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Research article

Extended MABAC method based on 2-tuple linguistic T-spherical fuzzy sets and Heronian mean operators: An application to alternative fuel selection

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Abstract: In recent years, fossil fuel resources have become increasingly rare and caused a variety of problems, with a global impact on economy, society and environment. To tackle this challenge, we must promote the development and diffusion of alternative fuel technologies. The use of cleaner fuels can reduce not only economic cost but also the emission of gaseous pollutants that deplete the ozone layer and accelerate global warming. To select an optimal alternative fuel, different fuzzy decision analysis methodologies can be utilized. In comparison to other extensions of fuzzy sets, the T-spherical fuzzy set is an emerging tool to cope with uncertainty by quantifying acceptance, abstention and rejection jointly. It provides a general framework to unify various fuzzy models including fuzzy sets, picture fuzzy sets, spherical fuzzy sets, intuitionistic fuzzy sets, Pythagorean fuzzy sets and generalized orthopair fuzzy sets. Meanwhile, decision makers prefer to employ linguistic terms when expressing qualitative evaluation in real-life applications. In view of these facts, we develop an extended multi-attributive border approximation area comparison (MABAC) method for solving multiple attribute group decision-making problems in this study. Firstly, the combination of T-spherical fuzzy sets with 2-tuple linguistic representation is presented, which provides a general framework for expressing and computing qualitative evaluation. Secondly, we put forward four kinds of 2-tuple linguistic T-spherical fuzzy aggregation operators by considering the Heronian mean operator. We investigate some fundamental properties of the proposed 2-tuple linguistic T-spherical fuzzy aggregation operators. Lastly, an extended MABAC method based on the 2-tuple linguistic T-spherical fuzzy generalized weighted Heronian mean and the 2-tuple linguistic T-spherical fuzzy weighted geometric Heronian mean operators is developed. For illustration, a case study on fuel technology selection with 2-tuple linguistic T-spherical fuzzy information is also conducted.

Moreover, we show the validity and feasibility of our approach by comparing it with several existing approaches.

Keywords: 2-tuple linguistic T-spherical fuzzy set; Heronian mean operator; MAGDM; MABAC; fuel technology selection
Mathematics Subject Classification: 03E72, 03E75, 90B50

1. Introduction

The conventional production and consumption relying on fossil fuel resources can lead to a number of serious problems including greenhouse gas emission, energy shortage, oil price fluctuation and so forth. The idea of sustainable production and consumption is gradually becoming a universally accepted goal of human society [13]. Policy makers in various countries around the world are paying increasingly more attention for seeking cheaper and cleaner alternative fuels. Relevant research reveals that the use of alternative fuel technologies can be crucial in gaseous pollutant control, biomass waste management, socioeconomic growth and sustainable development. Alternative fuels can be chosen from a large variety of different options such as ethanol, natural gas, propane, bio-diesel, hydrogen and others. In the selection of an optimal alternative fuel technology, one should take into account multiple criteria such as economic viability, quality of performance and environmental safety. In fact, alternative fuel technology selection can be viewed as a multiple attribute decision-making (MADM) problem [24, 44].

As a prominent branch of MADM research, multiple attribute group decision-making (MAGDM) refers to a process in which alternatives are assessed by a group of decision makers (DMs) based on several conflicting criteria to reach a consensus on ranking all alternatives or selecting the optimal one from them. Much attention has been paid to the methodological approaches to tackle the generalized fuzzy circumstances in recent publications on MAGDM. Because of the high intensity of MAGDM issues, it is not easy for DMs to obtain all the feasible information of alternatives accurately. Therefore, how to deal with vagueness and ambiguity becomes crucial for choosing the most acceptable alternative in realistic decision-making [55, 63] issues. In the MAGDM framework, many researchers and scientists have committed themselves to explore methodologies that can effectively describe fuzziness in decision-making information. However, it is very difficult to communicate the assessment details in various group decision-making [19, 34] problems because of the fuzziness of human perception and the ambiguity of the environment. To resolve this issue fuzzy set (FS) theory was proposed by Zadeh [61] as an extension of classical sets. Researchers have paid interest to FS and its various applications, but the limitation of FS is that it only describes the information about membership degree (MD). To enhance the ability of uncertainty modeling, scholars put forth some more sophisticated fuzzy theories such as the intuitionistic fuzzy set (IFS) [12], the Pythagorean fuzzy set (PyFS) [59], and the q-rung orthopair fuzzy set (q-ROFS) [60], with a restriction that the sum, square sum, and q-th sum of both MD and non-membership degree (NMD) on a scale of 0 to 1, respectively. These advanced FSs have shown great power in decision-making under uncertainty. For instance, Zhao et al. [64] took advantage of IFSs to capture the uncertain information, which is useful for stock investment selection. Liang et al. [30] applied three-way decision theory to research and development project selection with Pythagorean fuzzy information. Rani and Mishra [50] established a weighted aggregated sum product based assessment scheme to choose optimal fuel technology with the aid of q-ROFSs.

Maji et al. [38] defined the intuitionistic fuzzy soft set (IFSS) by combining soft sets with IFSs. Agarwal et al. [1] further introduced the generalized intuitionistic fuzzy soft set (GIFSS). Feng et al. [17] improved the GIFSS and simplified this notion as a synthesis of a basic IFSS and a parametric IFS. Using IFSS information, Feng et al. [18] further extended the preference ranking organization method for enrichment evaluation method. Liu et al. [32] proposed the q-ROF linguistic family of point aggregation operators (AOs) for linguistic q-ROFSs and designed a novel MAGDM methodology to process the linguistic q-ROF information. But all of the above described FSs have duplet forms such as MD and NMD, these approaches are unable to address the degrees of abstinence and refusal of human opinion. To handle that situation the concept of picture fuzzy set (PFS), as the direct extension of FS and IFS was introduced. Cuong and Kreinovich [15] introduced the concept of PFS in the form of triplets using MD, abstinence degree (AD), and NMD with a restriction, their sum must not exceed 1. To overcome the limitation of PFS, Gündogdu and Kahraman [21] presented the idea of spherical fuzzy sets (SFSs) which is the generalization of PyFS and PFS in which the square sum of MD, AD, and NMD lies between [0,1]. Further, Kahraman et al. [27] studied the TOPSIS method in spherical fuzzy circumstances and select the best hospital location. When a decision maker assigns a positive grade of 0.9, an abstinence grade of 0.85, and a negative grade of 0.8, the PFS and SFS are unable to deal with that case. Therefore, to tackle the sum and square sum limitation of PFS and SFS, Mahmood et al. [39] proposed the T-spherical fuzzy set (T-SFS), whose structure is a generalization of q-ROFS and SFS, with a great ability to deal with uncertainties. In certain scenarios, because of the non-applicability of the PFS and SFS, the T-SFS was introduced. The T-SFSs, based on three characteristic functions known as MD, AD, and NMD with the restriction that the sum of q-th powers of all three degrees must not exceed 1. The structure of T-SFSs is diverse in nature but, similar to q-ROFSs. T-SFSs can manage all circumstances in which the theories of FSs, IFSs, PyFSs, PFSs, and SFSs are invalid. The AOs in the T-SFS environment have been effectively addressed by many researchers. Garg et al. [20] defined several T-spherical fuzzy power AOs and explored their application to MADM based on T-SFSs. Guleria and Bajaj [22] studied the idea of T-SF soft set. The generalized T-SF weighted AO was introduced by Quek et al. [49]. Ullah et al. [52] investigated T-SFS-based correlation coefficients and their applicability in two real-life applications regarding clustering and decision-making. Munir et al. [40] developed the Einstein hybrid AOs for the aggregation of T-spherical fuzzy information. Ullah et al. [53] proposed several T-spherical fuzzy Hamacher AOs and considered how to use them to estimate the performance of search and rescue robots. Liu et al. [33] proposed the linguistic T-spherical fuzzy numbers, the linguistic T-spherical fuzzy weighted averaging operator, and extended the MABAC method to the linguistic spherical fuzzy environment.

