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Research article

An intuitionistic fuzzy entropy-based gained and lost dominance score decision-making method to select and assess sustainable supplier selection

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Abstract: Sustainable supplier selection (SSS) is recognized as a prime aim in supply chain because of its impression on profitability, adorability, and agility of the organization. This work introduces a multi-phase intuitionistic fuzzy preference-based model with which decision experts are authorized to choose the suitable supplier using the sustainability "triple bottom line (TBL)" attributes. To solve this issue, an intuitionistic fuzzy gained and lost dominance score (IF-GLDS) approach is proposed using the developed IF-entropy. To make better use of experts' knowledge and fully represent the uncertain information, the evaluations of SSS are characterized in the form of intuitionistic fuzzy set (IFS). To better distinguish fuzziness of IFSs, new entropy for assessing criteria weights is proposed with the help of an improved score function. By considering the developed entropy and improved score function, a weight-determining process for considered criterion is presented. A case study concerning the iron and steel industry in India for assessing and ranking the SSS is taken to demonstrate the practicability of the developed model. The efficacy of the developed model is certified with the comparison by diverse extant models.

Keywords: intuitionistic fuzzy sets; supply chain; sustainable supplier selection; entropy; multiplecriteria decision-making; gained and lost dominance score **Mathematics Subject Classification:** 90B50

1. Introduction

Owing to the direct impacts on profitability, flexibility and affordability, a "sustainable supplier selection (SSS)" is distinguished as a vital concerns that enterprises unquestionably encounter. The inflexible government guidelines concerning "sustainable development (SD)" force the enterprises to bearing their business with the sustainability perspective taking three facets/pillars, comprising the "environmental, economic and social (EES)" [1,2]. In spite of the desire for executing the SDs, the aims of enterprise and legislation may struggle in handling the SSS problem. Henceforth, various studies have focused on the procedure for selecting supplier to propose unified models with both "multi-criteria decision-making (MCDM)" and soft computing approaches. Meanwhile, the assessment of suitable weight values for sustainability "triple bottom line (TBL)" attributes still challenges scholars whereas there are abundant procedures explained to obtain preferences [1,3,4].

The procedure of supplier selection is finding, assessing, and accordingly selecting the appropriate suppliers [5] to show the most perilous character in the region of the "supply chain (SC)" under the enterprise schemes. In the meantime, the SSS states to choosing the appropriate supplier over the sustainability TBL standards to observe supplier's social, economic, and environmental performance [6]. In recent years, growing awareness of sustainability has encouraged both experts and scholars to tackle a SSS problem within the soft computing discipline [1,7]. In order to the need in different enterprises, numerous MCDM tools were proposed to support the decision experts (DEs) and scholars in taking the appropriate choice by finding weights. As a notion, the "best-worst method (BWM)", given by Rezaei [8], and the "method using the removal effects of criteria (MEREC)", discussed by Keshavarz-Ghorabaee et al. [9], have been presented as a reliable tools by which DEs can employ less input information associating to further models.

Eliciting all the details aforesaid and the wide-ranging literature (discussed in Section 2), several challenging issues can be recognized; (1) Though there are stunning works on SSS, most of them utilize the fuzzy decision-making models that comprise complex computations procedure in practice. These intricacies decline the need of SC executives to implement those models in realistic problems. (2) Diverse extant models are exposed in circumstances where there is a requirement to add/eliminate various supplier options. (3) Most extant models expected that the criteria weights are given randomly, which create the MCDM procedure poor convincing. Based on these issues, there is still scope for a more precise and flexible model that is accomplished of treating an SSS problem with an MCDM procedure. This paper aims to achieve the following outcomes:

A new intuitionistic fuzzy gained and lost dominance score (IF-GLDS) framework is presented, which is more reasonable and in line with realistic results.

To compute the significance degree of criteria, new procedure is developed based on new inverse trigonometric entropy and generalized improved score function for IFSs to choose the best supplier with more precisely.

A basic model is applied for summarizing the sustainability TBL attributes from the prolonged

literature of SSS which primarily focuses on the environmental and social pillars.

The developed framework is implemented to solve the SSS of iron and steel industries in India. Also, a comparison is made of proposed framework with IF-TOPSIS and IF-COPRAS methods to validate the results.

2. Literature review

Here, we review the literature on SSS approaches, IFSs and GLDS method to find the extant gaps and elucidate the need of utilizing the developed model. It is worth mentioning that we attempt to review the recent studies of multi-attribute SSSs as there are various studies about the SSS, IFSs and MCDM.

2.1. The GLDS approach

The ranking models are very crucial in handling the MCDM problems. Mainly, these models fall into following groups: utility degree-based models, reference point-based models and outranking models [10]. In utility degree-based models, diverse "aggregation operators (AOs)" are utilized to obtain the criterion ratings with associated criteria weights to find the final degrees of options [11–13]. The reference point-based models are broadly used to treat the MCDM issues under IFSs, namely the IF-TOPSIS [14,15] and IF-VIKOR [16]. The key notion of these models is to obtain a compromise degree which is closest to the ideal solution. The other kind of MCDM models are outranking models which consider the pairwise comparisons of options over different criteria [13,17]. To evade these limitations, Wu and Liao [13] presented the "gained and lost dominance score (GLDS)" model and to find the "comprehensive score (CS)" of each alternative. Moreover, the GLDS model cannot be utilized to treat the MCDM problems with uncertain settings characterized by IFSs. Hence, motivated by the conventional GLDS model, we develop a new prioritization model under the IFSs. Some recent descriptions related to the GLDS method are presented in Table 1.

2.2. Sustainable supplier selection

The conception of sustainability has been raised because of increased adversarial impacts on social and environmental conditions [30,31]. The prime cause for these adverse impacts is emerging economic regions without observing other regions of the nation. Henceforth, to use sustainability in the nation, the administration should do balancing with the "economic, environmental, and social (EES)" pillars. The meanings of these pillars have been elucidated as follows. Economic: A sustainable financial structure must tackle the production and supply of materials during the given time. Environmental: A sustainable structure should endeavor to preserve prime assets. Alternatively, it should not inhibit with the steady process of the environment by over mining renewable and non-renewable resources. Social: A sustainable structure should highlight the unbiased sharing of social rights, comprising health, education, and political solidity. Speciously, in the relationship with researches in a SSS setting, Noci [32] firstly discussed a supplier preference structure based on environmental perspective. Afterward, this notion was called as "green supplier selection (GSS)", which has been discussed by several researchers [33–35]. For the government interest and

competitive issues, the GSS was shifted by SSS to contain social and economic performances as well [36].

Author(s)	Year	Objective	Fuzzy environments	Benchmark	GDM
Wu and Liao [13]	2019	Optimal green enterprise selection	Probabilistic linguistic term sets (PLTSs)	Consensus-based GLDS method	Yes
Fu et al. [18]	2018	Underground mining procedure assessment problem	Hesitant fuzzy linguistic term sets (HFLTSs)	Score function-based GLDS method	Yes
Liao et al. [19]	2019	Life satisfaction assessment of earthquake-hit victims in Wenchuan, China	PLTSs	Logarithm- multiplicative AHP- based GLDS method	No
Fang et al. [20]	2019	Screening the high-risk population of lung cancer	Generalized probabilistic linguistic term sets (G- PLTSs)	Evidential reasoning- based GLDS method	No
Liao et al. [21]	2020	Investment assessment of BE angle capital China	q-rung orthopair fuzzy sets (q-ROFSs)	Distance measure-based GLDS method	Yes
Liu et al. [22]	2020	Social capital assessment of a public–private-partnership (PPP) project	q-ROFSs	Hamacher aggregation operator-based GLDS method	Yes
Liao et al. [23]	2020	Green supplier selection problem	Hesitant fuzzy sets (HFSs)	Choquet integral-based GLDS method	Yes
Ming et al. [24]	2020	Manage the patient satisfaction in the blood collection room	PLTSs	BWM-GLDS method	Yes
Liang et al. [25]	2020	Electric vehicle charging stations site assessment	Continuous interval- valued linguistic term sets (CIVLTSs)	Normal distribution- based GLDS method	Yes
Fan et al. [26]	2021	Selection of the best low-carbon logistics park site	2-tuple linguistic neutrosophic numbers (2TLNNs)	score function and distance measure-based GLDS	Yes
Wang et al. [27]	2022	Performance evaluation of Chengdu public transport group in Sichuan province	Continuous interval- valued double hierarchy linguistic term set (CIVDHL)	CIVDHL-GLDS model	Yes
Zhai et al. [28]	2022	Evaluation of the agriculture supply chain risks for investments of agricultural small and medium-sized enterprises (SMEs)	Pythagorean fuzzy sets (PFSs)	PF-MEREC-rank sum (RS)-GLDS Approach	Yes
Mishra et al. [29]	2023	To evade the shortcomings of the existing MCDM methods in the FFNs	Fermatean fuzzy numbers (FFNs)	FF-CRITIC-GLDS model	No

Table 1. Extant studied on GLDS model.

Additional, Xu et al. [7] assessed the GSS problem with the hybrid MCDM. Meksavang et al. [37] discussed the improved VIKOR tool to deal with the SSS problem on "picture fuzzy sets (PiFSs)". Lu et al. [38] gave an integrated tool to select the suitable supplier in straw biomass power plant. Memari et al. [1] discussed TOPSIS model for evaluating the SSS on IFSs. Mishra et al. [34] initiated the "weighted aggregates sum product assessment (WASPAS)" tool on HFSs to assess the GSS problem. Stevic et al. [39] gave the MARCOS model to choose the suitable supplier in the healthcare enterprise. Peng et al. [40] discussed a model with the VIKOR tool on PiFSs to treat the SSS problem. Kumari and Mishra [41] proposed the COPRAS model with IF-parametric information measures to treat GSS problem. Rani et al. [42] proposed an integrated procedure with COPRAS approaches to choose the SSS on HFSs. Mishra et al. [43] discussed a model with the "combined compromise solution (CoCoSo)" on HFSs to find the suitable "sustainable third party reverses logistic provider (S3PRLP)". Mishra and Rani [44] presented the CoCoSo and the "criteria importance through intercriteria correlation (CRITIC)" tools on "single-valued neutrosophic sets (SVNSs)" to find the best S3PRLP. Chen et al. [45] proposed a model with the projection model on interval-valued IFSs to evaluate S3PRLPs. Alrasheedi et al. [46] introduced a framework with the "stepwise weight assessment ratio analysis (SWARA)" and the WASPAS tools on PFSs to evaluate SSS over diverse sustainable criteria in the manufacturing enterprises.

