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Research article

Breathers, resonant multiple waves and complexiton solutions of a (2+1)-dimensional nonlinear evolution equation

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Abstract: Based on the Hirota bilinear form of a (2+1)-dimensional equation, breathers and resonant multiple waves as well as complexiton solutions are considered in this paper. First, the breather waves are constructed via employing the extend homoclinic test method. By calculation, two kinds of solutions are obtained. Through analysis, three pairs of breathers consisting of hyperbolic functions and trigonometric functions are derived. Furthermore, a rouge wave solution is deduced by applying the Taylor expansion method to a obtained breather wave. In addition, related figures are plotted to illustrate the dynamical features of these obtained solutions. Then, two types of the resonant multi-soliton solutions are obtained by applying the linear superposition principle to the the Hirota bilinear form. At the same time, 3D profiles and 2D density plots are presented to depict the intersection progression of wave motion. Finally, the complexiton solutions are constructed according to the yielded resonant multi-soliton solutions by further utilizing the linear superposition principle. By considering different domain fields, several types of complexiton solutions including the positive ones are derived. Moreover, related 3D and 2D figures are plotted for the obtained results in order to vividly exhibit their dynamics properties.

Keywords: exact solutions; breather solutions; resonant multiple wave solutions; linear superposition principle; complexiton solutions
Mathematics Subject Classification: 35A25, 35G50, 35Q35, 37K10

1. Introduction

As the exact solutions of nonlinear partial differential equations are beneficial for researchers to better understand the physical phenomena described by the equation, it has been one of the hotspots in the nonlinear field. With the aid of symbolic computation, many methods for searching exact solutions are proposed, such as the Painlevé analysis method [1], the multiple exp-function method [2, 3], three waves method [4, 5] etc. By employing these methods, we can obtain various forms of accurate solutions, such as traveling wave solutions [6], bright wave solutions [7], breathers [8–10], lump solutions [11–13], resonant solutions [14, 15], complexiton solutions [16] and other types of soliton solutions [17, 18]. By analyzing these solutions, researchers can well explain the physical phenomena on different dynamic characteristics represented by the original equation [19–21].

Among many types of exact solutions, breathers and resonant multi-soliton solutions have been receiving growing attention. As early as in 2009, the homoclinic test method and extended homoclinic test method were proposed by Professor Dai to derive breather solutions for high dimensional nonlinear equation in [22]. Then the homoclinic breather limit method was proposed by Xu for searching rogue wave solution to nonlinear evolution equation in [23]. Since then, many results on breathers and rouge waves have been obtained [24, 25]. In 2011, Ma [26] introduced the linear superposition principle to construct the resonant multi-soliton solutions of nonlinear equations. Afterwards the linear superposition principle has been widely used to establish the resonant multiple wave solutions [27–29]. Many results revealed that the linear superposition principle is much more effective in constructing resonant multiple wave solutions for nonlinear evolution equations.

Complexiton solution is another kind of special exact solution which is composed of exponential function and trigonometric function. By using Wronskian techniques, the complexiton solutions of KdV equation have been achieved for the first time [30]. By using the extended transformation rational function method, complexiton solutions of several nonlinear differential equations have been derived in Ref. [31]. The (3+1) dimensional Boiti-Leon-Manna-Pempinell has been investigated in [32] by utilizing the linear superposition principle. Up to now, linear superposition principle and Hirota bilinear method are the main ways for constructing complexiton solutions.

Recently, a (2 + 1)-dimensional nonlinear evolution equation has been considered, which was written by [33]

$$-4u_t + 3\int_{-\infty}^{x} u_{yy}\mathrm{d}x + u_{xxx} + 3(u^2)_x = 0. \tag{1.1}$$

If replacing u in Eq (1.1) by u_x, Eq (1.1) can be reduced to a (2+1)-dimensional equation, which is similar to the (2+1)-dimensional Boussinesq equation in Ref. [34]. So Eq (1.1) may be used to describe the motion of shallow water waves. In Ref. [33], several types of soliton solutions and singular solutions have been achieved by employing Hirota direct method, such as one soliton solution, two soliton solution, one singular and two singular solution. While breathers and resonant multiple waves as well as complexiton solutions of Eq (1.1) haven't been investigated yet.

In this paper, our main purpose is to investigate breather waves, resonant multiple waves and complexiton solutions for Eq (1.1). The structure of this paper is organized as follows. In Section 2, the breather wave solutions of Eq (1.1) are derived by employing the extend homoclinic test method and a rouge wave solution is deduce by using Taylor expansion method. Their dynamical behaviors are depicted in some figures. In Section 3, the resonant multiple waves of Eq (1.1) are presented by using linear superposition principle. Their dynamical behaviors are also shown in some three-dimensional plots and corresponding density plots. In Section 4, the complexiton solutions of Eq (1.1) are derived by using linear superposition principle and Hirota bilinear method based on the results in Section3. Their dynamical behaviors are depicted in two figures. Finally, some remarks are given.

2. Breather wave aolutions of the (2 + 1)-dimensional Eq (1.1)

First of all, we aim to construct the breather solutions of the Eq (1.1). It is easy to verify that there exists an equilibrium solution u_0 for Eq (1.1), so we suppose

$$u = u_0 + 2(\ln\varphi)_{xx}, \varphi = \varphi(x, y, t). \tag{2.1}$$

Substituting Eq (2.1) into Eq (1.1), then Eq (1.1) can be transformed into the following form

$$-8(\ln\varphi)_{xxt} + 6(\ln\varphi)_{xyy} + 2(\ln\varphi)_{xxxxx} + 12\left((\ln\varphi)_{xx}^2\right)_x + 12u_0(\ln\varphi)_{xxx} = 0. \tag{2.2}$$