Researchers have proposed multiple research theories on the FS's extension and these theories can be divided into two categories. The first is based on quantitative FSs, and the second is based on qualitative FSs, which are usually represented by linguistic variables. Zadeh [62] firstly introduced the notion of linguistic variables. To better reflect the human perception in MADM problems, linguistic information processing approaches can effectively prevent distortion and loss of data. One of the most important approaches to deal with linguistic decision-making issues is the 2-tuple linguistic (2TL) representation model [45], firstly proposed by Herrera and Martinez [25]. Several

2TL AOs and decision-making approaches have been proposed. Deng et al. [16] proposed the generalized and geometric 2TL Pythagorean fuzzy Heronian mean (HM) AOs by combining the generalized, geometric HM AOs and their weighted forms with 2TL Pythagorean fuzzy numbers. As many kinds of research have been done to become conscious about the correlation of arguments, which is a crucial aspect of aggregated results. The HM [14] operator proves to be an effective tool in this regard, which gains too much attention from researchers. The three-parameter HM operator and the three-parameter weighted HM operator defined by Liu and Chin [31] and extended to a linguistic environment. On the basis of Archimedean t-norm and t-conorm, Mo and Huang [41] developed the dual hesitant fuzzy geometric HM operator and dual hesitant fuzzy geometric weighted HM operator. Jiang et al. [26] devised a set of dual hesitant fuzzy power HM AOs with interval values. Yang and Li [56] presented the multiple-valued picture fuzzy linguistic generalized weighted geometric HM aggregating operator, which extends the traditional generalized HM operators to a multiple-valued picture fuzzy linguistic environment. Yu et al. [57] suggested some linguistic hesitant fuzzy HM AOs after generalizing the HM in a linguistic hesitant fuzzy environment.

Further, Akram et al. [4–7] introduced several decision-making methods under generalized fuzzy scenarios. In addition, there are two basic types of approaches for making decisions that are frequently used. The first is the information AOs through which many data can be compiled into a single consistent value. Operators that are commonly used include weighted averaging AOs, ordered weighted averaging AOs, power AOs, etc. The conventional MADM method is the second method, which mainly includes TOPSIS, VIKOR, TODIM, MOORA, and MABAC methods, etc. The theory of different decision-making methods [35, 36] is widely used in MADM problems according to requirements. It is worth noting that all of the above-mentioned methods have some deficiencies to select an optimal alternative from the given set of finite alternatives. To overcome that issue, the MABAC method was proposed by Pamučar and Ćirović [46] which is a very suitable and informative method for solving the MAGDM problems. They used the MABAC method for the transportation and processing resource selection in the logistics center and explained its effectiveness by comparison with the SAW, COPRAS, TOPSIS, MOORA, and the VIKOR methods. Pamučar et al. [47] later modified the original MABAC approach. Mishra et al. [42] present a novel interval-valued IFS-based multi-criteria MABAC approach. Many scholars have studied and expanded the MABAC method [23, 48, 51], for example, Xue et al. [54] applied the MABAC method to do a material selection.

1.1. Motivation and innovation of this study

The overall aim of this study is to identify the alternative fuel that may help some undeveloped countries to reduce their economic cost. After executing, various data from DMs, the MAGDM approach is used to determine the most acceptable alternative fuel. DMs who prioritize membership, abstinence, and non-membership degrees think clearly when they use the 2-tuple linguistic T-spherical fuzzy set (2TLT-SFS) [2] in this type of MAGDM. Furthermore, adopting the 2TLT-SF generalized weighted Heronian mean (2TLT-SFGWHM) operator and the 2TLT-SF weighted geometric Heronian mean (2TLT-SFWJHM) operator allows DMs to make more informed judgments on their significant and 2TLT-SF ideas. The MABAC method is a useful tool as it calculates the distance between each alternative and border approximation area (BAA), and also provides information about lower approximation area and upper approximation area. Furthermore, the

MABAC method has some merits which other methods have not been endowed with: (1) computing results are stable; (2) calculating equations are not complicated; (3) it evaluates the hidden values of gains and losses into account; (4) the model can combine with other approaches. As a consequence, we deduce that the MABAC framework is a valuable tool in the modern decision-making environment. In particular, the outcomes of the 2TLT-SF-MABAC approach are comparable to those of the 2TLT-SF-EDAS and the 2TLT-SF-CODAS approaches in this article. All these techniques are significant MAGDM fundamental approaches.

The innovation of this research can be summarized as follows:

(1) The 2TLT-SF generalized Heronian mean (2TLT-SFGHM) operator, the 2TLT-SFGWHM operator, the 2TLT-SF geometric Heronian mean (2TLT-SFJHM) operator, and the 2TLT-SFWJHM operator are proposed by the integration of 2TLT-SFS and HM operator to deal with group decision-making problems in which the attributes have interrelationships.

(2) Some fundamental properties of the proposed 2TLT-SF aggregation operators are obtained.

(3) The 2TLT-SF-MABAC method is proposed based on the 2TLT-SFGWHM and the 2TLT-SFWJHM operators to rank the alternative fuels. A novel MAGDM model is used to fuse the evaluation preferences of DMs.

(4) An assessment framework for the selection of alternative fuel to control the impact of greenhouse gas emissions is presented to show the usefulness and effectiveness of the proposed study.

1.2. Organization of the proposed study

The structure of this paper is organized as follows: Section 2 briefly recalls some fundamental concepts relevant to the 2TL representation model, the T-SFS, HM AOs, and the notion of 2TLT-SFS. In Section 3, the 2TLT-SF weighted averaging (2TLT-SFWA) operator, the 2TLT-SF weighted geometric (2TLT-SFWJ) operator and four different 2TLT-SFHM AOs are presented along with their basic properties. An extended MABAC approach based on the 2TLT-SFGWHM and the 2TLT-SFWJHM operators is developed for MAGDM in Section 4. By virtue of the proposed 2TLT-SF-MABAC method, we address a practical decision-making problem regarding the selection of the best alternative fuel in Section 5. The influence of parameters on the ranking results, comparative analysis with existing approaches and advantages of our method are discussed as well. Finally, Section 6 summarizes this research study and points out some future directions.

2. Preliminaries

In this section, certain related fundamental ideas of the T-SFS, HM, generalized HM, geometric HM, and the 2TLT-SFS are summarized in order to ease the following sections.

Definition 1. *[39] For any universal set* L*, a T-SFS in* L *is of the form*

$$\mathrm{T} = \{(\mathfrak{b}, \phi(\mathfrak{b}), \psi(\mathfrak{b}), \gamma(\mathfrak{b})) \mid \mathfrak{b} \in \mathrm{L}\},$$

where $\phi(\mathfrak{b}), \psi(\mathfrak{b}), \gamma(\mathfrak{b}) \in [0,1]$ *denote the MD, AD, and NMD of* $\mathfrak{b} \in \mathrm{L}$*, respectively. It is required that* $0 \leq \phi^q(\mathfrak{b}) + \psi^q(\mathfrak{b}) + \gamma^q(\mathfrak{b}) \leq 1$ *for* $q \geq 1$*. We refer to* $r(\mathfrak{b}) = \sqrt[q]{1 - (\phi^q(\mathfrak{b}) + \psi^q(\mathfrak{b}) + \gamma^q(\mathfrak{b}))}$ *as the refusal degree of* $\mathfrak{b}$ *in* T*. For simplicity, the triplet* $(\phi(\mathfrak{b}), \psi(\mathfrak{b}), \gamma(\mathfrak{b}))$ *is also known as a T-SFN.*

