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Research article

New applications of various distance techniques to multi-criteria decision-making challenges for ranking vague sets

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Abstract: Using the Fermatean vague normal set (FVNS), problems requiring multiple attribute decision making (MADM) have been resolved in this article. This article focuses on the log Fermatean vague normal weighted averaging (log FVNWA), logarithmic Fermatean vague normal weighted geometric (log FVNWG), log generalized Fermatean vague normal weighted averaging (log GFVNWA) and log generalized Fermatean vague normal weighted geometric (log GFVNWG) operators. Described the scoring function, accuracy function and operational laws of the log FVNS. The Euclidean and Humming distance are extended with numerical examples. The features of the log FVNS based on the algebraic operations, including idempotency, boundedness, commutativity and monotonicity are also examined. A field of applied engineering called agricultural robotics has been compared to computer science and machine tool technology. Five distinct agricultural robotics including autonomous mobile robots, articulated robots, humanoid robots, cobot robots, and hybrid robots are randomly chosen. Findings can be compared to established criteria to determine which robotics are the most successful. The results of the models are expressed as a natural number α. We contrast several existing with those that have been developed in order to show the effectiveness and accuracy of the models.

Keywords: vague set; Fermatean vague set; aggregating operators; Hamming distance
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1. Introduction

Real world systems are gradually becoming more complex, making it difficult for decision-makers to identify the optimal solution. Even though deciding between the alternatives is difficult, it is feasible to select the best option. Many firms; find it challenging to create the opportunity, objectives, and viewpoint constraints. Therefore, during the decision making (DM) process an individual or a group must simultaneously consider multiple objectives. Every day, we deal with a wide range of MADM-related issues. As a result, we must improve our DM abilities. Many researchers have contributed to this field of study using different methods. In order to deal with the uncertainties, various uncertain theories have been proposed by them, including the fuzzy set (FS) [1], intuitionistic FS (IFS) [2], interval valued FS (IVFS) [3], vague set [4], Pythagorean FS (PyFS) [5], IVPyFS [6], spherical FS (SFS) [7] and neutrosophic FS (NFS) [8]. Membership grade (MG) refers to a fuzzy set having degrees of belongingness ranging from 0 to 1 in the specified set. Atanassov [2] first presented the concept of an IFS, which is characterized by the fact that the total of its membership grade (MG) and non-membership grade (NMG) does not exceed 1. Occasionally, when applying a DM method, we communicate a single problem scenario in which the sum of the MG and NMG exceeds 1. In order to generalize IFS, Yager [5] developed the new notion of PyFS, which is characterized by the fact that the square sum of its MG and NMG is not greater than 1. Senapati et al. [9] proposed the concept of a Fermatean fuzzy set (FFS). As reported, the cubic sum of the MG and NMG must not be more than 1. Fermatean fuzzy number is a generalization of Pythagorean fuzzy number and intuitionistic fuzzy number.

These ideas are inadequate to demonstrate the neutral state (neither favor nor disfavor). The concept of picture FS was developed by Cuong et al. [10] using three pointers: positive, neutral and negative with a total grade no more than 1. It provides more benefits than IFS and PyFS for selective applications [11–17], and hence encourages the use of these sets in the DM method. The concept of a generalized PyFS with aggregation operator (AO) and its applications was first presented by Liu et al. [18] PyIVFS with AOs [6, 19–21] has the features where the sum of the truth membership grade (TMG), indeterminacy membership grade (IMG), and false membership grade (FMG) is greater than 1. Ashraf et al. [7] suggested the concept of SFS in which the sum of the squares of the TMG, IMG, and FMG is less than 1. The TOPSIS technique was used by Fatmaa et al. [22] to analyze the idea of SFS. Different notions of q-Rung picture FS with aggregation operator (AO) for DM were studied by Liu et al. [23]. The vague set was developed by Gau et al. [4] VS is subjected to the two functions TMG t_v and a FMG f_v. Let $t_v(x)$ denote TMG of x derived from the evidence for x, and $f_v(x)$ denotes FMG of x derived from the evidence against x. Both of these functions fall within the interval $[0, 1]$, where their sum does not exceed 1. Extensions of the VS include the IVFS and the FS [24–26]. This idea of extending PyFS to multi criteria DM (MCDM) using TOPSIS was initially proposed by Zhang et al. [27]. Applications were studied by Jana et al. [28] to discover how to broaden the bipolar fuzzy soft set (BFSS). Jana [29] presented a method for DM that works on an extended bipolar FS with MABAC. Jana et al. [30] also proposed an original method for a robust single valued NS aggregating operator (AO) using MCDM with BFSS [31]. Jana et al. [32] introduced PyFS with dombi AOs. Ullah et al. [33], discussed the practical uses of pattern recognition applications for complex PyFS distance measurement. The AOs based on MADM utilizing a trapezoidal NS method was introduced by Jana et al. [34] MCDM was discussed utilizing the NS and dombi power AOs [35]. Yang et al. [36] discussed interval-valued Pythagorean normal fuzzy information AOs for MADM. Yang et al. [37] discussed the notion of fuzzy c-numbers clustering procedures for fuzzy informations. Rong et al. [38] discussed MARCOS approach for cubic Fermatean fuzzy set and its application in evaluation and selecting cold chain logistics distribution center. Rong et al. [39] discussed the hybrid group decision approach based on MARCOS and regret theory for pharmaceutical enterprises assessment under a single-valued neutrosophic scenario.

A logarithmic operation, as a feasible alternative to algebraic operations, can offer results that are similar to algebraic in terms of smooth estimate quality. In contrast, only limited research has been conducted on logarithmic operations on IFSs, PyFSs and FFS. By using logarithmic operations within vague sets (VSs), we develop a method of vague set MADM based on logarithmic aggregation operators. In 2019, Spherical fuzzy logarithmic AOs based on entropy and their application to DM were discussed in Yun et al. [40] Logarithmic hybrid AOs for single-valued NSSs were proposed by Ashraf [41]. Dragan et al. [42] proposed the selecting power generation technologies using combinative distance-based assessment (CODAS). Recently, many authors discussed new operators such as, Bairagi [43] use extended topology for integrating subjective and objective factors in homogenous group DM for selecting robotic systems. The rational resilience DM model was discussed by Said et al. [44] in an uncertain context. According to Khan et al. [45] MADM is carried out using the Archimedean AO in T-SFS environments. In Riazand et al. [46], fuzzy AO is employed for the selection of third-party logistics providers. The MCDM approach using fuca method was discussed in Do et al. [47]. An emerging market stock selection framework based on MCDM was discussed by Biswas et al. [48] It was discussed in Hasan et al. [49] what some picture fuzzy mean AOs are and how they can be applied to DM. In their paper, Abbasth et al. [50] discussed a method of minimizing the mortality rate due to COVID-19 using a fuzzy soft Bonferroni Mean operator on a q-rung orthopair. Using neutrosophic information in DM is an important component of many approaches discussed by Liu et al. [51] The novel logarithmic operational laws and their AOs are discussed by Garg in [15]. Using SVTrN dombi AOs, Jana et al. [52] presented the MCDM approach. The concept of PyIVNNS with AO was addressed by Palanikumar et al. [54]. Recently, Adak et al. [53] discussed the spherical distance measurement method for solving MCDM problems based on Pythagorean fuzzy environment. Hasan [49] interacted the concept for some picture fuzzy mean operators and their applications in DM. The AO was essential in resolving MADM problems. Averaging and geometric AOs based on PyFS weighted, ordered weighted, and weighted power cases were also presented by Yager [5]. Later, Peng et al. [55] discussed a number of basic PyFS features using AOs. Liu et al. [18] established the generalized PyFS under AOs. We utilized OAs to get log FVNS information. The following Section 2 describes FS and VS information. The definition and various operations of log FVNs are presented in Section 3. Section 4 discusses ED and HD utilizing log FVNNs. Section 5 connects MADM based on log FVNN. An application of log FVNS, the insert algorithm, and a numerical example are given in Section 6. The conclusion is provided in Section 7. Below is a summary of the key aspects made in conducting this research.

(1) We introduced ED and HD based on log FVNSs.
(2) The log FVNWA, log FVNWG, log GFVNWA, and log GFVNWG operators were our suggestions.
(3) The MADM technique is explored by using log FVNSs.
(4) To ascertain the various ideal values for log FVNWA, log FVNWG, log GFVNWA and

log GFVNWG.
(5) Coverage of comparative analyses of the suggested and early investigations.
(6) DM outcomes for natural number α.

2. Basic concepts

The PyFS and VS ideas are discussed in this session.

Definition 2.1. *[5] Let $\mathscr{U}$ be universal. The PyFS $\mathscr{O}$ in $\mathscr{U}$ is $\mathscr{O} = \left\{\varepsilon, \langle\zeta_{\mathscr{O}}^{\mathscr{T}}(\varepsilon), \zeta_{\mathscr{O}}^{\mathscr{F}}(\varepsilon)\rangle \middle| \varepsilon \in \mathscr{U}\right\}$, $\zeta_{\mathscr{O}}^{\mathscr{T}} : \mathscr{U} \to [0,1]$ and $\zeta_{\mathscr{O}}^{\mathscr{F}} : \mathscr{U} \to [0,1]$ denote the MG and NMG of $\varepsilon \in \mathscr{U}$ to $\mathscr{O}$, respectively and $0 \leq (\zeta_{\mathscr{O}}^{\mathscr{T}}(\varepsilon))^2 + (\zeta_{\mathscr{O}}^{\mathscr{F}}(\varepsilon))^2 \leq 1$. To make things easier, $\mathscr{O} = \langle\zeta_{\mathscr{O}}^{\mathscr{T}}, \zeta_{\mathscr{O}}^{\mathscr{F}}\rangle$ is called a Pythagorean fuzzy number(PyFN).*

Definition 2.2. *The FFS $\mathscr{O}$ in $\mathscr{U}$ is $\mathscr{O} = \left\{\varepsilon, \langle\zeta_{\mathscr{O}}^{\mathscr{T}}(\varepsilon), \zeta_{\mathscr{O}}^{\mathscr{F}}(\varepsilon)\rangle \middle| \varepsilon \in \mathscr{U}\right\}$, $\zeta_{\mathscr{O}}^{\mathscr{T}} : \mathscr{U} \to [0,1]$ and $\zeta_{\mathscr{O}}^{\mathscr{F}} : \mathscr{U} \to [0,1]$ denote the MG and NMG of $\varepsilon \in \mathscr{U}$ to $\mathscr{O}$, respectively and $0 \leq (\zeta_{\mathscr{O}}^{\mathscr{T}}(\varepsilon))^3 + (\zeta_{\mathscr{O}}^{\mathscr{F}}(\varepsilon))^3 \leq 1$. To make things easier, $\mathscr{O} = \langle\zeta_{\mathscr{O}}^{\mathscr{T}}, \zeta_{\mathscr{O}}^{\mathscr{F}}\rangle$ is called a Fermatean fuzzy number(FFN).*

Definition 2.3. *[6] The Pythagorean IVFS (PyIVFS) $\mathscr{O}$ in $\mathscr{U}$ is $\mathscr{O} = \left\{\varepsilon, \left\langle\widetilde{\zeta_{\mathscr{O}}^{\mathscr{T}}}(\varepsilon), \widetilde{\zeta_{\mathscr{O}}^{\mathscr{F}}}(\varepsilon)\right\rangle \middle| \varepsilon \in \mathscr{U}\right\}$, where $\widetilde{\zeta_{\mathscr{O}}^{T}} : \mathscr{U} \to Int([0,1])$ and $\widetilde{\zeta_{\mathscr{O}}^{F}} : \mathscr{U} \to Int([0,1])$ denote the MG and NMG of $\varepsilon \in \mathscr{U}$ to $\mathscr{O}$, respectively, and $0 \leq (\zeta_{\mathscr{O}}^{\mathscr{T}U}(\varepsilon))^2 + (\zeta_{\mathscr{O}}^{\mathscr{F}U}(\varepsilon))^2 \leq 1$. To make things easier, $\mathscr{O} = \left\langle\left[\zeta_{\mathscr{O}}^{\mathscr{T}L}, \zeta_{\mathscr{O}}^{\mathscr{T}U}\right], \left[\zeta_{\mathscr{O}}^{\mathscr{F}L}, \zeta_{\mathscr{O}}^{\mathscr{F}U}\right]\right\rangle$ is called a PyIVFN.*

Definition 2.4. *The Fermatean IVFS (FIVFS) $\mathscr{O}$ in $\mathscr{U}$ is $\mathscr{O} = \left\{\varepsilon, \left\langle\widetilde{\zeta_{\mathscr{O}}^{\mathscr{T}}}(\varepsilon), \widetilde{\zeta_{\mathscr{O}}^{\mathscr{F}}}(\varepsilon)\right\rangle \middle| \varepsilon \in \mathscr{U}\right\}$, where $\widetilde{\zeta_{\mathscr{O}}^{T}} : \mathscr{U} \to Int([0,1])$ and $\widetilde{\zeta_{\mathscr{O}}^{F}} : \mathscr{U} \to Int([0,1])$ denote the MG and NMG of $\varepsilon \in \mathscr{U}$ to $\mathscr{O}$, respectively, and $0 \leq (\zeta_{\mathscr{O}}^{\mathscr{T}U}(\varepsilon))^3 + (\zeta_{\mathscr{O}}^{\mathscr{F}U}(\varepsilon))^3 \leq 1$. To make things easier, $\mathscr{O} = \left\langle\left[\zeta_{\mathscr{O}}^{\mathscr{T}L}, \zeta_{\mathscr{O}}^{\mathscr{T}U}\right], \left[\zeta_{\mathscr{O}}^{\mathscr{F}L}, \zeta_{\mathscr{O}}^{\mathscr{F}U}\right]\right\rangle$ is called a FIVFN.*

Definition 2.5. *[4] (i) A VS $\mathscr{O}$ in $\mathscr{U}$ is a pair $(\mathscr{T}_{\mathscr{O}}, \mathscr{F}_{\mathscr{O}})$, $\mathscr{T}_{\mathscr{O}} : \mathscr{U} \to [0,1], \mathscr{F}_{\mathscr{O}} : \mathscr{U} \to [0,1]$ are mappings such that $\mathscr{T}_{\mathscr{O}}(\varepsilon)+\mathscr{F}_{\mathscr{O}}(\varepsilon) \leq 1, \forall\varepsilon \in \mathscr{U}$, $\mathscr{T}_{\mathscr{O}}$ and $\mathscr{F}_{\mathscr{O}}$ are called the truth and false membership function, respectively. (ii) $\mathscr{O}(\varepsilon) = [\mathscr{T}_{\mathscr{O}}(\varepsilon), 1 - \mathscr{F}_{\mathscr{O}}(\varepsilon)]$ is called the vague value of ε in $\mathscr{O}$.*

Definition 2.6. *[4] (i) A VS $\mathscr{O}$ is contained in VS $\mathscr{O}_1$, $\mathscr{O} \subseteq \mathscr{O}_1$ if and only if $\mathscr{O}(\varepsilon) \leq \mathscr{O}_1(\varepsilon)$. That is, $\mathscr{T}_{\mathscr{O}}(\varepsilon) \leq \mathscr{T}_{\mathscr{O}_1}(\varepsilon)$ and $1 - \mathscr{F}_{\mathscr{O}}(\varepsilon) \leq 1 - \mathscr{F}_{\mathscr{O}_1}(\varepsilon), \forall\varepsilon \in \mathscr{U}$. (ii) The union of two VSs $\mathscr{O}$ and $\mathscr{O}_1$, as $X = \mathscr{O} \cup \mathscr{O}_1$, $\mathscr{T}_X = max\{\mathscr{T}_{\mathscr{O}}, \mathscr{T}_{\mathscr{O}_1}\}$ and $1 - \mathscr{F}_X = max\{1 - \mathscr{F}_{\mathscr{O}}, 1 - \mathscr{F}_{\mathscr{O}_1}\} = 1 - min\{\mathscr{F}_{\mathscr{O}}, \mathscr{F}_{\mathscr{O}_1}\}$. (iii) The intersection of two VSs $\mathscr{O}$ and $\mathscr{O}_1$ as $X = \mathscr{O} \cap \mathscr{O}_1$, $\mathscr{T}_X = min\{\mathscr{T}_{\mathscr{O}}, \mathscr{T}_{\mathscr{O}_1}\}$ and $1 - \mathscr{F}_X = min\{1 - \mathscr{F}_{\mathscr{O}}, 1 - \mathscr{F}_{\mathscr{O}_1}\} = 1 - max\{\mathscr{F}_{\mathscr{O}}, \mathscr{F}_{\mathscr{O}_1}\}$.*

Definition 2.7. *[4] A VS $\mathscr{O}$ of a set $\mathscr{U}$, $\forall\varepsilon \in \mathscr{U}$, then*
(i) $\mathscr{T}_{\mathscr{O}}(\varepsilon) = 0$ and $\mathscr{F}_{\mathscr{O}}(\varepsilon) = 1$ is called a zero VS of $\mathscr{U}$.
(ii) $\mathscr{T}_{\mathscr{O}}(\varepsilon) = 1$ and $\mathscr{F}_{\mathscr{O}}(\varepsilon) = 0$ is called a unit VS of $\mathscr{U}$.

Definition 2.8. *[37] The membership function of fuzzy number $M(x) = exp^{-\frac{(x-\kappa)^2}{\upsilon^2}}, (\upsilon > 0)$ and $M = (\kappa, \upsilon)$ is called a normal fuzzy number (NFN), where R is a real numbers.*

Definition 2.9. *[36] Let $L_1 = (\kappa_1, \upsilon_1) \in N$ and $L_2 = (\kappa_2, \upsilon_2) \in N$, $(\upsilon_1, \upsilon_2 > 0)$, then the distance between L_1 and L_2 is defined as $\Xi(L_1, L_2) = \sqrt{(\kappa_1 - \kappa_2)^2 + \frac{1}{2}(\upsilon_1 - \upsilon_2)^2}$.*

3. Log FVNN and its operations

The log FVNN was defined, along with a few of its important fundamental operations.