Integrating with respect x once, then one obtains

$$(-4D_xD_t + 3D_y^2 + D_x^4 + 6u_0D_x^2)\varphi\cdot\varphi = 0. \tag{2.3}$$

In order to obtain breather waves of Eq (1.1), according to the idea of the extend homoclinic test method, we search for the solution of Eq (2.3) as the following form

$$\varphi = \exp(-\lambda(x + a_1y - \alpha t)) + \mu_1\cos(\lambda(x + a_2y - \beta t)) + \mu_2\exp(\lambda(x + a_1y - \alpha t)), \tag{2.4}$$

in which $\lambda, \alpha, \beta, a_1, a_2, \mu_1, \mu_2$ are undetermined real constants. Inserting Eq (2.4) into Eq (2.3) leads to an algebraic equation and equating each coefficient for the powers of $\exp(\pm\lambda(x + a_1y - \alpha t)), \sin(\lambda(x + a_2y - \beta t)), \cos(\lambda(x + a_2y - \beta t))$ to be zero, some algebraic equations can be obtained

$$\begin{cases} (4\lambda^2\mu_1^2 - 3a_2^2\mu_1^2 - 4\mu_1^2\beta + 16\lambda^2\mu_2 - 6u_0\mu_1^2 + 12a_1^2\mu_2 + 16\mu_2\alpha + 24u_0\mu_2)\lambda^2 = 0, \\ (4\lambda^2 - 3a_1^2 + 3a_2^2 - 4\alpha + 4\beta)\lambda^2\mu_1\mu_2 = 0, \\ (4\lambda^2 - 3a_1^2 + 3a_2^2 - 4\alpha + 4\beta)\lambda^2\mu_1 = 0, \\ (3a_1a_2 + 2\alpha + 2\beta + 6u_0)\lambda^2\mu_1\mu_2 = 0, \\ (3a_1a_2 + 2\alpha + 2\beta + 6u_0)\lambda^2\mu_1 = 0. \end{cases} \tag{2.5}$$

Solving the obtained Eq (2.5), two cases are obtained.

Case 1:

$$\alpha = -\frac{3}{4}a_2^2 + \frac{1}{2}\lambda^2 - \frac{3}{2}u_0, \beta = -\frac{3}{4}a_2^2 - \frac{1}{2}\lambda^2 - \frac{3}{2}u_0, \mu_1^2 = -4\mu_2, \tag{2.6}$$

in which u_0, a_1, a_2, λ are arbitrary real numbers while μ_2 is negative.

Case 2:

$$\begin{aligned} &\alpha = -\frac{3}{8}a_1^2 - \frac{3}{4}a_1a_2 + \frac{3}{8}a_2^2 + \frac{1}{2}\lambda^2 - \frac{3}{2}u_0, \beta = \frac{3}{8}a_1^2 - \frac{3}{4}a_1a_2 - \frac{3}{8}a_2^2 - \frac{1}{2}\lambda^2 - \frac{3}{2}u_0, \\ &\mu_1^2 = \frac{4\mu_2(a_1^2 - 2a_1a_2 + a_2^2 + 4\lambda^2)}{(a_1^2 - 2a_1a_2 + a_2^2 - 4\lambda^2)}, \end{aligned} \tag{2.7}$$

in which $\lambda, u_0, a_1, a_2, \mu_2$ are arbitrary real numbers.

For the first case, owing to $\mu_2 < 0$, the expression (2.4) can be written as

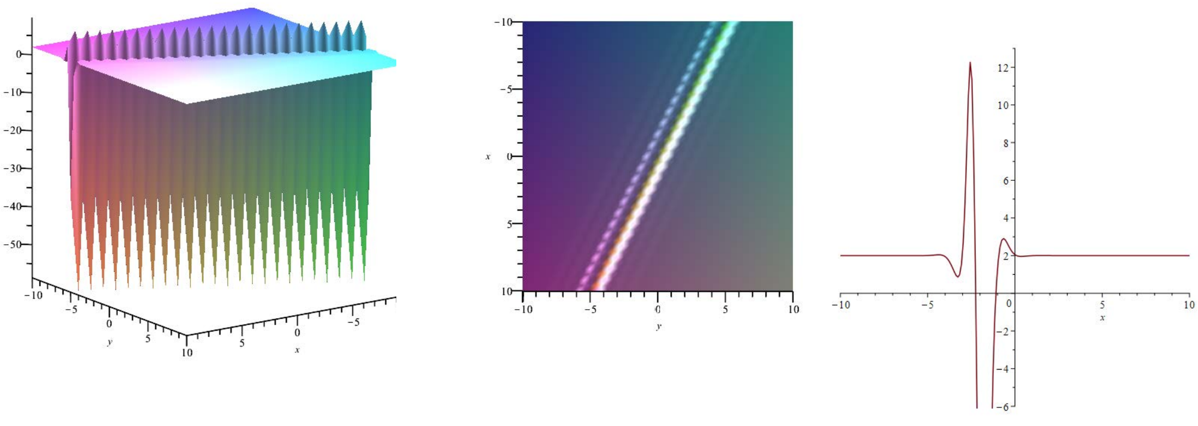
$$
\begin{aligned}
\varphi &= \exp(-\lambda(x + a_1 y + (\frac{3}{4}a_2^2 - \frac{1}{2}\lambda^2 + \frac{3}{2}u_0)t)) \\
&\pm 2\sqrt{-\mu_2}\cos(\lambda(x + a_2 y + (\frac{3}{4}a_2^2 + \frac{1}{2}\lambda^2 + \frac{3}{2}u_0)t)) \\
&+ \mu_2 \exp(\lambda(x + a_1 y + (\frac{3}{4}a_2^2 - \frac{1}{2}\lambda^2 + \frac{3}{2}u_0)t)) \\
&= -2\sqrt{-\mu_2}\sinh\left[\lambda(x + a_1 y + (\frac{3}{4}a_2^2 - \frac{1}{2}\lambda^2 + \frac{3}{2}u_0)t) + \ln\sqrt{-\mu_2}\right] \\
&\pm 2\sqrt{-\mu_2}\cos(\lambda(x + a_2 y + (\frac{3}{4}a_2^2 + \frac{1}{2}\lambda^2 + \frac{3}{2}u_0)t)).
\end{aligned}
\tag{2.8}
$$

