4], atmospheric space science [5], optical fiber systems [6], and others [7,8]. Searching for exact solutions or approximate solutions of these models plays an important role in the study of dynamical behavior and inner structures of those nonlinear phenomena [9]. Up to now, many authors have presented various powerful methods for this purpose, such as Bäcklund transformation method [10], Darboux transformation [11], Hirota bilinear method [12], projective Riccati equations method [13], Jacobi elliptic function expansion method [14], sine-Gordon method [15], exponential function method [16], improved (m+G'/G)-expansion method [17], PINN method [18], G'/G^2-expansion method [19], improved extended Tanh technique [20], the sub-equation technique [21], ect [22–24]. However, due to the complexity of nonlinear systems, it is often difficult for us to obtain the exact solutions; thus, people turn to look for their approximate solutions. So far, many approximate methods for effective convergence have been established, including finite element method [25], finite difference method [26], multiple-scale method [27], improved Adomian decomposition method [28], modified fractional variational iteration method [29], He-Laplace variational iteration method [30], and homotopy analysis method etc [31], among which the homotopy perturbation method (HPM), which was first proposed by Ji-Huan He in 1998 [32,33], is the most promising technology for fractal calculus, besides the applications for traditional differential equation [34,35]. As we all know, the Laplace transformation method is a powerful tool for us to solve a wide variety of initial-value problems, especially for the differential equation problems, and this method played an extremely important role in mathematical physics [36–38].

As we all know, the famous nonlinear Schrödinger equations (NLS) and coupled nonlinear Schrödinger equations (CNLS) are widely used in optical fiber, ocean engineering, plasma physics, quantum mechanics, etc [39–41]. In this article, we will utilize the modified Kudryashov method [42] to find exact solitary solutions of a class of fractional coupled nonlinear Schrödinger equation (FCNLS) first and then apply the Laplace transformation method combined with HPM to obtain the approximate solution, with the aid of adomian polynomials, we obtain many good results. The main advantages of these two methods are the efficient convergence of the iterative sequences and that the exact solutions can be easily obtained.

Consider the following FCNLS:

$$\begin{cases} iD_t^{\alpha}u + iaD_x^{\beta}u + bD_x^{2\beta}u + \delta(|u|^2 + \gamma|v|^2)u = 0, \quad 0<\alpha,\beta \le 1, \\ iD_t^{\alpha}v - iaD_x^{\beta}v + bD_x^{2\beta}v + \delta(\gamma|u|^2 + |v|^2)v = 0, \end{cases} \tag{1}$$

where $D_t^{\alpha}, D_x^{\beta}, D_x^{2\beta} = D_x^{\beta}(D_x^{\beta})$ represent the Caputo fractional derivative operator [43–45]. The coefficients a,b,δ,γ are real constants, $u = u(x,t), v = v(x,t)$ are two complex valued functions with respect to the time t and the propagation distance x. Equation (1) occurs in many fields including nonlinear optics, ocean engineering and plasma waves. If we select $a = 0$, functions u,v represent the amplitudes of circularly polarized waves in a nonlinear optical fiber, nonzero constant δ represents self-focusing and self-defocusing nonlinearity, nonzero constan γ represents cross-phase modulation and self-phase modulation [46]. Much literature on Eq (1) was available. See, for examples, [47–53], but the critically important mechanism of the fractal solitary traveling waves has not yet revealed, and the research on this topic has been preliminary.

Now, we review some basic definitions and properties of the Laplace transform for fractional calculus, some elementary introduction can be found in [36–38,43–45].

Definition 1. For a function $f(t):[0,\infty) \to R.$ The Riemann Liouville fractional integral operator and Caputo fractional derivative operator of order α are defined as [35]

$$J_t^{\alpha} f(t)=\frac{1}{\Gamma(\alpha)}\int_0^t (t-\tau)^{\alpha-1} f(\tau)d\tau, \alpha>0, t>0, J_t^0 f(t)=f(t).$$

$$D^{\alpha} f(t)=J^{n-\alpha}D^n f(x)=\begin{cases}\frac{1}{\Gamma(n-\alpha)}\int_0^t (t-\tau)^{n-\alpha-1} f^{(n)}(\tau)d\tau, n-1<\alpha<n, n\in N.\\ \frac{d^{(n)}f(t)}{dt^n}, \alpha=n\in N.\end{cases}$$

Definition 2. The Laplace transform and inverse transformation of function $f(t)$ is defined as [36–38]

$$F(s)=\mathcal{L}\left[f(t)\right]=\int_0^{\infty} e^{-st} f(t)dt\,, f(t)=\frac{1}{2\pi i}\int_{\lambda-i\infty}^{\lambda+i\infty} F(s)e^{st}ds, s=\lambda+i\varpi, t>0.$$

Definition 3. The Laplace transform of $D_t^{\alpha}u(x,t)$ is defined as [36–38]

$$\mathcal{L}\left[D_t^{\alpha}u(x,t)\right]=s^{\alpha}\mathcal{L}\left[u(x,t)\right]-\sum_{k=0}^{n-1}u^{(k)}(x,0)s^{\alpha-1-k},\quad n-1<\alpha<n. \tag{2}$$

Definition 4. The Adomian polynomials of $N(u=\sum_{i=0}^{\infty} p^i u_i)$ is defined as [54,55]

$$A=\sum_{n=0}^{\infty}A_n=\sum_{n=0}^{\infty}\frac{1}{n!}\frac{d^n}{dp^n}[N(\sum_{i=0}^{\infty}p^i u_i)]_{p=0}. \tag{3}$$

2. Description of the two methods

2.1. The modified Kudryashov method [42]

Consider the following differential equation

$$E(f, f', f'', f''', \cdots)=0\,, \tag{4}$$

where $f'=\frac{df}{d\xi}$, we assume that the solutions of Eq (4) can be presented as follows

$$f=f(\xi)=\sum_{i=0}^{N}a_i F^i(\xi), \qquad F(\xi)=\frac{1}{1+da^{\xi}}, \tag{5}$$

where N is a balance number, the coefficients $a_i (i=0,1,\cdots,N)$ and the variable function

$\xi = \xi(x,t)$ are evaluated later. The function $F(\xi)$ satisfies the following form and constraint condition:

$$F' = \frac{dF(\xi)}{d\xi} = F(\xi)(F(\xi)-1)\ln a = F(F-1)\ln a. \tag{6}$$

Substituting Eqs (5) and (6) into Eq (4) and collecting the coefficients of $F^i (i = 0,1,2,\cdots)$ to zero yields algebraic equations (AEs) for $a_0, a_1, \cdots, a_N$ and ξ. Utilizing mathematical software to solve the AEs, we can obtain the solutions of Eq (4).