Definition 2. *[2] Let* L *be a universal set. A 2TLT-SFS* $\mathcal{N}$ *in* L *is:*

$$\mathcal{N} = \{\langle \mathfrak{b}, ((\mathbf{s}_{\phi}(\mathfrak{b}), \Phi(\mathfrak{b})), (\mathbf{s}_{\psi}(\mathfrak{b}), \Psi(\mathfrak{b})), (\mathbf{s}_{\gamma}(\mathfrak{b}), \Upsilon(\mathfrak{b})))\rangle \mid \mathfrak{b} \in \mathrm{L}\}, \tag{2.1}$$

where $(\mathbf{s}_{\phi}(\mathfrak{b}), \Phi(\mathfrak{b})), (\mathbf{s}_{\psi}(\mathfrak{b}), \Psi(\mathfrak{b}))$ *and* $(\mathbf{s}_{\gamma}(\mathfrak{b}), \Upsilon(\mathfrak{b}))$ *represent the positive, neutral, and negative membership degrees, respectively. It is required that* $-0.5 \leq \Phi(\mathfrak{b}), \Psi(\mathfrak{b}), \Upsilon(\mathfrak{b}) < 0.5$, $0 \leq \Delta^{-1}(\mathbf{s}_{\phi}(\mathfrak{b}), \Phi(\mathfrak{b})) \leq \Gamma$, $0 \leq \Delta^{-1}(\mathbf{s}_{\psi}(\mathfrak{b}), \Psi(\mathfrak{b})) \leq \Gamma$, $0 \leq \Delta^{-1}(\mathbf{s}_{\gamma}(\mathfrak{b}), \Upsilon(\mathfrak{b})) \leq \Gamma$ *and*

$$0 \leq (\Delta^{-1}(\mathbf{s}_{\phi}(\mathfrak{b}), \Phi(\mathfrak{b})))^q + (\Delta^{-1}(\mathbf{s}_{\psi}(\mathfrak{b}), \Psi(\mathfrak{b})))^q + (\Delta^{-1}(\mathbf{s}_{\gamma}(\mathfrak{b}), \Upsilon(\mathfrak{b})))^q \leq \Gamma^q.$$

Regarding ease of use, $\mathcal{N} = ((\mathbf{s}_{\phi}, \Phi), (\mathbf{s}_{\psi}, \Psi), (\mathbf{s}_{\gamma}, \Upsilon))$ is called the 2TLT-SFN, where

$$0 \leq \Delta^{-1}(\mathbf{s}_{\phi}, \Phi),\ \Delta^{-1}(\mathbf{s}_{\psi}, \Psi),\ \Delta^{-1}(\mathbf{s}_{\gamma}, \Upsilon) \leq \Gamma,$$

and

$$0 \leq (\Delta^{-1}(\mathbf{s}_{\phi}, \Phi))^q + (\Delta^{-1}(\mathbf{s}_{\psi}, \Psi))^q + (\Delta^{-1}(\mathbf{s}_{\gamma}, \Upsilon))^q \leq \Gamma^q.$$

For comparing two 2TLT-SFNs, the score and accuracy values can be computed as described in the following:

Definition 3. *[2] Let* $\mathcal{N} = ((\mathbf{s}_{\phi}, \Phi), (\mathbf{s}_{\psi}, \Psi), (\mathbf{s}_{\gamma}, \Upsilon))$ *be a 2TLT-SFN. Then the score function* $\mathcal{S}$ *is given by:*

$$\mathcal{S}(\mathcal{N}) = \Delta\left(\frac{\Gamma}{2}\left(1 + \left(\frac{\Delta^{-1}(\mathbf{s}_{\phi}, \Phi)}{\Gamma}\right)^q - \left(\frac{\Delta^{-1}(\mathbf{s}_{\gamma}, \Upsilon)}{\Gamma}\right)^q\right)\right), \tag{2.2}$$

and the Ac accuracy value is constructed as described in the following:

$$Ac(\mathcal{N}) = \Delta\left(\Gamma\left(\left(\frac{\Delta^{-1}(\mathbf{s}_{\phi}, \Phi)}{\Gamma}\right)^q + \left(\frac{\Delta^{-1}(\mathbf{s}_{\gamma}, \Upsilon)}{\Gamma}\right)^q\right)\right). \tag{2.3}$$

Definition 4. *[2] Let* $\mathcal{N}_1 = ((\mathbf{s}_{\phi_1}, \Phi_1), (\mathbf{s}_{\psi_1}, \Psi_1), (\mathbf{s}_{\gamma_1}, \Upsilon_1))$ *and* $\mathcal{N}_2 = ((\mathbf{s}_{\phi_2}, \Phi_2), (\mathbf{s}_{\psi_2}, \Psi_2), (\mathbf{s}_{\gamma_2}, \Upsilon_2))$ *be two 2TLT-SFNs. The two 2TLT-SFNs can then be evaluated employing their score and accuracy values as follows:*

- *If* $\mathcal{S}(\mathcal{N}_1) < \mathcal{S}(\mathcal{N}_2)$, *then* $\mathcal{N}_1 \prec \mathcal{N}_2$;
- *If* $\mathcal{S}(\mathcal{N}_1) > \mathcal{S}(\mathcal{N}_2)$, *then* $\mathcal{N}_1 \succ \mathcal{N}_2$;
- *If* $\mathcal{S}(\mathcal{N}_1) = \mathcal{S}(\mathcal{N}_2)$, *then*
  (a) *If* $Ac(\mathcal{N}_1) < Ac(\mathcal{N}_2)$, *then* $\mathcal{N}_1 \prec \mathcal{N}_2$;
  (b) *If* $Ac(\mathcal{N}_1) > Ac(\mathcal{N}_2)$, *then* $\mathcal{N}_1 \succ \mathcal{N}_2$;
  (c) *If* $Ac(\mathcal{N}_1) = Ac(\mathcal{N}_2)$, *then* $\mathcal{N}_1 \sim \mathcal{N}_2$.

Example 1. *Let* $\mathcal{N}_1 = ((\mathbf{s}_5, 0.3), (\mathbf{s}_3, 0.4), (\mathbf{s}_2, -0.1))$, *and* $\mathcal{N}_2 = ((\mathbf{s}_7, -0.2), (\mathbf{s}_2, 0.1), (\mathbf{s}_1, -0.5))$ *be two 2TLT-SFNs and* $S = \{\mathbf{s}_0, \mathbf{s}_1, \mathbf{s}_2, \ldots, \mathbf{s}_8\}$ *be an LTS. Taking* $q = 4$, *by Eq (2.2) we can get the score function values of* $\mathcal{N}_1$ *and* $\mathcal{N}_2$ *as follows:*

$$\mathcal{S}(\mathcal{N}_1) = \Delta\left(\tfrac{8}{2}\left(1 + \left(\tfrac{\Delta^{-1}(\mathbf{s}_5, 0.3)}{8}\right)^4 - \left(\tfrac{\Delta^{-1}(\mathbf{s}_2, -0.1)}{8}\right)^4\right)\right) = (\mathbf{s}_5, -0.2422),$$

$$\mathcal{S}(\mathcal{N}_2)=\Delta\left(\frac{8}{2}\left(1+\left(\frac{\Delta^{-1}(\mathbf{s}_7,-0.2)}{8}\right)^4-\left(\frac{\Delta^{-1}(\mathbf{s}_1,-0.5)}{8}\right)^4\right)\right)=(\mathbf{s}_6,0.0880).$$

Moreover, according to Eq (2.3) the accuracy function values of $\mathcal{N}_1$ and $\mathcal{N}_2$ are two 2TLT-SFNs as follows:

$$Ac(\mathcal{N}_1)=\Delta\left(8\left(\left(\frac{\Delta^{-1}(\mathbf{s}_5,0.3)}{8}\right)^4+\left(\frac{\Delta^{-1}(\mathbf{s}_2,-0.1)}{8}\right)^4\right)\right)=(\mathbf{s}_2,-0.4336),$$
$$Ac(\mathcal{N}_2)=\Delta\left(8\left(\left(\frac{\Delta^{-1}(\mathbf{s}_7,-0.2)}{8}\right)^4+\left(\frac{\Delta^{-1}(\mathbf{s}_1,-0.5)}{8}\right)^4\right)\right)=(\mathbf{s}_4,0.1762).$$

According to Definition 4, we have $\mathcal{N}_1 \prec \mathcal{N}_2$ since $\mathcal{S}(\mathcal{N}_1)<\mathcal{S}(\mathcal{N}_2)$.

