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Research article

Decision support algorithm under SV-neutrosophic hesitant fuzzy rough information with confidence level aggregation operators

Muhammad Kamran[1], **Rashad Ismail**[2,3,*], **Shahzaib Ashraf**[1], **Nadeem Salamat**[1,*], **Seyma Ozon Yildirim**[4] **and Ismail Naci Cangul**[4]

[1] Institute of Mathematics, Khwaja Fareed University of Engineering and Information Technology, Rahim Yar Khan 64200, Pakistan

[2] Department of Mathematics, Faculty of Science and Arts, King Khalid University, Muhayl Assir 61913, Saudi Arabia

[3] Department of Mathematics and Computer, Faculty of Science, Ibb University, Ibb 70270, Yemen

[4] Department of Mathematics, Bursa Uludag University, Gorukle 16059, Turkey

* **Correspondence:** Email: nadeem.salamat@kfueit.edu.pk, rismail@kku.edu.sa.

Abstract: To deal with the uncertainty and ensure the sustainability of the manufacturing industry, we designed a multi criteria decision-making technique based on a list of unique operators for single-valued neutrosophic hesitant fuzzy rough (SV-NHFR) environments with a high confidence level. We show that, in contrast to the neutrosophic rough average and geometric aggregation operators, which are unable to take into account the level of experts' familiarity with examined objects for a preliminary evaluation, the neutrosophic average and geometric aggregation operators have a higher level of confidence in the fundamental idea of a more networked composition. A few of the essential qualities of new operators have also been covered. To illustrate the practical application of these operators, we have given an algorithm and a practical example. We have also created a manufacturing business model that takes sustainability into consideration and is based on the neutrosophic rough model. A symmetric comparative analysis is another tool we use to show the feasibility of our proposed enhancements.

Keywords: confidence level; neutrosophic information; aggregation operators; hesitant information; rough sets; decision-making
Mathematics Subject Classification: 03B52, 03E72

List of abbreviation used in manuscript

Name	Abbreviation
Confidence Level	CL
single-valued neutrosophic set	NMG
Fuzzy Set	FS
Membership degree	MD
intuitionistic fuzzy set	IFS
intuitionistic fuzzy number	IFN
Non Membership degree	NMD
Hesitant Fuzzy Set	HFS
Pythagorean fuzzy sets	PyFSs
Single-Valued neutrosophic weighted geometric	SV-NWG
Neutrosophic rough weighted averaging	NRWA
Neutrosophic rough weighted geometric	NRWG
Neutrosophic hesitant fuzzy rough weighted geometric	NHFRG
Neutrosophic hesitant fuzzy rough weighted averaging	NHFRWA
Abstention degree	AD
Decision Making	DM
Aggregation Operators	AOs
classical set theory	CST
Indeterminacy membership degree	IMD
Rough Set	RS
Single-Valued neutrosophic weighted averaging	SV-NWA
Multiple criteria decision making	MCDM
Neutrosophic Rough set	NRS
T-Spherical hesitant fuzzy weighted averaging	T-SHFWA
T-Spherical hesitant fuzzy weighted geometric	T-SHFWG
Picture FSs	PFSs
Neutrosophic set	NS

1. Introduction

Environmental challenges are now more prevalent in our daily lives than ever before. One of several subcategories that fall under the broad heading of sustainable development is sustainable manufacturing. Throughout the production process, other social and environmental issues also surface. These production-related challenges may be overcome via sustainable manufacturing techniques. Manufacturing that is environmentally responsible and resource-efficient is the goal of sustainable manufacturing. As a result of their good financial standing, these lodgings are also secure for their residents, employees, and customers. The choice of appropriate indicators for tracking the sustainability of manufacturing, an assessment method for detecting weak regions, and system upgrades to reinforce the sustainable manufacturing process are the four main elements of a manufacturing plan. A technology system's total performance can be assessed, or at least two technology systems can be compared, using indicators for technology evaluation. Instead of compiling a general set of indications appropriate for all purposes. As a method for making decisions, multi-criteria analysis collects information on a range of indicators or criteria to determine how various goals might be most effectively achieved. With different units placed next to one another, indicators can be evaluated. Multi-criteria analysis has a well-established field called fuzzy set theory, which provides answers to issues that traditional multi-criteria analysis has hitherto been unable to address. The techniques described in [1, 2] have been crucial for managing information in practical situations. Moreover, the frameworks in [3, 4] only define objects using membership degree (MD) and non membership degree (NMD). The information in many real-life issues, however, could not be fully characterised by only MD and NMD due to the occurrence of various types of abstention and refusal circumstances, such as when voting or expressing one's viewpoint. Because of this, these intuitionistic fuzzy sets (IFSs) [5], Pythagorean fuzzy sets (PyFSs) [6], and q rung orthopair fuzzy sets (qROFSs) [7] either couldn't entertain these events or did so while suffering significant information loss. Cuong obtained four degrees: an MD, an abstention degree (AD), an NMD, and a refusal degree (RD) [8] in order to formalise picture FSs (PFSs) and explicate a situation with greater accuracy and less information loss [9]. Although while PFSs could identify more information loss, MD, AD, and NMD still had restrictions that prevented the decision-makers from openly expressing their opinions. To overcome these restrictions, Mahmood et al. developed the idea of PFSs into spherical FSs (SFSs) and finally T-spherical FSs (TSFSs) [10, 11]. Decision-makers could therefore assign these MD, AD, and NMD according to their own preferences. Fuzzy sets (FSs) [12] are an efficient approach that generalise classical set theory (CST) [13], in which items have an MD that belongs to [0, 1]. Similar to CST [14], FSs' functions and relations can be explained. Since its introduction in 1965, FSs has been used in a range of situations and industries. Among the disciplines where FSs are applied include artificial intelligence [15], medicine [16], statistics [17], medical diagnosis [18, 19], and clustering [20, 21].

Researchers who study aggregation operators (AOs) such as Fahmi et al. [22] proposed cubic fuzzy Einstein AOs and their application to DM problems. An intuitionistic fuzzy set (IFS) with the structure of MD and NMD was created by Atanassov [5]. IFS applies the limitation that the sum (MD, NMD) belongs to [0, 1]. IFS is a highly helpful framework that can provide a two-dimensional scenario in problem-solving scenarios, it has been highlighted. Based on this concept, numerous scholars have developed IFS methodologies and applications in a variety of fields [23–25]. Although there are many theories for dealing with ambiguous information and knowledge [26], they are only partially successful in handling complex real-world situations. Smarandache [27] as stating that by combining unconventional analysis and a tri-component set, which sparked the creation of the neutrosophic set theorizing. The three membership functions that make up a NS are MD, depending on whether the output of each function is a real standard subset or a non-standard subset of the input data that is subset of the nonstandard unit interval $]0, 1^{+}[$. indeterminacy membership degree (IMD) and NMD are used. The use of NSs in applications possess a track record of achievement in the disciplines of cluster analysis and image processing [32, 33]. By condensing NSs, Wang et al. [27] proposed single-valued NS (SV-NS). Instead, SV-NSs can be viewed as a development of intuitionistic fuzzy sets with three independent membership functions and function values contained in the unit closed interval [5]. SV-NSs [34–36] raise a novel, well-liked research issue. In [37], a generalisation of fuzzy logics, the neutrosophic idea was applied to logics and numerous crucial components were looked explored. Neutrosophic rough sets (NRSs) [38] are a brand-new hybrid mathematical structure that deals with ambiguous and incomplete information and investigates certain operations and their attributes using combined rough set (RS) theory [39]. The constructive technique [40] was used to create a variety of rough set models, such as arbitrary binary relation-based rough sets, covering-based rough sets, and rough fuzzy sets. Shao et al. [41] defined the single-valued neutrosophic hesitant fuzzy set (SV-NHFS) principle, which is a generalisation of the SV-NFSs. There are three other techniques for handling incorrect information: rough sets, HFS, and SV-NSs. To simultaneously take advantage of the advantages of both, a hybrid model of SV-NSs and rough sets is needed. The approximate representation of each SV-NS in the system, as well as the extension and reduction of the single-valued neutrosophic information system, have all been the focus of numerous studies. In order to do this, we examine a broad framework used in the current research to analyse single-valued neutrosophic rough sets and suggest creating such single-valued neutrosophic rough sets by combining rough sets and HFS. We will formally investigate the hybrid model using axiomatic and proactive methods [42, 43]. The goal of this study is to use multicriteria decision-making to improve the sustainability of manufacturing work cells. To do this, two actions have been identified:

(a) As a component of a decision-making process, define and characterise a matrix, decide on and put into practise an acceptable weighting mechanism, and decide on and put into practise a suitable ranking system.

(b) Describe a representative work cell and use the environmental and sustainable analytical approach to illustrate the procedure.

There are various sorts, but the weighted and un-weighted categories are the two most common. The weighted decision matrix assigns different weights, whereas the un-weighted one assumes that all criteria are equally important. A set of options can be compared against a set of criteria using the potent quantitative tool known as the weighted decision matrix. When you have to select the best alternative and must carefully analyze a wide variety of variables, it is a really helpful tool that you may utilize. But the percentage of times you anticipate coming close to the same estimate if you repeat your experiment or re-sample the population in the same way is known as the confidence level. The upper and lower bounds of the estimate you anticipate finding at a particular level of confidence make up the confidence interval. We propose many new aggregation operators (AOs), including the CL-SV-NHFRG and CL-SV-NHFRA, for the underlying pretexts:

(1) Decision-makers are given more leeway with SV-NHFRS due to the combined idea of SV-NS, HFS, and RS.

(2) SV-NHFRS makes use of upper and lower approximation spaces, in contrast to SV-NRS.

(3) The familiarity of specialists with the objects under investigation cannot be taken into account for first review by SV-NWA and SV-NWG aggregation operators, but it can be by CL-SV-NHFRA and CL-SV-NHFRG AOs.

(4) This article seeks to address more complex and advanced data due to the clarity of the CL-SV-NHFRA and CL-SV-NHFRG operators and the fact that they cover the decision-making technique.

(5) All shortcomings are addressed in the suggested work.

Consequently, the following are the research's findings:

(i) To begin the formation of new AOs such as CL-SV-NHFRG and CL-SV-NHFRA.

(ii) We've specified attributes for the suggested aggregating operations.

(iii) Multi-criteria decision-making (MCDM) is a technique developed to handle the increasingly complicated data.

(iv) A real-world implementation of the algorithm has been provided, along with an SV-NHFRS-based method to enhance the sustainability of manufacturing operations that has also been demonstrated.

The structure of this article is as follows. In part 2, the fundamental ideas behind FS, NS, RS, SV-NRS, as well as a few fundamental operational laws, are reviewed. We introduce two brand-new aggregation operators in Section 3: CL-SV-NHFRWA and CL-SV-NHFRWG. In Section 4, a decision-making approach based on the proposed AOs is built, along with a solution to a numerical problem and numerical examples. We arrive at a conclusion in Section 5.

2. Preliminaries

The section will go over the core ideas for NSs, SV-NSs, SV-NRSs, scoring functions (SF), and accuracy functions (AF).

Definition 1. *Addressing a certain set ω. A FS [12] Z in ω is presented as*

$$Z = \{\langle \xi, \Lambda_Z(\xi)\rangle \,|\xi \in \omega\},$$

for each $\xi \in \omega$, the MD $\Lambda_Z : \omega \to \omega$ specifies the degree to which the element $\xi \in Z$, where $\Lambda_Z \in [0, 1]$.

Definition 2. *Addressing a certain set ω. An IFS [5] B in ω is presented as*

$$B = \{\langle \flat, \xi(\flat), \eta(\flat)\rangle \,|\flat \in \omega\},$$

for each $\flat \in \omega, \xi$ is the MD and η is the NMD to the IFS B, respectively, where $(\xi(\flat), \eta(\flat)) \in [0, 1]$ be the unit interval. Moreover, it is required that $0 \leq (\xi(\flat) + \eta(\flat)) \leq 1$, for each $\flat \in \omega$.

Definition 3. *Addressing a certain set ω and $\vartheta \in \omega$. A NS [27] $\flat$ in ω is denoted as MD $\xi_\flat(\vartheta)$, an IMD $\eta_\flat(\vartheta)$ and a NMD $£_\flat(\vartheta)$. $\xi_\flat(\vartheta)$, $\eta_\flat(\vartheta)$ and $£_\flat(\vartheta)$ are real standard and non-standard subset of $]0^{-}, 1^{+}[$ and*

$$\xi_\flat(\vartheta), \eta_\flat(\vartheta), £_\flat(\vartheta) : \omega \longrightarrow]0^{-}, 1^{+}[\,.$$

Definition 4. *The representation of neutrosophic set (NS) $\flat$ is mathematically defined as:*

$$\flat = \{\langle \vartheta, \xi_\flat(\vartheta), \eta_\flat(\vartheta), £_\flat(\vartheta))\rangle \,|\vartheta \in \omega\},$$

where

$$0^{-} < \xi_\flat(\vartheta) + \eta_\flat(\vartheta) + £_\flat(\vartheta) \leq 3^{+}.$$

Definition 5. *(See [28]) Addressing a certain set ω and $\vartheta \in \omega$. A SV-NS $\flat$ in ω is defined as MD $\xi_\flat(\vartheta)$, an IMD $\eta_\flat(\vartheta)$ and a NMD $£_\flat(\vartheta)$.*

$\xi_{\flat}(\vartheta)$, $\eta_{\flat}(\vartheta)$ and $\pounds_{\flat}(\vartheta)$ are real standard and non-standard subset of $[0, 1]$, and

$$\xi_{\flat}(\vartheta), \eta_{\flat}(\vartheta), \pounds_{\flat}(\vartheta) : \omega \longrightarrow [0, 1].$$

The representation of SV-NS $\flat$ is mathematically defined as:

$$\flat = \{\langle \vartheta, \xi_{\flat}(\vartheta), \eta_{\flat}(\vartheta), \pounds_{\flat}(\vartheta)) \rangle \, |\vartheta \in \omega\},$$

where

$$0 < \xi_{\flat}(\vartheta) + \eta_{\flat}(\vartheta) + \pounds_{\flat}(\vartheta) \leq 3.$$

Definition 6. *[30] For a fixed set Y, the $SV-NHFS$ F is mathematically represented as follows:*

$$F = \{\left\langle \flat, \xi_{\hbar_{\lambda}}(\flat), \eta_{\hbar_{\lambda}}(\flat), \pounds_{\hbar_{\lambda}}(\flat) \right\rangle |\flat \in Y\},$$

where $\xi_{\hbar_{\lambda}}(\flat), \eta_{\hbar_{\lambda}}(\flat)$ and $\pounds_{\hbar_{\lambda}}(\flat)$ are sets of some values in $[0, 1]$, called the hesitant MD, hesitant IMD and hesitant NMD sequentially where $\hbar$ shows the hesitant grade that must be satisfied the following properties:

$$\forall \flat \in Y, \forall \mu_{\lambda}(\flat) \in \xi_{\hbar_{\lambda}}(\flat), \forall \lambda_{\lambda}(\flat) \in \xi_{\hbar_{\lambda}}(\flat),$$

and

$$\forall \nu_{\lambda}(\flat) \in \pounds_{\hbar_{\lambda}}(\flat) \textit{ with } (\max (\xi_{\hbar_{\lambda}}(\flat))) + (\min (\eta_{\hbar_{\lambda}}(\flat))) + (\min (\eta_{\hbar_{\lambda}}(\flat))) \leq 3,$$

and

$$(\min (\xi_{\hbar_{\lambda}}(\flat))) + (\min (\eta_{\hbar_{\lambda}}(\flat))) + (\max (\pounds_{\hbar_{\lambda}}(\flat))) \leq 3.$$

For simplicity, we will use a pair $\lambda = (\xi_{\hbar_{\lambda}}, \eta_{\hbar_{\lambda}}, \pounds_{\hbar_{\lambda}})$ to mean $SV-NHFS$.

Definition 7. *(See [44]) Assume η be a universal set and $\hbar$ is relation on η. A set valued mapping is defined as*

$$\hbar^{*} : \eta \to M(\eta) \textit{ by } \hbar^{*}(\rho) = \{a \in \eta | (\rho, a) \in \hbar\},$$

for $\rho \in \eta$ where $\hbar^{*}(\rho)$ is referred to as the element's ρ successor neighborhood in connection to relation $\hbar$. The pair $(\eta, \hbar)$ is called (crisp) space of resemblance. Now for any set $\kappa \subseteq \eta$, the lower approximation (LA) and upper approximation (UA) of κ with respect to space of resemblance $(\eta, \hbar)$ is defined as:

$$\begin{aligned} \underline{\hbar}(\kappa) &= \{\rho \in \eta | \hbar^{*}(\rho) \subseteq \kappa\}; \\ \overline{\hbar}(\kappa) &= \{\rho \in \eta | \hbar^{*}(\rho) \cap \kappa \neq \phi\}. \end{aligned}$$

The pair $\left(\underline{\hbar}(\kappa), \overline{\hbar}(\kappa)\right)$ is called fuzzy RS where both $\underline{\hbar}(\kappa), \overline{\hbar}(\kappa) : M(\eta) \to M(\eta)$ are upper and lower approximation operators.

Definition 8. *(See [45]) Assume universal set $\ddot{U}$ and let $\varrho \in SV-NHFRS(\ddot{U} \times \ddot{U})$ be $SV-NF$ relation. then*

(i) ϱ is reflexive if

$$\xi_{\varrho}(\psi, \psi) = 1, \eta_{\varrho}(\psi, \psi) = 1 \textit{ and } \pounds_{\varrho}(\psi, \psi) = 1, \forall \psi \in \ddot{U};$$

(ii) ϱ is symmetric if

$$\forall (\psi, \xi) \in (\ddot{U} \times \ddot{U}), \xi_{\varrho}(\psi, \xi) = \xi_{\varrho}(\xi, \psi), \; \eta_{\varrho}(\psi, \xi) = \eta_{\varrho}(\xi, \psi)$$

$$\text{and}$$
$$£_{\varrho}(\psi,\xi) = £_{\varrho}(\xi,\psi);$$

(iii) ϱ is transitive if $\forall(\psi,t)\in(\ddot{U}\times\ddot{U})$,

$$\begin{aligned}\xi_{\varrho}(\xi,t) &\geq \vee_{\psi\in\ddot{U}}[\xi_{\varrho}(\xi,\psi)\wedge\xi_{\varrho}(\psi,t)],\\ \eta_{\varrho}(\xi,t) &= \wedge_{\psi\in\ddot{U}}[\eta_{\varrho}(\xi,\psi)\vee\eta_{\varrho}(\psi,t)],\end{aligned}$$

and

$$£_{\varrho}(\xi,t) = \wedge_{\psi\in\ddot{U}}[£_{\varrho}(\xi,\psi)\vee £_{\varrho}(\psi,t)].$$

Definition 9. *Assume universal set $\ddot{U}$ and let $\varrho\in SV-NHFRS(\ddot{U}\times\ddot{U})$ be $SV-NF$ relation. the pair $(\ddot{U},\varrho)$ represent a $SV-NF$ space of resemblance. Assume ς be any subset of $SV-NS(\ddot{U})$ i.e., $\varsigma\subseteq SV-NS(\ddot{U})$. Then on the bases of $SV-NF$ approximation space $(\ddot{U},\varrho)$, then the lower and upper approximations of ς are represented as $\overline{\varrho}(\varsigma)$ and $\underline{\varrho}(\varsigma)$ given as following:*

$$\overline{\varrho}(\varsigma) = \{\left\langle\psi,\xi_{\overline{\varrho}(\varsigma)}(\psi),\eta_{\overline{\varrho}(\varsigma)}(\psi),£_{\overline{\varrho}(\varsigma)}(\psi)\right\rangle|\psi\in\ddot{U}\},$$

$$\underline{\varrho}(\varsigma) = \{\langle\psi,\xi_{\underline{\varrho}(\varsigma)}(\psi),\eta_{\underline{\varrho}(\varsigma)}(\psi),£_{\underline{\varrho}(\varsigma)}(\psi)\rangle|\psi\in\ddot{U}\},$$

where

$$\begin{aligned}\xi_{\overline{\varrho}(\varsigma)}(\psi) &= \vee_{t\in\ddot{U}}[\xi_{\varrho}(\psi,\top)\vee\xi_{\varrho}(\top)],\\ \eta_{\overline{\varrho}(\varsigma)}(\psi) &= \wedge_{\top\in\ddot{U}}[\eta_{\varrho}(\psi,\top)\wedge\eta_{\varrho}(\top)],\\ £_{\overline{\varrho}(\varsigma)}(\psi) &= \wedge_{\top\in\ddot{U}}[£_{\varrho}(\psi,\top)\wedge £_{\varrho}(\top)],\\ \xi_{\underline{\varrho}(\varsigma)}(\psi) &= \wedge_{\top\in\ddot{U}}[\xi_{\varrho}(\psi,\top)\wedge\xi_{\varrho}(\top)],\\ \eta_{\underline{\varrho}(\varsigma)}(\psi) &= \wedge_{\top\in\ddot{U}}[\eta_{\varrho}(\psi,\top)\wedge\eta_{\varrho}(\top)],\\ £_{\underline{\varrho}(\varsigma)}(\psi) &= \vee_{\top\in\ddot{U}}[£_{\varrho}(\psi,\top)\vee £_{\varrho}(\top)].\end{aligned}$$

Such that

$$0 < \xi_{\overline{\varrho}(\varsigma)}(\psi)+\eta_{\overline{\varrho}(\varsigma)}(\psi)+£_{\overline{\varrho}(\varsigma)}(\psi)\leq 3,$$

and

$$0 < \xi_{\underline{\varrho}(\varsigma)}(\psi)+\eta_{\underline{\varrho}(\varsigma)}(\psi)+£_{\underline{\varrho}(\varsigma)}(\psi)\leq 3.$$

As $\underline{\varrho}(\varsigma)$ and $\overline{\varrho}(\varsigma)$ are $SV-NFSs$, so $\overline{\varrho}(\varsigma),\underline{\varrho}(\varsigma): SV-NFS(\ddot{U})\longrightarrow SV-NFS(\ddot{U})$ are LA and UA operators. So the pair

$$\varrho(\varsigma) = (\underline{\varrho}(\varsigma),\overline{\varrho}(\varsigma)) = \{\psi,\langle(\xi_{\underline{\varrho}(\varsigma)}(\psi),\eta_{\underline{\varrho}(\varsigma)}(\psi),£_{\underline{\varrho}(\varsigma)}(\psi)),(\xi_{\overline{\varrho}(\varsigma)}(\psi),\eta_{\overline{\varrho}(\varsigma)}(\psi),£_{\overline{\varrho}(\varsigma)}(\psi))|\psi\in\ddot{U}\}$$

is called $SV-NF$ rough set. For simplicity it can be denoted as

$\varrho(\varsigma) = (\underline{\varrho}(\varsigma),\overline{\varrho}(\varsigma)) = ((\underline{\xi}_{\hbar_\varsigma},\underline{\eta}_{\hbar_\varsigma},\underline{£}_{\hbar_\varsigma}),(\overline{\xi}_{\hbar_\varsigma},\overline{\eta}_{\hbar_\varsigma},\overline{£}_{\hbar_\varsigma}))$ *are known as $SV-NF$ rough number ($SV-NFRN$).*

Definition 10. *Let $F = \{(\underline{\xi}_{\hbar_\varsigma},\underline{\eta}_{\hbar_\varsigma},\underline{£}_{\hbar_\varsigma}),(\overline{\xi}_{\hbar_\varsigma},\overline{\eta}_{\hbar_\varsigma},\overline{£}_{\hbar_\varsigma})\}$ be a SV-neutrosophic hesitant rough number (SV-NHFRN). Then, SF and AF are describe as:*

$$Sc = \frac{1}{6}\left\{3 + \underline{\xi}_{\hbar_\varsigma} + \overline{\xi}_{\hbar_\varsigma} - \underline{\eta}_{\hbar_\varsigma} - \overline{\eta}_{\hbar_\varsigma} - \underline{\pounds}_{\hbar_\varsigma} - \overline{\pounds}_{\hbar_\varsigma}\right\}, S \in [0,1]$$
$$Ac = \frac{1}{6}\left\{3 + \underline{\xi}_{\hbar_\varsigma} + \overline{\xi}_{\hbar_\varsigma} + \underline{\eta}_{\hbar_\varsigma} + \overline{\eta}_{\hbar_\varsigma} - \underline{\pounds}_{\hbar_\varsigma} + \overline{\pounds}_{\hbar_\varsigma}\right\}, A \in [0,1].$$

Definition 11. *For two SV-NHFRNs*

$$B = \left\{\left(\underline{\xi}_{\hbar_1}, \underline{\eta}_{\hbar_1}, \underline{\pounds}_{\hbar_1}\right), \left(\overline{\xi}_{\hbar_1}, \overline{\eta}_{\hbar_1}, \overline{\pounds}_{\hbar_1}\right)\right\}$$

and

$$Y = \left\{\left(\underline{\xi}_{\hbar_2}, \underline{\eta}_{\hbar_2}, \underline{\pounds}_{\hbar_2}\right), \left(\overline{\xi}_{\hbar_2}, \overline{\eta}_{\hbar_2}, \overline{\pounds}_{\hbar_2}\right)\right\}.$$

The outcomes are as follows:

(1) If $S(B) > S(Y)$ then $B > Y$;

(2) If $S(B) < S(Y)$ then $B < Y$;

(3) If $S(B) = S(Y)$ then;

(i) If $A(B) > A(Y)$ then $B > Y$;

(ii) If $A(B) < A(Y)$ then $B < Y$;

(iii) If $A(B) = A(Y)$ then $B = Y$.

