Mathematics

http://www.aimspress.com/journal/Math

AIMS Mathematics, 8(7): 15536–15552.
DOI: 10.3934/math.2023792
Received: 26 September 2022
Revised: 16 February 2023
Accepted: 10 March 2023
Published: 26 April 2023

Research article

Distributed adaptive event-triggered control for general linear singular multi-agent systems

Yuming Chen, Jie Gao* **and Luan Teng**

The School of Science, Southwest Petroleum University, Chengdu 610500, China

* **Correspondence:** Email: gaojiejr@foxmail.com.

Abstract: This paper investigates the leader-following consensus of general linear singular multi-agent systems. A fully distributed adaptive event-triggered control protocol is first proposed by using the relative state estimate information between neighboring agents. Moreover, the proposed protocol does not require continuous communication between neighbors, which greatly alleviate the negative impact of communication. In addition, a novel adaptive gain is proposed, which will avoid using the Laplace of communication graph. Zeno behavior is excluded by proving that the inter-event times are lower bounded by a positive constant. Finally, a numerical simulation is proposed to verify the effectiveness and reliability of the protocol.

Keywords: adaptive event-triggered control; fully distributed control; leader-following consensus; singular multi-agent systems
Mathematics Subject Classification: 37F99, 39A05, 39A06, 70E55, 70E60

1. Introduction

The consensus control is one of the most basic distributed coordination control problems in multi-agent networks. In the past decade, the study of consensus in multi-agent systems has attracted considerable attention from a great many scholars. It is mainly due to the potential applications of multi-agent systems in various fields, such as physics, biology, control theory, and engineering, etc. [1–4]. The leaderless consensus problem and the leader-following consensus problem are the research basis of consensus. Their purpose is to design a suitable controller so that a group of agents reach agreement on a quantity of particular interest [5]. In practical applications, the distributed control protocol can greatly reduce the cost of information transmission and centralized control because the controller designed for each agent only relates to available information of its neighbors. In recent decades, some remarkable achievements have been made based on distributed consensus control for different systems, such as first-order/second-order continuous multi-agent system [6–8], general linear continuous multi-agent system [9–11], discrete-time multi-agent system [12–14], heterogeneous linear multi-agent system [15, 16], and so on. It is worth noting that there are few results of singular multi-agent systems in the existing literature.

In general, singular multi-agent systems can be decomposed into the exponential mode part (i.e., the normal dynamics part) and the impulsive mode part (i.e., the specific part of the descriptor dynamics) [17]. Singular multi-agent systems are more felicitous to describe the complicated dynamics phenomenon than normal systems and have been extensively used to describe power systems, robot systems, economic systems and so on. As more and more scholars study singular systems, many excellent conclusions are drawn in this respect [18–21]. For example, the authors in [18] investigated the containment control problem of singular heterogeneous multi-agent systems by state feedback and output feedback control protocols. The containment control of singular heterogeneous multi-agent systems is studied under directed interaction topology in [19]. The bipartite output regulation problem for singular heterogeneous multi-agent systems with signed graph is considered by state feedback and output feedback control protocols in [21]. For singular multi-agent systems, it is worth noting that the existing distributed control strategies need to use the global information of communication topology, that is, the eigenvalue of the Laplace matrix L. In practical applications, it may be difficult to obtain for large-scale networks. Therefore, there are still many challenges to solve the consensus problem of the general liner singular multi-agent systems.

Meanwhile, too many communication resources will be used to update controllers and exchange information between neighbors. Therefore, we introduce an event-based mechanism to achieve the consensus of the singular multi-agent systems. However, there is no research on event-based distributed control strategy for singular systems in the existing literature, and most of the research is for general liner non-singular multi-agent systems [22–26]. Two distributed filters are introduced to solve the problem of distributed target tracking under cyber attacks for targets with the nonlinear dynamics [22]. In [24], the authors discussed the event-triggered distributed optimization problem of second-order nonlinear multi-agent systems under undirected and connected communication topologies. The authors in [25] put forward a dynamic event-triggered control law and discussed its application in smart grids and intelligent transportation systems in detail. Based on the above, the distributed event-triggered control strategies in those literatures all need the global information of the communication topology and thereby are not fully distributed.

Therefore, inspired by the above papers, we are interested in the leader-following consensus of singular multi-agent systems based on fully distributed adaptive event-triggered control law which is a challenging problem and needs further research. The main contributions of this paper are highlighted as follows. First, a novel fully distributed adaptive event-triggered controller is proposed to solve the leaderless consensus problem of singular multi-agent systems, which reduce the number of information transmission and the load and cost of the communication. Meanwhile, the proposed controller avoids using global information by introducing an adaptive gain. Second, we further extend it to solve the problem of leader-following consensus for general liner singular multi-agent systems. Third, it is proved that the inter-event times are lower bounded by a positive constant to exclude the Zeno behavior.

The rest of this paper is organized as follows. Section 2 presents some preliminary information as well as the problem formulation. Then, we propose two novel fully distributed event-triggered strategies for the leaderless consensus and leader-following consensus and present the corresponding results in Sections 3 and Section 4, respectively. Section 5 includes simulations that validate the effectiveness and feasibility of the designed control law. Section 6 contains the conclusions and the further study.

Notations: $\mathbb{R}^m$ and $\mathbb{R}^{n\times m}$ denote the set of m-dimensional real column vectors and $n\times m$ real matrices, respectively. Any matrix $A>0$ means A is positive definite. $\otimes$ denotes the Kronecker product. $\|\cdot\|$ denote Euclidean vector norm or induced 2-norm for matrix. For any matrix B, $\lambda_{min}(B)$ and $\lambda_2(B)$ refer to the minimum and minimum nonzero eigenvalues, respectively.

2. Problem formulation and preliminaries

2.1. Graph theory

The communication graph $\mathcal{G}$ among the N agents is presented by a fixed undirected graph $\mathcal{G} = (\mathcal{V}, \varepsilon, \mathcal{A})$ with a finite set of N nodes $\mathcal{V} = \{v_1, v_2, ..., v_N\}$, a set of edges $\varepsilon \subset \mathcal{V} \times \mathcal{V}$, and the adjacency matrix $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{N\times N}$. For $i, j \in \mathcal{V}$, where $a_{ij} > 0$, if $(i, j) \in \varepsilon$; otherwise, $a_{ij} = 0$. Define the in-degree matrix as $\mathcal{D} = diag(d_i) \in \mathbb{R}^{N\times N}$ with $d_i = \sum_{j=1}^{N} a_{ij}$. The Laplacian matrix is defined as $\mathcal{L} = \mathcal{D} - \mathcal{A}$.

2.2. Problem formulation

In the subsection, a group of N identical agents with continuous-time general linear singular dynamics are considered. The dynamics of the ith agent is given by

$$E\dot{x}_i(t) = Ax_i(t) + Bu_i(t), \quad i \in \{1, 2, ..., N\}, \tag{2.1}$$

where $x_i(t) \in \mathbb{R}^n$ and $u_i(t) \in \mathbb{R}^m$ are the state information and the control input of the ith agent, respectively. $E, A \in \mathbb{R}^{n\times n}$ and $B \in \mathbb{R}^{n\times m}$ are constant matrices with appropriate dimensions. The systems (2.1) is said to be singular, if $rank(E)<n$.

The main purpose of this article is to solve the adaptive event-triggered consensus problem for the singular agents in the systems (2.1). The critical of the adaptive event-triggered consensus problem is to design a fully distributed event-based consensus protocol that consist of the event-based control laws and the triggering functions for each agent. Specifically, the goal is to solve the adaptive event-triggered leaderless consensus problem for the singular agents in the systems (2.1) by ensuring that $\lim_{t\to+\infty} \|x_i(t) - x_j(t)\| = 0$, where $i, j \in \{1, 2, ..., N\}$ and excluding Zeno behavior, i.e., the event-triggered number in a limited time is finite.

