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Research article

A cosine similarity measures between hesitancy fuzzy graphs and its application to decision making

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Abstract: A new cosine similarity measure between hesitancy fuzzy graphs, which have been shown to have greater discriminating capacity than certain current ones in group decision making problems by example verification. This study proposes a novel method for estimating expert-certified repute scores by determining the ambiguous information of hesitancy fuzzy preference relations as well as the regular cosine similarity grades from one separable hesitancy fuzzy preference relation to some others. The new approach considers both "objective" and "subjective" information given by experts. We construct working procedures for assessing the eligible reputational scores of the experts by applying hesitancy fuzzy preference relations. In an evaluation in which multiple conflicting factors are taken into consideration, this can be applied to increase or reduce the relevancy of specified criteria. Applying the two effective methods, the newly developed cosine similarity measure, the energy of hesitancy fuzzy graph, and we provide a solution to a decisional issue. Finally, the two working procedures and examples are given to verify the practicality and dominance of the proposed techniques.

Keywords: hesitant fuzzy set; hesitancy fuzzy graph; cosine similarity measures; hesitancy fuzzy preference relations; decision making
Mathematics Subject Classification: 05C72, 03E72, 94D05

1. Introduction

The fuzzy set (FS) concept is one of the most essential techniques for dealing with complex and challenging data in the real world. Hesitant fuzzy set (HFS) was proposed to extend the FS and it was proposed by Torra [38] and HFS has fascinated much recognition. Dissimilar to previous extensions of FSs, HFSs enable the membership element to a set have various elements between 0 and 1. Torra and Narukawa [37] provided various rules and examined the connections between HFSs and other FSs expansions and also demonstrated that the cover of an HFS is an intuitionistic fuzzy set (IFS) [12]. Much research was done on IFSs, such as intuitionistic fuzzy operations [10, 13, 14].

The fuzzy graph has been extended into hesitant fuzzy graph. The remarks on fuzzy graphs were discussed by Bhattacharya [15], and also proviede on the properties of fuzzy graphs. In 2015, Pathinathan et al. [32] was introduced the concept of hesitancy fuzzy graph (HFG) and for the HFGs, the theoretical description of the different Cartesian products, order, size, and degree were taken into account. In 1987, Atanassov [9, 11] was established the concept of Index matrix (IM) and also expanded it himself. In 2017, Arockiaraj et al. [7] established the concept of IM representation for HFG, along with appropriate examples. Multiple mathematical operations, such as union and join of HFGs. Muhammad et al. introduced the concept of HFG and many HFG operations are determined for the idea of HFG. In 2016, Pathinathan et al. [8] were proposed Cartesian products for HFGs with an appropriate example and theoretical verification. Gong et al. [20] conceived the idea of a hesitant fuzzy hypergraph (HFH) based on HFSs and fuzzy hypergraphs. As part of the research, certain fundamental graph operations of HFHs are examined, as well as various equivalence relationships between HFHs, hesitant fuzzy formal concept analysis, and hesitant fuzzy information systems. The process of finding and selecting choices based on the decision maker's values and preferences is known as decision making. The decision-making (DM) process is the selection of the best choice from suitable alternatives. One of the most well-known DM approaches is the Technique for order preference by similarity to an ideal solution (TOPSIS), which was developed by Hwang and Yoon [22]. Xia and Xu [40] proposed a set of aggregation techniques for hesitant fuzzy data and also they discussed the relationship between IFS and HFSs, that they constructed various operations and aggregation operators for hesitant fuzzy elements. They demonstrated its use in solving DM problems. Xu et al. [44] presented and discussed the notions of entropy and cross-entropy for hesitant fuzzy information and similarity measures (SMs) are also investigated. They proposed two techniques for making multiattribute decisions in which attribute values are provided in the form of hesitant fuzzy sets that holistically replicate humans' hesitant thinking. HFS is quite helpful in avoiding problems like these, where each attribute could be represented as an HFS defined on the basis of the judgments of DMs. Akram et al. [4] presented an Elimination and selection Translating Reality-II approach under hesitant Pythagorean fuzzy information to affect divergent views of decision experts. Garg et al. [19] proposed the idea of the complex HFS, which combines the HFS with the complex fuzzy set to manage comprehensive and inconvenient information in the real-decision theory. Sarwar et al. [35] propose a novel approach termed bipolar fuzzy extending TOPSIS based on entropy weights to deal with multi-criteria DM issues including bipolar measurements with positive and negative values. They also explore innovative uses of bipolar fuzzy competition graphs in food webs and provide several techniques for calculating the degree of competitiveness between species. Akram et al. [1] proposed an m-polar hesitant fuzzy TOPSIS technique for multi-criteria group DM, which is a logical expansion of the TOPSIS technique to this framework.

The preference relation, which is both the most frequent and most important representation of data has garnered a significant amount of interest from research scientists and has been extensively employed, particularly in the criterion decision Analysis (MCDA). Later on, Xia and Xu [41] discovered the benefits of HFEs and presented the idea of hesitant FPRs (HFPRs). When confidentiality is needed to prevent decision makers from influencing one another or protecting their privacy, the HFPRs can be considered an effective tool for representing preference information over alternatives for a group of decision makers [45]. This is especially true in situations where it is important to avoid influencing one another. There have been several distinct forms of preference relations developed up to this moment, namely the intuitionistic fuzzy preference relation [36], the linguistic preference relation [21, 43]. Liao et al. presented the concept of a "hesitant fuzzy preference relation" (HFPR) and examined the unique aspects of this relation. This was done so as to resolve the drawback that was previously mentioned.

The similarity measure (SM) for the fuzzy system is critical in dealing with issues involving ambiguous data, but it will be unable to deal with the ambiguousness and awkwardness of issues involving standard information. Dengfeng et al. [26] examined several SMs on IFSs and presented an appropriate SM between IFSs, which will be the first one, used to pattern recognition issues. Xu et al. [42] were introduced the concept of a set of distance measurements for HFSs, from which the related SMs may be calculated. Xu and Chen provided a complete analysis of distance and SMs as well as many new continuous distance and SMs for IFS. Pythagorean fuzzy set (PFS) SMs proposed by Farhadinia [18] not only meet well-known theorems but also solve the division-by-zero problem. They proposed measures that combined the distance between PFSs with the t-norm and s-norm concepts. The m-PFS and m-PF soft set on SMs were established by Akram et al. [5] for medical diagnostics. Chinram et al. [17] was introduced the idea of a complex hesitant fuzzy set is a modified technique of the complex fuzzy set for dealing with uncomfortable and untrustworthy information in everyday life situations and also they determine the validity and competency of the studied measures based on CHFSs, the comparison of investigated measures with certain stated measures and also examined of their graphs. Bolturk et al. [16] proposed a novel analytic hierarchy process technique for neutrosophic sets using interval values based on a cosine similarity measure (CSM) and illustrated a suitable example for CSM.

In 2013, Anjali et al. [6] introduced the notion of the fuzzy graph (FG)'s energy. The energy of a FG is the sum of absolute values of its adjacency matrix eigenvalues. In addition to this, some limitations on the FGs energy are provided. The idea of the energy of a FG is extended to the energy of an intuitionistic fuzzy graph by Prabha et al. [31]. There have been some recent research done on the energy of different fuzzy graphs, and you can find them in [2, 3, 24, 25, 28, 30, 33]. In this article, we extended the concept of the energy of FG to the energy of HFG. Then we presented a technique for calculating the cosine similarity degrees between HFGs by extending the presented SMs based on the CSM and between HFGs [27] and [23]. Therefore, the emphasis of this research is on the CSM, the weighted CSM, and to illustrate the usefulness of the proposed CSM, all current SMs between HFGs proposed are comparing with the CSM between HFGs by numerical example problems.