3. CL-single-valued neutrosophic hesitant fuzzy rough (CL-SV-NHFR) aggregation operators

Here, we first talk about CI-SV-NHFRWA AOs. We also go over the fundamental characteristics of the operators.

3.1. CL-SV-NHFR weighted average (CI-SV-NHFRWA) aggregation operators

We first discuss CL-SV-NHFR weighted average (CI-SV-NHFRWA) AO.

Definition 12. *Let* $\mho_\varsigma = \left(\left(\underline{\xi}_{\hbar_\varsigma}, \underline{\eta}_{\hbar_\varsigma}, \underline{\pounds}_{\hbar_\varsigma}\right), \left(\overline{\xi}_{\hbar_\varsigma}, \overline{\eta}_{\hbar_\varsigma}, \overline{\pounds}_{\hbar_\varsigma}\right)\right)$, $\varsigma = 1, 2, \ldots, n$ *be a collection of SV-NHFRNs and* Ξ_ς *be the CL of* $\mho_\varsigma$ *with* $0 \le \Xi_\varsigma \le 1$.

Let $\omega = (\omega_1, \omega_2, \omega_3 \ldots, \omega_n)^T$ be the weight vectors (WVs) for SV-NHFRNs with the condition $\sum_{\varsigma=1}^{n} \omega_\varsigma = 1$. Then, the mapping $CL - SV - NHFRWA : F^n \to F$ operator is given as $CL - SV - NHFRWA$

$$\left\{\begin{array}{c}(\mho_1, \Xi_1), (\mho_2, \Xi_2), \\ \ldots, (\mho_n, \Xi_n)\end{array}\right\} = \oplus_{\varsigma=1}^{n} \omega_\varsigma (\Xi_\varsigma \mho_\varsigma)$$
$$= \left\{\begin{array}{c}\omega_1 (\Xi_1 \mho_1) \oplus \omega_2 (\Xi_2 \mho_2) \oplus \\ \omega_3 (\Xi_3 \mho_3) \oplus \ldots \oplus \omega_n (\Xi_n \mho_n)\end{array}\right\}.$$

It is called the CL-SV-NHFRWA operator.

Theorem 1. *Let* $\mho_\varsigma = \left(\left(\underline{\xi}_{\hbar_\varsigma}, \underline{\eta}_{\hbar_\varsigma}, \underline{\pounds}_{\hbar_\varsigma}\right), \left(\overline{\xi}_{\hbar_\varsigma}, \overline{\eta}_{\hbar_\varsigma}, \overline{\pounds}_{\hbar_\varsigma}\right)\right)$, $\varsigma = 1, 2, \ldots, n$ *be a collection of SV-NHFRNs and* Ξ_ς *be the CL of* $\mho_\varsigma$ *with* $0 \leq \Xi_\varsigma \leq 1$.

Let $\omega = (\omega_1, \omega_2, \omega_3 \ldots, \omega_n)^T$ *be the WVs for SV-NHFRNs with the condition* $\sum_\varsigma^n \omega_\varsigma = 1$. *Then* $CL-SV-NHFRWA$

$$\left\{ \begin{array}{c} (\mho_1, \Xi_1), (\mho_2, \Xi_2), \\ \ldots, (\mho_n, \Xi_n) \end{array} \right\} = \left\{ \begin{array}{c} \left\{ \begin{array}{c} \left(1 - \prod_{\varsigma=1}^n \left(1 - \underline{\xi}_{\hbar_\varsigma}\right)^{\Xi_\varsigma \omega_\varsigma}\right), \\ \left(\prod_{\varsigma=1}^n \left(\underline{\eta}_{\hbar_\varsigma}\right)^{\Xi_\varsigma \omega_\varsigma}\right), \left(\prod_{\varsigma=1}^n \left(\underline{\pounds}_{\hbar_\varsigma}\right)^{\Xi_\varsigma \omega_\varsigma}\right) \end{array} \right\} \\ \left\{ \begin{array}{c} \left(1 - \prod_{\varsigma=1}^n \left(1 - \overline{\xi}_{\hbar_\varsigma}\right)^{\Xi_\varsigma \omega_\varsigma}\right), \\ \left(\prod_{\varsigma=1}^n \left(\overline{\eta}_{\hbar_\varsigma}\right)^{\Xi_\varsigma \omega_\varsigma}\right), \left(\prod_{\varsigma=1}^n \left(\overline{\pounds}_{\hbar_\varsigma}\right)^{\Xi_\varsigma \omega_\varsigma}\right) \end{array} \right\} \end{array} \right\}$$

Proof. For n = 2, we have

$$CL-SV-NHFRWA\left((\mho_1, \Xi_1), (\mho_2, \Xi_2)\right) = \omega_1 \left(\Xi_1 \mho_1\right) \oplus \omega_2 \left(\Xi_2 \mho_2\right).$$

Using the SV-NHFRN operating laws, we obtain

$$\begin{aligned} \Xi_1 \mho_1 &= \left\{ \begin{array}{c} \left(1 - \left(1 - \underline{\xi}_{\hbar_1}\right)^{\Xi_1}, \left(\underline{\eta}_{\hbar_1}\right)^{\Xi_1}, \left(\underline{\pounds}_{\hbar_1}\right)^{\Xi_1}\right), \\ 1 - \left(1 - \overline{\xi}_{\hbar_1}\right)^{\Xi_1}, \left(\overline{\eta}_{\hbar_1}\right)^{\Xi_1}, \left(\overline{\pounds}_{\hbar_1}\right)^{\Xi_1} \end{array} \right\} \\ &= \left\{ \begin{array}{c} \left(\underline{\Upsilon}_1, \underline{\wp}_1, \underline{\Theta}_1\right), \\ \left(\overline{\vartheta}_1, \overline{\wp}_1, \overline{\Omega}_1\right) \end{array} \right\}. \end{aligned}$$

Then

$$\begin{aligned} \omega_1 \left(\Xi_1 \mho_1\right) &= \left\{ \begin{array}{c} \left(1 - \left(1 - \underline{\Upsilon}_1\right)^{\Xi_1}, \left(\underline{\wp}_1\right)^{\Xi_1}, \left(\underline{\Theta}_1\right)^{\Xi_1}\right), \\ \left(1 - \left(1 - \overline{\vartheta}_1\right)^{\Xi_1}, \left(\overline{\wp}_1\right)^{\Xi_1}, \left(\overline{\Omega}_1\right)^{\Xi_1}\right) \end{array} \right\} \\ &= \left\{ \begin{array}{c} \left\{ \begin{array}{c} \left(1 - \left[1 - \left\{1 - \left(1 - \underline{\xi}_{\hbar_1}\right)^{\Xi_1}\right\}\right]^{\omega_1}\right), \\ \left(\underline{\eta}_{\hbar_1}^{\Xi_1}\right)^{\omega_1}, \left(\underline{\pounds}_{\hbar_1}^{\Xi_1}\right)^{\omega_1} \end{array} \right\}, \\ \left\{ \begin{array}{c} \left(1 - \left[1 - \left\{1 - \left(1 - \overline{\xi}_{\hbar_1}^{\Xi_1}\right)^{\Xi_1}\right\}\right]^{\omega_1}\right), \\ \left(\overline{\eta}_{\hbar_1}^{\Xi_1}\right)^{\omega_1}, \left(\overline{\pounds}_{\hbar_1}^{\Xi_1}\right)^{\omega_1} \end{array} \right\} \end{array} \right\} \\ &= \left\{ \begin{array}{c} \left\{ \begin{array}{c} \left(1 - \left(1 - \underline{\xi}_{\hbar_1}\right)^{\Xi_1 \omega_1}\right), \\ \left(\underline{\eta}_{\hbar_1}\right)^{\omega_1 \Xi_1}, \left(\underline{\pounds}_{\hbar_1}\right)^{\omega_1 \Xi_1} \end{array} \right\}, \\ \left\{ \begin{array}{c} \left(1 - \left(1 - \overline{\xi}_{\hbar_1}\right)^{\Xi_1 \omega_1}\right), \\ \left(\overline{\eta}_{\hbar_1}\right)^{\omega_1 \Xi_1}, \left(\overline{\pounds}_{\hbar_1}\right)^{\omega_1 \Xi_1} \end{array} \right\} \end{array} \right\}. \end{aligned}$$

Likewise, we can observe that

$$\Xi_2\mho_2=\left\{\begin{array}{c}\left\{\begin{array}{c}\left(1-\left(1-\underline{\xi}_{\hbar_2}\right)^{\Xi_2\omega_2}\right),\\ \left(\underline{\eta}_{\hbar_2}\right)^{\omega_2\Xi_2},\left(\underline{\pounds}_{\hbar_2}\right)^{\omega_2\Xi_2}\end{array}\right\},\\ \left\{\begin{array}{c}\left(1-\left(1-\overline{\xi}_{\hbar_2}\right)^{\Xi_2\omega_2}\right),\\ \left(\overline{\eta}_{\hbar_2}\right)^{\omega_2\Xi_2},\left(\overline{\pounds}_{\hbar_2}\right)^{\omega_2\Xi_2}\end{array}\right\}\end{array}\right\}.$$

Then,

$$\begin{aligned}CL-SV-NHFRWA &= ((\mho_1,\Xi_1),(\mho_2,\Xi_2))=\omega_1\left(\Xi_1\mho_1\right)\oplus\omega_2\left(\Xi_2\mho_2\right)\\ &=\left\{\begin{array}{c}\left\{\begin{array}{c}\left\{\begin{array}{c}\left(1-\left(1-\underline{\xi}_{\hbar_1}\right)^{\Xi_1\omega_1}\right)+\left(1-\left(1-\underline{\xi}_{\hbar_2}\right)^{\Xi_2\omega_2}\right)\\ -\left(1-\left(1-\underline{\xi}_{\hbar_1}\right)^{\Xi_1\omega_1}\right)-\left(1-\left(1-\underline{\xi}_{\hbar_2}\right)^{\Xi_2\omega_2}\right)\end{array}\right\},\\ \left\{\left(\underline{\eta}_{\hbar_1}\right)^{\omega_1\Xi_1}\left(\underline{\eta}_{\hbar_2}\right)^{\omega_2\Xi_2}\right\},\left\{\left(\underline{\pounds}_{\hbar_1}\right)^{\omega_1\Xi_1}\left(\underline{\pounds}_{\hbar_2}\right)^{\omega_2\Xi_2}\right\}\end{array}\right\},\\ \left\{\begin{array}{c}\left\{\begin{array}{c}\left(1-\left(1-\overline{\xi}_{\hbar_1}\right)^{\Xi_1\omega_1}\right)+\left(1-\left(1-\overline{\xi}_{\hbar_2}\right)^{\Xi_2\omega_2}\right)\\ -\left(1-\left(1-\overline{\xi}_{\hbar_1}\right)^{\Xi_1\omega_1}\right)-\left(1-\left(1-\overline{\xi}_{\hbar_2}\right)^{\Xi_2\omega_2}\right)\end{array}\right\},\\ \left\{\left(\overline{\eta}_{\hbar_1}\right)^{\omega_1\Xi_1}\left(\overline{\eta}_{\hbar_2}\right)^{\omega_2\Xi_2}\right\},\left\{\left(\overline{\pounds}_{\hbar_1}\right)^{\omega_1\Xi_1}\left(\overline{\pounds}_{\hbar_2}\right)^{\omega_2\Xi_2}\right\}\end{array}\right\}\end{array}\right\}.\end{aligned}$$

Thus,

$$\begin{aligned}&CL-SV-NHFRWA\left\{(\mho_1,\Xi_1),(\mho_2,\Xi_2)\right\}\\ &=\left(\begin{array}{c}\left\{\left(1-\prod_{\varsigma=1}^{2}\left(1-\underline{\xi}_{\hbar_\varsigma}\right)^{\Xi_\varsigma\omega_\varsigma}\right),\left(\prod_{\varsigma=1}^{2}\left(\underline{\eta}_{\hbar_\varsigma}\right)^{\Xi_\varsigma\omega_\varsigma}\right),\left(\prod_{\varsigma=1}^{2}\left(\underline{\pounds}_{\hbar_\varsigma}\right)^{\Xi_\varsigma\omega_\varsigma}\right)\right\},\\ \left\{\left(1-\prod_{\varsigma=1}^{2}\left(1-\overline{\xi}_{\hbar_\varsigma}\right)^{\Xi_\varsigma\omega_\varsigma}\right),\left(\prod_{\varsigma=1}^{2}\left(\overline{\eta}_{\hbar_\varsigma}\right)^{\Xi_\varsigma\omega_\varsigma}\right),\left(\prod_{\varsigma=1}^{2}\left(\overline{\pounds}_{\hbar_\varsigma}\right)^{\Xi_\varsigma\omega_\varsigma}\right)\right\}\end{array}\right).\end{aligned}$$

Suppose that the result is valid for $n=\dagger$, that is

$$\begin{aligned}&CL-SV-NHFRWA\left\{(\mho_1,\Xi_1),(\mho_2,\Xi_2),\ldots,(\mho_\dagger,\Xi_\dagger)\right\}\\ &=\left(\begin{array}{c}\left\{\left(1-\prod_{\varsigma=1}^{\dagger}\left(1-\underline{\xi}_{\hbar_\varsigma}\right)^{\Xi_\varsigma\omega_\varsigma}\right),\left(\prod_{\varsigma=1}^{\dagger}\left(\underline{\eta}_{\hbar_\varsigma}\right)^{\Xi_\varsigma\omega_\varsigma}\right),\left(\prod_{\varsigma=1}^{\dagger}\left(\underline{\pounds}_{\hbar_\varsigma}\right)^{\Xi_\varsigma\omega_\varsigma}\right)\right\}\\ \left\{\left(1-\prod_{\varsigma=1}^{\dagger}\left(1-\overline{\xi}_{\hbar_\varsigma}\right)^{\Xi_\varsigma\omega_\varsigma}\right),\left(\prod_{\varsigma=1}^{\dagger}\left(\overline{\eta}_{\hbar_\varsigma}\right)^{\Xi_\varsigma\omega_\varsigma}\right),\left(\prod_{\varsigma=1}^{\dagger}\left(\overline{\pounds}_{\hbar_\varsigma}\right)^{\Xi_\varsigma\omega_\varsigma}\right)\right\}\end{array}\right).\end{aligned}$$

Then, for $n=\dagger+1$, we get

$$\begin{aligned}&CL-SV-NHFRWA\left\{(\mho_1,\Xi_1),(\mho_2,\Xi_2),\ldots,(\mho_\dagger,\Xi_\dagger),(\mho_{\dagger+1},\Xi_{\dagger+1})\right\}\\ &=\left\{\begin{array}{c}\left\{\begin{array}{c}\left\{\left(1-\prod_{\varsigma=1}^{\dagger}\left(1-\underline{\xi}_{\hbar_\varsigma}\right)^{\Xi_\varsigma\omega_\varsigma}\right),\left(\prod_{\varsigma=1}^{\dagger}\left(\underline{\eta}_{\hbar_\varsigma}\right)^{\Xi_\varsigma\omega_\varsigma}\right),\left(\prod_{\varsigma=1}^{\dagger}\left(\underline{\eta}_{\hbar_\varsigma}\right)^{\Xi_\varsigma\omega_\varsigma}\right)\right\},\\ \left\{\left(1-\prod_{\varsigma=1}^{\dagger}\left(1-\overline{\xi}_{\hbar_\varsigma}\right)^{\Xi_\varsigma\omega_\varsigma}\right),\left(\prod_{\varsigma=1}^{\dagger}\left(\overline{\eta}_{\hbar_\varsigma}\right)^{\Xi_\varsigma\omega_\varsigma}\right),\left(\prod_{\varsigma=1}^{\dagger}\left(\overline{\eta}_{\hbar_\varsigma}\right)^{\Xi_\varsigma\omega_\varsigma}\right)\right\}\end{array}\right\}\oplus\\ \left\{\left\{\begin{array}{c}\left(1-\left(1-\underline{\xi}_{\hbar_{\dagger+1}}\right)^{\Xi_{\dagger+1}\omega_{\dagger+1}}\right),\\ \left(\underline{\eta}_{\hbar_{\dagger+1}}\right)^{\omega_{\dagger+1}\Xi_{\dagger+1}},\left(\underline{\pounds}_{\hbar_\varsigma}\right)^{\omega_{\dagger+1}\Xi_{\dagger+1}}\end{array}\right\},\left\{\begin{array}{c}\left(1-\left(1-\overline{\xi}_{\hbar_{\dagger+1}}\right)^{\Xi_{\dagger+1}\omega_{\dagger+1}}\right),\\ \left(\overline{\eta}_{\hbar_{\dagger+1}}\right)^{\omega_{\dagger+1}\Xi_{\dagger+1}},\left(\overline{\pounds}_{\hbar_{\dagger+1}}\right)^{\omega_{\dagger+1}\Xi_{\dagger+1}}\end{array}\right\}\right\}\end{array}\right\}\end{aligned}$$

$$= \left\{ \begin{array}{c} \left\{ \begin{array}{c} \left\{ \begin{array}{c} \left(1-\prod_{\varsigma=1}^{\dagger}\left(1-\underline{\xi}_{\hbar_\varsigma}\right)^{\Xi_\varsigma\omega_\varsigma}\right)+\left(1-\left(1-\underline{\xi}_{\hbar_\varsigma}\right)^{\Xi_{\dagger+1}\omega_{\dagger+1}}\right) \\ -\left(1-\prod_{\varsigma=1}^{\dagger}\left(1-\underline{\xi}_{\hbar_\varsigma}\right)^{\Xi_\varsigma\omega_\varsigma}\right)-\left(1-\left(1-\underline{\xi}_{\hbar_\varsigma}\right)^{\Xi_{\dagger+1}\omega_{\dagger+1}}\right) \end{array} \right\}, \\ \left\{\prod_{\varsigma=1}^{\dagger}\left(\underline{\eta}_{\hbar_\varsigma}\right)^{\Xi_\varsigma\omega_\varsigma},\left(\underline{\eta}_{\hbar_\varsigma}\right)^{\omega_{\dagger+1}\Xi_{\dagger+1}}\right\},\left\{\prod_{\varsigma=1}^{\dagger}\left(\underline{£}_{\hbar_\varsigma}\right)^{\Xi_\varsigma\omega_\varsigma},\left(\underline{£}_{\hbar_\varsigma}\right)^{\omega_{\dagger+1}\Xi_{\dagger+1}}\right\} \end{array} \right\}, \\ \left\{ \begin{array}{c} \left\{ \begin{array}{c} \left(1-\prod_{\varsigma=1}^{\dagger}\left(1-\overline{\xi}_{\hbar_\varsigma}\right)^{\Xi_\varsigma\omega_\varsigma}\right)+\left(1-\left(1-\overline{\xi}_{\hbar_\varsigma}\right)^{\Xi_{\dagger+1}\omega_{\dagger+1}}\right) \\ -\left(1-\prod_{\varsigma=1}^{\dagger}\left(1-\overline{\xi}_{\hbar_\varsigma}\right)^{\Xi_\varsigma\omega_\varsigma}\right)-\left(1-\left(1-\overline{\xi}_{\hbar_\varsigma}\right)^{\Xi_{\dagger+1}\omega_{\dagger+1}}\right) \end{array} \right\}, \\ \left\{\prod_{\varsigma=1}^{\dagger}\left(\overline{\eta}_{\hbar_\varsigma}\right)^{\Xi_\varsigma\omega_\varsigma},\overline{\eta}_{\hbar_\varsigma}\right\},\left\{\prod_{\varsigma=1}^{\dagger}\left(\overline{£}_{\hbar_\varsigma}\right)^{\Xi_\varsigma\omega_\varsigma},\overline{£}_{\hbar_\varsigma}\right\} \end{array} \right\} \end{array} \right\}$$

$$= \left\{ \begin{array}{c} \left\{\left(1-\prod_{\varsigma=1}^{\dagger+1}\left(1-\underline{\xi}_{\hbar_\varsigma}\right)^{\Xi_\varsigma\omega_\varsigma}\right),\left(\prod_{\varsigma=1}^{\dagger+1}\left(\underline{\eta}_{\hbar_\varsigma}\right)^{\Xi_\varsigma\omega_\varsigma}\right),\left(\prod_{\varsigma=1}^{\dagger+1}\left(\underline{£}_{\hbar_\varsigma}\right)^{\Xi_\varsigma\omega_\varsigma}\right)\right\}, \\ \left\{\left\{1-\prod_{\varsigma=1}^{\dagger+1}\left(1-\overline{\xi}_{\hbar_\varsigma}\right)^{\Xi_\varsigma\omega_\varsigma}\right\},\left\{\prod_{\varsigma=1}^{\dagger+1}\left(\overline{\eta}_{\hbar_\varsigma}\right)^{\Xi_\varsigma\omega_\varsigma}\right\},\left\{\prod_{\varsigma=1}^{\dagger+1}\left(\overline{£}_{\hbar_\varsigma}\right)^{\Xi_\varsigma\omega_\varsigma}\right\}\right\} \end{array} \right\}.$$

Hence the result is valid for $n = \dagger + 1$. Therefore, the result is valid for any number of SV-NHFRNs. □

Theorem 2. *For the collection of SV-NHFRNs* $\mho_\varsigma = \left(\left(\underline{\xi}_{\hbar_\varsigma}, \underline{\eta}_{\hbar_\varsigma}, \underline{£}_{\hbar_\varsigma}\right), \left(\overline{\xi}_{\hbar_\varsigma}, \overline{\eta}_{\hbar_\varsigma}, \overline{£}_{\hbar_\varsigma}\right)\right)$ *where* $\varsigma = 1, 2, \ldots, n$ *and* Ξ_ς *be the CL of* $\mho_\varsigma$ *with* $0 \le \Xi_\varsigma \le 1$.