3. Fully distributed adaptive event-triggered control law for leaderless consensus

In the section, we will design fully distributed adaptive event-triggered laws for leaderless consensus of the singular multi-agent systems. The following assumptions and lemmas are needed.

Assumption 3.1. *(E, A) is regular and (E, A, B) is stabilizable.*

Remark 3.1. *Assumption 3.1 ensures that the adaptive event-triggered consensus problem is well posed. For the regularity of (E, A), it makes sure the existence and uniqueness of the solution of the system equations (2.1) of the agents. For the stabilizable of (E, A, B), it guarantees the existence of the distributed control law.*

Assumption 3.2. *The undirected graph $\mathcal{G}$ is connected, i.e., there exists a root agent that has directed paths to all other agents.*

Remark 3.2. *Similar to Assumption 3.1, Assumption 3.2 also guarantees that the adaptive event-triggered consensus problem is well posed. Without this assumption, at least one isolated agent can't get any information from other agents.*

Lemma 3.1. *[18] For $E, A \in \mathbb{R}^{n\times n}$, and $B \in \mathbb{R}^{n\times m}$, suppose that (E, A) is regular and impulse free, and (E, A, B) is R-controllable. Then, for any given positive definite matrices $Q > 0$, there exists a positive define matrix $P > 0$ which is the solution of the following equality:*

$$A^T PE + E^T PA - E^T PBB^T PE + E^T QE = 0. \tag{3.1}$$

Lemma 3.2. *(Cauchys covergence criterion) [27] The sequence $V(t_i)$, $i = 0, 1, 2, ...$ converges to something if and only if this holds: For any $\phi > 0$, there exists a positive number $\mathcal{W}_\phi$ such that $\forall s > \mathcal{W}_\phi$,*

$$|V(t_{s+1}) - V(t_s)| < \phi \quad or \quad |\int_{t_s}^{t_{s+1}} \dot{V}(t)\, dt| < \phi. \tag{3.2}$$

Compared with the traditional controller, we consider the following fully distributed adaptive event-triggered controller for the ith agent:

$$\begin{cases} u_i(t) &= c_i(t)K \sum_{j=1}^{N} a_{ij}(\tilde{x}_i(t) - \tilde{x}_j(t)), \\ \dot{c}_i(t) &= \frac{(1-\alpha)}{(1+\alpha)} \|K \sum_{j=1}^{N} a_{ij}(\tilde{x}_i(t) - \tilde{x}_j(t))\|^2, \end{cases} \tag{3.3}$$

where $\alpha \in (0, 1)$, the gain matrix K is to be designed; $\tilde{x}_i(t) \in \mathbb{R}^n$, which are described by (3.4), is the state estimate value of $x_i(t)$.

$$\begin{cases} E\dot{\tilde{x}}_i(t) = A\tilde{x}_i(t), & t \in [t_k^i, t_{k+1}^i), \\ \tilde{x}_i(t) = x_i(t), & t = t_k^i, \end{cases} \tag{3.4}$$

where the sampling instant t_k^i means the kth triggering time instant of the ith agent.

Remark 3.3. *The distributed control means that the ith agent only communicates with its neighbor agents, instead of communicating with all agents in (3.3). The event-triggered control laws of [28–31] require absolute real state information, namely $x_i(t_k^i)$ and $x_j(t_{k'}^j)$. Especially, the adaptive gains $c_{ij}(t)$ in [28] are required to satisfy $c_{ij}(t) = c_{ji}(t)$ for $\forall t \geq 0$. It undoubtedly complicates the triggering mechanism, raises the computing cost, and increases the communication load, implying that it is not feasible in some practical applications. However, the estimate value $\tilde{x}_i(t)$ of $x_i(t)$ only requires real state information at discrete event-triggered time instants, which greatly makes up these shortcomings. Meanwhile, in [32–35], the adaptive event-triggered consensus protocols require the global eigenvalue information of the communication graph $\mathcal{G}$. In this paper, the gain $c_i(t)$ in (3.3) is adaptively adjusted which is based on sampling relative estimate value information to avoid using the global information of the Laplace matrix.*

We further propose the following triggering condition to determine the sampling instants:

$$t_{k+1}^i = inf\{t > t_k^i | f_i(t) \geq 0\}, \; i = 1, 2, ..., N, \tag{3.5}$$

$$f_i(t) = \alpha\|K\sum_{j=1}^{N} a_{ij}(z_i(t) - z_j(t))\|^2 - \|K\sum_{j=1}^{N} a_{ij}(e_i(t) - e_j(t))\|^2 + \frac{\mu}{c_i(t)}e^{-\nu t}, \tag{3.6}$$

where $\mu > 0$, $\nu > 0$, $e_i(t) = \tilde{x}_i(t) - x_i(t)$ the measurement error and the error $z_i(t) = x_i(t) - \frac{1}{N}\sum_{k=1}^{N} x_k(t)$ between the state of each agent and the average state of all states, respectively. Obviously, $\tilde{x}_i(t)-\tilde{x}_j(t) = e_i(t) - e_j(t) + (z_i(t) - z_j(t))$. Then, the time derivative of $e_i(t)$ can be written as

$$\begin{aligned} E\dot{e}_i(t) =& A\tilde{x}_i(t) - Ex_i(t) \\ =& Ae_i(t) - c_i(t)BK\sum_{j=1}^{N} a_{ij}(\tilde{x}_i(t) - \tilde{x}_j(t)) \\ =& Ae_i(t) - c_i(t)BK\sum_{j=1}^{N} a_{ij}(e_i(t) - e_j(t)) - c_i(t)BK\sum_{j=1}^{N} a_{ij}(z_i(t) - z_j(t)). \end{aligned} \tag{3.7}$$

Therefore, (3.7) can be rewritten as

$$diag(E)\dot{e}(t) = (I_N \otimes A)e(t) - (c(t)L \otimes BK)e(t) - (c(t)L \otimes BK)z(t), \tag{3.8}$$

where $e(t) = col(e_1(t), e_2(t), ..., e_N(t))$, $c(t) = diag\{c_1(t), c_2(t), ..., c_N(t)\}$ and $z(t) = col(z_1(t), z_2(t), ..., z_N(t))$. Let $\varepsilon_i(t) = Ex_i(t)$.

Then, the time derivative of $\varepsilon(t)$ can be written as

$$\dot{\varepsilon}(t) = (I_N \otimes A)x(t) + (c(t)L \otimes BK)e(t) + (c(t)L \otimes BK)z(t). \tag{3.9}$$

Definition 3.1. *To simplify the calculation, define* $\acute{e}_i(t) = \sum_{j=1}^{N} a_{ij}(e_i(t) - e_j(t))$, $\acute{\tilde{x}}_i(t) = \sum_{j=1}^{N} a_{ij}(\tilde{x}_i(t) - \tilde{x}_j(t))$, $\acute{z}_i(t) = \sum_{j=1}^{N} a_{ij}(z_i(t) - z_j(t))$, $M = I_N - \frac{1}{N}\mathbf{1}\mathbf{1}^{\mathbf{T}}$.

Therefore,

$$\begin{aligned} diag(E)\dot{z}(t) =& (M \otimes I_N)\dot{\varepsilon}(t) \\ =& (M \otimes A)x(t) + (Mc(t)L \otimes BK)e(t) + (Mc(t)L \otimes BK)z(t) \\ =& (I_N \otimes A)z(t) + (Mc(t)L \otimes BK)e(t) + (Mc(t)L \otimes BK)z(t). \end{aligned} \tag{3.10}$$

We are now ready to present the main results of this section.