Definition 3.1. *The log FVS $\mathscr{O}$ in $\mathscr{U}$ is* $\mathscr{O} = \Big\{\varepsilon, \Big\langle\Big[\log\mathscr{T}_{\mathscr{O}}(\varepsilon), \log(1-\mathscr{F}_{\mathscr{O}}(\varepsilon))\Big], \Big[\log\mathscr{F}_{\mathscr{O}}(\varepsilon), \log(1-\mathscr{T}_{\mathscr{O}}(\varepsilon))\Big]\Big\rangle \Big| \varepsilon \in \mathscr{U}\Big\}$, $\widetilde{\zeta^{T}_{\mathscr{O}}} : \mathscr{U} \rightarrow Int([0,1])$ *and* $\widetilde{\zeta^{F}_{\mathscr{O}}} : \mathscr{U} \rightarrow Int([0,1])$ *denote the TMG, IMG and FMG of* $\varepsilon \in \mathscr{U}$ *to* $\mathscr{O}$, *respectively and* $0 \leq (\log_{\mho_i} 1-\mathscr{F}_{\mathscr{O}}(\varepsilon))^3 + (\log_{\mho_i} 1-\mathscr{T}_{\mathscr{O}}(\varepsilon))^3 \leq 1$, *where* $\mho = \prod[\mathscr{T}_{\mathscr{O}}, 1-\mathscr{F}_{\mathscr{O}}], [\mathscr{F}_{\mathscr{O}}, 1-\mathscr{T}_{\mathscr{O}}]$. *To make things easier,* $\mathscr{O} = \Big\langle\Big[\log\mathscr{T}_{\mathscr{O}}, \log(1-\mathscr{F}_{\mathscr{O}})\Big], \Big[\log\mathscr{F}_{\mathscr{O}}, \log(1-\mathscr{T}_{\mathscr{O}})\Big]\Big\rangle$ *is called a log FVN.*

Definition 3.2. *Let* $(\kappa, \upsilon) \in N$, $\mathscr{O} = \Big\langle(\kappa,\upsilon); [\log\mathscr{T}_{\mathscr{O}}, \log(1-\mathscr{F}_{\mathscr{O}})], [\log\mathscr{F}_{\mathscr{O}}, \log(1-\mathscr{T}_{\mathscr{O}})]\Big\rangle$ *be a log FVNN, TMG, IMG and FMG are defined as* $[\log_{\mho_i}\mathscr{T}_{\mathscr{O}}, \log_{\mho_i}(1-\mathscr{F}_{\mathscr{O}})] = \Big[\log_{\mho_i}\mathscr{T}_{\mathscr{O}} \cdot exp^{-\frac{(x-\kappa)^3}{\upsilon^3}}, \log_{\mho_i}(1-\mathscr{F}_{\mathscr{O}}) \cdot exp^{-\frac{(x-\kappa)^3}{\upsilon^3}}\Big]$, $[\log_{\mho_i}\mathscr{F}_{\mathscr{O}}, \log_{\mho_i}(1-\mathscr{T}_{\mathscr{O}})] = \Big[1-(1-\log_{\mho_i}\mathscr{F}_{\mathscr{O}}) \cdot exp^{-\frac{(x-\kappa)^3}{\upsilon^3}}, 1-(1-\log_{\mho_i}(1-\mathscr{T}_{\mathscr{O}})) \cdot exp^{-\frac{(x-\kappa)^3}{\upsilon^3}}\Big]$ *respectively, where* $x \in X$ *is a non-empty set and* $[\log\mathscr{T}_{\mathscr{O}}, \log(1-\mathscr{F}_{\mathscr{O}})]$, $[\log\mathscr{F}_{\mathscr{O}}, \log(1-\mathscr{T}_{\mathscr{O}})] \in Int([0,1])$ *and* $0 \leq (\log(1-\mathscr{F}_{\mathscr{O}})(\varepsilon))^{\alpha} + (\log(1-\mathscr{T}_{\mathscr{O}})(\varepsilon))^{\alpha} \leq 2$, *where* $\mho = \prod[\mathscr{T}_{\mathscr{O}}, 1-\mathscr{F}_{\mathscr{O}}], [\mathscr{F}_{\mathscr{O}}, 1-\mathscr{T}_{\mathscr{O}}]$.

Definition 3.3. *Let* $\mathscr{O} = \Big\langle(\kappa,\upsilon); \Big[\log\mathscr{T}_{\mathscr{O}}, \log(1-\mathscr{F}_{\mathscr{O}})\Big], \Big[\log\mathscr{F}_{\mathscr{O}}, \log(1-\mathscr{T}_{\mathscr{O}})\Big]\Big\rangle$ *be the log FVNN, the score function of* $\mathscr{O}$ *is* $\mathbb{S}(\mathscr{O}) = \frac{\frac{\kappa}{2}\left(\frac{\mathbb{X}_1}{2}+1-\frac{\mathbb{Z}_1}{2}\right)+\frac{\upsilon}{2}\left(\frac{\mathbb{X}_2}{2}+1-\frac{\mathbb{Z}_2}{2}\right)}{2}$, *where* $-1 \leq \mathbb{S}(\mathscr{O}) \leq 1$. *The accuracy function of* $\mathscr{O}$ *is* $\mathbb{A}(\mathscr{O}) = \frac{\frac{\kappa}{2}\left(\frac{\mathbb{X}_1}{2}+1+\frac{\mathbb{Z}_1}{2}\right)+\frac{\upsilon}{2}\left(\frac{\mathbb{X}_2}{2}+1+\frac{\mathbb{Z}_2}{2}\right)}{2}$, *where* $0 \leq \mathbb{A}(\mathscr{O}) \leq 1$. *Where* $\mathbb{X}_1 = (\log\mathscr{T}_{\mathscr{O}})^3, \mathbb{Z}_1 = (\log\mathscr{F}_{\mathscr{O}})^3$ *and* $\mathbb{X}_2 = (\log(1-\mathscr{F}_{\mathscr{O}}))^3, \mathbb{Z}_2 = (\log(1-\mathscr{T}_{\mathscr{O}}))^3$.

Definition 3.4. *Let* $\mathscr{O} = \Big\langle(\kappa,\upsilon); [\log\mathscr{T}_{\mathscr{O}}, \log(1-\mathscr{F}_{\mathscr{O}})], [\log\mathscr{F}_{\mathscr{O}}, \log(1-\mathscr{T}_{\mathscr{O}})]\Big\rangle$, $\mathscr{O}_1 = \Big\langle(\kappa_1,\upsilon_1); [\log\mathscr{T}_{\mathscr{O}1}, \log(1-\mathscr{F}_{\mathscr{O}1})], [\log\mathscr{F}_{\mathscr{O}1}, \log(1-\mathscr{T}_{\mathscr{O}1})]\Big\rangle$ *and* $\mathscr{O}_2 = \Big\langle(\kappa_2,\upsilon_2); [\log\mathscr{T}_{\mathscr{O}2}, \log(1-\mathscr{F}_{\mathscr{O}2})], [\log\mathscr{F}_{\mathscr{O}2}, \log(1-\mathscr{T}_{\mathscr{O}2})]\Big\rangle$ *be the three log FVNNs,* α *is a real number and* $\mho = \prod[\mathscr{T}_{\mathscr{O}_i}, 1-\mathscr{F}_{\mathscr{O}_i}], [\mathscr{F}_{\mathscr{O}_i}, 1-\mathscr{T}_{\mathscr{O}_i}]$, *then*

(1) $\mathscr{O}_1 \veebar \mathscr{O}_2$

$$= \left[\begin{array}{c} (\kappa_1+\kappa_2, \upsilon_1+\upsilon_2); \\ \left[\begin{array}{c} \sqrt[3\alpha]{(\log_{\mho_i}\mathscr{T}_{\mathscr{O}1})^{3\alpha} + (\log_{\mho_i}\mathscr{T}_{\mathscr{O}2})^{3\alpha} - (\log_{\mho_i}\mathscr{T}_{\mathscr{O}1})^{3\alpha} \cdot (\log_{\mho_i}\mathscr{T}_{\mathscr{O}2})^{3\alpha}}, \\ \sqrt[3\alpha]{(\log_{\mho_i}(1-\mathscr{F}_{\mathscr{O}1}))^{3\alpha} + (\log_{\mho_i}(1-\mathscr{F}_{\mathscr{O}2}))^{3\alpha} - (\log_{\mho_i}(1-\mathscr{F}_{\mathscr{O}1}))^{3\alpha} \cdot (\log_{\mho_i}(1-\mathscr{F}_{\mathscr{O}2}))^{3\alpha}} \end{array}\right], \\ \Big[\log_{\mho_i}\mathscr{F}_{\mathscr{O}1} \cdot \log_{\mho_i}\mathscr{F}_{\mathscr{O}2}, \log_{\mho_i}(1-\mathscr{T}_{\mathscr{O}1}) \cdot \log_{\mho_i}(1-\mathscr{T}_{\mathscr{O}2})\Big] \end{array}\right],$$

(2) $\mathscr{O}_1 \barwedge \mathscr{O}_2$

$$= \left[\begin{array}{c} (\kappa_1 \cdot \kappa_2, \upsilon_1 \cdot \upsilon_2); \\ \Big[\log_{\mho_i}\mathscr{T}_{\mathscr{O}1} \cdot \log_{\mho_i}\mathscr{T}_{\mathscr{O}2}, \log_{\mho_i}(1-\mathscr{F}_{\mathscr{O}1}) \cdot \log_{\mho_i}(1-\mathscr{F}_{\mathscr{O}2})\Big], \\ \left[\begin{array}{c} \sqrt[3\alpha]{(\log_{\mho_i}\mathscr{F}_{\mathscr{O}1})^{3\alpha} + (\log_{\mho_i}\mathscr{F}_{\mathscr{O}2})^{3\alpha} - (\log_{\mho_i}\mathscr{F}_{\mathscr{O}1})^{3\alpha} \cdot (\log_{\mho_i}\mathscr{F}_{\mathscr{O}2})^{3\alpha}}, \\ \sqrt[3\alpha]{(\log_{\mho_i}(1-\mathscr{T}_{\mathscr{O}1}))^{3\alpha} + (\log_{\mho_i}(1-\mathscr{T}_{\mathscr{O}2}))^{3\alpha} - (\log_{\mho_i}(1-\mathscr{T}_{\mathscr{O}1}))^{3\alpha} \cdot (\log_{\mho_i}(1-\mathscr{T}_{\mathscr{O}2}))^{3\alpha}} \end{array}\right] \end{array}\right],$$

$$(3)\ \Delta\cdot\mathscr{O}=\left[\begin{array}{c}(\Delta\cdot\kappa,\Delta\cdot\upsilon);\\ \left[\sqrt[3\alpha]{1-(1-(\log_{\mho_i}\mathscr{T}_{\mathscr{O}})^{3\alpha})^{\alpha}},\ \sqrt[3\alpha]{1-(1-(\log_{\mho_i}(1-\mathscr{F}_{\mathscr{O}}))^{3\alpha})^{\alpha}}\right],\\ \left[(\log_{\mho_i}\mathscr{F}_{\mathscr{O}})^{\alpha},(\log_{\mho_i}(1-\mathscr{T}_{\mathscr{O}}))^{\alpha}\right]\end{array}\right],$$

$$(4)\ \mathscr{O}^{\Delta}=\left[\begin{array}{c}(\kappa^{\Delta},\upsilon^{\Delta});\left[(\log_{\mho_i}\mathscr{T}_{\mathscr{O}})^{\alpha},(\log_{\mho_i}(1-\mathscr{F}_{\mathscr{O}}))^{\alpha}\right],\\ \left[\sqrt[3\alpha]{1-(1-(\log_{\mho_i}\mathscr{F}_{\mathscr{O}})^{3\alpha})^{\alpha}},\ \sqrt[3\alpha]{1-(1-(\log_{\mho_i}(1-\mathscr{T}_{\mathscr{O}}))^{3\alpha})^{\alpha}}\right]\end{array}\right].$$

4. Distance between log FVNNs

We measured ED and HD measurements and examined a number mathematical properties of log FVNNs.

Definition 4.1. *Let* $\mathscr{O}_1=\left\langle(\kappa_1,\upsilon_1);[\log\mathscr{T}_{\mathscr{O}1},\log(1-\mathscr{F}_{\mathscr{O}1})],[\log\mathscr{F}_{\mathscr{O}1},\ \log(1-\mathscr{T}_{\mathscr{O}1})]\right\rangle$ *and* $\mathscr{O}_2=\left\langle(\kappa_2,\upsilon_2);[\log\mathscr{T}_{\mathscr{O}2},\log(1-\mathscr{F}_{\mathscr{O}2})],[\log\mathscr{F}_{\mathscr{O}2},\ \log(1-\mathscr{T}_{\mathscr{O}2})]\right\rangle$ *be any two log FVNNs, then ED between* $\mathscr{O}_1$ *and* $\mathscr{O}_2$ *is*

$$\Xi_E\left(\mathscr{O}_1,\mathscr{O}_2\right)=\frac{1}{2}\sqrt[3]{\begin{array}{l}\left[\begin{array}{l}\frac{(\log_{\mho_i}\mathscr{T}_{\mathscr{O}1})^3+1-(\log_{\mho_i}\mathscr{F}_{\mathscr{O}1})^3+(\log_{\mho_i}(1-\mathscr{F}_{\mathscr{O}1}))^3+1-(\log_{\mho_i}(1-\mathscr{T}_{\mathscr{O}1}))^3}{2}\kappa_1\\ -\frac{(\log_{\mho_i}\mathscr{T}_{\mathscr{O}2})^3+1-(\log_{\mho_i}\mathscr{F}_{\mathscr{O}2})^3+(\log_{\mho_i}(1-\mathscr{F}_{\mathscr{O}2}))^3+1-(\log_{\mho_i}(1-\mathscr{T}_{\mathscr{O}2}))^3}{2}\kappa_2\end{array}\right]^3\\ +\frac{1}{2}\left[\begin{array}{l}\frac{(\log_{\mho_i}\mathscr{T}_{\mathscr{O}1})^3+1-(\log_{\mho_i}\mathscr{F}_{\mathscr{O}1})^3+(\log_{\mho_i}(1-\mathscr{F}_{\mathscr{O}1}))^3+1-(\log_{\mho_i}(1-\mathscr{T}_{\mathscr{O}1}))^3}{2}\upsilon_1\\ -\frac{(\log_{\mho_i}\mathscr{T}_{\mathscr{O}2})^3+1-(\log_{\mho_i}\mathscr{F}_{\mathscr{O}2})^3+(\log_{\mho_i}(1-\mathscr{F}_{\mathscr{O}2}))^3+1-(\log_{\mho_i}(1-\mathscr{T}_{\mathscr{O}2}))^3}{2}\upsilon_2\end{array}\right]^3\end{array}},$$

and HD between $\mathscr{O}_1$ *and* $\mathscr{O}_2$ *is defined as*

$$\Xi_H\left(\mathscr{O}_1,\mathscr{O}_2\right)=\frac{1}{2}\left[\begin{array}{l}\left|\begin{array}{l}\frac{(\log_{\mho_i}\mathscr{T}_{\mathscr{O}1})^3+1-(\log_{\mho_i}\mathscr{F}_{\mathscr{O}1})^3+(\log_{\mho_i}(1-\mathscr{F}_{\mathscr{O}1}))^3+1-(\log_{\mho_i}(1-\mathscr{T}_{\mathscr{O}1}))^3}{2}\kappa_1\\ -\frac{(\log_{\mho_i}\mathscr{T}_{\mathscr{O}2})^3+1-(\log_{\mho_i}\mathscr{F}_{\mathscr{O}2})^3+(\log_{\mho_i}(1-\mathscr{F}_{\mathscr{O}2}))^3+1-(\log_{\mho_i}(1-\mathscr{T}_{\mathscr{O}2}))^3}{2}\kappa_2\end{array}\right|\\ +\frac{1}{2}\left|\begin{array}{l}\frac{(\log_{\mho_i}\mathscr{T}_{\mathscr{O}1})^3+1-(\log_{\mho_i}\mathscr{F}_{\mathscr{O}1})^3+(\log_{\mho_i}(1-\mathscr{F}_{\mathscr{O}1}))^3+1-(\log_{\mho_i}(1-\mathscr{T}_{\mathscr{O}1}))^3}{2}\upsilon_1\\ -\frac{(\log_{\mho_i}\mathscr{T}_{\mathscr{O}2})^3+1-(\log_{\mho_i}\mathscr{F}_{\mathscr{O}2})^3+(\log_{\mho_i}(1-\mathscr{F}_{\mathscr{O}2}))^3+1-(\log_{\mho_i}(1-\mathscr{T}_{\mathscr{O}2}))^3}{2}\upsilon_2\end{array}\right|\end{array}\right].$$

Theorem 4.1. *Let* $\mathscr{O}_1=\left\langle(\kappa_1,\upsilon_1);[\log\mathscr{T}_{\mathscr{O}1},\log(1-\mathscr{F}_{\mathscr{O}1})],[\log\mathscr{F}_{\mathscr{O}1},\ \log(1-\mathscr{T}_{\mathscr{O}1})]\right\rangle$, $\mathscr{O}_2=\left\langle(\kappa_2,\upsilon_2);[\log\mathscr{T}_{\mathscr{O}2},\log(1-\mathscr{F}_{\mathscr{O}2})],[\log\mathscr{F}_{\mathscr{O}2},\log(1-\mathscr{T}_{\mathscr{O}2})]\right\rangle$ *and* $\mathscr{O}_3=\left\langle(\kappa_3,\upsilon_3);[\log\mathscr{T}_{\mathscr{O}3},\log(1-\mathscr{F}_{\mathscr{O}3})],[\log\mathscr{F}_{\mathscr{O}3},\ \log(1-\mathscr{T}_{\mathscr{O}3})]\right\rangle$ *be any three log FVNNs, then*

(1) $\Xi_E(\mathscr{O}_1,\mathscr{O}_2)=0$, *if and only if* $\mathscr{O}_1=\mathscr{O}_2$.
(2) $\Xi_E(\mathscr{O}_1,\mathscr{O}_2)$=$\Xi_E(\mathscr{O}_2,\mathscr{O}_1)$.
(3) $\Xi_E(\mathscr{O}_1,\mathscr{O}_3)\leq\Xi_E(\mathscr{O}_1,\mathscr{O}_2)+\Xi_E(\mathscr{O}_2,\mathscr{O}_3)$.

Proof. The proof of (1) and (2) is clear. We only provide proof of (3). Now,

$$\Big(\Xi_E(\mathscr{O}_1,\mathscr{O}_2)+\Xi_E(\mathscr{O}_2,\mathscr{O}_3)\Big)^3=\left[\begin{array}{l}\frac{1}{2}\sqrt[3]{\begin{array}{l}\left[\frac{(\log_{\mho_i}\mathscr{T}_{\mathscr{O}1})^3+1-(\log_{\mho_i}\mathscr{F}_{\mathscr{O}1})^3+(\log_{\mho_i}(1-\mathscr{F}_{\mathscr{O}1}))^3+1-(\log_{\mho_i}(1-\mathscr{T}_{\mathscr{O}1}))^3}{2}\kappa_1 \atop -\frac{(\log_{\mho_i}\mathscr{T}_{\mathscr{O}2})^3+1-(\log_{\mho_i}\mathscr{F}_{\mathscr{O}2})^3+(\log_{\mho_i}(1-\mathscr{F}_{\mathscr{O}2}))^3+1-(\log_{\mho_i}(1-\mathscr{T}_{\mathscr{O}2}))^3}{2}\kappa_2\right]^3\\+\frac{1}{2}\left[\frac{(\log_{\mho_i}\mathscr{T}_{\mathscr{O}1})^3+1-(\log_{\mho_i}\mathscr{F}_{\mathscr{O}1})^3+(\log_{\mho_i}(1-\mathscr{F}_{\mathscr{O}1}))^3+1-(\log_{\mho_i}(1-\mathscr{T}_{\mathscr{O}1}))^3}{2}\upsilon_1 \atop -\frac{(\log_{\mho_i}\mathscr{T}_{\mathscr{O}2})^3+1-(\log_{\mho_i}\mathscr{F}_{\mathscr{O}2})^3+(\log_{\mho_i}(1-\mathscr{F}_{\mathscr{O}2}))^3+1-(\log_{\mho_i}(1-\mathscr{T}_{\mathscr{O}2}))^3}{2}\upsilon_2\right]^3\end{array}}\\+\frac{1}{2}\sqrt[3]{\begin{array}{l}\left[\frac{(\log_{\mho_i}\mathscr{T}_{\mathscr{O}2})^3+1-(\log_{\mho_i}\mathscr{F}_{\mathscr{O}2})^3+(\log_{\mho_i}(1-\mathscr{F}_{\mathscr{O}2}))^3+1-(\log_{\mho_i}(1-\mathscr{T}_{\mathscr{O}2}))^3}{2}\kappa_2 \atop -\frac{(\log_{\mho_i}\mathscr{T}_{\mathscr{O}3})^3+1-(\log_{\mho_i}\mathscr{F}_{\mathscr{O}3})^3+(\log_{\mho_i}(1-\mathscr{F}_{\mathscr{O}3}))^3+1-(\log_{\mho_i}(1-\mathscr{T}_{\mathscr{O}3}))^3}{2}\mho_3\right]^3\\+\frac{1}{2}\left[\frac{(\log_{\mho_i}\mathscr{T}_{\mathscr{O}2})^3+1-(\log_{\mho_i}\mathscr{F}_{\mathscr{O}2})^3+(\log_{\mho_i}(1-\mathscr{F}_{\mathscr{O}2}))^3+1-(\log_{\mho_i}(1-\mathscr{T}_{\mathscr{O}2}))^3}{2}\upsilon_2 \atop -\frac{(\log_{\mho_i}\mathscr{T}_{\mathscr{O}3})^3+1-(\log_{\mho_i}\mathscr{F}_{\mathscr{O}3})^3+(\log_{\mho_i}(1-\mathscr{F}_{\mathscr{O}3}))^3+1-(\log_{\mho_i}(1-\mathscr{T}_{\mathscr{O}3}))^3}{2}\upsilon_3\right]^3\end{array}}\end{array}\right]^3$$