Let $\omega = (\omega_1, \omega_2, \omega_3 \ldots, \omega_n)^T$ be the WVs for SV-NHFRNs with the condition $\sum_\varsigma^n \omega_\varsigma = 1$.

CI-SV-NHFRWA AOs then possess the following characteristics:

(1) Idempotency If for all $(\mho_\varsigma, \Xi_\varsigma) = (\mho, \Xi)$, i.e., $\underline{\xi} = \underline{\xi}_{\hbar_\varsigma}$, $\overline{\xi} = \overline{\xi}_{\hbar_\varsigma}$, $\underline{\eta}_{\hbar_\varsigma} = \underline{\eta}$, $\overline{\eta}_{\hbar_\varsigma} = \overline{\eta}$, $\underline{£}_{\hbar_\varsigma} = \underline{£}$, and $\overline{£}_{\hbar_\varsigma} = \overline{£}$, $\Xi_\varsigma = \Xi$, then

$$CL-SV-NHFRWA\left((\mho_1, \Xi_1), (\mho_2, \Xi_2), \ldots, (\mho_n, \Xi_n)\right) = \Xi_\mho.$$

Proof. If $(\mho_\varsigma, \Xi_\varsigma) = (\mho, \Xi)$, then by using Theorem 1, we get

$$\begin{array}{l} CL-SV-NHFRWA\left\{(\mho_{\hbar_1}, \Xi_1), (\mho_{\hbar_2}, \Xi_2), \ldots, (\mho_{\hbar_n}, \Xi_n)\right\} \\ = \left\{ \begin{array}{c} \left\{\left(1-\prod_{\varsigma=1}^{n}\left(1-\underline{\xi}_{\hbar_\varsigma}\right)^{\Xi\omega_\varsigma}\right),\left(\prod_{\varsigma=1}^{n}\left(\underline{\eta}_{\hbar_\varsigma}\right)^{\Xi_\varsigma\omega_\varsigma}\right),\left(\prod_{\varsigma=1}^{n}\left(\underline{£}_{\hbar_\varsigma}\right)^{\Xi\omega_\varsigma}\right)\right\}, \\ \left\{\left(1-\prod_{\varsigma=1}^{n}\left(1-\overline{\xi}_{\hbar_\varsigma}\right)^{\Xi\omega_\varsigma}\right),\left(\prod_{\varsigma=1}^{n}\left(\overline{\eta}_{\hbar_\varsigma}\right)^{\Xi\omega_\varsigma}\right),\left(\prod_{\varsigma=1}^{n}\left(\overline{£}_{\hbar_\varsigma}\right)^{\Xi\omega_\varsigma}\right)\right\} \end{array} \right\} \\ = \left\{ \begin{array}{c} \left\{\left(1-\left(1-\underline{\xi}_{\hbar_\varsigma}\right)^{\Xi\sum_{\varsigma=1}^{n}\omega_\varsigma}\right),\left(\left(\underline{\eta}_{\hbar_\varsigma}\right)^{\Xi\sum_{\varsigma=1}^{n}\omega_\varsigma}\right),\left(\left(\underline{£}_{\hbar_\varsigma}\right)^{\Xi\sum_{\varsigma=1}^{n}\omega_\varsigma}\right)\right\}, \\ \left\{\left(1-\left(1-\overline{\xi}_{\hbar_\varsigma}\right)^{\Xi\sum_{\varsigma=1}^{n}\omega_\varsigma}\right),\left(\left(\overline{\eta}_{\hbar_\varsigma}\right)^{\Xi\sum_{\varsigma=1}^{n}\omega_\varsigma}\right),\left(\left(\overline{\eta}_{\hbar_\varsigma}\right)^{\Xi\sum_{\varsigma=1}^{n}\omega_\varsigma}\right)\right\} \end{array} \right\} \\ = \left\{ \begin{array}{c} \left\{\left(1-\left(1-\underline{\xi}_{\hbar_\varsigma}\right)^{\Xi}\right),\left(\underline{\eta}_{\hbar_\varsigma}\right)^{\Xi},\left(\underline{£}_{\hbar_\varsigma}\right)^{\Xi}\right\}, \\ \left\{\left(1-\left(1-\overline{\xi}_{\hbar_\varsigma}\right)^{\Xi}\right),\left(\overline{\eta}_{\hbar_\varsigma}\right)^{\Xi},\left(\overline{\eta}_{\hbar_\varsigma}\right)^{\Xi}\right\} \end{array} \right\} = \Xi_\mho. \end{array}$$

□

(2) Boundedness Let

$$\mho_\varsigma^- = \left\{ \begin{array}{c} \left(\min \underline{\xi}_{\hbar_\varsigma}, \min \underline{\eta}_{\hbar_\varsigma}, \min \underline{\pounds}_{\hbar_\varsigma}\right), \\ \left(\max \overline{\xi}_{\hbar_\varsigma}, \max \overline{\eta}_{\hbar_\varsigma}, \max \overline{\eta}_{\hbar_\varsigma}\right) \end{array} \right\}, \text{ and } \mho_\varsigma^+ = \left\{ \begin{array}{c} \left(\max \underline{\xi}_{\hbar_\varsigma}, \max \underline{\eta}_{\hbar_\varsigma}, \max \underline{\pounds}_{\hbar_\varsigma}\right), \\ \left(\min \overline{\xi}_{\hbar_\varsigma}, \min \overline{\eta}_{\hbar_\varsigma}, \min \overline{\pounds}_{\hbar_\varsigma}\right) \end{array} \right\}.$$

Then, for all ω_ς, we have

$$\mho_\varsigma^- \leq CL-SV-NHFRWA \left\{ \begin{array}{c} (\mho_1, \Xi_1), (\mho_2, \Xi_2), \\ \ldots, (\mho_n, \Xi_n) \end{array} \right\} \leq \mho_\varsigma^+.$$

Proof. For every ς,

$$\min(\underline{\xi}_{\hbar_\varsigma}) \leq \underline{\xi}_{\hbar_\varsigma} \leq \max(\underline{\xi}_{\hbar_\varsigma}) \Longrightarrow 1 - \max(\underline{\xi}_{\hbar_\varsigma}) \leq 1 - \underline{\xi}_{\hbar_\varsigma} \leq 1 - \min(\underline{\xi}_{\hbar_\varsigma}).$$

Now for every ω, we get

$$\begin{array}{l} \left\{ \left(\prod_{\varsigma=1}^{n} \left(1 - \max\left(\underline{\xi}_{\hbar_\varsigma}\right)\right)^{(\max \Xi_\varsigma)\omega_\varsigma} \right) \leq \left(\prod_{\varsigma=1}^{n} \left(1 - \underline{\xi}_{\hbar_\varsigma}\right)^{\Xi_\varsigma \omega_\varsigma} \right) \leq \left(\prod_{\varsigma=1}^{n} \left(1 - \min\left(\underline{\xi}_{\hbar_\varsigma}\right)\right)^{(\min \Xi_\varsigma)\omega_\varsigma} \right) \right\} \\ \Longrightarrow \left\{ \left(\left(1 - \max\left(\underline{\xi}_{\hbar_\varsigma}\right)\right)^{(\max \Xi_\varsigma) \sum_{\varsigma=1}^{n} \omega_\varsigma} \right) \leq \left(\prod_{\varsigma=1}^{n} \left(1 - \underline{\xi}_{\hbar_\varsigma}\right)^{\Xi_\varsigma \omega_\varsigma} \right) \leq \left(\left(1 - \min\left(\underline{\xi}_{\hbar_\varsigma}\right)\right)^{(\min \Xi_\varsigma) \sum_{\varsigma=1}^{n} \omega_\varsigma} \right) \right\} \\ \Longrightarrow \left\{ \begin{array}{c} \left(1 - \left(1 - \min\left(\underline{\xi}_{\hbar_\varsigma}\right)\right)^{(\min \Xi_\varsigma)}\right) \leq \left(1 - \prod_{\varsigma=1}^{n} \left(1 - \underline{\xi}_{\hbar_\varsigma}\right)^{\Xi_\varsigma \omega_\varsigma}\right) \\ \leq \left(1 - \left(1 - \max\left(\underline{\xi}_{\hbar_\varsigma}\right)\right)^{(\max \Xi_\varsigma)\omega_\varsigma} \underline{\xi}_{\hbar_\varsigma}\right) \leq \left(1 - \prod_{\varsigma=1}^{n} \left(1 - \underline{\xi}_{\hbar_\varsigma}\right)^{\Xi_\varsigma \omega_\varsigma}\right) \leq \left(\underline{\xi}_{\hbar_\varsigma}\right) \end{array} \right\}. \end{array}$$

Similarly, for every ς,

$$\min(\overline{\xi}_{\hbar_\varsigma}) \leq \overline{\xi}_{\hbar_\varsigma} \leq \max(\overline{\xi}_{\hbar_\varsigma}) \Longrightarrow 1 - \max(\overline{\xi}_{\hbar_\varsigma}) \leq 1 - \overline{\xi}_{\hbar_\varsigma} \leq 1 - \min(\overline{\xi}_{\hbar_\varsigma}).$$

Now for every ω, we get

$$\begin{array}{l} \left\{ \left(\prod_{\varsigma=1}^{n} \left(1 - \max\left(\overline{\xi}_{\hbar_\varsigma}\right)\right)^{(\max \Xi_\varsigma)\omega_\varsigma} \right) \leq \left(\prod_{\varsigma=1}^{n} \left(1 - \overline{\xi}_{\hbar_\varsigma}\right)^{\Xi_\varsigma \omega_\varsigma} \right) \leq \left(\prod_{\varsigma=1}^{n} \left(1 - \min\left(\overline{\xi}_{\hbar_\varsigma}\right)\right)^{(\min \Xi_\varsigma)\omega_\varsigma} \right) \right\} \\ \Longrightarrow \left\{ \left(\left(1 - \max\left(\overline{\xi}_{\hbar_\varsigma}\right)\right)^{(\max \Xi_\varsigma) \sum_{\varsigma=1}^{n} \omega_\varsigma} \right) \leq \left(\prod_{\varsigma=1}^{n} \left(1 - \xi_\varsigma\right)^{\Xi_\varsigma \omega_\varsigma} \right) \leq \left(\left(1 - \min\left(\overline{\xi}_{\hbar_\varsigma}\right)\right)^{(\min \Xi_\varsigma) \sum_{\varsigma=1}^{n} \omega_\varsigma} \right) \right\} \\ \Longrightarrow \left\{ \begin{array}{c} \left(1 - \left(1 - \min\left(\xi_\varsigma\right)\right)^{(\min \Xi_\varsigma)}\right) \leq \left(1 - \prod_{\varsigma=1}^{n} \left(1 - \xi_\varsigma\right)^{\Xi_\varsigma \omega_\varsigma}\right) \\ \leq \left(1 - \left(1 - \max\left(\xi_\varsigma\right)\right)^{(\max \Xi_\varsigma)\omega_\varsigma} \xi^{\min}_{\mho_\varsigma \Xi_\varsigma}\right) \leq \left(1 - \prod_{\varsigma=1}^{n} \left(1 - \xi_\varsigma\right)^{\Xi_\varsigma \omega_\varsigma}\right) \leq \xi^{\max}_{\mho_\varsigma \Xi_\varsigma} \end{array} \right\}. \end{array}$$

Similarly,

$$\left\{ \begin{array}{c} \left(\min(\underline{\eta}_{\hbar_\varsigma}) \leq \underline{\eta}_{\hbar_\varsigma} \leq \max(\underline{\eta}_{\hbar_\varsigma})\right) \\ \Leftrightarrow \min(\underline{\eta}_{\hbar_\varsigma})^{\min \Xi_\varsigma} \leq \prod_{\varsigma=1}^{n} \left(\underline{\eta}_{\hbar_\varsigma}\right)^{\Xi_\varsigma \omega_\varsigma} \leq \max(\underline{\eta}_{\hbar_\varsigma})^{\max \Xi_\varsigma} \end{array} \right.$$

$$\Longrightarrow \left\{\left(\underline{\eta}_{\hbar_\varsigma}\right)^{\mho_\varsigma \Xi_\varsigma} \leq \prod_{\varsigma=1}^{n}\left(\underline{\eta}_{\hbar_\varsigma}\right)^{\Xi_\varsigma \omega_\varsigma} \leq \left(\underline{\eta}_{\hbar_\varsigma}\right)^{\mho_\varsigma \Xi_\varsigma}\right\}$$

and

$$\left\{\begin{array}{c}\left(\min\left(\overline{\eta}_{\hbar_\varsigma}\right) \leq \overline{\eta}_{\hbar_\varsigma} \leq \max\left(\overline{\eta}_{\hbar_\varsigma}\right)\right) \\ \Leftrightarrow \min(\overline{\eta}_{\hbar_\varsigma})^{\min \Xi_\varsigma} \leq \prod_{\varsigma=1}^{n}\left(\overline{\eta}_{\hbar_\varsigma}\right)^{\Xi_\varsigma \omega_\varsigma} \leq \max(\overline{\eta}_{\hbar_\varsigma})^{\max \Xi_\varsigma}\end{array}\right\}$$

$$\Longrightarrow \left\{\left(\overline{\eta}_{\hbar_\varsigma}\right)^{\mho_\varsigma \Xi_\varsigma} \leq \prod_{\varsigma=1}^{n}\left(\overline{\eta}_{\hbar_\varsigma}\right)^{\Xi_\varsigma \omega_\varsigma} \leq \left(\overline{\eta}_{\hbar_\varsigma}\right)^{\mho_\varsigma \Xi_\varsigma}\right\}.$$

Also,

$$\left\{\begin{array}{c}\min(\underline{£}_{\hbar_\varsigma}) \leq \underline{£}_{\hbar_\varsigma} \leq \max(\underline{£}_{\hbar_\varsigma}) \\ \Leftrightarrow \min(\underline{£}_{\hbar_\varsigma})^{\min \Xi_\varsigma} \leq \prod_{\varsigma=1}^{n}\left(\underline{£}_{\hbar_\varsigma}\right)^{\Xi_\varsigma \omega_\varsigma} \leq \max(\underline{£}_{\hbar_\varsigma})^{\max \Xi_\varsigma}\end{array}\right\}$$

$$\Longrightarrow \left\{\left(\underline{£}_{\hbar_\varsigma}\right)^{\mho_\varsigma \Xi_\varsigma} \leq \prod_{\varsigma=1}^{n}\left(\underline{£}_{\hbar_\varsigma}\right)^{\Xi_\varsigma \omega_\varsigma} \leq \left(\underline{£}_{\hbar_\varsigma}\right)^{\mho_\varsigma \Xi_\varsigma}\right\}$$

and

$$\left\{\begin{array}{c}\min\left(\overline{£}_{\hbar_\varsigma}\right) \leq \overline{£}_{\hbar_\varsigma} \leq \max\left(\overline{£}_{\hbar_\varsigma}\right) \\ \Leftrightarrow \min(\overline{£}_{\hbar_\varsigma})^{\min \Xi_\varsigma} \leq \prod_{\varsigma=1}^{n}\left(\overline{£}_{\hbar_\varsigma}\right)^{\Xi_\varsigma \omega_\varsigma} \leq \max(\overline{£}_{\hbar_\varsigma})^{\max \Xi_\varsigma}\end{array}\right\}$$

$$\Longrightarrow \left\{\left(\overline{£}_{\hbar_\varsigma}\right)^{\mho_\varsigma \Xi_\varsigma} \leq \prod_{\varsigma=1}^{n}\left(\overline{£}_{\hbar_\varsigma}\right)^{\Xi_\varsigma \omega_\varsigma} \leq \left(\overline{£}_{\hbar_\varsigma}\right)^{\mho_\varsigma \Xi_\varsigma}\right\}.$$

If

$$CL-SV-NRWA\left\{\begin{array}{c}(\mho_1, \Xi_1), (\mho_2, \Xi_2), \\ \ldots, (\mho_n, \Xi_n)\end{array}\right\} = \mho = \left\{\begin{array}{c}\left(\underline{\xi}_{\hbar_\varsigma}, \underline{\eta}_{\hbar_\varsigma}, \underline{£}_{\hbar_\varsigma}\right), \\ \left(\overline{\xi}_{\hbar_\varsigma}, \overline{\eta}_{\hbar_\varsigma}, \overline{£}_{\hbar_\varsigma}\right)\end{array}\right\},$$

then from the above analysis, we get

$$\left\{\begin{array}{c}\left\{\left(\underline{\xi}_{\hbar_\varsigma} \leq \underline{\xi}_{\hbar_\varsigma} \leq \underline{\xi}_{\hbar_\varsigma}\right), \left(\left(\overline{\xi}_{\hbar_\varsigma}\right)^{\mho_\varsigma \Xi_\varsigma} \leq \left(\overline{\xi}_{\hbar_\varsigma}\right)^{\mho_\varsigma \Xi_\varsigma} \leq \left(\overline{\xi}_{\hbar_\varsigma}\right)^{\mho_\varsigma \Xi_\varsigma}\right)\right\}, \\ \left\{\left(\left(\underline{\eta}_{\hbar_\varsigma}\right)^{\mho_\varsigma \Xi_\varsigma} \leq \left(\underline{\eta}_{\hbar_\varsigma}\right)^{\mho_\varsigma \Xi_\varsigma} \leq \left(\underline{\eta}_{\hbar_\varsigma}\right)^{\mho_\varsigma \Xi_\varsigma}\right), \left(\left(\overline{\eta}_{\hbar_\varsigma}\right)^{\mho_\varsigma \Xi_\varsigma} \leq \left(\overline{\eta}_{\hbar_\varsigma}\right)^{\mho_\varsigma \Xi_\varsigma} \leq \left(\overline{\eta}_{\hbar_\varsigma}\right)^{\mho_\varsigma \Xi_\varsigma}\right)\right\}, \\ \left\{\left(\left(\underline{£}_{\hbar_\varsigma}\right)^{\mho_\varsigma \Xi_\varsigma} \leq \left(\underline{£}_{\hbar_\varsigma}\right)^{\mho_\varsigma \Xi_\varsigma} \leq \left(\underline{£}_{\hbar_\varsigma}\right)^{\mho_\varsigma \Xi_\varsigma}\right), \left(\left(\overline{£}_{\hbar_\varsigma}\right)^{\mho_\varsigma \Xi_\varsigma} \leq \left(\overline{£}_{\hbar_\varsigma}\right)^{\mho_\varsigma \Xi_\varsigma} \leq \left(\overline{£}_{\hbar_\varsigma}\right)^{\mho_\varsigma \Xi_\varsigma}\right)\right\}\end{array}\right\}.$$

Then, by using the definition of SF, we can conclude that

$$\mho_\varsigma^{-} \leq CL-SV-NHFRWA\left\{(\mho_1, \Xi_1), (\mho_2, \Xi_2), \ldots, (\mho_n, \Xi_n)\right\} \leq \mho_\varsigma^{+}.$$

$\square$

(3) Monotonicity Let

$$\mho^{*}_{\varsigma}=\left\{\left(\underline{\xi}_{\hbar_{\varsigma}},\underline{\eta}_{\hbar_{\varsigma}},\underline{£}_{\hbar_{\varsigma}}\right),\left(\overline{\xi}_{\hbar_{\varsigma}},\overline{\eta}_{\hbar_{\varsigma}},\overline{£}_{\hbar_{\varsigma}}\right)\right\}(\ \varsigma=1,2,3,...,n)$$

be another collection of SV-NHFRNs such that

$$\left\{\begin{array}{c}\left\{\left(\underline{\xi}_{\hbar_{\varsigma}}\leq\underline{\xi}_{\hbar_{\varsigma}}\right),\left(\left(\underline{\eta}_{\hbar_{\varsigma}}\right)\geq\left(\underline{\eta}_{\hbar_{\varsigma}}\right)\right),\left(\left(\underline{£}_{\hbar_{\varsigma}}\right)\geq\left(\underline{£}_{\hbar_{\varsigma}}\right)\right)\right\},\\ \left\{\left(\left(\overline{\xi}_{\hbar_{\varsigma}}\right)\leq\left(\overline{\xi}_{\hbar_{\varsigma}}\right)\right),\left(\left(\overline{\eta}_{\hbar_{\varsigma}}\right)\geq\left(\overline{\eta}_{\hbar_{\varsigma}}\right)\right),\left(\left(\overline{£}_{\hbar_{\varsigma}}\right)\geq\left(\overline{£}_{\hbar_{\varsigma}}\right)\right)\right\}\end{array}\right\}$$

for all ω_{ς}. Then

$$\begin{aligned}&CL-SV-NHFRWA\left\{(\mho_1,\Xi_1),(\mho_2,\Xi_2),\ldots,(\mho_n,\Xi_n)\right\}\\ \leq\ &CL-SV-NHFRWA\left\{(\mho^{*}_1,\Xi_1),(\mho^{*}_2,\Xi_2),\ldots,(\mho^{*}_n,\Xi_n)\right\}.\end{aligned}$$