Theorem 3.1. *Given Assumptions 3.1, 3.2 and Lemma 3.1, the leader-following multi-agent system (2.1) under the control protocol (3.3) with the dynamic event-triggered strategy i.e. described as (3.5) and (3.6), will achieve leaderless consensus if* $\alpha \in (0, 1)$ *and* $K = -B^T PE$.

Consider the Lyapunov-like function candidate

$$V_1(t) = (diag(E)z(t))^T(L \otimes P)diag(E)z(t) + \frac{1}{4}\sum_{i=1}^{N}(c_i(t) - c_0)^2, \tag{3.11}$$

where c_0 is a positive constant satisfying $c_0 \geq \frac{2(1+\alpha)(1+\kappa)}{\lambda_2(L)\kappa(1-\alpha)(1-\alpha-\kappa)}$ with $(0 < \alpha < 1, 0 < \kappa < \frac{1}{\alpha} - 1)$.

The time derivative of $V_1(t)$ along the trajectory of $z_i(t)$ in (3.1) with $K = -B^T PE$ is given by

$$\begin{aligned}\dot{V}_1(t) =& 2z^T(t)diag(E^T)(L \otimes P)diag(E)\dot{z}(t) + \frac{1-\alpha}{2(1+\alpha)}\sum_{i=1}^{N}(c_i(t) - c_0)\|K\acute{\tilde{x}}_i(t)\|^2 \\ =& 2z^T(t)diag(E^T)(L \otimes P)[(I_N \otimes A)z(t) + (Mc(t)L \otimes BK)e(t) + (Mc(t)L \otimes BK)z(t)] \\ &+ \frac{1-\alpha}{2(1+\alpha)}\sum_{i=1}^{N}(c_i(t) - c_0)\|K\acute{\tilde{x}}_i(t)\|^2.\end{aligned} \tag{3.12}$$

Obviously, $LM = L$. Then,

$$\begin{aligned}\dot{V}_1(t) \leq& 2z^T(t)(L \otimes E^T PA)z(t) - z^T(t)(L \otimes \Gamma)z(t) + e^T(t)(Lc(t)L \otimes \Gamma)e(t) \\ &+ \frac{1-\alpha}{2(1+\alpha)}\sum_{i=1}^{N}(c_i(t) - \bar{c}_0)\|K\acute{\tilde{x}}_i(t)\|^2.\end{aligned} \tag{3.13}$$

Moreover, let $\Gamma = E^T PBB^T PE$,

$$\begin{aligned}&\frac{1-\alpha}{2(1+\alpha)}\sum_{i=1}^{N} c_i(t)\|K\acute{\tilde{x}}_i(t)\|^2 \\ =& \frac{1-\alpha}{2(1+\alpha)}\sum_{i=1}^{N} c_i(t)\|K\acute{e}_i(t) + K\acute{z}_i(t)\|^2 \\ \leq& \frac{1-\alpha}{2(1+\alpha)}\sum_{i=1}^{N} c_i(t)(2\|K\acute{e}_i(t)\|^2 + 2\|K\acute{z}_i(t)\|^2) \\ =& \frac{1-\alpha}{1+\alpha}e^T(t)(Lc(t)L \otimes \Gamma)e(t) + \frac{1-\alpha}{1+\alpha}z^T(t)(Lc(t)L \otimes \Gamma)z(t).\end{aligned} \tag{3.14}$$

Then, by triggering condition (3.6), we can further imply

$$e^T(t)(Lc(t)L \otimes \Gamma)e(t) \leq \alpha z^T(t)(Lc(t)L \otimes \Gamma)z(t) + N\mu e^{-\nu t}. \tag{3.15}$$

Substituting (3.14) and (3.15) into (3.13) yields

$$\dot{V}_1(t) \leq 2z^T(t)(L \otimes E^T PA)z(t) + \frac{2N\mu}{1+\alpha}e^{-\nu t} - \frac{1-\alpha}{2(1+\alpha)}c_0\|K\acute{\tilde{x}}_i(t)\|^2. \tag{3.16}$$

It can be verified that

$$\begin{aligned}\|K\acute{z}_i(t)\|^2 =& \|K\acute{\tilde{x}}_i(t) - K\acute{e}_i(t)\|^2 \\ =& \|K\acute{\tilde{x}}_i(t)\|^2 + \|K\acute{e}_i(t)\|^2 - 2\acute{\tilde{x}}_i(t)^T\Gamma\acute{e}_i(t) \\ \leq& \|K\acute{\tilde{x}}_i(t)\|^2 + \|K\acute{e}_i(t)\|^2 + \frac{1}{\kappa}\|K\acute{\tilde{x}}_i(t)\|^2 + \kappa\|K\acute{e}_i(t)\|^2,\end{aligned} \tag{3.17}$$

where $0 < \kappa < 1/\alpha - 1$. According to (3.17) and the triggering condition (3.5), one has

$$\begin{aligned}-\|K\acute{\tilde{x}}_i(t)\|^2 \leq& \kappa\|K\acute{e}_i(t)\|^2 - \frac{\kappa}{1+\kappa}\|K\acute{z}_i(t)\|^2 \\ \leq& \kappa[\alpha\|K\acute{z}_i(t)\|^2 + \frac{\mu}{c_i(t)}e^{-\nu t}] - \frac{\kappa}{1+\kappa}\|K\acute{z}_i(t)\|^2 \\ \leq& \frac{\kappa\mu}{c_i(t)}e^{-\nu t} - \frac{\kappa(1-\alpha-\alpha\kappa)}{1+\kappa}\|K\acute{z}_i(t)\|^2.\end{aligned} \tag{3.18}$$

Thus, it further follows that

$$\begin{aligned}\dot{V}_1(t) \leq & 2z^T(t)(L\otimes E^TPA)z(t) + \frac{2N\mu}{1+\alpha}e^{-\nu t} + \frac{1-\alpha}{2(1+\alpha)}c_0[\frac{\kappa\mu}{c_i(t)}e^{-\nu t} - \frac{\kappa(1-\alpha-\alpha\kappa)}{1+\kappa}\|K\acute{z}_i(t)\|^2] \\ \leq & 2z^T(t)(L\otimes E^TPA)z(t) - c_0\varrho z^T(t)(L^2\otimes\Gamma)z(t) + \theta e^{-\nu t},\end{aligned} \tag{3.19}$$

where $\varrho = \frac{\kappa(1-\alpha)(1-\alpha-\alpha\kappa)}{2(1+\alpha)(1+\kappa)} > 0$ and $\theta = \frac{2N\mu}{1+\alpha} + \frac{c_0(1-\alpha)\kappa\mu}{2(1+\alpha)\min c_i(0)} > 0$.