The two interesting subfields of study of the existing body of research- namely HFGs and the proposed working procedures I and II to multi criteria decision making analysis- serve as the motivation for the investigation that is presented in this paper. As a consequence of their combination, a unique method is produced, which we have named working procedure-I and working procedure-II. This method enables us to carry out accurate assessments of the most effective models for television companies. Therefore, the primary motives of this study are described described in the following categories:

(1) In the way it shows evaluation data, HFG allows for multiple instances of membership degrees, non-membership degrees, and hesitant element degrees, which can be used to figure out how the ratings of TV company models are calculated.
(2) The hesitancy fuzzy cosine similarity measures include the benefits of hesitancy fuzzy similarity measures and HFGs to accommodate more complicated situations in the selection of television

company models.

(3) Additionally, the working procedures provide a highly effective framework for a variety of applications. These applications need the DMs to include at least three criteria, and there is variability between these criteria that is essential to the nature of the evaluations.

(4) Working procedures I and II are suitable for providing more reliable and accurate data when determining systems with multiple evaluation data values.

The application of the working procedures I and II to the decision making analysis is the primary emphasis of this research. In order to establish the weights for the decision-making issue, energy of HFG and a cosine similarity measure has been considered as a possible solution. The company operations that have been presented are applied to a process requiring the evaluation of a television company that involves three domain experts. Furthermore, the technique presented in this research will be validated to verify its accuracy. The primary addition that our work makes is the use of the new cosine similarity measure between the HFGs, which, when combined with the working procedure for television firm evaluation, makes our work very useful.

For the rest of this article, the structure is as follows. The fundamental concepts of hesitation fuzzy graphs and the cosine similarity measurements of hesitancy fuzzy graphs are presented in Section 2. Section 3 describes a numerical solution of Energy of hesitancy fuzzy graphs based on cosine similarity measures in decision-making problems with working procedures I and II and a flow chart for working procedures I and II is provided. And lastly, Section 4 concludes with the conclusions.

2. Preliminaries

This section describes fundamental HGF ideas and terminologies, as well as a cosine similarity measures for HFGs, which will be necessary for the systematic review.

Definition 2.1. *Suppose Y is a finite non-empty set. An HFS on Y can be expressed as a function h, when implemented to Y gives a subset of* $[0,1]$*, then the mathematical the symbol is written as follows:*

$$E = \{\langle y, h_E(Y)\rangle \; y \in Y\},$$

where $h_E(Y)$ is known as hesitant element and it is a collection of numbers in the range $[0,1]$ *indicating the membership degrees of the element $y \in Y$ to the set E.*

Definition 2.2. *A hesitancy fuzzy graph is of the $HG = (V, E, \mu, \gamma, \beta)$, where*

- *Consider $V = \{t_1, t_2, \dots, t_n\}$ such that $\mu_1 : V \to [0,1]$, $\gamma_1 : V \to [0,1]$ and $\beta_1 : V \to [0,1]$ are denotes the grade of membership, non-membership and hesitant of the elements $t_i \in V$ and $\mu_1(t_i) + \gamma_1(t_i) + \beta_1(t_i) = 1$, where $\beta_1(t_i) = 1 - [\mu_1(t_i) + \gamma_1(t_i)]$ and $0 \le \mu_1(t_i) + \gamma_1(t_i) \le 1$.*
- *Consider $E \subseteq V \times V$, where $\mu_2 : V \times V \to [0,1]$, $\gamma_2 : V \times V \to [0,1]$ and $\beta_2 : V \times V \to [0,1]$ are such that*

$$\mu_2(t_i, t_j) \le min\left[\mu_1(t_i),\ \mu_1(t_j)\right],$$

$$\gamma_2(t_i, t_j) \le max\left[\gamma_1(t_i),\ \gamma_1(t_j)\right],$$

$$\beta_2(t_i, t_j) \le min\left[\beta_1(t_i),\ \beta_1(t_j)\right],$$

and

$$0 \le \mu_2(t_i, t_j) + \gamma_2(t_i, t_j) + \beta_2(t_i, t_j) \le 1,\ \forall (t_i, t_j) \in E.$$

Definition 2.3. *Suppose $T = \{t_1, t_2, t_3, \ldots, t_n\}$ is a nonempty set, and then a hesitant fuzzy preference relation (HFPR) H on T is obtainable by a matrix $H = (h_{ij})_{(n\times n)} \subset Y \times Y$, where $h_{ij} = \xi^l_{ij}$ for all $l = 1, 2, \ldots, n$ is a hesitant fuzzy element indicating the entire possible preference grade (s) of the objective t_i over t_j. Furthermore, h_{ij} must satisfy the succeeding conditions:*

$$\xi_{ij}^{\sigma(l)} + \xi_{ji}^{\sigma(n-l+1)} \leq 1, \quad \xi_{ii} = 0, \quad i = j = 1, 2, \ldots, r,$$

where $\xi_{ij}^{\sigma(l)}$ is the l^{th} largest elements in h_{ij}.

Example 2.1. By defining the matrix $H = (h_{ij})_{4\times4}$ from the Figure 1, we get

$$H = (h_{ij})_{4\times4} = \begin{bmatrix} (0,0,0) & (0.2,0.2,0.1) & (0.2,0.3,0.1) & (0.3,0.3,0.1) \\ (0.2,0.2,0.1 & (0,0,0) & (0.2,0.3,0.4) & (0,0,0) \\ (0.2,0.3,0.1) & (0.2,0.3,0.4) & (0,0,0) & (0.3,0.3,0.2) \\ (0.3,0.3,0.1) & (0,0,0) & (0.3,0.3,0.2) & (0,0,0) \end{bmatrix}.$$

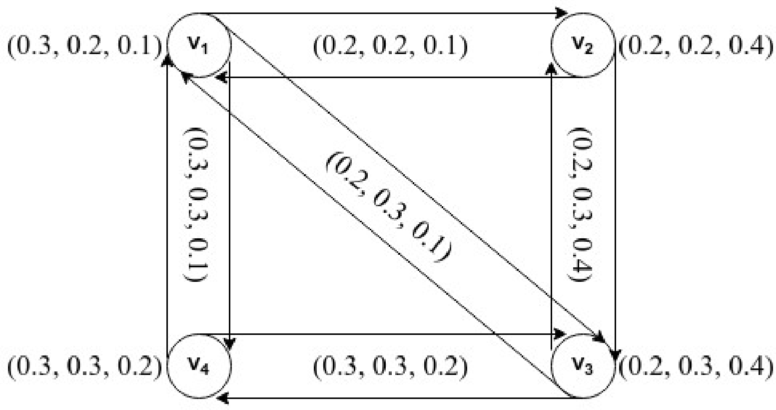

Figure 1. Hesitancy fuzzy graph with four alternatives.

Definition 2.4. *Let $M = (r_{ij})_{(n\times n)}$ is the fuzzy preference relation (FPR), and then*

$$r_{ij} = \sum_{b=1}^{l} C_b,\ \mu_{ij}^{(b)}, \quad \forall i, j = 1, 2, 3, \ldots, n,$$

where C_b is the subjective and objective weights of the expert e_b for the FPR $M = (r_{ij})_{(n\times n)}$ and $\sum_{i=1}^{n} C_b = 1$, $C_b > 0$, $b \in N$.

Definition 2.5. *If P and R are HFGs then the SMs from P to R be denoted as $S(P, R)$, it has the following characteristics:*

- $0 \leq S(P, R) \leq 1$,
- $S(P, R) = 1$, *iff* $P = R$,
- $S(P, R) = S(R, P)$,
- *If* $P \subseteq R \subseteq B$, *then* $S(P, B) \leq S(P, R)$ *and* $S(P, B) \leq S(R, B)$.