Next substituting the obtained results (2.8) into Eq (2.1) leads to the homoclinic breather waves of Eq (1.1) as follows

$$u_1 = u_0 - \frac{4\sqrt{-\mu_2}\lambda^2(\cosh(\Delta_1)\sin(\Delta_2) + 1)}{(\sinh(\Delta_1) - \cos(\Delta_2))^2}, \tag{2.9}$$

$$u_2 = u_0 + \frac{4\sqrt{-\mu_2}\lambda^2(\cosh(\Delta_1)\sin(\Delta_2) - 1)}{(\sinh(\Delta_1) + \cos(\Delta_2))^2}, \tag{2.10}$$

where $\Delta_1 = \lambda(x + a_1 y + (\frac{3}{4}a_2^2 - \frac{1}{2}\lambda^2 + \frac{3}{2}u_0)t) + \ln\sqrt{-\mu_2}, \Delta_2 = \lambda(x + a_2 y + (\frac{3}{4}a_2^2 + \frac{1}{2}\lambda^2 + \frac{3}{2}u_0)t)$. Here u_1, u_2 are two homoclinic breather wave solutions of Eq (1.1). Their expressions are much similar and their properties are also similar. There is a possibility that their denominator is close to zero. So their absolute values may be large. We choose u_1 for an example. The breather solution (2.9) is vividly shown in Figure 1.



(a) (b) (c)

Figure 1. (Color online) Profiles of breather wave (2.9) with the following parameters: $\lambda = 3, u_0 = 2, a_1 = 2, a_2 = 2, \mu_2 = -1$ at $t = 0$. (a) Perspective view of the wave. (b) Overhead view of the wave. (c) The wave propagation pattern along the x axis with $y = 1$.

For the second case, when $\mu_2 > 0$, the expression (2.4) can be written as

$$
\begin{aligned}
\varphi &= \exp(-\lambda(x + a_1 y + (\frac{3}{8}a_1^2 + \frac{3}{4}a_1a_2 - \frac{3}{8}a_2^2 - \frac{1}{2}\lambda^2 + \frac{3}{2}u_0)t))\pm \\
&\sqrt{\frac{4\mu_2(a_1^2 - 2a_1a_2 + a_2^2 + 4\lambda^2)}{(a_1^2 - 2a_1a_2 + a_2^2 - 4\lambda^2)}}\cos(\lambda(x + a_2y - (\frac{3}{8}a_1^2 - \frac{3}{4}a_1a_2 - \frac{3}{8}a_2^2 - \frac{1}{2}\lambda^2 - \frac{3}{2}u_0)t)) \\
&+ \mu_2\exp(\lambda(x + a_1y + (\frac{3}{8}a_1^2 + \frac{3}{4}a_1a_2 - \frac{3}{8}a_2^2 - \frac{1}{2}\lambda^2 + \frac{3}{2}u_0)t)) \\
&= 2\sqrt{\mu_2}\cosh\left[\lambda(x + a_1y + (\frac{3}{8}a_1^2 + \frac{3}{4}a_1a_2 - \frac{3}{8}a_2^2 - \frac{1}{2}\lambda^2 + \frac{3}{2}u_0)t) + \ln\sqrt{\mu_2}\right]\pm \\
&\sqrt{\frac{4\mu_2(a_1^2 - 2a_1a_2 + a_2^2 + 4\lambda^2)}{(a_1^2 - 2a_1a_2 + a_2^2 - 4\lambda^2)}}\cos(\lambda(x + a_2y - (\frac{3}{8}a_1^2 - \frac{3}{4}a_1a_2 - \frac{3}{8}a_2^2 - \frac{1}{2}\lambda^2 - \frac{3}{2}u_0)t)).
\end{aligned}
\tag{2.11}
$$

Then substituting the obtained results (2.11) into Eq (2.1) yields the homoclinic waves of Eq (1.1) as follows

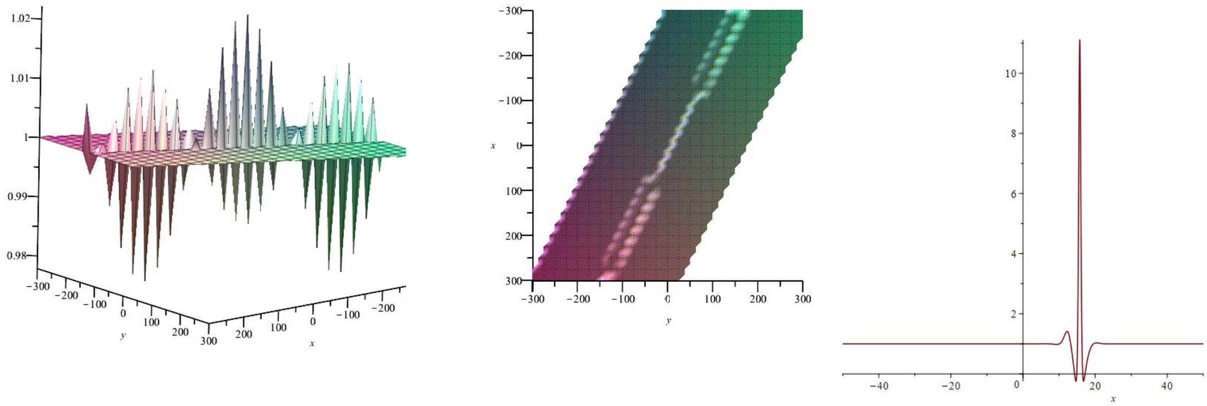
$$
u_3 = u_0 + \frac{2\lambda^2(4\mu_2 - P^2 + 4\sqrt{\mu_2}P\sinh(\Phi_1)\sin(\Phi_2))}{(2\sqrt{\mu_2}\cosh(\Phi_1) + P\cos(\Phi_2))^2}, \tag{2.12}
$$

$$
u_4 = u_0 + \frac{2\lambda^2(4\mu_2 - P^2 - 4\sqrt{\mu_2}P\sinh(\Phi_1)\sin(\Phi_2))}{(2\sqrt{\mu_2}\cosh(\Phi_1) - P\cos(\Phi_2))^2}, \tag{2.13}
$$

in which

$$
\begin{aligned}
P &= \sqrt{\frac{4\mu_2(a_1^2 - 2a_1a_2 + a_2^2 + 4\lambda^2)}{(a_1^2 - 2a_1a_2 + a_2^2 - 4\lambda^2)}}, \\
\Phi_1 &= \lambda(x + a_1y + (\frac{3}{8}a_1^2 + \frac{3}{4}a_1a_2 - \frac{3}{8}a_2^2 - \frac{1}{2}\lambda^2 + \frac{3}{2}u_0)t) + \ln\sqrt{\mu_2}, \\
\Phi_2 &= \lambda(x + a_2y - (\frac{3}{8}a_1^2 - \frac{3}{4}a_1a_2 - \frac{3}{8}a_2^2 - \frac{1}{2}\lambda^2 - \frac{3}{2}u_0)t).
\end{aligned}
\tag{2.14}
$$