Under Assumption 3.2, the Laplacian matrix L is symmetric and a M-matrix. Thus, there exists an orthogonal matrix U such that $U^TLU = \Lambda = diag\{\lambda_1, \lambda_2, ..., \lambda_N\}$, with $0 = \lambda_1 < \lambda_2 \leq \lambda_3 ... \leq \lambda_N$. Let $\bar{z}(t) = (U^T \otimes I_N)z(t) = col(\bar{z}_1(t), \bar{z}_2(t), ..., \bar{z}_N(t),)$ and $\hat{z}_i(t) = E\bar{z}_i(t)$. Then, one has

$$\begin{aligned}\dot{V}_1(t) \leq & \sum_{i=1}^{N}\lambda_i\bar{z}_i^T(t)[E^TPA + A^TPE - c_0\varrho\lambda_i\Gamma]\bar{z}_i(t) + \theta e^{-\nu t} \\ \leq & \sum_{i=1}^{N}\lambda_i\bar{z}_i^T(t)[E^TPA + A^TPE - \Gamma]\bar{z}_i(t) + \theta e^{-\nu t} \\ \leq & -\sum_{i=1}^{N}\lambda_i\bar{z}_i^T(t)E^TQE\bar{z}_i(t) + \theta e^{-\nu t} \\ = & -\sum_{i=1}^{N}\lambda_i\hat{z}_i^T(t)Q\hat{z}_i(t) + \theta e^{-\nu t} \\ \leq & -\lambda_2(L)\lambda_{min}(Q)\sum_{i=1}^{N}\hat{z}_i^T(t)\hat{z}_i(t) + \theta e^{-\nu t},\end{aligned} \tag{3.20}$$

where Q satisfies the equality (3.1) and $\lambda_{min}(Q)$ is the minimal eigenvalue of Q. Consider the following function:

$$W_1(t) = V_1(t) + \frac{1}{\nu}\theta e^{-\nu t}, \tag{3.21}$$

which is continuous on the interval $[0, +\infty)$. It follows from (3.20) that

$$\dot{W}_1(t) \leq -\lambda_2(L)\lambda_{min}(Q)\sum_{i=1}^{N}\hat{z}_i^T(t)\hat{z}_i(t). \tag{3.22}$$

Therefore, it can be seen from (3.22) that $W_1(t)$ is non-increasing on the interval $[0, +\infty)$. Because of $W_1(t) \geq 0$, $W_1(t)$ is bounded. It implies that $W_1(t)$ is bounded, i.e., $\lim_{t\to+\infty} W_1(t)$ exists. Thus, from the definition of $W_1(t)$, it can be seen that $z_i(t)$ and $c_i(t)$ are bounded over $[0, +\infty)$, for $i, j = 1, 2, ..., N$.

Since $\lim_{t\to+\infty} W_1(t)$ exists, by the Cauchy convergence criterion in Lemma 3.2, there exists a positive number T_0, such that for any $T_2 > T_1 > T_0$, $V_1(T_1) - V_1(T_2) < \varepsilon$. Suppose that the triggering time instants for all agents over (T_1, T_2) are given by $t_l^1 < t_l^2 < t_l^3 < ... < t_l^r$. Then, it follows from (3.22) that

$$
\begin{aligned}
&\lambda_2(L)\lambda_{min}(Q)\int_{T_1}^{T_2}\sum_{i=1}^{N}\hat{z}_i^T(s)\hat{z}_i(s)\,ds\\
=&\lambda_2(L)\lambda_{min}(Q)\times(\int_{T_1}^{t_l^1}\sum_{i=1}^{N}\hat{z}_i^T(s)\hat{z}_i(s)\,ds+\int_{t_l^1}^{t_l^2}\sum_{i=1}^{N}\hat{z}_i^T(s)\hat{z}_i(s)\,ds+...+\int_{t_l^r}^{T_2}\sum_{i=1}^{N}\hat{z}_i^T(t)\hat{z}_i(t)\,ds)\\
\leq&-(\int_{T_1}^{t_l^1}\dot{W}_1(s)\,ds+\int_{t_l^1}^{t_l^2}\dot{W}_1(s)\,ds+...+\int_{t_l^r}^{T_2}\dot{W}_1(s)\,ds)\\
=&(W_1(T_1)-W_1(T_2))<\varepsilon.
\end{aligned}
\tag{3.23}
$$

By the Cauchy criterion, we can conclude that $\lim_{t\to+\infty}\lambda_2(L)\lambda_{min}(Q)\int_0^t\sum_{i=1}^N\hat{z}_i^T(s)\hat{z}_i(s)\,ds$ exists, i.e., $\lim_{t\to+\infty}\int_0^t\hat{z}^T(s)\hat{z}(s)\,ds$ exists. Because of (3.9) and (3.10), $\int_0^t\hat{z}^T(s)\hat{z}(s)\,ds$ is twice differentiable on each interval $[t_k^i,t_{k+1}^i)$. Note that the boundedness of $\hat{z}(t)$ and $\dot{\hat{z}}(t)$ are bounded over the interval $[0,+\infty)$. Therefore, there exists a positive constant H, such that

$$\sup_{t\in[t_k^i,t_{k+1}^i)}|\hat{z}^T(t)\dot{\hat{z}}(t)|\leq H. \tag{3.24}$$

Then, by Lemma 1 in [36], i.e., the general Barbalat's lemma, $\lim_{t\to+\infty}\hat{z}^T(t)\hat{z}(t)=0$, which means that $\lim_{t\to+\infty}z^T(t)z(t)=0$.

The proof is thus completed.

The following theorem excludes the Zeno behavior.

Theorem 3.2. *Under the conditions in Theorem 3.1, the network does not exhibit the Zeno behavior and the interval between two consecutive triggering instants for any agent is strictly positive.*

Proof. For the ith agent at time interval $t\in[t_k^i,t_{k+1}^i)$, it follows from (3.8) that

$$
\begin{aligned}
\sum_{i=1}^{N}\frac{d\|K\acute{e}_i(t)\|^2}{dt}=&2(Ee(t))^T(LL\otimes PBB^TP)(E\dot{e}(t))\\
=&2(Ee(t))^T(LL\otimes PBB^TP)[(I_N\otimes A)e(t)-(c(t)L\otimes BK)e(t)-(c(t)L\otimes BK)z(t)].
\end{aligned}
\tag{3.25}
$$

From Theorem 3.1, we know that $z_i(t)$, $e_i(t)$ and $c_i(t)$ are bounded over $[0,+\infty)$, for $i=1,2,...,N$. Therefore, $\frac{d\|K\sum_{j=1}^N a_{ij}(e_i(t)-e_j(t))\|^2}{dt}$ is an upper bound, which is assumed to be ϖ_i. Note that $\|K\sum_{j=1}^N a_{ij}(e_i(t_k^i)-e_j(t_k^i))\|^2=0$. Using these two properties, one has

$$\|K\acute{e}_i(t)\|^2\leq\varpi_i(t-t_k^i),\ t\in[t_k^i,t_{k+1}^i). \tag{3.26}$$

When the triggering function (3.6) is at $t=t_{k+1}^i$, there is

$$\|K\acute{e}_i(t_{k+1}^i)\|^2=\alpha\|K\acute{z}_i(t_{k+1}^i))\|^2+\frac{\mu}{c_0}e^{-\nu t_{k+1}^i}. \tag{3.27}$$

Combining (3.26) and (3.27), one has

$$\frac{\mu}{c_0}e^{-\nu t_{k+1}^i}\leq\|K\acute{e}_i(t_{k+1}^i)\|^2\leq\varpi_i(t_{k+1}^i-t_k^i). \tag{3.28}$$

According to the above inequality, it is obvious that $\tau_k^i=t_{k+1}^i-t_k^i>0$ for any finite horizon. Thus, no agent exhibits Zeno behavior for each agent in the system for any limited time. The proof is over.

4. Fully distributed adaptive event-triggered control law for leader-follower consensus

In this section, we consider the event-triggered consensus problem in the presence of a leader. Without loss of generality, assume that the agent indexed by $x_1(t)$ is the leader whose control input $u_1(t) = 0$. The communication graph $\overline{\mathcal{G}}$ among the agents is assumed to satisfy the following assumption.

Assumption 4.1. *The pair (A, B) in (2.1) is stabilizable. The subgraph associated with the followers is undirected and the graph $\overline{\mathcal{G}}$ contains a directed spanning tree with the leader as the root.*

Because the leader has no neighbors, the Laplacian matrix L can be partitioned as $L = \begin{bmatrix} 0 & 0_{1\times(N-1)} \\ L_2 & L_1 \end{bmatrix}$, where $L_1 \in \mathbb{R}^{(N-1)\times(N-1)}$ is symmetric and $L_2 \in \mathbb{R}^{(N-1)\times 1}$. In light of Lemma 1 in [28], L_1 is positive definite.