$$\begin{aligned}
&= \frac{1}{8}\Big((\upsilon_1\kappa_1-\upsilon_2\kappa_2)^3+\frac{1}{2}(\upsilon_1\upsilon_1-\upsilon_2\upsilon_2)^3\Big)+\frac{1}{8}\Big((\upsilon_2\kappa_2-\upsilon_3\kappa_3)^3+\frac{1}{2}(\upsilon_2\upsilon_2-\upsilon_3\upsilon_3)^3\Big)\\
&\quad+\frac{3}{8}\left(\sqrt[3]{[(\upsilon_1\kappa_1-\upsilon_2\kappa_2)^3+\frac{1}{2}(\upsilon_1\upsilon_1-\upsilon_2\upsilon_2)^3]^2}\times\sqrt[3]{(\upsilon_2\kappa_2-\upsilon_3\kappa_3)^3+\frac{1}{2}(\upsilon_2\upsilon_2-\upsilon_3\upsilon_3)^3}\right)\\
&\quad+\frac{3}{8}\left(\sqrt[3]{(\upsilon_1\kappa_1-\upsilon_2\kappa_2)^3+\frac{1}{2}(\upsilon_1\upsilon_1-\upsilon_2\upsilon_2)^3}\times\sqrt[3]{[(\upsilon_2\kappa_2-\upsilon_3\kappa_3)^3+\frac{1}{2}(\upsilon_2\upsilon_2-\upsilon_3\upsilon_3)^3]^2}\right)\\
&\geq \frac{1}{4}(\upsilon_1\kappa_1-\upsilon_2\kappa_2+\upsilon_2\kappa_2-\upsilon_3\kappa_3)^3+\frac{1}{8}(\upsilon_1\upsilon_1-\upsilon_2\upsilon_2+\upsilon_2\upsilon_2-\upsilon_3\upsilon_3)^3\\
&= \frac{1}{4}(\upsilon_1\kappa_1-\upsilon_3\kappa_3)^3+\frac{1}{8}(\upsilon_1\upsilon_1-\upsilon_3\upsilon_3)^3\\
&= \Xi_E(\mathscr{O}_1,\mathscr{O}_3)^3.
\end{aligned}$$

where

$$\upsilon_1=\frac{1+(\log_{\mho_i}\mathscr{T}_{\mathscr{O}1})^3-(\log_{\mho_i}\mathscr{F}_{\mathscr{O}1})^3+1+(\log_{\mho_i}(1-\mathscr{F}_{\mathscr{O}1}))^3-(\log_{\mho_i}(1-\mathscr{T}_{\mathscr{O}1}))^3}{2},$$

$$\upsilon_2=\frac{1+(\log_{\mho_i}\mathscr{T}_{\mathscr{O}2})^3-(\log_{\mho_i}\mathscr{F}_{\mathscr{O}2})^3+1+(\log_{\mho_i}(1-\mathscr{F}_{\mathscr{O}2}))^3-(\log_{\mho_i}(1-\mathscr{T}_{\mathscr{O}2}))^3}{2},$$

$$\upsilon_3=\frac{1+(\log_{\mho_i}\mathscr{T}_{\mathscr{O}3})^3-(\log_{\mho_i}\mathscr{F}_{\mathscr{O}3})^3+1+(\log_{\mho_i}(1-\mathscr{F}_{\mathscr{O}3}))^3-(\log_{\mho_i}(1-\mathscr{T}_{\mathscr{O}3}))^3}{2}.$$

□

Corollary 4.1. *Let* $\mathscr{O}_1=\Big\langle(\kappa_1,\upsilon_1);[\log\mathscr{T}_{\mathscr{O}1},\log(1-\mathscr{F}_{\mathscr{O}1})],[\log\mathscr{F}_{\mathscr{O}1},\log(1-\mathscr{T}_{\mathscr{O}1})]\Big\rangle$, $\mathscr{O}_2=\Big\langle(\kappa_2,\upsilon_2);[\log\mathscr{T}_{\mathscr{O}2},\log(1-\mathscr{F}_{\mathscr{O}2})],[\log\mathscr{F}_{\mathscr{O}2},\log(1-\mathscr{T}_{\mathscr{O}2})]\Big\rangle$ *and* $\mathscr{O}_3=\Big\langle(\kappa_3,\upsilon_3);[\log\mathscr{T}_{\mathscr{O}3},\log(1-\mathscr{F}_{\mathscr{O}3})],[\log\mathscr{F}_{\mathscr{O}3},\log(1-\mathscr{T}_{\mathscr{O}3})]\Big\rangle$ *be any three log FVNNs, then* $\Xi_H(\mathscr{O}_1,\mathscr{O}_2)$ *satisfies the following properties.*

(1) $\Xi_H(\mathscr{O}_1, \mathscr{O}_2) = 0$ *if and only if* $\mathscr{O}_1 = \mathscr{O}_2$.
(2) $\Xi_H(\mathscr{O}_1, \mathscr{O}_2) = \Xi_H(\mathscr{O}_2, \mathscr{O}_1)$.
(3) $\Xi_H(\mathscr{O}_1, \mathscr{O}_3) \leq \Xi_H(\mathscr{O}_1, \mathscr{O}_2) + \Xi_H(\mathscr{O}_2, \mathscr{O}_3)$.

5. Log FVNS using AOs

This section introduces the novel concepts of log FVNWA, log FVNWG, log GFVNWA, and log GFVNWG operators utilizing log FVNS.

5.1. Log FVNWA operator

Definition 5.1. *Let* $\mathscr{O}_i = \left\langle (\kappa_i, \upsilon_i); [\log \mathscr{T}_{\mathscr{O}i}, \log(1-\mathscr{F}_{\mathscr{O}i})], [\log \mathscr{F}_{\mathscr{O}i}, \log(1-\mathscr{T}_{\mathscr{O}i})] \right\rangle$ *be the family of log FVNNs,* $\Lambda = (\Lambda_1, \Lambda_2, ..., \Lambda_n)$ *be the weight of* $\mathscr{O}_i$*,* $\Lambda_i \geq 0$ *and* $\oslash_{i=1}^{n}\Lambda_i = 1$ *and* $\mho = \prod[\mathscr{T}_{\mathscr{O}_i}, 1-\mathscr{F}_{\mathscr{O}_i}], [\mathscr{F}_{\mathscr{O}_i}, 1-\mathscr{T}_{\mathscr{O}_i}]$*, then log FVNWA operator is log FVNWA* $(\mathscr{O}_1, \mathscr{O}_2, ..., \mathscr{O}_n) = \oslash_{i=1}^{n}\Lambda_i\mathscr{O}_i$*, for i=1,2,...,n.*

Theorem 5.1. *Let* $\mathscr{O}_i = \left\langle (\kappa_i, \upsilon_i); [\log \mathscr{T}_{\mathscr{O}i}, \log(1-\mathscr{F}_{\mathscr{O}i})], [\log \mathscr{F}_{\mathscr{O}i}, \log(1-\mathscr{T}_{\mathscr{O}i})] \right\rangle$ *be the family of log FVNNs, then log FVNWA* $(\mathscr{O}_1, \mathscr{O}_2, ..., \mathscr{O}_n)$

$$= \left[\begin{array}{c} \left(\oslash_{i=1}^{n}\Lambda_i\kappa_i, \oslash_{i=1}^{n}\Lambda_i\upsilon_i\right); \\ \left[\sqrt[3\alpha]{1-\circledcirc_{i=1}^{n}\left(1-(\log_{\mho_i}\mathscr{T}_{\mathscr{O}i})^{3\alpha}\right)^{\Lambda_i}}, \sqrt[3\alpha]{1-\circledcirc_{i=1}^{n}\left(1-(\log_{\mho_i}(1-\mathscr{F}_{\mathscr{O}i}))^{3\alpha}\right)^{\Lambda_i}}\right], \\ \left[\circledcirc_{i=1}^{n}(\log_{\mho_i}\mathscr{F}_{\mathscr{O}i})^{\Lambda_i}, \circledcirc_{i=1}^{n}(\log_{\mho_i}(1-\mathscr{T}_{\mathscr{O}i}))^{\Lambda_i}\right] \end{array}\right].$$

Proof. We use induction method to prove the theorem.

If $n = 2$, then log FVNWA $(\mathscr{O}_1, \mathscr{O}_2) = \Lambda_1\mathscr{O}_1 \veebar \Lambda_2\mathscr{O}_2$, where

$$\Lambda_1\mathscr{O}_1 = \left[\begin{array}{c} \left(\Lambda_1\kappa_1, \Lambda_1\upsilon_1\right); \\ \left[\sqrt[3\alpha]{1-\left(1-(\log_{\mho_i}\mathscr{T}_{\mathscr{O}1})^{3\alpha}\right)^{\Lambda_1}}, \sqrt[3\alpha]{1-\left(1-(\log_{\mho_i}(1-\mathscr{F}_{\mathscr{O}1}))^{3\alpha}\right)^{\Lambda_1}}\right], \\ \left[(\log_{\mho_i}\mathscr{F}_{\mathscr{O}1})^{\Lambda_1}, (\log_{\mho_i}(1-\mathscr{T}_{\mathscr{O}1}))^{\Lambda_1}\right] \end{array}\right]$$

and

$$\Lambda_2\mathscr{O}_2 = \left[\begin{array}{c} \left(\Lambda_2\kappa_2, \Lambda_2\upsilon_2\right); \\ \left[\sqrt[3\alpha]{1-\left(1-(\log_{\mho_i}\mathscr{T}_{\mathscr{O}2})^{3\alpha}\right)^{\Lambda_2}}, \sqrt[3\alpha]{1-\left(1-(\log_{\mho_i}(1-\mathscr{F}_{\mathscr{O}2}))^{3\alpha}\right)^{\Lambda_2}}\right], \\ \left[(\log_{\mho_i}\mathscr{F}_{\mathscr{O}2})^{\Lambda_2}, (\log_{\mho_i}(1-\mathscr{T}_{\mathscr{O}2}))^{\Lambda_2}\right] \end{array}\right].$$

Hence,

$$\Lambda_1\mathscr{O}_1 \veebar \Lambda_2\mathscr{O}_2 = \left[\begin{array}{c} \left(\Lambda_1\kappa_1+\Lambda_2\kappa_2, \Lambda_1\upsilon_1+\Lambda_2\upsilon_2\right); \\ \left[\begin{array}{c} \sqrt[3\alpha]{\begin{array}{c}\left(1-\left(1-(\log_{\mho_i}\mathscr{T}_{\mathscr{O}1})^{3\alpha}\right)^{\Lambda_1}\right)+\left(1-\left(1-(\log_{\mho_i}\mathscr{T}_{\mathscr{O}2})^{3\alpha}\right)^{\Lambda_2}\right) \\ -\left(1-\left(1-(\log_{\mho_i}\mathscr{T}_{\mathscr{O}1})^{3\alpha}\right)^{\Lambda_1}\right)\cdot\left(1-\left(1-(\log_{\mho_i}\mathscr{T}_{\mathscr{O}2})^{3\alpha}\right)^{\Lambda_2}\right),\end{array}} \\ \sqrt[3\alpha]{\begin{array}{c}\left(1-\left(1-(\log_{\mho_i}(1-\mathscr{F}_{\mathscr{O}1}))^{3\alpha}\right)^{\Lambda_1}\right)+\left(1-\left(1-(\log_{\mho_i}(1-\mathscr{F}_{\mathscr{O}2}))^{3\alpha}\right)^{\Lambda_2}\right) \\ -\left(1-\left(1-(\log_{\mho_i}(1-\mathscr{F}_{\mathscr{O}1}))^{3\alpha}\right)^{\Lambda_1}\right)\cdot\left(1-\left(1-(\log_{\mho_i}(1-\mathscr{F}_{\mathscr{O}2}))^{3\alpha}\right)^{\Lambda_2}\right)\end{array}} \end{array}\right], \\ \left[(\log_{\mho_i}\mathscr{F}_{\mathscr{O}1})^{\Lambda_1}\cdot(\log_{\mho_i}\mathscr{F}_{\mathscr{O}2})^{\Lambda_2}, (\log_{\mho_i}(1-\mathscr{T}_{\mathscr{O}1}))^{\Lambda_1}\cdot(\log_{\mho_i}(1-\mathscr{T}_{\mathscr{O}2}))^{\Lambda_2}\right] \end{array}\right]$$

$$= \left[\begin{array}{c} \left(\Lambda_1\kappa_1+\Lambda_2\kappa_2, \Lambda_1\upsilon_1+\Lambda_2\upsilon_2\right); \\ \left[\begin{array}{c} \sqrt[3\alpha]{1-\left(1-(\log_{\mho_i}\mathscr{T}_{\mathscr{O}1})^{3\alpha}\right)^{\Lambda_1}\cdot\left(1-(\log_{\mho_i}\mathscr{T}_{\mathscr{O}2})^{3\alpha}\right)^{\Lambda_2}}, \\ \sqrt[3\alpha]{1-\left(1-(\log_{\mho_i}(1-\mathscr{F}_{\mathscr{O}1}))^{3\alpha}\right)^{\Lambda_1}\cdot\left(1-(\log_{\mho_i}(1-\mathscr{F}_{\mathscr{O}2}))^{3\alpha}\right)^{\Lambda_2}} \end{array}\right], \\ \left[(\log_{\mho_i}\mathscr{F}_{\mathscr{O}1})^{\Lambda_1}\cdot(\log_{\mho_i}\mathscr{F}_{\mathscr{O}2})^{\Lambda_2}, (\log_{\mho_i}(1-\mathscr{T}_{\mathscr{O}1}))^{\Lambda_1}\cdot(\log_{\mho_i}(1-\mathscr{T}_{\mathscr{O}2}))^{\Lambda_2}\right] \end{array}\right].$$

Thus, log FVNWA $(\mathscr{O}_1, \mathscr{O}_2)$

$$= \left[\begin{array}{c} \left(\oslash_{i=1}^{2}\Lambda_i\kappa_i, \oslash_{i=1}^{2}\Lambda_i\upsilon_i\right); \\ \left[\sqrt[3\alpha]{1-\circledcirc_{i=1}^{2}\left(1-(\log_{\mho_i}\mathscr{T}_{\mathscr{O}i})^{3\alpha}\right)^{\Lambda_i}}, \sqrt[3\alpha]{1-\circledcirc_{i=1}^{2}\left(1-(\log_{\mho_i}(1-\mathscr{F}_{\mathscr{O}i}))^{3\alpha}\right)^{\Lambda_i}}\right], \\ \left[\circledcirc_{i=1}^{2}(\log_{\mho_i}\mathscr{F}_{\mathscr{O}i})^{\Lambda_i}, \circledcirc_{i=1}^{2}(\log_{\mho_i}(1-\mathscr{T}_{\mathscr{O}i}))^{\Lambda_i}\right] \end{array}\right].$$

Suppose that it is true for $n = l$ and $l \geq 3$.

Thus, log FVNWA $(\mathscr{O}_1, \mathscr{O}_2, ..., \mathscr{O}_l)$

$$= \left[\begin{array}{c} \left(\oslash_{i=1}^{l}\Lambda_i\kappa_i, \oslash_{i=1}^{l}\Lambda_i\upsilon_i\right); \\ \left[\sqrt[3\alpha]{1-\circledcirc_{i=1}^{l}\left(1-(\log_{\mho_i}\mathscr{T}_{\mathscr{O}i})^{3\alpha}\right)^{\Lambda_i}}, \sqrt[3\alpha]{1-\circledcirc_{i=1}^{l}\left(1-(\log_{\mho_i}(1-\mathscr{F}_{\mathscr{O}i}))^{3\alpha}\right)^{\Lambda_i}}\right], \\ \left[\circledcirc_{i=1}^{l}(\log_{\mho_i}\mathscr{F}_{\mathscr{O}i})^{\Lambda_i}, \circledcirc_{i=1}^{l}(\log_{\mho_i}(1-\mathscr{T}_{\mathscr{O}i}))^{\Lambda_i}\right] \end{array}\right].$$

If $n = l + 1$ and we apply, then log FVNWA $(\mathscr{O}_1, \mathscr{O}_2, ..., \mathscr{O}_l, \mathscr{O}_{l+1})$

$$= \left[\begin{array}{c} \left(\oslash_{i=1}^{l}\Lambda_i\kappa_i+\Lambda_{l+1}\mho_{l+1}, \oslash_{i=1}^{l}\Lambda_i\upsilon_i+\Lambda_{l+1}\upsilon_{l+1}\right); \\ \left[\begin{array}{c} \sqrt[3\alpha]{\begin{array}{c}\oslash_{i=1}^{l}\left(1-\left(1-(\log_{\mho_i}\mathscr{T}_{\mathscr{O}i})^{3\alpha}\right)^{\Lambda_i}\right)+\left(1-\left(1-(\log_{\mho_i}\mathscr{T}_{\mathscr{O}l+1})^{3\alpha}\right)^{\Lambda_{l+1}}\right) \\ -\circledcirc_{i=1}^{l}\left(1-\left(1-(\log_{\mho_i}\mathscr{T}_{\mathscr{O}i})^{3\alpha}\right)^{\Lambda_i}\right)\cdot\left(1-\left(1-(\log_{\mho_i}\mathscr{T}_{\mathscr{O}l+1})^{3\alpha}\right)^{\Lambda_{l+1}}\right),\end{array}} \\ \sqrt[3\alpha]{\begin{array}{c}\oslash_{i=1}^{l}\left(1-\left(1-(\log_{\mho_i}(1-\mathscr{F}_{\mathscr{O}i}))^{3\alpha}\right)^{\Lambda_i}\right)+\left(1-\left(1-(\log_{\mho_i}(1-\mathscr{F}_{\mathscr{O}l+1}))^{3\alpha}\right)^{\Lambda_{l+1}}\right) \\ -\circledcirc_{i=1}^{l}\left(1-\left(1-(\log_{\mho_i}(1-\mathscr{F}_{\mathscr{O}i}))^{3\alpha}\right)^{\Lambda_i}\right)\cdot\left(1-\left(1-(\log_{\mho_i}(1-\mathscr{F}_{\mathscr{O}l+1}))^{3\alpha}\right)^{\Lambda_{l+1}}\right)\end{array}} \end{array}\right], \\ \left[\circledcirc_{i=1}^{l}(\log_{\mho_i}\mathscr{F}_{\mathscr{O}i})^{\Lambda_i}\cdot(\log_{\mho_i}\mathscr{F}_{\mathscr{O}l+1})^{\Lambda_{l+1}}, \circledcirc_{i=1}^{l}(\log_{\mho_i}(1-\mathscr{T}_{\mathscr{O}i}))^{\Lambda_i}\cdot(\log_{\mho_i}(1-\mathscr{T}_{\mathscr{O}l+1}))^{\Lambda_{l+1}}\right] \end{array}\right]$$

$$= \left[\begin{array}{c} \left(\oslash_{i=1}^{l+1}\Lambda_i\kappa_i, \oslash_{i=1}^{l+1}\Lambda_i\upsilon_i\right); \\ \left[\sqrt[3\alpha]{1-\circledcirc_{i=1}^{l+1}\left(1-(\log_{\mho_i}\mathscr{T}_{\mathscr{O}i})^{3\alpha}\right)^{\Lambda_i}}, \sqrt[3\alpha]{1-\circledcirc_{i=1}^{l+1}\left(1-(\log_{\mho_i}(1-\mathscr{F}_{\mathscr{O}i}))^{3\alpha}\right)^{\Lambda_i}}\right], \\ \left[\circledcirc_{i=1}^{l+1}(\log_{\mho_i}\mathscr{F}_{\mathscr{O}i})^{\Lambda_i}, \circledcirc_{i=1}^{l+1}(\log_{\mho_i}(1-\mathscr{T}_{\mathscr{O}i}))^{\Lambda_i}\right] \end{array}\right].$$

□

Theorem 5.2. *(Idempotency property) If all* $\mathscr{O}_i = \left\langle(\kappa_i, \upsilon_i); [\log\mathscr{T}_{\mathscr{O}i}, \log(1-\mathscr{F}_{\mathscr{O}i})], [\log\mathscr{F}_{\mathscr{O}i}, \log(1-\mathscr{T}_{\mathscr{O}i})]\right\rangle (i = 1, 2, ..., n)$ *are equal and* $\mathscr{O}_i = \mathscr{O}$, *then log FVNWA* $(\mathscr{O}_1, \mathscr{O}_2, ..., \mathscr{O}_n) = \mathscr{O}$.