Proof. Since

$$\left\{\begin{array}{c}\left\{\left(\left(\underline{\xi}_{\hbar_{\varsigma}}\right)\leq\left(\underline{\xi}_{\hbar_{\varsigma}}\right)\right),\left(\left(\underline{\eta}_{\hbar_{\varsigma}}\right)\geq\left(\underline{\eta}_{\hbar_{\varsigma}}\right)\right),\left(\left(\underline{£}_{\hbar_{\varsigma}}\right)\geq\left(\underline{£}_{\hbar_{\varsigma}}\right)\right)\right\},\\ \left\{\left(\left(\overline{\xi}_{\hbar_{\varsigma}}\right)\leq\left(\overline{\xi}_{\hbar_{\varsigma}}\right)\right),\left(\left(\overline{\eta}_{\hbar_{\varsigma}}\right)\geq\left(\overline{\eta}_{\hbar_{\varsigma}}\right)\right),\left(\left(\overline{£}_{\hbar_{\varsigma}}\right)\geq\left(\overline{£}_{\hbar_{\varsigma}}\right)\right)\right\}\end{array}\right\}$$

for all ς,

$$\left\{\begin{array}{c}\left\{\left(1-\underline{\xi}_{\hbar_{\varsigma}}\leq 1-\underline{\xi}_{\hbar_{\varsigma}}\right)\right\}\\ \Longrightarrow\left\{\left(\prod_{\varsigma=1}^{n}\left(1-\underline{\xi}_{\hbar_{\varsigma}}\right)^{\Xi_{\varsigma}\omega_{\varsigma}}\right)\leq\left(\prod_{\varsigma=1}^{n}\left(1-\underline{\xi}_{\hbar_{\varsigma}}\right)^{\Xi_{\varsigma}\omega_{\varsigma}}\right)\right\}\\ \Longrightarrow\left\{\left(1-\prod_{\varsigma=1}^{n}\left(1-\underline{\xi}_{\hbar_{\varsigma}}\right)^{\Xi_{\varsigma}\omega_{\varsigma}}\right)\leq\left(1-\prod_{\varsigma=1}^{n}\left(1-\underline{\xi}_{\hbar_{\varsigma}}\right)^{\Xi_{\varsigma}\omega_{\varsigma}}\right)\right\}\end{array}\right\},$$

similarly,

$$\left\{\begin{array}{c}\left\{\left(1-\overline{\xi}_{\hbar_{\varsigma}}\leq 1-\overline{\xi}_{\hbar_{\varsigma}}\right)\right\}\\ \Longrightarrow\left\{\left(\prod_{\varsigma=1}^{n}\left(1-\overline{\xi}_{\hbar_{\varsigma}}\right)^{\Xi_{\varsigma}\omega_{\varsigma}}\right)\leq\left(\prod_{\varsigma=1}^{n}\left(1-\overline{\xi}_{\hbar_{\varsigma}}\right)^{\Xi_{\varsigma}\omega_{\varsigma}}\right)\right\}\\ \Longrightarrow\left\{\left(1-\prod_{\varsigma=1}^{n}\left(1-\overline{\xi}_{\hbar_{\varsigma}}\right)^{\Xi_{\varsigma}\omega_{\varsigma}}\right)\leq\left(1-\prod_{\varsigma=1}^{n}\left(1-\overline{\xi}_{\hbar_{\varsigma}}\right)^{\Xi_{\varsigma}\omega_{\varsigma}}\right)\right\}\end{array}\right\},$$

also

$$\left\{\prod_{\varsigma=1}^{n}\left(\underline{\eta}_{\hbar_{\varsigma}}\right)^{\Xi_{\varsigma}\omega_{\varsigma}}\geq\prod_{\varsigma=1}^{n}\left(\underline{\eta}_{\hbar_{\varsigma}}\right)^{\Xi_{\varsigma}\omega_{\varsigma}}\right\},\left\{\prod_{\varsigma=1}^{n}\left(\overline{\eta}_{\hbar_{\varsigma}}\right)^{\Xi_{\varsigma}\omega_{\varsigma}}\geq\prod_{\varsigma=1}^{n}\left(\overline{\eta}_{\hbar_{\varsigma}}\right)^{\Xi_{\varsigma}\omega_{\varsigma}}\right\},$$

and

$$\left\{\prod_{\varsigma=1}^{n}\left(\underline{£}_{\hbar_{\varsigma}}\right)^{\Xi_{\varsigma}\omega_{\varsigma}}\geq\prod_{\varsigma=1}^{n}\left(\underline{£}_{\hbar_{\varsigma}}\right)^{\Xi_{\varsigma}\omega_{\varsigma}}\right\},\left\{\prod_{\varsigma=1}^{n}\left(\overline{£}_{\hbar_{\varsigma}}\right)^{\Xi_{\varsigma}\omega_{\varsigma}}\geq\prod_{\varsigma=1}^{n}\left(\overline{£}_{\hbar_{\varsigma}}\right)^{\Xi_{\varsigma}\omega_{\varsigma}}\right\}.$$

If

$$CL-SV-NHFRWA\left\{\begin{array}{c}(\mho_1,\Xi_1),(\mho_2,\Xi_2),\\ \ldots,(\mho_n,\Xi_n)\end{array}\right\}=\left\{\begin{array}{c}\left(\underline{\xi}_{\hbar_{\varsigma}},\underline{\eta}_{\hbar_{\varsigma}},\underline{£}_{\hbar_{\varsigma}}\right),\\ \left(\overline{\xi}_{\hbar_{\varsigma}},\overline{\eta}_{\hbar_{\varsigma}},\overline{£}_{\hbar_{\varsigma}}\right)\end{array}\right\}=\mho$$

and

$$CL-SV-NHFRWA\left\{\begin{array}{c}(\mho^{*}_1,\Xi_1),(\mho^{*}_2,\Xi_2),\\ \ldots,(\mho^{*}_n,\Xi_n)\end{array}\right\}=\left\{\begin{array}{c}\left(\underline{\xi}_{\hbar_{\varsigma}},\underline{\eta}_{\hbar_{\varsigma}},\underline{£}_{\hbar_{\varsigma}}\right),\\ \left(\overline{\xi}_{\hbar_{\varsigma}},\overline{\eta}_{\hbar_{\varsigma}},\overline{£}_{\hbar_{\varsigma}}\right)\end{array}\right\}=\mho.$$

Then we get $SF(\mho) \leq SF(\mho^*)$.

We have two cases: $\square$

Case 1: If $SF(\mho) < SF(\mho^*)$, applying SF we obtained

$$\begin{aligned} & CL-SV-NHFRWA\{(\mho_1,\Xi_1),(\mho_2,\Xi_2),\ldots,(\mho_n,\Xi_n)\} \\ < \; & CL-SV-NHFRWA\{(\mho_1^*,\Xi_1),(\mho_2^*,\Xi_2),\ldots,(\mho_n^*,\Xi_n)\}. \end{aligned}$$

Case 2: If $SF(\mho) = SF(\mho^*)$, applying SF we obtained

$$SF(\mho) = \left\{ \frac{1}{6} \left(\begin{array}{c} 3 + \left(1 - \prod_{\varsigma=1}^{n}\left(1-\underline{\xi}_{\hbar_\varsigma}\right)^{\Xi_\varsigma\omega_\varsigma}\right) + \left(1 - \prod_{\varsigma=1}^{n}\left(1-\overline{\xi}_{\hbar_\varsigma}\right)^{\Xi_\varsigma\omega_\varsigma}\right) \\ -\left(\prod_{\varsigma=1}^{n}\left(\underline{\eta}_{\hbar_\varsigma}\right)^{\Xi_\varsigma\omega_\varsigma}\right) - \left(\prod_{\varsigma=1}^{n}\left(\overline{\eta}_{\hbar_\varsigma}\right)^{\Xi_\varsigma\omega_\varsigma}\right) - \left(\prod_{\varsigma=1}^{n}\left(\underline{£}_{\hbar_\varsigma}\right)^{\Xi_\varsigma\omega_\varsigma}\right) - \left(\prod_{\varsigma=1}^{n}\left(\overline{£}_{\hbar_\varsigma}\right)^{\Xi_\varsigma\omega_\varsigma}\right) \end{array} \right) \right\}.$$

$$SF(\mho^*) = \left\{ \frac{1}{6} \left(\begin{array}{c} 3 + \left(1 - \prod_{\varsigma=1}^{n}\left(1-\underline{\xi}_{\hbar_\varsigma}\right)^{\Xi_\varsigma\omega_\varsigma}\right) + \left(1 - \prod_{\varsigma=1}^{n}\left(1-\overline{\xi}_{\hbar_\varsigma}\right)^{\Xi_\varsigma\omega_\varsigma}\right) \\ -\left(\prod_{\varsigma=1}^{n}\left(\underline{\eta}_{\hbar_\varsigma}\right)^{\Xi_\varsigma\omega_\varsigma}\right) - \left(\prod_{\varsigma=1}^{n}\left(\overline{\eta}_{\hbar_\varsigma}\right)^{\Xi_\varsigma\omega_\varsigma}\right) - \left(\prod_{\varsigma=1}^{n}\left(\underline{\eta}_{\hbar_\varsigma}\right)^{\Xi_\varsigma\omega_\varsigma}\right) - \left(\prod_{\varsigma=1}^{n}\left(\overline{\eta}_{\hbar_\varsigma}\right)^{\Xi_\varsigma\omega_\varsigma}\right) \end{array} \right) \right\}.$$

Since we have

$$\left\{\left(\underline{\xi}_{\hbar_\varsigma} \leq \underline{\xi}_{\hbar_\varsigma}\right), \left(\underline{\eta}_{\hbar_\varsigma} \geq \underline{\eta}_{\hbar_\varsigma}\right), \left(\underline{£}_{\hbar_\varsigma} \geq \underline{£}_{\hbar_\varsigma}\right), \left(\overline{\xi}_{\hbar_\varsigma} \leq \overline{\xi}_{\hbar_\varsigma}\right), \left(\overline{\eta}_{\hbar_\varsigma} \geq \overline{\eta}_{\hbar_\varsigma}\right), \left(\overline{£}_{\hbar_\varsigma} \geq \overline{£}_{\hbar_\varsigma}\right)\right\}$$

for all ς, we have

$$\begin{aligned} \left(1-\prod_{\varsigma=1}^{n}\left(1-\underline{\xi}_{\hbar_\varsigma}\right)^{\Xi_\varsigma\omega_\varsigma}\right) &= \left(1-\prod_{\varsigma=1}^{n}\left(1-\underline{\xi}_{\hbar_\varsigma}\right)^{\Xi_\varsigma\omega_\varsigma}\right), \\ \left(1-\prod_{\varsigma=1}^{n}\left(1-\overline{\xi}_{\hbar_\varsigma}\right)^{\Xi_\varsigma\omega_\varsigma}\right) &= \left(1-\prod_{\varsigma=1}^{n}\left(1-\overline{\xi}_{\hbar_\varsigma}\right)^{\Xi_\varsigma\omega_\varsigma}\right), \\ \left(\prod_{\varsigma=1}^{n}\left(\underline{\eta}_{\hbar_\varsigma}\right)^{\Xi_\varsigma\omega_\varsigma}\right) &= \left(\prod_{\varsigma=1}^{n}\left(\underline{\eta}_{\hbar_\varsigma}\right)^{\Xi_\varsigma\omega_\varsigma}\right), \left(\prod_{\varsigma=1}^{n}\left(\overline{\eta}_{\hbar_\varsigma}\right)^{\Xi_\varsigma\omega_\varsigma}\right) = \left(\prod_{\varsigma=1}^{n}\left(\overline{\eta}_{\hbar_\varsigma}\right)^{\Xi_\varsigma\omega_\varsigma}\right), \\ \left(\prod_{\varsigma=1}^{n}\left(\underline{£}_{\hbar_\varsigma}\right)^{\Xi_\varsigma\omega_\varsigma}\right) &= \left(\prod_{\varsigma=1}^{n}\left(\underline{£}_{\hbar_\varsigma}\right)^{\Xi_\varsigma\omega_\varsigma}\right), \left(\prod_{\varsigma=1}^{n}\left(\overline{£}_{\hbar_\varsigma}\right)^{\Xi_\varsigma\omega_\varsigma}\right) = \left(\prod_{\varsigma=1}^{n}\left(\overline{£}_{\hbar_\varsigma}\right)^{\Xi_\varsigma\omega_\varsigma}\right). \end{aligned}$$

Now applying the definition of AF, we get

$$AC(\mho) = \left\{ \frac{1}{6} \left(\begin{array}{c} 3 + \left(1 - \prod_{\varsigma=1}^{n}\left(1-\underline{\xi}_{\hbar_\varsigma}\right)^{\Xi_\varsigma\omega_\varsigma}\right) + \left(1 - \prod_{\varsigma=1}^{n}\left(1-\overline{\xi}_{\hbar_\varsigma}\right)^{\Xi_\varsigma\omega_\varsigma}\right) + \left(\prod_{\varsigma=1}^{n}\left(\underline{\eta}_{\hbar_\varsigma}\right)^{\Xi_\varsigma\omega_\varsigma}\right) \\ + \prod_{\varsigma=1}^{n}\left(\left(\overline{\eta}_{\hbar_\varsigma}\right)^{\Xi_\varsigma\omega_\varsigma}\right) + \left(\prod_{\varsigma=1}^{n}\left(\underline{£}_{\hbar_\varsigma}\right)^{\Xi_\varsigma\omega_\varsigma}\right) + \prod_{\varsigma=1}^{n}\left(\left(\overline{£}_{\hbar_\varsigma}\right)^{\Xi_\varsigma\omega_\varsigma}\right) \end{array} \right) \right\}$$

$$= \left\{\frac{1}{6}\left(\begin{array}{c} 3+\left(1-\prod_{\varsigma=1}^{n}\left(1-\underline{\xi}_{\hbar_\varsigma}\right)^{\Xi_\varsigma\omega_\varsigma}\right)+\left(1-\prod_{\varsigma=1}^{n}\left(1-\overline{\xi}_{\hbar_\varsigma}\right)^{\Xi_\varsigma\omega_\varsigma}\right)+\left(\prod_{\varsigma=1}^{n}\left(\underline{\eta}_{\hbar_\varsigma}\right)^{\Xi_\varsigma\omega_\varsigma}\right) \\ +\left(\prod_{\varsigma=1}^{n}\left(\overline{\eta}_{\hbar_\varsigma}\right)^{\Xi_\varsigma\omega_\varsigma}\right)+\left(\prod_{\varsigma=1}^{n}\left(\underline{\eta}_{\hbar_\varsigma}\right)^{\Xi_\varsigma\omega_\varsigma}\right)+\left(\prod_{\varsigma=1}^{n}\left(\overline{\eta}_{\hbar_\varsigma}\right)^{\Xi_\varsigma\omega_\varsigma}\right)\end{array}\right)\right\}$$
$$= AC\left(\mho^*\right).$$

Thus,

$$\begin{aligned} & CL-SV-NHFRWA\{(\mho_1,\Xi_1),(\mho_2,\Xi_2),\ldots,(\mho_n,\Xi_n)\} \\ < \; & CL-SV-NHFRWA\{(\mho_1^*,\Xi_1),(\mho_2^*,\Xi_2),\ldots,(\mho_n^*,\Xi_n)\}. \end{aligned}$$

3.1.1. CL-SV-NHFR ordered weighted average (CL-SV-NHFROWA) aggregation operators

In this part, a CL-SV-NHFROWA operator's fundamental definition is given. We'll also go into great detail about the main attributes of this operator.

Definition 13. *Let* $\mho_\varsigma = \left\{\left(\underline{\xi}_{\hbar_\varsigma}, \underline{\eta}_{\hbar_\varsigma}, \underline{£}_{\hbar_\varsigma}\right), \left(\overline{\xi}_{\hbar_\varsigma}, \overline{\eta}_{\hbar_\varsigma}, \overline{£}_{\hbar_\varsigma}\right)\right\}$, $\varsigma = 1, 2, \ldots, n$ *be a family of SV-NHFRNs and* Ξ_ς *be the CL of* $\mho_\varsigma$ *with* $0 \le \Xi_\varsigma \le 1$.

Let $\omega = (\omega_1, \omega_2, \omega_3 \ldots, \omega_n)^T$ be the WVs for SV-NHFRNs with the condition $\sum_{\varsigma}^{n} \omega_\varsigma = 1$.
Then, the mapping $CL-SV-NHFROWA : F^n \to F$ operator is given as

$$\begin{aligned} & CL-SV-NHFROWA\{(\mho_1,\Xi_1),(\mho_2,\Xi_2),\ldots,(\mho_n,\Xi_n)\} \\ = \; & \left\{\omega_1\left(\Xi_{\varepsilon(1)}\mho_{\varepsilon(1)}\right)\oplus\omega_2\left(\Xi_{\varepsilon(2)}\mho_{\varepsilon(2)}\right)\oplus\omega_3\left(\Xi_{\varepsilon(3)}\mho_{\varepsilon(3)}\right)\ldots\oplus\omega_n\left(\Xi_{\varepsilon(n)}\mho_{\varepsilon(n)}\right)\right\}, \end{aligned}$$

where $(\varepsilon(1), \varepsilon(1)\varepsilon(2), \varepsilon(3), ..., \varepsilon(n))$ is the permutation of $(\varsigma = 1, 2, \ldots, n)$ such that for all ς, $\mho_{\varepsilon(\varsigma-1)} \ge \mho_{\varepsilon(\varsigma)}$.

Theorem 3. *Let*

$$\mho_\varsigma = \left(\left(\underline{\xi}_{\hbar_\varsigma}, \underline{\eta}_{\hbar_\varsigma}, \underline{£}_{\hbar_\varsigma}\right), \left(\overline{\xi}_{\hbar_\varsigma}, \overline{\eta}_{\hbar_\varsigma}, \overline{£}_{\hbar_\varsigma}\right)\right), \varsigma = 1, 2, \ldots, n$$

be a collection of SV-NHFRNs and Ξ_ς *be the CL of* $\mho_\varsigma$ *with* $0 \le \Xi_\varsigma \le 1$.
Let $\omega = (\omega_1, \omega_2, \omega_3 \ldots, \omega_n)^T$ *be the* WVs *for SV-NHFRNs with the condition* $\sum_{\varsigma}^{n} \omega_\varsigma = 1$.
Then

$$\begin{aligned} & CL-SV-NHFROWA\{(\mho_1,\Xi_1),(\mho_2,\Xi_2),\ldots,(\mho_n,\Xi_n)\} \\ = \; & \left\{\begin{array}{c} \left\{\left(1-\prod_{\varsigma=1}^{n}\left(1-\underline{\xi}_{\hbar_\varsigma}\right)^{\Xi_{\varepsilon(\varsigma)}\omega_\varsigma}\right),\left(\prod_{\varsigma=1}^{n}\left(\underline{\eta}_{\hbar_\varsigma}\right)^{\Xi_{\varepsilon(\varsigma)}\omega_\varsigma}\right),\left(\prod_{\varsigma=1}^{n}\left(\underline{£}_{\hbar_\varsigma}\right)^{\Xi_{\varepsilon(\varsigma)}\omega_\varsigma}\right)\right\}, \\ \left\{\left(1-\prod_{\varsigma=1}^{n}\left(1-\xi_{\varepsilon(\varsigma)}\right)^{\Xi_{\varepsilon(\varsigma)}\omega_\varsigma}\right),\left(\prod_{\varsigma=1}^{n}\left(\overline{\eta}_{\hbar_\varsigma}\right)^{\Xi_{\varepsilon(\varsigma)}\omega_\varsigma}\right),\left(\prod_{\varsigma=1}^{n}\left(\overline{£}_{\hbar_\varsigma}\right)^{\Xi_{\varepsilon(\varsigma)}\omega_\varsigma}\right)\right\} \end{array}\right\}. \end{aligned} \tag{3.1}$$

Proof. The proof is comparable to the Theorem 1 demonstration. □

Here, we examine the traits of the CI-SV-NHFROWA operator.

(1) Idempotency

If for all ς $(\mho_\varsigma, \Xi_\varsigma) = (\mho, \Xi)$, i.e., $\overline{\xi}_{\hbar_\varsigma} = \overline{\xi}, \underline{\xi}_{\hbar_\varsigma} = \underline{\xi}, \underline{\eta}_{\hbar_\varsigma} = \underline{\eta}, \overline{\eta}_{\hbar_\varsigma} = \overline{\eta}$, $, \underline{\pounds}_{\hbar_\varsigma} = \underline{\pounds}$ and $\overline{\pounds}_{\hbar_\varsigma} = \overline{\pounds}$, $\Xi_\varsigma = \Xi$, then

$$CL-SV-NHFROWA\{(\mho_1, \Xi_1), (\mho_2, \Xi_2), \ldots, (\mho_n, \Xi_n)\} = \Xi_\mho.$$

(2) Boundedness: Let $\mho_\varsigma^- = \left\{ \begin{array}{c} \left(\underline{\xi}_{\hbar_\varsigma}, \underline{\eta}_{\hbar_\varsigma}, \underline{\pounds}_{\hbar_\varsigma}\right), \\ \left(\overline{\xi}_{\hbar_\varsigma}, \overline{\eta}_{\hbar_\varsigma}, \overline{\pounds}_{\hbar_\varsigma}\right) \end{array} \right\}$ and $\mho_\varsigma^+ = \left\{ \begin{array}{c} \left(\underline{\xi}_{\hbar_\varsigma}, \underline{\eta}_{\hbar_\varsigma}, \underline{\pounds}_{\hbar_\varsigma}\right), \\ \left(\overline{\xi}_{\hbar_\varsigma}, \overline{\eta}_{\hbar_\varsigma}, \overline{\pounds}_{\hbar_\varsigma}\right) \end{array} \right\}$.

Then, for all ω_ς,

$$\mho_\varsigma^- \le CL-SV-NHFROWA\{(\mho_1, \Xi_1), (\mho_2, \Xi_2), \ldots, (\mho_n, \Xi_n)\} \le \mho_\varsigma^+.$$

(3) Monotonicity: Let $\mho_\varsigma^* = \left\{\left(\underline{\xi}_{\hbar_\varsigma}, \underline{\eta}_{\hbar_\varsigma}, \underline{\pounds}_{\hbar_\varsigma}\right), \left(\overline{\xi}_{\hbar_\varsigma}, \overline{\eta}_{\hbar_\varsigma}, \overline{\pounds}_{\hbar_\varsigma}\right)\right\}$ ($\varsigma = 1, 2, 3, ..., n$) be another collection of SV-NHFRNs such that

$$\left\{\left(\underline{\xi}_{\hbar_\varsigma} \le \underline{\xi}_{\hbar_\varsigma}, \underline{\eta}_{\hbar_\varsigma} \mho_\varsigma \ge \underline{\eta}_{\hbar_\varsigma} \mho_\varsigma^*, \underline{\pounds}_{\hbar_\varsigma} \ge \underline{\pounds}_{\hbar_\varsigma}\right), \left(\overline{\xi}_{\hbar_\varsigma} \le \overline{\xi}_{\hbar_\varsigma}, \overline{\eta}_{\hbar_\varsigma} \ge \overline{\eta}_{\hbar_\varsigma}, \overline{\pounds}_{\hbar_\varsigma} \ge \overline{\pounds}_{\hbar_\varsigma}\right)\right\}$$

for all ω_ς. Then

$$\begin{aligned} & CL-SV-NHFROWA\{(\mho_1, \Xi_1), (\mho_2, \Xi_2), \ldots, (\mho_n, \Xi_n)\} \\ \le \; & CL-SV-NHFROWA\{(\mho_1^*, \Xi_1), (\mho_2^*, \Xi_2), \ldots, (\mho_n^*, \Xi_n)\}. \end{aligned}$$

3.2. CL-single-valued neutrosophic hesitant fuzzy rough geometric aggregation operators

In this section, we discuss CI-SV-NR geometric AOs. We'll look into the essential qualities of the operators as well.

3.2.1. CL-SV-NHFR weighted geometric (CL-SV-NHFRWG) aggregation operator

Definition 14. $\mho_\varsigma = \left(\left(\underline{\xi}_{\hbar_\varsigma}, \underline{\eta}_{\hbar_\varsigma}, \underline{\pounds}_{\hbar_\varsigma}\right), \left(\overline{\xi}_{\hbar_\varsigma}, \overline{\eta}_{\hbar_\varsigma}, \overline{\pounds}_{\hbar_\varsigma}\right)\right)$, $\varsigma = 1, 2, \ldots, n$ *be a collection of SV-NHFRNs and* Ξ_ς *be the CL of* $\mho_\varsigma$ *with* $0 \le \Xi_\varsigma \le 1$.

Let $\omega = (\omega_1, \omega_2, \omega_3 \ldots, \omega_n)^T$ be the WVs for SV-NRNs with the condition $\sum_\varsigma^n \omega_\varsigma = 1$. Then, the mapping $CL-SV-NHFRWA : F^n \to F$ operator is given as CL-SV-NRWG operator,

$$\{(\mho_1, \Xi_1), (\mho_2, \Xi_2), \ldots, (\mho_n, \Xi_n)\} = \oplus_{\varsigma=1}^n \left(\mho_\varsigma^{\Xi_\varsigma}\right)^{\omega_\varsigma} = \left(\mho_1^{\Xi_1}\right)^{\omega_1} \oplus \left(\mho_2^{\Xi_2}\right)^{\omega_2} \oplus \left(\mho_3^{\Xi_3}\right)^{\omega_3} ... \oplus \left(\mho_n^{\Xi_n}\right)^{\omega_n}.$$

It is called the CL-SV-NHFRWG operator.