2.2. The procedure of Laplace-Homotopy perturbation method (Laplace-HPM)

The basic idea and specific steps of the Laplace transformation and HPM can be seen in [56–58]. Here, we will utilize the efficient method to Eq (1) for finding the approximate solution.

If we let $D_t^\alpha u = u_t^\alpha, D_x^\beta u = u_x^\beta, D_x^{2\beta} u = u_{xx}^{2\beta}$ and apply the Laplace transform about t on both sides of the first equation of Eq (1), we have:

$$i\mathcal{L}\{u_t^\alpha\} + i\mathcal{L}\{au_x^\beta\} + \mathcal{L}\{bu_{xx}^{2\beta}\} + \mathcal{L}\{\delta\left(|u|^2 + \gamma|v|^2\right)u\} = 0, \tag{7}$$

$$s^\alpha \mathcal{L}[u(x,t)] - s^{\alpha\text{-}1}u(x,0) = i\mathcal{L}[iau_x^\beta + bu_{xx}^{2\beta} + \delta(|u|^2 + \gamma|v|^2)u], \tag{8}$$

$$\mathcal{L}[u(x,t)] = \frac{1}{s}u(x,0) + \frac{i}{s^\alpha}\mathcal{L}[iau_x^\beta + bu_{xx}^{2\beta} + \delta(|u|^2 + \gamma|v|^2)u]. \tag{9}$$

Applying inverse Laplace transform to both sides of (9), we obtain

$$u(x,t) = u(x,0) + \mathcal{L}^{-1}[\frac{i}{s^\alpha}\mathcal{L}[iau_x^\beta + bu_{xx}^{2\beta} + \delta(|u|^2 + \gamma|v|^2)u]]. \tag{10}$$

Generally, we can construct the homotopy equation as follows:

$$(1-p)(u-u_0) + p\{(u-u_0) - \mathcal{L}^{-1}[\frac{i}{S^\alpha}\mathcal{L}[iau_x^\beta + bu_{xx}^{2\beta} + \delta(|u|^2 + \gamma|v|^2)u]]\} = 0, \tag{11}$$

where $p \in [0,1]$ is a homotopy parameter. u_0 is an initial guess of $u(x,t)$ that satisfies the boundary conditions of Eq (1). Obviously, $u : u_0 \to u$ since $p : 0 \to 1$.

Assuming that the solution for Eq (1) can be written as

$$u(x,t) = \sum_{n=0}^{\infty} u_n p^n. \tag{12}$$

Substituting Eq (12) into Eq (11) yields

$$\sum_{n=0}^{\infty} u_n p^n = u_0 + p\{\mathcal{L}^{-1}[\frac{i}{S^{\alpha}}\mathcal{L}[iau_x^{\beta} + bu_{xx}^{2\beta} + \delta\sum_{n=0}^{\infty} p^n A_n + \delta\gamma\sum_{n=0}^{\infty} p^n A_n^{'}]]\}, \tag{13}$$

where A_n and $A_n^{'}$ are the n-th term of the Adomian polynomials of nonlinear terms in Eq (11). Now, equating the coefficients of the identical powers of p on both sides, we get the following iterations:

$$p^0 : u_0 = u_0(x), \cdots, p^{n+1} : u_{n+1} = \mathcal{L}^{-1}[\frac{i}{s^{\alpha}}\mathcal{L}(iau_{nx}^{\beta} + bu_{nxx}^{2\beta} + \delta A_n + \delta\gamma A_n^{'})], n \geq 0. \tag{14}$$

Using the same method, we can give following approximations for Eq (1):

$$p^0 : v_0 = v_0(x), \cdots, p^n : v_{n+1} = \mathcal{L}^{-1}[\frac{i}{s^{\alpha}}\mathcal{L}(-iav_{nx}^{\beta} + bv_{nxx}^{2\beta} + \delta A_n^{''} + \delta\gamma A_n^{'''})], n \geq 0, \tag{15}$$

where $A_n^{''}$ and $A_n^{'''}$ are the n-th term of Adomian polynomials of nonlinear terms for Eq (1). From Definition 4, we have

$$A_0 = u_0^2\overline{u_0}, A_1 = u_0^2\overline{u_1} + 2u_0u_1\overline{u_0}, A_2 = u_0^2\overline{u_2} + u_1^2\overline{u_0} + 2u_0u_1\overline{u_1} + 2u_0u_2\overline{u_0}, \cdots,$$

$$\begin{aligned} A_n = {} & u_0^2\overline{u_n} + u_1^2\overline{u_{n-2}} + \cdots + u_k^2\overline{u_{n-2k}} + 2u_0u_1\overline{u_{n-1}} + 2u_0u_2\overline{u_{n-2}} + \cdots + 2u_0u_{2k}\overline{u_{n-2k}} \\ & + \cdots + 2u_ku_{n-k}\overline{u_0}, \quad n > 2k, k > 1. \end{aligned}$$

$$\begin{aligned} & A_0^{'} = u_0v_0\overline{v_0}, A_1^{'} = v_0\overline{v_0}u_1 + v_0\overline{v_1}u_0 + v_1\overline{v_0}u_0, \\ & A_2^{'} = v_0\overline{v_0}u_2 + v_0\overline{v_1}u_1 + v_0\overline{v_2}u_0 + v_1\overline{v_0}u_1 + v_1\overline{v_1}u_0 + v_2\overline{v_0}u_0, \cdots, \\ & A_n^{'} = v_0\overline{v_0}u_n + v_1\overline{v_1}u_{n-2} + \cdots + v_k\overline{v_k}u_{n-2k} + v_0\overline{v_1}u_{n-1} + v_1\overline{v_0}u_{n-1} + \cdots + v_0\overline{v_n}u_0 \\ & \quad + \cdots + v_n\overline{v_0}u_0, \quad n > 2k, k > 1. \end{aligned}$$

$$\begin{aligned} & A_0^{''} = u_0\overline{u_0}v_0, A_1^{''} = u_0\overline{u_0}v_1 + u_0\overline{u_1}v_0 + u_1\overline{u_0}v_0, \\ & A_2^{''} = u_0\overline{u_0}v_2 + u_0\overline{u_1}v_1 + u_0\overline{u_2}v_0 + u_1\overline{u_0}v_1 + u_1\overline{u_1}v_0 + u_2\overline{u_0}v_0, \cdots, \\ & A_n^{''} = u_0\overline{u_0}v_n + u_1\overline{u_1}v_{n-2} + \cdots + u_k\overline{u_k}v_{n-2k} + u_0\overline{u_1}v_{n-1} + u_1\overline{u_0}v_{n-1} + \cdots + u_0\overline{u_n}v_0 \\ & \quad + \cdots + u_n\overline{u_0}v_0, \quad n > 2k, k > 1. \end{aligned}$$