Definition 4.1. *To simplify the calculation, define $\grave{e}_i(t) = \sum_{j=2}^N a_{ij}(e_i(t) - e_j(t)) + a_{i1}e_i(t)$; $\grave{\tilde{x}}_i(t) = \sum_{j=2}^N a_{ij}(\tilde{x}_i(t) - \tilde{x}_j(t)) + a_{i1}(\tilde{x}_i(t) - x_1(t))$; $\grave{z}_i(t) = \sum_{j=2}^N a_{ij}(z_i(t) - z_j(t)) + a_{i1}z_i(t)$; $\tilde{z}_i(t) = \tilde{x}_i(t) - x_1(t)$.*

For each follower, we propose the following adaptive event-based control law:

$$\begin{cases} u_i(t) = & c_i(t)K\grave{e}_i(t), \\ \dot{c}_i(t) = & \frac{(1-\alpha)}{(1+\alpha)}\|K\grave{e}_i(t)\|^2, \end{cases} \tag{4.1}$$

where $i = 2, 3, ..., N$.

We further propose the following triggering condition for determining the sampling instants:

$$t_{k+1}^i = inf\{t > t_k^i | f_i(t) \geq 0\},\ i = 2, ..., N, \tag{4.2}$$

where

$$f_i(t) = \alpha\|K\grave{z}_i(t)\|^2 - \|K\grave{e}_i(t)\|^2 + \frac{\mu}{c_i(t)}e^{-\nu t}. \tag{4.3}$$

For the ith follower, we define the measurement error $e_i(t) = \tilde{x}_i(t) - x_i(t)$ and the error $z_i(t) = x_i(t) - x_1(t)$, respectively. Then, the time derivative of $e_i(t)$ can be written as

$$\begin{aligned} E\dot{e}_i(t) =& A\tilde{x}_i(t) - Ex_i(t) \\ =& Ae_i(t) - c_i(t)BK\grave{\tilde{x}}_i(t) \\ =& Ae_i(t) - c_i(t)BK\sum_{j=2}^N h_{ij}\tilde{z}_j(t), \end{aligned} \tag{4.4}$$

where $H = L_1 + \Delta$ and $\Delta = diag\{a_{11}, a_{21}, ..., a_{N-11}\}$. Then, (4.4) can be rewritten as

$$diag(E)\dot{e}(t) = (I_N \otimes A)e(t) - (c(t)H \otimes BK)e(t) - (c(t)H \otimes BK)z(t). \tag{4.5}$$

Therefore, the time derivative of $z_i(t)$ can be written as

$$\begin{aligned} E\dot{z}_i(t) =& Ax_i(t) + Bu_i(t) - Ex_1(t) \\ =& Az_i(t) + c_i(t)BK\grave{\tilde{x}}_i(t) \\ =& Az_i(t) + c_i(t)BK\sum_{j=2}^N h_{ij}\tilde{z}_j(t). \end{aligned} \tag{4.6}$$

Then, (4.6) can be rewritten in a compact form as

$$diag(E)\dot{z}(t) = (I_N \otimes A)z(t) + (c(t)H \otimes BK)e(t) + (c(t)H \otimes BK)z(t). \tag{4.7}$$

Theorem 4.1. *Given Assumption 4.1 and Lemma 3.1, the leader-following multi-agent system (2.1) under the control protocol (4.1) with the dynamic event-triggered strategy (4.2), will achieve consensus if* $\alpha \in (0, 1)$ *and* $K = -B^T PE$.

Proof. Consider the Lyapunov-like function candidate

$$V_2(t) = (diag(E)z(t))^T (H \otimes P)diag(E)z(t) + \frac{1}{4}\sum_{i=2}^{N}(c_i(t) - c_0)^2, \tag{4.8}$$

where $c_0 \geq \frac{2(1+\alpha)(1+\kappa)}{\lambda_2(L_1)\kappa(1-\alpha)(1-\alpha-\alpha\kappa)}$ with $0 < \alpha < 1,\ 0 < \kappa < \frac{1}{\alpha} - 1$.

The time derivative of $V_2(t)$ along the trajectory of $Ez_i(t)$ in (4.7) with $K = -B^T PE$ is given by

$$\begin{aligned}\dot{V}_2(t) =&2z^T(t)diag(E^T)(H \otimes P)diag(E)\dot{z}(t) + \frac{1-\alpha}{2(1+\alpha)}\sum_{i=2}^{N}(c_i(t) - c_0)\|K\tilde{\grave{x}}_i(t)\|^2\\ =&2z^T(t)diag(E^T)(H \otimes P)[(I_N \otimes A)z(t) + (c(t)H \otimes BK)e(t) + (c(t)H \otimes BK)z(t)]\\ &+ \frac{1-\alpha}{2(1+\alpha)}\sum_{i=2}^{N}(c_i(t) - c_0)\|K\tilde{\grave{x}}_i(t)\|^2.\end{aligned} \tag{4.9}$$

Then, one has

$$\begin{aligned}\dot{V}_2(t) \leq&2z^T(t)(H \otimes E^T PA)z(t) - z^T(t)(H \otimes 2\Gamma)e(t) - z^T(t)(Hc(t)H \otimes 2\Gamma)z(t)\\ &+ \frac{1-\alpha}{2(1+\alpha)}\sum_{i=2}^{N}(c_i(t) - c_0)\|K\tilde{\grave{x}}_i(t)\|^2\\ \leq&2z^T(t)(H \otimes E^T PA)z(t) - z^T(t)(Hc(t)H \otimes \Gamma)e(t) - z^T(t)(Hc(t)H \otimes \Gamma)z(t)\\ &+ \frac{1-\alpha}{2(1+\alpha)}\sum_{i=2}^{N}(c_i(t) - c_0)\|K\tilde{\grave{x}}_i(t)\|^2.\end{aligned} \tag{4.10}$$

Moreover,

$$\begin{aligned}&\frac{1-\alpha}{2(1+\alpha)}\sum_{i=2}^{N}c_i(t)\|K\tilde{\grave{x}}_i(t)\|^2\\ &= \frac{1-\alpha}{2(1+\alpha)}\sum_{i=2}^{N}c_i(t)\|K\grave{e}_i(t) + K\grave{z}_i(t)\|^2\\ &\leq \frac{1-\alpha}{2(1+\alpha)}\sum_{i=2}^{N}c_i(t)(2\|K\grave{e}_i(t)\|^2 + 2\|K\grave{z}_i(t)\|^2)\\ &= \frac{1-\alpha}{1+\alpha}e^T(t)(Hc(t)H \otimes \Gamma)e(t) + \frac{1-\alpha}{1+\alpha}z^T(t)(Hc(t)H \otimes \Gamma)z(t).\end{aligned} \tag{4.11}$$

The triggering condition (4.3) implies

$$e^T(t)(Hc(t)H \otimes \Gamma)e(t) \leq \alpha z^T(t)(Hc(t)H \otimes \Gamma)z(t) + (N-1)\mu e^{-\nu t}. \tag{4.12}$$

Substituting (4.11) and (4.12) into (4.10), one gets

$$\dot{V}_2(t) \le 2z^T(t)(H \otimes E^T PA)z(t) + \frac{2(N-1)\mu}{1+\alpha}e^{-\nu t} - \frac{1-\alpha}{2(1+\alpha)}c_0\|K\grave{\tilde{x}}_i(t)\|^2. \tag{4.13}$$