2.3. Intuitionistic fuzzy sets

The concept of "fuzzy sets (FSs)", established by Zadeh [47], has broadly acknowledged from "decision experts (DEs)" in MCDM process. For FSs, the "membership degree (MD)" of an object is described as the interval [0, 1] and the "non-membership degree (ND)" is solely its complement. But, realistically, this assumption does not meet with human perception. To avoid the limitations of FSs, Atanassov [48] prolonged the idea of FSs to IFSs by ranging the value into three functions: the MD, ND and the "hesitancy degree (HD)" in which the sum of the MD and ND is ≤ 1 [49,50]. Hence, the IFSs have more prospective than FSs to treat the uncertain issues, so that, various scholars have concentrated on IFSs and their implementations.

Motivated by the idea of information doctrine, the entropy enumerates the amount of uncertainty on FSs [51]. Afterwards, an axiomatic definition of entropy on IFSs was discussed by Szmidt and Kacprzyk [52]. Accordingly, diverse entropies on IFSs have been developed in [53-57] and utilized in various disciplines. Mishra and Rani [16] presented weighted divergence measurebased VIKOR tool on IFSs to choose a "cloud service provider (CSP)". Rani and Mishra [58] proposed integrated SWARA-VIKOR method for assessing decision-making problem under SVNSs. Mishra et al. [49] presented the "additive ratio assessment (ARAS)" tool to choose the best IT personnel. Mishra et al. [59] presented the parametric divergence measure-based EDAS tool to evaluate the "health-care waste disposal (HCWD)" procedure on IFSs. Mishra et al. [60] discussed "combinative distance-based assessment (CODAS)" tool on IFSs to evaluate "low-carbon sustainable suppliers (LCSS)". Recently, Yan et al. [61] put forward a hybridized MCDM framework for urban rail transit system selection under intuitionistic fuzzy environment. A multi-objective tool with the Markowitz and DEA cross-efficiency approached has been developed to evaluate the portfolios on IFSs [62]. Zhan et al. [63] presented intuitionistic fuzzy rough graphs, and described some kinds of intuitionistic fuzzy rough graphs with applications in decision-making problems. Akram et al. [64] gave the intuitionistic fuzzy N-soft sets (IFNSSs) and intuitionistic fuzzy N-soft

rough sets (IFNSRSs)-based approaches to solve the realistic decision-making problems. Akram et al. [65] discussed the complex intuitionistic fuzzy Hamacher aggregation operator to treat the MCDM tool to find the best source for generation of electricity with the aid of the proposed operators. Feng et al. [66] proposed the regular Minkowski distance of q-rung orthopair membership grades which strengthens or extends some useful distance measures in the literature and developed a flexible model to support multiple attribute decision making with generalized orthopair fuzzy soft information. Akram et al. [67] introduced a WASPAS tool with a 2-tuple linguistic Fermatean fuzzy (2TLFF) set for the solid waste disposal location selection (SWDLS) problem by using the Hamacher aggregation operators. Tripathi et al. [68] proposed a distance measure for IFSs to introduce a modified intuitionistic fuzzy complex proportional assessment method. In the past few years, many theories and applications related to IFSs have been presented [69–71]. In this study, we present entropy using the inverse trigonometric function for the first time under IFSs. Further, the developed entropy and new score function are applied to find the criteria weight of SSS problem.

3. New entropy for IFS

This section discusses the notion of the MCDM procedure implemented in the paper. Particularly, we concentrate on the demonstration of the decision data by the IFSs. Then, we turn to develop new entropy for IFS.

3.1. Preliminaries

Initially, we show the concepts related to the IFSs. **Definition 3.1 [48].** An IFS S on a fixed set $Z = \{z_1, z_2, ..., z_n\}$ is given by

$$S = \{ \langle z_i, \mu_S(z_i), \nu_S(z_i) \rangle : z_i \in Z \},$$

$$(1)$$

where $\mu_S: Z \to [0,1]$ and $\nu_S: Z \to [0,1]$ show the MD and ND of z_i to S in Z, with the condition

$$0 \le \mu_S(z_i) \le 1, 0 \le \nu_S(z_i) \le 1 \text{ and } 0 \le \mu_S(z_i) + \nu_S(z_i) \le 1, \forall z_i \in \mathbb{Z}.$$
 (2)

The intuitionistic index of $z_i \in Z$ to S is defined by

$$\pi_S(z_i) = 1 - \mu_S(z_i) - \nu_S(z_i) \text{ and } 0 \le \pi_S(z_i) \le 1.$$

For simplicity, Xu [72] described the "intuitionistic fuzzy number (IFN)" $\zeta = (\mu_{\zeta}, \nu_{\zeta})$ which holds $\mu_{\zeta}, \nu_{\zeta} \in [0,1]$ and $0 \le \mu_{\zeta} + \nu_{\zeta} \le 1$.

Definition 3.2 [72]: For a IFN $\zeta = (\mu_{\zeta}, \nu_{\zeta})$. Then

$$\mathbb{S}(\zeta) = (\mu_{\zeta} - \nu_{\zeta}), h(\zeta) = (\mu_{\zeta} + \nu_{\zeta}), \tag{3}$$

are called score and accuracy values, respectively. Here, $S(\zeta) \in [-1,1]$ and $h(\zeta) \in [0,1]$.

Definition 3.3 [72]: Let $\zeta_j = (\mu_{\zeta_j}, \nu_{\zeta_j}), j = 1(1)n$ be IFNs. Then the weighted averaging and geometric operators on IFNs are given by

$$IFWA_{w}(\zeta_{1},\zeta_{2},\ldots,\zeta_{n}) = \bigoplus^{j=1}_{m} w_{j}\zeta_{j} = \left[1 - \prod_{j=1}^{n} \left(1 - \mu_{\zeta_{j}}\right)^{w_{j}}, \prod_{j=1}^{n} v_{\zeta_{j}}^{w_{j}}\right],$$
(4)

$$IFWG_{w}(\zeta_{1},\zeta_{2},...,\zeta_{n}) = \bigotimes^{j=1}_{m} w_{j}\zeta_{j} = \left[\prod_{j=1}^{n} \mu_{\zeta_{j}}^{w_{j}}, 1 - \prod_{j=1}^{n} \left(1 - \nu_{\zeta_{j}}\right)^{w_{j}}\right],$$
(5)

where $w_j = (w_1, w_2, ..., w_n)^T$ is a weight value of $\zeta_j, j = 1, 2, ..., n$, with $\sum_{j=1}^n w_j = 1, w_j \in [0, 1]$.

Definition 3.4 [71]: Let $\zeta = (\mu_{\zeta}, \nu_{\zeta})$ be IFNs. A generalized score value of an IFN is given by

$$\mathbb{S}^{*}(\zeta) = \mu_{\zeta} \Big[1 + (\gamma_{1} + \gamma_{2}) \Big(1 - \mu_{\zeta} - \nu_{\zeta} \Big) \Big], \tag{6}$$

where $\gamma_1 + \gamma_2 = 1$, $\gamma_1, \gamma_2 > 0$ signifies the attitudinal performances of $S^*(\zeta)$, as it shows the weighted average of HD between the MD and ND.

Definition 3.5 [52]: A mapping $e: IFS(Z) \to [0,1]$ is called IF-entropy, if it fulfills the postulates as (P1). e(S) = 0, iff *S* is a crisp set; (P2). e(S) = 1, iff $\mu_S(z_i) = \nu_S(z_i)$ for all $z_i \in Z$; (P2). e(S) = c(T) if *S* is large furging then *T* is a

(P3). $e(S) \leq e(T)$ if *S* is less fuzzier than *T*, *i*. *e*. $\mu_S(z_i) \leq \mu_T(z_i)$ and $\nu_S(z_i) \geq \nu_T(z_i)$ for $\mu_T(z_i) \leq \nu_T(z_i)$ or $\mu_S(z_i) \geq \mu_T(z_i)$ and $\nu_S(z_i) \leq \nu_T(z_i)$ for $\mu_T(z_i) \geq \nu_T(z_i)$ for any $z_i \in Z$; (P4). $e(S) = e(S^c)$.

3.2. Proposed inverse tangent entropy for IFS

Corresponding to Hooda and Mishra [73], the entropy for determining the fuzziness degree of an object on IFS called 'IF-entropy' is defined by

Definition 3.6: Consider that $S \in IFS(Z)$, then the IF-entropy is given by

$$e(S) = \frac{2}{n(1-\alpha)\ln 2} \sum_{i=1}^{n} \left(\tan^{-1} \left\{ \left(\mu_{S}^{\alpha}(z_{i}) + \nu_{S}^{\alpha}(z_{i}) \right) \left(\mu_{S}(z_{i}) + \nu_{S}(z_{i}) \right)^{1-\alpha} + 2^{1-\alpha} \pi_{S}(z_{i}) \right\} - \frac{\pi}{4} \right), \tag{7}$$

where $\alpha > 0$ and $\alpha \neq 1$.

Theorem 3.1: The mapping e(S), given by Eq (7), is valid IF-entropy. *Proof.* The proof is same as the Theorem 4 in Ansari et al [54].