$$A_0^{'''} = v_0^2\overline{v_0}, A_1^{'''} = v_0^2\overline{v_1} + 2v_0v_1\overline{v_0}, A_2^{'''} = v_0^2\overline{v_2} + v_1^2\overline{v_0} + 2v_0v_1\overline{v_1} + 2v_0v_2\overline{v_0}, \cdots,$$

$$\begin{aligned} A_n^{'''} = {} & v_0^2\overline{v_n} + v_1^2\overline{v_{n-2}} + \cdots + v_k^2\overline{v_{n-2k}} + 2v_0v_1\overline{v_{n-1}} + 2v_0v_2\overline{v_{n-2}} + \cdots + 2v_0v_{2k}\overline{v_{n-2k}} \\ & + \cdots + 2v_kv_{n-k}\overline{v_0}, \quad n > 2k, k > 1, \end{aligned}$$

where $\overline{(\cdot)}$ indicates the conjugation of $(\cdot)$.

When $p \to 1$, it yields the n-th approximate solution and exact solution for Eq (1) as follows

$$u_{(n)} = u_0 + u_1 + u_2 + \cdots + u_n, v_{(n)} = v_0 + v_1 + v_2 + \cdots + v_n,$$

$$u_{exact} = u = \lim_{n\to\infty} u_{(n)}, v_{exact} = v = \lim_{n\to\infty} v_{(n)}.$$

3. Exact and approximate solutions of the FCNLS

3.1. Exact solutions

We can give the following function and traveling wave transformation:

$$u = P(\xi)e^{i\eta_1}, v = Q(\xi)e^{i\eta_2}, \tag{16}$$

$$\begin{aligned} &\xi = \frac{k}{\Gamma(\beta+1)}x^\beta + \frac{\omega}{\Gamma(\alpha+1)}t^\alpha, \\ &\eta_1 = \frac{k_1}{\Gamma(\beta+1)}x^\beta + \frac{c_1}{\Gamma(\alpha+1)}t^\alpha, \eta_2 = \frac{k_2}{\Gamma(\beta+1)}x^\beta + \frac{c_2}{\Gamma(\alpha+1)}t^\alpha, \end{aligned} \tag{17}$$

where constants k, k_1, k_2 and ω, c_1, c_2 are to be determined latter.

Substituting Eqs (16) and (17) into Eq (1) and separating the real part and the imaginary part, thus we have

$$\begin{cases} bk^2 P_{\xi\xi} - (k_1^2 b + ak_1 + c_1)P + \delta(P^2 + \gamma Q^2)P = 0, & (18.1) \\ bk^2 Q_{\xi\xi} - (k_2^2 b - ak_2 + c_2)Q + \delta(\gamma P^2 + Q^2)Q = 0, & (18.2) \\ (\omega + ak + 2bkk_1)P_\xi = 0, & (18.3) \\ (\omega - ak + 2bkk_2)Q_\xi = 0. & (18.4) \end{cases} \tag{18}$$

From Eq (18.3) and Eq (18.4), we obtain the following exact solution

$$u = me^{i(\frac{k_1}{\Gamma(\beta+1)}x^\beta + \frac{\delta(m^2+\gamma n^2) - k_1^2 b - ak_1}{\Gamma(\alpha+1)}t^\alpha)}, v = ne^{i(\frac{k_2}{\Gamma(\beta+1)}x^\beta + \frac{\delta(\gamma m^2 + n^2) - k_2^2 b + ak_2}{\Gamma(\alpha+1)}t^\alpha)}. \tag{19}$$

From Eq (18.3) and Eq (18.2), we obtain

$$\omega = -bk(k_1 + k_2). \tag{20}$$

According to the homogeneous balance principle and the modified Kudryashov method [47], we assume Eq (18) have the following solutions

$$\begin{cases} P(\xi) = a_0 + a_1 F = a_0 + \dfrac{a_1}{1 + da^\xi}, d \in R, \\ Q(\xi) = b_0 + b_1 F = b_0 + \dfrac{b_1}{1 + da^\xi}, d \in R. \end{cases} \tag{21}$$

Substituting Eqs (6) and (17) into Eq (18), and setting the coefficients of F^i to zero yield a set of AEs for the unknowns $a_0, a_1, a_2, b_0, b_1, b_2, k, k_1, k_2, c_1, c_2$ and ω.

$$
\begin{aligned}
&F^0 : -ak_1a_0 - bk_1{}^2a_0 + \delta a_0^3 + \gamma\delta a_0 b_0^2 - a_0c_1 = 0,\\
&F^1 : -ak_1a_1 - bk_1{}^2a_1 + bk^2(lna)^2 a_1 + 3\delta a_0^2 a_1 + \gamma\delta a_1 b_0^2 + 2\gamma\delta a_0 b_0 b_1 - a_1c_1 = 0,\\
&F^2 : -3bk^2(lna)^2 a_1 + 3\delta a_0 a_1^2 + 2\gamma\delta a_1 b_0 b_1 + \gamma\delta a_0 b_1^2 = 0,\\
&F^3 : 2bk^2(lna)^2 a_1 + \delta a_1^3 + \gamma\delta a_1 b_1^2 = 0,
\end{aligned}
$$

$$
\begin{aligned}
&F^0 : ak_2b_0 - bk_2{}^2b_0 + \gamma\delta a_0^2 b_0 + \delta b_0^3 - b_0c_2 = 0,\\
&F^1 : 2\gamma\delta a_0 a_1 b_0 + ak_2b_1 - bk_2{}^2b_1 + bk^2(lna)^2 b_1 + \gamma\delta a_0^2 b_1 + 3\delta b_0^2 b_1 - b_1c_2 = 0,\\
&F^2 : \gamma\delta a_1^2 b_0 - 3bk^2(lna)^2 b_1 + 2\gamma\delta a_0 a_1 b_1 + 3\delta b_0 b_1^2 = 0,\\
&F^3 : 2bk^2(lna)^2 b_1 + \gamma\delta a_1^2 b_1 + \delta b_1^3 = 0.
\end{aligned}
$$