It can be verified that

$$\begin{aligned}\|K\grave{z}_i(t)\|^2 =& \|K\grave{\tilde{x}}_i(t) - K\grave{e}_i(t)\|^2 \\ =& \|K\grave{\tilde{x}}_i(t)\|^2 + \|K\grave{e}_i(t)\|^2 - 2\grave{\tilde{x}}_i(t)^T\Gamma\grave{e}_i(t) \\ \le& \|K\grave{\tilde{x}}_i(t)\|^2 + \|K\grave{e}_i(t)\|^2 + \frac{1}{\kappa}\|K\grave{\tilde{x}}_i(t)\|^2 + \kappa\|K\grave{e}_i(t)\|^2,\end{aligned} \tag{4.14}$$

where κ is a positive constant which is determined later. By using of the triggering condition (4.3), one has

$$\begin{aligned}-\|K\grave{\tilde{x}}_i(t)\|^2 \le& \kappa\|K\grave{e}_i(t)\|^2 - \frac{\kappa}{1+\kappa}\|K\grave{z}_i(t)\|^2 \\ \le& \kappa[\alpha\|K\grave{z}_i(t)\|^2 + \frac{\mu}{c_i(t)}e^{-\nu t}] - \frac{\kappa}{1+\kappa}\|K\grave{z}_i(t)\|^2 \\ \le& \frac{\kappa\mu}{c_i(t)}e^{-\nu t} - \frac{\kappa(1-\alpha-\alpha\kappa)}{1+\kappa}\|K\grave{z}_i(t)\|^2.\end{aligned} \tag{4.15}$$

Thus, it further follows that

$$\begin{aligned}\dot{V}_2(t) \le& 2z^T(t)(H \otimes E^T PA)z(t) + \frac{2(N-1)\mu}{1+\alpha}e^{-\nu t} + \frac{1-\alpha}{2(1+\alpha)}c_0[\frac{\kappa\mu}{c_i(t)}e^{-\nu t} - \frac{\kappa(1-\alpha-\alpha\kappa)}{1+\kappa}\|K\grave{z}_i(t)\|^2] \\ \le& 2z^T(t)(H \otimes E^T PA)z(t) - c_0\varrho z^T(t)(H^2 \otimes \Gamma)z(t) + \theta e^{-\nu t},\end{aligned} \tag{4.16}$$

where $\varrho = \frac{\kappa(1-\alpha)(1-\alpha-\alpha\kappa)}{2(1+\alpha)(1+\kappa)} > 0$ and $\theta = \frac{2(N-1)\mu}{1+\alpha} + \frac{c_0(1-\alpha)\kappa\mu}{2(1+\alpha)\min c_i(0)} > 0$.

Under Assumption 3.3, the Laplacian matrix H is symmetric. Thus, there exists an orthogonal matrix U such that $U^T HU = \Lambda = diag\{\lambda_1, \lambda_2, ..., \lambda_{N-1}\}$, with $0 = \lambda_1 < \lambda_2 \le ... \le \lambda_{N-1}$. Let $\bar{z}(t) = (U^T \otimes I_N)z(t) = col(\bar{z}_1(t), \bar{z}_2(t), ..., \bar{z}_{N-1}(t),)$. Then, it follows from condition in Theorem 3.1, $\hat{z}_i(t) = E\bar{z}_i(t)$ and (3.1) that

$$\begin{aligned}\dot{V}_2(t) \le& \sum_{i=2}^{N}\lambda_i\bar{z}_i^T(t)[E^T PA + A^T PE - c_0\varrho\lambda_i\Gamma]\bar{z}_i(t) + \theta e^{-\nu t} \\ \le& \sum_{i=2}^{N}\lambda_i\bar{z}_i^T(t)[E^T PA + A^T PE - \Gamma]\bar{z}_i(t) + \theta e^{-\nu t} \\ \le& -\sum_{i=2}^{N}\lambda_i\bar{z}_i^T(t)E^T QE\bar{z}_i(t) + \theta e^{-\nu t} \\ =& \sum_{i=2}^{N}\lambda_i\hat{z}_i^T(t)Q\hat{z}_i(t) + \theta e^{-\nu t} \\ \le& -\lambda_{nonz}(L_1)\lambda_{min}(Q)\sum_{i=2}^{N}\hat{z}_i^T(t)\hat{z}_i(t) + \theta e^{-\nu t}.\end{aligned} \tag{4.17}$$

Then, the rest of the proof is similar with in Theorem 3.1.

Theorem 4.2. *Under the conditions of Theorem 4.1, the Zeno behavior can be excluded in the leader-following system.*

The proof of Theorem 4.2 is similar with Theorem 3.2 and it is omitted the details of the proof for brevity.

5. Illustrative examples

In this section, an example is provided to demonstrate the effectiveness of the results obtained. Consider the singular multi-agent systems with six agents, Its communication topology is $\mathcal{G}$, which is described in Figure 1. The dynamic model of the ith agent is based on (2.1), where

$$E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, A = \begin{bmatrix} -2 & 3 & 0 \\ 3 & -4 & 2 \\ -1 & -3 & -2 \end{bmatrix}, B = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}.$$

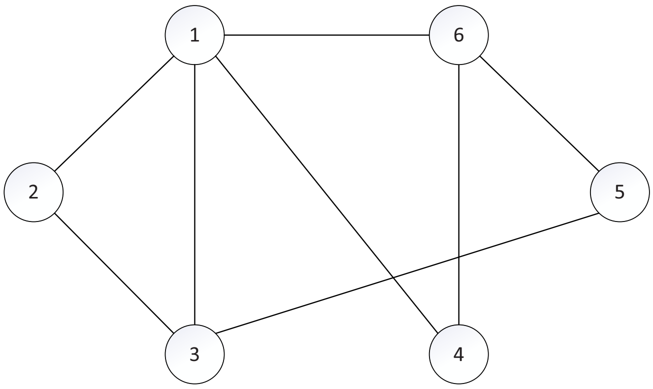

Figure 1. Communication topology $\overline{\mathcal{G}}$.

In the trigger function (3.15), the parameter is selected as $\beta = 0.5$, $\gamma = 0.95$, $\mu = 8$ and $\nu = 3$. Later, to satisfy (3.1), we use the tool of the LMI toolbox to get

$$P = \begin{bmatrix} 1.3276 & 0.6912 & 0.4233 \\ 0.6912 & 0.6931 & 0.2017 \\ 0.4233 & 0.2017 & 0.5181 \end{bmatrix},$$

$$Q = \begin{bmatrix} 1.0051 & 6.7933 & 0 \\ -7.2368 & 1.3037 & 0 \\ 0 & 0 & 1.3356 \end{bmatrix}.$$

Then, for the initial conditions, $c_i(0) = 0$ and $x_i(0)$, $\tilde{x}_i(0)$ can be selected randomly, $i = 1, 2, ..., 6$. Finally, various initial conditions are simulated. The results corresponding to an initial condition are shown in the Figures 2–4. Adaptive parameters $c_i(t)$ is described in Figure 2, which ultimately converge to some positive values. Triggered instants under adaptive event-triggered control law in (3.6) is presented in Figure 3, which shows that the Zeno behavior can be excluded. Figure 4 shows that the errors of all agents converges to zero, which proves the effectiveness and authenticity of the proposed fully distributed adaptive event-triggered control strategy.