Definition 2.6. *Suppose that P and R are two HFGs in $T = \{t_1, t_2, t_3, \ldots, t_n\}$. Based on the extension of the CSMs for HFGs, then the weighted CSM between the HFGs P and R are defined as follows:*

$$CS(P,R) = \frac{1}{n}\sum_{i=1}^{n} \frac{\mu_P(t_i)\mu_R(t_i) + \gamma_P(t_i)\gamma_R(t_i) + \beta_P(t_i)\beta_R(t_i)}{\sqrt{\mu_P^2(t_i) + \gamma_P^2(t_i) + \beta_P^2(t_i)}\sqrt{\mu_R^2(t_i) + \gamma_R^2(t_i) + \beta_R^2(t_i)}},$$

where $\mu_P(t_i)$ is the grade of the membership element, $\gamma_P(t_i)$ be the grade of the non-membership element and $\beta_P(t_i)$ be the grade of the hesitant elements. Thus, the CSM between HFGs P and R are satisfies the following conditions:

$$0 \le CS(P,R) \le 1,$$
$$CS(P,R) = CS(R,P),$$
$$CS(P,R) = 1,\ iff\ P = R.$$

Then,

$$\mu_P(t_i) = \mu_R(t_i),\ \gamma_P(t_i) = \gamma_R(t_i),$$

and

$$\beta_P(t_i) = \beta_R(t_i), \quad \forall i = 1, 2, 3, \ldots, n.$$

The value of $CS(P,R)$ lies in between 0 and 1, it will not exceeds 0 and 1. Also, CSM satisfies the symmetry property.

3. The group decision making problems (GDMP) by hesitancy fuzzy preference relations

Suppose that $T = \{t_1, t_2, \ldots, t_n\}$ be the replacement set, and $Y = \{y_1, y_2, \ldots, y_n\}$ be the expert set. The expert y_l deals the evidence of optimal to all replacements and forms HFPRs

$$M^{(l)} = \left(a_{ij}^{(l)}\right)_{m\times m},$$

where $a_{ij}^{(l)} = \left(\mu_{ij}^{(l)}, \gamma_{ij}^{(l)}, \beta_{ij}^{(l)}\right)$, $0 \le \mu_{ij}^{(l)} + \gamma_{ij}^{(l)} + \beta_{ij}^{(l)} \le 1$ and $\mu_{ij}^{(l)} = \gamma_{ij}^{(l)} = \beta_{ij}^{(l)} = 0$, $\forall\ i, j = 1, 2, 3, \ldots, n$.

3.1. Weighted sum model

In this sector, weighted sum model (WSM) working procedure is constructed for GDMP concentrated on HFPRs. We define an impartial scoring vector as $C = \{c_1, c_2, c_3, \ldots, c_m\}$ of experts for GDMP based on HFPRs, where $C_b > 0$, $b = 1, 2, 3, \ldots, l$, and the entire scoring values of the experts is equal to one is denoted as $\sum_{i=1}^{l} C_i = 1$.

Stage I. Evaluate the energy $E(M^{(b)})$ of $M^{(b)}$:

$$E(M^{(k)}) = \det \sum_{i=1}^{n} \kappa_i. \tag{3.1}$$

Stage II. Evaluate the scores C_b^1, determined by $E(M^{(k)})$, of the expert e_b:

$$C_b^1 = \left((C_\mu)_i, (C_\gamma)_i, (C_\beta)_i\right) = \left[\frac{E\left((D_\mu)_i\right)}{\sum_{r=1}^{l} E((D_\mu)_r)}, \frac{E\left((D_\gamma)_i\right)}{\sum_{r=1}^{l} E((D_\gamma)_r)}, \frac{E\left((D_\beta)_i\right)}{\sum_{r=1}^{l} E((D_\beta)_r)}\right]. \tag{3.2}$$

Stage III. Evaluate the CSM $CS\left(M^{(b)}, M^{(d)}\right)$ between $M^{(b)}$ and $M^{(d)}$ for every $b \neq d$,

$$CS(P, R) = \frac{1}{n}\sum_{i=1}^{n} \frac{\mu_P(t_i)\mu_R(t_i) + \gamma_P(t_i)\gamma_R(t_i) + \beta_P(t_i)\beta_R(t_i)}{\sqrt{\mu_P^2(t_i) + \gamma_P^2(t_i) + \beta_P^2(t_i)}\sqrt{\mu_R^2(t_i) + \gamma_R^2(t_i) + \beta_R^2(t_i)}}. \tag{3.3}$$

Here, $P = M^{(b)}$ and $R = M^{(d)}$.

The mean cosine similarity degree $CS(M^{(b)})$ of $M^{(b)}$ to the others is calculated by

$$CS(M^{(b)}) = \frac{1}{m-1}\sum_{i=1, b\neq d}^{n} CS(M^{(b)}, M^{(d)}),\ b = 1, 2, 3, \ldots, l. \tag{3.4}$$

Stage IV. Evaluate the scores C_b^a, determined by $CS(M^{(b)})$ of the expert e_b:

$$C_b^a = \frac{CS\left(M^{(b)}\right)}{\sum_{i=1}^{l} CS\left(M^{(i)}\right)},\ b = 1, 2, 3, \ldots, l. \tag{3.5}$$

Stage V. Evaluate the "objective" scores C_b^2 of the expert e_b:

$$C_b^2 = \eta\, C_b^1 + (1-\eta)\, C_b^a, \quad \forall\, \eta \in [0, 1], \quad b = 1, 2, 3, \ldots, l. \tag{3.6}$$

Stage VI. Evaluate the subjective and objective scores C_b^1 and C_b^2 of the expert e_b:

$$C_b = \gamma\, C_b^1 + (1-\gamma)\, C_b^2, \quad \forall\, \gamma \in [0, 1], \quad b = 1, 2, 3, \ldots, l. \tag{3.7}$$

3.2. Working procedure I

Stage I. Evaluate the mean hesitancy fuzzy values (HFVs) $r_i^{(k)}$ of replacements t_i to the others replacements:

$$r_i^{(k)} = \frac{1}{n}\sum_{j=1}^{n} r_{ij}{}^{(k)}, \quad j = 1, 2, 3, \ldots, n. \tag{3.8}$$

Stage II. Calculate the values of $r_i^{(k)}$ equivalent to m experts in to a collection of HFVs of the replacements t_i to other replacements:

$$r_i^{(k)} = \sum_{b=1}^{l} C_b\, r_{ij}^{(k)}, \tag{3.9}$$

Stage III. Calculate the score function of r_i:

$$CS(r_i) = \frac{\mu_i - \gamma_i + \beta_i}{\sqrt{\mu_i^2 + \gamma_i^2 + \beta_i^2}}, \tag{3.10}$$

where the highest value of the score function is the greater of the replacement t_i and then build a ranking order of the replacements.

3.3. Working procedure II

Stage I. Evaluate the cooperative HFPR $M = (r_{ij})_{n\times n}$ by

$$r_{ij} = \left(\sum_{b=1}^{l} C_b\, \mu_{ij}^{(b)}, \sum_{b=1}^{l} C_b\, \gamma_{ij}^{(b)}, \sum_{b=1}^{l} C_b\, \beta_{ij}^{(b)}\right), \quad \forall\, i, j = 1, 2, 3, \ldots, n. \tag{3.11}$$

Stage II. Calculate the CSMs $CS\,(M^{(i)}, M^{(+)})$ between M^i and M^+ for every replacement t_i:

$$CS\left(M^{(i)}, M^{(+)}\right) = \frac{1}{n}\left(\sum_{j=i}^{n} \det\left\{\frac{\mu_{ij}(1) - \gamma_{ij}(0) + \beta_{ij}(1)}{(\mu_{ij}^2 + \gamma_{ij}{}^2 + \beta_{ij}^2)}\right\}\right). \tag{3.12}$$

Stage III. Calculate the CSMs $CS\,(M^{(i)}, M^{(-)})$ between M^i and M^- for every replacement t_i:

$$CS\left(M^{(i)}, M^{(-)}\right) = \frac{1}{n}\left(\sum_{j=i}^{n} \det\left\{\frac{\mu_{ij}(0) - \gamma_{ij}(1) + \beta_{ij}(0)}{(\mu_{ij}^2 + \gamma_{ij}{}^2 + \beta_{ij}^2)}\right\}\right). \tag{3.13}$$

Stage IV. Evaluate the values of $g(t_i)$, for every replacement t_i:

$$g(t_i) = \frac{CS\left(M^{(i)}, M^{(+)}\right)}{CS\,(M^{(i)}, M^{(+)}) + CS\,(M^{(i)}, M^{(-)})}. \tag{3.14}$$

The highest value of $g(t_i)$ is greater to the replacements t_i. And we estimate the rank of the replacements. Procedures I and II are given to illustrate how to achieve absorbed scores to classify replacements in the two following instances. Now the order of ranking of the replacements is conformed.