Definition 5. *[3] Let $\mathcal{N}_1=((\mathbf{s}_{\phi_1},\Phi_1),(\mathbf{s}_{\psi_1},\Psi_1),(\mathbf{s}_{\gamma_1},\Upsilon_1))$ and $\mathcal{N}_2=((\mathbf{s}_{\phi_2},\Phi_2),(\mathbf{s}_{\psi_2},\Psi_2),(\mathbf{s}_{\gamma_2},\Upsilon_2))$ be two 2TLT-SFNs. The 2TLT-SF normalized Hamming distance is defined as:*

$$d(\mathcal{N}_1,\mathcal{N}_2)=\Delta\left(\frac{\Gamma}{3}\left(\begin{array}{l}\left|\left(\frac{\Delta^{-1}(\mathbf{s}_{\phi_1},\Phi_1)}{\Gamma}\right)^q-\left(\frac{\Delta^{-1}(\mathbf{s}_{\phi_2},\Phi_2)}{\Gamma}\right)^q\right|+\left|\left(\frac{\Delta^{-1}(\mathbf{s}_{\psi_1},\Psi_1)}{\Gamma}\right)^q-\left(\frac{\Delta^{-1}(\mathbf{s}_{\psi_2},\Psi_2)}{\Gamma}\right)^q\right|\\+\left|\left(\frac{\Delta^{-1}(\mathbf{s}_{\gamma_1},\Upsilon_1)}{\Gamma}\right)^q-\left(\frac{\Delta^{-1}(\mathbf{s}_{\gamma_2},\Upsilon_2)}{\Gamma}\right)^q\right|\end{array}\right)\right). \tag{2.4}$$

Definition 6. *[2] Let $\mathcal{N}=((\mathbf{s}_\phi,\Phi),(\mathbf{s}_\psi,\Psi),(\mathbf{s}_\gamma,\Upsilon))$, $\mathcal{N}_1=((\mathbf{s}_{\phi_1},\Phi_1),(\mathbf{s}_{\psi_1},\Psi_1),(\mathbf{s}_{\gamma_1},\Upsilon_1))$ $\mathcal{N}_2=((\mathbf{s}_{\phi_2},\Phi_2),(\mathbf{s}_{\psi_2},\Psi_2),(\mathbf{s}_{\gamma_2},\Upsilon_2))$ be three 2TLT-SFNs, $q\geq 1$ and $\lambda>0$. Then*

(1) $\mathcal{N}_1\oplus\mathcal{N}_2=\left(\begin{array}{l}\Delta\left(\Gamma\sqrt[q]{1-\left(1-\left(\frac{\Delta^{-1}(\mathbf{s}_{\phi_1},\Phi_1)}{\Gamma}\right)^q\right)\left(1-\left(\frac{\Delta^{-1}(\mathbf{s}_{\phi_2},\Phi_2)}{\Gamma}\right)^q\right)}\right),\\\Delta\left(\Gamma\left(\frac{\Delta^{-1}(\mathbf{s}_{\psi_1},\Psi_1)}{\Gamma}\right)\left(\frac{\Delta^{-1}(\mathbf{s}_{\psi_2},\Psi_2)}{\Gamma}\right)\right),\Delta\left(\Gamma\left(\frac{\Delta^{-1}(\mathbf{s}_{\gamma_1},\Upsilon_1)}{\Gamma}\right)\left(\frac{\Delta^{-1}(\mathbf{s}_{\gamma_2},\Upsilon_2)}{\Gamma}\right)\right)\end{array}\right)$;

(2) $\mathcal{N}_1\otimes\mathcal{N}_2=\left(\begin{array}{l}\Delta\left(\Gamma\left(\frac{\Delta^{-1}(\mathbf{s}_{\phi_1},\Phi_1)}{\Gamma}\right)\left(\frac{\Delta^{-1}(\mathbf{s}_{\phi_2},\Phi_2)}{\Gamma}\right)\right),\\\Delta\left(\Gamma\sqrt[q]{1-\left(1-\left(\frac{\Delta^{-1}(\mathbf{s}_{\psi_1},\Psi_1)}{\Gamma}\right)^q\right)\left(1-\left(\frac{\Delta^{-1}(\mathbf{s}_{\psi_2},\Psi_2)}{\Gamma}\right)^q\right)}\right),\\\Delta\left(\Gamma\sqrt[q]{1-\left(1-\left(\frac{\Delta^{-1}(\mathbf{s}_{\gamma_1},\Upsilon_1)}{\Gamma}\right)^q\right)\left(1-\left(\frac{\Delta^{-1}(\mathbf{s}_{\gamma_2},\Upsilon_2)}{\Gamma}\right)^q\right)}\right)\end{array}\right)$;

(3) $\lambda\mathcal{N}=\left(\Delta\left(\Gamma\sqrt[q]{1-\left(1-\left(\frac{\Delta^{-1}(\mathbf{s}_\phi,\Phi)}{\Gamma}\right)^q\right)^\lambda}\right),\Delta\left(\Gamma\left(\frac{\Delta^{-1}(\mathbf{s}_\psi,\Psi)}{\Gamma}\right)^\lambda\right),\Delta\left(\Gamma\left(\frac{\Delta^{-1}(\mathbf{s}_\gamma,\Upsilon)}{\Gamma}\right)^\lambda\right)\right)$;

(4) $\mathcal{N}^\lambda=\left(\Delta\left(\Gamma\left(\frac{\Delta^{-1}(\mathbf{s}_\phi,\Phi)}{\Gamma}\right)^\lambda\right),\Delta\left(\Gamma\sqrt[q]{1-\left(1-\left(\frac{\Delta^{-1}(\mathbf{s}_\psi,\Psi)}{\Gamma}\right)^q\right)^\lambda}\right),\Delta\left(\Gamma\sqrt[q]{1-\left(1-\left(\frac{\Delta^{-1}(\mathbf{s}_\gamma,\Upsilon)}{\Gamma}\right)^q\right)^\lambda}\right)\right)$.

Definition 7. *[14] Let $a_\theta(\theta=1,2,\ldots,\mathfrak{R})$ be a group with positive numbers. Then*

$$HM(a_1,a_2,\ldots,a_{\mathfrak{R}})=\frac{2}{\mathfrak{R}(\mathfrak{R}+1)}\sum_{\theta=1}^{\mathfrak{R}}\sum_{\vartheta=\theta}^{\mathfrak{R}}\sqrt{a_\theta a_\vartheta}, \tag{2.5}$$

is known as HM operator.

Based on Definition 7, the generalized HM operator was introduced by Yu [58] as follows:

Definition 8. *[58] Let* $\kappa, \varepsilon > 0$ *and* $a_\theta(\theta = 1, 2, \ldots, \mathfrak{R})$ *be a group with positive numbers. Then*

$$GHM^{\kappa,\varepsilon}(a_1, a_2, \ldots, a_{\mathfrak{R}}) = \left(\frac{2}{\mathfrak{R}(\mathfrak{R}+1)} \sum_{\theta=1}^{\mathfrak{R}} \sum_{\vartheta=\theta}^{\mathfrak{R}} a_\theta^\kappa a_\vartheta^\varepsilon \right)^{\frac{1}{\kappa+\varepsilon}}, \tag{2.6}$$

is known as the generalized HM operator. It is worth noting that the generalized HM operator converted into the HM operator when $\kappa = \varepsilon = \frac{1}{2}$*.*

Subsequently, Yu [58] proposed the geometric HM operator as follows:

Definition 9. *[58] Let* $\kappa, \varepsilon > 0$ *and* $a_\theta(\theta = 1, 2, \ldots, \mathfrak{R})$ *be a group with positive numbers. Then*

$$JHM^{\kappa,\varepsilon}(a_1, a_2, \ldots, a_{\mathfrak{R}}) = \frac{1}{\kappa+\varepsilon} \left(\prod_{\theta=1,\vartheta=\theta}^{\mathfrak{R}} (\kappa a_\theta + \varepsilon a_\vartheta) \right)^{\frac{2}{\mathfrak{R}(\mathfrak{R}+1)}}, \tag{2.7}$$

is called the geometric HM operator.

Nomenclature

The terminologies and notations used in this paper are listed in Table 1.

Table 1. Terminologies and notations.