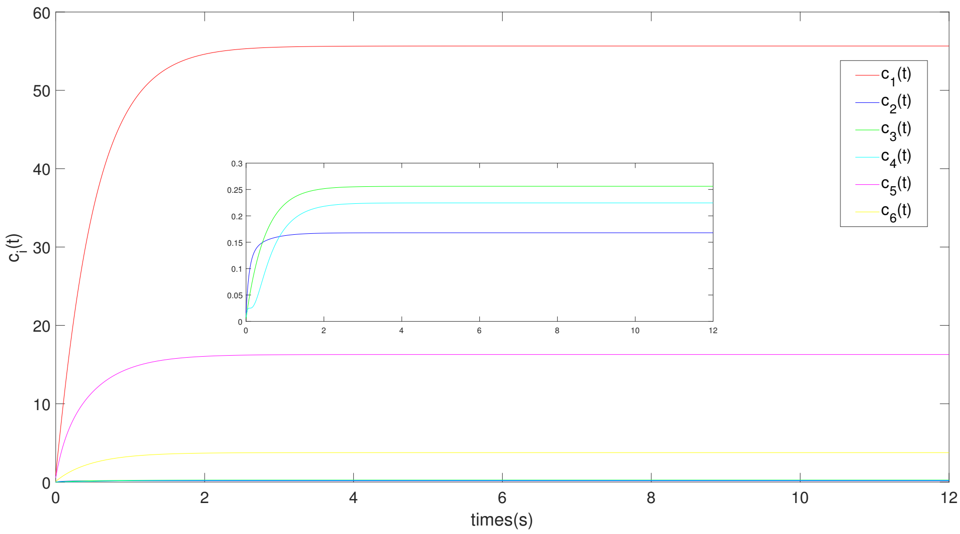

Figure 2. Adaptive parameters $c_i(t), i = 1, 2, ..., 6$.

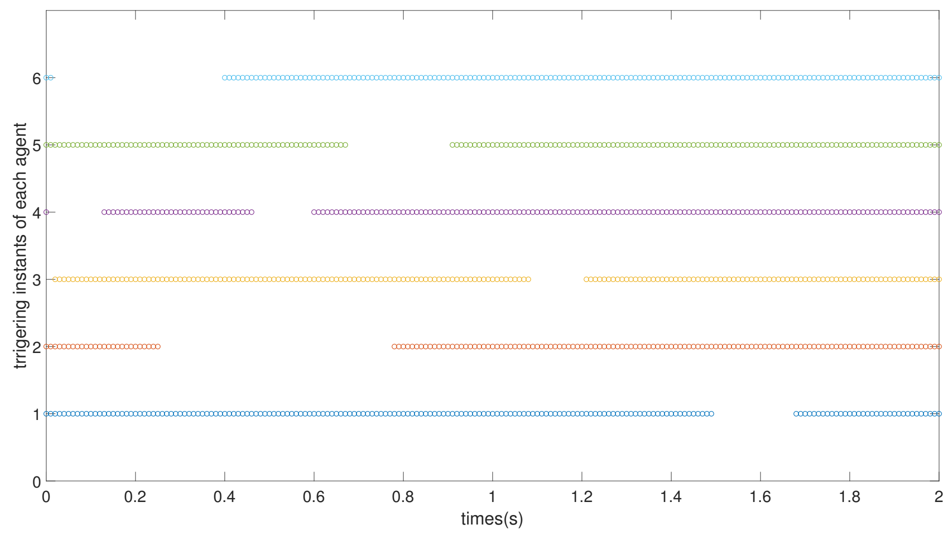

Figure 3. Triggered instants under adaptive event-triggered control law.

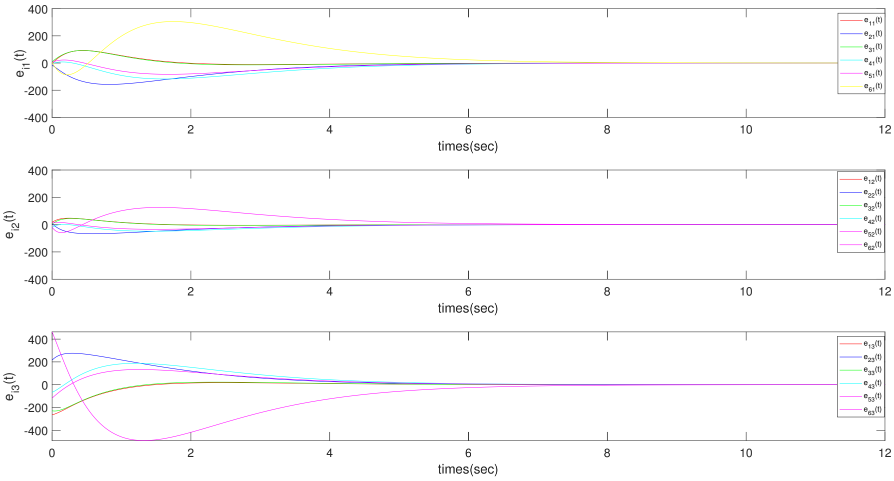

Figure 4. The errors of all agents under adaptive event-triggered control law.

6. Conclusions and future work

In the paper, we propose the fully distributed adaptive event-based control laws to solve the leaderless consensus problem and the leader-following consensus problem of the general linear singular multi-agent systems. The triggering conditions and the adaptive gains depend on the relative state estimate information between adjacent agents for each agent. Therefore, the proposed event-triggered protocol does not require continuous communication between neighbors, which greatly alleviate the negative impact of communication. Based on the designed adaptive gain, the protocol does not need to know the Laplace of the communication graph which can be designed in a fully distributed. It has been shown that Zeno behavior is avoided during triggering process. In addition, numerical examples show the effectiveness and reliability of the method. Extending the results to heterogeneous singular multi-agent systems is a promising future direction.

Acknowledgments

This research was supported by the Sichuan Petroleum and Natural Gas Development Research Center under Grant No. SKB19-06.

Conflict of interest

The authors declare that there are no conflicts of interest.

References

1. J. Mei, W. Ren, J. Chen, Distributed consensus of second-order multi-agent systems with heterogeneous unknown inertias and control gains under a directed graph, *IEEE Trans. Automat. Control*, **61** (2016), 2019–2034. https://doi.org/10.1109/tac.2015.2480336

2. L. Ding, Q. L. Han, X. H. Ge, X. M. Zhang, An overview of recent advances in event-triggered consensus of multiagent systems, *IEEE Trans. Cybernet.*, **48** (2018), 1110–1123. https://doi.org/10.1109/tcyb.2017.2771560

3. J. H. Qin, Q. C. Ma, Y. Shi, L. Wang, Recent advances in consensus of multi-agent systems: a brief survey, *IEEE Trans. Ind. Electron.*, **64** (2017), 4972–4983. https://doi.org/10.1109/tie.2016.2636810

4. E. Ephrati, J. S. Rosenschein, Deriving consensus in multiagent systems, *Artif. Intell.*, **87** (1996), 21–74. https://doi.org/10.1016/0004-3702(95)00105-0

5. Y. Y. Qian, L. Liu, G. Feng, Output consensus of heterogeneous linear multi-agent systems with adaptive event-triggered control, *IEEE Trans. Automat. Control*, **64** (2019), 2606–2613. https://doi.org/10.1109/tac.2018.2868997

6. Z. J. Ma, Y. Wang, X. M. Li, Cluster-delay consensus in first-order multi-agent systems with nonlinear dynamics, *Nonlinear Dyn.*, **83** (2016), 1303–1310. https://doi.org/10.1007/s11071-015-2403-8

7. W. W. Yu, H. Wang, F. Cheng, X. H. Yu, G. H. Wen, Second-order consensus in multiagent systems via distributed sliding mode control, *IEEE Trans. Cybernet.*, **47** (2017), 1872–1881. https://doi.org/10.1109/tcyb.2016.2623901

8. J. Y. Yu, Y. Shi, Scaled group consensus in multiagent systems with first/second-order continuous dynamics, *IEEE Trans. Cybernet.*, **48** (2018), 2259–2271. https://doi.org/10.1109/TCYB.2017.2731601