Proof. Given that $(\kappa_i, \upsilon_i) = (\kappa, \upsilon)$, $[\log\mathscr{T}_{\mathscr{O}i}, \log(1-\mathscr{F}_{\mathscr{O}i})] = [\log\mathscr{T}_{\mathscr{O}}, \log(1-\mathscr{F}_{\mathscr{O}})]$, and $[\log\mathscr{F}_{\mathscr{O}i}, \log(1-\mathscr{T}_{\mathscr{O}i})] = [\log\mathscr{F}_{\mathscr{O}}, \log(1-\mathscr{T}_{\mathscr{O}})]$ and $\oslash_{i=1}^{n}\Lambda_i = 1$.

Now, $logFVNWA(\mathscr{O}_1, \mathscr{O}_2, ..., \mathscr{O}_n)$

$$\begin{aligned} &= \left[\begin{array}{c} \left(\oslash_{i=1}^{n}\Lambda_i\kappa_i, \oslash_{i=1}^{n}\Lambda_i\upsilon_i\right); \\ \left[\sqrt[3\alpha]{1-\circledcirc_{i=1}^{n}\left(1-(\log_{\mho_i}\mathscr{T}_{\mathscr{O}i})^{3\alpha}\right)^{\Lambda_i}}, \sqrt[3\alpha]{1-\circledcirc_{i=1}^{n}\left(1-(\log_{\mho_i}(1-\mathscr{F}_{\mathscr{O}i}))^{3\alpha}\right)^{\Lambda_i}}\right], \\ \left[\circledcirc_{i=1}^{n}(\log_{\mho_i}\mathscr{F}_{\mathscr{O}i})^{\Lambda_i}, \circledcirc_{i=1}^{n}(\log_{\mho_i}(1-\mathscr{T}_{\mathscr{O}i}))^{\Lambda_i}\right] \end{array}\right] \\ &= \left[\begin{array}{c} \left(\kappa\oslash_{i=1}^{n}\Lambda_i, \upsilon\oslash_{i=1}^{n}\Lambda_i\right); \\ \left[\sqrt[3\alpha]{1-\left(1-(\log_{\mho_i}\mathscr{T}_{\mathscr{O}})^{3\alpha}\right)^{\oslash_{i=1}^{n}\Lambda_i}}, \sqrt[3\alpha]{1\left(1-(\log_{\mho_i}(1-\mathscr{F}_{\mathscr{O}}))^{3\alpha}\right)^{\oslash_{i=1}^{n}\Lambda_i}}\right], \\ \left[(\log_{\mho_i}\mathscr{F}_{\mathscr{O}})^{\oslash_{i=1}^{n}\Lambda_i}, (\log_{\mho_i}(1-\mathscr{T}_{\mathscr{O}}))^{\oslash_{i=1}^{n}\Lambda_i}\right] \end{array}\right] \\ &= \left[\begin{array}{c} (\kappa, \upsilon); \\ \left[\sqrt[3\alpha]{1-\left(1-(\log_{\mho_i}\mathscr{T}_{\mathscr{O}})^{3\alpha}\right)}, \sqrt[3\alpha]{1-\left(1-(\log_{\mho_i}(1-\mathscr{F}_{\mathscr{O}}))^{3\alpha}\right)}\right], \\ \left[(\log_{\mho_i}\mathscr{F}_{\mathscr{O}}), (\log_{\mho_i}(1-\mathscr{T}_{\mathscr{O}}))\right] \end{array}\right] \\ &= \mathscr{O}. \end{aligned}$$

□

Theorem 5.3. *(Boundedness property) Let* $\mathscr{O}_i = \left\langle(\kappa_{ij}, \upsilon_{ij}); [\log\mathscr{T}_{\mathscr{O}ij}, \log(1-\mathscr{F}_{\mathscr{O}ij})], [\log\mathscr{F}_{\mathscr{O}ij}, \log(1-\mathscr{T}_{\mathscr{O}ij})]\right\rangle (i = 1, 2, ..., n); (j = 1, 2, ..., i_j)$ *be the collection of log FVNWA, where* $\underbrace{\kappa} = \min\kappa_{ij}$, $\overbrace{\kappa} = \max\kappa_{ij}$, $\underbrace{\upsilon} = \max\upsilon_{ij}$, $\overbrace{\upsilon} = \min\upsilon_{ij}$, $\underbrace{\log_{\mho_i}\mathscr{T}_{\mathscr{O}}} = \min\log_{\mho_i}\mathscr{T}_{\mathscr{O}ij}$,

$\overbrace{\log_{\mho_i}\mathscr{T}_{\mathscr{O}}} = \max\log_{\mho_i}\mathscr{T}_{\mathscr{O}ij}$, $\underbrace{\log_{\mho_i}(1-\mathscr{F}_{\mathscr{O}})} = \min\log_{\mho_i}(1-\mathscr{F}_{\mathscr{O}ij})$,

$\overbrace{\log_{\mho_i}(1-\mathscr{F}_{\mathscr{O}})} = \max\log_{\mho_i}(1-\mathscr{F}_{\mathscr{O}ij})$, $\underbrace{\log_{\mho_i}\mathscr{F}_{\mathscr{O}}} = \min\log_{\mho_i}\mathscr{F}_{\mathscr{O}ij}$,

$\overbrace{\log_{\mho_i}\mathscr{F}_{\mathscr{O}}} = \max\log_{\mho_i}\mathscr{F}_{\mathscr{O}ij}$, $\underbrace{\log_{\mho_i}(1-\mathscr{T}_{\mathscr{O}})} = \min\log_{\mho_i}(1-\mathscr{T}_{\mathscr{O}ij})$,

$\overbrace{\log_{\mho_i}(1-\mathscr{T}_{\mathscr{O}})} = \max\log_{\mho_i}(1-\mathscr{T}_{\mathscr{O}ij})$.

Then,

$$\begin{aligned}
&\Big\langle (\underbrace{\kappa}, \underbrace{\upsilon}); [\underbrace{\log_{\mho_i} \mathscr{T}_{\mathscr{O}}}, \underbrace{\log_{\mho_i}(1-\mathscr{F}_{\mathscr{O}})}], [\overbrace{\log_{\mho_i} \mathscr{F}_{\mathscr{O}}}, \overbrace{\log_{\mho_i}(1-\mathscr{T}_{\mathscr{O}})}] \Big\rangle \\
\le\ & logFVNWA(\mathscr{O}_1, \mathscr{O}_2, ..., \mathscr{O}_n) \\
\le\ & \Big\langle (\overbrace{\kappa}, \overbrace{\upsilon}); [\overbrace{\log_{\mho_i} \mathscr{T}_{\mathscr{O}}}, \overbrace{\log_{\mho_i}(1-\mathscr{F}_{\mathscr{O}})}], [\underbrace{\log_{\mho_i} \mathscr{F}_{\mathscr{O}}}, \underbrace{\log_{\mho_i}(1-\mathscr{T}_{\mathscr{O}})}] \Big\rangle.
\end{aligned}$$

where $1 \le i \le n,\ \ j = 1, 2, ..., i_j$.

Proof. Since, $\underbrace{\log_{\mho_i} \mathscr{T}_{\mathscr{O}}} = \min \log_{\mho_i} \mathscr{T}_{\mathscr{O}ij}$, $\overbrace{\log_{\mho_i} \mathscr{T}_{\mathscr{O}}} = \max \log_{\mho_i} \mathscr{T}_{\mathscr{O}ij}$, $\underbrace{\log_{\mho_i}(1-\mathscr{F}_{\mathscr{O}})} = \min \log_{\mho_i}(1-\mathscr{F}_{\mathscr{O}ij})$, $\overbrace{\log_{\mho_i}(1-\mathscr{F}_{\mathscr{O}})} = \max \log_{\mho_i}(1-\mathscr{F}_{\mathscr{O}ij})$ and $\underbrace{\log_{\mho_i} \mathscr{T}_{\mathscr{O}}} \le \log_{\mho_i} \mathscr{T}_{\mathscr{O}ij} \le \overbrace{\log_{\mho_i} \mathscr{T}_{\mathscr{O}}}$ and $\underbrace{\log_{\mho_i}(1-\mathscr{F}_{\mathscr{O}})} \le \log_{\mho_i}(1-\mathscr{F}_{\mathscr{O}})_{ij} \le \overbrace{\log_{\mho_i}(1-\mathscr{F}_{\mathscr{O}})}$.

Now,

$$\begin{aligned}
&\underbrace{\log_{\mho_i} \mathscr{T}_{\mathscr{O}}} + \underbrace{\log_{\mho_i}(1-\mathscr{F}_{\mathscr{O}})} \\
=\ & \sqrt[3\alpha]{1 - \circledcirc_{i=1}^{n}\Big(1 - (\underbrace{\log_{\mho_i} \mathscr{T}_{\mathscr{O}}})^{3\alpha}\Big)^{\Lambda_i}} + \sqrt[3\alpha]{1 - \circledcirc_{i=1}^{n}\Big(1 - (\underbrace{\log_{\mho_i}(1-\mathscr{F}_{\mathscr{O}})})^{3\alpha}\Big)^{\Lambda_i}} \\
\le\ & \sqrt[3\alpha]{1 - \circledcirc_{i=1}^{n}\Big(1 - (\log_{\mho_i} \mathscr{T}_{\mathscr{O}ij})^{3\alpha}\Big)^{\Lambda_i}} + \sqrt[3\alpha]{1 - \circledcirc_{i=1}^{n}\Big(1 - (\log_{\mho_i}(1-\mathscr{F}_{\mathscr{O}ij}))^{3\alpha}\Big)^{\Lambda_i}} \\
\le\ & \sqrt[3\alpha]{1 - \circledcirc_{i=1}^{n}\Big(1 - (\overbrace{\log_{\mho_i} \mathscr{T}_{\mathscr{O}}})^{3\alpha}\Big)^{\Lambda_i}} + \sqrt[3\alpha]{1 - \circledcirc_{i=1}^{n}\Big(1 - (\overbrace{\log_{\mho_i}(1-\mathscr{F}_{\mathscr{O}})})^{3\alpha}\Big)^{\Lambda_i}} \\
=\ & \overbrace{\log_{\mho_i} \mathscr{T}_{\mathscr{O}}} + \overbrace{\log_{\mho_i}(1-\mathscr{F}_{\mathscr{O}})}.
\end{aligned}$$

Since, $\underbrace{\log_{\mho_i} \mathscr{F}_{\mathscr{O}}} = \min \log_{\mho_i} \mathscr{F}_{\mathscr{O}ij}$, $\overbrace{\log_{\mho_i} \mathscr{F}_{\mathscr{O}}} = \max \log_{\mho_i} \mathscr{F}_{\mathscr{O}ij}$, $\underbrace{\log_{\mho_i}(1-\mathscr{T}_{\mathscr{O}})} = \min \log_{\mho_i}(1-\mathscr{T}_{\mathscr{O}ij})$, $\overbrace{\log_{\mho_i}(1-\mathscr{T}_{\mathscr{O}})} = \max \log_{\mho_i}(1-\mathscr{T}_{\mathscr{O}ij})$ and $\underbrace{\log_{\mho_i} \mathscr{F}_{\mathscr{O}}} \le \log_{\mho_i} \mathscr{F}_{\mathscr{O}ij} \le \overbrace{\log_{\mho_i} \mathscr{F}_{\mathscr{O}}}$ and $\underbrace{\log_{\mho_i}(1-\mathscr{T}_{\mathscr{O}})} \le \log_{\mho_i}(1-\mathscr{T}_{\mathscr{O}ij}) \le \overbrace{\log_{\mho_i}(1-\mathscr{T}_{\mathscr{O}})}$.

Now,

$$\begin{aligned}
\underbrace{\log_{\mho_i} \mathscr{F}_{\mathscr{O}}} + \underbrace{\log_{\mho_i}(1-\mathscr{T}_{\mathscr{O}})} &= \circledcirc_{i=1}^{n}(\underbrace{\log_{\mho_i} \mathscr{F}_{\mathscr{O}}})^{\Lambda_i} + \circledcirc_{i=1}^{n}(\underbrace{\log_{\mho_i}(1-\mathscr{T}_{\mathscr{O}})})^{\Lambda_i} \\
&\le \circledcirc_{i=1}^{n}(\log_{\mho_i} \mathscr{F}_{\mathscr{O}ij})^{\Lambda_i} + \circledcirc_{i=1}^{n}(\log_{\mho_i}(1-\mathscr{T}_{\mathscr{O}ij}))^{\Lambda_i} \\
&\le \circledcirc_{i=1}^{n}(\overbrace{\log_{\mho_i} \mathscr{F}_{\mathscr{O}}})^{\Lambda_i} + \circledcirc_{i=1}^{n}(\overbrace{\log_{\mho_i}(1-\mathscr{T}_{\mathscr{O}})})^{\Lambda_i} \\
&= \overbrace{\log_{\mho_i} \mathscr{F}_{\mathscr{O}}} + \overbrace{\log_{\mho_i}(1-\mathscr{T}_{\mathscr{O}})}.
\end{aligned}$$

Since, $\underbrace{\kappa} = \min \kappa_{ij}$, $\overbrace{\kappa} = \max \kappa_{ij}$, $\underbrace{\upsilon} = \max \upsilon_{ij}$, $\overbrace{\upsilon} = \min \upsilon_{ij}$ and $\underbrace{\kappa} \le \kappa_{ij} \le \overbrace{\kappa}$ and $\overbrace{\upsilon} \le \upsilon_{ij} \le \underbrace{\upsilon}$.

Hence, $\oslash_{i=1}^{n}\Lambda_i \underbrace{\kappa} \leq \oslash_{i=1}^{n}\Lambda_i\kappa_{ij} \leq \oslash_{i=1}^{n}\Lambda_i \overbrace{\kappa}$ and $\oslash_{i=1}^{n}\Lambda_i \overbrace{\upsilon} \leq \oslash_{i=1}^{n}\Lambda_i\upsilon_{ij} \leq \oslash_{i=1}^{n}\Lambda_i \underbrace{\upsilon}$. Therefore,

$$\frac{\oslash_{i=1}^{n}\Lambda_i \underbrace{\kappa}}{2} \times \left[\frac{\left(\sqrt[3\alpha]{1-\circledcirc_{i=1}^{n}\left(1-(\underbrace{\log_{\mho_i}\mathscr{T}_{\mathscr{O}}})^{3\alpha}\right)^{\Lambda_i}}\right)^2 + \left(\sqrt[3\alpha]{1-\circledcirc_{i=1}^{n}\left(1-(\underbrace{\log_{\mho_i}(1-\mathscr{F}_{\mathscr{O}})})^{3\alpha}\right)^{\Lambda_i}}\right)^2}{2} + 1 - \frac{\left(\sqrt[q]{1-\circledcirc_{i=1}^{n}\left(1-(\underbrace{\log_{\mho_i}\mathscr{I}_{\mathscr{O}}})^{\alpha}\right)^{\Lambda_i}}\right)^2 + \left(\sqrt[q]{1-\circledcirc_{i=1}^{n}\left(1-(\underbrace{\log_{\mho_i}\mathscr{I}_{\mathscr{O}}})^{\alpha}\right)^{\Lambda_i}}\right)^2}{2} + 1 - \frac{\left(\circledcirc_{i=1}^{n}(\overbrace{\log_{\mho_i}\mathscr{F}_{\mathscr{O}}})^{\Lambda_i}\right)^2 + \left(\circledcirc_{i=1}^{n}(\overbrace{\log_{\mho_i}(1-\mathscr{T}_{\mathscr{O}})})^{\Lambda_i}\right)^2}{2}\right]$$

$$\leq \frac{\oslash_{i=1}^{n}\Lambda_i\kappa_{ij}}{2} \times \left[\frac{\left(\sqrt[3\alpha]{1-\circledcirc_{i=1}^{n}\left(1-(\log_{\mho_i}\mathscr{T}_{\mathscr{O}ij})^{3\alpha}\right)^{\Lambda_i}}\right)^2 + \left(\sqrt[3\alpha]{1-\circledcirc_{i=1}^{n}\left(1-(\log_{\mho_i}(1-\mathscr{F}_{\mathscr{O}ij}))^{3\alpha}\right)^{\Lambda_i}}\right)^2}{2} + 1 - \frac{\left(\circledcirc_{i=1}^{n}(\log_{\mho_i}\mathscr{F}_{\mathscr{O}ij})^{\Lambda_i}\right)^2 + \left(\circledcirc_{i=1}^{n}(\log_{\mho_i}(1-\mathscr{T}_{\mathscr{O}ij}))^{\Lambda_i}\right)^2}{2}\right]$$

$$\leq \frac{\oslash_{i=1}^{n}\Lambda_i \overbrace{\kappa}}{2} \times \left[\frac{\left(\sqrt[3\alpha]{1-\circledcirc_{i=1}^{n}\left(1-(\overbrace{\log_{\mho_i}\mathscr{T}_{\mathscr{O}}})^{3\alpha}\right)^{\Lambda_i}}\right)^2 + \left(\sqrt[3\alpha]{1-\circledcirc_{i=1}^{n}\left(1-(\overbrace{\log_{\mho_i}(1-\mathscr{F}_{\mathscr{O}})})^{3\alpha}\right)^{\Lambda_i}}\right)^2}{2} + 1 - \frac{\left(\circledcirc_{i=1}^{n}(\underbrace{\log_{\mho_i}\mathscr{F}_{\mathscr{O}}})^{\Lambda_i}\right)^2 + \left(\circledcirc_{i=1}^{n}(\underbrace{\log_{\mho_i}(1-\mathscr{T}_{\mathscr{O}})})^{\Lambda_i}\right)^2}{2}\right].$$

Hence,

$$\begin{aligned} & \left\langle(\underbrace{\kappa}, \underbrace{\upsilon}); [\underbrace{\log\mathscr{T}_{\mathscr{O}}}, \underbrace{\log(1-\mathscr{F}_{\mathscr{O}})}], [\overbrace{\log\mathscr{F}_{\mathscr{O}}}, \overbrace{\log(1-\mathscr{T}_{\mathscr{O}})}]\right\rangle \\ \leq\ & FVNWA(\mathscr{O}_1, \mathscr{O}_2, ..., \mathscr{O}_n) \\ \leq\ & \left\langle(\overbrace{\kappa}, \overbrace{\upsilon}); [\overbrace{\log\mathscr{T}_{\mathscr{O}}}, \overbrace{\log(1-\mathscr{F}_{\mathscr{O}})}], [\underbrace{\log\mathscr{F}_{\mathscr{O}}}, \underbrace{\log(1-\mathscr{T}_{\mathscr{O}})}]\right\rangle. \end{aligned}$$

□

Theorem 5.4. *(Monotonicity property) Let* $\mathscr{O}_i = \left\langle(\kappa_{t_{ij}}, \upsilon_{t_{ij}}); [\log\mathscr{T}_{\mathscr{O}t_{ij}}, \log(1-\mathscr{F}_{\mathscr{O}t_{ij}})], [\log\mathscr{F}_{\mathscr{O}t_{ij}}, \log(1-\mathscr{T}_{\mathscr{O}t_{ij}})]\right\rangle$ *and* $\Lambda_i = \left\langle(\kappa_{h_{ij}}, \upsilon_{h_{ij}}); [\log\mathscr{T}_{\mathscr{O}h_{ij}}, \log(1-\mathscr{F}_{\mathscr{O}h_{ij}})], [\log\mathscr{F}_{\mathscr{O}h_{ij}}, \log(1-\mathscr{T}_{\mathscr{O}h_{ij}})]\right\rangle (i = 1, 2, ..., n); (j = 1, 2, ..., i_j)$ *be the families of log FVNWAs. For any i, if there is* $\kappa_{t_{ij}} \leq \upsilon_{h_{ij}}$, $\left(\log_{\mho_i}\mathscr{T}_{\mathscr{O}t_{ij}}\right)^2 + \left(\log_{\mho_i}(1-\mathscr{F}_{\mathscr{O}t_{ij}})\right)^2 \leq \left(\log_{\mho_i}\mathscr{T}_{\mathscr{O}h_{ij}}\right)^2 + \left(\log_{\mho_i}(1-\mathscr{F}_{\mathscr{O}h_{ij}})\right)^2$ $\left(\log_{\mho_i}\mathscr{F}_{\mathscr{O}t_{ij}}\right)^2 + \left(\log_{\mho_i}(1-\mathscr{T}_{\mathscr{O}})_{t_{ij}}\right)^2 \geq \left(\log_{\mho_i}\mathscr{F}_{\mathscr{O}h_{ij}}\right)^2 + \left(\log_{\mho_i}(1-\mathscr{T}_{\mathscr{O}h_{ij}})\right)^2$ *or* $\mathscr{O}_i \leq \mathscr{W}_i$*, then log FVNWA* $(\mathscr{O}_1, \mathscr{O}_2, ..., \mathscr{O}_n) \leq$ *q-Rung log FVNWA* $(\mathscr{W}_1, \mathscr{W}_2, ..., \mathscr{W}_n)$.