Here u_3, u_4 are two homoclinic breather wave solutions, and when $t \to \pm\infty$, they tend to a fixed point u_0. In the following, Figure 2 presents the breather solutions (2.12).



(a) (b) (c)

Figure 2. (Color online) Breather wave (2.12) for Eq (1.1) with the following selected parameters: $\lambda = 1, u_0 = 1, a_1 = 2, a_2 = -4, \mu_2 = 1$ at $t = 2$. (a) Perspective view. (b) The overhead view . (c) The wave propagation pattern of the wave along the x axis with $y = 2$.

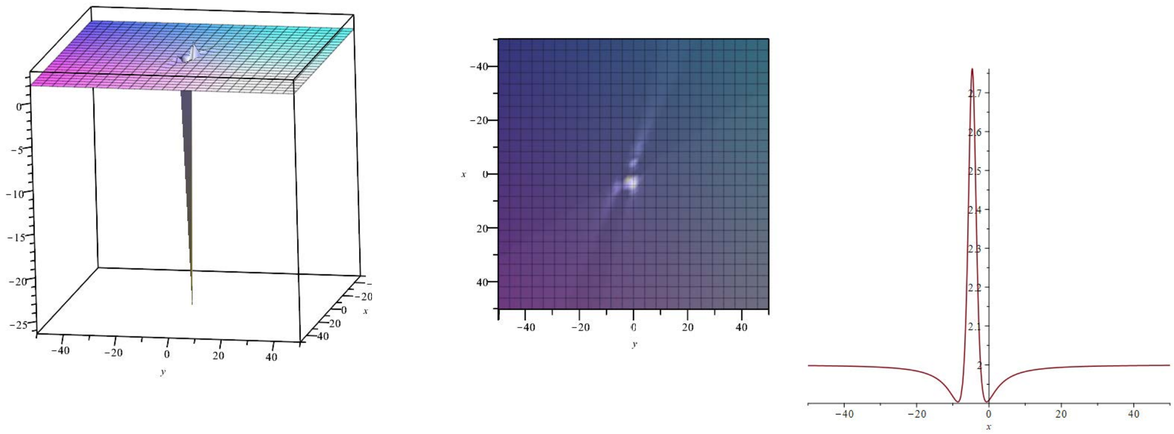
Equation (2.12) is similar to Eq (2.13). Since u_3 and u_4 are almost the same, in the rest of this section, only u_4 will be taken into consideration. In consideration of Eq (2.12), taking $\delta_2 = 1$, along with the following Taylor expansions at $\lambda = 0$.

$$\begin{aligned}
\sinh(\Phi_1) &= \lambda(x + a_1 y + (\frac{3}{8}a_1^2 + \frac{3}{4}a_1 a_2 - \frac{3}{8}a_2^2 - \frac{1}{2}\lambda^2 + \frac{3}{2}u_0)t) + O(\lambda^2),\\
\sinh(\Phi_2) &= \lambda(x + a_2 y - (\frac{3}{8}a_1^2 - \frac{3}{4}a_1 a_2 - \frac{3}{8}a_2^2 - \frac{1}{2}\lambda^2 - \frac{3}{2}u_0)t) + O(\lambda^2),\\
\cosh(\Phi_1) &= 1 + \frac{1}{2}\lambda^2(x + a_1 y + (\frac{3}{8}a_1^2 + \frac{3}{4}a_1 a_2 - \frac{3}{8}a_2^2 - \frac{1}{2}\lambda^2 + \frac{3}{2}u_0)t)^2 + O(\lambda^3),\\
\cos(\Phi_2) &= 1 - \frac{1}{2}\lambda^2(x + a_2 y - (\frac{3}{8}a_1^2 - \frac{3}{4}a_1 a_2 - \frac{3}{8}a_2^2 - \frac{1}{2}\lambda^2 - \frac{3}{2}u_0)t)^2 + O(\lambda^3).
\end{aligned} \tag{2.15}$$

We obtain rogue wave solutions of Eq (1.1)

$$u_5 = u_0 - \frac{16\left[4B + (x + a_1 y - \alpha t)(x + a_2 y - \beta t)\right]}{\left((x + a_1 y - \alpha t)^2 + (x + a_2 y - \beta t)^2) - 8B\right)^2}. \tag{2.16}$$

Where $B = \frac{1}{(a_1^2 - 2a_1 a_2 + a_2^2)}$. Relevant dynamical behaviors are shown in Figure 3.



(a) (b) (c)

Figure 3. Rouge wave (2.16) for Eq (1.1) with the following selected parameters: $u_0 = 2, a_1 = 3, a_2 = 1, \alpha = -\frac{33}{4}, \beta = -\frac{9}{4}$ at $t = \frac{1}{2}$. (a) Perspective view of the real part of the wave. (b) The overhead view of the wave. (c) The wave propagation pattern of the wave along the x axis with $y = 1$.