Solving the AEs along with Eqs (5), (16) and (20) results in the following solutions:

$$
\begin{aligned}
&b_0 = \mp a_0, b_1 = \pm 2a_0, a_1 = -2a_0, k = \pm\frac{a_0}{\ln a}\sqrt{\frac{-2(1+\gamma)\delta}{b}}, \omega = -bk(k_1+k_2),\\
&c_1 = (1+\gamma)\delta a_0^2 - ak_1 - bk_1^2, c_2 = (1+\gamma)\delta a_0^2 + ak_2 - bk_2^2.
\end{aligned}
$$

Thus, the new exact solutions of the CFNLS can be identified as

$$
u = \left(a_0 - \frac{2a_0}{1+da^{\pm\frac{a_0}{\ln a}\sqrt{\frac{-2(1+\gamma)\delta}{b}}\left[\frac{1}{\Gamma(\beta+1)}x^\beta - \frac{b(k_1+k_2)}{\Gamma(\alpha+1)}t^\alpha\right]}}\right)e^{i\left[\frac{k_1}{\Gamma(\beta+1)}x^\beta + \frac{(1+\gamma)\delta a_0^2 - ak_1 - bk_1^2}{\Gamma(\alpha+1)}t^\alpha\right]},
$$

$$
v = \left(\mp a_0 \pm \frac{2a_0}{1+da^{\pm\frac{a_0}{\ln a}\sqrt{\frac{-2(1+\gamma)\delta}{b}}\left[\frac{1}{\Gamma(\beta+1)}x^\beta - \frac{b(k_1+k_2)}{\Gamma(\alpha+1)}t^\alpha\right]}}\right)e^{i\left[\frac{k_2}{\Gamma(\beta+1)}x^\beta + \frac{(1+\gamma)\delta a_0^2 + ak_2 - bk_2^2}{\Gamma(\alpha+1)}t^\alpha\right]}.
$$

If we select the corresponding values of some parameters, some simulations of 3D plots, 2D plots and contour plots are given in Figures 1 and 2.

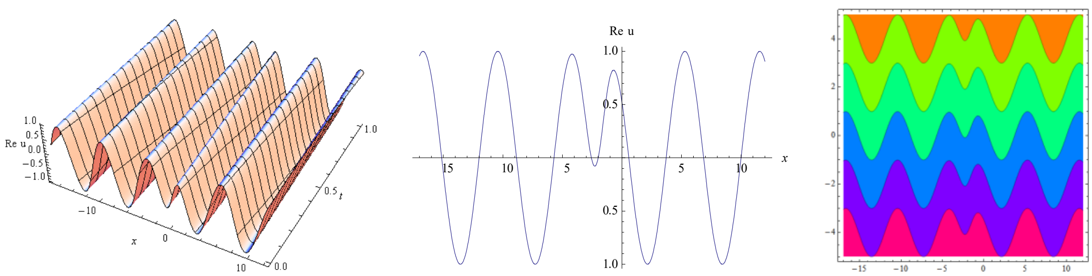

Figure 1. The 3D plot, 2D plot and contour plot of u with $a = 2, a_0 = d = \gamma = \delta = 1$, $b = -1, k_1 = k_2 = 1, \alpha = \beta = 1$.

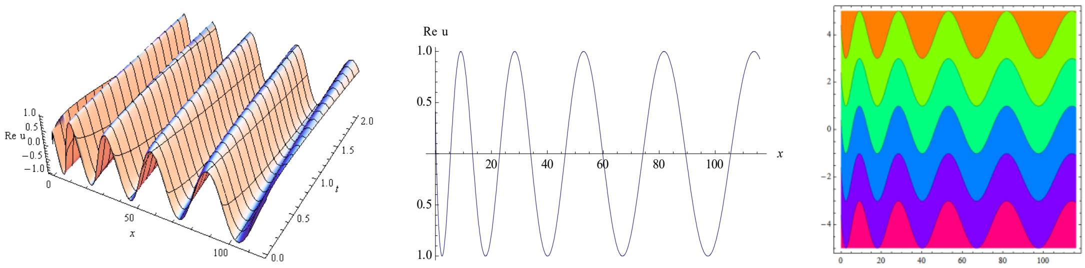

Figure 2. The 3D plot, 2D plot and contour plot of u with $a=2, a_0=d=\gamma=\delta=1$, $b=-1, k_1=k_2=1, \alpha=0.5, \beta=0.7$.

3.2. Approximate solutions

Let us consider Eq (1) with the initial condition $u_0=e^{ix}, v_0=e^{ix}$. By applying the aforesaid method in the first iteration, we have:

$$u_1=\mathcal{L}^{-1}[\frac{i}{s^{\alpha}}\mathcal{L}(iae^{i\left(x+\frac{\pi}{2}\beta\right)}+be^{i(x+\pi\beta)}+\delta e^{ix}(1+\gamma))]=\mathcal{L}^{-1}[\frac{ie^{ix}}{s^{\alpha+1}}(iae^{i\frac{\pi}{2}\beta}+be^{i\pi\beta}+\delta(1+\gamma))]$$
$$=(iae^{i\frac{\pi}{2}\beta}+be^{i\pi\beta}+\delta(1+\gamma))\frac{ie^{ix}t^{\alpha}}{\Gamma(\alpha+1)},$$

$$v_1=\mathcal{L}^{-1}[\frac{i}{S^{\alpha}}\mathcal{L}(-iav_{0x}^{\beta}+bv_{0xx}^{2\beta}+\delta A_0^{''}+\delta\gamma A_0^{'''})]=\mathcal{L}^{-1}[\frac{ie^{ix}}{s^{\alpha+1}}(-iae^{i\frac{\pi}{2}\beta}+be^{i\pi\beta}+\delta(1+\gamma))]$$
$$=(-iae^{i\frac{\pi}{2}\beta}+be^{i\pi\beta}+\delta(1+\gamma))\frac{ie^{ix}t^{\alpha}}{\Gamma(\alpha+1)}.$$