3.4. Application 1: the selection of finest television company models

A television (TV) channel allows for one-way communication between the sender and recipient. This approach may be used by the sender to inform, entertain, and persuade. TV is an important method of communicating information and plays a key part in our lives today. In today's modern world, we would not be where we are today without some sort of media. The purpose of television is to inform, educate, and amuse the general public. It serves as a link between the public and the government. There are hundreds of channels devoted to movies, music, fashion, sports, and news. As well as offering entertainment, it also provides links to a range of information and happenings across the globe. Mr. Punarv wishes to purchase a smart TV from a recognised brand for a variety of purposes. Various smart TVs are available nowadays, such as the Toshiba, Samsung, One Plus, MI, etc. There are several firms offering smart TVs for sale. However, he needs to choose a firm that sells the cheapest and best quality smart TVs. We assume that $A = \{v_1, v_2, v_3, v_4\}$ be the set of four smart TV firms (alternatives $v_1 = Toshiba$, $v_2 = Samsung$, $v_3 = OnePlus$, and $v_4 = MI$), and $C = \{c_1, c_2, c_3\}$ be the set of three criteria for specifying value and quality in relation to "Price", "Picture quality" and "Audio" with preference information provided in the form of HFPR $R = (r_{ij})$, where $R = (\mu_{ij}, \gamma_{ij}, \beta_{ij})$ the hesitancy fuzzy element allotted by Mr. Punarv expert with μ_{ij} as the degree to which the firm v_i is chosen over the firm v_1 with respect to the specified criteria. In terms of the specified criterion, γ_{ij} and β_{ij} are the degrees at which the firm v_i is not favoured above the firm v_1. The HFPR $R = (r_{ij})$ for the specified constraints appears in the matrices below, respectively. We consider that in the GDM issue, we have four replacements t_i and three experts e_b $(b = 1, 2, 3)$. Assume each expert's scores are $0, 0.3, 0.5, 0.8$ and 1.0. Furthermore, the ranking orders of the alternatives using the criteria prepared from an experts are in the framework shown in Figure 2. The four replacement units are composed of the following every expert e_b $(b = 1, 2, 3)$ and the HFPRs $M^{(b)} = r_{ij}^{(b)}$ $(b = 1, 2, 3)$ are constructed individually, as shown below.

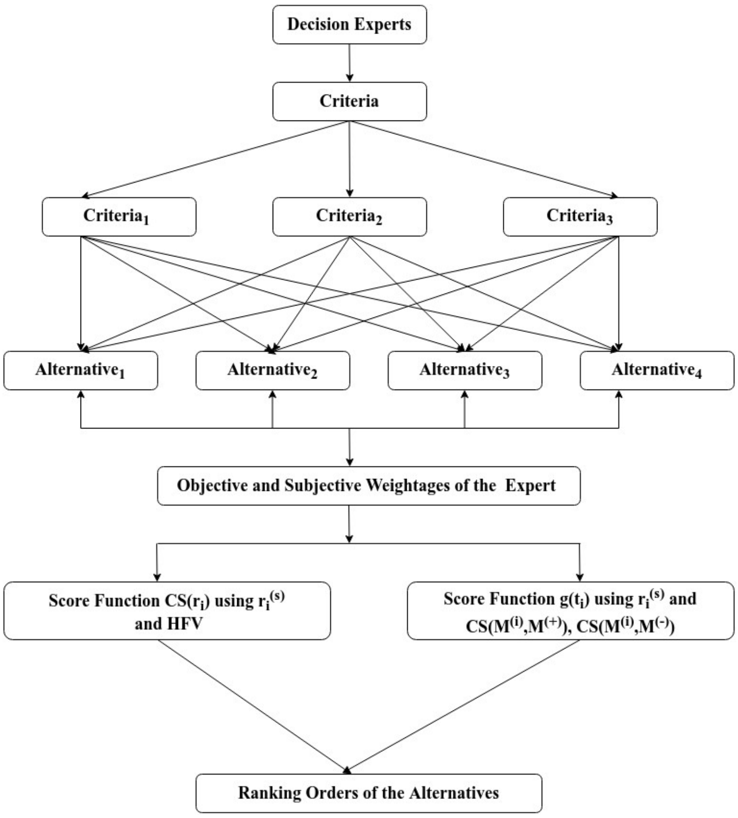

Figure 2. The Framework of evaluation ranking order for the alternatives.

Adjacency matrix from Figure 3, we get

$$M^{(1)} = A(HG) = \begin{bmatrix} (0,0,0) & (0.2,0.4,0.3) & (0.2,0.4,0.2) & (0.2,0.4,0.3) \\ (0.2,0.4,0.3) & (0,0,0) & (0.5,0.2,0.2) & (0.5,0.1,0.3) \\ (0.2,0.4,0.2) & (0.5,0.2,0.2) & (0,0,0) & (0.6,0.1,0.2) \\ (0.2,0.4,0.3) & (0.5,0.1,0.3) & (0.6,0.1,0.2) & (0,0,0) \end{bmatrix}.$$

Adjacency matrix from Figure 4, we get

$$M^{(2)} = A(HG) = \begin{bmatrix} (0,0,0) & (0.2,0.5,0.3) & (0.2,0.5,0.2) & (0.1,0.6,0.2) \\ (0.2,0.5,0.3) & (0,0,0) & (0.2,0.5,0.2) & (0.1,0.5,0.3) \\ (0.2,0.5,0.2) & (0.2,0.5,0.2) & (0,0,0) & (0.1,0.6,0.2) \\ (0.1,0.6,0.2) & (0.1,0.5,0.3) & (0.1,0.6,0.2) & (0,0,0) \end{bmatrix}.$$

Adjacency matrix from Figure 5, we get

$$M^{(3)} = A(HG) = \begin{bmatrix} (0,0,0) & (0.4,0.4,0.2) & (0.2,0.4,0.2) & (0.4,0.4,0.1) \\ (0.4,0.4,0.2) & (0,0,0) & (0.2,0.3,0.2) & (0.5,0.3,0.1) \\ (0.2,0.4,0.2) & (0.2,0.3,0.2) & (0,0,0) & (0.2,0.3,0.1) \\ (0.4,0.4,0.1) & (0.5,0.3,0.1) & (0.2,0.3,0.1) & (0,0,0) \end{bmatrix}.$$

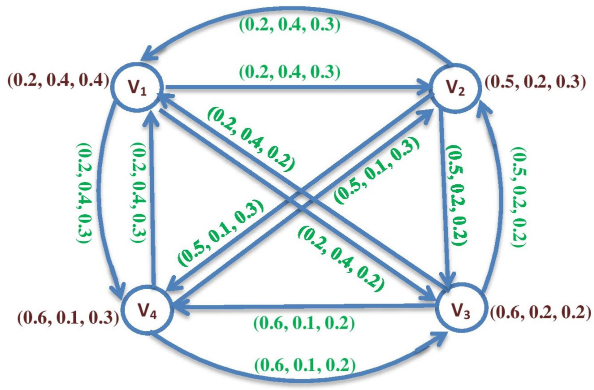

Figure 3. HFPR for the criteria price.