Particular and Limiting Cases

- When $\alpha \rightarrow 1$, then Eq (7) reduces to measures in Vlachos and Sergiadis [55] entropy.
- It may be noticed that if an IFS is an ordinary FS, that is, for $all_{z_i} \in Z$, $\mu_S(z_i) = 1 \nu_S(z_i)$, then the IF-entropy of order- α changes to fuzzy entropy of order- α [73]. Next, we take some extant entropies as Bustince and Burillo [53] entropy $e_{bb}(S)$, Szmidt and

Kacprzyk [52] entropy $e_{sk}(S)$, Hung and Yang [56] entropies $e_{hy}^2(S)$ and $e_s(S)$, Vlachos and

Sergiadis [55] entropy $e_{vs}(S)$, Zhang and Jiang [74] entropy $e_{zj}(S)$, Wei et al. [57] entropy $e_w(S)$, Mishra et al [50] entropy $e_m(S)$ and Ansari et al. [54] entropy $e_a(S)$, which can be listed as

$$e_{bb}(S) = \frac{1}{n} \sum_{i=1}^{n} (1 - \mu_{S}(z_{i}) - \nu_{S}(z_{i}))$$

$$e_{sk}(S) = \frac{1}{n} \sum_{i=1}^{n} \left(\frac{(1 - \nu_{S}(z_{i})) \wedge (1 - \mu_{S}(z_{i}))}{(1 - \nu_{S}(z_{i})) \vee (1 - \mu_{S}(z_{i}))} \right)$$

$$e_{zj}(S) = \frac{1}{n} \sum_{i=1}^{n} \left(\frac{\mu_{S}(z_{i}) \wedge \nu_{S}(z_{i})}{\mu_{S}(z_{i}) \vee \nu_{S}(z_{i})} \right)$$

$$e_{hc}^{2}(S) = \frac{1}{n} \sum_{i=1}^{n} (1 - \mu_{S}^{2}(z_{i}) - \nu_{S}^{2}(z_{i}) - \pi_{S}^{2}(z_{i}))$$

$$e_{s}(S) = -\frac{1}{n} \sum_{i=1}^{n} (\mu_{S}(z_{i}) \ln \mu_{S}(z_{i}) + \nu_{S}(z_{i}) \ln \nu_{S}(z_{i}) + \pi_{S}(z_{i}) \ln \pi_{S}(z_{i}))$$

$$e_{\nu S}(S) = -\frac{1}{n \ln 2} \sum_{i=1}^{n} (\mu_{S}(z_{i}) \ln \mu_{S}(z_{i}) + \nu_{S}(z_{i}) \ln \nu_{S}(z_{i}) + (\mu_{S}(z_{i}) + \nu_{S}(z_{i})) \ln (\mu_{S}(z_{i}) + \nu_{S}(z_{i}) \ln 2)$$

$$e_{w}(S) = \frac{1}{n} \sum_{i=1}^{n} \left[\left\{ \sqrt{2} \cos \left(\frac{\mu_{S}(z_{i}) - \nu_{S}(z_{i})}{4} \right) \pi - 1 \right\} \times \frac{1}{\sqrt{2} - 1} \right]$$
(S)

 $e_m(S)$

$$= \frac{1}{n\sqrt{e}(\sqrt{e}-1)} \sum_{i=1}^{n} \left[e -\left(\frac{\mu_{S}(z_{i})+1-\nu_{S}(z_{i})}{2}\right) e^{\left(\frac{\mu_{S}(z_{i})+1-\nu_{S}(z_{i})}{2}\right)} - \left(\frac{\nu_{S}(z_{i})+1-\mu_{S}(z_{i})}{2}\right) e^{\left(\frac{\nu_{S}(z_{i})+1-\mu_{S}(z_{i})}{2}\right)} \right]$$
$$e_{a}(S) = \frac{1}{2^{1-\alpha}-1} \left[exp\left\{ (\alpha-1)\sum_{i=1}^{n} (\mu_{S}(z_{i}) \ln\left(\frac{\mu_{S}(z_{i})}{\mu_{S}(z_{i})+\nu_{S}(z_{i})}\right) + \nu_{S}(z_{i}) \ln\left(\frac{\nu_{S}(z_{i})}{\mu_{S}(z_{i})+\nu_{S}(z_{i})}\right) - \pi_{S}(z_{i}) \ln 2 \right) \right\} - 1 \right].$$

Example 3.1 [54,74]: Let $S \in IFS(Z)$. For any positive real value *n*, De et al. [75] introduced $IFS(Z)S^n$ as follow:

$$S^{n} = \{ \langle z_{i}, [\mu_{S}(z_{i})]^{n}, 1 - [1 - \nu_{S}(z_{i})]^{n} \rangle : z_{i} \in Z \}.$$

We assume $S \in IFS(Z)$ given by

 $S = \{(6,0.1,0.8), (7,0.3,0.5), (8,0.5,0.4), (9,0.9,0), (10,1.0,0)\}.$

By defining the classification of "linguistic terms (LTs)", De et al. [75] obtained S as "LARGE" on Z. Therefore, using the $IFS(Z)S^n$ as

- $S^{\frac{1}{2}}$ defined as "More or less LARGE",
- S^2 defined as "Very LARGE",
- S^3 defined as "Quite very LARGE",
- S^4 defined as "Very very LARGE".

Now, we use the abovementioned IFSs to compute the IF-entropies and outcomes are illustrated in Table 2. From mathematical logical pattern, it is significant that the IF-entropies of abovementioned IFSs have the following pattern:

$$e\left(S^{\frac{1}{2}}\right) \ge e(S) \ge e(S^2) \ge e(S^3) \ge e(S^4).$$
(8)

Entropies	$S^{\frac{1}{2}}$	S	S^2	<i>S</i> ³	S^4
$e_{bb}(S)$	0.4090	0.5000	0.4900	0.4670	0.4670
$e_{sk}(S)$	0.3450	0.3740	0.1970	0.1310	0.1090
$e_{hy}^2(S)$	0.3420	0.3440	0.2610	0.1990	0.1610
$e_s(S)$	0.4330	0.4310	0.3270	0.2530	0.2080
$e_{vs}(S)$	0.5518	0.5217	03491	0.2354	0.1417
$e_{zj}(S)$	0.2851	0.3050	0.1042	0.0383	0.0161
$e_w(S)$	0.4545	0.4377	0.3029	0.2159	0.1709
$e_m(S)$	0.5522	0.5333	0.3758	0.2719	0.2149
$e_a(S)$	0.4659	0.4356	0.2737	0.1775	0.1032
<i>e</i> (<i>S</i>)	0.5732	0.5634	0.4587	0.3648	0.2970

Table 2. Results of developed IF-entropy with diverse extant entropies.

According to Table 2, the performances of $e_s(S)$, $e_{vs}(S)$, $e_w(S)$, $e_m(S)$, $e_a(S)$ and e(S) are good, but the performance of IF-entropies $e_{bb}(S)$, $e_{sk}(S)$, $e_{hy}^2(S)$ and $e_{zj}(S)$ are poor, these entropies do not fulfill the necessary pattern in Eq (8). From Example 3.1, we can observe that the performance of e(S), is better than diverse extant entropies.

4. Proposed IF-GLDS method

In the section, we present the IF-GLDS model to the IFSs environment, which has many remarkable merits to find the ranking of options.

The fundamental idea of the GLDS model is to consider pairwise comparisons of options, so as to establish the dominance flows among options. Then, the "gained dominance score (GDS)" and "lost dominance score (LDS)" values are computed by the summation operator and the maximum operator, respectively. Furthermore, the "overall gained dominance score (OGDS)" and "overall lost dominance score (OLDS)" values of options are obtained with associated criteria weights. The final preference of options is derived by an aggregation operator, which considers both the subordinate ranks and the OGDS and OLDS simultaneously. This outranking method was based on the assumption that the optimal options should have the largest gained dominance flow and the smallest lost dominance flow. Under the intuitionistic fuzzy context, we need to define the dominance flow of alternatives, which signifies that an option is higher rank to the other one under a given criterion.



Figure 1. The flowchart of the IF-GLDS method for MCDM problems.

Table 3. Performance rating of DEs, criteria and SSS [7]	76].
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LVs	IFNs
Extremely good (EG)	(0.95,0.05)
Very very good (VVG)	(0.85,0.10)
Very good (VG)	(0.80,0.15)
Good (G)	(0.70,0.20)
moderate good (MG)	(0.60,0.30)
Average (A)	(0.50,0.40)
Moderate bad (MB)	(0.40,0.50)
Bad (B)	(0.30,0.60)
Very bad (VB)	(0.20,0.70)
Very very bad (VVB)	(0.10,0.80)
Extremely bad (EB)	(0.05,0.95)

Step 1. Create the "linguistic decision matrix (LDM)" about a group of DEs

Construct the LDM for each DE over considered the criterion set then we convert the LDM into intuitionistic fuzzy decision matrix (IF-DM) $Y_k = (y_{ij}^{(k)})_{m \times n}$ for each DE using Table 3 as follows:

$$C_{1} \cdots C_{n} Y_{k} = \left(y_{ij}^{(k)}\right)_{m \times n} = \begin{bmatrix}G_{1} \\ \vdots \\ G_{m} \end{bmatrix} \begin{bmatrix}(\mu_{11k}, \nu_{11k}) & \cdots & (\mu_{1nk}, \nu_{1nk}) \\ \vdots & \ddots & \vdots \\ (\mu_{m1k}, \nu_{m1k}) & \cdots & (\mu_{mnk}, \nu_{mnk})\end{bmatrix}_{m \times n}, k = 1, 2, \dots, \ell,$$
(9)

wherein $y_{ij}^{(k)}$ describes the IFN corresponding to the "linguistic value (LV)" given in Table 3 of an

option G_i over criterion p_j for k^{th} expert.

Step 2. Calculate the weight of DEs

To find the weight of DEs, let $\xi = (\mu_k, \nu_k), k = 1, 2, ..., \ell$ be significance rating by DE based on their knowledge then numeric DE's weight is obtained as

$$\varpi_k = \frac{\mu_k (2 - \mu_k - \nu_k)}{\sum_{k=1}^{\ell} [\mu_k (2 - \mu_k - \nu_k)]}, k = 1, 2, \dots, \ell.$$
(10)

Here, $\varpi_k \ge 0$ and $\sum_{k=1}^{\ell} \varpi_k = 1$. The number ℓ denotes the number of DEs, $k = 1, 2, ..., \ell$. Step 3. Make the "aggregated intuitionistic fuzzy decision-matrix (A-IF-DM)

Let $Y_k = (y_{ij}^k)$ be the LDM of k^{th} DE into the "linguistic variables (LVs)" and then change to

the IFN as $y_{ij}^k = (\mu_{ijk}, \nu_{ijk}), k = 1, 2, ..., \ell$. Then the A-IF-DM $\mathbb{R} = (y_{ij})_{m \times n}$, where $y_{ij} = \frac{1}{2}$

 $(\mu_{ij}, \nu_{ij}), i = 1, 2, ..., m, j = 1, 2, ..., n$, is computed as $\mathbb{R} = \bigoplus_{i=1}^{\ell} \varpi_k y_{ij}^k$ such that

$$y_{ij} = (\mu_{ij}, \nu_{ij}) = \left(1 - \prod_{k=1}^{\ell} (1 - \mu_{ijk})^{\varpi_k}, \prod_{k=1}^{\ell} (\nu_{ijk})^{\varpi_k}\right).$$
(11)

Step 4. Evaluate the criteria weights

Let $w = (w_1, w_2, ..., w_n)^T$ be a weight value of criterion with $\sum_{j=1}^n w_j = 1$ and $w_j \in [0,1]$.