Theorem 4. *Let* $\mho_\varsigma = \left(\left(\underline{\xi}_{\hbar_\varsigma}, \underline{\eta}_{\hbar_\varsigma}, \underline{\pounds}_{\hbar_\varsigma}\right), \left(\overline{\xi}_{\hbar_\varsigma}, \overline{\eta}_{\hbar_\varsigma}, \overline{\pounds}_{\hbar_\varsigma}\right)\right)$, $\varsigma = 1, 2, \ldots, n$ *be a collection of SV-NRNs and* Ξ_ς *be the CL of* $\mho_\varsigma$ *with* $0 \le \Xi_\varsigma \le 1$.

Let $\omega = (\omega_1, \omega_2, \omega_3 \ldots, \omega_n)^T$ *be the* WVs *for SV-NHFRNs with the condition* $\sum_\varsigma^n \omega_\varsigma = 1$. *Then* $CL-SV-NHFRWG$

$$\begin{aligned} & CL-SV-NHFRWG((\mho_1, \Xi_1), (\mho_2, \Xi_2), \ldots, (\mho_n, \Xi_n)) \\ = & \left\{ \begin{array}{c} \left\{\left(\prod_{\varsigma=1}^n \left(\underline{\xi}_{\hbar_\varsigma}\right)^{\Xi_\varsigma \omega_\varsigma}\right), \left(1 - \prod_{\varsigma=1}^n \left(1 - \underline{\eta}_{\hbar_\varsigma}\varsigma\right)^{\Xi_\varsigma \omega_\varsigma}\right), \left(1 - \prod_{\varsigma=1}^n \left(1 - \underline{\pounds}_{\hbar_\varsigma}\right)^{\Xi_\varsigma \omega_\varsigma}\right)\right\} \\ \left\{\left(\prod_{\varsigma=1}^n \left(\overline{\xi}_{\hbar_\varsigma}\right)^{\Xi_\varsigma \omega_\varsigma}\right), \left(1 - \prod_{\varsigma=1}^n \left(1 - \overline{\eta}_{\hbar_\varsigma}\right)^{\Xi_\varsigma \omega_\varsigma}\right), \left(1 - \prod_{\varsigma=1}^n \left(1 - \overline{\pounds}_{\hbar_\varsigma}\right)^{\Xi_\varsigma \omega_\varsigma}\right)\right\} \end{array} \right\}. \end{aligned} \tag{3.2}$$

Proof. For n = 2, we have

$$CL-SV-NHFRWG\left((\mho_1,\Xi_1),(\mho_2,\Xi_2)\right)=\left(\mho_{\varsigma}^{\Xi_\varsigma}\right)^{\omega_1}\oplus\left(\mho_{2}^{\Xi_\varsigma}\right)^{\omega_2}.$$

By using the operational laws for SV-NHFRNs, we get

$$\begin{aligned}\mho_1^{\Xi_1} &= \left(\begin{array}{c}\left(\underline{\xi}_{\hbar_\varsigma},\left(1-\left(1-\underline{\eta}_{\hbar_\varsigma 1}\right)^{\Xi_1}\right),\left(1-\left(1-\underline{\pounds}_{\hbar_\varsigma}\right)^{\Xi_1}\right)\right),\\ \left(\overline{\xi}_{\hbar_\varsigma}\right)^{\Xi_1},\left(1-\left(-1-\overline{\eta}_{\hbar_\varsigma}\right)^{\Xi_1}\right),\left(1-\left(-1-\overline{\pounds}_{\hbar_\varsigma}\right)^{\Xi_1}\right)\end{array}\right)\\ &= \left\{\left(\underline{\Upsilon}_1,\underline{\xi}_1,\underline{\Theta}_1\right),\left(\overline{\vartheta}_1,\overline{\xi}_1,\overline{\Omega}_1\right)\right\}.\end{aligned}$$

Then

$$\begin{aligned}\left(\mho_1^{\Xi_1}\right)^{\omega_1} &= \left(\begin{array}{c}\left(\left(\underline{\Upsilon}_1^{\Xi_1}\right),\left(1-\left(1-\underline{\xi}_1\right)^{\Xi_1}\right),\left(1-\left(1-\underline{\Theta}_1\right)^{\Xi_1}\right)\right),\\ \left(\left(\overline{\vartheta}_1^{\Xi_1}\right),\left(1-\left(1-\overline{\xi}_1\right)^{\Xi_1}\right),\left(1-\left(1-\overline{\Omega}_1\right)^{\Xi_1}\right)\right)\end{array}\right)\\ &= \left(\begin{array}{c}\left\{\begin{array}{c}\left(\underline{\xi}_{\hbar_1}\right)^{\omega_1},\left(1-\left[1-\left\{1-\left(1-\underline{\eta}_{\hbar_1}\right)^{\Xi_1}\right\}\right]^{\omega_1}\right),\\ \left(1-\left[1-\left\{1-\left(1-\underline{\pounds}_{\hbar_1}\right)^{\Xi_1}\right\}\right]^{\omega_1}\right)\end{array}\right\},\\ \left\{\begin{array}{c}\left(\left(\overline{\xi}_{\hbar_1}\right)^{\Xi_1}\right)^{\omega_1},\left(1-\left[1-\left\{1-\left(1-\overline{\eta}_{\hbar_1}\right)^{\Xi_1}\right\}\right]^{\omega_1}\right),\\ \left(1-\left[1-\left\{1-\left(1-\overline{\pounds}_{\hbar_1}\right)^{\Xi_1}\right\}\right]^{\omega_1}\right)\end{array}\right\}\end{array}\right)\\ &= \left(\begin{array}{c}\left(\left(\underline{\xi}_{\hbar_1}\right)^{\Xi_1\omega_1},\left(1-\left(1-\underline{\eta}_{\hbar_1}\right)^{\Xi_1\omega_1}\right),\left(1-\left(1-\underline{\pounds}_{\hbar_1}\right)^{\Xi_1\omega_1}\right)\right),\\ \left(\left(\overline{\xi}_{\hbar_1}\right)^{\Xi_1\omega_1},\left(1-\left(1-\overline{\eta}_{\hbar_1}\right)^{\Xi_1\omega_1}\right),\left(1-\left(1-\overline{\pounds}_{\hbar_1}\right)^{\Xi_1\omega_1}\right)\right)\end{array}\right).\end{aligned}$$

Likewise, we can observe that

$$\left(\mho_2^{\Xi_2}\right)^{\omega_2}=\left(\begin{array}{c}\left(\left(\underline{\xi}_{\hbar_\varsigma}\right),\left(1-\left(1-\underline{\eta}_{\hbar_\varsigma 2}\right)^{\Xi_2\omega_2}\right),\left(1-\left(1-\underline{\pounds}_{\hbar_\varsigma}\right)^{\Xi_2\omega_2}\right)\right),\\ \left(\left(\overline{\xi}_{\hbar_\varsigma}\right),\left(1-\left(1-\overline{\eta}_{\hbar_\varsigma}\right)^{\Xi_2\omega_2}\right),\left(1-\left(1-\overline{\pounds}_{\hbar_\varsigma}\right)^{\Xi_2\omega_2}\right)\right)\end{array}\right).$$

Now,

$$\begin{aligned}CL-SV-NHFRWG &= \left((\mho_1,\Xi_1),(\mho_2,\Xi_2)\right)=\left(\mho_1^{\Xi_1}\right)^{\omega_1}\oplus\left(\mho_2^{\Xi_2}\right)^{\omega_2}\\ &= \left(\begin{array}{c}\left\{\begin{array}{c}\left(\underline{\xi}_{\hbar_1}\right)^{\Xi_1\omega_1}\left(\underline{\xi}_{\hbar_2}\right)^{\Xi_2\omega_2},\left(1-\left(1-\underline{\eta}_{\hbar_1}\right)^{\Xi_1\omega_1}\right)\left(1-\left(1-\underline{\eta}_{\hbar_2}\right)^{\Xi_2\omega_2}\right),\\ \left(1-\left(1-\underline{\pounds}_{\hbar_1}\right)^{\Xi_1\omega_1}\right)\left(1-\left(1-\underline{\pounds}_{\hbar_2}\right)^{\Xi_2\omega_2}\right)\end{array}\right\},\\ \left\{\begin{array}{c}\left(\overline{\xi}_{\hbar_\varsigma}\right)\left(\overline{\xi}_{\hbar_\varsigma}\right),\left(1-\left(1-\overline{\eta}_{\hbar_1}\right)^{\Xi_1\omega_1}\right)\left(1-\left(1-\overline{\eta}_{\hbar_2}\right)^{\Xi_2\omega_2}\right),\\ \left(1-\left(1-\overline{\pounds}_{\hbar_1}\right)^{\Xi_1\omega_1}\right)\left(1-\left(1-\overline{\pounds}_{\hbar_2}\right)^{\Xi_2\omega_2}\right)\end{array}\right\}\end{array}\right).\end{aligned}$$

Thus, In light of that,

$$
\begin{aligned}
&CL-SV-NHFRWG\{(\mho_1,\Xi_1),(\mho_2,\Xi_2)\}\\
=&\left(\begin{array}{c}\left\{\left(\prod_{\varsigma=1}^{2}\left(\underline{\xi}_{\hbar_\varsigma}\right)^{\Xi_\varsigma\omega_\varsigma}\right),\left(1-\prod_{\varsigma=1}^{2}\left(1-\underline{\eta}_{\hbar_\varsigma}\varsigma\right)^{\Xi_\varsigma\omega_\varsigma}\right),\left(1-\prod_{\varsigma=1}^{2}\left(1-\underline{\pounds}_{\hbar_\varsigma}\right)^{\Xi_\varsigma\omega_\varsigma}\right)\right\},\\ \left\{\left(\prod_{\varsigma=1}^{2}\left(\overline{\xi}_{\hbar_\varsigma}\right)^{\Xi_\varsigma\omega_\varsigma}\right),\left(1-\prod_{\varsigma=1}^{2}\left(1-\overline{\eta}_{\hbar_\varsigma}\right)^{\Xi_\varsigma\omega_\varsigma}\right),\left(1-\prod_{\varsigma=1}^{2}\left(1-\overline{\pounds}_{\hbar_\varsigma}\right)^{\Xi_\varsigma\omega_\varsigma}\right)\right\}\end{array}\right).
\end{aligned}
$$

Suppose that the result is valid for $n=\dagger$, that is

$$
\begin{aligned}
&CL-SV-NHFRWG\{(\mho_1,\Xi_1),(\mho_2,\Xi_2),\ldots,(\mho_\dagger,\Xi_\dagger)\}\\
=&\left(\begin{array}{c}\left\{\left(\prod_{\varsigma=1}^{\dagger}\left(\underline{\xi}_{\hbar_\varsigma}\right)^{\Xi_\varsigma\omega_\varsigma}\right),\left(1-\prod_{\varsigma=1}^{\dagger}\left(1-\underline{\eta}_{\hbar_\varsigma}\right)^{\Xi_\varsigma\omega_\varsigma}\right),\left(1-\prod_{\varsigma=1}^{\dagger}\left(1-\underline{\pounds}_{\hbar_\varsigma}\right)^{\Xi_\varsigma\omega_\varsigma}\right)\right\},\\ \left\{\left(\prod_{\varsigma=1}^{\dagger}\left(\overline{\xi}_{\hbar_\varsigma}\right)^{\Xi_\varsigma\omega_\varsigma}\right),\left(1-\prod_{\varsigma=1}^{\dagger}\left(1-\overline{\eta}_{\hbar_\varsigma}\right)^{\Xi_\varsigma\omega_\varsigma}\right),\left(1-\prod_{\varsigma=1}^{\dagger}\left(1-\overline{\pounds}_{\hbar_\varsigma}\right)^{\Xi_\varsigma\omega_\varsigma}\right)\right\}\end{array}\right).
\end{aligned}
$$

Then, for $n=\dagger+1$, we get

$$
\begin{aligned}
&CL-SV-NHFRWG\{(\mho_1,\Xi_1),(\mho_2,\Xi_2),\ldots,(\mho_\dagger,\Xi_\dagger),(\mho_{\dagger+1},\Xi_{\dagger+1})\}\\
=&\left(\begin{array}{c}\left\{\left(\prod_{\varsigma=1}^{\dagger}\left(\underline{\xi}_{\hbar_\varsigma}\right)^{\Xi_\varsigma\omega_\varsigma}\right),\left(1-\prod_{\varsigma=1}^{\dagger}\left(1-\underline{\eta}_{\hbar_\varsigma}\right)^{\Xi_\varsigma\omega_\varsigma}\right),\left(1-\prod_{\varsigma=1}^{\dagger}\left(1-\underline{\pounds}_{\hbar_\varsigma}\right)^{\Xi_\varsigma\omega_\varsigma}\right)\right\},\\ \left\{\left(\prod_{\varsigma=1}^{\dagger}\left(\overline{\xi}_{\hbar_\varsigma}\right)^{\Xi_\varsigma\omega_\varsigma}\right),\left(1-\prod_{\varsigma=1}^{\dagger}\left(1-\overline{\eta}_{\hbar_\varsigma}\right)^{\Xi_\varsigma\omega_\varsigma}\right),\left(1-\prod_{\varsigma=1}^{\dagger}\left(1-\overline{\pounds}_{\hbar_\varsigma}\right)^{\Xi_\varsigma\omega_\varsigma}\right)\right\}\end{array}\right)\oplus\\
&\left(\begin{array}{c}\left\{\left(\left(\underline{\xi}_{\hbar_{\dagger+1}}\right)^{\Xi_{\dagger+1}\omega_{\dagger+1}}\right),\left(1-\left(1-\underline{\eta}_{\hbar_{\dagger+1}}\right)^{\Xi_{\dagger+1}\omega_{\dagger+1}}\right),\left(1-\left(1-\underline{\pounds}_{\hbar_\varsigma}\right)^{\Xi_{\dagger+1}\omega_{\dagger+1}}\right)\right\},\\ \left\{\left(\left(\overline{\xi}_{\hbar_{\dagger+1}}\right)^{\Xi_{\dagger+1}\omega_{\dagger+1}}\right),\left(1-\left(1-\overline{\eta}_{\hbar_{\dagger+1}}\right)^{\Xi_{\dagger+1}\omega_{\dagger+1}}\right),\left(1-\left(1-\overline{\pounds}_{\hbar_{\dagger+1}}\right)^{\Xi_{\dagger+1}\omega_{\dagger+1}}\right)\right\}\end{array}\right)\\
=&\left(\begin{array}{c}\left(\left(\prod_{\varsigma=1}^{\dagger}\left(\left(\underline{\xi}_{\hbar_\varsigma}\right)^{\Xi_\varsigma\omega_\varsigma}\right)\right)+\left(\left(\underline{\xi}_{\hbar_{\dagger+1}}\right)^{\Xi_{\dagger+1}\omega_{\dagger+1}}\right)\left(\prod_{\varsigma=1}^{\dagger}\left(\left(\overline{\xi}_{\hbar_\varsigma}\right)^{\Xi_\varsigma\omega_\varsigma}\right)\right)\left(\left(\overline{\xi}_{\hbar_{\dagger+1}}\right)^{\Xi_{\dagger+1}\omega_{\dagger+1}}\right)\right),\\
\left(\begin{array}{c}\left(\left(1-\prod_{\varsigma=1}^{\dagger}\left(1-\underline{\eta}_{\hbar_\varsigma}\right)^{\Xi_\varsigma\omega_\varsigma}\right)+\left(1-\left(1-\underline{\eta}_{\hbar_{\dagger+1}}\right)^{\Xi_{\dagger+1}\omega_{\dagger+1}}\right)\right)\\ \left(\left(1-\prod_{\varsigma=1}^{\dagger}\left(1-\overline{\eta}_{\hbar_\varsigma}\right)^{\Xi_\varsigma\omega_\varsigma}\right)+\left(1-\left(1-\overline{\eta}_{\hbar_{\dagger+1}}\right)^{\Xi_{\dagger+1}\omega_{\dagger+1}}\right)\right)\end{array}\right),\\
\left(\begin{array}{c}\left(\left(1-\prod_{\varsigma=1}^{\dagger}\left(1-\underline{\pounds}_{\hbar_\varsigma}\right)^{\Xi_\varsigma\omega_\varsigma}\right)+\left(1-\left(1-\underline{\pounds}_{\hbar_\varsigma}\right)^{\Xi_{\dagger+1}\omega_{\dagger+1}}\right)\right)\\ \left(\left(1-\prod_{\varsigma=1}^{\dagger}\left(1-\overline{\pounds}_{\hbar_\varsigma}\right)^{\Xi_\varsigma\omega_\varsigma}\right)+\left(1-\left(1-\overline{\pounds}_{\hbar_\varsigma}\right)^{\Xi_{\dagger+1}\omega_{\dagger+1}}\right)\right)\end{array}\right)\end{array}\right)\\
=&\left(\begin{array}{c}\left\{\left(\prod_{\varsigma=1}^{\dagger+1}\left(\left(\underline{\xi}_{\hbar_\varsigma}\right)^{\Xi_\varsigma\omega_\varsigma}\right)\right),\left(1-\prod_{\varsigma=1}^{\dagger+1}\left(1-\underline{\eta}_{\hbar_\varsigma}\right)^{\Xi_\varsigma\omega_\varsigma}\right),\left(1-\prod_{\varsigma=1}^{\dagger+1}\left(1-\underline{\pounds}_{\hbar_\varsigma}\right)^{\Xi_\varsigma\omega_\varsigma}\right)\right\},\\ \left\{\left(\prod_{\varsigma=1}^{\dagger+1}\left(\left(\overline{\xi}_{\hbar_\varsigma}\right)^{\Xi_\varsigma\omega_\varsigma}\right)\right),\left(1-\prod_{\varsigma=1}^{\dagger+1}\left(1-\overline{\eta}_{\hbar_\varsigma}\right)^{\Xi_\varsigma\omega_\varsigma}\right),\left(1-\prod_{\varsigma=1}^{\dagger+1}\left(1-\overline{\pounds}_{\hbar_\varsigma}\right)^{\Xi_\varsigma\omega_\varsigma}\right)\right\}\end{array}\right).
\end{aligned}
$$

As a result, the statement holds true for $n=\dagger+1$. The results are therefore generalizable to any number of SV-NHFRNs. □

3.2.2. CL-single-valued neutrosophic hesitant fuzzy rough ordered weighted geometric (CL-SV-NHFROWG) aggregation operator

In this section, we go over a CL-SV-NHFROWG operator's fundamental definition. We also go into great detail about the fundamental characteristics of these operators.

Definition 15. *Let* $\mho_\varsigma = \left\{\left(\underline{\xi}_{\hbar_\varsigma}, \underline{\eta}_{\hbar_\varsigma}, \underline{\pounds}_{\hbar_\varsigma}\right), \left(\overline{\xi}_{\hbar_\varsigma}, \overline{\eta}_{\hbar_\varsigma}, \overline{\pounds}_{\hbar_\varsigma}\right)\right\}$*,* $\varsigma = 1, 2, \ldots, n$ *be a family of SV-NHFRNs and* Ξ_ς *be the CL of* $\mho_\varsigma$ *with* $0 \le \Xi_\varsigma \le 1$.

Let $\omega = (\omega_1, \omega_2, \omega_3 \ldots, \omega_n)^T$ be the WVs for SV-NHFRNs with the condition $\sum_\varsigma^n \omega_\varsigma = 1$. Then, the mapping $CL - SV - NHFROWA : F^n \to F$ operator is given as

$$\begin{aligned} & CL - SV - NHFROWG\{(\mho_1, \Xi_1), (\mho_2, \Xi_2), \ldots, (\mho_n, \Xi_n)\} \\ = & \left\{\left(\mho_{\varepsilon(1)}^{\Xi_{\varepsilon(1)}}\right)^{\omega_1} \oplus \left(\mho_{\varepsilon(2)}^{\Xi_{\varepsilon(2)}}\right)^{\omega_2} \oplus \left(\mho_{\varepsilon(2)}^{\Xi_{\varepsilon(2)}}\right)^{\psi_2} ... \oplus \left(\mho_{\varepsilon(n)}^{\Xi_{\varepsilon(n)}}\right)^{\omega_n}\right\}, \end{aligned}$$

where where $(\varepsilon(1), \varepsilon(1)\, \varepsilon(2), \varepsilon(3), ..., \varepsilon(n))$ is the permutation of $(\varsigma = 1, 2, \ldots, n)$ such that for all ς, $\mho_{\varepsilon(\varsigma-1)} \ge \mho_{\varepsilon(\varsigma)}$.

Definition 16. *Let* $\mho_\varsigma = \left\{\left(\underline{\xi}_{\hbar_\varsigma}, \underline{\eta}_{\hbar_\varsigma}, \underline{\pounds}_{\hbar_\varsigma}\right), \left(\overline{\xi}_{\hbar_\varsigma}, \overline{\eta}_{\hbar_\varsigma}, \overline{\pounds}_{\hbar_\varsigma}\right)\right\}$*,* $\varsigma = 1, 2, \ldots, n$ *be a family of SV-NHFRNs and* Ξ_ς *be the CL of* $\mho_\varsigma$ *with* $0 \le \Xi_\varsigma \le 1$.

Theorem 5. *Let* $\omega = (\omega_1, \omega_2, \omega_3 \ldots, \omega_n)^T$ *be the* WVs *for SV-NHFRNs with the condition* $\sum_\varsigma^n \omega_\varsigma = 1$. *Then,*

$$\begin{aligned} & CL - SV - NHFROWG\{(\mho_1, \Xi_1), (\mho_2, \Xi_2), \ldots, (\mho_n, \Xi_n)\} \\ = & \left(\begin{array}{c} \left\{\left(\prod_{\varsigma=1}^n \left(\underline{\xi}_{\hbar_\varsigma}\right)^{\Xi_{\varepsilon(\varsigma)}\omega_\varsigma}\right), \left(1 - \prod_{\varsigma=1}^n \left(1 - \underline{\eta}_{\hbar_\varsigma \varepsilon(\varsigma)}\right)^{\Xi_{\varepsilon(\varsigma)}\omega_\varsigma}\right), \left(1 - \prod_{\varsigma=1}^n \left(1 - \underline{\pounds}_{\hbar_\varsigma}\right)^{\Xi_{\varepsilon(\varsigma)}\omega_\varsigma}\right)\right\}, \\ \left\{\left(\prod_{\varsigma=1}^n \left(\overline{\xi}_{\hbar_\varsigma}\right)^{\Xi_{\varepsilon(\varsigma)}\omega_\varsigma}\right), \left(1 - \prod_{\varsigma=1}^n \left(1 - \overline{\eta}_{\hbar_\varsigma}\right)^{\Xi_{\varepsilon(\varsigma)}\omega_\varsigma}\right), \left(1 - \prod_{\varsigma=1}^n \left(1 - \overline{\pounds}_{\hbar_\varsigma}\right)^{\Xi_{\varepsilon(\varsigma)}\omega_\varsigma}\right)\right\} \end{array}\right). \end{aligned} \quad (3.3)$$

Proof. Proof is similar as the proof of Theorem 3. □

We next discuss the properties of the CL-SV-NHFROWG operator.