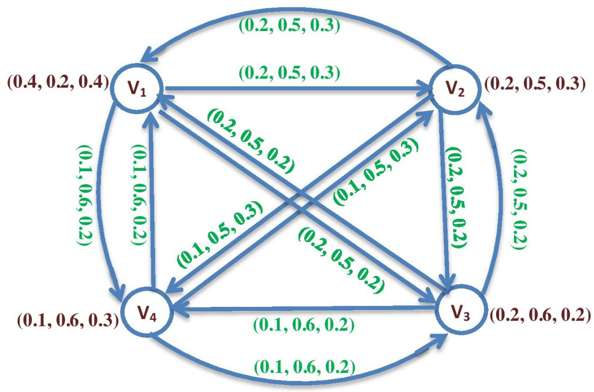

Figure 4. HFPR for the criteria price.

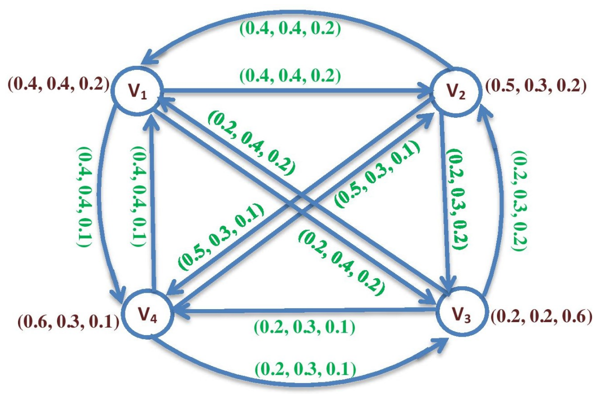

Figure 5. HFPR for the criteria price.

Stage I. The energy of the adjacency matrices $M^{(1)}$, $M^{(2)}$ and $M^{(3)}$ were calculated using Eq (3.1):

$$E(M^{(1)}) = (2.3410, 1.6806, 1.5166),$$

$$E(M^{(2)}) = (0.9292, 3.2047, 1.4111),$$

$$E(M^{(3)}) = (1.9792, 2.1100, 0.9292).$$

Stage II. The scores C_b^1 of each expert e_b is determined by using Eq (3.2), we get

$$C_1^1 = [0.4227, 0.3035, 0.2738],$$

$$C_2^1 = [0.1676, 0.5779, 0.2546],$$

$$C_3^1 = [0.3944, 0.4205, 0.1852].$$

Stage III. The CSMs $CS(M^{(b)}),\ CS(M^{(d)})$ between $M^{(b)}$ and $M^{(d)}$ is determined by using Eq (3.3), we get

$$CS\left(M^{(1)}, M^{(2)}\right) = 2.2887,$$

$$CS\left(M^{(2)}, M^{(3)}\right) = 2.6381,$$

$$CS\left(M^{(1)}, M^{(3)}\right) = 2.6205.$$

Using Eq (3.4), the mean CS degree (CSD) $CS(M^{(b)})$ of $M^{(b)}$ is determined as below:

$$CS(M^{(1)}) = 2.4546,$$

$$CS(M^{(2)}) = 2.4634,$$

$$CS(M^{(3)}) = 2.6293.$$

Stage IV. The values of the scores C_b^a of each expert e_b is determined by using Eq (3.5), we get

$$C^b = (0.3253, 0.3484, 0.3264).$$

Stage V. The objective scores C_b^2 of every expert e_b is determined by using Eq (3.6) and $\eta = 0.5$, we get

$$C_1^2 = [0.3740, 0.3260, 0.3001],$$

$$C_2^2 = [0.2645, 0.4632, 0.2905],$$

$$C_3^2 = [0.3624, 0.3845, 0.2558].$$

Stage VI. Using Eq (3.7), we find the scores of subjective and objective C_b^1 and C_b^2 of each expert e_b and substitute $\gamma = 0.3$, we have

$$C_1 = [0.3899, 0.3193, 0.2922],$$

$$C_2 = [0.2228, 0.4976, 0.2797],$$

$$C_3 = [0.3720, 0.3953, 0.2346].$$

According to working procedure I

Stage I. The mean HFVs $r_i^{(b)}$ of the replacements t_i to the other replacements is calculated by using Eq (3.8), we get

$$r_1^{(1)} = (0.2000, 0.4000, 0.2667), \quad r_2^{(1)} = (0.4000, 0.2333, 0.2667),$$

$$r_3^{(1)} = (0.4333, 0.2333, 0.2000), \quad r_4^{(1)} = (0.4333, 0.2000, 0.2667),$$

$$r_1^{(2)} = (0.1667, 0.5333, 0.2333), \quad r_2^{(2)} = (0.1667, 0.5000, 0.2667),$$

$$r_3^{(2)} = (0.1667, 0.5333, 0.2000), \quad r_4^{(2)} = (0.1000, 0.5667, 0.2333),$$

$$r_1^{(3)} = (0.3333, 0.3000, 0.1667), \quad r_2^{(3)} = (0.3667, 0.3333, 0.1667),$$

$$r_3^{(3)} = (0.2000, 0.3333, 0.1667), \quad r_4^{(3)} = (0.3667, 0.3333, 0.1000).$$

Stage II. Using Eq (3.9), we find the values of r_i, we get

$$r_1 = (0.2391, 0.5117, 0.1823),$$

$$r_2 = (0.3295, 0.4550, 0.1916),$$

$$r_3 = (0.2805, 0.4716, 0.1535),$$

$$r_4 = (0.3276, 0.4776, 0.1666).$$

Stage III. The values of the score function $CS(r_i)$ of r_i is determined by using Eq (3.10), we get

$$CS(r_1) = -0.1521,$$

$$CS(r_2) = 0.1114,$$

$$CS(r_3) = -0.0660,$$

$$CS(r_4) = 0.0275.$$

Therefore,

$$CS(r_2) > CS(r_4) > CS(r_3) > CS(r_1).$$

Hence,

$$t_2 > t_4 > t_3 > t_1.$$

Therefore, t_2 is the top place, while t_1 be the last place, as a final point t_3 and t_4 be the centre place ranking order.

According to working procedure II

In this part, we offer the ranking conclusions ability by our relative CS technique.

Stage I. The HFPR's cooperative $M = (r_{ij})_{n\times n}$ are calculated by using Eq (3.11), we get

$$M = \begin{bmatrix} (0,0,0) & (0.2713, 0.5346, 0.2185) & (0.1969, 0.5346, 0.1613) & (0.2491, 0.5844, 0.1671) \\ (0.2713, 0.5346, 0.2185) & (0,0,0) & (0.3139, 0.4313, 0.1613) & (0.4032, 0.3993, 0.1950) \\ (0.1969, 0.5346, 0.1613) & (0.3139, 0.4313, 0.1613) & (0,0,0) & (0.3306, 0.4491, 0.1378) \\ (0.2491, 0.5844, 0.1671) & (0.4032, 0.3993, 0.1950) & (0.3306, 0.4491, 0.1378) & (0,0,0) \end{bmatrix}.$$

Stage II. The CSMs $CS(M^i, M^+)$ between M^i and M^+ for every replacement t_i is calculated by using Eq (3.12), we get

$$CS(M^1, M^+) = 0.4995,$$

$$CS(M^2, M^+) = 0.6543,$$

$$CS(M^3, M^+) = 0.5683,$$

$$CS(M^4, M^+) = 0.6094.$$

Stage III. The CSMs $CS(M^i, M^-)$ between M^i and M^- for every replacement t_i is calculated by using Eq (3.13), we get

$$CS(M^1, M^-) = 0.6576,\ CS(M^2, M^-) = 0.5693,$$

$$CS(M^3, M^-) = 0.6147,\ CS(M^4, M^-) = 0.5843.$$

Stage IV. The values of $g(t_i)$, for every replacement t_i is determined by using Eq (3.14):

$$g(t_1) = 0.4317,$$

$$g(t_2) = 0.5347,$$

$$g(t_3) = 0.4804,$$

$$g(t_4) = 0.5105.$$

Hence,

$$g(t_2) > g(t_4) > g(t_3) > g(t_1).$$

Therefore,

$$t_2 > t_4 > t_3 > t_1,$$

where t_2 is the top place, while t_1 be the last place, as a final point t_3 and t_4 be the centre place ranking order.