Proof. For any i, $\kappa_{t_{ij}} \leq \upsilon_{h_{ij}}$. Therefore, $\oslash_{i=1}^{n}\kappa_{t_{ij}} \leq \oslash_{i=1}^{n}\upsilon_{h_{ij}}$. For any i,

$$\left(\log_{\mho_i}\mathscr{T}_{\mathscr{O}t_{ij}}\right)^2 + \left(\log_{\mho_i}(1-\mathscr{F}_{\mathscr{O}t_{ij}})\right)^2 \leq \left(\log_{\mho_i}\mathscr{T}_{\mathscr{O}h_{ij}}\right)^2 + \left(\log_{\mho_i}(1-\mathscr{F}_{\mathscr{O}h_{ij}})\right)^2.$$

Therefore,

$$1 - \left(\log_{\mho_i}\mathscr{T}_{\mathscr{O}t_i}\right)^2 + 1 - \left(\log_{\mho_i}(1-\mathscr{F}_{\mathscr{O}t_i})\right)^2 \geq 1 - \left(\log_{\mho_i}\mathscr{T}_{\mathscr{O}h_i}\right)^2 + 1 - \left(\log_{\mho_i}(1-\mathscr{F}_{\mathscr{O}h_i})\right)^2.$$

Hence,

$$\circledcirc_{i=1}^{n}\left(1-\left(\log_{\mho_i}\mathscr{T}_{\mathscr{O}t_i}\right)^2\right)^{\Lambda_i}+\circledcirc_{i=1}^{n}\left(1-\left(\log_{\mho_i}(1-\mathscr{F}_{\mathscr{O}t_i})\right)^2\right)^{\Lambda_i}$$
$$\geq\circledcirc_{i=1}^{n}\left(1-\left(\log_{\mho_i}\mathscr{T}_{\mathscr{O}h_i}\right)^2\right)^{\Lambda_i}+\circledcirc_{i=1}^{n}\left(1-\left(\log_{\mho_i}(1-\mathscr{F}_{\mathscr{O}h_i})\right)^2\right)^{\Lambda_i},$$

and

$$\sqrt[3\alpha]{1-\circledcirc_{i=1}^{n}\left(1-\left(\log_{\mho_i}\mathscr{T}_{\mathscr{O}t_i}\right)^{3\alpha}\right)^{\Lambda_i}}+\sqrt[3\alpha]{1-\circledcirc_{i=1}^{n}\left(1-\left(\log_{\mho_i}(1-\mathscr{F}_{\mathscr{O}t_i)})\right)^{3\alpha}\right)^{\Lambda_i}}$$
$$\leq\sqrt[3\alpha]{1-\circledcirc_{i=1}^{n}\left(1-\left(\log_{\mho_i}\mathscr{T}_{\mathscr{O}h_i}\right)^{3\alpha}\right)^{\Lambda_i}}+\sqrt[3\alpha]{1-\circledcirc_{i=1}^{n}\left(1-\left(\log_{\mho_i}(1-\mathscr{F}_{\mathscr{O}h_i)})\right)^{3\alpha}\right)^{\Lambda_i}}.$$

For any i,

$$\left(\log_{\mho_i}\mathscr{F}_{\mathscr{O}t_{ij}}\right)^2+\left(\log_{\mho_i}(1-\mathscr{T}_{\mathscr{O}t_{ij})})\right)^2\geq\left(\log_{\mho_i}\mathscr{F}_{\mathscr{O}h_{ij}}\right)^2+\left(\log_{\mho_i}(1-\mathscr{T}_{\mathscr{O}h_{ij})})\right)^2.$$

Therefore,

$$1-\frac{\left(\circledcirc_{i=1}^{n}\log_{\mho_i}\mathscr{F}_{\mathscr{O}t_{ij}}\right)^2+\left(\circledcirc_{i=1}^{n}\log_{\mho_i}(1-\mathscr{T}_{\mathscr{O}t_{ij})})\right)^2}{2}\leq 1-\frac{\left(\circledcirc_{i=1}^{n}\log_{\mho_i}\mathscr{F}_{\mathscr{O}h_{ij}}\right)^2+\left(\circledcirc_{i=1}^{n}\log_{\mho_i}(1-\mathscr{T}_{\mathscr{O}h_{ij})})\right)^2}{2}.$$

$$\frac{\oslash_{i=1}^{n}\kappa_{tij}}{2}\times\left[\begin{array}{c}\frac{\left(\sqrt[3\alpha]{1-\circledcirc_{i=1}^{n}\left(1-(\log_{\mho_i}\mathscr{T}_{\mathscr{O}ti})^{3\alpha}\right)^{\Lambda_i}}\right)^2+\left(\sqrt[3\alpha]{1-\circledcirc_{i=1}^{n}\left(1-(\log_{\mho_i}(1-\mathscr{F}_{\mathscr{O}ti}))^{3\alpha}\right)^{\Lambda_i}}\right)^2}{2}\\+1-\frac{\left(\circledcirc_{i=1}^{n}(\log_{\mho_i}\mathscr{F}_{\mathscr{O}tij})\right)^2+\left(\circledcirc_{i=1}^{n}(\log_{\mho_i}(1-\mathscr{T}_{\mathscr{O}tij}))\right)^2}{2}\end{array}\right]$$

$$\leq\frac{\oslash_{i=1}^{n}\kappa_{hij}}{2}\times\left[\begin{array}{c}\frac{\left(\sqrt[3\alpha]{1-\circledcirc_{i=1}^{n}\left(1-(\log_{\mho_i}\mathscr{T}_{\mathscr{O}hi})^{3\alpha}\right)^{\Lambda_i}}\right)^2+\left(\sqrt[3\alpha]{1-\circledcirc_{i=1}^{n}\left(1-(\log_{\mho_i}(1-\mathscr{F}_{\mathscr{O}hi}))^{3\alpha}\right)^{\Lambda_i}}\right)^2}{2}\\+1-\frac{\left(\circledcirc_{i=1}^{n}(\log_{\mho_i}\mathscr{F}_{\mathscr{O}hij})\right)^2+\left(\circledcirc_{i=1}^{n}(\log_{\mho_i}(1-\mathscr{T}_{\mathscr{O}hij}))\right)^2}{2}\end{array}\right].$$

Hence, log FVNWA $(\mathscr{O}_1,\mathscr{O}_2,...,\mathscr{O}_n)\leq logFVNWA(\mathscr{W}_1,\mathscr{W}_2,...,\mathscr{W}_n)$. □

5.2. Log FVNWG operator

Definition 5.2. *Let* $\mathscr{O}_i=\left\langle(\kappa_i,\upsilon_i);[\log\mathscr{T}_{\mathscr{O}i},\log(1-\mathscr{F}_{\mathscr{O}i})],[\log\mathscr{F}_{\mathscr{O}i},\log(1-\mathscr{T}_{\mathscr{O}i})]\right\rangle$ *be the family of log FVNNs, then log FVNWG operator is log FVNWG* $(\mathscr{O}_1,\mathscr{O}_2,...,\mathscr{O}_n)=\circledcirc_{i=1}^{n}\mathscr{O}_i^{\Lambda_i}$ $(i=1,2,...,n)$.

Theorem 5.5. *Let* $\mathscr{O}_i=\left\langle(\kappa_i,\upsilon_i);[\log\mathscr{T}_{\mathscr{O}i},\log(1-\mathscr{F}_{\mathscr{O}i})],[\log\mathscr{F}_{\mathscr{O}i},\log(1-\mathscr{T}_{\mathscr{O}i})]\right\rangle$ *be the family of log FVNNs, then*

$$logFVNWG(\mathscr{O}_1,\mathscr{O}_2,...,\mathscr{O}_n)$$
$$=\left[\begin{array}{c}\left(\circledcirc_{i=1}^{n}\kappa_i^{\Lambda_i},\circledcirc_{i=1}^{n}\upsilon_i^{\Lambda_i}\right);\left[\circledcirc_{i=1}^{n}(\log_{\mho_i}\mathscr{T}_{\mathscr{O}i})^{\Lambda_i},\circledcirc_{i=1}^{n}(\log_{\mho_i}(1-\mathscr{F}_{\mathscr{O}i}))^{\Lambda_i}\right],\\\left[\sqrt[3\alpha]{1-\circledcirc_{i=1}^{n}\left(1-(\log_{\mho_i}\mathscr{F}_{\mathscr{O}i})^{3\alpha}\right)^{\Lambda_i}},\sqrt[3\alpha]{1-\circledcirc_{i=1}^{n}\left(1-(\log_{\mho_i}(1-\mathscr{T}_{\mathscr{O}i}))^{3\alpha}\right)^{\Lambda_i}}\right]\end{array}\right].$$

Theorem 5.6. *If all* $\mathscr{O}_i=\left\langle(\kappa_i,\upsilon_i);[\log\mathscr{T}_{\mathscr{O}i},\log(1-\mathscr{F}_{\mathscr{O}i})][\log\mathscr{F}_{\mathscr{O}i},\log(1-\mathscr{T}_{\mathscr{O}i})]\right\rangle$ *are equal and* $\mathscr{O}_i=\mathscr{O}$, *for* $i=1,2,...,n$, *then log FVNWG* $(\mathscr{O}_1,\mathscr{O}_2,...,\mathscr{O}_n)=\mathscr{O}$.

Corollary 5.1. *Boundedness and monotonicity properties are satisfied using the log FVNWG operator.*

5.3. Log GFVNWA operator

Definition 5.3. *Let* $\mathscr{O}_i = \left\langle(\kappa_i, \upsilon_i); [\log \mathscr{T}_{\mathscr{O}i}, \log(1-\mathscr{F}_{\mathscr{O}i})], [\log \mathscr{F}_{\mathscr{O}i}, \log(1-\mathscr{T}_{\mathscr{O}i})]\right\rangle$ *be the family of log FVNN, then log GFVNWA* $(\mathscr{O}_1, \mathscr{O}_2, ..., \mathscr{O}_n) = \left(\oslash_{i=1}^n \Lambda_i \mathscr{O}_i^\Delta\right)^{1/\Delta}$.

Theorem 5.7. *Let* $\mathscr{O}_i = \left\langle(\kappa_i, \upsilon_i); [\log \mathscr{T}_{\mathscr{O}i}, \log(1-\mathscr{F}_{\mathscr{O}i})], [\log \mathscr{F}_{\mathscr{O}i}, \log(1-\mathscr{T}_{\mathscr{O}i})]\right\rangle$ *be the family of log FVNNs, then log GFVNWA* $(\mathscr{O}_1, \mathscr{O}_2, ..., \mathscr{O}_n)$

$$= \left[\begin{array}{c} \left(\left(\oslash_{i=1}^n \Lambda_i \kappa_i^\Delta\right)^{1/\Delta}, \left(\oslash_{i=1}^n \Lambda_i \upsilon_i^\Delta\right)^{1/\Delta}\right); \\ \left[\left(\sqrt[3\alpha]{1-\circledcirc_{i=1}^n\left(1-\left((\log_{\mho_i} \mathscr{T}_{\mathscr{O}i})^\alpha\right)^{3\alpha}\right)^{\Lambda_i}}\right)^{1/q}, \left(\sqrt[3\alpha]{1-\circledcirc_{i=1}^n\left(1-\left((\log_{\mho_i}(1-\mathscr{F}_{\mathscr{O}i}))^\alpha\right)^{3\alpha}\right)^{\Lambda_i}}\right)^{1/q}\right], \\ \left[\begin{array}{c} \sqrt[3\alpha]{1-\left(1-\left(\circledcirc_{i=1}^n\left(\sqrt[3\alpha]{1-\left(1-(\log_{\mho_i}\mathscr{F}_{\mathscr{O}i})^{3\alpha}\right)^\alpha}\right)^{\Lambda_i}\right)^{3\alpha}\right)^{1/q}}, \\ \sqrt[3\alpha]{1-\left(1-\left(\circledcirc_{i=1}^n\left(\sqrt[3\alpha]{1-\left(1-(\log_{\mho_i}(1-\mathscr{T}_{\mathscr{O}i}))^{3\alpha}\right)^\alpha}\right)^{\Lambda_i}\right)^{3\alpha}\right)^{1/q}} \end{array}\right] \end{array}\right].$$

Proof. We prove that,

$$\oslash_{i=1}^n \Lambda_i \mathscr{O}_i^\Delta = \left[\begin{array}{c} \left(\left(\oslash_{i=1}^n \Lambda_i \kappa_i^\Delta\right), \left(\oslash_{i=1}^n \Lambda_i \upsilon_i^\Delta\right)\right); \\ \left[\sqrt[3\alpha]{1-\circledcirc_{i=1}^n\left(1-\left((\log_{\mho_i} \mathscr{T}_{\mathscr{O}i})^\alpha\right)^{3\alpha}\right)^{\Lambda_i}}, \sqrt[3\alpha]{1-\circledcirc_{i=1}^n\left(1-\left((\log_{\mho_i}(1-\mathscr{F}_{\mathscr{O}i}))^\alpha\right)^{3\alpha}\right)^{\Lambda_i}}\right], \\ \left[\circledcirc_{i=1}^n\left(\sqrt[3\alpha]{1-\left(1-(\log_{\mho_i}\mathscr{F}_{\mathscr{O}i})^{3\alpha}\right)^\alpha}\right)^{\Lambda_i}, \circledcirc_{i=1}^n\left(\sqrt[3\alpha]{1-\left(1-(\log_{\mho_i}(1-\mathscr{T}_{\mathscr{O}i}))^{3\alpha}\right)^\alpha}\right)^{\Lambda_i}\right] \end{array}\right].$$

The proof based on inductive approach. □

If $n = 2$, then

$$\Lambda_1\mathscr{O}_1 \veebar \Lambda_2\mathscr{O}_2$$

$$= \left[\begin{array}{c} \left(\Lambda_1\kappa_1^{\Delta} + \Lambda_2\kappa_2^{\Delta}, \Lambda_1\upsilon_1^{\Delta} + \Lambda_2\upsilon_2^{\Delta}\right); \\ \left[\begin{array}{c} \sqrt[3\alpha]{\begin{array}{c}\left(\sqrt[3\alpha]{1-\left(1-\left((\log_{\mho_i}\mathscr{T}_{\mathscr{O}1})^{\alpha}\right)^{3\alpha}\right)^{\Lambda_1}}\right)^{3\alpha} + \left(\sqrt[3\alpha]{1-\left(1-\left((\log_{\mho_i}\mathscr{T}_{\mathscr{O}2})^{\alpha}\right)^{3\alpha}\right)^{\Lambda_1}}\right)^{3\alpha}, \\ -\left(\sqrt[3\alpha]{1-\left(1-\left((\log_{\mho_i}\mathscr{T}_{\mathscr{O}1})^{\alpha}\right)^{3\alpha}\right)^{\Lambda_1}}\right)^{3\alpha} \cdot \left(\sqrt[3\alpha]{1-\left(1-\left((\log_{\mho_i}\mathscr{T}_{\mathscr{O}2})^{\alpha}\right)^{3\alpha}\right)^{\Lambda_1}}\right)^{3\alpha}\end{array}} \\ \sqrt[3\alpha]{\begin{array}{c}\left(\sqrt[3\alpha]{1-\left(1-\left((\log_{\mho_i}(1-\mathscr{F}_{\mathscr{O}1}))^{\alpha}\right)^{3\alpha}\right)^{\Lambda_1}}\right)^{3\alpha} + \left(\sqrt[3\alpha]{1-\left(1-\left((\log_{\mho_i}(1-\mathscr{F}_{\mathscr{O}2}))^{\alpha}\right)^{3\alpha}\right)^{\Lambda_1}}\right)^{3\alpha} \\ -\left(\sqrt[3\alpha]{1-\left(1-\left((\log_{\mho_i}(1-\mathscr{F}_{\mathscr{O}1}))^{\alpha}\right)^{3\alpha}\right)^{\Lambda_1}}\right)^{3\alpha} \cdot \left(\sqrt[3\alpha]{1-\left(1-\left((\log_{\mho_i}(1-\mathscr{F}_{\mathscr{O}2}))^{\alpha}\right)^{3\alpha}\right)^{\Lambda_1}}\right)^{3\alpha}\end{array}} \end{array}\right], \\ \left[\begin{array}{c} \left(\sqrt[3\alpha]{1-\left(1-(\log_{\mho_i}\mathscr{F}_{\mathscr{O}1})^{3\alpha}\right)^{\alpha}}\right)^{\Lambda_1} \cdot \left(\sqrt[3\alpha]{1-\left(1-(\log_{\mho_i}\mathscr{F}_{\mathscr{O}2})^{3\alpha}\right)^{\alpha}}\right)^{\Lambda_1}, \\ \left(\sqrt[3\alpha]{1-\left(1-(\log_{\mho_i}(1-\mathscr{T}_{\mathscr{O}1}))^{3\alpha}\right)^{\alpha}}\right)^{\Lambda_1} \cdot \left(\sqrt[3\alpha]{1-\left(1-(\log_{\mho_i}(1-\mathscr{T}_{\mathscr{O}2}))^{3\alpha}\right)^{\alpha}}\right)^{\Lambda_1} \end{array}\right] \end{array}\right]$$

$$= \left[\begin{array}{c} \left(\oslash_{i=1}^{2}\Lambda_i\kappa_i^{\Delta}, \oslash_{i=1}^{2}\Lambda_i\upsilon_i^{\Delta}\right); \\ \left[\sqrt[3\alpha]{1-\circledcirc_{i=1}^{2}\left(1-\left((\log_{\mho_i}\mathscr{T}_{\mathscr{O}1})^{\alpha}\right)^{3\alpha}\right)^{\Lambda_i}}, \sqrt[3\alpha]{1-\circledcirc_{i=1}^{2}\left(1-\left((\log_{\mho_i}(1-\mathscr{F}_{\mathscr{O}1}))^{\alpha}\right)^{3\alpha}\right)^{\Lambda_i}}\right], \\ \left[\circledcirc_{i=1}^{2}\left(\sqrt[3\alpha]{1-\left(1-(\log_{\mho_i}\mathscr{F}_{\mathscr{O}i})^{3\alpha}\right)^{\alpha}}\right)^{\Lambda_i}, \circledcirc_{i=1}^{2}\left(\sqrt[3\alpha]{1-\left(1-(\log_{\mho_i}(1-\mathscr{T}_{\mathscr{O}i}))^{3\alpha}\right)^{\alpha}}\right)^{\Lambda_i}\right] \end{array}\right].$$