Notation	Description
HM	Heronian mean
2TLT-SFS	2-Tuple linguistic T-spherical fuzzy set
2TLT-SFGHM	2TLT-SF generalized Heronian mean
2TLT-SFGWHM	2TLT-SF generalized weighted Heronian mean
2TLT-SFJHM	2TLT-SF geometric Heronian mean
2TLT-SFWJHM	2TLT-SF weighted geometric Heronian mean
2TLT-SF-MABAC	2TLT-SF multi-attributive border approximation area comparison
$\mathcal{N}_\vartheta = ((\mathbf{s}_{\phi_\vartheta}, \Phi_\vartheta), (\mathbf{s}_{\psi_\vartheta}, \Psi_\vartheta), (\mathbf{s}_{\gamma_\vartheta}, \Upsilon_\vartheta))$	2TLT-SF number
$S = \{\mathbf{s}_\vartheta \mid \vartheta = 0, 1, \ldots, \Gamma\}$	2-Tuple linguistic term
$(\mathbf{s}_{\phi_\vartheta}, \Phi_\vartheta)$	Membership degree of 2TLT-SFN
$(\mathbf{s}_{\psi_\vartheta}, \Psi_\vartheta)$	Abstinence degree of 2TLT-SFN
$(\mathbf{s}_{\gamma_\vartheta}, \Upsilon_\vartheta)$	Non-membership degree of 2TLT-SFN
κ, ε	Parameters of HM operators
F	Alternatives
$\aleph$	Attributes
$F(\mathcal{N})$	2TLT-SFN scoring function
$Ac(\mathcal{N})$	2TLT-SFN accuracy function

3. The 2TLT-SF aggregation operators

3.1. The 2TLT-SFWA and 2TLT-SFWJ operators

In this subsection, we introduce two types of weighted information AOs such as 2TLT-SFWA and 2TLT-SFWJ operators.

Definition 10. *Let* $\mathcal{N}_\vartheta = ((\mathbf{s}_{\phi_\vartheta}, \Phi_\vartheta), (\mathbf{s}_{\psi_\vartheta}, \Psi_\vartheta), (\mathbf{s}_{\gamma\vartheta}, \Upsilon_\vartheta))(\vartheta = 1, 2, \ldots, \mathfrak{N})$ *be 2TLT-SFNs. The 2TLT-SFWA operator is a mapping* $T^{\mathfrak{N}} \to T$ *such that*

$$2TLT\text{-}SFWA(\mathcal{N}_1, \mathcal{N}_2, \ldots, \mathcal{N}_{\mathfrak{N}}) = \oplus_{\vartheta=1}^{\mathfrak{N}} \xi_\vartheta \mathcal{N}_\vartheta, \tag{3.1}$$

in which T is the set of 2TLT-SFNs, $\xi = (\xi_1, \xi_2, \ldots, \xi_{\mathfrak{N}})^T$ *is the vector of weights of* $\mathcal{N}_\vartheta(\vartheta = 1, 2, \ldots, \mathfrak{N})$*, such that* $\xi_\vartheta \in [0, 1]$ *and* $\sum_{\vartheta=1}^{\mathfrak{N}} \xi_\vartheta = 1$.

Theorem 1. *Let* $\mathcal{N}_\vartheta = ((\mathbf{s}_{\phi_\vartheta}, \Phi_\vartheta), (\mathbf{s}_{\psi_\vartheta}, \Psi_\vartheta), (\mathbf{s}_{\gamma\vartheta}, \Upsilon_\vartheta))(\vartheta = 1, 2, \ldots, \mathfrak{N})$ *be 2TLT-SFNs with vector of weights* $\xi = (\xi_1, \xi_2, \ldots, \xi_{\mathfrak{N}})^T$*, such that* $\xi_\vartheta \in [0, 1]$ *and* $\sum_{\vartheta=1}^{\mathfrak{N}} \xi_\vartheta = 1$*, then*

$$2TLT\text{-}SFWA(\mathcal{N}_1, \mathcal{N}_2, \ldots, \mathcal{N}_{\mathfrak{N}}) = \begin{pmatrix} \Delta\left(\Gamma\left(1 - \prod_{\vartheta=1}^{\mathfrak{N}}\left(1 - \left(\frac{\Delta^{-1}(\mathbf{s}_{\phi_\vartheta}, \Phi_\vartheta)}{\Gamma}\right)^q\right)^{\xi_\vartheta}\right)^{\frac{1}{q}}\right), \\ \Delta\left(\Gamma\prod_{\vartheta=1}^{\mathfrak{N}}\left(\frac{\Delta^{-1}(\mathbf{s}_{\psi_\vartheta}, \Psi_\vartheta)}{\Gamma}\right)^{\xi_\vartheta}\right), \Delta\left(\Gamma\prod_{\vartheta=1}^{\mathfrak{N}}\left(\frac{\Delta^{-1}(\mathbf{s}_{\gamma_\vartheta}, \Upsilon_\vartheta)}{\Gamma}\right)^{\xi_\vartheta}\right) \end{pmatrix}. \tag{3.2}$$

Proof. We prove that the Eq (3.2) holds by using the mathematical induction method for positive integer $\mathfrak{N}$.

(a) When $\mathfrak{N} = 1$, we have

$$\xi_1 \mathcal{N}_1 = \left(\Delta\left(\Gamma\left(1 - \left(1 - \left(\frac{\Delta^{-1}(\mathbf{s}_{\phi_1}, \Phi_1)}{\Gamma}\right)^q\right)^{\xi_1}\right)^{\frac{1}{q}}\right), \Delta\left(\Gamma\left(\frac{\Delta^{-1}(\mathbf{s}_{\psi_1}, \Psi_1)}{\Gamma}\right)^{\xi_1}\right), \Delta\left(\Gamma\left(\frac{\Delta^{-1}(\mathbf{s}_{\gamma_1}, \Upsilon_1)}{\Gamma}\right)^{\xi_1}\right) \right).$$

Thus, Eq (3.2) holds for $\mathfrak{N} = 1$.

(b) Suppose that Eq (3.2) holds for $\mathfrak{N} = \mathfrak{M}$,

$$\begin{aligned} &\text{2TL}T\text{-SFWA}(\mathcal{N}_1, \mathcal{N}_2, \ldots, \mathcal{N}_{\mathfrak{N}}) \\ &= \begin{pmatrix} \Delta\left(\Gamma\left(1 - \prod_{\vartheta=1}^{\mathfrak{M}}\left(1 - \left(\frac{\Delta^{-1}(\mathbf{s}_{\phi_\vartheta}, \Phi_\vartheta)}{\Gamma}\right)^q\right)^{\xi_\vartheta}\right)^{\frac{1}{q}}\right), \Delta\left(\Gamma\prod_{\vartheta=1}^{\mathfrak{M}}\left(\frac{\Delta^{-1}(\mathbf{s}_{\psi_\vartheta}, \Psi_\vartheta)}{\Gamma}\right)^{\xi_\vartheta}\right), \\ \Delta\left(\Gamma\prod_{\vartheta=1}^{\mathfrak{M}}\left(\frac{\Delta^{-1}(\mathbf{s}_{\gamma_\vartheta}, \Upsilon_\vartheta)}{\Gamma}\right)^{\xi_\vartheta}\right) \end{pmatrix}. \end{aligned}$$

Then, when $\mathfrak{N} = \mathfrak{M} + 1$, by inductive assumption, we have

$$\text{2TL}T\text{-SFWA}(\mathcal{N}_1, \mathcal{N}_2, \ldots, \mathcal{N}_{\mathfrak{M}}, \mathcal{N}_{\mathfrak{M}+1}) = \text{2TL}T\text{-SFWA}(\mathcal{N}_1, \mathcal{N}_2, \ldots, \mathcal{N}_{\mathfrak{M}}) \oplus \xi_{\mathfrak{M}+1}\mathcal{N}_{\mathfrak{M}+1}$$

$$= \begin{pmatrix} \Delta\left(\Gamma\left(1 - \prod_{\vartheta=1}^{\mathfrak{M}}\left(1 - \left(\frac{\Delta^{-1}(\mathbf{s}_{\phi_\vartheta}, \Phi_\vartheta)}{\Gamma}\right)^q\right)^{\xi_\vartheta}\right)^{\frac{1}{q}}\right), \Delta\left(\Gamma\prod_{\vartheta=1}^{\mathfrak{M}}\left(\frac{\Delta^{-1}(\mathbf{s}_{\psi_\vartheta}, \Psi_\vartheta)}{\Gamma}\right)^{\xi_\vartheta}\right), \\ \Delta\left(\Gamma\prod_{\vartheta=1}^{\mathfrak{M}}\left(\frac{\Delta^{-1}(\mathbf{s}_{\gamma_\vartheta}, \Upsilon_\vartheta)}{\Gamma}\right)^{\xi_\vartheta}\right) \end{pmatrix}$$