Further,

$$A_1=\frac{ie^{ix}t^{\alpha}}{\Gamma(\alpha+1)}(iae^{-i\frac{\pi}{2}\beta}-be^{-i\pi\beta}-\delta(1+\gamma))+2(iae^{i\frac{\pi}{2}\beta}+be^{i\pi\beta}+\delta(1+\gamma))\frac{ie^{ix}t^{\alpha}}{\Gamma(\alpha+1)},$$

$$A_1^{'}=[2be^{i\pi\beta}+\delta(1+\gamma)-iae^{-i\frac{\pi}{2}\beta}-be^{-i\pi\beta}]\frac{ie^{ix}t^{\alpha}}{\Gamma(\alpha+1)},$$

$$A_1^{''}=[2be^{i\pi\beta}+\delta(1+\gamma)+iae^{-i\frac{\pi}{2}\beta}-be^{-i\pi\beta}]\frac{ie^{ix}t^{\alpha}}{\Gamma(\alpha+1)},$$

$$A_1^{'''}=[2(-iae^{i\frac{\pi}{2}\beta}+be^{i\pi\beta}+\delta(1+\gamma))-(iae^{-i\frac{\pi}{2}\beta}+be^{-i\pi\beta}+\delta(1+\gamma))]\frac{ie^{ix}t^{\alpha}}{\Gamma(\alpha+1)}.$$

Thus,

$$u_2=\mathcal{L}^{-1}[\frac{i}{s^{\alpha}}\mathcal{L}(iau_{1x}^{\beta}+bu_{1xx}^{2\beta}+\delta A_1+\delta\gamma A_1^{'})]$$
$$=\frac{e^{ix}(it^{\alpha})^2}{\Gamma(2\alpha+1)}[b^2e^{2i\pi\beta}+2iabe^{\frac{3i\pi\beta}{2}}+e^{i\pi\beta}(-a^2+3b(1+\gamma)\delta)+e^{\frac{i\pi\beta}{2}}(ia(3+\gamma)\delta)$$
$$+e^{-\frac{1}{2}i\pi\beta}(-ia(-1+\gamma)\delta)+e^{-i\pi\beta}(-b(1+\gamma)\delta)+(1+\gamma)^2\delta^2]$$

$$v_2=\mathcal{L}^{-1}[\frac{i}{s^{\alpha}}\mathcal{L}(-iav_{1x}^{\beta}+bv_{1xx}^{2\beta}+\delta A_1^{''}+\delta\gamma A_1^{'''})]$$
$$=\frac{e^{ix}(it^{\alpha})^2}{\Gamma(2\alpha+1)}[b^2e^{2i\pi\beta}-2iabe^{\frac{3i\pi\beta}{2}}+e^{i\pi\beta}(-a^2+3b(1+\gamma)\delta)+e^{\frac{i\pi\beta}{2}}(-ia(1+3\gamma)\delta)$$
$$+e^{-\frac{1}{2}i\pi\beta}(-ia(-1+\gamma)\delta)+e^{-i\pi\beta}(-b(1+\gamma)\delta)+(1+\gamma)^2\delta^2].$$

With the same process, after substituting $u_0,\overline{u_0},v_0,\overline{v_0},u_1,\overline{u_1},v_1,\overline{v_1},u_2,\overline{u_2},v_2,\overline{v_2}$ into $A_2,A_2^{'},\ A_2^{''},A_2^{'''}$ and using the iterative equations (14) and (15), we can obtain

$$u_3=\mathcal{L}^{-1}[\frac{i}{s^{\alpha}}\mathcal{L}(iau_{2x}^{\beta}+bu_{2xx}^{2\beta}+\delta A_2+\delta\gamma A_2^{'})]$$
$$=\frac{e^{ix}t^{3\alpha}}{\Gamma(1+3\alpha)}\{-ib^3e^{3i\pi\beta}+3ab^2e^{\frac{5i\pi\beta}{2}}-ib[5b(1+\gamma)\delta-3a^2]e^{2i\pi\beta}$$
$$-a[a^2-2b(5+2\gamma)\delta]e^{\frac{3i\pi\beta}{2}}-i\delta[6b(1+\gamma)^2\delta-a^2(5+3\gamma)]e^{i\pi\beta}$$
$$+a\delta[b(1-\gamma)+6(1+\gamma)\delta]e^{\frac{i\pi\beta}{2}}-a\delta(1+\gamma)(b+\delta-\gamma\delta)e^{-\frac{1}{2}i\pi\beta}$$
$$-i\delta(1+\gamma)[-a^2+b(1+\gamma)\delta]e^{-i\pi\beta}+2ab\delta(\gamma-1)e^{-\frac{3}{2}i\pi\beta}-ib^2\delta(1+\gamma)e^{-2i\pi\beta}$$
$$-i\delta[a^2(\gamma-1)+(1+\gamma)(3(1+\gamma)^2\delta^2-b^2)]\}+\frac{\Gamma(1+2\alpha)e^{ix}t^{3\alpha}}{\Gamma(1+\alpha)^2\Gamma(1+3\alpha)}\{2ab\delta e^{\frac{3i\pi\beta}{2}}$$
$$-ib^2\delta(1+\gamma)e^{2i\pi\beta}-ia^2(\gamma-1)\delta e^{i\pi\beta}-2ab(\gamma-1)\delta e^{\frac{i\pi\beta}{2}}-2a\delta(b+(\gamma^2-1)\delta)e^{-\frac{1}{2}i\pi\beta}$$
$$+2ib(1+\gamma)^2\delta^2e^{-i\pi\beta}+i\delta[2a^2+(1+\gamma)(2b^2+(1+\gamma)^2\delta^2)]\},$$

$$
\begin{aligned}
v_3 &= \mathcal{L}^{-1}[\frac{i}{s^{\alpha}}\mathcal{L}(-iav_{2x}^{\beta}+bv_{2xx}^{2\beta}+\delta A_2^{''}+\delta\gamma A_2^{'''})]\\
&=\frac{e^{ix}t^{3\alpha}}{\Gamma(1+3\alpha)}\{-ib^3e^{3i\pi\beta}-3ab^2e^{\frac{5i\pi\beta}{2}}+ib[3a^2-5b(1+\gamma)\delta]e^{2i\pi\beta}\\
&+a[a^2-2b\delta(2+5\gamma)]e^{\frac{3i\pi\beta}{2}}+i\delta[a^2(3+5\gamma)-6b\delta(1+\gamma)^2]e^{i\pi\beta}\\
&+a\delta[b-b\gamma-6\gamma\delta(1+\gamma)]e^{\frac{i\pi\beta}{2}}+a\delta(1+\gamma)[b+\delta(\gamma-1)]e^{-\frac{1}{2}i\pi\beta}\\
&+i\delta(1+\gamma)[a^2-b(1+\gamma)\delta]e^{-i\pi\beta}+2ab\delta(\gamma-1)e^{-\frac{3}{2}i\pi\beta}-ib^2\delta(1+\gamma)e^{-2i\pi\beta}\\
&+i\delta[a^2(\gamma-1)+(1+\gamma)[b^2-3(1+\gamma)^2\delta^2]]\}+\frac{\Gamma(1+2\alpha)e^{ix}t^{3\alpha}}{\Gamma(1+\alpha)^2\Gamma(1+3\alpha)}\{-ib^2\delta(1+\gamma)e^{2i\pi\beta}\\
&-2ab\gamma\delta e^{\frac{3i\pi\beta}{2}}+ia^2\delta(\gamma-1)e^{i\pi\beta}-2ab\delta(\gamma-1)e^{\frac{i\pi\beta}{2}}+2a\delta(b\gamma+\delta-\gamma^2\delta)e^{-\frac{1}{2}i\pi\beta}\\
&+2ib\delta^2(1+\gamma)^2e^{-i\pi\beta}+i\delta[2a^2\gamma+2b^2(1+\gamma)+(1+\gamma)^3\delta^2]\},
\end{aligned}
$$