9. Y. Y. Chen, Y. Shi, Distributed consensus of linear multiagent systems: Laplacian spectra-based method, *IEEE Trans. Syst. Man Cybernet. Syst.*, **50** (2020), 700–706. https://doi.org/10.1109/tsmc.2017.2774841

10. J. X. Xi, C. Wang, H. Liu, Z. Wang, Dynamic output feedback guaranteed-cost synchronization for multiagent networks with given cost budgets, *IEEE Access*, **6** (2018), 28923–28935. https://doi.org/10.1109/access.2018.2819989

11. X. Lin, Y. S. Zheng, Finite-time consensus of switched multiagent systems, *IEEE Trans. Syst. Man Cybernet. Syst.*, **47** (2017), 1535–1545. https://doi.org/10.1109/tsmc.2016.2631659

12. P. Lin, Y. M. Jia, Consensus of second-order discrete-time multi-agent systems with nonuniform time-delays and dynamically changing topologies, *Automatica*, **45** (2009), 2154–2158. https://doi.org/10.1016/j.automatica.2009.05.002

13. F. Xiao, L. Wang, Consensus protocols for discrete-time multi-agent systems with time-varying delays, *Automatica*, **44** (2008), 2577–2582. https://doi.org/10.1016/j.automatica.2008.02.017

14. K. Y. You, L. H. Xie, Network topology and communication data rate for consensusability of discrete-time multi-agent systems, *IEEE Trans. Automat. Control*, **56** (2011), 2262–2275. https://doi.org/10.1109/tac.2011.2164017

15. H. Haghshenas, M. A. Badamchizadeh, M. Baradarannia, Containment control of heterogeneous linear multi-agent systems, *Automatica*, **54** (2015), 210–216. https://doi.org/10.1016/j.automatica.2015.02.002

16. W. F. Hu, L. Liu, G. Feng, Output consensus of heterogeneous linear multi-agent systems by distributed event-triggered/self-triggered strategy, *IEEE Trans. Cybernet.*, **47** (2017), 1914–1924. https://doi.org/10.1109/tcyb.2016.2602327
17. X. F. Liu, Y. F. Xie, F. B. Li, W. H. Gui, Admissible consensus for homogenous descriptor multiagent systems, *IEEE Trans. Syst. Man Cybernet. Syst.*, **51** (2021), 965–974. https://doi.org/10.1109/TSMC.2018.2889681
18. Y. R. Cong, Z. G. Feng, H. W. Song, S. M. Wang, Containment control of singular heterogeneous multi-agent systems, *J. Franklin Inst.*, **355** (2018), 4629–4643. https://doi.org/10.1016/J.JFRANKLIN.2018.04.009
19. X. X. Zhang, X. P. Liu, Z. G. Feng, Distributed containment control of singular heterogeneous multi-agent systems, *J. Franklin Inst.*, **357** (2020), 1378–1399. https://doi.org/10.1016/j.jfranklin.2019.10.025
20. T. Zheng, M. He, J. X. Xi, G. B. Liu, Leader-following guaranteed-performance consensus design for singular multi-agent systems with Lipschitz nonlinear dynamics, *Neurocomputing*, **266** (2017), 651–658. https://doi.org/10.1016/j.neucom.2017.05.073
21. J. Wu, Q. Deng, T. Han, H. C. Yan, Bipartite output regulation for singular heterogeneous multi-agent systems on signed graph, *Asian J. Control*, **24** (2022), 2452–2460. https://doi.org/10.1002/asjc.2654
22. T. X. Zhang, H. Yu, F. Hao, A novel distributed event-triggered control with time-varying thresholds, *J. Franklin Inst.*, **357** (2020), 4132–4153. https://doi.org/10.1016/j.jfranklin.2020.01.019
23. X. H. Ge, Q. L. Han, L. Ding, Y. L. Wang, X. M. Zhang, Dynamic event-triggered distributed coordination control and its applications: a survey of trends and techniques, *IEEE Trans. Syst. Man Cybernet. Syst.*, **50** (2020), 3112–3125. https://doi.org/10.1109/TSMC.2020.3010825
24. S. L. Li, X. H. Nian, Z. H. Deng, Distributed optimization of second-order nonlinear multiagent systems with event-triggered communication, *IEEE Trans. Control Network Syst.*, **8** (2021), 1954–1963. https://doi.org/10.1109/TCNS.2021.3092832
25. A. Jenabzadeh, B. Safarinejadian, Y. Lu, W. D. Zhang, Distributed event-triggered target tracking under cyber attacks, *J. Franklin Inst.*, **359** (2022), 2377–2402. https://doi.org/10.1016/j.jfranklin.2021.12.020
26. K. Mohammadi, E. Azizi, J. Choi, M. T. Hamidi-Beheshti, A. Bidram, S. Bolouki, Asynchronous periodic distributed event-triggered voltage and frequency control of microgrids, *IEEE Trans. Power Syst.*, **36** (2021), 4524–4538. https://doi.org/10.1109/TPWRS.2021.3059158
27. Z. G. Wu, Y. Xu, R. Q. Lu, Y. Q. Wu, T. W. Huang, Event-triggered control for consensus of multiagent systems with fixed/switching topologies, *IEEE Trans. Syst. Man Cybernet. Syst.*, **48** (2018), 1736–1746. https://doi.org/10.1109/TSMC.2017.2744671
28. B. Cheng, Z. K. Li, Fully distributed event-triggered protocols for linear multiagent networks, *IEEE Trans. Automat. Control*, **64** (2019), 1655–1662. https://doi.org/10.1109/TAC.2018.2857723

29. D. V. Dimarogonas, E. Frazzoli, K. H. Johansson, Distributed event-triggered control for multi-agent systems, *IEEE Trans. Automat. Control*, **57** (2012), 1291–1297. https://doi.org/10.1109/TAC.2011.2174666

30. D. D. Wang, Q. H. Zhou, W. Zhu, Adaptive event-based consensus of multi-agent systems with general linear dynamics, *J. Syst. Sci. Complexity*, **31** (2018), 120–129. https://doi.org/10.1007/s11424-018-7360-0

31. W. B. Zhang, Y. Tang, Y. R. Liu, J. Kurths, Event-triggering containment control for a class of multi-agent networks with fixed and switching topologies, *IEEE Trans. Circuits Syst. I. Regul. Pap.*, **64** (2017), 619–629. https://doi.org/10.1109/TCSI.2016.2618944

32. Z. K. Li, Z. S. Duan, *Cooperative control of multi-agent systems: a consensus region approach*, Boca Raton: CRC Press, 2015. https://doi.org/10.1201/b17571

33. R. Gao, J. S. Huang, L. Wang, Leaderless consensus control of uncertain multi-agents systems with sensor and actuator attacks, *Inform. Sci.*, **505** (2019), 144–156. https://doi.org/10.1016/j.ins.2019.07.075

34. S. L. Du, J. F. Qiao, W. Li, Leader-following consensus for nonlinear multiagent systems with large delays, *Trans. Inst. Meas. Control*, **41** (2019), 2223–2235. https://doi.org/10.1177/0142331218796121

35. Z. K. Li, G. H. Wen, Z. S. Duan, W. Ren, Designing fully distributed consensus protocols for linear multi-agent systems with directed graphs, *IEEE Trans. Automat. Control*, **60** (2015), 1152–1157. https://doi.org/10.1109/TAC.2014.2350391

36. X. W. Li, Y. Tang, H. R. Karimi, Consensus of multi-agent systems via fully distributed event-triggered control, *Automatica*, **116** (2020), 108898. https://doi.org/10.1016/j.automatica.2020.108898