$$\oplus\left(\begin{array}{l}\Delta\left(\Gamma\left(1-\left(1-\left(\frac{\Delta^{-1}(s_{\phi_{\mathfrak{M}+1}},\Phi_{\mathfrak{M}+1})}{\Gamma}\right)^q\right)^{\xi_{\mathfrak{M}+1}}\right)^{\frac{1}{q}}\right),\Delta\left(\Gamma\left(\frac{\Delta^{-1}(s_{\psi_{\mathfrak{M}+1}},\Psi_{\mathfrak{M}+1})}{\Gamma}\right)^{\xi_{\mathfrak{M}+1}}\right),\\ \Delta\left(\Gamma\left(\frac{\Delta^{-1}(s_{\gamma_{\mathfrak{M}+1}},\Upsilon_{\mathfrak{M}+1})}{\Gamma}\right)^{\xi_{\mathfrak{M}+1}}\right)\end{array}\right)$$

$$=\left(\begin{array}{l}\Delta\left(\Gamma\left(1-\prod_{\vartheta=1}^{\mathfrak{M}+1}\left(1-\left(\frac{\Delta^{-1}(s_{\phi_\vartheta},\Phi_\vartheta)}{\Gamma}\right)^q\right)^{\xi_\vartheta}\right)^{\frac{1}{q}}\right),\Delta\left(\Gamma\prod_{\vartheta=1}^{\mathfrak{M}+1}\left(\frac{\Delta^{-1}(s_{\psi_\vartheta},\Psi_\vartheta)}{\Gamma}\right)^{\xi_\vartheta}\right),\\ \Delta\left(\Gamma\prod_{\vartheta=1}^{\mathfrak{M}+1}\left(\frac{\Delta^{-1}(s_{\gamma_\vartheta},\Upsilon_\vartheta)}{\Gamma}\right)^{\xi_\vartheta}\right)\end{array}\right).$$

Therefore, we can deduce that Eq (3.2) holds for positive integer $\mathfrak{N}=\mathfrak{M}+1$. Thus, by the mathematical induction method, we know that Eq (3.2) holds for any $\mathfrak{N}\geq 1$. □

Theorem 2. *Let* $\mathcal{N}_\vartheta = ((s_{\phi_\vartheta},\Phi_\vartheta),(s_{\psi_\vartheta},\Psi_\vartheta),(s_{\gamma\vartheta},\Upsilon_\vartheta))$ *and* $\mathcal{N}'_\vartheta = ((s'_{\phi_\vartheta},\Phi'_\vartheta),(s'_{\psi_\vartheta},\Psi'_\vartheta),(s'_{\gamma\vartheta},\Upsilon'_\vartheta))(\vartheta = 1,2,\ldots,\mathfrak{N})$ *be two sets of 2TLT-SFNs; then the 2TLT-SFWA operator possesses the essential properties:*

(1) (Idempotency) If all $\mathcal{N}_\vartheta = ((s_{\phi_\vartheta},\Phi_\vartheta),(s_{\psi_\vartheta},\Psi_\vartheta),(s_{\gamma\vartheta},\Upsilon_\vartheta))(\vartheta = 1,2,\ldots,\mathfrak{N})$ *are equal, for all* ϑ*, then*

$$2TLT\text{-}SFWA(\mathcal{N}_1,\mathcal{N}_2,\ldots,\mathcal{N}_{\mathfrak{N}}) = \mathcal{N}.$$

(2) (Monotonicity) If $\mathcal{N}_\vartheta \leq \mathcal{N}'_\vartheta$*, for all* ϑ*, then*

$$2TLT\text{-}SFWA(\mathcal{N}_1,\mathcal{N}_2,\ldots,\mathcal{N}_{\mathfrak{N}}) \leq 2TLT\text{-}SFWA(\mathcal{N}'_1,\mathcal{N}'_2,\ldots,\mathcal{N}'_{\mathfrak{N}}).$$

(3) (Boundedness) Let $\mathcal{N}_\vartheta = ((s_{\phi_\vartheta},\Phi_\vartheta),(s_{\psi_\vartheta},\Psi_\vartheta),(s_{\gamma\vartheta},\Upsilon_\vartheta))(\vartheta = 1,2,\ldots,\mathfrak{N})$ *be 2TLT-SFNs, and let* $\mathcal{N}^- = (\min_\vartheta(s_{\phi_\vartheta},\Phi_\vartheta),\max_\vartheta(s_{\psi_\vartheta},\Psi_\vartheta),\max_\vartheta(s_{\gamma\vartheta},\Upsilon_\vartheta))$ *and* $\mathcal{N}^+ = (\max_\vartheta(s_{\phi_\vartheta},\Phi_\vartheta),\min_\vartheta(s_{\psi_\vartheta},\Psi_\vartheta),\min_\vartheta(s_{\gamma\vartheta},\Upsilon_\vartheta))$*, then*

$$\mathcal{N}^- \leq 2TLT\text{-}SFWA(\mathcal{N}_1,\mathcal{N}_2,\ldots,\mathcal{N}_{\mathfrak{N}}) \leq \mathcal{N}^+.$$

Definition 11. *Let* $\mathcal{N}_\vartheta = ((s_{\phi_\vartheta},\Phi_\vartheta),(s_{\psi_\vartheta},\Psi_\vartheta),(s_{\gamma\vartheta},\Upsilon_\vartheta))(\vartheta = 1,2,\ldots,\mathfrak{N})$ *be 2TLT-SFNs. The 2TLT-SFWJ operator is a mapping* $T^{\mathfrak{N}} \to T$ *such that*

$$2TLT\text{-}SFWJ(\mathcal{N}_1,\mathcal{N}_2,\ldots,\mathcal{N}_{\mathfrak{N}}) = \otimes_{\vartheta=1}^{\mathfrak{N}}\mathcal{N}_\vartheta^{\xi_\vartheta}, \tag{3.3}$$

in which T *is the set of 2TLT-SFNs,* $\xi = (\xi_1,\xi_2,\ldots,\xi_{\mathfrak{N}})^T$ *is the vector of weights of* $\mathcal{N}_\vartheta(\vartheta = 1,2,\ldots,\mathfrak{N})$*, such that* $\xi_\vartheta \in [0,1]$ *and* $\sum_{\vartheta=1}^{\mathfrak{N}}\xi_\vartheta = 1$.

Theorem 3. *Let* $\mathcal{N}_\vartheta = ((s_{\phi_\vartheta},\Phi_\vartheta),(s_{\psi_\vartheta},\Psi_\vartheta),(s_{\gamma\vartheta},\Upsilon_\vartheta))(\vartheta = 1,2,\ldots,\mathfrak{N})$ *be 2TLT-SFNs with vector of weights* $\xi = (\xi_1,\xi_2,\ldots,\xi_{\mathfrak{N}})^T$*, such that* $\xi_\vartheta \in [0,1]$ *and* $\sum_{\vartheta=1}^{\mathfrak{N}}\xi_\vartheta = 1$*. Then their aggregation value by the 2TLT-SFWJ operator is still a 2TLT-SFN, and*

$$2TLT\text{-}SFWJ(\mathcal{N}_1,\mathcal{N}_2,\ldots,\mathcal{N}_{\mathfrak{N}})$$

$$= \begin{pmatrix} \Delta\left(\Gamma \prod_{\vartheta=1}^{\Re} \left(\frac{\Delta^{-1}(\mathbf{s}_{\phi_\vartheta}, \Phi_\vartheta)}{\Gamma}\right)^{\xi_\vartheta}\right), \Delta\left(\Gamma\left(1 - \prod_{\vartheta=1}^{\Re} \left(1 - \left(\frac{\Delta^{-1}(\mathbf{s}_{\psi_\vartheta}, \Psi_\vartheta)}{\Gamma}\right)^q\right)^{\xi_\vartheta}\right)^{\frac{1}{q}}\right), \\ \Delta\left(\Gamma\left(1 - \prod_{\vartheta=1}^{\Re} \left(1 - \left(\frac{\Delta^{-1}(\mathbf{s}_{\gamma_\vartheta}, \Upsilon_\vartheta)}{\Gamma}\right)^q\right)^{\xi_\vartheta}\right)^{\frac{1}{q}}\right) \end{pmatrix}. \tag{3.4}$$

Proof. The proof is analogous to that of Theorem 1. □

The 2TLT-SFWJ operator has the same properties, idempotency, monotonicity, and boundedness as of the 2TLT-SFWA operator.