Now, we apply the IF-entropy and generalized score value-base model to find the criteria weight as

$$w_{j} = \frac{\sum_{i=1}^{m} \left(e(y_{ij}) + \mathbb{S}^{*}(y_{ij}) \right)}{\sum_{j=1}^{n} \left(\sum_{i=1}^{m} \left(e(y_{ij}) + \mathbb{S}^{*}(y_{ij}) \right) \right)}, j = 1, 2, \dots, n.$$
(12)

Step 5. Generate the normalized A-IF-DM

To normalize the A-IF-DM $\mathbb{R} = (y_{ij})_{m \times n}$, we consider the different types of criteria and obtain the normalized A-IF-DM $\overline{\mathbb{R}} = (\overline{y}_{ij})_{m \times n}$ as follows:

$$\bar{y}_{ij} = \begin{cases} y_{ij} = (\mu_{ij}, \nu_{ij}), \text{forbenefitcriterion} \\ (y_{ij})^c = (\nu_{ij}, \mu_{ij}), \text{forcostcriterion} \end{cases}$$
(13)

where $(y_{ij})^c$ is the complementation of y_{ij} .

Step 6: Construct the "dominance flows (DFs)"

Let $y_{ij} = (\mu_{ij}, \nu_{ij})$ and $y_{\nu j} = (\mu_{\nu j}, \nu_{\nu j})$ be two IFNs for two alternatives G_i and G_ν under the

criterion p_i , respectively.

The dominance flow of the alternative G_i over G_v with respect to the criterion p_j is defined as

$$DF_{j}(G_{i}, G_{v}) = \begin{cases} \mathbb{S}^{*}(y_{ij}) - \mathbb{S}^{*}(y_{vj}), if \mathbb{S}^{*}(y_{ij}) \ge \mathbb{S}^{*}(y_{vj}) \\ 0, if \mathbb{S}^{*}(y_{ij}) < \mathbb{S}^{*}(y_{vj}) \end{cases},$$
(14)

where $S^*(y_{ij})$ is the generalized score value for IFNs.

We find the DF using the vector normalization as follows:

$$DF_{j}^{N}(G_{i}, G_{v}) = \frac{DF_{j}(G_{i}, G_{v})}{\sqrt{\sum_{\nu=1}^{m} \sum_{i=1}^{m} \left[DF_{j}(G_{i}, G_{v}) \right]^{2}}}.$$
(15)

Step 7. Calculate the GDS of each option

The "unicriterion GDS (UGDS)" of option G_i outranking the other options G_v ($v = 1, 2, ..., mand v \neq i$) over criterion p_i is estimated as

$$UGDS_{j}(G_{i}) = \sum_{\nu=1}^{m} DF_{j}^{N}(G_{i}, G_{\nu}).$$
(16)

The "overall gained dominance score (OGDS)" of option G_i is determined by

$$OGDS(G_i) = \sum_{\nu=1}^m w_j \cdot UGDS_j(G_i).$$
(17)

Then, a subordinate preference set (SPS) $\rho_1 = \{r_1(G_1), r_1(G_2), \dots, r_1(G_m)\}$ is found in decreasing values of $OGDS(G_i)$ $(i = 1, 2, \dots, m)$.

Step 8. Calculate the LDS of each option

To illustrate the amount that option G_i does not always dominate G_v , the "unicriterion LDS (ULDS)" of option G_i is calculated using the maximizing operator as

$$ULDS_j(G_i) = \max_{v} \left(DF_j^N(G_v, G_i) \right).$$
(18)

The "overall lost dominance score (OLDS)" of option G_i is computed as

$$OLDS(G_i) = \max_j \left(w_j \cdot DF_j^N(G_v, G_i) \right).$$
(19)

Then, a "subordinate preference rank set (SPS)" $\rho_2 = \{r_2(G_1), r_2(G_2), \dots, r_2(G_m)\}$ is obtained in increasing values of $OLDS(G_i)$ $(i = 1, 2, \dots, m)$.

Step 9. Compute the "collective score (CS)" of each option

First, Normalizing the OGDS and OLDS of each option using Eq (15) and obtain the $OGDS^{N}(G_{i})$ and $OLDS^{N}(G_{i})$. The final preference set $\rho = \{r(G_{1}), r(G_{2}), \dots, r(G_{m})\}$ is derived in decreasing values of CS_{i} , where CS_{i} indicates the collective score of alternative G_{i} as

$$CS_i = OGDS^N(G_i) \cdot \frac{m - r_1(G_i) + 1}{(m(m+1)/2)} - OLDS^N(G_i) \cdot \frac{r_2(G_i)}{(m(m+1)/2)}, i = 1, 2, \dots, m.$$
(20)

Step 10. End.

5. Implementation of case study

In the section, we implement a realistic case study to reveal the confirmation and application of the developed IF-GLDS model.



Figure 2. A systematic model of the proposed framework.

India is a developing nation with an emerging economy in which the "iron and steel (I-S)" industry contributes considerably. Indian I-S industry has been key producers of rails, plates, wires, rods, wheel, and catering to essential for global consumers. Being a large scale enterprise convoluted in steel production, the enterprise conveys out several actions such as excavation of minerals from mines, GHG emission, and wastes disposal in water and land which influences all pillars of sustainability. Thus, to have proficient and useful SSC in the industry, sustainable suppliers need to be assessed and designated. Hence, there is a requirement for recognizing and cataloguing of the sustainability attributes for SSS for I-S industry in India. To achieve this objective and choose the suitable supplier, eight suppliers chosen and are being considered according to the performance of sustainable TBL criteria. Figure 2 shows the descriptions of criteria which are taken for obtaining weights based on the entropy and generalized score value-method. To recognize the key criteria to

assess the SSS, in this paper, a survey with the DE's interview and literature review has been made. In first phase, we recognize 25 criteria to evaluate and select the best sustainable suppliers from the previous literature [1,7,37–46,77,78] and presented in Table 4. In second phase, the considered criteria are categorized into three key pillars of sustainability, comprising social, economic and environmental and depicted in Figure 2. In the following phase of the developed model, a team of DEs from these companies are made to conduct the whole process.

Goal	Criteria	Sub-criteria	Cost/benefit
	Environmental	Environment-related certificates (p1)	Cost
		Internal control process (p2)	Cost
		Environmental protection plans (p3)	Cost
		Environmental protection policies (p4)	Cost
		Staff environmental training (p5)	Cost
e		Air emissions (p6)	Cost
tio		Wastewater (p7)	Cost
elec		Use of harmful and environmental-friendly materials (p8)	Cost
ers		Green packaging (p9)	Benefit
ppli		Recycling capability (p10)	Benefit
[ns a	Social	Occupational health and safety management (p11)	Benefit
able		Standardized health and safety conditions (p12)	Benefit
ain		Health and safety incidents (p13)	Cost
sust		Health and safety practices (p14)	Benefit
nal		Job stability (p15)	Benefit
ptin		Job opportunities (p16)	Benefit
le oj		Child labor (p17)	Cost
it th		Flexible working arrangements (p18)	Benefit
elec		Employee welfare (p19)	Benefit
SO .		The interests and rights of employee (p20)	Benefit
	Economic	Product price (p21)	Cost
		Fright cost (p22)	Cost
		Technical capability (p23)	Benefit
		Reputation (p24)	Benefit
		Delivery schedule (p25)	Cost

Table 4. Assessment of TBL criteria for SSS.

Next, Table 3 reveals the LVs and corresponding IFNs for the DEs, criteria weights and the SSS option. Table 5 evaluates the weight of DE using Eq (10) and Table 3. Table 6 shows the LDM for DE's ratings as (e1, e2, e3) in form of LVs.

Table 5. DEs	'weights	for	SSS.
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DEs	<i>e</i> ₁	<i>e</i> ₂	<i>e</i> ₃
LVs	Very good (0.80, 0.15)	Good (0.70, 0.20)	Average (0.50, 0.40)
Weight	0.4000	0.3500	0.2500

Table 6. The LDM for SSS.