therefore, the Laplace-HPM series approximate solution for Eq (1) is

$$u_{appr}=u_0+u_1+u_2+u_3\cdots,v_{appr}=v_0+v_1+v_2+v_3\cdots.$$

4. Convergence and numerical results

To study the convergence of the Laplace-HPM method, let us state the following theorem.

Theorem. (Sufficient condition of convergence).

Suppose that X and Y are Banach spaces, and $N:X\to Y$ is a contract nonlinear mapping, that is,

$$\forall u,u^*\in X:\|N(u)-N(u^*)\|\le\gamma\|u-u^*\|,0<\gamma<1. \tag{22}$$

Then, according to Banach's fixed point theorem, N has a unique fixed point u, that is, $N(u)=u$. Assume that the sequence generated by the homotopy perturbation method can be written as

$$U_n=N(U_{n-1}),U_n=\sum_{i=0}^{n}u_i,u_i\in X,n=1,2,3,\cdots, \tag{23}$$

and suppose that

$$U_0=u_0\in B_r(u),B_r(u)=\{u^*\in X\|\|u^*-u\|<\gamma\}. \tag{24}$$

Then, we have

$$\text{(i) } U_n\in B_r(u),\quad \text{(ii) } \lim_{n\to\infty}U_n=u. \tag{25}$$

Proof. (i) By the inductive approach, for $n=1$ we have

$$\|U_1-u\|=\|N(U_0)-N(u)\|\le\gamma\|U_0-u\|,$$

and then

$$\|U_n-u\|=\|N(U_{n-1})-N(u)\|\le\gamma^n\|U_0-u\|\le\gamma^n r\Rightarrow U_n\in B_r(u).$$

(ii) Because of $0<\gamma<1$, we have $\lim_{n\to\infty}\|U_n-u\|=0$, that is, $\lim_{n\to\infty}U_n=u$.

Now, we investigate the error analysis between exact and approximate solutions, as stated by Tables 1 and 2, indicating that the series solution quickly converges to exact solution. The absolute errors at various values of t and x demonstrate the simplicity and great accuracy of the Laplace-HPM. The numerical results for u_{exact} and $u_{(1)},u_{(2)},u_{(3)}$ at $m=\delta=k_1=a=b=1$, $\gamma=-1,\alpha=\beta=1$ are shown in Figures 3 and 4, and we can find that the real part of $u_{(1)},u_{(2)},u_{(3)}$ converge to u_{exact} and increases very rapidly with increases in t, while the imaginary part is opposite. The comparison of real part and modulus between the exact solution v and approximate solution $v_{(1)},v_{(2)},v_{(3)}$ at $n=\delta=k_2=a=b=1,\gamma=1,\alpha=\beta=1$ and $\alpha=\beta=0.9$ are simulated through Figures 5–8, which clearly show that the values of real part decreases as t increases, and $v_{(3)}$ is closer to v than $v_{(1)}$ and $v_{(2)}$. These approximate values give excellent agreement with exact solutions, and the values of absolute errors are few. We also note that when the time is small, the accuracy of obtained solution increase, and the absolute errors decrease. This means that our equation highly relies on instant time. From the results obtained and presented in figures and tables, we can prove the efficiency of proposed method.

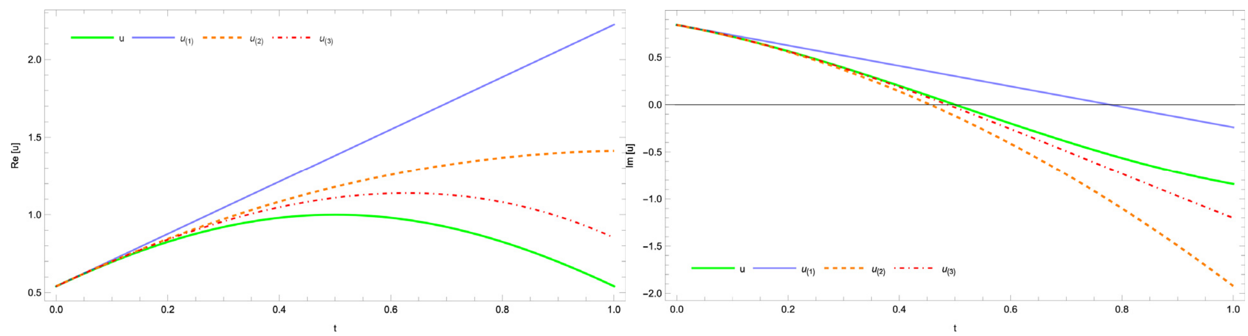

Figure 3. Comparison of real part and imaginary part between the exact solution u and approximate solution $u_{(1)},u_{(2)},u_{(3)}$ at $m=\delta=k_1=a=b=1,\gamma=-1,\ \alpha=\beta=1$.