Likewise, we calculate the place position conclusions of the values $\gamma = 0, 0.3, 0.5, 0.8$ and 1.0 by using the above working procedure I and II in Tables 1–4.

Table 1. The ranking order of the replacements for distinct values of γ using working procedure I.

γ	**C**	**r**
0	$C_1 = (0.3740, 0.3260, 0.3001)$	$r_1 = (0.2397, 0.4928, 0.1905)$
	$C_2 = (0.2645, 0.4632, 0.2905)$	$r_2 = (0.3266, 0.4358, 0.2002)$
	$C_3 = (0.3624, 0.3845, 0.2558)$	$r_3 = (0.2786, 0.4512, 0.1608)$
		$r_4 = (0.3214, 0.4558, 0.1734)$
0.3	$C_1 = (0.3255, 0.3764, 0.2667)$	$r_1 = (0.2391, 0.5117, 0.1823)$
	$C_2 = (0.2971, 0.4670, 0.2359)$	$r_2 = (0.3295, 0.4550, 0.1916)$
	$C_3 = (0.3090, 0.3595, 0.3905)$	$r_3 = (0.2805, 0.4716, 0.1535)$
		$r_4 = (0.3276, 0.4776, 0.1666)$
0.5	$C_1 = (0.3616, 0.3821, 0.2564)$	$r_1 = (0.2423, 0.5297, 0.1623)$
	$C_2 = (0.2925, 0.4866, 0.2692)$	$r_2 = (0.3351, 0.4739, 0.1714)$
	$C_3 = (0.3062, 0.3626, 0.3734)$	$r_3 = (0.2854, 0.4912, 0.1377)$
		$r_4 = (0.3341, 0.4981, 0.1476)$
0.8	$C_1 = (0.3527, 0.3878, 0.2461)$	$r_1 = (0.2439, 0.5432, 0.1687)$
	$C_2 = (0.2878, 0.4662, 0.2060)$	$r_2 = (0.3403, 0.4871, 0.1775)$
	$C_3 = (0.3034, 0.3657, 0.3563)$	$r_3 = (0.2895, 0.5056, 0.1414)$
		$r_4 = (0.3417, 0.5139, 0.1554)$
1.0	$C_1 = (0.4277, 0.3035, 0.2738)$	$r_1 = (0.2449, 0.5557, 0.1633)$
	$C_2 = (0.1676, 0.5779, 0.2546)$	$r_2 = (0.3436, 0.4999, 0.1718)$
	$C_3 = (0.3944, 0.4205, 0.1852)$	$r_3 = (0.2921, 0.5192, 0.1366)$
		$r_4 = (0.3467, 0.5283, 0.1509)$

Table 2. The ranking order of the replacements by using working procedure.

γ	**CS($\mathbf{r_1}$)**	**CS($\mathbf{r_2}$)**	**CS($\mathbf{r_3}$)**	**CS($\mathbf{r_4}$)**	**Ranking**
0	−0.1079	0.1568	−0.0213	0.0390	$t_2 > t_4 > t_3 > t_1$
0.3	−0.1521	0.1114	−0.0660	0.0275	$t_2 > t_4 > t_3 > t_1$
0.5	−0.2069	0.0539	−0.1165	−0.0266	$t_2 > t_4 > t_3 > t_1$
0.8	−0.2110	0.0495	−0.1246	−0.0264	$t_2 > t_4 > t_3 > t_1$
1.0	−0.2346	0.0246	−0.1559	−0.0473	$t_2 > t_4 > t_3 > t_1$

Table 3. The ranking order of the replacements for distinct values of γ using working procedure I.

γ	**C**	$\mathbf{CS}(\mathbf{M^i}, \mathbf{M^+})$	$\mathbf{CS}(\mathbf{M^i}, \mathbf{M^-})$
0	(0.3740, 0.3260, 0.3001) (0.2645, 0.4632, 0.2905) (0.3624, 0.3845, 0.2558)	(0.5137, 0.6612, 0.5819, 0.6197)	(0.6490, 0.5577, 0.6045, 0.5752)
0.3	(0.3255, 0.3764, 0.2667) (0.2971, 0.4670, 0.2359) (0.3090, 0.3595, 0.3905)	(0.4995, 0.6543, 0.5683, 0.6094)	(0.6576, 0.5693, 0.6147, 0.5843)
0.5	(0.3616, 0.3821, 0.2564) (0.2925, 0.4866, 0.2692) (0.3062, 0.3626, 0.3734)	(0.4713, 0.6240, 0.5402, 0.5821)	(0.6672, 0.5815, 0.6241, 0.5946)
0.8	(0.3527, 0.3878, 0.2461) (0.2878, 0.4662, 0.2060) (0.3034, 0.3657, 0.3563)	(0.5029, 0.6567, 0.5662, 0.6143)	(0.6579, 0.5721, 0.6186, 0.5872)
1.0	(0.4277, 0.3035, 0.2738) (0.1676, 0.5779, 0.2546) (0.3944, 0.4205, 0.1852)	(0.4580, 0.6087, 0.5230, 0.5716)	(0.6721, 0.5892, 0.6312, 0.6001)

Table 4. The ranking order of the replacements for distinct values of γ using working procedure II.

γ	$\mathbf{CS}(\mathbf{r_1})$	$\mathbf{CS}(\mathbf{r_2})$	$\mathbf{CS}(\mathbf{r_3})$	$\mathbf{CS}(\mathbf{r_4})$	**Ranking**
0	0.4418	0.5425	0.4905	0.5186	$t_2 > t_4 > t_3 > t_1$
0.3	0.4317	0.5347	0.4804	0.5105	$t_2 > t_4 > t_3 > t_1$
0.5	0.4140	0.5176	0.4640	0.4947	$t_2 > t_4 > t_3 > t_1$
0.8	0.4332	0.5344	0.4779	0.5113	$t_2 > t_4 > t_3 > t_1$
1.0	0.4053	0.5081	0.4531	0.4878	$t_2 > t_4 > t_3 > t_1$

According to the working procedure and Xu's technique, by substituting the values of $\gamma = 0, 0.3, 0.5, 0.8$ and 1.0, we get the same results for all the values. Therefore,

$$t_2 > t_4 > t_3 > t_1.$$

Hence, t_2 place the highest position, while t_1 place the last position, finally t_3 and t_4 places the centre position orders and which is mentioned in the above Tables 2 and 4. According to the reults of the study shown above, we believe that the selection of televisions manufactured by the Samsung Company is the best.

Based on the findings of the study shown above, we believe that Mr. Punarv would benefit most from purchasing a television manufactured by the Samsung company.