3.2. The 2TLT-SF Heronian mean aggregation operators

In this subsection, we extend the generalized HM to the 2TLT-SFSs environment and propose the 2TLT-SFGHM, the 2TLT-SFGWHM, the 2TLT-SFJHM and the 2TLT-SFWJHM operators for aggregating the 2TLT-SFNs since 2TLT-SFS is an effective tool for communicating uncertain data in real decision-making framework.

Definition 12. *Let* $\mathcal{N}_\vartheta = ((\mathbf{s}_{\phi_\vartheta}, \Phi_\vartheta), (\mathbf{s}_{\psi_\vartheta}, \Psi_\vartheta), (\mathbf{s}_{\gamma_\vartheta}, \Upsilon_\vartheta))(\vartheta = 1, 2, \ldots, \Re)$ *be 2TLT-SFNs. The 2TLT-SFGHM is a mapping* $P^{\Re} \to P$ *such that*

$$\text{2TLT-SFGHM}^{\kappa,\varepsilon}(\mathcal{N}_1, \mathcal{N}_2, \ldots, \mathcal{N}_{\Re}) = \left(\frac{2}{\Re(\Re+1)} \oplus_{\theta=1}^{\Re} \oplus_{\vartheta=\theta}^{\Re} (\mathcal{N}_\theta^\kappa \otimes \mathcal{N}_\vartheta^\varepsilon)\right)^{\frac{1}{\kappa+\varepsilon}}, \tag{3.5}$$

where $\kappa, \varepsilon \geq 0$.

It can be shown that the aggregated value by using 2TLT-SFGHM operator is actually a 2TLT-SFN. By Definition 6, we can deduce the following outcome:

Theorem 4. *Let* $\mathcal{N}_\vartheta = ((\mathbf{s}_{\phi_\vartheta}, \Phi_\vartheta), (\mathbf{s}_{\psi_\vartheta}, \Psi_\vartheta), (\mathbf{s}_{\gamma_\vartheta}, \Upsilon_\vartheta))(\vartheta = 1, 2, \ldots, \Re)$ *be 2TLT-SFNs. Then*

$$\begin{aligned} &\text{2TLT-SFGHM}^{\kappa,\varepsilon}(\mathcal{N}_1, \mathcal{N}_2, \ldots, \mathcal{N}_{\Re}) \\ &= \begin{pmatrix} \Delta\left(\Gamma\left(\sqrt[q]{1 - \prod_{\theta=1,\vartheta=\theta}^{\Re} \left(1 - \left(\frac{\Delta^{-1}(\mathbf{s}_{\phi_\theta}, \Phi_\theta)}{\Gamma}\right)^{q\kappa} \left(\frac{\Delta^{-1}(\mathbf{s}_{\phi_\vartheta}, \Phi_\vartheta)}{\Gamma}\right)^{q\varepsilon}\right)^{\frac{2}{\Re(\Re+1)}}}\right)^{\frac{1}{\kappa+\varepsilon}}\right), \\ \Delta\left(\Gamma \sqrt[q]{1 - \left(1 - \prod_{\theta=1,\vartheta=\theta}^{\Re} \left(1 - \left(1 - \left(\frac{\Delta^{-1}(\mathbf{s}_{\psi_\theta}, \Psi_\theta)}{\Gamma}\right)^q\right)^\kappa \left(1 - \left(\frac{\Delta^{-1}(\mathbf{s}_{\psi_\vartheta}, \Psi_\vartheta)}{\Gamma}\right)^q\right)^\varepsilon\right)^{\frac{2}{\Re(\Re+1)}}\right)^{\frac{1}{\kappa+\varepsilon}}}\right), \\ \Delta\left(\Gamma \sqrt[q]{1 - \left(1 - \prod_{\theta=1,\vartheta=\theta}^{\Re} \left(1 - \left(1 - \left(\frac{\Delta^{-1}(\mathbf{s}_{\gamma_\theta}, \Upsilon_\theta)}{\Gamma}\right)^q\right)^\kappa \left(1 - \left(\frac{\Delta^{-1}(\mathbf{s}_{\gamma_\vartheta}, \Upsilon_\vartheta)}{\Gamma}\right)^q\right)^\varepsilon\right)^{\frac{2}{\Re(\Re+1)}}\right)^{\frac{1}{\kappa+\varepsilon}}}\right) \end{pmatrix}, \end{aligned} \tag{3.6}$$

where $\kappa, \varepsilon \geq 0$.

Proof. According to Definition 6, we can derive

$$\mathcal{N}_\theta^\kappa = \left(\Delta\left(\Gamma\left(\frac{\Delta^{-1}(\mathbf{s}_{\phi_\theta}, \Phi_\theta)}{\Gamma}\right)^\kappa\right), \Delta\left(\Gamma \sqrt[q]{1 - \left(1 - \left(\frac{\Delta^{-1}(\mathbf{s}_{\psi_\theta}, \Psi_\theta)}{\Gamma}\right)^q\right)^\kappa}\right), \Delta\left(\Gamma \sqrt[q]{1 - \left(1 - \left(\frac{\Delta^{-1}(\mathbf{s}_{\gamma_\theta}, \Upsilon_\theta)}{\Gamma}\right)^q\right)^\kappa}\right)\right),$$

$$\mathcal{N}_{\vartheta}^{\varepsilon}=\left(\Delta\left(\Gamma\left(\frac{\Delta^{-1}(\mathbf{s}_{\phi_{\vartheta}},\Phi_{\vartheta})}{\Gamma}\right)^{\varepsilon}\right),\Delta\left(\Gamma\sqrt[q]{1-\left(1-\left(\frac{\Delta^{-1}(\mathbf{s}_{\psi_{\vartheta}},\Psi_{\vartheta})}{\Gamma}\right)^{q}\right)^{\varepsilon}}\right),\Delta\left(\Gamma\sqrt[q]{1-\left(1-\left(\frac{\Delta^{-1}(\mathbf{s}_{\gamma_{\vartheta}},\Upsilon_{\vartheta})}{\Gamma}\right)^{q}\right)^{\varepsilon}}\right)\right).$$

Thus

$$(\mathcal{N}_{\theta}^{\kappa})\otimes(\mathcal{N}_{\vartheta}^{\varepsilon})=\begin{pmatrix}\Delta\left(\Gamma\left(\frac{\Delta^{-1}(\mathbf{s}_{\phi_{\theta}},\Phi_{\theta})}{\Gamma}\right)^{\kappa}\left(\frac{\Delta^{-1}(\mathbf{s}_{\phi_{\vartheta}},\Phi_{\vartheta})}{\Gamma}\right)^{\varepsilon}\right),\\ \Delta\left(\Gamma\sqrt[q]{1-\left(1-\left(\frac{\Delta^{-1}(\mathbf{s}_{\psi_{\theta}},\Psi_{\theta})}{\Gamma}\right)^{q}\right)^{\kappa}\left(1-\left(\frac{\Delta^{-1}(\mathbf{s}_{\psi_{\vartheta}},\Psi_{\vartheta})}{\Gamma}\right)^{q}\right)^{\varepsilon}}\right),\\ \Delta\left(\Gamma\sqrt[q]{1-\left(1-\left(\frac{\Delta^{-1}(\mathbf{s}_{\gamma_{\theta}},\Upsilon_{\theta})}{\Gamma}\right)^{q}\right)^{\kappa}\left(1-\left(\frac{\Delta^{-1}(\mathbf{s}_{\gamma_{\vartheta}},\Upsilon_{\vartheta})}{\Gamma}\right)^{q}\right)^{\varepsilon}}\right)\end{pmatrix}.$$