(1) Idempotency If $\forall\, \varsigma$, $(\mho_\varsigma, \Xi_\varsigma) = (\mho, \Xi)$, i.e., $\underline{\xi}_{\hbar_\varsigma} = \underline{\xi}_{\hbar_\varsigma}, \overline{\xi}_{\hbar_\varsigma} = \overline{\xi}_{\hbar_\varsigma}, \underline{\eta}_{\hbar_\varsigma} = \underline{\eta}_{\hbar_\varsigma}, \overline{\eta}_{\hbar_\varsigma} = \overline{\eta}_{\hbar_\varsigma}, \underline{\pounds}_{\hbar_\varsigma} = \underline{\pounds}_{\hbar_\varsigma}$ and $\overline{\pounds}_{\hbar_\varsigma} = \overline{\pounds}_{\hbar_\varsigma}$, $\Xi_\varsigma = \Xi$, then

$$CL - SV - NHFROWG\{(\mho_1, \Xi_1), (\mho_2, \Xi_2), \ldots, (\mho_n, \Xi_n)\} = \Xi_\mho.$$

(2) Boundedness Let

$$\mho_\varsigma^- = \left\{ {}^{\min}_{\mho_\varsigma \Xi_\varsigma}, \left(\underline{\xi}_{\hbar_\varsigma}, \underline{\eta}_{\hbar_\varsigma}, \underline{\pounds}_{\hbar_\varsigma}\right), \left(\overline{\xi}_{\hbar_\varsigma}, \overline{\eta}_{\hbar_\varsigma}, \overline{\pounds}_{\hbar_\varsigma}\right)\right\}$$

and

$$\mho_\varsigma^+ = \left\{ {}^{\max}_{\mho_\varsigma \Xi_\varsigma}, \left(\underline{\xi}_{\hbar_\varsigma}, \underline{\eta}_{\hbar_\varsigma}, \underline{\pounds}_{\hbar_\varsigma}\right), \left(\overline{\xi}_{\hbar_\varsigma}, \overline{\eta}_{\hbar_\varsigma}, \overline{\pounds}_{\hbar_\varsigma}\right)\right\}.$$

Then, for all ω_ς,

$$\mho_\varsigma^- \le CL - SV - NHFROWG\{(\mho_1, \Xi_1), (\mho_2, \Xi_2), \ldots, (\mho_n, \Xi_n)\} \le \mho_\varsigma^+.$$

(3) Monotonicity Let

$$\mho_\varsigma^* = \left(\left(\underline{\xi}_{\hbar_\varsigma}, \underline{\eta}_{\hbar_\varsigma} \mho_\varsigma^*, \underline{\pounds}_{\hbar_\varsigma}\right), \left(\overline{\xi}_{\hbar_\varsigma}, \overline{\eta}_{\hbar_\varsigma}, \overline{\pounds}_{\hbar_\varsigma}\right)\right)(\ \varsigma = 1, 2, 3, ..., n)$$

be another family of SV-NHFRNs such that

$$\left\{\left(\underline{\xi}_{\hbar_\varsigma} \le \underline{\xi}_{\hbar_\varsigma}\right), \left(\underline{\eta}_{\hbar_\varsigma} \mho_\varsigma \ge \underline{\eta}_{\hbar_\varsigma} \mho_\varsigma^*\right), \left(\underline{\pounds}_{\hbar_\varsigma} \ge \underline{\pounds}_{\hbar_\varsigma}\right)\left(\overline{\xi}_{\hbar_\varsigma} \le \overline{\xi}_{\hbar_\varsigma}\right), \left(\overline{\eta}_{\hbar_\varsigma} \ge \overline{\eta}_{\hbar_\varsigma}\right), \left(\overline{\pounds}_{\hbar_\varsigma} \ge \overline{\pounds}_{\hbar_\varsigma}\right)\right\}$$

for all ω_ς. Then

$$
\begin{aligned}
& CL-SV-NHFROWG\{(\mho_1,\Xi_1),(\mho_2,\Xi_2),\ldots,(\mho_n,\Xi_n)\} \\
\leq\ & CL-SV-NHFROWG\{(\mho_1^*,\Xi_1),(\mho_2^*,\Xi_2),\ldots,(\mho_n^*,\Xi_n)\}.
\end{aligned}
$$

4. Decision-making strategy based on CL-SV-NHFR AOs

With the help of MCDM, the optimal solution that satisfies their criteria can be selected successfully. In this section, we'll look at how the most current operators are used. We therefore created an MCDM algorithm to illustrate the usefulness and effectiveness of the proposed work.

Suppose that $\Omega^* = \{\Omega_1,\Omega_2,\Omega_3,...,\Omega_n\}$ indicate the assortment of options and $\hat{C} = \left\{\hat{C}_1,\hat{C}_2,\hat{C}_3,...,\hat{C}_n\right\}$ indicate the group of criteria. Also, Assume that $\omega = (\omega_1,\omega_2,\omega_3\ldots,\omega_n)^T$ be the SV-NHFRNs' WVs with the restriction $\sum_\varsigma^n \omega_\varsigma = 1$.

Assume experts give SV-NHFRNs their CLs detailing how they rank each alternative in relation to each criterion.

$$\left(\mho_{\varsigma j}^s\right)_{m\times n} = \left\{\left(\left(\underline{\xi}_{\hbar_\varsigma},\underline{\eta}_{\hbar_\varsigma},\underline{\pounds}_{\hbar_\varsigma}\right),\left(\overline{\xi}_{\hbar_{\varsigma_j}},\overline{\eta}_{\hbar_{\varsigma_j}}/\overline{\Lambda}_{\hbar_{\varsigma_j}}^s,\overline{\pounds}_{\hbar_\varsigma}\right),\Xi_{\varsigma j}^s\right)\right\}.$$

In order to employ the concept of CL, experts must state that they are familiarized with the assessed alternatives and must assign the CL with the value $\Xi_{\varsigma j}^\iota\left(0\leq\Xi_{\varsigma j}^\iota\leq 1\right)$. We must now take the following actions:

Step 1: Construct the SV-NHFRNs and CL data that the expert has provided, and then determine their assessment of the presence as

$$[M^\iota]_{m\times n} = \left(\left(\underline{\xi}_{\hbar_\varsigma},\underline{\eta}_{\hbar_\varsigma}{}^{\iota}_{\varsigma},\underline{\pounds}_{\hbar_\varsigma}\right),\left(\overline{\xi}_{\hbar_\varsigma},\overline{\eta}_{\hbar_\varsigma},\overline{\pounds}_{\hbar_\varsigma}\right),\Xi_{\varsigma j}^\iota\right).$$

Step 2: Utilizing the CL-SV-NHFRWA or CL-SV-NHFRWG concept to integrate each expert's individual matrix into a collective judgement matrix $[M]_{m\times n}$. That is,

$$
\begin{aligned}
\mho_{\varsigma j} &= CL-SV-NHFRWA\left\{(\mho_{\varsigma j}^1,\Xi_{\varsigma j}^1),(\mho_{\varsigma j}^2,\Xi_{\varsigma j}^2),\ldots,(\mho_{\varsigma j}^\iota,\Xi_{\varsigma j}^\iota)\right\} \\
&= \left(\begin{array}{c}
\left\{\left(1-\prod_{\iota=1}^{\flat}\left(1-\underline{\xi}_{\hbar_\varsigma}\right)^{\Xi_{\varsigma j}^\iota\dagger_\varsigma}\right),\left(\prod_{\iota=1}^{\flat}\left(\underline{\eta}_{\hbar_\varsigma}\right)^{\Xi_{\varsigma j}^\iota\dagger_\varsigma}\right),\left(\prod_{\iota=1}^{\flat}\left(\underline{\pounds}_{\hbar_\varsigma}\right)^{\Xi_{\varsigma j}^\iota\dagger_\varsigma}\right)\right\}, \\
\left\{\left(1-\prod_{\varsigma=1}^{\flat}\left(1-\overline{\xi}_{\hbar_\varsigma}\right)^{\Xi_{\varsigma j}^\iota\dagger_\varsigma}\right),\left(\prod_{\iota=1}^{\flat}\left(\overline{\eta}_{\hbar_\varsigma}\right)^{\Xi_{\varsigma j}^\iota\dagger_\varsigma}\right),\left(\prod_{\iota=1}^{\flat}\left(\overline{\pounds}_{\hbar_\varsigma}\right)^{\Xi_{\varsigma j}^\iota\dagger_\varsigma}\right)\right\}
\end{array}\right)
\end{aligned}
$$

or

$$
\begin{aligned}
\mho_{\varsigma j} &= CL-SV-NHFRWG\left\{(\mho_{\varsigma j}^1,\Xi_{\varsigma j}^1),(\mho_{\varsigma j}^2,\Xi_{\varsigma j}^2),\ldots,(\mho_{\varsigma j}^\iota,\Xi_{\varsigma j}^\iota)\right\} \\
&= \left(\begin{array}{c}
\left\{\left(\prod_{\iota=1}^{\flat}\left(\underline{\xi}_{\hbar_\varsigma}\right)^{\Xi_{\varsigma j}^\iota\dagger_\varsigma}\right),\left(1-\prod_{\iota=1}^{\flat}\left(1-\underline{\eta}_{\hbar_\varsigma}\right)^{\Xi_{\varsigma j}^\iota\dagger_\varsigma}\right),\left(1-\prod_{\iota=1}^{\flat}\left(1-\underline{\pounds}_{\hbar_\varsigma}\right)^{\Xi_{\varsigma j}^\iota\dagger_\varsigma}\right)\right\}, \\
\left\{\left(\prod_{\iota=1}^{\flat}\left(\overline{\xi}_{\hbar_\varsigma}\right)^{\Xi_{\varsigma j}^\iota\dagger_\varsigma}\right),\left(1-\prod_{\varsigma=1}^{\flat}\left(1-\overline{\eta}_{\hbar_\varsigma}\right)^{\Xi_{\varsigma j}^\iota\dagger_\varsigma}\right),\left(1-\prod_{\varsigma=1}^{\flat}\left(1-\overline{\pounds}_{\hbar_\varsigma}\right)^{\Xi_{\varsigma j}^\iota\dagger_\varsigma}\right)\right\}
\end{array}\right).
\end{aligned}
$$

Step 3: Aggregating the matrix's alternate execution using the SV-NHFRWA or SV-NHFRWG operator $[M]_{m\times n}$ as

$$\begin{aligned}
\mho_{\varsigma} &= SV-NHFRWA\left(\mho_{\varsigma 1},\mho_{\varsigma 2},...,\mho_{\varsigma n}\right)\\
&= \left\{\begin{array}{c} \left\{\left(1-\prod_{j=1}^{n}\left(1-\underline{\xi}_{\hbar_{\varsigma}}\right)^{\omega_j}\right),\left(\prod_{j=1}^{n}\left(\underline{\eta}_{\hbar_{\varsigma}}\right)^{\omega_j}\right),\left(\prod_{j=1}^{n}\left(\underline{\pounds}_{\hbar_{\varsigma}}\right)^{\omega_j}\right)\right\}, \\ \left\{\left(1-\prod_{j=1}^{n}\left(1-\overline{\xi}_{\hbar_{\varsigma}}\right)^{\omega_j}\right),\left(\prod_{j=1}^{n}\left(\overline{\eta}_{\hbar_{\varsigma}}\right)^{\omega_j}\right),\left(\prod_{j=1}^{n}\left(\overline{\pounds}_{\hbar_{\varsigma}}\right)^{\omega_j}\right)\right\} \end{array}\right\}
\end{aligned}$$

or

$$\begin{aligned}
\mho_{\varsigma} &= SV-NHFRWG\left(\mho_{\varsigma 1},\mho_{\varsigma 2},...,\mho_{\varsigma n}\right)\\
&= \left\{\begin{array}{c} \left\{\left(\prod_{j=1}^{n}\left(\underline{\xi}_{\hbar_{\varsigma}}\right)^{\omega_j}\right),\left(1-\prod_{j=1}^{n}\left(1-\underline{\eta}_{\hbar_{\varsigma}}\right)^{\omega_j}\right),\left(1-\prod_{j=1}^{n}\left(1-\underline{\pounds}_{\hbar_{\varsigma}}\right)^{\omega_j}\right)\right\}, \\ \left\{\left(\prod_{j=1}^{n}\left(\overline{\xi}_{\hbar_{\varsigma}}\right)^{\omega_j}\right),\left(1-\prod_{j=1}^{n}\left(1-\overline{\eta}_{\hbar_{\varsigma}}\right)^{\omega_j}\right),\left(1-\prod_{j=1}^{n}\left(1-\overline{\pounds}_{\hbar_{\varsigma}}\right)^{\omega_j}\right)\right\} \end{array}\right\}.
\end{aligned}$$

Step 4: Calculate the score values for each choice using SF, and then rank the results.

Then, score function (SF) and accuracy function (AF) are given by

$$\begin{aligned}
Sc &= \frac{1}{6}\left\{3+\underline{\xi}_{\hbar_{\varsigma}}+\overline{\xi}_{\hbar_{\varsigma}}-\underline{\eta}_{\hbar_{\varsigma}}-\overline{\eta}_{\hbar_{\varsigma}}-\underline{\pounds}_{\hbar_{\varsigma}}-\overline{\pounds}_{\hbar_{\varsigma}}\right\}, S\in[0,1]\\
Ac &= \frac{1}{6}\left\{3+\underline{\xi}_{\hbar_{\varsigma}}+\overline{\xi}_{\hbar_{\varsigma}}+\underline{\eta}_{\hbar_{\varsigma}}+\overline{\eta}_{\hbar_{\varsigma}}-\underline{\pounds}_{\hbar_{\varsigma}}+\overline{\pounds}_{\hbar_{\varsigma}}\right\}, A\in[0,1].
\end{aligned}$$

All the steps of the algorithm are shown in Figure 1.

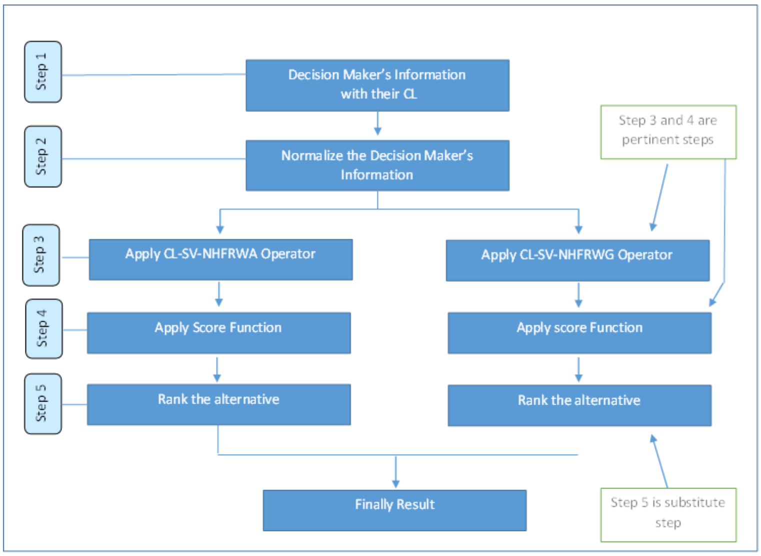

Figure 1. Flow chart of the decision-making algorithm.

4.1. Numerical example

Leaders can arrange their thoughts to make long-term decisions by using strategies such as analysis of strengths, weaknesses, opportunities, and threats. Formal approaches of decision-making assist leaders in avoiding typical fallacies like extrapolation or sunk-cost bias. Every level of a corporation makes decisions, from the routine ones that lower-level employees make every day to the more significant management decisions that may take years to consider. Every manufacturer's existence revolves around planning, which is also the secret to efficient resource and inventory management. Even lean and engineer-to-order manufacturers need to establish plans for the materials and resources (equipment, capacity, people, and skills) to be on hand to meet client demands. Demand management, forecasting, master scheduling, material planning (MRP), and capacity planning are all incorporated into the manufacturing planning process, which also includes production control, inventory management, and procurement. Applications for execution in purchasing and production make sure that all tasks are coordinated and finished on time, maximizing the effective use of resources. On-time delivery, content customers, and low prices are the end results. This 'closed loop' of coordinated planning and execution also maintains coordination in the face of shifting demand, unforeseen disruptions, and other difficulties.

Here, we talk about a planning issue for the manufacturing sector and choose the best, longest-lasting aspect. The weight values are

$\omega = (0.28, 0.14, 0.36, 0.22)^T$ that are effected on the informations. Here are some of the most crucial things to take into account before starting a manufacturing business. We will choose the most important Thing that is necessary for the first stage, and we will also determine how these factors can be ranked. Demand for your product; Business location; Competition from other manufacturers; Setup costs.

Waste reduction, material efficiency, resource efficiency, and eco-efficiency were identified to be the four main strategies and know as hesitant points. The key traits of these sustainable manufacturing strategies were identified through an analysis of the literature.

There are four major techniques that are as given below:

Ω_1 = **Demand for your product:** Over the past ten years, on-demand manufacturing, also known as cloud manufacturing, has evolved and is starting to transform the supply chain sector. The global cloud manufacturing market is anticipated to reach roughly \$112 billion by 2024, rising at an incredible 19.8 percent yearly, according to MarketWatch [46], driven by the explosive rise of e-commerce and altering customer preferences. In a technique known as "on-demand manufacturing" products are only made in the quantities and at the times needed. Contrarily, traditional manufacturing necessitates the production of vast numbers of goods, which must then be held in facilities until they are prepared for export. On-demand manufacturing generally eliminates the need to retain expensive inventory and provides more options to manufacture unique, specialized items because of its increased flexibility and capacity to produce one-off orders. The amount of people looking for your goods, their willingness to pay for it, and the quantity of your product that is offered to customers by both your business and your rivals are some of the elements that affect market demand. Demand for goods and services overall may change throughout time; typically, it does.

Ω_2 = **Business location:** When choosing a site for their firm, manufacturers and producers of physical goods should take into consideration a wide range of criteria. The efficiency with which a firm develops, produces, and reaches its prospective client base while minimizing costs and maintaining the greatest quality of production and distribution is, as in every business, the key to success. The size and scope of the manufacturing plant they foresee, the manufacturing process, and what local and environmental implications the manufacturing process would include, would be a starting point for businesses involved in production and manufacturing. These organizations should also take into account the strength of the local supply chain, the availability of human resources, and other support services needed to guarantee constant, continuous, and sustainable production with the fewest interruptions.

Ω_3 = **Competition from other manufacturers:** Over the past few years, American managers have been increasingly cognizant of the critical role that top-notch manufacturing competence plays in the country's competitive performance. The need to increase productivity, product quality, and new product innovation is currently at the top of the list of priorities for many corporations. All of this is advantageous. To comprehend how their manufacturing organizations are contributing to overarching strategy goals and the numerous types of contributions those firms may be required to make, managers still lack a powerful descriptive framework.

Ω_4 = **Setup costs:** Setup costs in manufacturing are the expenses needed to prepare machinery to handle a different batch of products. Therefore, in activity-based costing, setup costs are viewed as batch-level costs. Setup fees are seen as non-value-added expenses that have to be kept to a minimum. Materials, labour, and overhead are the three main areas of costs in the manufacturing industry. There are no indirect costs. In other words, while the foreman's salary and supplies are included, neither the corporate accountant's salary nor those of the accountant's office are.

The various hesitancies are listed above. The information is gathered from experts. The information gathered from the four specialists is presented in Table 1 through Table 4. The combined table is Table 5. The information that was gathered by using the innovative operators is shown in Tables 6 and 8. Tables 7 and 9 include the data that was acquired after the scoring function was used to determine the outcomes for the current MADM. The Table 10 shows the aggregated values for validity test and Table 11 is alternative ranking for validity test.

Step 1.

Table 1. Expert-1 information.

(a)

	$\hat{C}_1$	$\hat{C}_2$
Ω_1	$\left\{\begin{array}{c}\left[\left\{\begin{array}{c}(0.5,0.7,0.4),(0.4,0.5),\\(0.3,0.7,0.2,0.6)\end{array}\right\},0.4\right],\\\left[\left\{\begin{array}{c}(0.2,0.7),(0.5,0.9,0.2,0.5),\\(0.3,0.9,0.3)\end{array}\right\},0.8\right]\end{array}\right\}$	$\left\{\begin{array}{c}\left[\left\{\begin{array}{c}(0.4,0.7,0.1),\\(0.5,0.7),(0.4)\end{array}\right\},0.7\right],\\\left[\left\{\begin{array}{c}(0.1,0.4,0.9),\\(0.5,0.7,0.3),(0.4,0.5)\end{array}\right\},0.6\right]\end{array}\right\}$
Ω_2	$\left\{\begin{array}{c}\left[\left\{\begin{array}{c}(0.2),(0.9,0.1,0.4,0.4),\\(0.3,0.5)\end{array}\right\},0.7\right],\\\left[\left\{\begin{array}{c}(0.9,0.4,0.5),\\(0.6,0.9,0.4),(0.8)\end{array}\right\},0.9\right]\end{array}\right\}$	$\left\{\begin{array}{c}\left[\left\{\begin{array}{c}(0.5,0.7,0.8),\\(0.7,0.8),(0.3,0.6)\end{array}\right\},0.3\right],\\\left[\left\{\begin{array}{c}(0.4,0.6,0.2),\\(0.5),(0.9,0.4)\end{array}\right\},0.8\right]\end{array}\right\}$
Ω_3	$\left\{\begin{array}{c}\left[\left\{\begin{array}{c}(0.3,0.6,0.7),(0.9),\\(0.7,0.7,0.3)\end{array}\right\},0.6\right],\\\left[\left\{\begin{array}{c}(0.8,0.1),(0.3,0.7,0.7),\\(0.3,0.5,0.7)\end{array}\right\},0.3\right]\end{array}\right\}$	$\left\{\begin{array}{c}\left[\left\{\begin{array}{c}(0.3),(0.5,0.9),\\(0.8,0.2)\end{array}\right\},0.6\right],\\\left[\left\{\begin{array}{c}(0.5),(0.3,0.6,0.7),\\(0.2,0.5,0.8)\end{array}\right\},0.3\right]\end{array}\right\}$
Ω_4	$\left\{\begin{array}{c}\left[\left\{\begin{array}{c}(0.4,0.2),(0.5),\\(0.4,0.7,0.3)\end{array}\right\},0.6\right],\\\left[\left\{\begin{array}{c}(0.8),(0.3,0.5,0.6),\\(0.3,0.1)\end{array}\right\},0.9\right]\end{array}\right\}$	$\left\{\begin{array}{c}\left[\left\{\begin{array}{c}(0.6,0.8,0.2),\\(0.5),(0.2,0.4)\end{array}\right\},0.7\right],\\\left[\left\{\begin{array}{c}(0.4,0.7),(0.4,0.9),\\(0.6,0.8,0.3)\end{array}\right\},0.8\right]\end{array}\right\}$

(b)

	$\hat{C}_3$	$\hat{C}_4$
Ω_1	$\left\{ \left[\left\{ (0.9,0.2,0.6), (0.7), (0.4,0.6) \right\}, 0.7 \right], \left[\left\{ (0.8,0.6), (0.4,0.5,0.9), (0.4,0.9) \right\}, 0.8 \right] \right\}$	$\left\{ \left[\left\{ (0.8,0.5), (0.4,0.4), (0.5,0.3) \right\}, 0.7 \right], \left[\left\{ (0.4,0.6), (0.4,0.9,0.6), (0.5,0.5) \right\}, 0.6 \right] \right\}$
Ω_2	$\left\{ \left[\left\{ (0.5,0.6,0.5), (0.4,0.9,0.4), (0.4) \right\}, 0.5 \right], \left[\left\{ (0.3,0.4,0.7), (0.4,0.6,0.3), (0.4,0.8,0.1) \right\}, 0.9 \right] \right\}$	$\left\{ \left[\left\{ (0.7,0.1), (0.3,0.4), (0.4,0.5,0.9) \right\}, 0.7 \right], \left[\left\{ (0.4,0.1,0.4,0.5), (0.5,0.1,0.8), (0.4,0.5) \right\}, 0.5 \right] \right\}$
Ω_3	$\left\{ \left[\left\{ (0.4,0.9,0.7), (0.4), (0.7,0.4,0.9) \right\}, 0.9 \right], \left[\left\{ (0.5,0.7,0.5), (0.6), (0.3,0.7,0.7) \right\}, 0.4 \right] \right\}$	$\left\{ \left[\left\{ (0.3,0.9,0.1), (0.2,0.4,0.7), (0.6) \right\}, 0.8 \right], \left[\left\{ (0.3,0.5,0.4), (0.3,0.7), (0.2) \right\}, 0.9 \right] \right\}$
Ω_4	$\left\{ \left[\left\{ (0.9,0.1), (0.7,0.7,0.4), (0.9) \right\}, 0.5 \right], \left[\left\{ (0.1,0.4,0.7), (0.9,0.7,0.5), (0.5,0.7) \right\}, 0.8 \right] \right\}$	$\left\{ \left[\left\{ (0.3,0.7,0.5), (0.3), (0.4,0.6,0.3) \right\}, 0.7 \right], \left[\left\{ (0.6,0.4,0.4,0.9), (0.3,0.2), (0.3) \right\}, 0.6 \right] \right\}$

Table 2. Expert-2 information.