Suppose that it is true for $n = l$ and $l \geq 3$. Thus,

$$\oslash_{i=1}^{l}\Lambda_i\mathscr{O}_i^{\Delta} = \left[\begin{array}{c} \left(\oslash_{i=1}^{l}\Lambda_i\kappa_i^{\Delta}, \oslash_{i=1}^{l}\Lambda_i\upsilon_i^{\Delta}\right); \\ \left[\sqrt[3\alpha]{1-\circledcirc_{i=1}^{l}\left(1-\left((\log_{\mho_i}\mathscr{T}_{\mathscr{O}1})^{\alpha}\right)^{3\alpha}\right)^{\Lambda_i}}, \sqrt[3\alpha]{1-\circledcirc_{i=1}^{l}\left(1-\left((\log_{\mho_i}(1-\mathscr{F}_{\mathscr{O}1}))^{\alpha}\right)^{3\alpha}\right)^{\Lambda_i}}\right], \\ \left[\circledcirc_{i=1}^{l}\left(\sqrt[3\alpha]{1-\left(1-(\log_{\mho_i}\mathscr{F}_{\mathscr{O}i})^{3\alpha}\right)^{\alpha}}\right)^{\Lambda_i}, \circledcirc_{i=1}^{l}\left(\sqrt[3\alpha]{1-\left(1-(\log_{\mho_i}\mathscr{F}_{\mathscr{O}i})^{3\alpha}\right)^{\alpha}}\right)^{\Lambda_i}\right] \end{array}\right].$$

If $n = l + 1$ and we apply, then,

$$\oslash_{i=1}^{l}\Lambda_i\mathscr{O}_i^{\Delta} + \Lambda_{l+1}\mathscr{O}_{l+1}^{\Delta} = \oslash_{i=1}^{l+1}\Lambda_i\mathscr{O}_i^{\Delta}.$$

Now,

$$\oslash_{i=1}^{l}\Lambda_i\mathscr{O}_i^{\Delta}+\Lambda_{l+1}\mathscr{O}_{l+1}^{\Delta}=\Lambda_1\mathscr{O}_1^{\Delta}\veebar\Lambda_2\mathscr{O}_2^{\Delta}\veebar\ldots\veebar w_l\mathscr{O}_l^{\Delta}\veebar\Lambda_{l+1}\mathscr{O}_{l+1}^{\Delta}$$

$$=\left[\begin{array}{c}\left(\oslash_{i=1}^{l}\Lambda_i\kappa_i^{\Delta}+\Lambda_{l+1}\mho_{l+1}^{\Delta},\oslash_{i=1}^{l}\Lambda_i\upsilon_i^{\Delta}+\Lambda_{l+1}\upsilon_{l+1}^{\Delta}\right);\\ \left[\begin{array}{c}\sqrt[3\alpha]{\begin{array}{c}\left(\sqrt[3\alpha]{1-\circledcirc_{i=1}^{l}\left(1-\left((\log_{\mho_i}\mathscr{T}_{\mathscr{O}i})^{\alpha}\right)^{3\alpha}\right)^{\Lambda_i}}\right)^{3\alpha}+\left(\sqrt[3\alpha]{1-\left(1-\left((\log_{\mho_i}\mathscr{T}_{\mathscr{O}l+1})^{\alpha}\right)^{3\alpha}\right)^{\Lambda_1}}\right)^{3\alpha}\\-\left(\sqrt[3\alpha]{1-\circledcirc_{i=1}^{l}\left(1-\left((\log_{\mho_i}\mathscr{T}_{\mathscr{O}i})^{\alpha}\right)^{3\alpha}\right)^{\Lambda_i}}\right)^{3\alpha}\cdot\left(\sqrt[3\alpha]{1-\left(1-\left((\log_{\mho_i}\mathscr{T}_{\mathscr{O}l+1})^{\alpha}\right)^{3\alpha}\right)^{\Lambda_1}}\right)^{3\alpha}\end{array}},\\ \sqrt[3\alpha]{\begin{array}{c}\left(\sqrt[3\alpha]{1-\circledcirc_{i=1}^{l}\left(1-\left((\log_{\mho_i}(1-\mathscr{F}_{\mathscr{O}i}))^{\alpha}\right)^{3\alpha}\right)^{\Lambda_i}}\right)^{3\alpha}+\left(\sqrt[3\alpha]{1-\left(1-\left((\log_{\mho_i}(1-\mathscr{F}_{\mathscr{O}l+1}))^{\alpha}\right)^{3\alpha}\right)^{\Lambda_1}}\right)^{3\alpha}\\-\left(\sqrt[3\alpha]{1-\circledcirc_{i=1}^{l}\left(1-\left((\log_{\mho_i}(1-\mathscr{F}_{\mathscr{O}i}))^{\alpha}\right)^{3\alpha}\right)^{\Lambda_i}}\right)^{3\alpha}\cdot\left(\sqrt[3\alpha]{1-\left(1-\left((\log_{\mho_i}(1-\mathscr{F}_{\mathscr{O}l+1}))^{\alpha}\right)^{3\alpha}\right)^{\Lambda_1}}\right)^{3\alpha}\end{array}}\end{array}\right],\\ \left[\begin{array}{c}\circledcirc_{i=1}^{l}\left(\sqrt[3\alpha]{1-\left(1-(\log_{\mho_i}\mathscr{F}_{\mathscr{O}i})^{3\alpha}\right)^{\alpha}}\right)^{\Lambda_i}\cdot\left(\sqrt[3\alpha]{1-\left(1-(\log_{\mho_i}\mathscr{F}_{\mathscr{O}l+1})^{3\alpha}\right)^{\alpha}}\right)^{\Lambda_1},\\ \circledcirc_{i=1}^{l}\left(\sqrt[3\alpha]{1-\left(1-(\log_{\mho_i}(1-\mathscr{T}_{\mathscr{O}i}))^{3\alpha}\right)^{\alpha}}\right)^{\Lambda_i}\cdot\left(\sqrt[3\alpha]{1-\left(1-(\log_{\mho_i}(1-\mathscr{T}_{\mathscr{O}l+1}))^{3\alpha}\right)^{\alpha}}\right)^{\Lambda_1}\end{array}\right]\end{array}\right].$$

Thus,

$$\oslash_{i=1}^{l+1}\Lambda_i\mathscr{O}_i^{\Delta}=\left[\begin{array}{c}\left(\oslash_{i=1}^{l+1}\Lambda_i\kappa_i^{\Delta},\oslash_{i=1}^{l+1}\Lambda_i\upsilon_i^{\Delta}\right);\\ \left[\sqrt[3\alpha]{1-\circledcirc_{i=1}^{l+1}\left(1-\left((\log_{\mho_i}\mathscr{T}_{\mathscr{O}1})^{\alpha}\right)^{3\alpha}\right)^{\Lambda_i}},\sqrt[3\alpha]{1-\circledcirc_{i=1}^{l+1}\left(1-\left((\log_{\mho_i}(1-\mathscr{F}_{\mathscr{O}1}))^{\alpha}\right)^{3\alpha}\right)^{\Lambda_i}}\right],\\ \left[\circledcirc_{i=1}^{l+1}\left(\sqrt[3\alpha]{1-\left(1-(\log_{\mho_i}\mathscr{F}_{\mathscr{O}i})^{3\alpha}\right)^{\alpha}}\right)^{\Lambda_i},\circledcirc_{i=1}^{l+1}\left(\sqrt[3\alpha]{1-\left(1-(\log_{\mho_i}\mathscr{F}_{\mathscr{O}i})^{3\alpha}\right)^{\alpha}}\right)^{\Lambda_i}\right]\end{array}\right].$$

Hence, $\left(\oslash_{i=1}^{l+1}\Lambda_i\mathscr{O}_i^{\Delta}\right)^{1/\Delta}$

$$=\left[\begin{array}{c}\left(\left(\oslash_{i=1}^{l+1}\Lambda_i\kappa_i^{\Delta}\right)^{1/\Delta},\left(\oslash_{i=1}^{l+1}\Lambda_i\upsilon_i^{\Delta}\right)^{1/\Delta}\right);\\ \left[\left(\sqrt[3\alpha]{1-\circledcirc_{i=1}^{l+1}\left(1-\left((\log_{\mho_i}\mathscr{T}_{\mathscr{O}i})^{\alpha}\right)^{3\alpha}\right)^{\Lambda_i}}\right)^{1/q},\left(\sqrt[3\alpha]{1-\circledcirc_{i=1}^{l+1}\left(1-\left((\log_{\mho_i}(1-\mathscr{F}_{\mathscr{O}i}))^{\alpha}\right)^{3\alpha}\right)^{\Lambda_i}}\right)^{1/q}\right],\\ \left[\begin{array}{c}\sqrt[3\alpha]{1-\left(1-\left(\circledcirc_{i=1}^{l+1}\left(\sqrt[3\alpha]{1-\left(1-(\log_{\mho_i}\mathscr{F}_{\mathscr{O}i})^{3\alpha}\right)^{\alpha}}\right)^{\Lambda_i}\right)\right)^{2}}^{1/q},\\ \sqrt[3\alpha]{1-\left(1-\left(\circledcirc_{i=1}^{l+1}\left(\sqrt[3\alpha]{1-\left(1-(\log_{\mho_i}(1-\mathscr{T}_{\mathscr{O}i}))^{3\alpha}\right)^{\alpha}}\right)^{\Lambda_i}\right)\right)^{2}}^{1/q}\end{array}\right]\end{array}\right].$$

The above expression is true for $l\geq 1$. Put $\alpha=1$, the log GFVNWA operator is modified to the log FVNWA operator.

Theorem 5.8. *If all* $\mathscr{O}_i=\left\langle(\kappa_i,\upsilon_i);[\log\mathscr{T}_{\mathscr{O}i},\log(1-\mathscr{F}_{\mathscr{O}i})],[\log\mathscr{F}_{\mathscr{O}i},\log(1-\mathscr{T}_{\mathscr{O}i})]\right\rangle(i=1,2,...,n)$ *are equal and* $\mathscr{O}_i=\mathscr{O}$*, then log GFVNWA* $(\mathscr{O}_1,\mathscr{O}_2,...,\mathscr{O}_n)=\mathscr{O}$.

Corollary 5.2. *Boundedness and monotonicity properties are satisfied using log GFVNWA operator.*

5.4. Log GFVNWG operator

Definition 5.4. *Let* $\mathscr{O}_i = \left\langle (\kappa_i, \upsilon_i); [\log \mathscr{T}_{\mathscr{O}i}, \log(1-\mathscr{F}_{\mathscr{O}i})], \log \mathscr{I}_{\mathscr{O}i}], [\log \mathscr{F}_{\mathscr{O}i}, \log(1-\mathscr{T}_{\mathscr{O}i})] \right\rangle$ *be the family of log FVNNs, then log GFVNWG* $(\mathscr{O}_1, \mathscr{O}_2, ..., \mathscr{O}_n) = \frac{1}{\Delta}\left(\circledcirc_{i=1}^{n} (\Delta\mathscr{O}_i)^{\Lambda_i} \right)$ $(i = 1, 2, ..., n)$ *is called a log GFVNWG operator.*

Theorem 5.9. *Let* $\mathscr{O}_i = \left\langle (\kappa_i, \upsilon_i); [\log \mathscr{T}_{\mathscr{O}i}, \log(1-\mathscr{F}_{\mathscr{O}i})], [\log \mathscr{F}_{\mathscr{O}i}, \log(1-\mathscr{T}_{\mathscr{O}i})] \right\rangle$ *be the family of log FVNNs, then log GFVNWG*$(\mathscr{O}_1, \mathscr{O}_2, ..., \mathscr{O}_n)$

$$= \left[\begin{array}{c} \left(\frac{1}{\Delta} \circledcirc_{i=1}^{n} (\Delta\kappa_i)^{\Lambda_i}, \frac{1}{\Delta} \circledcirc_{i=1}^{n} (\Delta\upsilon_i)^{\Lambda_i} \right); \\ \left[\begin{array}{c} \sqrt[3\alpha]{1 - \left(1 - \left(\circledcirc_{i=1}^{n} \left(\sqrt[3\alpha]{1 - \left(1 - (\log_{\mho_i} \mathscr{T}_{\mathscr{O}i})^{3\alpha}\right)^{\alpha}} \right)^{\Lambda_i} \right)^{3\alpha} \right)^{1/q}}, \\ \sqrt[3\alpha]{1 - \left(1 - \left(\circledcirc_{i=1}^{n} \left(\sqrt[3\alpha]{1 - \left(1 - (\log_{\mho_i} (1-\mathscr{F}_{\mathscr{O}i}))^{3\alpha}\right)^{\alpha}} \right)^{\Lambda_i} \right)^{3\alpha} \right)^{1/q}} \end{array} \right], \\ \left[\left[\left(\sqrt[3\alpha]{1 - \circledcirc_{i=1}^{n} \left(1 - \left((\log_{\mho_i} \mathscr{F}_{\mathscr{O}i})^{\alpha}\right)^{3\alpha}\right)^{\Lambda_i}} \right)^{1/q}, \left(\sqrt[3\alpha]{1 - \circledcirc_{i=1}^{n} \left(1 - \left((\log_{\mho_i} (1-\mathscr{T}_{\mathscr{O}i}))^{\alpha}\right)^{3\alpha}\right)^{\Lambda_i}} \right)^{1/q} \right] \right] \end{array} \right].$$

Put $\alpha = 1$, the log GFVNWG operator is modified to the log FVNWG operator.

Corollary 5.3. *Boundedness and monotonicity properties are satisfied by log GFVNWG operator.*

Corollary 5.4. *If all* $\mathscr{O}_i = \left\langle (\kappa_i, \upsilon_i); [\log \mathscr{T}_{\mathscr{O}i}, \log(1-\mathscr{F}_{\mathscr{O}i})], [\log \mathscr{F}_{\mathscr{O}i}, \log(1-\mathscr{T}_{\mathscr{O}i})] \right\rangle$ *are equal and* $\mathscr{O}_i = \mathscr{O}$*, for* $i = 1, 2, ..., n$*, then log GFVNWG*$(\mathscr{O}_1, \mathscr{O}_2, ..., \mathscr{O}_n) = \mathscr{O}$.

6. Log FVNN based on MADM

We proposed log FVNN with AOs for MADM. A score function based on log FVNN was used to select the most appropriate option from a set of possibilities in the MADM. Let $\mathscr{O} = \{\mathscr{O}_1, \mathscr{O}_2, ..., \mathscr{O}_n\}$ be the set of n-alternatives, $\xi = \{\xi_1, \xi_2, ..., \xi_m\}$ be the set of m-attributes and weights $\Lambda = \{\Lambda_1, \Lambda_2, ..., \Lambda_m\}$, where $\Lambda_i \in [0, 1]$ and $\sum_i^m \Lambda_i = 1$.

Let $\mathscr{O}_{ij} = \left\langle (\kappa_{ij}, \upsilon_{ij}); [\log_{\mho_i} \mathscr{T}_{\mathscr{O}ij}, \log_{\mho_i}(1-\mathscr{F}_{\mathscr{O}ij})], [\log_{\mho_i} \mathscr{F}_{\mathscr{O}ij}, \log_{\mho_i}(1-\mathscr{T}_{\mathscr{O}ij})] \right\rangle$ denote log FVNN of alternative $\mathscr{O}_i$ in attribute ξ_j, $i = 1, 2, ..., n$ and $j = 1, 2, ..., m$. Since $\left[\log_{\mho_i} \mathscr{T}_{\mathscr{O}ij}, \log_{\mho_i}(1-\mathscr{F}_{\mathscr{O}ij})\right], \left[\log_{\mho_i} \mathscr{F}_{\mathscr{O}ij}, \log_{\mho_i}(1-\mathscr{T}_{\mathscr{O}ij})\right] \in [0, 1]$ and $0 \leq (\log_{\mho_i} 1 - \mathscr{F}_{\mathscr{O}ij}(\varepsilon))^{\alpha} + (\log_{\mho_i} 1 - \mathscr{T}_{\mathscr{O}ij}(\varepsilon))^{\alpha} \leq 1$.

6.1. Algorithm

Step-1: Input the log FVNN decision values.

Step-2: Compute the normalize decision values. The decision matrix $n \times m$ is given by $\Xi = (\widetilde{\xi}_{ij})_{n\times m}$ is normalized into $\widehat{\Xi} = (\breve{\xi}_{ij})_{n\times m}$; where $\breve{\xi}_{ij} = \left\langle (\widehat{\kappa_{ij}}, \widehat{\upsilon_{ij}}); [\widehat{\log_{\mho_i} \mathscr{T}_{\mathscr{O}ij}}, \log_{\mho_i} \widehat{(1-\mathscr{F}_{\mathscr{O}ij})}], [\widehat{\log_{\mho_i} \mathscr{F}_{\mathscr{O}ij}}, \log_{\mho_i} \widehat{(1-\mathscr{T}_{\mathscr{O}ij})}] \right\rangle$ and $\widehat{\kappa_{ij}} = \frac{\mho_{ij}}{\max_i(\kappa_{ij})}, \widehat{\upsilon_{ij}} = \frac{\upsilon_{ij}}{\max_i(\upsilon_{ij})} \cdot \frac{\upsilon_{ij}}{\kappa_{ij}}, \widehat{\log_{\mho_i} \mathscr{T}_{\mathscr{O}ij}} = \log_{\mho_i} \mathscr{T}_{\mathscr{O}ij}$,
$\log_{\mho_i} \widehat{(1-\mathscr{F}_{\mathscr{O}ij})} = \log_{\mho_i}(1-\mathscr{F}_{\mathscr{O}ij})$, where $\mho_i = \prod[\mathscr{T}_{\mathscr{O}_i}, 1-\mathscr{F}_{\mathscr{O}_i}], [\mathscr{F}_{\mathscr{O}_i}, 1-\mathscr{T}_{\mathscr{O}_i}]$.

Step-3: Find the aggregate values for each alternative using log FVNN AOs, attribute ξ_j in $\widetilde{\xi_i}$, $\breve{\xi}_{ij} = \left\langle (\widehat{\kappa_{ij}}, \widehat{\upsilon_{ij}}); [\widehat{\log_{\mho_i} \mathscr{T}_{\mathscr{O}ij}}, \log_{\mho_i} \widehat{(1-\mathscr{F}_{\mathscr{O}ij})}], [\widehat{\log_{\mho_i} \mathscr{F}_{\mathscr{O}ij}}, \log_{\mho_i} \widehat{(1-\mathscr{T}_{\mathscr{O}ij})}] \right\rangle$ is aggregated into $\breve{\xi}_i = \left\langle (\widehat{\kappa_i}, \widehat{\upsilon_i}); [\widehat{\log_{\mho_i} \mathscr{T}_{\mathscr{O}i}}, \log_{\mho_i} \widehat{(1-\mathscr{F}_{\mathscr{O}i})}], [\widehat{\log_{\mho_i} \mathscr{F}_{\mathscr{O}i}}, \log_{\mho_i} \widehat{(1-\mathscr{T}_{\mathscr{O}i})}] \right\rangle$.