	G 1	G ₂	G ₃	G 4	G 5	G ₆	G 7	G 8
p_1	(B, VB, VB)	(B,MB,VB)	(G, MG,A)	(MG,MG,A)	(A,A,MB)	(VG,G,MG)	(MG,A,MG)	(B,B,B)
p_2	(MB,B,A)	(VB,VB,VB	(MG,A,MG)	(VG,MG,VG	(VG,G,G)	(A,MB,MG)	(A,A,A)	(VB,B,MB)
))				
<i>p</i> ₃	(G,VG,G)	(VG,VG,VG	(A,MG,A)	(MG,G,A)	(A,A,MG)	(B,MB,MB)	(VB,B,A)	(G,G,MG)
)						
p_4	(A,MG,MG	(G,G,G)	(MG,MG,A)	(B,MB,VB)	(MB,A,MB)	(MG,G,MG)	(VG,G,MG)	(VG,VG,G)
)							
p_5	(MG,G,A)	(VG,G,VG)	(MB,MB,A)	(VB,MB,B)	(MG,MG,G)	(VG,G,VG)	(G,MG,MG)	(G,G,MB)
p_6	(B, B, VB)	(B,VB,VB)	(G, VG,A)	(B,MG,A)	(MG,A,MB)	(VG,B,A)	(B,A,MG)	(A,B,VB)
p_7	(MG,B,A)	(VB,B,VB)	(MG,A,VG)	(VVG,G,A)	(VVG,A,G)	(A,MB,G)	(A,VB,B)	(VB,B,A)
p_8	(VG,VG,M	(G,VG,VG)	(A,MG,VB)	(MG,B,A)	(A,MB,MG)	(B,A,VVB)	(VB,MB,A)	(G,B,MB)
	G)							
p_9	(A,G,MG)	(VVG,G,G)	(MG,G,A)	(VB,MB,VB	(MB,B,MB)	(MB,G,A)	(VB,G,A)	(VG,G,B)
)				
p_{10}	(VVG,G,A)	(MG,G,VG)	(MB,MB,A)	(VVB,MB,B	(MG,G,VG)	(G,A,VVG)	(G,MB,A)	(G,VG,A)
)				
p_{11}	(B, B, VB)	(B,B,VB)	(G, A,B)	(MB,B,A)	(A,B,MB)	(B,G,MB)	(G,A,MB)	(B,VB,A)
p_{12}	(B,B,A)	(VB,MB,VB	(VG,A,MB)	(MB,MG,V	(VG,VG,G)	(A,MB,G)	(A,A,VVB)	(A,B,MG)
)		G)				
p_{13}	(VG,G,VG)	(G,VG,VG)	(MG,MB,A)	(MG,VG,B)	(A,MB,MG)	(B,MG,B)	(B,VVB,A)	(VB,A,MG)
p_{14}	(A,G,MG)	(G,G,VVG)	(VG,MG,A)	(VVB,VB,B	(MB,A,MG)	(A,MB,MG)	(G,VG,VVG	(VG,G,G)
))	
p_{15}	(G,MG,A)	(VG,G,MG)	(MB,G,A)	(VB,VVB,B	(MG,VB,G)	(A,G,MG)	(A,MG,VG)	(G,VG,B)
		(D I D I D)				(1.0.110)		
p_{16}	(B, VB,B)	(B,VB,VB)	(G, VG, A)	(MB,MG,A)	(G.A,MB)	(A,G,VG)	(A,A,MB)	(B,A,B)
p_{17}	(MB,A,A)	(MB,MB,V	(MB,A,MG)	(VG,MG,VB	(G,G,MG)	(A,B,VG)	(A,MB,MB)	(MB,B,VB)
		$\mathbf{B})$	$(\mathbf{A} \mathbf{D} \mathbf{A})$) (VC D A)	$(A \mathbf{D} \mathbf{M} \mathbf{C})$			
p_{18}	(VG,G,VV	(10,0,10)	(A,B,A)	(VG,B,A)	(A,B,MG)	(VG,B,MB)	(B,MB,A)	(G,B,MB)
D LO	$(\Lambda G MG)$	(G VVG G)	(MB G A)	(PMPMP)		(GAMB)	(G G MB)	(VB G B)
<i>p</i> ₁₉	(A, G, MG)	(U, V, U, U)	(MB VVB A	(MR VR R)	$(VG \land B)$	$(\mathbf{O}, \mathbf{A}, \mathbf{M}\mathbf{D})$	(G, VB, G)	(VB, O, B)
p_{20}	(A,0,A)	(10,4,10))	(1110, 10,0)	(10,A,D)	(0,0,00)	(0, 10,0)	(0,0,10)
<i>n</i> 21	(BAVB)	(B MB MB)) (G. G. B)	(MB MG B)	(VB G MB)	(G VVG A)	(VB A G)	(BAMB)
<i>P</i> ₂₁	$(\mathbf{B},\mathbf{N},\mathbf{V},\mathbf{B})$	(VR VR A)	$(\mathbf{O}, \mathbf{O}, \mathbf{D})$ (MG VB G)	(VB MB VG	(R VB G)	(B, MG G)	$(VB \land A)$	$(\mathbf{VB} \mathbf{A} \mathbf{B})$
P 22	(B, (B, I)	(12,12,11)	(110, 12,0))	(2,12,3)	(2,110,0)	(,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,	(,,,,,,,,)
<i>1</i> 223	(VG.A.G)	(VG.MG.G)	(A.MB.B)) (MB.VVG.A	(VG.VB.MG	(A.MG.MB)	(VB.MB.A)	(B.A.MG)
P 23	(* 0,1,0)	(* 0,110,0)	(11,112,2)))	(11,110,112)	(*2,02,1)	(2,1,110)
<i>D</i> 24	(A.G.MG)	(G.A.G)	(MG.MB.B)	, (VVB.MB.B	(VB.A.VB)	(A.MG.G)	(B.VB.MG)	(G.MG.B)
r -7	(,-,	(-,-,-)	(,,,))	(-, -, -, -, -, -, -, -, -, -, -, -, -,	((_, _,	(-,,)
D 25	(VVG.G.A)	(MG,G.VG)	(MG,B.A)	, (VB,B.VVB	(A,VVG.G)	(B,A,A)	(G,MB.VG)	(G,VVG.B)
1)				

Further, using Eq (11) and Table 6, we have created the A-IF-DM in Table 7. From Table 7 and Eq (12), we find the criteria weight using the developed inverse tangent parametric entropy and generalized score value as follow:

 $w_j = (0.0389, 0.0386, 0.0406, 0.0411, 0.0410, 0.0387, 0.0391, 0.0403, 0.0384, 0.0408, 0.0382, 0.0399, 0.0403, 0.0399, 0.0407, 0.0389, 0.0403, 0.0411, 0.0409, 0.0402, 0.0401, 0.0405, 0.0412, 0.0396, 0.0408).$

Since all the criteria are not same types (as categorizes cost-type and benefit-type), therefore, normalized A-IF-DM is given in Table 8 using Eq (13).

Using Eq (6), the normalized A-IF-DM $\mathbb{\bar{R}} = (\bar{y}_{ij})_{m \times n}$, is changed into the generalized score-

matrix $S^*(y_{ij})$ as displayed in Table 9. Using the score-matrix, the $OGDS(G_i)$ is estimated by

Eqs (15)–(17), while the $OLDS(G_i)$ is obtained by Eqs (15), (18), and (19), which are displayed in Tables 10–12. Also, we can get two SPSs ρ_1 and ρ_2 . From Eq (20), the collective scores CS_i of each option is derived as CS1 = -0.0206, CS2 = -0.0189, CS3 = 0.0074, CS4 = -0.0162, CS5 = 0.0014, CS6 = 0.0065, CS7 = 0.0091 and CS8 = 0.0280 and mentioned in Table 12. Therefore, $G_8 > G_7 > G_3 > G_6 > G_5 > G_4 > G_2 > G_1$. That is to say, the best sustainable supplier alternative is G8.

	<i>G</i> ₁	G ₂	G3	G 4	G 5	G ₆	G 7	<i>G</i> ₈
p_1	(0.242,	(0.314,	(0.623,	(0.577,	(0.477,	(0.726,	(0.568,	(0.300,
	0.658)	0.585)	0.274)	0.322)	0.423)	0.197)	0.332)	0.600)
p_2	(0.395,	(0.200,	(0.568,	(0.745,	(0.745,	(0.496,	(0.500,	(0.290,
	0.504)	0.700)	0.332)	0.191)	0.178)	0.402)	0.400)	0.610)
p_3	(0.740,	(0.800,	(0.538,	(0.618,	(0.527,	(0.362,	(0.321,	(0.678,
	0.181)	0.150)	0.362)	0.280)	0.372)	0.538)	0.577)	0.221)
p_4	(0.563,	(0.700,	(0.577,	(0.314,	(0.437,	(0.638,	(0.726,	(0.779,
	0.337)	0.200)	0.322)	0.585)	0.462)	0.260)	0.197)	0.161)
p_5	(0.618,	(0.770,	(0.427,	(0.300,	(0.628,	(0.770,	(0.643,	(0.643,
	0.280)	0.166)	0.473)	0.599)	0.271)	0.166)	0.255)	0.251)
p_6	(0.276,	(0.242,	(0.704,	(0.471,	(0.521,	(0.610,	(0.459,	(0.367,
	0.624)	0.658)	0.215)	0.425)	0.377)	0.311)	0.438)	0.530)
p_7	(0.486,	(0.237,	(0.636,	(0.742,	(0.728,	(0.531,	(0.359,	(0.321,
	0.411)	0.663)	0.279)	0.180)	0.193)	0.364)	0.538)	0.577)
p_8	(0.762,	(0.765,	(0.480,	(0.486,	(0.496,	(0.337,	(0.357,	(0.520,
	0.178)	0.168)	0.416)	0.411)	0.402)	0.559)	0.541)	0.369)
p_9	(0.326,	(0.289,	(0.526,	(0.434,	(0.407,	(0.534,	(0.397,	(0.344,
	0.572)	0.609)	0.365)	0.455)	0.481)	0.358)	0.500)	0.554)
p_{10}	(0.742,	(0.696,	(0.427,	(0.267,	(0.696,	(0.698,	(0.566,	(0.704,
	0.180)	0.219)	0.473)	0.632)	0.219)	0.214)	0.328)	0.215)
p_{11}	(0.276,	(0.276,	(0.557,	(0.395,	(0.411,	(0.499,	(0.573,	(0.326,
	0.624)	0.624)	0.335)	0.504)	0.487)	0.390)	0.321)	0.572)

Table 7. The A-IF-DM for SSS.

Continued on next page

	G 1	G ₂	G 3	G 4	G 5	G_6	G 7	G_8
<i>p</i> ₁₂	(0.356,	(0.277,	(0.637,	(0.604,	(0.779,	(0.531,	(0.421,	(0.468,
	0.542)	0.622)	0.286)	0.309)	0.161)	0.364)	0.476)	0.429)
<i>p</i> ₁₃	(0.770,	(0.765,	(0.513,	(0.639,	(0.496,	(0.425,	(0.297,	(0.429,
	0.166)	0.168)	0.385)	0.280)	0.402)	0.471)	0.600)	0.466)
p_{14}	(0.605,	(0.748,	(0.679,	(0.189,	(0.491,	(0.496,	(0.781,	(0.745,
	0.292)	0.168)	0.244)	0.710)	0.407)	0.402)	0.152)	0.178)
<i>p</i> 15	(0.623,	(0.726,	(0.550,	(0.194,	(0.526,	(0.605,	(0.632,	(0.678,
	0.274)	0.197)	0.343)	0.706)	0.365)	0.292)	0.283)	0.238)
p_{16}	(0.267,	(0.242,	(0.704,	(0.503,	(0.573,	(0.667,	(0.477,	(0.378,
	0.633)	0.658)	0.215)	0.395)	0.321)	0.246)	0.423)	0.521)
p_{17}	(0.462,	(0.355,	(0.491,	(0.639,	(0.678,	(0.553,	(0.442,	(0.320,
	0.437)	0.544)	0.407)	0.281)	0.221)	0.361)	0.457)	0.580)
p_{18}	(0.786,	(0.770,	(0.438,	(0.610,	(0.468,	(0.592,	(0.390,	(0.520,
	0.150)	0.166)	0.461)	0.311)	0.429)	0.329)	0.509)	0.369)
<i>p</i> 19	(0.605,	(0.765,	(0.550,	(0.362,	(0.368,	(0.573,	(0.643,	(0.451,
	0.292)	0.157)	0.343)	0.538)	0.529)	0.321)	0.251)	0.434)
p_{20}	(0.742,	(0.696,	(0.486,	(0.214,	(0.711,	(0.428,	(0.654,	(0.709,
	0.180)	0.219)	0.411)	0.686)	0.207)	0.470)	0.256)	0.207)
p_{21}	(0.357,	(0.362,	(0.629,	(0.459,	(0.472,	(0.733,	(0.469,	(0.401,
	0.541)	0.538)	0.263)	0.438)	0.415)	0.187)	0.421)	0.497)
p_{22}	(0.605,	(0.773,	(0.618,	(0.277,	(0.367,	(0.550,	(0.495,	(0.685,
	0.292)	0.152)	0.280)	0.622)	0.533)	0.343)	0.393)	0.235)
<i>p</i> ₂₃	(0.695,	(0.718,	(0.420,	(0.647,	(0.614,	(0.516,	(0.357,	(0.459,
	0.227)	0.205)	0.479)	0.269)	0.306)	0.382)	0.541)	0.438)
p_{24}	(0.605,	(0.641,	(0.470,	(0.267,	(0.321,	(0.593,	(0.362,	(0.576,
	0.292)	0.255)	0.427)	0.632)	0.575)	0.304)	0.533)	0.315)
p_{25}	(0.582,	(0.724,	(0.339,	(0.310,	(0.623,	(0.548,	(0.577,	(0.520,

Table 8. The normalized A-IF-DM for SSS.