(a)

	$\hat{C}_1$	$\hat{C}_2$
Ω_1	$\left\{ \left[\left\{ (0.9,0.4,0.7), (0.3,0.4), (0.6) \right\}, 0.9 \right], \left[\left\{ (0.8,0.2,0.4), (0.4,0.4), (0.4,0.9,0.5) \right\}, 0.5 \right] \right\}$	$\left\{ \left[\left\{ (0.4,0.6,0.8,0.5), (0.6,0.4), (0.8,0.5) \right\}, 0.9 \right], \left[\left\{ (0.7), (0.5,0.9,0.6,0.4), (0.10.5,0.9) \right\}, 0.8 \right] \right\}$
Ω_2	$\left\{ \left[\left\{ (0.4,0.8), (0.4,0.9,0.7,0.3), (0.4,0.2) \right\}, 0.7 \right], \left[\left\{ (0.6,0.6,0.7), (0.4,0.9), (0.5,0.9,0.1,0.6) \right\}, 0.9 \right] \right\}$	$\left\{ \left[\left\{ (0.6,0.4,0.1), (0.1,0.3,0.8), (0.7,0.6,0.8) \right\}, 0.6 \right], \left[\left\{ (0.8), (0.2,0.1,0.9), (0.7,0.5,0.4) \right\}, 0.9 \right] \right\}$
Ω_3	$\left\{ \left[\left\{ (0.1,0.9,0.8), (0.4,0.3,0.4), (0.7,0.8,0.1) \right\}, 0.9 \right], \left[\left\{ (0.4,0.7,0.6), (0.5,0.6), (0.4,0.7,0.9) \right\}, 0.5 \right] \right\}$	$\left\{ \left[\left\{ (0.4,0.8,0.1), (0.5,0.4,0.9), (0.6,0.7) \right\}, 0.4 \right], \left[\left\{ (0.8,0.7), (0.9,0.3,0.7), (0.2,0.5) \right\}, 0.2 \right] \right\}$
Ω_4	$\left\{ \left[\left\{ (0.8,0.1,0.5,0.3), (0.7,0.7,0.4), (0.9) \right\}, 0.6 \right], \left[\left\{ (0.2,0.5,0.7), (0.9,0.3,0.7,0.5), (0.4,0.4,0.7) \right\}, 0.8 \right] \right\}$	$\left\{ \left[\left\{ (0.8,0.9,0.6), (0.3,0.4), (0.6,0.7,0.3) \right\}, 0.9 \right], \left[\left\{ (0.4,0.4), (0.5,0.2,0.6), (0.3,0.5,0.7) \right\}, 0.2 \right] \right\}$

(b)

	$\hat{C}_3$	$\hat{C}_4$
Ω_1	$\left\{\begin{array}{c}\left[\left\{\begin{array}{c}(0.8,0.2,0.7,0.3),(0.3,0.2,0.7),\\(0.9,0.1,0.4,0.5)\end{array}\right\},0.9\right],\\\left[\left\{\begin{array}{c}(0.2,0.7,0.6),(0.4,0.6,0.9,0.7),\\(0.7,0.9,0.6,0.5)\end{array}\right\},0.7\right]\end{array}\right\}$	$\left\{\begin{array}{c}\left[\left\{\begin{array}{c}(0.5,0.7,0.9),\\(0.8,0.6,0.4),(0.8,0.6,0.3)\end{array}\right\},0.9\right],\\\left[\left\{\begin{array}{c}(0.5,0.6),(0.9,0.6),\\(0.7,0.6,0.5)\end{array}\right\},0.6\right]\end{array}\right\}$
Ω_2	$\left\{\begin{array}{c}\left[\left\{\begin{array}{c}(0.8,0.4,0.6,0.5),\\(0.5,0.4,0.8),(0.4,0.7)\end{array}\right\},0.7\right],\\\left[\left\{\begin{array}{c}(0.9,0.3,0.5,0.7),(0.4,0.6),\\(0.4,0.8,0.3,0.6)\end{array}\right\},0.9\right]\end{array}\right\}$	$\left\{\begin{array}{c}\left[\left\{\begin{array}{c}(0.8,0.1,0.3),(0.9,0.3,0.4),\\(0.6,0.5)\end{array}\right\},0.8\right],\\\left[\left\{\begin{array}{c}(0.5),(0.5,0.1),\\(0.4,0.5,0.7)\end{array}\right\},0.5\right]\end{array}\right\}$
Ω_3	$\left\{\begin{array}{c}\left[\left\{\begin{array}{c}(0.3,0.4,0.5,0.7),(0.4,0.5,0.8),\\(0.3,0.6,0.5,0.9)\end{array}\right\},0.2\right],\\\left[\left\{\begin{array}{c}(0.6,0.8,0.6),(0.5,0.7,0.6),\\(0.4,0.7,0.5,0.6)\end{array}\right\},0.6\right]\end{array}\right\}$	$\left\{\begin{array}{c}\left[\left\{\begin{array}{c}(0.7,0.8),(0.1,0.4,0.7),\\(0.6,0.5,0.3)\end{array}\right\},0.5\right],\\\left[\left\{\begin{array}{c}(0.9,0.4),(0.6,0.7,0.9),\\(0.2,0.7,0.5)\end{array}\right\},0.9\right]\end{array}\right\}$
Ω_4	$\left\{\begin{array}{c}\left[\left\{\begin{array}{c}(0.5,0.9,0.9,0.3),\\(0.4,0.7,0.7),(0.9,0.3)\end{array}\right\},0.8\right],\\\left[\left\{\begin{array}{c}(0.3,0.4,0.9),\\(0.4,0.7,0.7),(0.7,0.6,0.8)\end{array}\right\},0.6\right]\end{array}\right\}$	$\left\{\begin{array}{c}\left[\left\{\begin{array}{c}(0.1,0.3,0.7,0.5),\\(0.3,0.4,0.7),(0.4)\end{array}\right\},0.8\right],\\\left[\left\{\begin{array}{c}(0.1,0.7,0.4,0.9),\\(0.3,0.9),(0.3,0.2)\end{array}\right\},0.3\right]\end{array}\right\}$

Table 3. Expert-3 information.

(a)

	$\hat{C}_1$	$\hat{C}_2$
Ω_1	$\left\{\begin{array}{c}\left[\left\{\begin{array}{c}(0.9),(0.6,0.4),\\(0.5,0.4,0.6)\end{array}\right\},0.1\right],\\\left[\left\{\begin{array}{c}(0.7,0.6,0.2),\\(0.3,0.5,0.9),(0.6,0.9,0.5)\end{array}\right\},0.9\right]\end{array}\right\}$	$\left\{\begin{array}{c}\left[\left\{\begin{array}{c}(0.5,0.8,0.7),\\(0.4),(0.9)\end{array}\right\},0.9\right],\\\left[\left\{\begin{array}{c}(0.6,0.6),\\(0.6),(0.5)\end{array}\right\},0.6\right]\end{array}\right\}$
Ω_2	$\left\{\begin{array}{c}\left[\left\{\begin{array}{c}(0.7,0.6,0.5,0.4),\\(0.4),(0.4,0.5)\end{array}\right\},0.7\right],\\\left[\left\{\begin{array}{c}(0.4,0.7),(0.3),\\(0.4,0.8)\end{array}\right\},0.5\right]\end{array}\right\}$	$\left\{\begin{array}{c}\left[\left\{\begin{array}{c}(0.8,0.1),\\(0.4),(0.4,0.5,0.9)\end{array}\right\},0.7\right],\\\left[\left\{\begin{array}{c}(0.4,0.5),(0.5,0.8,0.8),\\(0.7,0.4,0.5)\end{array}\right\},0.9\right]\end{array}\right\}$
Ω_3	$\left\{\begin{array}{c}\left[\left\{\begin{array}{c}(0.4,0.9,0.7),\\(0.4),(0.7,0.4,0.9)\end{array}\right\},0.4\right],\\\left[\left\{\begin{array}{c}(0.5,0.7,0.5),\\(0.6),(0.3,0.7,0.7)\end{array}\right\},0.9\right]\end{array}\right\}$	$\left\{\begin{array}{c}\left[\left\{\begin{array}{c}(0.9,0.1),(0.2,0.4),\\(0.6,0.5,0.4)\end{array}\right\},0.7\right],\\\left[\left\{\begin{array}{c}(0.7,0.7),(0.7,0.3,0.7),\\(0.6,0.2)\end{array}\right\},0.9\right]\end{array}\right\}$
Ω_4	$\left\{\begin{array}{c}\left[\left\{\begin{array}{c}(0.9,0.1,0.3),\\(0.7,0.8,0.4,0.7),(0.9,0.3)\end{array}\right\},0.9\right],\\\left[\left\{\begin{array}{c}(0.4,0.9,0.4,0.7),\\(0.6,0.7,0.9),(0.9,0.5,0.7)\end{array}\right\},0.4\right]\end{array}\right\}$	$\left\{\begin{array}{c}\left[\left\{\begin{array}{c}(0.3,0.7,0.5),\\(0.3),(0.4,0.6,0.3)\end{array}\right\},0.3\right],\\\left[\left\{\begin{array}{c}(0.4,0.9),(0.6,0.3,0.2),\\(0.3,0.2)\end{array}\right\},0.8\right]\end{array}\right\}$

(b)

	$\hat{C}_3$	$\hat{C}_4$
Ω_1	$\left\{\begin{array}{c}\left[\left\{\begin{array}{c}(0.6,0.6),\\(0.7,0.9,0.2,0.4),(0.6)\end{array}\right\},0.8\right],\\\left[\left\{\begin{array}{c}(0.8,0.6,0.6),\\(0.5),(0.7)\end{array}\right\},0.8\right]\end{array}\right\}$	$\left\{\begin{array}{c}\left[\left\{\begin{array}{c}(0.9,0.8,0.7),(0.7,0.5,0.4),\\(0.80.5,0.3)\end{array}\right\},0.5\right],\\\left[\left\{\begin{array}{c}(0.3,0.5,0.6),\\(0.1,0.9,0.6),(0.5,0.7,0.2)\end{array}\right\},0.6\right]\end{array}\right\}$
Ω_2	$\left\{\begin{array}{c}\left[\left\{\begin{array}{c}(0.9,0.5,0.6,0.5),\\(0.9,0.4),(0.4)\end{array}\right\},0.8\right],\\\left[\left\{\begin{array}{c}(0.7),(0.6,0.3),\\(0.4,0.8)\end{array}\right\},0.7\right]\end{array}\right\}$	$\left\{\begin{array}{c}\left[\left\{\begin{array}{c}(0.3,0.7,0.2),\\(0.9,0.3,0.4),(0.5,0.9)\end{array}\right\},0.7\right],\\\left[\left\{\begin{array}{c}(0.5),(0.8),\\(0.6,0.5,0.4)\end{array}\right\},0.8\right]\end{array}\right\}$
Ω_3	$\left\{\begin{array}{c}\left[\left\{\begin{array}{c}(0.5,0.4,0.9,0.7),\\(0.4,0.6,0.9),(0.4,0.9)\end{array}\right\},0.8\right],\\\left[\left\{\begin{array}{c}(0.7,0.5),(0.3,0.6),\\(0.8,0.8,0.7)\end{array}\right\},0.5\right]\end{array}\right\}$	$\left\{\begin{array}{c}\left[\left\{\begin{array}{c}(0.9,0.6),(0.2,0.7,0.9),\\(0.8,0.6)\end{array}\right\},0.8\right],\\\left[\left\{\begin{array}{c}(0.5,0.9),(0.8,0.7),\\(0.2,0.7)\end{array}\right\},0.9\right]\end{array}\right\}$
Ω_4	$\left\{\begin{array}{c}\left[\left\{\begin{array}{c}(0.5,0.9,0.1),\\(0.7,0.4),(0.1,0.9)\end{array}\right\},0.7\right],\\\left[\left\{\begin{array}{c}(0.5,0.4,0.9),(0.9,0.8,0.5),\\(0.5,0.7,0.6)\end{array}\right\},0.4\right]\end{array}\right\}$	$\left\{\begin{array}{c}\left[\left\{\begin{array}{c}(0.5),(0.3,0.9),\\(0.6,0.3)\end{array}\right\},0.9\right],\\\left[\left\{\begin{array}{c}(0.6,0.9),(0.3,0.2,0.5),\\(0.3,0.7)\end{array}\right\},0.6\right]\end{array}\right\}$

Table 4. Expert-4 information.

(a)

	$\hat{C}_1$	$\hat{C}_2$
Ω_1	$\left\{\begin{array}{c}\left[\left\{\begin{array}{c}(0.7,0.2,0.2,0.6),(0.7,0.4),\\(0.3,0.9,0.6)\end{array}\right\},0.2\right],\\\left[\left\{\begin{array}{c}(0.9,0.7,0.6),\\(0.7,0.9),(0.4,0.8)\end{array}\right\},0.9\right]\end{array}\right\}$	$\left\{\begin{array}{c}\left[\left\{\begin{array}{c}(0.3,0.5,0.7),\\(0.7,0.4),(0.3)\end{array}\right\},0.8\right],\\\left[\left\{\begin{array}{c}(0.5,0.4,0.6),(0.9,0.6),\\(0.8,0.5,0.4)\end{array}\right\},0.6\right]\end{array}\right\}$
Ω_2	$\left\{\begin{array}{c}\left[\left\{\begin{array}{c}(0.3,0.5,0.8,0.5),\\(0.6,0.9,0.8),(0.8,0.4)\end{array}\right\},0.9\right],\\\left[\left\{\begin{array}{c}(0.7),(0.4),\\(0.5,0.8,0.3)\end{array}\right\},0.9\right]\end{array}\right\}$	$\left\{\begin{array}{c}\left[\left\{\begin{array}{c}(0.5,0.3),\\(0.1,0.9,0.4),(0.5,0.9)\end{array}\right\},0.9\right],\\\left[\left\{\begin{array}{c}(0.4,0.5),\\(0.5,0.7,0.8),(0.4)\end{array}\right\},0.7\right]\end{array}\right\}$
Ω_3	$\left\{\begin{array}{c}\left[\left\{\begin{array}{c}(0.9,0.5),(0.4,0.8),\\(0.7,0.9)\end{array}\right\},0.8\right],\\\left[\left\{\begin{array}{c}(0.5,0.9,0.8),(0.6,0.6),\\(0.4,0.8,0.7)\end{array}\right\},0.5\right]\end{array}\right\}$	$\left\{\begin{array}{c}\left[\left\{\begin{array}{c}(0.7,0.9,0.9),\\(0.3,0.8,0.7),(0.6,0.7)\end{array}\right\},0.9\right],\\\left[\left\{\begin{array}{c}(0.3,0.5),(0.3,0.7,0.3),\\(0.2,0.6,0.7)\end{array}\right\},0.5\right]\end{array}\right\}$
Ω_4	$\left\{\begin{array}{c}\left[\left\{\begin{array}{c}(0.7,0.9,0.1),\\(0.7,0.7,0.4),(0.9)\end{array}\right\},0.5\right],\\\left[\left\{\begin{array}{c}(0.3,0.8,0.7),\\(0.4,0.3,0.5),(0.2,0.5,0.7)\end{array}\right\},0.9\right]\end{array}\right\}$	$\left\{\begin{array}{c}\left[\left\{\begin{array}{c}(0.7,0.7,0.9),\\(0.3,0.6),(0.8,0.6,0.3)\end{array}\right\},0.3\right],\\\left[\left\{\begin{array}{c}(0.6,0.9),(0.3,0.2,0.6),\\(0.3,0.7)\end{array}\right\},0.8\right]\end{array}\right\}$

(b)

	$\hat{C}_3$	$\hat{C}_4$
Ω_1	$\left\{\begin{array}{c}\left[\left\{\begin{array}{c}(0.6),(0.8),\\(0.5,0.6,0.3)\end{array}\right\},0.5\right],\\\left[\left\{\begin{array}{c}(0.9,0.6),(0.4,0.1,0.9),\\(0.1,0.9,0.5)\end{array}\right\},0.8\right]\end{array}\right\}$	$\left\{\begin{array}{c}\left[\left\{\begin{array}{c}(0.6,0.8,0.5),\\(0.5,0.7,0.4),(0.8,0.3,0.6)\end{array}\right\},0.9\right],\\\left[\left\{\begin{array}{c}(0.6,0.7,0.6),(0.7,0.9),\\(0.5,0.8,0.9)\end{array}\right\},0.7\right]\end{array}\right\}$
Ω_2	$\left\{\begin{array}{c}\left[\left\{\begin{array}{c}(0.3,0.6,0.1),\\(0.2,0.9,0.9),(0.4,0.7)\end{array}\right\},0.7\right],\\\left[\left\{\begin{array}{c}(0.9,0.6,0.7),\\(0.5,0.6,0.3),(0.7,0.8,0.1,0.9)\end{array}\right\},0.9\right]\end{array}\right\}$	$\left\{\begin{array}{c}\left[\left\{\begin{array}{c}(0.4,0.7,0.1),\\(0.9,0.3,0.4),(0.5,0.9)\end{array}\right\},0.7\right],\\\left[\left\{\begin{array}{c}(0.4,0.5),(0.1,0.8),\\(0.4,0.5,0.6)\end{array}\right\},0.2\right]\end{array}\right\}$
Ω_3	$\left\{\begin{array}{c}\left[\left\{\begin{array}{c}(0.8,0.2,0.7),\\(0.4,0.1),(0.4,0.9)\end{array}\right\},0.9\right],\\\left[\left\{\begin{array}{c}(0.7,0.6,0.5),(0.1,0.6),\\(0.4,0.7,0.4)\end{array}\right\},0.7\right]\end{array}\right\}$	$\left\{\begin{array}{c}\left[\left\{\begin{array}{c}(0.9,0.3,0.9),(0.9,0.2,0.7),\\(0.6,0.3,0.6)\end{array}\right\},0.8\right],\\\left[\left\{\begin{array}{c}(0.3,0.4),(0.3,0.7,0.5),\\(0.2,0.6,0.4)\end{array}\right\},0.8\right]\end{array}\right\}$
Ω_4	$\left\{\begin{array}{c}\left[\left\{\begin{array}{c}(0.8,0.9,0.1),(0.7,0.8),\\(0.9,0.3)\end{array}\right\},0.7\right],\\\left[\left\{\begin{array}{c}(0.6,0.4,0.7),(0.7,0.5),\\(0.3,0.5,0.7)\end{array}\right\},0.8\right]\end{array}\right\}$	$\left\{\begin{array}{c}\left[\left\{\begin{array}{c}(0.9,0.5,0.7,0.5),\\(0.3,0.7),(0.6,0.6,0.3)\end{array}\right\},0.8\right],\\\left[\left\{\begin{array}{c}(0.3,0.4,0.6,0.9),\\(0.7,0.3,0.2),(0.3,0.1,0.9)\end{array}\right\},0.9\right]\end{array}\right\}$

Step 2.

Table 5. Integrated matrix of experts evaluations.

(a)

	$\hat{C}_1$	$\hat{C}_2$
Ω_1	$\left\{\begin{array}{c}\{0.2487,0.2896,0.3271\},\\\{0.2874,0.2769,0.3542\}\end{array}\right\}$	$\left\{\begin{array}{c}\{0.4301,0.2773,0.3492\},\\\{0.2896,0.3662,0.3849\}\end{array}\right\}$
Ω_2	$\left\{\begin{array}{c}\{0.3453,0.2978,0.3476\},\\\{0.3245,0.3336,0.3478\}\end{array}\right\}$	$\left\{\begin{array}{c}\{0.3151,0.3708,0.3956\},\\\{0.2977,0.3796,0.2740\}\end{array}\right\}$
Ω_3	$\left\{\begin{array}{c}\{0.4153,0.4605,0.3789\},\\\{0.3958,0.3793,0.2874\}\end{array}\right\}$	$\left\{\begin{array}{c}\{0.2384,0.3164,0.2068\},\\\{0.4316,0.2952,0.4146\}\end{array}\right\}$
Ω_4	$\left\{\begin{array}{c}\{0.3654,0.3564,0.3367\},\\\{0.3056,0.3297,0.3749\}\end{array}\right\}$	$\left\{\begin{array}{c}\{0.2839,0.3192,0.2966\},\\\{0.3627,0.2749,0.3867\}\end{array}\right\}$

(b)

	$\hat{C}_3$	$\hat{C}_4$
Ω_1	$\left\{\begin{array}{c}\{0.2868,0.2904,0.3716\},\\\{0.2791,0.3759,0.2908\}\end{array}\right\}$	$\left\{\begin{array}{c}\{0.2962,0.2118,0.4084\},\\\{0.3298,0.2895,0.2864\}\end{array}\right\}$
Ω_2	$\left\{\begin{array}{c}\{0.4239,0.5364,0.2797\},\\\{0.3679,0.3056,0.2708\}\end{array}\right\}$	$\left\{\begin{array}{c}\{0.2779,0.2619,0.2796\},\\\{0.2473,0.2978,0.3567\}\end{array}\right\}$
Ω_3	$\left\{\begin{array}{c}\{0.3167,0.2855,0.3716\},\\\{0.2867,0.1915,0.3827\}\end{array}\right\}$	$\left\{\begin{array}{c}\{0.4318,0.2839,0.2563\},\\\{0.2879,0.3729,0.3919\}\end{array}\right\}$
Ω_4	$\left\{\begin{array}{c}\{0.2194,0.2556,0.3756\},\\\{0.3488,0.2763,0.2695\}\end{array}\right\}$	$\left\{\begin{array}{c}\{0.3592,0.2247,0.3716\},\\\{0.2376,0.2639,0.2769\}\end{array}\right\}$

Step 3a.

Table 6. Aggregated values appyling CL-SV-NHFRWA operator.