Step-4: Calculate the positive and negative ideal values for each case:

$$\breve{\xi}^+ = \begin{bmatrix} \left(\max_{1\le i\le n}(\widehat{\kappa_{ij}}), \min_{1\le i\le n}(\widehat{\upsilon_{ij}}) \right); \\ [1,1],[0,0] \end{bmatrix} \text{ and } \breve{\xi}^- = \begin{bmatrix} \left(\min_{1\le i\le n}(\widehat{\kappa_{ij}}), \max_{1\le i\le n}(\widehat{\upsilon_{ij}}) \right); \\ [0,0],[1,1] \end{bmatrix}$$

Step-5: Determine the EDs between each alternative with positive and negative ideal values are $\Xi_i^+ = \Xi_E\left(\breve{\xi}_i, \breve{\xi}^+\right)$; $\Xi_i^- = \Xi_E\left(\breve{\xi}_i, \breve{\xi}^-\right)$.

Step-6: To calculate the relative closeness values by $\Xi_i^* = \frac{\Xi_i^-}{\Xi_i^+ + \Xi_i^-}$.

Step-7: The maximum value is $\max \Xi_i^*$.

6.2. *Selection process robotics*

This section demonstrates a practical use of MADM utilizing the AOs of log FVNNs. In recent years, many countries' agriculture industry has undergone a substantial transition from traditional farming methods to modern smart farming; we have come a long way, technology has risen to the challenge and is developing cutting-edge methods to increase rural agriculture productivity. The ultimate goal is to raise farmer productivity and food yields in order to sustain the world's expanding population. The combination of AI with technical instruments, such as drones and moisture sensors, is key to achieving this type of expansion in a sustainable manner. One such instance is the usage of robots in agriculture. Farmers can concentrate more on increasing total output yields because of the automation of laborious, slow, and boring duties performed by agricultural robots. Machines are designed to lessen some of the daily labor that people must perform. Due to technological advancement, robots are now capable of doing the labor-intensive, complex, and repetitive activities found in all disciplines of science and technology. Some robots that use artificial intelligence can complete given tasks perfectly and constantly. In order to make the application of robots in agriculture feasible over the long run, a multidisciplinary strategy is needed that will connects engineering technology with agricultural concepts. In Addition to advancements in processor speed and AI capabilities, we can now utilize robots in a number of ways to carry out essential tasks. There are numerous applications for robots in the present day, including directing traffic, managing supplies, welding metal in hostile conditions, and much more. However, robots may generally be grouped into following five categories.

(1) Autonomous Mobile Robot(AMR) ($\mathscr{Z}_1$):
AMRs move around the world while making judgments almost immediately. They are able to gather information about their surroundings with the aid of technologies like sensors and cameras. Having processing equipment on board allows them to analyze the information and make an informed decision, regardless of whether they are evading an approaching employee, selecting the optimal packaging, or determining the surface to disinfect.

(2) Articulated Robot ($\mathscr{Z}_2$):
Robotic arms and articulated robots are designed to resemble the actions of a human arm. These usually have between two and ten rotary joints. All those are perfect for arc welding, material handling, machine tending, and packaging because each extra joint or pivot enables a wider range of motion.

(3) Humanoids Robot($\mathscr{Z}_3$):
Although many mobile humanoid robots could be potentially considered AMRs, the word is typically applied to robots that perform tasks that are human-centric and frequently have human-like shapes. They use many of the same technology components as AMRs in order to perceive, plan, and act when performing tasks such as giving instructions or providing customized service.

(4) Cobot Robot ($\mathscr{Z}_4$):
Cobots are machines that function independently or in tandem with humans. In contrast to the majority of other types of robots, cobots can share workplaces with humans to help them work more efficiently. They are frequently utilized to remove laborious, risky, or taxing operations from regular work flows. Cobots are occasionally able to detect and respond to human movement.

(5) Hybrid Robot ($\mathscr{Z}_5$):
Robots of different kinds are frequently joined to construct hybrid systems that can perform more difficult jobs. A robot for handling packages inside a warehouse might well be made using an AMR and a robotic arm. The ability to compute is being concentrated as more complexity is included in single solutions.

Description and classification for agriculture:

(1) Subsistence Farming (ξ_1):
Subsistence farming is performed by the majority of farmers in the state. It is characterized by small, dispersed landholdings and the use of simple tools. With high levels of poverty, the farmers do not utilize as much fertilizer and high yielding types in their crops as they could. They typically lack necessary amenities like irrigation and electricity.

(2) Shifting Agriculture (ξ_2):
In order to start this sort of agriculture, a piece of forestland needs to be cleared by cutting down trees and burning their trunks and branches. After the ground has been cleaned, crops are planted there for two to three years before being abandoned as the soil loses its fertility. The process is then repeated as the farmers move to new locations. This method of farming typically involves the cultivation of dry paddy, maize, millets, and vegetables.

(3) Plantation Agriculture robot (ξ_3):
Bush or tree cultivation is known as plantation farming. Rubber, tea, chocolate, coconut, and fruit crops, grapes, and oranges are all grown as a single crop. It is capital demanding and requires strong managerial skills, technical expertise, high-end equipment, fertilizers, irrigation systems, and transportation infrastructure. Agriculture on plantations is an export focused industry. The majority of the plants used in plantation agriculture have a lifespan of three years or longer. Tree crops like natural rubber, cocoa in tutus, oil palm, tea, cocoa, and coffee require years to grow but are thereafter fruitful for extended periods of time. Both sides of the equator are considered tropical regions for plantation agriculture. On each continent with a tropical climate, plantations exit. Some plantations, specially those for tea, coffee, and rubber, have on-site or nearby processing facilities.

(4) Intensive Farming (ξ_4):
Farmers use fertilizers and insects extensively in regions where irrigation has been possible. Additionally, they planted high yielding seed varieties on their land. Agricultural automation has resulted from the extensive use of machinery in farming. Its traits include a low fallow ratio

and a larger utilization of inputs like capital and labour per unit of land area, also give it the name industrial agriculture.

Suppose that five robots as $\mathscr{X} = \{\mathscr{X}_1, \mathscr{X}_2, \mathscr{X}_3, \mathscr{X}_4, \mathscr{X}_5\}$. Four attributes are considered as $\xi = \{\xi_1, \xi_2, \xi_3, \xi_4\}$ and their weights are $\Lambda = \{0.4, 0.3, 0.2, 0.1\}$. Our goal is to select the best option for each alternative. Table 1 shows the DM informations.

Step-1: DM information are

Table 1. DM information.

	ξ_1	ξ_2	ξ_3	ξ_4
$\mathscr{X}_1$	$\begin{bmatrix}(0.85, 0.8);\\ [0.5, 0.6],\\ [0.4, 0.5],\end{bmatrix}$	$\begin{bmatrix}(0.8, 0.5);\\ [0.5, 0.55],\\ [0.45, 0.5]\end{bmatrix}$	$\begin{bmatrix}(0.75, 0.65);\\ [0.8, 0.85],\\ [0.15, 0.2]\end{bmatrix}$	$\begin{bmatrix}(0.85, 0.65);\\ [0.7, 0.85],\\ [0.15, 0.3]\end{bmatrix}$
$\mathscr{X}_2$	$\begin{bmatrix}(0.65, 0.6);\\ [0.45, 0.5],\\ [0.5, 0.55],\end{bmatrix}$	$\begin{bmatrix}(0.85, 0.45);\\ [0.58, 0.6],\\ [0.4, 0.42],\end{bmatrix}$	$\begin{bmatrix}(0.65, 0.6);\\ [0.6, 0.65],\\ [0.35, 0.4],\end{bmatrix}$	$\begin{bmatrix}(0.8, 0.6);\\ [0.8, 0.85],\\ [0.15, 0.2]\end{bmatrix}$
$\mathscr{X}_3$	$\begin{bmatrix}(0.9, 0.65);\\ [0.7, 0.75],\\ [0.25, 0.3],\end{bmatrix}$	$\begin{bmatrix}(0.75, 0.65);\\ [0.89, 0.9],\\ [0.1, 0.11],\end{bmatrix}$	$\begin{bmatrix}(0.85, 0.7);\\ [0.7, 0.85],\\ [0.15, 0.3],\end{bmatrix}$	$\begin{bmatrix}(0.75, 0.55);\\ [0.7, 0.75],\\ [0.25, 0.3],\end{bmatrix}$
$\mathscr{X}_4$	$\begin{bmatrix}(0.7, 0.6);\\ [0.6, 0.8],\\ [0.2, 0.4],\end{bmatrix}$	$\begin{bmatrix}(0.65, 0.5);\\ [0.7, 0.75],\\ [0.25, 0.3],\end{bmatrix}$	$\begin{bmatrix}(0.8, 0.75);\\ [0.6, 0.9],\\ [0.1, 0.4],\end{bmatrix}$	$\begin{bmatrix}(0.85, 0.7);\\ [0.85, 0.9],\\ [0.1, 0.15],\end{bmatrix}$
$\mathscr{X}_5$	$\begin{bmatrix}(0.75, 0.7);\\ [0.5, 0.55],\\ [0.45, 0.5],\end{bmatrix}$	$\begin{bmatrix}(0.7, 0.6);\\ [0.72, 0.77],\\ [0.23, 0.28],\end{bmatrix}$	$\begin{bmatrix}(0.75, 0.65);\\ [0.7, 0.75],\\ [0.25, 0.3],\end{bmatrix}$	$\begin{bmatrix}(0.8, 0.75);\\ [0.6, 0.95],\\ [0.05, 0.4],\end{bmatrix}$

Table 2 shows the normalized decision matrix informations.

Step-2: Obtain normalized decision matrix:

Table 2. Normalized decision values.

	ξ_1	ξ_2	ξ_3	ξ_4
$\mathscr{Z}_1$	$\begin{bmatrix}(0.9444, 0.9412);\\ [0.5, 0.6],\\ [0.4, 0.5],\end{bmatrix}$	$\begin{bmatrix}(0.9412, 0.4808);\\ [0.5, 0.55],\\ [0.45, 0.5]\end{bmatrix}$	$\begin{bmatrix}(0.8824, 0.7511);\\ [0.8, 0.85],\\ [0.15, 0.2]\end{bmatrix}$	$\begin{bmatrix}(1, 0.6627);\\ [0.7, 0.85],\\ [0.15, 0.3]\end{bmatrix}$
$\mathscr{Z}_2$	$\begin{bmatrix}(0.7222, 0.6923);\\ [0.45, 0.5],\\ [0.5, 0.55],\end{bmatrix}$	$\begin{bmatrix}(1, 0.3665);\\ [0.58, 0.6],\\ [0.4, 0.42],\end{bmatrix}$	$\begin{bmatrix}(0.7647, 0.7385);\\ [0.6, 0.65],\\ [0.35, 0.4],\end{bmatrix}$	$\begin{bmatrix}(0.9412, 0.6);\\ [0.8, 0.85],\\ [0.15, 0.2]\end{bmatrix}$
$\mathscr{Z}_3$	$\begin{bmatrix}(1, 0.5868);\\ [0.7, 0.75],\\ [0.25, 0.3],\end{bmatrix}$	$\begin{bmatrix}(0.8824, 0.8667);\\ [0.89, 0.9],\\ [0.1, 0.11],\end{bmatrix}$	$\begin{bmatrix}(1, 0.7686);\\ [0.7, 0.85],\\ [0.15, 0.3],\end{bmatrix}$	$\begin{bmatrix}(0.8824, 0.5378);\\ [0.7, 0.75],\\ [0.25, 0.3],\end{bmatrix}$
$\mathscr{Z}_4$	$\begin{bmatrix}(0.7778, 0.6429);\\ [0.6, 0.8],\\ [0.2, 0.4],\end{bmatrix}$	$\begin{bmatrix}(0.7647, 0.5917);\\ [0.7, 0.75],\\ [0.25, 0.3],\end{bmatrix}$	$\begin{bmatrix}(0.9412, 0.9375);\\ [0.6, 0.9],\\ [0.1, 0.4],\end{bmatrix}$	$\begin{bmatrix}(1, 0.7686);\\ [0.85, 0.9],\\ [0.1, 0.15],\end{bmatrix}$
$\mathscr{Z}_5$	$\begin{bmatrix}(0.8333, 0.8167);\\ [0.5, 0.55],\\ [0.45, 0.5],\end{bmatrix}$	$\begin{bmatrix}(0.8235, 0.7912);\\ [0.72, 0.77],\\ [0.23, 0.28],\end{bmatrix}$	$\begin{bmatrix}(0.8824, 0.7511);\\ [0.7, 0.75],\\ [0.25, 0.3],\end{bmatrix}$	$\begin{bmatrix}(0.9412, 0.9375);\\ [0.6, 0.95],\\ [0.05, 0.4],\end{bmatrix}$

Table 3 shows the log FVNWA operator for every alternative.

Step-3: Aggregate information based on log FVNWA operator for every alternative ($\alpha = 1$).

Table 3. Log FVNWA operator.

$\breve{\mathscr{Z}}_1$	$\breve{\mathscr{Z}}_2$	$\breve{\mathscr{Z}}_3$	$\breve{\mathscr{Z}}_4$	$\breve{\mathscr{Z}}_5$
$\begin{bmatrix}(0.9366, 0.7372);\\ [0.3166, 0.3439],\\ [0.1985, 0.2064]\end{bmatrix}$	$\begin{bmatrix}(0.8359, 0.5946);\\ [0.3157, 0.3204],\\ [0.1988, 0.2014]\end{bmatrix}$	$\begin{bmatrix}(0.9529, 0.7022);\\ [0.2686, 0.2804],\\ [0.2465, 0.2482]\end{bmatrix}$	$\begin{bmatrix}(0.8288, 0.699);\\ [0.2955, 0.3177],\\ [0.2254, 0.2168]\end{bmatrix}$	$\begin{bmatrix}(0.851, 0.808);\\ [0.2919, 0.3855],\\ [0.1838, 0.2338]\end{bmatrix}$

Step-4: The both ideal values of each alternatives are

$$\breve{\mathscr{Z}}^{+} = \left[(0.9529, 0.5946); [1, 1], [0, 0]\right],$$

$$\breve{\mathscr{Z}}^{-} = \left[(0.8288, 0.808); [0, 0], [1, 1]\right].$$

Step-5: The HD between every alternative with different ideal values are

$$\Xi_1^{+} = 0.6871, \Xi_2^{+} = 0.7558, \Xi_3^{+} = 0.6171, \Xi_4^{+} = 0.7150, \Xi_5^{+} = 0.7099,$$

and

$$\Xi_1^{-} = 0.5631, \Xi_2^{-} = 0.4944, \Xi_3^{-} = 0.6331, \Xi_4^{-} = 0.5352, \Xi_5^{-} = 0.5403.$$

Step-6: Relative closeness values are

$$\Xi_1^{*} = 0.4504, \Xi_2^{*} = 0.3955, \Xi_3^{*} = 0.5064, \Xi_4^{*} = 0.4281, \Xi_5^{*} = 0.4322.$$

Step-7: Ranking of alternatives are

$$\mathscr{Z}_3 \geq \mathscr{Z}_1 \geq \mathscr{Z}_5 \geq \mathscr{Z}_4 \geq \mathscr{Z}_2.$$

The humanoid robot $\mathscr{Z}_3$ is the best one.

6.3. Comparison between the suggested and existing methods

The comparison between the suggested models and a few of the existing models were made in this subsection. This demonstrates its value and advantages. Yang et al. discussed the new notion of interval-valued Pythagorean normal fuzzy information aggregation operators for MADM approach [21]. Recently, Palanikumar et al. [54] interacted MADM approach for Pythagorean neutrosophic normal interval-valued aggregation operators. with the use of the log FVNWA, log FVNWG, log GFVNWA, and log GFVNWG methods respectively, using ED and HD. Table 4 shows the comparison between existing and proposed methods.

Table 4. Comparison table.

$\alpha = 1$	FVNWA	FVNWG	GFVNWA	GFVNWG
TOPSIS- HD Proposed	$\mathscr{Z}_3 \geq \mathscr{Z}_1 \geq \mathscr{Z}_5$ $\mathscr{Z}_4 \geq \mathscr{Z}_2$	$\mathscr{Z}_3 \geq \mathscr{Z}_1 \geq \mathscr{Z}_5$ $\mathscr{Z}_4 \geq \mathscr{Z}_2$	$\mathscr{Z}_3 \geq \mathscr{Z}_1 \geq \mathscr{Z}_5$ $\mathscr{Z}_4 \geq \mathscr{Z}_2$	$\mathscr{Z}_3 \geq \mathscr{Z}_1 \geq \mathscr{Z}_5$ $\mathscr{Z}_4 \geq \mathscr{Z}_2$
TOPSIS-HD [21]	$\mathscr{Z}_3 \geq \mathscr{Z}_4 \geq \mathscr{Z}_5$ $\mathscr{Z}_1 \geq \mathscr{Z}_2$	$\mathscr{Z}_3 \geq \mathscr{Z}_4 \geq \mathscr{Z}_5$ $\mathscr{Z}_1 \geq \mathscr{Z}_2$	$\mathscr{Z}_3 \geq \mathscr{Z}_4 \geq \mathscr{Z}_5$ $\mathscr{Z}_1 \geq \mathscr{Z}_2$	$\mathscr{Z}_3 \geq \mathscr{Z}_4 \geq \mathscr{Z}_5$ $\mathscr{Z}_1 \geq \mathscr{Z}_2$
Score (Proposed)	$\mathscr{Z}_3 \geq \mathscr{Z}_5 \geq \mathscr{Z}_4$ $\mathscr{Z}_1 \geq \mathscr{Z}_2$	$\mathscr{Z}_3 \geq \mathscr{Z}_5 \geq \mathscr{Z}_4$ $\mathscr{Z}_1 \geq \mathscr{Z}_2$	$\mathscr{Z}_3 \geq \mathscr{Z}_5 \geq \mathscr{Z}_4$ $\mathscr{Z}_1 \geq \mathscr{Z}_2$	$\mathscr{Z}_3 \geq \mathscr{Z}_5 \geq \mathscr{Z}_4$ $\mathscr{Z}_1 \geq \mathscr{Z}_2$
Score [54]	$\mathscr{Z}_3 \geq \mathscr{Z}_5 \geq \mathscr{Z}_4$ $\mathscr{Z}_1 \geq \mathscr{Z}_2$	$\mathscr{Z}_3 \geq \mathscr{Z}_5 \geq \mathscr{Z}_4$ $\mathscr{Z}_1 \geq \mathscr{Z}_2$	$\mathscr{Z}_3 \geq \mathscr{Z}_5 \geq \mathscr{Z}_4$ $\mathscr{Z}_1 \geq \mathscr{Z}_2$	$\mathscr{Z}_3 \geq \mathscr{Z}_5 \geq \mathscr{Z}_4$ $\mathscr{Z}_1 \geq \mathscr{Z}_2$

Comparison of the merits of the MADM method to those of competing approaches. The nearest positions and values are listed below. The various values are derived using the log FVNWA technique. Generate data through using log FVNWA operator for the alternatives ($\alpha = 2$). Table 5 shows the log FVNWA operator for every alternative.

Step-3: Aggregate information based on log FVNWA operator for every alternatives ($\alpha = 2$).

Table 5. Log FVNWA operator.