For the second case, when $\mu_2 < 0$, the expression (2.4) can be written as

$$\begin{aligned}
\varphi &= \exp(-\lambda(x + a_1 y + (\frac{3}{8}a_1^2 + \frac{3}{4}a_1 a_2 - \frac{3}{8}a_2^2 - \frac{1}{2}\lambda^2 + \frac{3}{2}u_0)t))\pm \\
&\sqrt{\frac{4\mu_2(a_1^2 - 2a_1 a_2 + a_2^2 + 4\lambda^2)}{(a_1^2 - 2a_1 a_2 + a_2^2 - 4\lambda^2)}}\cos(\lambda(x + a_2 y - (\frac{3}{8}a_1^2 - \frac{3}{4}a_1 a_2 - \frac{3}{8}a_2^2 - \frac{1}{2}\lambda^2 - \frac{3}{2}u_0)t)) \\
&+ \mu_2\exp(\lambda(x + a_1 y + (\frac{3}{8}a_1^2 + \frac{3}{4}a_1 a_2 - \frac{3}{8}a_2^2 - \frac{1}{2}\lambda^2 + \frac{3}{2}u_0)t)) \\
&= -2\sqrt{-\mu_2}\sinh\left[\lambda(x + a_1 y + (\frac{3}{8}a_1^2 + \frac{3}{4}a_1 a_2 - \frac{3}{8}a_2^2 - \frac{1}{2}\lambda^2 + \frac{3}{2}u_0)t) + \ln\sqrt{-\mu_2}\right]\pm \\
&\sqrt{\frac{4\mu_2(a_1^2 - 2a_1 a_2 + a_2^2 + 4\lambda^2)}{(a_1^2 - 2a_1 a_2 + a_2^2 - 4\lambda^2)}}\cos(\lambda(x + a_2 y - (\frac{3}{8}a_1^2 - \frac{3}{4}a_1 a_2 - \frac{3}{8}a_2^2 - \frac{1}{2}\lambda^2 - \frac{3}{2}u_0)t)).
\end{aligned} \tag{2.17}$$

Then inserting the obtained results (2.17) into Eq (2.1) yields the homoclinic waves of Eq (1.1) as follows

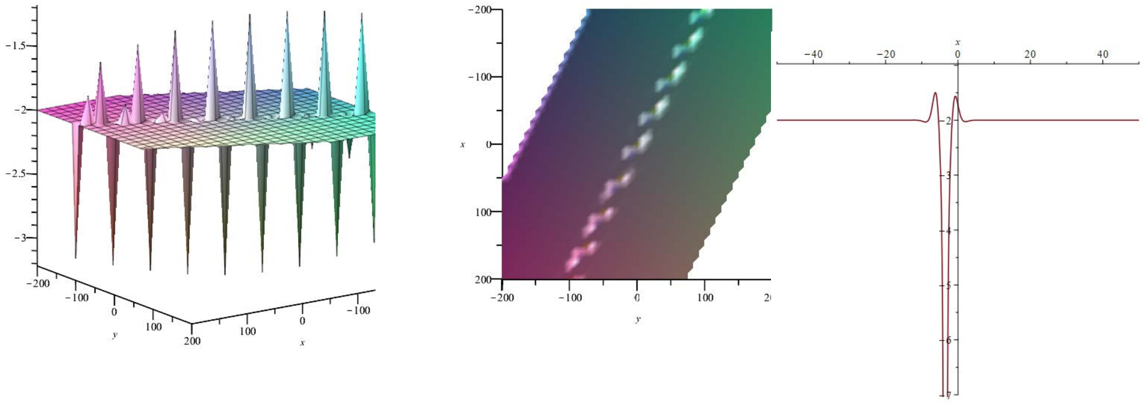
$$u_6 = u_0 + \frac{2\lambda^2(-4\mu_2 + Q^2 + 4\sqrt{-\mu_2}Q\cosh(\Theta_1)\sin(\Theta_2))}{(2\sqrt{-\mu_2}\sinh(\Theta_1) - Q\cos(\Theta_2))^2}, \tag{2.18}$$

$$u_7 = u_0 + \frac{2\lambda^2(4\mu_2 - Q^2 + 4\sqrt{-\mu_2}Q\cosh(\Theta_1)\sin(\Theta_2))}{(2\sqrt{-\mu_2}\sinh(\Theta_1) + Q\cos(\Theta_2))^2}, \tag{2.19}$$

in which

$$Q = \sqrt{\frac{4\mu_2(a_1^2 - 2a_1a_2 + a_2^2 + 4\lambda^2)}{(a_1^2 - 2a_1a_2 + a_2^2 - 4\lambda^2)}},$$
$$\Theta_1 = \lambda(x + a_1y + (\frac{3}{8}a_1^2 + \frac{3}{4}a_1a_2 - \frac{3}{8}a_2^2 - \frac{1}{2}\lambda^2 + \frac{3}{2}u_0)t) + \ln\sqrt{-\mu_2}, \tag{2.20}$$
$$\Theta_2 = \lambda(x + a_2y - (\frac{3}{8}a_1^2 - \frac{3}{4}a_1a_2 - \frac{3}{8}a_2^2 - \frac{1}{2}\lambda^2 - \frac{3}{2}u_0)t).$$

Here u_6, u_7 are two homoclinic breather wave solutions. Obviously, when $t \to \pm\infty$, they tend to a fixed point u_0. In the following, Figure 4 presents the breather solutions (2.18) .



(a) (b) (c)

Figure 4. (Color online) Profiles of breather wave (2.18) for Eq (1.1) with the following selected parameters: $\lambda = 1, u_0 = -2, a_1 = 2, a_2 = 1, \alpha = \frac{7}{8}, \beta = \frac{17}{8}, \mu_2 = -\frac{1}{4}$ at $t = 2$. (a) Perspective view. (b) Overhead view. (c) The propagation pattern of the wave along the x axis with $y = 1$.