0.299)

0.343)

0.310)

0.369)

0.589)

	G 1	G ₂	G ₃	<i>G</i> ₄	G5	<i>G</i> ₆	G ₇	<i>G</i> ₈
p_1	(0.658,	(0.585,	(0.274,	(0.322,	(0.423,	(0.197,	(0.332,	(0.600,
1	0.242)	0.314)	0.623)	0.577)	0.477)	0.726)	0.568)	0.300)
p_2	(0.504,	(0.700,	(0.332,	(0.191,	(0.178,	(0.402,	(0.400,	(0.610,
1	0.395)	0.200)	0.568)	0.745)	0.745)	0.496)	0.500)	0.290)
p_3	(0.181,	(0.150,	(0.362,	(0.280,	(0.372,	(0.538,	(0.577,	(0.221,
	0.740)	0.800)	0.538)	0.618)	0.527)	0.362)	0.321)	0.678)
p_4	(0.337,	(0.200,	(0.322,	(0.585,	(0.462,	(0.260,	(0.197,	(0.161,
-	0.563)	0.700)	0.577)	0.314)	0.437)	0.638)	0.726)	0.779)
p_5	(0.280,	(0.166,	(0.473,	(0.599,	(0.271,	(0.166,	(0.255,	(0.251,
	0.618)	0.770)	0.427)	0.300)	0.628)	0.770)	0.643)	0.643)
p_6	(0.624,	(0.658,	(0.215,	(0.425,	(0.377,	(0.311,	(0.438,	(0.530,
_	0.276)	0.242)	0.704)	0.471)	0.521)	0.610)	0.459)	0.367)

Continued on next page

0.314)

0.211)

0.557)

12026

	G 1	G_2	G3	G 4	G 5	G 6	G 7	G 8
p_7	(0.411,	(0.663,	(0.279,	(0.180,	(0.193,	(0.364,	(0.538,	(0.577,
	0.486)	0.237)	0.636)	0.742)	0.728)	0.531)	0.359)	0.321)
p_8	(0.178,	(0.168,	(0.416,	(0.411,	(0.402,	(0.559,	(0.541,	(0.369,
	0.762)	0.765)	0.480)	0.486)	0.496)	0.337)	0.357)	0.520)
p_9	(0.326,	(0.289,	(0.526,	(0.434,	(0.407,	(0.534,	(0.397,	(0.344,
	0.572)	0.609)	0.365)	0.455)	0.481)	0.358)	0.500)	0.554)
p_{10}	(0.742,	(0.696,	(0.427,	(0.267,	(0.696,	(0.698,	(0.566,	(0.704,
	0.180)	0.219)	0.473)	0.632)	0.219)	0.214)	0.328)	0.215)
p_{11}	(0.276,	(0.276,	(0.557,	(0.395,	(0.411,	(0.499,	(0.573,	(0.326,
	0.624)	0.624)	0.335)	0.504)	0.487)	0.390)	0.321)	0.572)
p_{12}	(0.356,	(0.277,	(0.637,	(0.604,	(0.779,	(0.531,	(0.421,	(0.468,
	0.542)	0.622)	0.286)	0.309)	0.161)	0.364)	0.476)	0.429)
p_{13}	(0.166,	(0.168,	(0.385,	(0.280,	(0.402,	(0.471,	(0.600,	(0.466,
	0.770)	0.765)	0.513)	0.639)	0.496)	0.425)	0.297)	0.429)
p_{14}	(0.605,	(0.748,	(0.679,	(0.189,	(0.491,	(0.496,	(0.781,	(0.745,
	0.292)	0.168)	0.244)	0.710)	0.407)	0.402)	0.152)	0.178)
p_{15}	(0.623,	(0.726,	(0.550,	(0.194,	(0.526,	(0.605,	(0.632,	(0.678,
	0.274)	0.197)	0.343)	0.706)	0.365)	0.292)	0.283)	0.238)
p_{16}	(0.267,	(0.242,	(0.704,	(0.503,	(0.573,	(0.667,	(0.477,	(0.378,
	0.633)	0.658)	0.215)	0.395)	0.321)	0.246)	0.423)	0.521)
p_{17}	(0.437,	(0.544,	(0.407,	(0.281,	(0.221,	(0.361,	(0.457,	(0.580,
	0.462)	0.355)	0.491)	0.639)	0.678)	0.553)	0.442)	0.320)
p_{18}	(0.786,	(0.770,	(0.438,	(0.610,	(0.468,	(0.592,	(0.390,	(0.520,
	0.150)	0.166)	0.461)	0.311)	0.429)	0.329)	0.509)	0.369)
p_{19}	(0.605,	(0.765,	(0.550,	(0.362,	(0.368,	(0.573,	(0.643,	(0.451,
	0.292)	0.157)	0.343)	0.538)	0.529)	0.321)	0.251)	0.434)
p_{20}	(0.742,	(0.696,	(0.486,	(0.214,	(0.711,	(0.428,	(0.654,	(0.709,
	0.180)	0.219)	0.411)	0.686)	0.207)	0.470)	0.256)	0.207)
p_{21}	(0.541,	(0.538,	(0.263,	(0.438,	(0.415,	(0.187,	(0.421,	(0.497,
	0.357)	0.362)	0.629)	0.459)	0.472)	0.733)	0.469)	0.401)
p_{22}	(0.292,	(0.152,	(0.280,	(0.622,	(0.533,	(0.343,	(0.393,	(0.235,
	0.605)	0.773)	0.618)	0.277)	0.367)	0.550)	0.495)	0.685)
p_{23}	(0.695,	(0.718,	(0.420,	(0.647,	(0.614,	(0.516,	(0.357,	(0.459,
	0.227)	0.205)	0.479)	0.269)	0.306)	0.382)	0.541)	0.438)
p_{24}	(0.605,	(0.641,	(0.470,	(0.267,	(0.321,	(0.593,	(0.362,	(0.576,
	0.292)	0.255)	0.427)	0.632)	0.575)	0.304)	0.533)	0.315)
<i>p</i> 25	(0.314,	(0.211,	(0.557,	(0.589,	(0.299,	(0.343,	(0.310,	(0.369,
	0.582)	0.724)	0.339)	0.310)	0.623)	0.548)	0.577)	0.520)

	G 1	G ₂	G3	G 4	G 5	G 6	G 7	G 8
p_1	0.724	0.644	0.302	0.355	0.465	0.212	0.365	0.660
p_2	0.555	0.770	0.365	0.203	0.192	0.443	0.440	0.671
<i>p</i> ₃	0.195	0.157	0.398	0.309	0.410	0.592	0.636	0.243
p_4	0.371	0.220	0.355	0.644	0.509	0.287	0.212	0.171
p_5	0.309	0.177	0.520	0.659	0.298	0.177	0.281	0.278
p_6	0.686	0.724	0.232	0.469	0.415	0.336	0.483	0.585
p_7	0.453	0.729	0.303	0.194	0.208	0.402	0.593	0.636
p_8	0.189	0.179	0.459	0.453	0.443	0.617	0.596	0.410
p_9	0.359	0.318	0.583	0.482	0.452	0.592	0.437	0.379
p_{10}	0.800	0.755	0.470	0.294	0.755	0.759	0.626	0.761
p_{11}	0.304	0.304	0.617	0.435	0.453	0.554	0.634	0.359
p_{12}	0.393	0.305	0.686	0.656	0.826	0.587	0.464	0.516
p_{13}	0.177	0.179	0.424	0.303	0.443	0.520	0.662	0.515
p_{14}	0.667	0.811	0.731	0.208	0.541	0.546	0.833	0.802
<i>p</i> 15	0.687	0.782	0.609	0.213	0.583	0.667	0.686	0.735
p_{16}	0.293	0.266	0.761	0.554	0.634	0.725	0.525	0.416
p_{17}	0.481	0.599	0.449	0.303	0.243	0.392	0.503	0.638
p_{18}	0.836	0.819	0.482	0.658	0.516	0.639	0.430	0.578
p_{19}	0.667	0.825	0.609	0.398	0.406	0.634	0.711	0.503
p_{20}	0.800	0.755	0.536	0.235	0.769	0.471	0.713	0.769
p_{21}	0.596	0.592	0.291	0.483	0.462	0.202	0.467	0.548
<i>p</i> ₂₂	0.322	0.163	0.309	0.685	0.586	0.380	0.437	0.254
<i>p</i> ₂₃	0.749	0.773	0.463	0.701	0.663	0.568	0.393	0.506
p_{24}	0.667	0.708	0.518	0.294	0.354	0.654	0.400	0.639
p_{25}	0.347	0.225	0.615	0.648	0.322	0.380	0.345	0.410

Table 9. Score values of normalized A-IF-DM for SSS.

Table 10. The GDSs of each option for SSS.