Table 1. Comparison of exact solution u and approximate solution obtained by Laplace-HPM with $\alpha = \beta = 1$.

x	t	$\mathrm{Re}[u_{exact}]$	$\mathrm{Re}[u_{(3)}]$	$\mathrm{Re}[u_{(2)}]$	$\mathrm{Re}[u_{(1)}]$	$\lvert \mathrm{Re}[u_{exact}] - \mathrm{Re}[u_{(3)}] \rvert$
1	0.1	0.69671	0.69993	0.70049	0.70860	0.0032243
	0.2	0.82534	0.83998	0.84447	0.87689	0.0146490
	0.3	0.92106	0.95710	0.97224	1.04518	0.0360370
	0.4	0.98007	1.04790	1.08381	1.21348	0.0678370
	0.5	1.00000	1.10904	1.17916	1.38180	0.1090370
2	0.1	-0.22720	-0.22865	-0.22805	-0.23429	0.0014493
	0.2	-0.02920	-0.03231	-0.02746	-0.05243	0.0031090
	0.3	0.16997	0.16924	0.18561	0.12943	0.0007230
	0.4	0.36236	0.37237	0.41117	0.31129	0.0100120
	0.5	0.54030	0.57343	0.64921	0.49320	0.0331290
x	t	$\mathrm{Im}[u_{exact}]$	$\mathrm{Im}[u_{(3)}]$	$\mathrm{Im}[u_{(2)}]$	$\mathrm{Im}[u_{(1)}]$	$\lvert \mathrm{Im}[u_{exact}] - \mathrm{Im}[u_{(3)}] \rvert$
1	0.1	0.71736	0.72115	0.72079	0.73341	0.0037926
	0.2	0.56464	0.57774	0.57486	0.62535	0.0131010
	0.3	0.38942	0.41342	0.40369	0.51729	0.0239980
	0.4	0.19867	0.23033	0.20728	0.40923	0.0316600
	0.5	0.00000	0.03064	-0.01438	0.30120	0.0306420
2	0.1	0.97385	0.97861	0.97889	0.99253	0.0047623
	0.2	0.99957	1.01898	1.02120	1.07576	0.0194050
	0.3	0.98545	1.02874	1.03623	1.15899	0.0432900
	0.4	0.93204	1.00623	1.02398	1.24221	0.0741890
	0.5	0.84147	0.94978	0.98446	1.32540	0.1083080

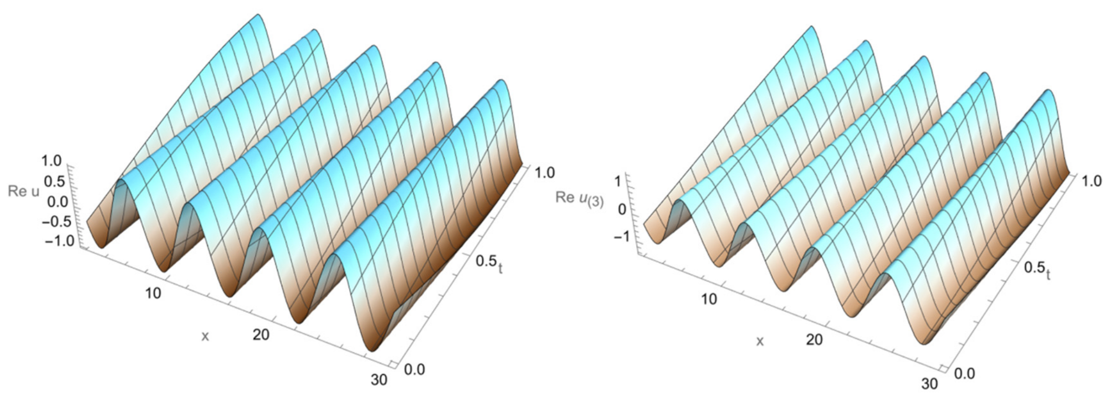

Figure 4. The 3D plot of the real part of the exact solution u and the approximate solution $u_{(3)}$ at $m = \delta = k_1 = a = b = 1, \gamma = -1, \alpha = \beta = 1$.

Table 2. Comparison of exact solution v and approximate solution obtained by Laplace-HPM with $\alpha=\beta=1$.

x	t	$\mathrm{Re}[v_{exact}]$	$\mathrm{Re}[v_{(3)}]$	$\mathrm{Re}[v_{(2)}]$	$\mathrm{Re}[v_{(1)}]$	$\lvert \mathrm{Re}[v_{exact}]-\mathrm{Re}[v_{(3)}]\rvert$
	0.1	0.36236	0.36289	0.36120	0.37201	0.0005272
	0.2	0.16997	0.17395	0.16049	0.20371	0.0039861
1	0.3	-0.02920	-0.01640	-0.06183	0.03542	0.0128040
	0.4	-0.22720	-0.19806	-0.30577	-0.13287	0.0291390
	0.5	-0.41615	-0.36095	-0.57132	-0.30120	0.0551950
	0.1	-0.58850	-0.58786	-0.58968	-0.59801	0.0006363
	0.2	-0.73739	-0.73203	-0.74657	-0.77987	0.0053684
2	0.3	-0.85689	-0.83772	-0.88682	-0.96173	0.0191720
	0.4	-0.94222	-0.89403	-1.01042	-1.14358	0.0481950
	0.5	-0.98999	-0.89005	-1.11737	-1.32540	0.0999460
x	t	$Abs[v_{exact}]$	$Abs[v_{(3)}]$	$Abs[v_{(2)}]$	$Abs[v_{(1)}]$	$\lvert Abs[v_{exact}]-Abs[v_{(3)}]\rvert$
	0.10	1.00000	0.999802	1.000200	1.019804	0.0001978
	0.15	1.00000	0.999010	1.001012	1.044031	0.0009899
	0.20	1.00000	0.996923	1.003195	1.077033	0.0030765
	0.25	1.00000	0.992649	1.007782	1.118034	0.0073508
1	0.30	1.00000	0.985148	1.016071	1.166190	0.0148510
	0.35	1.00000	0.973308	1.029575	1.220656	0.0266910
	0.40	1.00000	0.956025	1.049952	1.280625	0.0439748
	0.45	1.00000	0.932304	1.078900	1.345360	0.0676955
	0.50	1.00000	0.901388	1.118030	1.414210	0.0986117

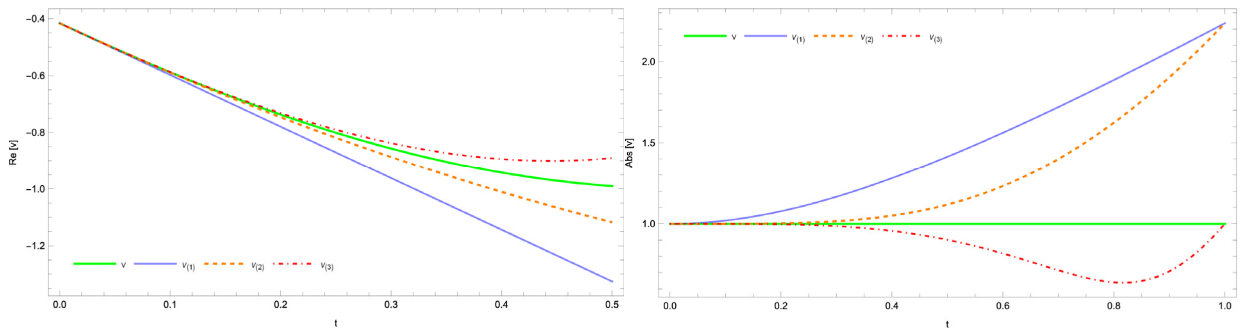

Figure 5. The comparison of real part and modulus between the exact solution v and approximate solution $v_{(1)}, v_{(2)}, v_{(3)}$ at $n=\delta=k_2=a=b=1, \gamma=1, \alpha=\beta=1$.