3. Resonant multiple wave solutions of Eq (1.1)

In order to obtain the resonant multiple wave solutions of the (2+1)-dimensional Eq (1.1), taking $u_0 = 0$ in (2.3), then the bilinear form of (1.1) can be written as

$$(-4D_xD_t + 3D_y^2 + D_x^4)\varphi \cdot \varphi = 0. \tag{3.1}$$

By introducing the following N-wave variables

$$\varphi_i = h_ix + k_iy + l_it + \eta_i, 1 \le i \le n, \tag{3.2}$$

where h_i, k_i, l_i are real constants to be determined, η_i is an arbitrary constant. According to the linear superposition principle, the following results can be derived

$$-4(h_i - h_j)(l_i - l_j) + 3(k_i - k_j)^2 + (h_i - h_j)^4 = 0. \tag{3.3}$$

We can assume that

$$h_i = h_i, k_i = ah_i^2, l_i = bh_i^3, \tag{3.4}$$

where a, b are arbitrary real constants. By substituting Eq (3.4) into Eq (3.3), collecting all the coefficients of polynomials on h_i and h_j, then equating each term to be zero, therefore the parameters a, b satisfy

$$\begin{cases} 4b - 4 = 0, \\ -6a^2 + 6 = 0, \\ 3a^2 - 4b + 1 = 0. \end{cases} \tag{3.5}$$

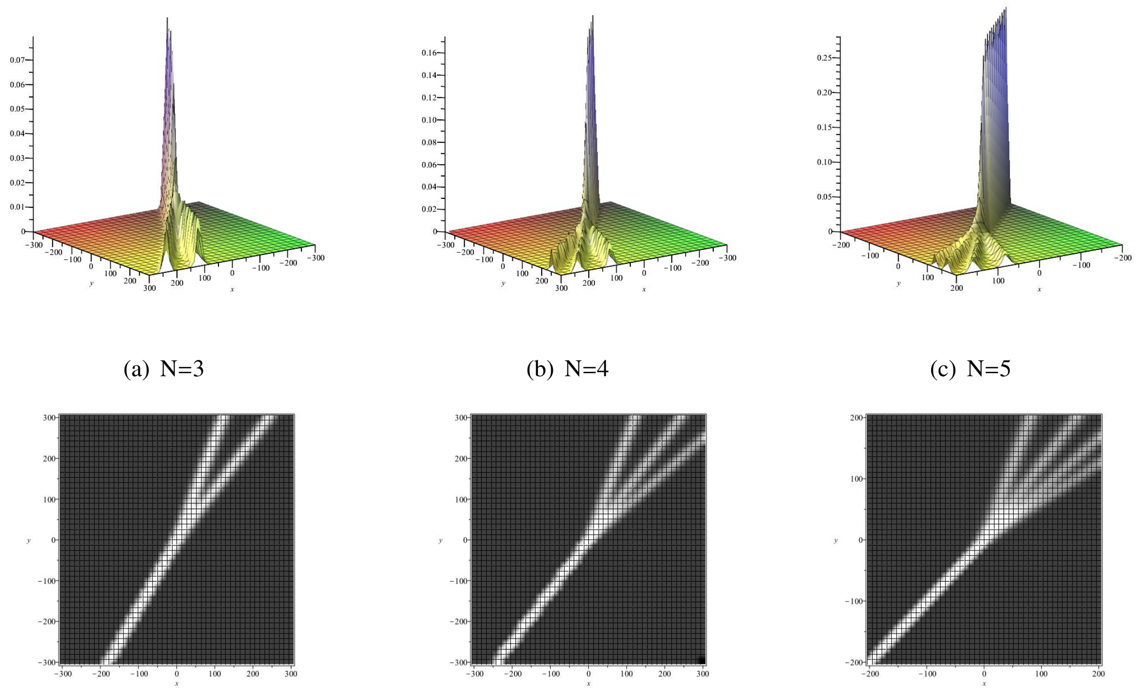
By solving Eq (3.5), we get

$$a = \pm 1, b = 1. \tag{3.6}$$

Therefore, the multiple resonant wave solutions of Eq (1.1) can be written as the following the expression (3.2)

$$\varphi = \sum_{i=1}^{N} \lambda_i e^{h_i x \pm h_i^2 y + h_i^3 t + \eta_i}, u = 2(\ln \varphi)_{xx}, \tag{3.7}$$

where h_i and $\lambda_i (1 < i < N, i \in N^+)$ are arbitrary real constants. Profiles and density plots of (3.7) are shown in Figure 5.



(a) N=3 (b) N=4 (c) N=5

(d) N=3 (e) N=4 (f) N=5

Figure 5. (Color online) 3D profiles (a)–(c) and 2D density plots (d)–(f) of resonant three-, four-, five-wave solutions for (3.7) at $t = 1$ with $\lambda_i = 1 (i = 1 \leq i \leq 5), a = -1, b = 1, h_1 = 0.1, h_2 = 0.3$. (a) and (d) $N = 3, k_3 = 0.5$; (b) and (e) $N = 4, h_4 = 0.7$; (c) and (f) $N = 5, h_5 = 0.9$.

By observation and analysis, we can conclude that for any positive integer $N > 1$ in Eq (3.7), $N - 1$ cross marks will be appeared in their density plots.

4. Complexiton solutions of Eq (1.1)

In what follows, we aim to construct the complexiton solutions of Eq (1.1) under the real field and the complex field. Assume N_1, N_2 are natural numbers, $N = N_1 + N_2, \varphi_i = h_i x - h_i^2 y + h_i^3 t$, where $h_i (1 \le i \le N_1)$ are real numbers. When $1 \le i \le N_1$, the following expression

$$\varphi = \sum_{i=1}^{N_1} \lambda_i e^{h_i x - h_i^2 y + h_i^3 t} \tag{4.1}$$

is a linear superposition solution for the bilinear equation (3.1). For $N_1 + 1 \le i \le N$, taking $h_i = p_i + I q_i$, where $I = \sqrt{-1}$ and p_i, q_i are real numbers, then

$$\begin{aligned} \varphi_i &= h_i x - h_i^2 y + h_i^3 t \\ &= p_i x - (p_i^2 - q_i^2) y + (p_i^2 - 3 p_i q_i^2 t) + \eta_i + I[q_i x + 2 p_i q_i y + (3 p_i^2 q_i - q_i^3) t] \\ &= \varphi_{i,1} + I \varphi_{i,2}, \end{aligned} \tag{4.2}$$

where $\varphi_{i,1} = p_i x - (p_i^2 - q_i^2) y + (p_i^2 - 3 p_i q_i^2 t), \varphi_{i,2} = q_i x + 2 p_i q_i y + (3 p_i^2 q_i - q_i^3) t$. Obviously, the conjugate function of φ_i is given by