	G 1	G_2	G3	G 4	G 5	G ₆	G 7	G 8
p_1	1.6217	1.0759	0.0637	0.1387	0.4428	0.0000	0.1599	1.1437
p_2	0.8509	1.6354	0.2173	0.0071	0.0000	0.3710	0.3633	1.1859
<i>p</i> ₃	0.0288	0.0000	0.5207	0.2513	0.5661	1.3925	1.6271	0.1014
p_4	0.5033	0.0470	0.4249	1.9665	1.1867	0.2129	0.0338	0.0000
p_5	0.2660	0.0000	1.2735	2.0482	0.2261	0.0000	0.1680	0.1608
p_6	1.2663	1.4773	0.0000	0.3363	0.2079	0.0825	0.3808	0.7855
p_7	0.4601	1.5103	0.1331	0.0000	0.0091	0.3270	0.9171	1.1951
p_8	0.0082	0.0000	0.5112	0.4865	0.4536	1.3090	1.1880	0.3721
p_9	0.0549	0.0000	1.4334	0.6222	0.4216	1.5176	0.3413	0.1084
p_{10}	0.8729	0.6472	0.1302	0.0000	0.6472	0.6622	0.3610	0.6710
p_{11}	0.0000	0.0000	1.3020	0.3403	0.4128	0.9213	1.4219	0.1108
p_{12}	0.0695	0.0000	0.9438	0.8016	1.7179	0.5291	0.1817	0.3048
<i>p</i> ₁₃	0.0000	0.0016	0.4768	0.1945	0.5360	0.8393	1.6124	0.7600
p_{14}	0.4505	0.8748	0.6138	0.0000	0.2125	0.2189	0.9730	0.8403

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	G 1	G_2	G ₃	G 4	G 5	G 6	G 7	G 8
p_{15}	0.5130	0.9805	0.3198	0.0000	0.2804	0.4517	0.5093	0.7313
p_{16}	0.0194	0.0000	1.3780	0.5156	0.8036	1.1967	0.4321	0.1966
p_{17}	0.5284	1.2034	0.4024	0.0590	0.0000	0.2342	0.6365	1.4719
p_{18}	1.5496	1.4430	0.0466	0.5777	0.1075	0.4926	0.0000	0.2741
p 19	0.7022	1.6520	0.4651	0.0000	0.0072	0.5546	0.9383	0.1807
p_{20}	0.9030	0.7114	0.2444	0.0000	0.7581	0.1576	0.5991	0.7581
p_{21}	1.0719	1.0452	0.0846	0.4851	0.4099	0.0000	0.4242	0.7941
p_{22}	0.1857	0.0000	0.1554	1.8131	1.2771	0.3652	0.5856	0.0704
<i>p</i> ₂₃	1.1431	1.3033	0.0667	0.8688	0.6878	0.3258	0.0000	0.1486
p_{24}	0.9487	1.1870	0.4199	0.0000	0.0498	0.8840	0.1262	0.8218
<i>p</i> 25	0.1361	0.0000	1.5177	1.7289	0.0886	0.2248	0.1306	0.3938

Table 11. The LDSs of each option for SSS.

	G 1	G ₂	G ₃	G 4	G 5	G ₆	G 7	G_8
p_1	0.3622	0.3056	0.0637	0.1012	0.1790	0.0000	0.1082	0.3169
p_2	0.2283	0.3750	0.1122	0.0071	0.0000	0.1628	0.1609	0.3107
p_3	0.0288	0.0000	0.1824	0.1150	0.1915	0.3292	0.3625	0.0651
p_4	0.1650	0.0404	0.1518	0.3903	0.2789	0.0957	0.0338	0.0000
p_5	0.1051	0.0000	0.2730	0.3837	0.0963	0.0000	0.0828	0.0804
p_6	0.3602	0.3904	0.0000	0.1880	0.1452	0.0825	0.1991	0.2801
p_7	0.1690	0.3492	0.0711	0.0000	0.0091	0.1358	0.2604	0.2885
p_8	0.0082	0.0000	0.2305	0.2256	0.2173	0.3606	0.3433	0.1902
p_9	0.0549	0.0000	0.3547	0.2195	0.1793	0.3667	0.1593	0.0816
p_{10}	0.3743	0.3410	0.1302	0.0000	0.3410	0.3440	0.2456	0.3455
p_{11}	0.0000	0.0000	0.3152	0.1319	0.1500	0.2517	0.3323	0.0554
p_{12}	0.0695	0.0000	0.3009	0.2772	0.4115	0.2227	0.1256	0.1666
p_{13}	0.0000	0.0016	0.1921	0.0980	0.2069	0.2668	0.3773	0.2629
p_{14}	0.2929	0.3848	0.3337	0.0000	0.2125	0.2157	0.3988	0.3790
<i>p</i> 15	0.3592	0.4312	0.3001	0.0000	0.2804	0.3440	0.3584	0.3956
p_{16}	0.0194	0.0000	0.3564	0.2074	0.2650	0.3305	0.1865	0.1080
p_{17}	0.2342	0.3503	0.2027	0.0590	0.0000	0.1466	0.2558	0.3886
p_{18}	0.3637	0.3484	0.0466	0.2042	0.0770	0.1872	0.0000	0.1326
p_{19}	0.2406	0.3819	0.1887	0.0000	0.0072	0.2111	0.2800	0.0939
p_{20}	0.3774	0.3473	0.2010	0.0000	0.3567	0.1576	0.3193	0.3567
p_{21}	0.3747	0.3709	0.0846	0.2673	0.2473	0.0000	0.2520	0.3291
p_{22}	0.1230	0.0000	0.1129	0.4038	0.3272	0.1679	0.2119	0.0704
p_{23}	0.3391	0.3620	0.0667	0.2934	0.2572	0.1667	0.0000	0.1076
p_{24}	0.3096	0.3437	0.1859	0.0000	0.0498	0.2988	0.0880	0.2864
<i>p</i> 25	0.1115	0.0000	0.3564	0.3865	0.0886	0.1416	0.1096	0.1690

	OGDS	$ ho_1$	OGDS ^N	OLDS	ρ_2	OLDS ^N	CS _i	Final Ranking	
G_1	0.5668	7	0.3633	0.2034	4	0.3669	-0.0206	8	
G_2	0.6710	8	0.4301	0.2051	3	0.3700	-0.0189	7	
G_3	0.5235	2	0.3356	0.1923	6	0.3469	0.0074	3	
G_4	0.5354	4	0.3432	0.1593	8	0.2874	-0.0162	6	
G_5	0.4619	1	0.2961	0.1835	7	0.3310	0.0014	5	
G_6	0.5285	3	0.3388	0.1993	5	0.3596	0.0065	4	
G_7	0.5630	6	0.3609	0.2097	2	0.3783	0.0091	2	
G_8	0.5405	5	0.3465	0.2101	1	0.3790	0.0280	1	

Table 12. The collective scores of each option for SSS.

Comparison and Discussion 6.

To elucidate the efficacy and irreplaceable merits of the IF-GLDS model, the IF-TOPSIS [14] and IF-COPRAS [79] tools are utilized to treat the abovementioned MCDM problem.

6.1. IF-TOPSIS model

Steps 1-4. Similar to aforesaid model

Step 5. Determine the "ideal solution (IS)" and "anti-ideal solution (A-IS)"

Let ϕ^+ and ϕ^- symbolize the IF-IS and IF-A-IS and are estimated as

$$\phi^{+} = \begin{cases} \max_{i} \mu_{ij}, \text{ for benefit} - typep_{j} \\ \min_{i} \nu_{ij}, \text{ for cost} - typep_{j} \end{cases} \text{ for } j = 1, 2, \dots, n,$$
(21)

$$\phi^{-} = \begin{cases} \min_{i} \mu_{ij}, \text{ for benefit} - typep_{j} \\ \max_{i} \nu_{ij}, \text{ for cost} - typep_{j} \end{cases} \text{ for } j = 1, 2, \dots, n.$$

$$(22)$$

Step 6. Compute the degree of similarities from IF-IS and IFA-IS

Here, we find weighted similarity $S(G_i, \phi^+)$ among the options G_i and the IF-IS ϕ^+ .

$$S(G_{i},\phi^{+}) = 1 - \sum_{i=1}^{n} w_{j} \sin\left[\left\{\frac{\left|\mu_{\varsigma_{ij}} - \mu_{\phi^{+}}\right| + \left|\nu_{\varsigma_{ij}} - \nu_{\phi^{+}}\right|\right\}}{4\left(1 + \left|\pi_{\varsigma_{ij}} - \pi_{\phi^{+}}\right|\right)}\right\}\pi\right],\tag{23}$$

and the weighted similarity $S(G_i, \phi^-)$ among the options G_i and the IFA-IS ϕ^- is given as follows:

$$S(G_i, \phi^-) = 1 - \sum_{i=1}^n w_i \sin\left[\left\{\frac{\left|\mu_{y_{ij}} - \mu_{\phi^-}\right| + \left|\nu_{y_{ij}} - \nu_{\phi^-}\right|\right}{4\left(1 + \left|\pi_{y_{ij}} - \pi_{\phi^-}\right|\right)}\right\}\pi\right].$$
(24)

Step 7. Obtain the "relative closeness coefficient (RCC)"

The expression for computing of RCC of each option with the IF-IS is given by

$$\mathbb{C}(G_i) = \frac{S(G_i, \phi^+)}{S(G_i, \phi^+) + S(G_i, \phi^-)}, i = 1, 2, \dots, m.$$
(25)

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Step 8. Select the maximum degree, $\mathbb{C}(G_k)$, among the $\mathbb{C}(G_i)$, i = 1, 2, ..., m. Hence, G_k is the best option.

From Table 7, Eqs (21) and (22), IF-IS and IFA-IS are obtained. Now, the computational results are depicted in Table 13.

$$\begin{split} \phi^+ &= \{(0.242, 0.658), (0.200, 0.700), (0.321, 0.577), (0.314, 0.585), (0.300, 0.599), (0.242, 0.658), \\ (0.237, 0.663), (0.337, 0.559), (0.534, 0.358), (0.742, 0.180), (0.573, 0.321), (0.779, 0.161), (0.297, 0.600), (0.781, 0.152), (0.726, 0.197), (0.704, 0.215), (0.320, 0.580), (0.786, 0.150), (0.765, 0.157), \\ (0.742, 0.180), (0.357, 0.541), (0.277, 0.622), (0.718, 0.205), (0.641, 0.255), (0.310, 0.589) \} \\ \phi^- &= \{(0.726, 0.197), (0.745, 0.178), (0.740, 0.181), (0.779, 0.161), (0.770, 0.166), (0.704, 0.215), \\ (0.742, 0.180), (0.765, 0.168), (0.289, 0.609), (0.267, 0.632), (0.276, 0.624), (0.277, 0.622), (0.770, 0.166), (0.189, 0.710), (0.194, 0.706), (0.242, 0.658), (0.678, 0.221), (0.390, 0.509), (0.362, 0.538), \\ (0.214, 0.686), (0.733, 0.187), (0.773, 0.152), (0.357, 0.541), (0.267, 0.632), (0.724, 0.211)\}. \end{split}$$

SS Options	$S(G_i, \phi^+)$	$S(G_i, \phi^-)$	$\mathbb{C}(G_i)$	Ranking	
G_1	0.714	0.654	0.522	4	
G_2	0.728	0.652	0.528	3	
G_3	0.681	0.677	0.502	6	
G_4	0.627	0.747	0.456	8	
G_5	0.677	0.686	0.497	7	
G_6	0.696	0.664	0.512	5	
G_7	0.724	0.640	0.531	2	
G_8	0.729	0.637	0.534	1	

Table 13. Ranking orders of IF- TOPSIS method for SSS.