$\check{\mathscr{Z}}_1$	$\check{\mathscr{Z}}_2$	$\check{\mathscr{Z}}_3$	$\check{\mathscr{Z}}_4$	$\check{\mathscr{Z}}_5$
$\begin{bmatrix}(0.9366, 0.7372); \\ [0.3324, 0.3659], \\ [0.1985, 0.2064],\end{bmatrix}$	$\begin{bmatrix}(0.8359, 0.5946); \\ [0.3358, 0.3388], \\ [0.1988, 0.2014]\end{bmatrix}$	$\begin{bmatrix}(0.9529, 0.7022); \\ [0.2833, 0.3047], \\ [0.2465, 0.2482]\end{bmatrix}$	$\begin{bmatrix}(0.8288, 0.6990); \\ [0.3074, 0.3405], \\ [0.2254, 0.2168]\end{bmatrix}$	$\begin{bmatrix}(0.8510, 0.8080); \\ [0.3179, 0.4294], \\ [0.1838, 0.2338]\end{bmatrix}$

Step-4: The positive and negative ideal values of each alternative are

$$\check{\mathscr{Z}}^{+} = \Big[(0.9529, 0.5946); [1, 1], [0, 0]\Big]$$

and

$$\check{\mathscr{Z}}^{-} = \Big[(0.8288, 0.8080); [0, 0], [1, 1]\Big].$$

Step-5: The HD between every alternative with different ideal values are

$$\Xi_1^+ = 0.6997, \Xi_2^+ = 0.7670, \Xi_3^+ = 0.6305, \Xi_4^+ = 0.7255, \Xi_5^+ = 0.7324,$$

and

$$\Xi_1^- = 0.5505, \Xi_2^- = 0.4833, \Xi_3^- = 0.6198, \Xi_4^- = 0.5247, \Xi_5^- = 0.5178.$$

Step-6: Relative closeness values are

$$\Xi_1^* = 0.4404, \Xi_2^* = 0.3865, \Xi_3^* = 0.4957, \Xi_4^* = 0.4197, \Xi_5^* = 0.4142.$$

Step-7: Ranking of alternatives are

$$\mathscr{Z}_3 \geq \mathscr{Z}_1 \geq \mathscr{Z}_4 \geq \mathscr{Z}_5 \geq \mathscr{Z}_2.$$

Figure 1 shows the different α values for all alternative.

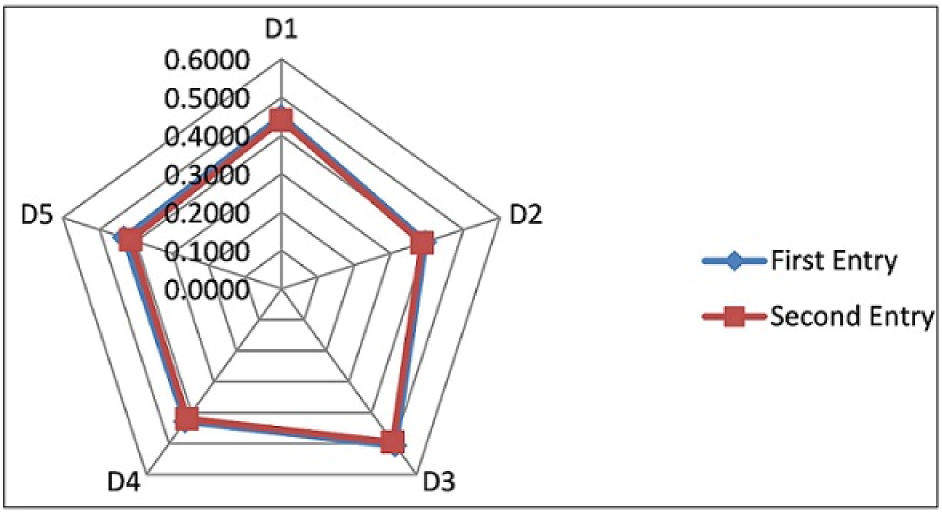

Figure 1. Different α values.

6.4. Critical analysis

The ranking of the alternative, based on the log FVNWA method, is $\mathscr{Z}_3 \geq \mathscr{Z}_1 \geq \mathscr{Z}_5 \geq \mathscr{Z}_4 \geq \mathscr{Z}_2$. In the event when $\alpha = 2$, the ranking of the alternatives in a new order is as follows: $\mathscr{Z}_3 \geq \mathscr{Z}_1 \geq \mathscr{Z}_4 \geq \mathscr{Z}_5 \geq \mathscr{Z}_2$. Hence, the robotic $\mathscr{Z}_5$ is replaced by the robotic $\mathscr{Z}_4$ as the best option. The log FVNWG, log GFVNWA, and log GFVNWG operators can be used in a similar manner.

6.5. Advantages

The applications in the numerous advantages, in accordance with the study previously presented. It presents the concept of log FVNN by combining the concepts of FVNS. The log FVNN analyses human behavior and natural events that, in real life, follow a normal distribution. It explains complex information with the total of TMG, IMG, and FMG being greater than 1, but the square total of its TMG, IMG, and FMG being less than 1. We find the most suitable alternative based on a set of options provided by the decision maker using the proposed log FVNS AOs. Therefore, the proposed MADM technique based on log FVNS AOs provides another approach for finding the most effective alternative in DM. Depending on α and their own preferences, the decision maker is free to select the outcome. Different ranking outcomes of each alternative could be produced dynamically with the aid of operators like log FVNWA, log FVNWG, log GFVNWA, and log GFVNWA.

7. Conclusions

This article focused on the log FVN using MADM problems that arise in various DM domains. We reached a number of findings in our discussion from several AOs' log FVNs that have been important to their log FVNs. We have recommended AO criteria for log FVNWA, log FVNWG, log GFVNWA, and log GFVNWG. In scenarios with unclear and contradictory facts, the application of the log FVN based on the MADM methodology can assist individuals in selecting the appropriate action from the available alternatives. We have applied the operators for log FVNWA, log FVNWG, log GFVNWA, and log GFVNWG to the MADM problem based on α. The different rankings can be estimated to use the operators for log FVNWA, log FVNWG, log GFVNWA, and log GFVNWG based on α. As a conclusion, the α criteria with the strongest effect on the rank of alternatives have also been examined at. By setting the values of α in line with the real scenario, the decision-makers can choose the most appropriate ranking. Consequently, the decision-maker can base their method selection on the actual values of α. In order to show the applicability and benefits of the suggested models, we finally compared them to many currently employed models. Further discussions will be held on the following topics are:

(1) Soft sets and expert sets are explored in terms of log FVN.
(2) Based on log FVN, we investigate Pythagorean cubic FSs and spherical cubic FSs.
(3) A generalized Fermatean cubic FS and complex FFS can be used to solve the problem of MADM.

Conflict of interest

The authors declare no conflict of interest.

References

1. L. A. Zadeh, Fuzzy sets, *Inform. Control*, **8** (1965), 338–353. https://doi.org/10.1016/S0019-9958(65)90241-X

2. K. T. Atanassov, Intuitionistic fuzzy sets, *Fuzzy Sets Syst.*, **20** (1986), 87–96. https://doi.org/10.1016/S0165-0114(86)80034-3

3. M. B. Gorzalczany, A method of inference in approximate reasoning based on interval-valued fuzzy sets, *Fuzzy Set Syst.*, **21** (1987), 1–17. https://doi.org/10.1016/0165-0114(87)90148-5

4. R. Biswas, Vague groups, *Int. J. Comput. Cognition*, **4** (2006), 20–23.

5. R. R. Yager, Pythagorean membership grades in multi criteria decision making, *IEEE T. Fuzzy Sys.*, **22** (2014), 958–965. https://doi.org/10.1109/TFUZZ.2013.2278989

6. X. D. Peng, Y. Yang, Fundamental properties of interval valued Pythagorean fuzzy aggregation operators, *Int. J. Intell. Syst.*, **31** (2015), 444–487. https://doi.org/10.1002/INT.21790
7. S. Ashraf, S. Abdullah, T. Mahmood, F. Ghani, T. Mahmood, Spherical fuzzy sets and their applications in multi-attribute decision making problems, *J. Intell. Fuzzy Syst.*, **36** (2019), 2829–2844. https://doi.org/10.3233/JIFS-172009
8. F. Smarandache, Neutrosophic set-a generalization of the intuitionstic fuzzy set, *Int. J. Pure Appl. Math.*, **24** (2005), 287–297.
9. T. Senapati, R. R. Yager, Fermatean fuzzy sets, *J. Amb. Intell. Hum. Comp.*, **11** (2020), 663–674.
10. B. C. Cuong, V. Kreinovich, Picture fuzzy sets-A new concept for computational intelligence problems, *2013 Third World Congress on Information and Communication Technologies (WICT 2013)*, 2013. https://doi.org/10.1109/WICT.2013.7113099
11. M. Akram, W. A. Dudek, F. Ilyas, Group decision-making based on Pythagorean fuzzy TOPSIS method, *Int. J. Intell. Syst.*, **34** (2019), 1455–1475. https://doi.org/10.1002/int.22103
12. M. Akram, W. A. Dudek, J. M. Dar, Pythagorean Dombi fuzzy aggregation operators with application in multicriteria decision-making, *Int. J. Intell. Syst.*, **34** (2019), 3000–3019. https://doi.org/10.1002/int.22183
13. M. Akram, X. D. Peng, A. N. Al-Kenani, A. Sattar, Prioritized weighted aggregation operators under complex Pythagorean fuzzy information, *J. Intell. Fuzzy Syst.*, **39** (2020), 4763–4783. https://doi.org/10.3233/JIFS-200684
14. T. M. Athira, S. J. John, H. Garg, Entropy and distance measures of Pythagorean fuzzy soft sets and their applications, *J. Intell. Fuzzy Syst.*, **37** (2019), 4071–4084. https://doi.org/10.3233/JIFS-190217
15. H. Garg, Some picture fuzzy aggregation operators and their applications to multi criteria decision-making, *Arab. J. Sci. Eng.*, **42** (2017), 5275–5290.
16. H. Garg, New logarithmic operational laws and their aggregation operators for Pythagorean fuzzy set and their applications, *Int. J. Intell. Syst.*, **34** (2019), 82–106. https://doi.org/10.1002/int.22043
17. Z. Xu, Intuitionistic fuzzy aggregation operators, *IEEE T. Fuzzy Syst.*, **15** (2007), 1179–1187. https://doi.org/10.1109/TFUZZ.2006.890678
18. W. F. Liu, J. Chang, X. He, Generalized Pythagorean fuzzy aggregation operators and applications in decision making, *Control Decis.*, **31** (2016), 2280–2286. http://doi.org/10.13195/j.kzyjc.2015.1537
19. K. Rahman, S. Abdullah, M. Shakeel, M. S. A. Khan, M. Ullah, Interval-valued Pythagorean fuzzy geometric aggregation operators and their application to group decision making problem, *Cogent Math.*, **4** (2017), 1338638. https://doi.org/10.1080/23311835.2017.1338638
20. K. Rahman, A. Ali, S. Abdullah, F. Amin, Approaches to multi-attribute group decision making based on induced interval valued Pythagorean fuzzy Einstein aggregation operator, *New Math. Nat. Comput.*, **14** (2018), 343–361. http://doi.org/10.1142/S1793005718500217
21. Z. L. Yang, J. P. Chang, Interval-valued Pythagorean normal fuzzy information aggregation operators for multi-attribute decision making, *IEEE Access*, **8** (2020), 51295–51314. https://doi.org/10.1109/ACCESS.2020.2978976

22. F. K. Gündoğdu, C. Kahraman, Spherical fuzzy sets and spherical fuzzy TOPSIS method, *J. Intell. Fuzzy Syst.*, **36**, (2019), 337–352. http://doi.org/10.3233/JIFS-181401

23. P. D. Liu, G. Shahzadi, M. Akram, Specific types of q-Rung picture fuzzy Yager aggregation operators for decision-making, *Int. J. Comput. Intell. Syst.*, **13** (2020), 1072–1091. https://doi.org/10.2991/ijcis.d.200717.001

24. H. Bustince, P. Burillo, Vague sets are intuitionistic fuzzy sets, *Fuzzy Set Syst.*, **79** (1996), 403–405. https://doi.org/10.1016/0165-0114(95)00154-9

25. A. Kumar, S.P. Yadav, S. Kumar, Fuzzy system reliability analysis using Tw (the weakest t-norm) based arithmetic operations on $L-R$ type interval valued vague sets, *Int. J. Qual. Reliab. Manag.*, **24** (2007), 846–860. https://doi.org/10.1108/02656710710817126

26. J. Wang, S. Y. Liu, J. Zhang, S. Y. Wang, On the parameterized OWA operators for fuzzy MCDM based on vague set theory, *Fuzzy Optim. Decis. Mak.*, **5** (2006), 5–20. http://doi.org/10.1007/s10700-005-4912-2

27. X. L. Zhang, Z. S. Xu, Extension of TOPSIS to multiple criteria decision making with Pythagorean fuzzy sets, *Int. J. Intell. Syst.*, **29** (2014), 1061–1078. https://doi.org/10.1002/int.21676

28. C. Jana, M. Pal, Application of bipolar intuitionistic fuzzy soft sets in decision-making problem, *Int. J. Fuzzy Syst. Appl.*, **7** (2018), 32–55. https://doi.org/10.4018/IJFSA.2018070103

29. C. Jana, Multiple attribute group decision-making method based on extended bipolar fuzzy MABAC approach, *Comput. Appl. Math.*, **40** (2021), 227.

30. C. Jana, M. Pal, A Robust single-valued neutrosophic soft aggregation operators in multi-criteria decision making, *Symmetry*, **11** (2019), 110. https://doi.org/10.3390/sym11010110

31. C. Jana, M. Pal, J. Q. Wang, A robust aggregation operator for multi-criteria decision making method with bipolar fuzzy soft environment, *Iran. J. Fuzzy Syst.*, 2019.

32. C. Jana, T. Senapati, M. Pal, Pythagorean fuzzy Dombi aggregation operators and its applications in multiple attribute decision-making, *Int. J. Intell. Syst.*, **34** (2019), 2019–2038. http://doi.org/10.1002/int.22125

33. K. Ullah, T. Mahmood, Z. Ali, N. Jan, On some distance measures of complex Pythagorean fuzzy sets and their applications in pattern recognition, *Complex Intell. Syst.*, **6** (2019), 15–27.

34. C. Jana, M. Pal, F. Karaaslan, J. Q. Wang, Trapezoidal neutrosophic aggregation operators and their application to the multi-attribute decision-making process, *Sci. Iran.*, **27** (2020), 1655–1673. http://doi.org/10.24200/sci.2018.51136.20

35. C. Jana, M. Pal, Multi-criteria decision-making process based on some single-valued neutrosophic Dombi power aggregation operators, *Soft Comput.*, **25** (2021), 5055–5072.

36. Z.Yang, J.Chang, Interval-valued Pythagorean normal fuzzy information aggregation operators for multi-attribute decision making, *IEEE Acess*, **8** (2020), 51295–51314. https://doi.org/10.1109/ACCESS.2020.2978976

37. M. S. Yang, C. H. Ko, On a class of fuzzy c-numbers clustering procedures for fuzzy data, *Fuzzy Set Syst.*, **84** (1996), 49–60. https://doi.org/10.1016/0165-0114(95)00308-8

38. Y. Rong, L. Y. Yu, W. Y. Niu, Y. Liu, T. Senapati, A. R. Mishra, MARCOS approach based upon cubic Fermatean fuzzy set and its application in evaluation and selecting cold chain logistics distribution center, *Eng. Appl. Artif. Intell.*, **116** (2022), 105401. https://doi.org/10.1016/j.engappai.2022.105401

39. Y. Rong, W. Y. Niu, H. Garg, Y. Liu, L. Y. Yu, A hybrid group decision approach based on MARCOS and regret theory for pharmaceutical enterprises assessment under a single-valued neutrosophic scenario, *Systems*, **10** (2022), 106. https://doi.org/10.3390/systems10040106

40. Y. Jin, S. Ashraf, S. Abdullah, Spherical fuzzy logarithmic aggregation operators based on entropy and their application in decision support systems, *Entropy*, **21** (2019), 628. https://doi.org/10.3390/e21070628

41. S. Ashraf, S. Abdullah, F. Smarandache, N. ul Amin, Logarithmic hybrid aggregation operators based on single valued neutrosophic sets and their applications in decision support systems, *Symmetry*, **11** (2019), 364. https://doi.org/10.3390/sym11030364

42. D. Pamucar, I. Badi, K. Sanja, R. Obradovic, A novel approach for the selection of power generation technology using an linguistic neutrosophic combinative distance-based assessment (CODAS) method: A case study in Libya, *Energies*, **11** (2018), 2489. https://doi.org/10.3390/en11092489

43. B. Bairagi, A homogeneous group decision making for selection of robotic systems using extended TOPSIS under subjective and objective factors, *Decis. Mak. Appl. Manag. Eng.*, **5** (2022), 300–315. https://doi.org/10.31181/dmame0304052022b

44. S. Said, H. Bouloiz, M. Gallab, New model for making resilient decisions in an uncertain context: The rational resilience based decision making model (R2DM), *Decis. Mak. Appli. Manag. Eng.*, **6** (2023), 34–57. https://doi.org/10.31181/dmame0601051229022s

45. M. R. Khan, K. Ullah, Q. Khan, Multi-attribute decision-making using Archimedean aggregation operator in T-spherical fuzzy environment, *Rep. Mech. Eng.*, **4** (2023). https://doi.org/10.31181/rme20031012023k

46. M. Riazand, H. M. A. Farid, Picture fuzzy aggregation approach with application to third-partylogistic provider selection process, *Rep. Mech. Eng.*, **3** (2022), 227–236. https://doi.org/10.31181/rme20023062022r

47. D. T. Do, Application of fuca method for multi-criteria decision making in mechanical machining processes, *Oper. Res. Eng. Sci. Theory Appl.*, **5** (2022), 131–152. https://https://doi.org/10.31181/oresta051022061d

48. S. Biswas, G. Bandyopadhyay, D. Pamucar, N. Joshi, A multi-criteria based stock selection frameworkin emerging market, *Oper. Res. Eng. Sci. Theory Appl.*, 2022. https://doi.org/10.31181/oresta161122121b

49. M. K. Hasan, M. Y. Ali, A. Sultana, N. K. Mitra, Some picture fuzzy mean operators and their applications in decision-making, *J. Fuzzy. Ext. Appl.*, **3** (2022), 349–361.

50. M. Abbas, M. W. Asghar, Y. H. Guo, Decision-making analysis of minimizing the death rate due to COVID-19 by using q-rung orthopair fuzzy soft bonferroni mean operator, *J. Fuzzy Ext. Appl.*, **3** (2022), 231–248. https://doi.org/10.22105/jfea.2022.335045.1214

51. F. Liu, G. Aiwu, V. Lukovac, M. Vukic, A multicriteria model for the selection of the transport service provider: A single valued neutrosophic DEMATEL multicriteria model, *Decis. Mak. Appl. Manag. Eng.*, **1** (2018), 121–130.
52. C. Jana, G. Muhiuddin, M. Pal, Multi-criteria decision making approach based on SVTrN Dombi aggregation functions, *Artif. Intell. Rev.*, **54** (2021), 3685–3723.
53. A. K. Adak, G. Kumar, Spherical distance measurement method for solving MCDM problems under Pythagorean fuzzy environment, *J. Fuzzy Ext. Appl.*, **4** (2023), 28–39. https://doi.org/10.22105/jfea.2022.351677.1224
54. M. Palanikumar, K. Arulmozhi, C. Jana, Multiple attribute decision-making approach for Pythagorean neutrosophic normal interval-valued fuzzy aggregation operators, *Comput. Appl. Math.*, **41** (2022), 90.
55. X. D. Peng, H. Y. Yuan, Fundamental properties of Pythagorean fuzzy aggregation operators, *Fund. Inform.*, **147** (2016), 415–446. https://doi.org/10.3233/FI-2016-1415