$$\overline{\varphi_i} = \varphi_{i,1} - I \varphi_{i,2}. \tag{4.3}$$

It can be verified that both e^{φ_i} and $e^{\overline{\varphi_i}}$ are solutions of Eq (3.1). By using the linear superposition principle, the following expression

$$\sum_{i=N_1}^{N} (\delta_i e^{\varphi_i} + \widetilde{\delta_i} e^{\overline{\varphi_i}}) = \sum_{i=N_1}^{N} e^{\varphi_{i,1}} \left[(\delta_i + \widetilde{\delta_i}) \cos(\varphi_{i,2}) + I(\delta_i - \widetilde{\delta_i}) \sin(\varphi_{i,2}) \right] \tag{4.4}$$

is also a solution of Eq (3.1), where $\delta_i, \widetilde{\delta_i}$ are complex numbers.

By employing the linear superposition principle again, the complex value solution of (2+1)-dimensional Eq (1.1) can be obtained

$$u = 2(\ln \varphi)_{xx}, \tag{4.5}$$

where

$$\begin{aligned} \varphi &= \sum_{i=1}^{N_1} \lambda_i e^{\varphi_i} + \sum_{i=N_1}^{N} (\delta_i e^{\varphi_i} + \widetilde{\delta_i} e^{\overline{\varphi_i}}) \\ &= \sum_{i=1}^{N_1} \lambda_i e^{\varphi_i} + \sum_{i=N_1}^{N} e^{\varphi_{i,1}} \left[(\delta_i + \widetilde{\delta_i}) \cos(\varphi_{i,2}) + I(\delta_i - \widetilde{\delta_i}) \sin(\varphi_{i,2}) \right], \end{aligned} \tag{4.6}$$

in which $\delta_i, \widetilde{\delta_i}$ are complex numbers while $\varphi_{i,1}, \varphi_{i,2}$ are determined by Eq (4.2).

In order to obtain the real complexiton solutions of Eq (1.1), taking

$$\delta_i = \frac{\delta_{i,1} - I \delta_{i,2}}{2}, \widetilde{\delta_i} = \frac{\delta_{i,1} + I \delta_{i,2}}{2}, \tag{4.7}$$

Substituting Eq (4.7) along with Eq (4.6) into Eq (4.4), one real complexiton solutions of Eq (1.1) can be written as

$$u = 2(\ln\varphi)_{xx} \tag{4.8}$$

with

$$\varphi = \sum_{i=1}^{N_1} \lambda_i e^{\varphi_i} + \sum_{i=N_1}^{N} e^{\varphi_{i,1}} \left[\delta_{i,1}\cos(\varphi_{i,2}) + \delta_{i,2}\sin(\varphi_{i,2})\right], \tag{4.9}$$

in which $\lambda_i, \delta_{i,1}, \delta_{i,2}$ are real numbers,$\varphi_{i,1}, \varphi_{i,2}$ are determined by Eq (4.2).

According to the obtained results in Eq (3.6), if $e^{\varphi_i} = e^{h_i x - h_i^2 y + h_i^3 t}$ is one solution of Eq (3.1) for $1 \le i \le N_1$, it can be verified that $e^{-\varphi_i} = e^{-h_i x + h_i^2 y - h_i^3 t} = e^{(-h_i)x + (-h_i)^2 y + (-h_i)^3 t}$ is also one solution of Eq (3.1). By employing the linear superposition principle, $\frac{e^{\varphi_i}+e^{-\varphi_i}}{2}$ and $\frac{e^{\varphi_i}-e^{-\varphi_i}}{2}$ are also solutions of Eq (3.1), so the real complexiton solutions of Eq (1.1) can also be written as the following expressions:

$$u = 2(\ln\varphi)_{xx} \tag{4.10}$$

with

$$\varphi = \sum_{i=1}^{N_1} \lambda_i \cosh(\varphi_i) + \sum_{i=N_1}^{N} e^{\varphi_{i,1}} \left[\delta_{i,1}\cos(\varphi_{i,2}) + \delta_{i,2}\sin(\varphi_{i,2})\right] \tag{4.11}$$

or

$$\varphi = \sum_{i=1}^{N_1} \lambda_i \sinh(\varphi_i) + \sum_{i=N_1}^{N} e^{\varphi_{i,1}} \left[\delta_{i,1}\cos(\varphi_{i,2}) + \delta_{i,2}\sin(\varphi_{i,2})\right]. \tag{4.12}$$

In general, the complexiton solution Eq (4.10) is singular. If $e^{\varphi_i} = e^{(Ih_i)x + (Ih_i)^2 y + (Ih_i)^3 t} = e^{I(h_i x - h_i^3 t) - h_i^2 y} = e^{I(h_i x - h_i^3 t) - h_i^2 y}$ is one solution of Eq (3.1) for $N_1 \le i \le N$, it can be verified that $e^{\overline{\varphi_i}} = e^{-I(h_i x - h_i^3 t) - h_i^2 y} = e^{(-Ih_i)x + (-Ih_i)^2 y + (-Ih_i)^3 t}$ is also one solution of Eq (3.1). By utilizing the linear superposition principle, $\frac{e^{\varphi_i}+e^{\overline{\varphi_i}}}{2} = e^{-h_i^2 y}\cos(h_i x - h_i^3 t)$ is also a solution of Eq (3.1), which can be written as

$$u = 2(\ln\varphi)_{xx}, \tag{4.13}$$

where

$$\varphi = \sum_{i=1}^{N_1} \lambda_i \cosh(\varphi_i) + \sum_{i=N_1}^{N} e^{-h_i^2 y}\cos(h_i x - h_i^3 t). \tag{4.14}$$

If coefficients $\lambda_i (1 \le i \le N_1)$ are positive real constants and satisfy the following condition:

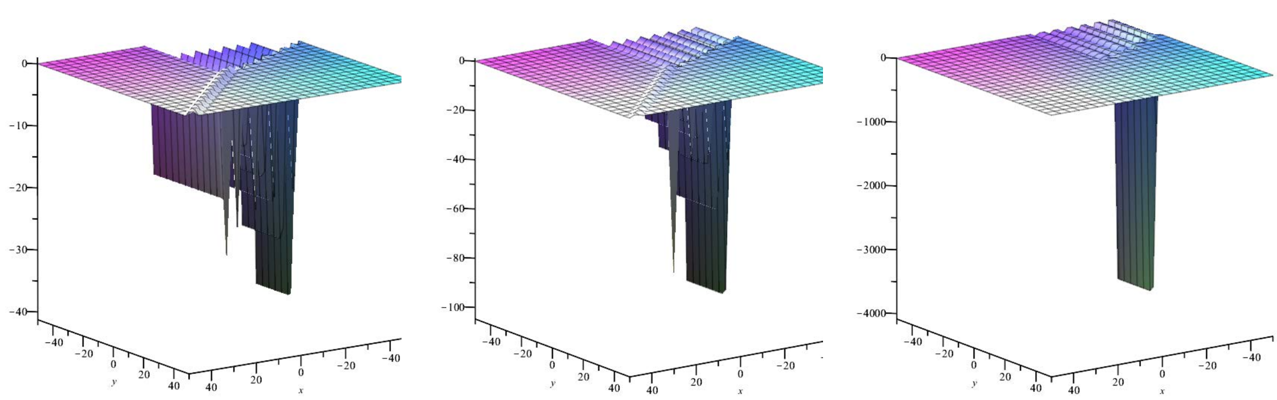
$$\sum_{i=1}^{N_1} \lambda_i \cosh(\varphi_i) > \sum_{i=N_1}^{N} \lambda_i e^{\varphi_{i,1}} \delta_{i,1}, \tag{4.15}$$

then φ will always be greater than zero. By transformation $u = 2(\ln\varphi)_{xx}$, the positive complexiton solution of (2+1) dimensional equation (1.1) can be obtained, where φ is determined by Eq (4.14).

Taking $N = 2, N_1 = 1, h_1 = 1, h_2 = -1, \lambda_1 = 4, \lambda_2 = 1$ in Eq (4.14), the positive complexiton solution can be written as

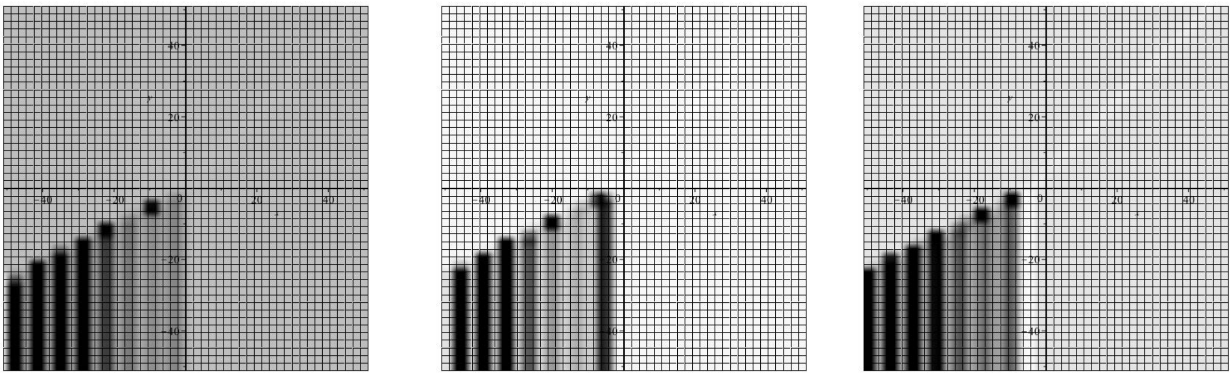
$$u = \frac{8\cosh(t + x - y) - 2\,\mathrm{e}^{-y}\cos(-x + t)}{4\cosh(t + x - y) + \mathrm{e}^{-y}\cos(-x + t)} - \frac{2\left(4\sinh(t + x - y) + \mathrm{e}^{-y}\sin(-x + t)\right)^2}{\left(4\cosh(t + x - y) + \mathrm{e}^{-y}\cos(-x + t)\right)^2}. \tag{4.16}$$

Profiles and density plots of (4.16) are shown in Figures 6 and 7 respectively.



(a) t=1 (b) t=3 (c) t=5

Figure 6. (Color online) Evolution for complexiton solutions Eq (4.16) of (2+1) dimensional equation Eq (1.1) at different times.



(a) t=1 (b) t=3 (c) t=5

Figure 7. (Color online) Density plots for complexiton solutions Eq (4.16) of (2+1) dimensional equation Eq (1.1) at different times.

By observation, the shape of completion solution Eq (4.16) seems to change little with time in Figure 6, but their changes are obvious in the density plots.

5. Conclusions

In summary, a (2+1)-dimensional equation Eq (1.1) has been systematically investigated in this paper and three types of solutions were obtained. First, the breather solutions were constructed by using the extend homoclinic test method and symbolic calculation. By analysis, three pairs of breathers consisting of hyperbolic functions and trigonometric functions were derived. Meanwhile, homoclinic limit method and the Taylor expansion method were employed to derive a rogue wave solution of Eq (1.1). Their dynamical behaviors were depicted in four figures. Second, the resonant multiple solutions of Eq (1.1) were derived by using linear superposition principle, which didn't depend on the dispersion relation. In addition, 3D profiles and 2D density plots were shown in order to depict their dynamical properties. Third, the complexiton solutions were constructed under the real number field and the complex number field respectively by using linear superposition principle many times. As a result, several types of complexiton solutions were derived. Among these solutions, a positive complexiton solution Eq (4.16) was deduced. Its dynamical behaviors were depicted in Figures 6 and 7. As exponential functions and trigonometric functions are not separated in the representation of Eq (4.16), how to construct a perfect positive complexiton solutions of Eq (1.1) is an interesting problem. In a sense, all results in this paper are different types of hybrid solutions. In our future work, we will explore more types of hybrid solutions of higher order nonlinear evolution equations.

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Conflict of interest

The authors declare that they have no competing interests.

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