3.5. *Application 2: the selection of maize seeds for agriculture maize farming*

In this part, we utilize the hesitancy fuzzy cosine similarity measure in the weighted sum model (WSM) for evaluating the four varieties of agricultural maize crops seeds (AMCSs). Assume that $A = \{V_1, V_2, V_3, V_4\}$ is the set of four varieties of AMCSs (Alternatives V_1 = Pioneer Seeds-P3396, V_2 = Dekalb-DKC 9178, V_3 = Syngenta NK7328, and V_4 = Tata seeds-DMH 8255). An agricultural scientist examines the AMCS for particular aspects, making use of a short priority sheet for evaluating the AMCS. In this research, the priority sheet was assigned by an agricultural scientist and used as the criteria for determining the standard level (decision-maker). The wide category of the priority list has been chosen to be a soil, seedling growth, crop yield, and market price. The four alternatives are composed of the following agri scientists, and the HFPRs $M(b) = {r_{ij}}^{(b)}$ $(b = 1, 2, 3, 4)$ are constructed individually, as shown below.

The adjacency matrix is obtained from Figure 6,

$$M^{(1)} = A(HG) = \begin{bmatrix} (0,0,0) & (0.60,0.06,0.04) & (0.80,0.05,0.05) & (0.10,0.15,0.00) \\ (0.60,0.06,0.04) & (0,0,0) & (0.22,0.12,0.01) & (0.54,0.08,0.03) \\ (0.80,0.05,0.05) & (0.22,0.12,0.01) & (0,0,0) & (0.33,0.10,0.02) \\ (0.10,0.15,0.00) & (0.54,0.08,0.03) & (0.33,0.10,0.02) & (0,0,0) \end{bmatrix}.$$

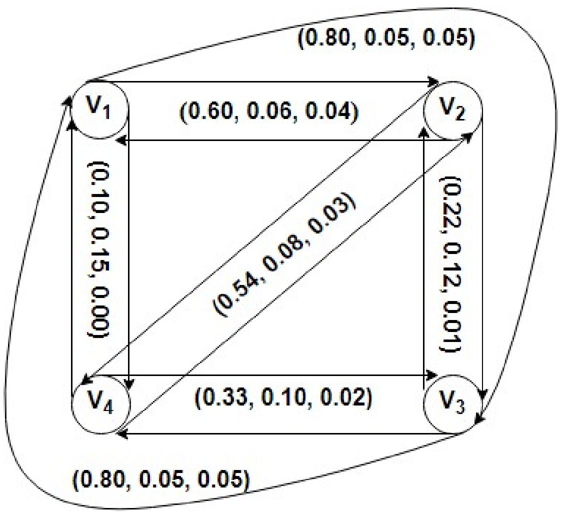

Figure 6. The hesitancy fuzzy preference relation for soil criteria.

The adjacency matrix is obtained from Figure 7,

$$M^{(2)} = A(HG) = \begin{bmatrix} (0,0,0) & (0.54,0.08,0.03) & (0.33,0.10,0.02) & (0.22,0.12,0.01) \\ (0.54,0.08,0.03) & (0,0,0) & (0.10,0.15,0.00) & (0.60,0.06,0.04) \\ (0.33,0.10,0.02) & (0.10,0.15,0.00) & (0,0,0) & (0.80,0.05,0.05) \\ (0.22,0.12,0.01) & (0.60,0.06,0.04) & (0.80,0.05,0.05) & (0,0,0) \end{bmatrix}.$$

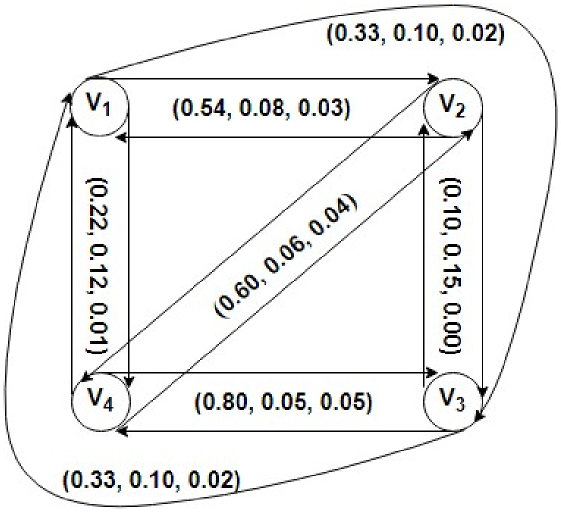

Figure 7. The hesitancy fuzzy preference relation for seedling growth criteria.

The adjacency matrix is obtained from Figure 8,

$$M^{(3)} = A(HG) = \begin{bmatrix} (0,0,0) & (0.10,0.15,0.00) & (0.54,0.08,0.03) & (0.33,0.10,0.02) \\ (0.10,0.15,0.00) & (0,0,0) & (0.22,0.12,0.01) & (0.80,0.05,0.05) \\ (0.54,0.08,0.03) & (0.22,0.12,0.01) & (0,0,0) & (0.60,0.06,0.04) \\ (0.33,0.10,0.02) & (0.80,0.05,0.05) & (0.60,0.06,0.04) & (0,0,0) \end{bmatrix}.$$

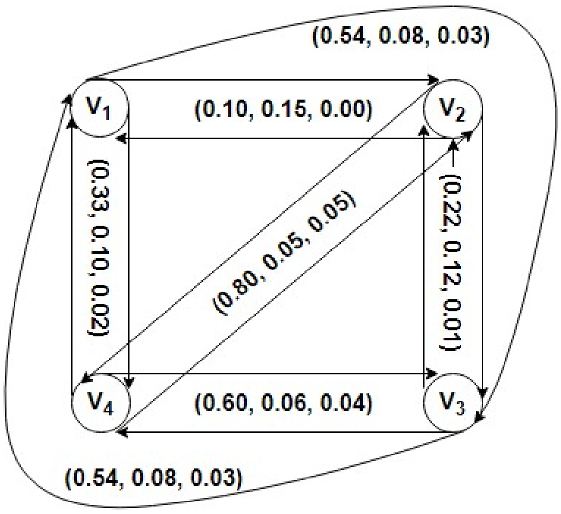

Figure 8. The hesitancy fuzzy preference relation for crop yield criteria.

The adjacency matrix is obtained from Figure 9,

$$M^{(4)} = A(HG) = \begin{bmatrix} (0,0,0) & (0.33,0.10,0.02) & (0.10,0.15,0.00) & (0.54,0.08,0.03) \\ (0.33,0.10,0.02) & (0,0,0) & (0.80,0.05,0.05) & (0.22,0.12,0.01) \\ (0.10,0.15,0.00) & (0.80,0.05,0.05) & (0,0,0) & (0.60,0.06,0.04) \\ (0.54,0.08,0.03) & (0.22,0.12,0.01) & (0.60,0.06,0.04) & (0,0,0) \end{bmatrix}.$$

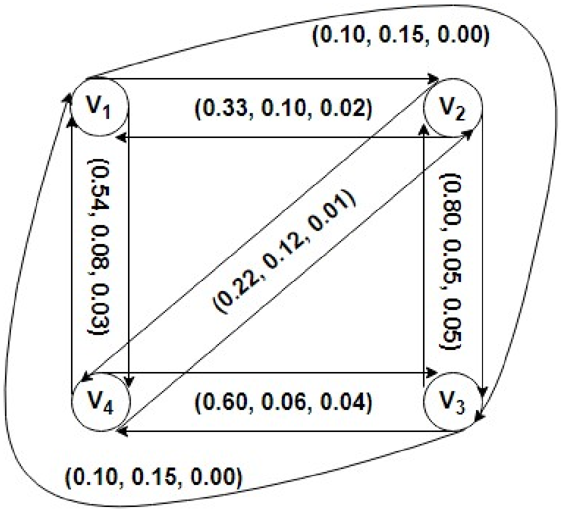

Figure 9. The hesitancy fuzzy preference relation for market price criteria.