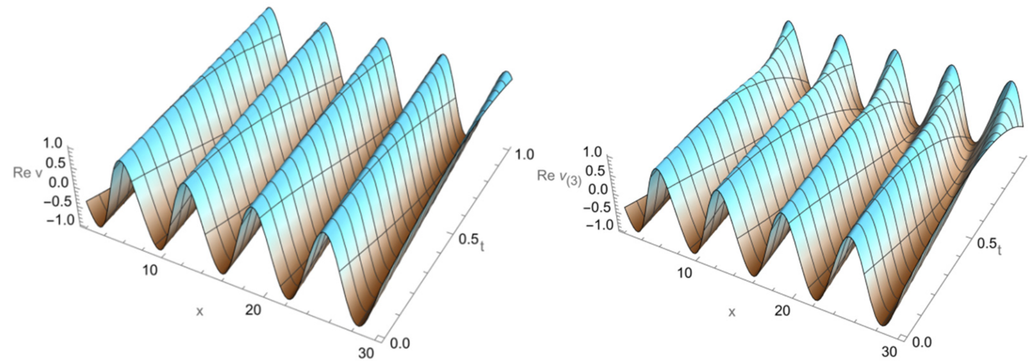

Figure 6. The 3D plot of the real part of the exact solution v and the approximate solution $v_{(3)}$ at $n=\delta=k_2=a=b=1, \gamma=1, \alpha=\beta=1$.

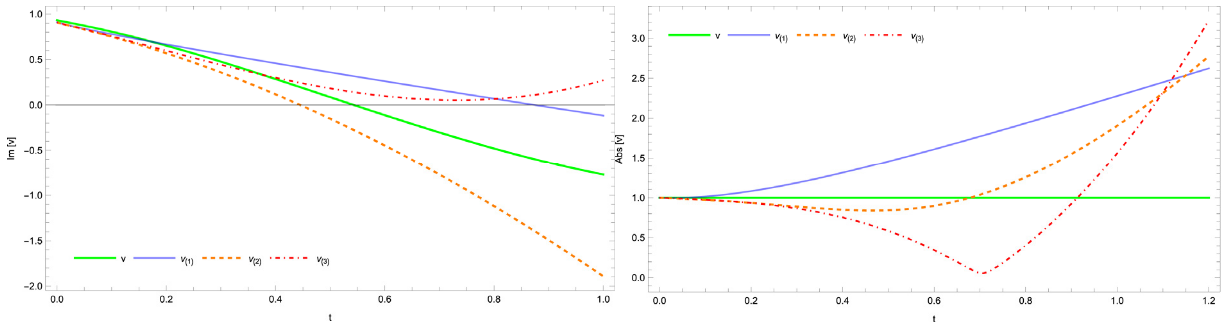

Figure 7. The comparison of imaginary part and modulus between the exact solution v and approximate solution $v_{(1)}, v_{(2)}, v_{(3)}$ at $n=\delta=k_2=a=b=1, \gamma=1, \alpha=\beta=0.9$.

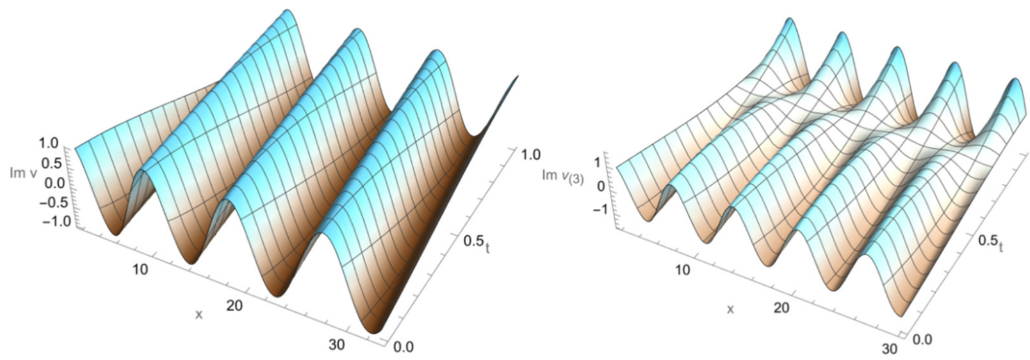

Figure 8. The 3D plot of the imaginary part of the exact solution v and the approximate solution $v_{(3)}$ at $n=\delta=k_2=a=b=1, \gamma=1, \alpha=\beta=0.9$.

5. Discussion and conclusions

In this research, the modified Kudryashov method and Laplace transform method combined with homotopy perturbation have been successfully applied to solve space-time coupled fractional nonlinear Schrödinger equation (FCNLS) Eq (1), and some propagation behavior of these obtained solutions are simulated. The graphs are important for revealing the internal structure of Eq (1). For example, Figures 1 and 2 show that the waveform of $\mathrm{Re}\,u$ produced a jitter between the intervals $(-4,0)$ in the case of $\alpha=\beta=1$ while it presents periodic in the right half plane if we selected $\alpha=0.5,\beta=0.7$. The comparison diagram and error analysis diagram of approximate solutions are given to study the accuracy of the approximate solution. These results show that Laplace homotopy perturbation method is an effective and reliable method, and a more accurate approximate solution can be obtained through a few iterations. This paper will open up a flood of opportunities for solving fractional differential equations such as KP equation, Ginzburg-Landau equation, KdV-Burgers equation, etc. The novel Laplace-HPM is extremely promising and will be useful for fractional differential equations. The current definition of the Caputo fractional derivative still has great limitations, and it is difficult to characterize the necessary connection between two real number order derivatives. On the other hand, the unified definition of fractional derivatives needs to be further explored and developed for use in the future.

Acknowledgments

This work is supported by the practical innovation training program projects for the university students of Jiangsu Province (Grant No. 202211276054Y), Natural science research projects of Institutions in Jiangsu Province (Grant No. 18KJB110013).

Conflicts of interest

The authors declare no conflicts of interest.

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