From **Table 13**, G_8 is the best SSS alternative and prioritization of SSS option is obtained as $G_8 > G_7 > G_2 > G_1 > G_3 > G_6 > G_5 > G_4$.

6.2. IF-COPRAS model

Steps 1–4. Similar to aforesaid model Step 5. Sum of ratings of criteria for benefit and cost-type

At this point, each option is expressed with the sum of maximizing attribute, $\alpha_i^{(1)}$, for benefit-

type, and minimizing attribute, $\alpha_i^{(2)}$, for cost-type. To obtain the degree of $\alpha_i^{(1)}$ and $\alpha_i^{(2)}$, we used the following AOs as

$$\alpha_{i}^{(1)} = \bigoplus^{j=1}_{l} w_{j} y_{ij}, i = 1, 2, \dots, m,$$
(26)

$$\alpha_{i}^{(2)} = \bigoplus^{n} w_{j} y_{ij}, i = 1, 2, \dots, m.$$
⁽²⁷⁾

Step 6. Calculate the "relative weight (RW)" of each option

The expression for computing the RW (γ_i) of each option using

$$\gamma_{i} = \mathbb{S}^{*}\left(\alpha_{i}^{(1)}\right) + \frac{\min_{i}\mathbb{S}^{*}\left(\alpha_{i}^{(2)}\right)\sum_{i=1}^{m}\mathbb{S}^{*}\left(\alpha_{i}^{(2)}\right)}{\mathbb{S}^{*}\left(\alpha_{i}^{(2)}\right)\sum_{i=1}^{m}\frac{\min_{i}\mathbb{S}^{*}\left(\alpha_{i}^{(2)}\right)}{\mathbb{S}^{*}\left(\alpha_{i}^{(2)}\right)}}, i = 1, 2, \dots, m.$$
(28)

Here, $\mathbb{S}^*(\alpha_i^{(1)})$ and $\mathbb{S}^*(\alpha_i^{(2)})$ symbolize the generalized scores of $\alpha_i^{(1)}$ and $\alpha_i^{(2)}$, respectively.

Formula (28) can also be prearranged as

$$\gamma_{i} = \mathbb{S}^{*}(\alpha_{i}^{(1)}) + \frac{\sum_{i=1}^{m} \mathbb{S}^{*}(\alpha_{i}^{(2)})}{\mathbb{S}^{*}(\alpha_{i}^{(2)}) \sum_{i=1}^{m} \frac{1}{\mathbb{S}^{*}(\alpha_{i}^{(2)})}}, i = 1, 2, \dots, m.$$
(29)

Step 7. Prioritize the options

Corresponding to the RWs of each option, the preference of alternatives is obtained. The highest RW of option has been considered as superior one, and hence, it is the best choice.

$$G^* = \max_i \gamma_i, i = 1, 2, \dots, m.$$
(30)

Step 8. Compute the "utility degree (UD)" of each option

By assessing the examined options with the best choice, the UD of each option is calculated. The expression for the computing the UD δ_i of each option as

$$\delta_i = \frac{\gamma_i}{\gamma_{max}} \tag{31}$$

Here, γ_i and γ_{max} are the RWs estimated from Eq (28).

Now, the results of IF-COPRAS are mentioned in Table 14. From Table 7 and Eqs (26)–(31), the degrees of $\alpha_i^{(1)}$ and $\alpha_i^{(2)}$, γ_i , preferences of options and δ_i , i = 1, 2, ..., m of each option are obtained. Based on the UD, G_7 is the best SSS option because it has the maximum RW degree (0.756) of SSS alternative.

SS Options	$\alpha_i^{(1)}$	$\mathbb{S}^*(\alpha_i^{(1)})$	$\alpha_i^{(2)}$	$\mathbb{S}^*(\alpha_i^{(2)})$	γ_i	δ_i	Ranking
G_1	(0.350, 0.571)	0.378	(0.349, 0.572)	0.377	0.737	97.48%	3
G_2	(0.372, 0.548)	0.402	(0.384, 0.540)	0.413	0.730	96.51%	4
G_3	(0.315, 0.609)	0.339	(0.346, 0.571)	0.375	0.700	92.58%	6
G_4	(0.227, 0.707)	0.242	(0.326, 0.597)	0.351	0.628	83.02%	8
G_5	(0.322, 0.603)	0.346	(0.354, 0.564)	0.383	0.700	92.50%	7
G_6	(0.331, 0.589)	0.358	(0.364, 0.556)	0.393	0.703	92.90%	5
G_7	(0.313, 0.610)	0.337	(0.299, 0.621)	0.323	0.756	100.00%	1
G_8	(0.322, 0.600)	0.347	(0.313, 0.608)	0.338	0.748	98.86%	2

Table 14. The outcomes of IF-COPRAS model for SSS.



Figure 3. Preference order comparison of proposed and existing methods.

The preference order of SSS alternative obtained by developed framework show a great conformity with IF-TOPSIS [14] approach. As compared with the IF-TOPSIS approach, we obtain same optimal sustainable supplier alternative SSS-8 (G_8), whereas by comparing with the IF-COPRAS [79] approach, the optimal sustainable supplier alternative SSS-7 (G_7). The preference orders of proposed method, IF-TOPSIS and IF-COPRAS method are slightly totally different. The comparison results proposed and existing models are presented in Figure 3. From Figure 3, we observe that the prioritizations of options are different from the ordering of the developed IF-GLDS model. While the best SSS alternative is G_8 from the IF-GLDS and IF-TOPSIS models and IF-COPRAS model provides the best option is G_7 . Subsequently, relating the computation processes of some extant models for the same case study, we can find some outcomes as follows:

- Based on to the computation processes of these models, we find that the GDS and LDS estimated by the IF-GLDS can not only certify that the designated option executes remarkably in whole, but also evade the bad performance over each criterion. Thus, the developed IF-GLDS can offer managers with an effective reference as compared with the IF-TOPSIS [14]. While, the IF-VIKOR model assumes both "group utility" and "individual regret" of options, the outcomes occurred by it may be unreliable concerning the diverse weights [12].
- We can use the GDS and LDS and corresponding subordinate preferences obtained by the developed IF-GLDS to find collective score each option over diverse criteria. Whereas, the IF-TOPSIS and IF-COPRAS can only deliver the only final preference of options. The IF-GLDS diminishes the workload of researchers and supports them catch issues to improve. In this viewpoint, the computation outcomes of the developed IF-GLDS model are more practical.
- In the developed IF-GLDS model, we find the weights of DEs and criteria, respectively. It would exclude the impact of subjective issues as much as possible.
- Both the DFs and SPSs of options are measured in the developed model. In this way, the result determined by the IF-GLDS model is not only the nearest one to the IF-IS but also dominates other solutions.

Also, the benefits of proposed IF-GLDS model over the Sen et al. [78] approach are given by

• The IF-GLDS tool uses the vector and linear normalization process, and entropy-score functionbased model, whereas IF-COPRAS tool applied only vector normalization process. IF-TOPSIS and IF-GRA utilize linear normalization process. Hence, the developed tool evades the information loss and offers more precise decision outcomes utilizing diverse attributes.

- The IF-GLDS tool computes the criteria weight using the proposed entropy and score function based model. A new weighting formula by combining the score function and entropy to derive the weights of criteria is reasonable, smoothness, has lesser number of computational steps and confirms the effectiveness and usefulness of the developed method. Whereas in Sen et al. [78], weight of attribute is obtained randomly.
- For the IF-TOPSIS [14] model, it is compulsory to estimate the similarity between each option and that of the IF-IS, which is a complicated procedure and decreases the accurateness of the outcomes, whilst IF-GRA [78] can be derived from the correlation of complicated relationships between components of an IF-DM using IF-distances. The IF-TOPSIS, IF-GRA and IF-COPRAS approaches have two significant limitations [77]: (a) the prioritizations of options may vary under possible transformations of the initial criteria values, in the measurement-theoretic sense of the term; and (b) the prioritization ordering of options may change if a new candidate added to the considered set of options or a preceding one is deleted from it or swapped it. Here, we introduce an integrated IF-GLDS approach to consider pairwise comparisons of options, so as to establish the DFs among alternatives. Then, the gained and lost dominance scores are computed by the summation operator and the maximum operator, respectively. Furthermore, the OGDS and OLDS of options are obtained with associated weights of criteria. The final priority of options is derived by an aggregation function, which considers both the subordinate orders and the OGDS and OLDS simultaneously.

7. Conclusions

In this paper, the IFNs are used to represent uncertainty and fuzzy arguments in assessing sustainable supplier. The evaluation criteria are established, which comprise many qualitative and quantitative inducing factors from technology to society. To make a scientific decision, the IF-GLDS method is proposed based on the proposed IF-entropy. In the context of IFSs setting, how to compute the criteria and DEs' weights is a critical topic, which is the basis of ranking the options in MCDM process and has attracted the interest of many scholars in the domain. For this reason, we conducted a survey on the ranking models for IFNs in the existing literature. It is proven that the exiting methods work well in comparing IFNs, but in some particular cases they have some limitations. Thus, the inverse tangent entropy measure is proposed and a weighting procedure with IF-score function is presented to find the criteria weights. Finally, we have developed the IF-GLDS model to prioritize sustainable supplier selection of iron and steel enterprise. The efficacy and practicality of the developed model is justified with the comparative discussions. The main benefit of the developed model is valid for handling realistic decision-making problem.

In future works, we aim to extend the model under different disciplines such as the PFSs, SVNSs, "complex fuzzy sets (CFSs)" and others for group decision making to eradicate the limitations. Moreover, we can increase the number of sustainability pillars and attributes considered in this study in the light of experience so then the researches can assess the suppliers more proficiently.

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Conflict of interest

The authors declare there is no conflict of interest.

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