The energy of the adjacency matrices $M^{(1)}, M^{(2)}$ and $M^{(3)}$ of HFGby using the Eq (3.1), we get

$$E(M^{(1)}) = (2.7590, 0.5641, 0.1676),$$

$$E(M^{(2)}) = (2.7534, 0.6190, 0.1818),$$

$$E(M^{(3)}) = (2.8790, 0.5641, 0.1676),$$

$$E(M^{(4)}) = (2.7590, 0.5641, 0.1676).$$

The scores C_b^1 of each expert e_b is determined by using Eq (3.2), we get

$$C_1^1 = [0.7904, 0.1616, 0.0480],\ C_2^1 = [0.7747, 0.1742, 0.0512],$$

$$C_3^1 = [0.7974, 0.1562, 0.0464],\ C_4^1 = [0.7904, 0.1616, 0.0480].$$

The CSMs $CS(M^{(b)}),\ CS(M^{(d)})$ between $M^{(b)}$ and $M^{(d)}$ is determined by using Eq (3.3), we get

$$CS\left(M^{(1)}, M^{(2)}\right) = 2.8568,\ CS\left(M^{(1)}, M^{(3)}\right) = 2.6891,$$

$$CS\left(M^{(1)}, M^{(4)}\right) = 2.3400,\ CS\left(M^{(2)}, M^{(3)}\right) = 2.7417,$$

$$CS\left(M^{(2)}, M^{(4)}\right) = 2.6117,\ CS\left(M^{(3)}, M^{(4)}\right) = 2.7732.$$

Using Eq (3.4), the mean CS degree (CSD) $CS(M^{(b)})$ of $M^{(b)}$ is determined as below:

$$CS(M^{(1)}) = 2.6286,\ CS(M^{(2)}) = 2.7367,$$

$$CS(M^{(3)}) = 2.7347,\ CS(M^{(4)}) = 2.5750.$$

The values of the scores C_b^a of each expert e_b is determined by using Eq (3.5), we get

$$C^b = (0.2462, 0.2564, 0.2562, 0.2412).$$

The objective scores C_b^2 of every expert e_b is determined by using Eq (3.6) and $\eta = 0.5$, we get

$$C_1^2 = [0.5183, 0.2039, 0.1471],\ C_2^2 = [0.5156, 0.2153, 0.1538],$$

$$C_3^2 = [0.5268, 0.2062, 0.1513],\ C_4^2 = [0.5158, 0.2014, 0.1446].$$

Using Eq (3.7), we find the scores of subjective and objective C_b^1 and C_b^2 of each expert e_b and substitute $\gamma = 0.3$, we have

$$C_1 = [0.6544, 0.1828, 0.0976],$$
$$C_2 = [0.6452, 0.1948, 0.1025],$$
$$C_3 = [0.6621, 0.1812, 0.0989],$$
$$C_4 = [0.6531, 0.1815, 0.0979].$$

According to working procedure I

The mean HFVs $r_i^{(b)}$ of the replacements t_i to the other replacements is calculated by using Eq (3.8), we get

$$r_1^{(1)} = (0.5000, 0.0867, 0.0300),\ r_2^{(1)} = (0.4533, 0.0867, 0.0267),$$
$$r_3^{(1)} = (0.4500, 0.0900, 0.0267),\ r_4^{(1)} = (0.3233, 0.1100, 0.0167),$$
$$r_1^{(2)} = (0.3633, 0.1000, 0.0600),\ r_2^{(2)} = (0.4133, 0.0967, 0.0233)$$
$$r_3^{(2)} = (0.4100, 0.1000, 0.0233),\ r_4^{(2)} = (0.5400, 0.0767, 0.0333),$$
$$r_1^{(3)} = (0.3233, 0.1100, 0.0167),\ r_2^{(3)} = (0.3733, 0.1067, 0.0200),$$
$$r_3^{(3)} = (0.4067, 0.0867, 0.0267),\ r_4^{(3)} = (0.5767, 0.0700, 0.0367),$$
$$r_1^{(4)} = (0.3233, 0.1100, 0.0167),\ r_2^{(4)} = (0.4500, 0.0900, 0.0267),$$
$$r_3^{(4)} = (0.5000, 0.0867, 0.0300),\ r_4^{(4)} = (0.4533, 0.0867, 0.0267).$$

Using Eq (3.9), we find the values of r_i, we get

$$r_1 = (1.1288, 0.0690, 0.0099),\ r_2 = (1.1285, 0.0692, 0.0138),$$

$$r_3 = (1.0983, 0.0693, 0.0099),\ r_4 = (1.1290, 0.0691, 0.0099).$$

The values of the score function $CS(r_i)$ of r_i is determined by using Eq (3.10), we get

$$CS(r_1) = 0.9458\ ,\ CS(r_2) = 0.9491,$$

$$CS(r_3) = 0.9440,\ CS(r_4) = 0.9420.$$

Therefore, $CS(r_2) > CS(r_4) > CS(r_3) > CS(r_1)$.

Hence,

$$t_2 > t_1 > t_3 > t_4.$$

Therefore, t_2 is the top place, while t_4 be the last place, as a final point t_1 and t_3 be the centre place ranking order.

According to working procedure II

In this part, we offer the ranking conclusions ability by our relative CS technique.

The HFPR's cooperative $M = (r_{ij})_{n\times n}$ are calculated by using Eq (3.11), we get

$$M = \begin{bmatrix} (0,0,0) & (1.0228,0.0719,0.0089) & (1.1593,0.0528,0.0148) & (0.7786,0.0834,0.0059) \\ (1.0228,0.0719,0.0089) & (0,0,0) & (0.8766,0.0820,0.0069) & (1.4139,0.0572,0.0130) \\ (1.1593,0.0528,0.0148) & (0.8766,0.0820,0.0069) & (0,0,0) & (1.5212,0.0498,0.0149) \\ (0.7786,0.0834,0.0059) & (1.4139,0.0572,0.0130) & (1.5212,0.0498,0.0149) & (0,0,0) \end{bmatrix}.$$

The CSMs $CS(M^i, M^+)$ between M^i and M^+ for every replacement t_i is calculated by using Eq (3.12), we get

$$CS(M^1, M^+) = 0.7529,\ CS(M^2, M^+) = 0.7545,$$
$$CS(M^3, M^+) = 0.7541,\ CS(M^4, M^+) = 0.7548.$$

The CSMs $CS(M^i, M^-)$ between M^i and M^- for every replacement t_i is calculated by using Eq (3.13), we get

$$CS(M^1, M^-) = 0.0449,\ CS(M^2, M^-) = 0.0427,$$
$$CS(M^3, M^-) = 0.0509,\ CS(M^4, M^-) = 0.0554.$$

The values of $g(t_i)$, for every replacement t_i, is determined by using Eq (3.14),

$$g(t_1) = 0.9437,\ g(t_2) = 0.9464,$$
$$g(t_3) = 0.9368,\ g(t_4) = 0.9315.$$

Hence,

$$g(t_2) > g(t_1) > g(t_3) > g(t_4).$$

Therefore,

$$t_2 > t_1 > t_3 > t_4.$$

Hence, t_2 place the highest position, while t_1 place the last position, finally t_3 and t_4 places the centre position orders and which is mentioned in the above tables.

4. Conclusions

This research introduced an innovative process to evaluating the relative reputational scores of an expert by calculating the unclear information of HFPRs and the mean similarity grade of a particular HFPR to all the remaining. Also, this article established the CSMs, and energy on the undetermined signs of HFPRs. This research constructed a tool for evaluating the score values of experts that takes both the subjective and objective scores of the experts into consideration. The scored CSMs was implemented to decision-making issues, and the outcomes are explained in more detail. This research illustrated the real time numerical examples to find out the best television from television firms and the selection of maize seeds for agriculture maize farming by applying the WSM working procedure I and II, after applied these WSM working procedures we got the finest one in both the cases.

Forthcoming, we implement the TOPSIS technique based on the correlation coefficient using HFGs information and its application to decision-making issues [23,29,34,39,45,46].

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Conflict of interest

The authors declare no potential conflicts of interest.

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