	CL-PSV-NHFRWA AO Values	Score Function Values
Ω_1	$\left\{ \begin{array}{c} \{0.2098, 0.3718, 0.2849\}, \\ \{0.2965, 0.6178, 0.5993\} \end{array} \right\}$	0.2467
Ω_2	$\left\{ \begin{array}{c} \{0.3659, 0.6713, 0.7289\}, \\ \{0.5472, 0.6549, 0.8572\} \end{array} \right\}$	0.2792
Ω_3	$\left\{ \begin{array}{c} \{0.4387, 0.7583, 0.6475\}, \\ \{0.5493, 0.6207, 0.7498\} \end{array} \right\}$	0.3841
Ω_4	$\left\{ \begin{array}{c} \{0.6523, 0.4596, 0.4196\}, \\ \{0.3976, 0.6594, 0.6757\} \end{array} \right\}$	0.3079

Step 4a.

Table 7. Ranking of the alternatives.

AO	Score	Ranking
CL-PSV-NHFRWA	$S(\Omega_3) > S(\Omega_4) > S(\Omega_2) > S(\Omega_1)$	Ω_3

Step 3b.

Table 8. Aggregated values applying CL-SV-NHFRWG AO.

	CL-PSV-NHFRWG AO Values	Score Function Values
Ω_1	$\left[\begin{array}{c} \{0.2098, 0.3718, 0.2849\}, \\ \{0.2965, 0.6178, 0.5993\} \end{array} \right]$	0.4893
Ω_2	$\left[\begin{array}{c} \{0.3659, 0.6713, 0.7289\}, \\ \{0.5472, 0.6549, 0.8572\} \end{array} \right]$	0.6276
Ω_3	$\left[\begin{array}{c} \{0.4387, 0.7583, 0.6475\}, \\ \{0.5493, 0.6207, 0.7498\} \end{array} \right]$	0.7936
Ω_4	$\left[\begin{array}{c} \{0.6523, 0.4596, 0.4196\}, \\ \{0.3976, 0.6594, 0.6757\} \end{array} \right]$	0.6816

Step 4b.

Table 9. Ranking of the alternatives.

AOs	Score	Ranking
CL-PSV-NHFRWG	$S(\Omega_3) > S(\Omega_4) > S(\Omega_2) > S(\Omega_1)$	Ω_3

4.2. Validity test

The proficiency and validity of the aforementioned test are used in this part [47] to evaluate the sufficiency and validity of our established strategy. The following details about the SV-NPHFR are enclosed:

Step 1. At this point, we provide the appropriate alternative in place of the less desirable portion of the alternative, maintaining the same similar positions for each selection criterion.

Step 2. We now determine the total preference values for each alternative under the weighted criterion using the provided set of SV-neutrosophic hesitant fuzzy rough aggregation operators.

Step 3. After then, use the score function to get the outcome.

Step 4. Rank the alternative.

Table 10. Aggregated values of updated information in validity test.

Aggrigated Values		CL-PSV-NHFRWG AO	CL-PSV-NHFRWA AO
Ω_1	$\begin{bmatrix} \{0.2294, 0.2938, 0.2347\}, \\ \{0.3264, 0.5193, 0.4153\} \end{bmatrix}$	0.3978	3426
Ω_2	$\begin{bmatrix} \{0.3144, 0.4523, 0.5419\}, \\ \{0.2442, 0.4289, 0.6272\} \end{bmatrix}$	0.4583	3242
Ω_3	$\begin{bmatrix} \{0.3457, 0.6173, 0.5465\}, \\ \{0.4253, 0.4157, 0.5468\} \end{bmatrix}$	0.5642	4754
Ω_4	$\begin{bmatrix} \{0.4324, 0.3286, 0.2916\}, \\ \{0.2646, 0.5654, 0.5472\} \end{bmatrix}$	0.4726	4126

Table 11. Ranking of the alternatives in validity test

AOs	Score	Ranking
CL-PSV-NHFRWG	$S(\Omega_3) > S(\Omega_4) > S(\Omega_2) > S(\Omega_1)$	Ω_3
CL-PSV-NHFRWA	$S(\Omega_3) > S(\Omega_4) > S(\Omega_1) > S(\Omega_2)$	Ω_3

4.3. Compression evaluation

The goal of this section of the essay is to contrast our innovative research with some existing procedures in order to show how better and reliable it is. The following part will compare and contrast our work with that of operators from the SV-NWA [48], SV-NWG [48], SV-NWA Dombi (SV-NDWA) [49], SV-NWG Dombi (SV-NDWG) [49], SV-NRWA [50], SV-NRWG [50], T-SHFWA [51], T-SHFWG [51], SV-NHFRA and SV-NHFRG [35] AOs. Table 12 provides the overall analysis of the comparison study. Since both positive and negative features are present in our data, all other methodologies are unable to provide us with adequate findings. If the CL is ignored, we are unable to compare our findings to those of earlier research. Due the this resin our comparison part is so simple. Decision confidence, or the sense of having made a choice correctly or erroneously, is a crucial component of subjective experience during decision-making. It rises for right selections and falls for wrong ones as the difficulty of the task lowers. We add the validity test to test our validation. We can see from the analysis that

Table 12. Comparative analysis.

AOs	Result	Ranking
SV-NWA	Incapable of access	No outcome
SV-NWG	Incapable of access	No outcome
SV-NDWA	Incapable of access	No outcome
SV-NDWG	Incapable of access	No outcome
SV-NRWA	Incapable of access	No outcome
SV-NRWG	Incapable of access	No outcome
T-SHFWA	Incapable of access	No outcome
T-SHFWG	Incapable of access	No outcome
SV-NHFRWA	Outcomes	$S(\Omega_3) > S(\Omega_2) > S(\Omega_4) > S(\Omega_1)$
SV-HFRWG	Outcomes	$S(\Omega_4) > S(\Omega_3) > S(\Omega_1) > S(\Omega_2)$
CL-SV-NHFRWA	Outcomes	$S(\Omega_3) > S(\Omega_4) > S(\Omega_2) > S(\Omega_1)$
CL-SV-NHFRWG	Outcomes	$S(\Omega_3) > S(\Omega_4) > S(\Omega_2) > S(\Omega_1)$

4.4. Advantages

This section explains the benefits of the suggested work over the current work. These are the benefits of our work:

(i) It is said and demonstrated that SV-NHFRS is preferable to IFS, PyFS, and SV-NS as an illustration of how several CL are taken into account while determining the optimal choice.

(ii) CL-SV-NHFRS AOs are more adaptable than IFS, PyFS, and SV-NS Einstein, Dombi aggregation operators.

(iii) Every concern raised in the literature can be addressed by proposed operators, however current operators are unable to do so when the data is presented in SV-NHFRSs.

(iv) As a realistic and helpful technique for modeling various uncertainties in typical MCDM scenarios, SV-NHFRSs were used. By dividing the concept of CL into three parts, it is possible to define indefinite and incomplete MCDM information properly.

(v) The application of CL could significantly increase the computational effectiveness of information fusion in MCDM information fusion approaches. Moreover, it may be possible to model choice risks using information fusion techniques.

(vi) A recent advancement in fuzzy set theory is the use of SV-NHFR sets, which can more properly handle uncertainty in practical settings. As a result, the suggested approach is better suited than current ways to address real-world and engineering decision-making issues.

(vii) Also, Table 10 shows that the results derived using the various methodologies available do not take the qualities into consideration during the study. In other words, all of these methods tested their hypotheses with the assumption that decision-makers have total faith in the items under study. These kinds of requirements are, however, only partially realized in practise.

(viii) A specific instance of the proposed operators is the existing aggregation operators. As a result, we draw the conclusion that compared to the existing aggregation operators, the ones provided are more broad in nature and more suited to address real-world problems.

5. Conclusions

We deal with sophisticated and difficult data every day. To work more efficiently and compute thorough information, we developed methodologies and tools for this type of data. Aggregation incurs expenditures just to reduce the volume of data to a single value. The SV-NHFRS was created as a potent combination of an SV-NRS and HFS for situations where each item has a range of possible values that are dictated by MD, indeterminacy, and non-MD. The operators CL-SV-NHFRWA, CL-SV-NHFRWG, CL-SV-NHFROWG, and CL-SV-NHFROWA are given advice on this page. Also proposed was a novel MADM approach based on the CL-SV-NHFRWA and CL-SV-NHFRWG operators. Further details about the advantages of these tactics are provided below.

(a) At first, idempotency, commutativity, boundedness, and monotonicity are covered as basic principles and characteristics of the CL-SV-NHFRWG and CL-SV-NHFRWA operators.

(b) Second, the flexibility of the suggested AOs is demonstrated by the conversion of the CL-SV-NHFRWA and CL-SV-NHFRWG operators to the earlier AOs for SV-NHFSs.

(c) Third, when compared to existing methods for solving MADM problems in an SV-NHF context, the results produced by the CL-SV-NHFRWA and CL-SV-NHFRWG operators are reliable and accurate, proving their applicability in real-world situations.

(d) Fourth, compared to existing methodologies, which are unable to account for the interrelationships of attributes in real-world applications, the MADM techniques proposed in this paper are also capable of recognising more correlation between attributes and alternatives. This shows that they have a higher accuracy and a larger setpoint. This demonstrates that even more linkages between features may be found utilizing the MADM approaches described in this research.

(e) Fifth, in order to discover a practical method, the suggested AOs are also employed in practise to look at symmetrical analysis.

(f) Sixth, the proposed AOs may be used in future studies on customised individual consistency control consensus problems, consensus building with non-cooperative behaviour management decision-making problems, and two-sided matching decision-making with multi-granular and incomplete criteria weight information. The levels of participation, abstention, and nonmembership are irrelevant for the purpose of this investigation of the limitations imposed by suggested AOs. A novel hybrid structure of prioritised, interactive AOs is being implemented on this side of the proposed AOs.

(g) Seventh, we will examine the theoretical basis of CL-SV-NHFSs for Einstein operations in upcoming work using advanced decision-making methodologies like as TOPSIS, VIKOR, TODAM, GRA, and EDAS. We'll also go over the ways in which these techniques are used in a number of disciplines, including soft computing, robotics, horticulture, intelligent systems, social sciences, finance, and human resource management.

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Conflicts of interest

The authors declare no conflicts of interest.

References

1. L. A. Zadeh, Similarity relations and fuzzy orderings, *Inform. Sci.*, **3** (1971), 177–200. https://doi.org/10.1016/s0020-0255(71)80005-1
2. S. T. Chang, K. P. Lu, M. S. Yang, Fuzzy change-point algorithms for regression models, *IEEE T. Fuzzy Syst.*, **23** (2015), 2343–2357. https://doi.org/10.1109/TFUZZ.2015.2421072
3. R. M. Zulqarnain, X. L. Xin, M. Saeed, Extension of TOPSIS method under intuitionistic fuzzy hypersoft environment based on correlation coefficient and aggregation operators to solve decision making problem, *AIMS Math.*, **6** (2021), 2732–2755. https://doi.org/10.3934/math.2021167
4. A. Satirad, R. Chinram, A. Iampan, Pythagorean fuzzy sets in UP-algebras and approximations, *AIMS Math.*, **6** (2021), 6002–6032. https://doi.org/10.3934/math.2021354
5. A. U. Rahman, M. Saeed, H. A. E. W. Khalifa, W. A. Afifi, Decision making algorithmic techniques based on aggregation operations and similarity measures of possibility intuitionistic fuzzy hypersoft sets, *AIMS Math.*, **7** (2022), 3866–3895. http://dx.doi.org/10.3934/math.2022214
6. Q. Han, W. Li, Y. Song, T. Zhang, R. Wang, A new method for MAGDM based on improved TOPSIS and a novel pythagorean fuzzy soft entropy, *Symmetry*, **11** (2019), 905. https://doi.org/10.3390/sym11070905
7. M. Akram, G. Shahzadi, A hybrid decision-making model under q-rung orthopair fuzzy Yager aggregation operators, *Granular Comput.*, **6** (2021), 763–777. https://doi.org/10.1007/s41066-020-00229-z
8. M. J. Khan, P. Kumam, W. Deebani, W. Kumam, Z. Shah, Bi-parametric distance and similarity measures of picture fuzzy sets and their applications in medical diagnosis, *Egypt. Inform. J.*, **22** (2021), 201–212. https://doi.org/10.2307/j.ctv1cftj6q.24
9. S. Ashraf, S. Abdullah, T. Mahmood, M. Aslam, Cleaner production evaluation in gold mines using novel distance measure method with cubic picture fuzzy numbers, *Int. J. Fuzzy Syst.*, **21** (2019), 2448–2461. https://doi.org/10.1007/s40815-019-00681-3
10. K. Ullah, H. Garg, T. Mahmood, N. Jan, Z. Ali, Correlation coefficients for T-spherical fuzzy sets and their applications in clustering and multi-attribute decision making, *Soft Comput.*, **24** (2020), 1647–1659. https://doi.org/10.1007/s00500-019-03993-6

11. M. Riaz, M. Saba, M. A. Khokhar, M. Aslam, Novel concepts of m-polar spherical fuzzy sets and new correlation measures with application to pattern recognition and medical diagnosis, *AIMS Math.*, **6** (2021), 11346–11379. https://doi.org/10.3934/math.2021659
12. L. A. Zadeh, Fuzzy sets as a basis for a theory of possibility, *Fuzzy Set. Syst.*, **1** (1978), 3–28. https://doi.org/10.1016/0165-0114(78)90029-5
13. A. Kechris, *Classical descriptive set theory*, Springer Science & Business Media, **156** (2012).
14. F. Smarandache, Neutrosophic set is a generalization of intuitionistic fuzzy set, inconsistent intuitionistic fuzzy set (picture fuzzy set, ternary fuzzy set), pythagorean fuzzy set, spherical fuzzy set, and q-rung orthopair fuzzy set, while neutrosophication is a generalization of regret theory, grey system theory, and three-ways decision (revisited), *J. New Theory*, **29** (2019), 1–31.
15. Z. Shahbazi, Y. C. Byun, A procedure for tracing supply chains for perishable food based on blockchain, machine learning and fuzzy logic, *Electronics*, **10** (2020), 41. https://doi.org/10.3390/electronics10010041
16. A. Si, S. Das, S. Kar, Picture fuzzy set-based decision-making approach using Dempster-Shafer theory of evidence and grey relation analysis and its application in COVID-19 medicine selection, *Soft Comput.*, 2021, 1–15. https://doi.org/10.1007/s00500-021-05909-9
17. R. Costache, M. C. Popa, D. T. Bui, D. C. Diaconu, N. Ciubotaru, G. Minea, Spatial predicting of flood potential areas using novel hybridizations of fuzzy decision-making, bivariate statistics, and machine learning, *J. Hydrology*, **585** (2020), 124808. https://doi.org/10.1016/j.jhydrol.2020.124808
18. M. Riaz, M. Saba, M. A. Khokhar, M. Aslam, Medical diagnosis of nephrotic syndrome using m-polar spherical fuzzy sets, *Int. J. Biomath.*, **15** (2022), 2150094. https://doi.org/10.1142/S1793524521500947
19. Y. Jin, M. Kamran, N. Salamat, S. Zeng, R. H. Khan, Novel distance measures for single-valued neutrosophic fuzzy sets and their applications to multicriteria group decision-making problem, *J. Funct. Space.*, **2022** (2022). https://doi.org/10.1155/2022/7233420
20. K. Ullah, H. Garg, T. Mahmood, N. Jan, Z. Ali, Correlation coefficients for T-spherical fuzzy sets and their applications in clustering and multi-attribute decision making, *Soft Comput.*, **24** (2020), 1647–1659. https://doi.org/10.1007/s00500-019-03993-6
21. J. B. Liu, N. Salamat, M. Kamran, S. Ashraf, R. H. Khan, Single valued neutrosophic sets: A promising approach to assess image quality, *Fractals*, 2023. https://doi.org/10.1142/S0218348X23400741
22. A. Fahmi, S. Abdullah, F. Amin, M. S. A. Khan, Trapezoidal cubic fuzzy number Einstein hybrid weighted averaging operators and its application to decision making, *Soft Comput.*, **23** (2019), 5753–5783. https://doi.org/10.1007/s00500-018-3242-6
23. T. Alsboui, R. Hill, H. Al-Aqrabi, H. M. A. Farid, M. Riaz, S. Iram, A dynamic multi-mobile agent itinerary planning approach in wireless sensor networks via intuitionistic fuzzy set, *Sensors*, **22** (2022), 8037. https://doi.org/10.3390/s22208037
24. R. M. Zulqarnain, X. L. Xin, I. Siddique, W. A. Khan, M. A. Yousif, TOPSIS method based on correlation coefficient under pythagorean fuzzy soft environment and its application towards green supply chain management, *Sustainability*, **13** (2021), 1642. https://doi.org/10.3390/su13041642

25. M. J. Khan, P. Kumam, P. Liu, W. Kumam, S. Ashraf, A novel approach to generalized intuitionistic fuzzy soft sets and its application in decision support system, *Mathematics*, **7** (2019), 742. https://doi.org/10.3390/math7080742

26. S. Zeng, A. Hussain, T. Mahmood, M. I. Ali, S. Ashraf, M. Munir, Covering-based spherical fuzzy rough set model hybrid with TOPSIS for multi-attribute decision-making, *Symmetry*, **11** (2019), 547. https://doi.org/10.3390/sym11040547

27. H. Wang, F. Smarandache, Y. Zhang, *Single valued neutrosophic sets*, Infinite study, **12** (2010).

28. M. Saqlain, N. Jafar, S. Moin, M. Saeed, S. Broumi, Single and multi-valued neutrosophic hypersoft set and tangent similarity measure of single valued neutrosophic hypersoft sets, *Neutrosophic Sets Sy.*, **32** (2020), 317–329.

29. M. A. Khan, S. Ashraf, S. Abdullah, F. Ghani, Applications of probabilistic hesitant fuzzy rough set in decision support system, *Soft Comput.*, **24** (2020), 16759–16774. https://doi.org/10.1007/s00500-020-04971-z

30. R. Sahin, P. Liu, Correlation coefficient of single-valued neutrosophic hesitant fuzzy sets and its applications in decision making, *Neural Comput. Appl.*, **28** (2017), 1387–1395. https://doi.org/10.1007/s00521-015-2163-x

31. C. Zhang, D. Li, S. Broumi, A. K. Sangaiah, Medical diagnosis based on single-valued neutrosophic probabilistic rough multisets over two universes, *Symmetry*, **10** (2018), 213. https://doi.org/10.3390/sym10060213

32. R. Sahin, M. Karabacak, *A novel similarity measure for single-valued neutrosophic sets and their applications in medical diagnosis, taxonomy, and clustering analysis, Optimization theory based on neutrosophic and plithogenic sets*, Academic Press, 2020, 315–341. https://doi.org/10.1016/B978-0-12-819670-0.00014-7

33. Y. Guo, A. S. Ashour, *Neutrosophic sets in dermoscopic medical image segmentation*, Neutrosophic Set in Medical Image Analysis, Academic Press, 2019, 229–243. https://doi.org/10.1016/B978-0-12-818148-5.00011-4

34. H. M. A. Farid, M. Riaz, Single-valued neutrosophic Einstein interactive aggregation operators with applications for material selection in engineering design: Case study of cryogenic storage tank, *Complex Intell. Syst.*, **8** (2022), 2131–2149. https://doi.org/10.1007/s40747-021-00626-0

35. M. Kamran, S. Nadeem, S. Ashraf, A. Alam, Novel decision modeling for manufacturing sustainability under single-valued neutrosophic hesitant fuzzy rough aggregation information, *Comput. Intell. Neurosci.*, **2022** (2022). https://doi.org/10.1155/2022/7924094

36. M. Kamran, S. Ashraf, N. Salamat, M. Naeem, T. Botmart, Cyber security control selection based decision support algorithm under single valued neutrosophic hesitant fuzzy Einstein aggregation information, *AIMS Math.*, **8** (2023), 5551–5573. https://doi.org/10.3934/math.2023280

37. A. Yolcu, A. Benek, T. Y. Öztürk, A new approach to neutrosophic soft rough sets, *Knowl. Inform. Syst.*, 2023, 1–18. https://doi.org/10.1007/s10115-022-01824-z

38. L. Jiao, H. L. Yang, S. G. Li, Three-way decision based on decision-theoretic rough sets with single-valued neutrosophic information, *Int. J. Mach. Learn. Cyb.*, **11** (2020), 657–665. https://doi.org/10.1007/s13042-019-01023-3

39. R. Chinram, A. Hussain, T. Mahmood, M. I. Ali, EDAS method for multi-criteria group decision making based on intuitionistic fuzzy rough aggregation operators, *IEEE Access*, **9** (2021), 10199–10216. https://doi.org/10.1109/ACCESS.2021.3049605
40. M. Z. Hanif, N. Yaqoob, M. Riaz, M. Aslam, Linear Diophantine fuzzy graphs with new decision-making approach, *AIMS Math.*, **7** (2022), 14532–14556. https://doi.org/10.3934/math.2022801
41. S. Shao, X. Zhang, Y. Li, C. Bo, Probabilistic single-valued (interval) neutrosophic hesitant fuzzy set and its application in multi-attribute decision making, *Symmetry*, **10** (2018), 419. https://doi.org/10.3390/sym10090419
42. J. R. Trillo, F. J. Cabrerizo, F. Chiclana, M. Á. Martínez, F. Mata, E. Herrera-Viedma, Theorem verification of the quantifier-guided dominance degree with the mean operator for additive preference relations, *Mathematics*, **10** (2022), 2035. https://doi.org/10.3390/math10122035
43. M. Kamran, R. Ismail, E. H. A. Al-Sabri, N. Salamat, M. Farman, S. Ashraf, An optimization strategy for MADM framework with confidence level aggregation operators under probabilistic neutrosophic hesitant fuzzy rough environment, *Symmetry*, **15** (2023), 578. https://doi.org/10.3390/sym15030578
44. B. Sun, X. Zhou, N. Lin, Diversified binary relation-based fuzzy multigranulation rough set over two universes and application to multiple attribute group decision making, *Inform. Fusion*, **55** (2020), 91–104. https://doi.org/10.1016/j.inffus.2019.07.013
45. C. Wang, Y. Huang, W. Ding, Z. Cao, Attribute reduction with fuzzy rough self-information measures, *Inform. Sci.*, **549** (2021), 68–86. https://doi.org/10.1016/j.ins.2020.11.021
46. *Economic report of the president, transmitted to the congress February 2010 together with the annual report of the Council of Economic Advisors*, Council of Economic Advisers, 2010.
47. X. Wang, E. Triantaphyllou, Ranking irregularities when evaluating alternatives by using some ELECTRE methods, *Omega*, **36** (2008), 45–63. https://doi.org/10.1016/j.omega.2005.12.003
48. A. R. Mishra, P. Rani, R. S. Prajapati, Multi-criteria weighted aggregated sum product assessment method for sustainable biomass crop selection problem using single-valued neutrosophic sets, *Appl. Soft Comput.*, **113** (2021), 108038. https://doi.org/10.1016/j.asoc.2021.108038
49. C. Jana, M. Pal, Multi-criteria decision making process based on some single-valued neutrosophic Dombi power aggregation operators, *Soft Comput.*, **25** (2021), 5055–5072. https://doi.org/10.1007/s00500-020-05509-z
50. S. B. Said, M. Lathamaheswari, P. K. Singh, A. A. Ouallane, A. Bakhouyi, A. Bakali, et al., An intelligent traffic control system using neutrosophic sets, rough sets, graph theory, fuzzy sets and its extended approach: A literature review, *Neutrosophic Set. Syst.*, **50** (2022).
51. A. Al-Quran, A new multi attribute decision making method based on the T-spherical hesitant fuzzy sets, *IEEE Access*, **9** (2021), 156200–156210. https://doi.org/10.1109/ACCESS.2021.3128953

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