Therefore,

$$\oplus_{\theta=1}^{\Re}\oplus_{\vartheta=\theta}^{\Re}(\mathcal{N}_{\theta}^{\kappa}\otimes\mathcal{N}_{\vartheta}^{\varepsilon})=\begin{pmatrix}\Delta\left(\Gamma\sqrt[q]{1-\prod_{\theta=1,\vartheta=\theta}^{\Re}\left(1-\left(\frac{\Delta^{-1}(\mathbf{s}_{\phi_{\theta}},\Phi_{\theta})}{\Gamma}\right)^{q\kappa}\left(\frac{\Delta^{-1}(\mathbf{s}_{\phi_{\vartheta}},\Phi_{\vartheta})}{\Gamma}\right)^{q\varepsilon}\right)}\right),\\ \Delta\left(\Gamma\sqrt[q]{1-\left(1-\prod_{\theta=1,\vartheta=\theta}^{\Re}\left(1-\left(1-\left(\frac{\Delta^{-1}(\mathbf{s}_{\psi_{\theta}},\Psi_{\theta})}{\Gamma}\right)^{q}\right)^{\kappa}\left(1-\left(\frac{\Delta^{-1}(\mathbf{s}_{\psi_{\vartheta}},\Psi_{\vartheta})}{\Gamma}\right)^{q}\right)^{\varepsilon}\right)\right)}\right),\\ \Delta\left(\Gamma\sqrt[q]{1-\left(1-\prod_{\theta=1,\vartheta=\theta}^{\Re}\left(1-\left(1-\left(\frac{\Delta^{-1}(\mathbf{s}_{\gamma_{\theta}},\Upsilon_{\theta})}{\Gamma}\right)^{q}\right)^{\kappa}\left(1-\left(\frac{\Delta^{-1}(\mathbf{s}_{\gamma_{\vartheta}},\Upsilon_{\vartheta})}{\Gamma}\right)^{q}\right)^{\varepsilon}\right)\right)}\right)\end{pmatrix}.$$

Furthermore,

$$\frac{2}{\Re(\Re+1)}\oplus_{\theta=1}^{\Re}\oplus_{\vartheta=\theta}^{\Re}(\mathcal{N}_{\theta}^{\kappa}\otimes\mathcal{N}_{\vartheta}^{\varepsilon})$$
$$=\begin{pmatrix}\Delta\left(\Gamma\sqrt[q]{1-\prod_{\theta=1,\vartheta=\theta}^{\Re}\left(1-\left(\frac{\Delta^{-1}(\mathbf{s}_{\phi_{\theta}},\Phi_{\theta})}{\Gamma}\right)^{q\kappa}\left(\frac{\Delta^{-1}(\mathbf{s}_{\phi_{\vartheta}},\Phi_{\vartheta})}{\Gamma}\right)^{q\varepsilon}\right)^{\frac{2}{\Re(\Re+1)}}}\right),\\ \Delta\left(\Gamma\sqrt[q]{1-\left(1-\prod_{\theta=1,\vartheta=\theta}^{\Re}\left(1-\left(1-\left(\frac{\Delta^{-1}(\mathbf{s}_{\psi_{\theta}},\Psi_{\theta})}{\Gamma}\right)^{q}\right)^{\kappa}\left(1-\left(\frac{\Delta^{-1}(\mathbf{s}_{\psi_{\vartheta}},\Psi_{\vartheta})}{\Gamma}\right)^{q}\right)^{\varepsilon}\right)^{\frac{2}{\Re(\Re+1)}}\right)}\right),\\ \Delta\left(\Gamma\sqrt[q]{1-\left(1-\prod_{\theta=1,\vartheta=\theta}^{\Re}\left(1-\left(1-\left(\frac{\Delta^{-1}(\mathbf{s}_{\gamma_{\theta}},\Upsilon_{\theta})}{\Gamma}\right)^{q}\right)^{\kappa}\left(1-\left(\frac{\Delta^{-1}(\mathbf{s}_{\gamma_{\vartheta}},\Upsilon_{\vartheta})}{\Gamma}\right)^{q}\right)^{\varepsilon}\right)^{\frac{2}{\Re(\Re+1)}}\right)}\right)\end{pmatrix}.$$

Therefore,

$$2TLT\text{-SFGHM}^{\kappa,\varepsilon}(\mathcal{N}_{1},\mathcal{N}_{2},\ldots,\mathcal{N}_{\Re})$$
$$=\begin{pmatrix}\Delta\left(\Gamma\left(\sqrt[q]{1-\prod_{\theta=1,\vartheta=\theta}^{\Re}\left(1-\left(\frac{\Delta^{-1}(\mathbf{s}_{\phi_{\theta}},\Phi_{\theta})}{\Gamma}\right)^{q\kappa}\left(\frac{\Delta^{-1}(\mathbf{s}_{\phi_{\vartheta}},\Phi_{\vartheta})}{\Gamma}\right)^{q\varepsilon}\right)^{\frac{2}{\Re(\Re+1)}}}\right)^{\frac{1}{\kappa+\varepsilon}}\right),\\ \Delta\left(\Gamma\sqrt[q]{1-\left(1-\prod_{\theta=1,\vartheta=\theta}^{\Re}\left(1-\left(1-\left(\frac{\Delta^{-1}(\mathbf{s}_{\psi_{\theta}},\Psi_{\theta})}{\Gamma}\right)^{q}\right)^{\kappa}\left(1-\left(\frac{\Delta^{-1}(\mathbf{s}_{\psi_{\vartheta}},\Psi_{\vartheta})}{\Gamma}\right)^{q}\right)^{\varepsilon}\right)^{\frac{2}{\Re(\Re+1)}}\right)^{\frac{1}{\kappa+\varepsilon}}}\right),\\ \Delta\left(\Gamma\sqrt[q]{1-\left(1-\prod_{\theta=1,\vartheta=\theta}^{\Re}\left(1-\left(1-\left(\frac{\Delta^{-1}(\mathbf{s}_{\gamma_{\theta}},\Upsilon_{\theta})}{\Gamma}\right)^{q}\right)^{\kappa}\left(1-\left(\frac{\Delta^{-1}(\mathbf{s}_{\gamma_{\vartheta}},\Upsilon_{\vartheta})}{\Gamma}\right)^{q}\right)^{\varepsilon}\right)^{\frac{2}{\Re(\Re+1)}}\right)^{\frac{1}{\kappa+\varepsilon}}}\right)\end{pmatrix}.$$

□

The 2TLT-SFGHM operator possesses the same properties as Theorem 2.

It can be seen in Definition 12 that the 2TLT-SFGHM aggregation operator doesn't show the weighting values of attributes. Attribute weights, expert weights, and attribute evaluation values are all crucial components in solving MAGDM problems. We utilize the proposed operators to solve MAGDM problems in which attribute weights are known, and the evaluation values are represented by 2TLT-SFSs. To get around the restriction of 2TLT-SFGHM, we propose the 2TLT-SFGWHM operator.

Definition 13. *Let $\kappa, \varepsilon > 0, \mathcal{N}_\vartheta = ((\mathbf{s}_{\phi_\vartheta}, \Phi_\vartheta), (\mathbf{s}_{\psi_\vartheta}, \Psi_\vartheta), (\mathbf{s}_{\gamma_\vartheta}, \Upsilon_\vartheta))(\vartheta = 1, 2, \ldots, \mathfrak{N})$ be 2TLT-SFNs, $\xi = (\xi_1, \xi_2, \ldots, \xi_{\mathfrak{N}})^T$ is the vector of weights of $\mathcal{N}_\vartheta(\vartheta = 1, 2, \ldots, \mathfrak{N})$, where ξ_ϑ indicates the importance degree of $\mathcal{N}_\vartheta$, satisfying $\xi_\vartheta > 0(\vartheta = 1, 2, \ldots, \mathfrak{N})$ and $\sum_{\vartheta=1}^{\mathfrak{N}} \xi_\vartheta = 1$. If*

$$\text{2TLT-SFGWHM}_{\xi}^{\kappa,\varepsilon}(\mathcal{N}_1, \mathcal{N}_2, \ldots, \mathcal{N}_{\mathfrak{N}}) = \left(\oplus_{\theta=1}^{\mathfrak{N}} \oplus_{\vartheta=\theta}^{\mathfrak{N}} \left(\xi_\theta \xi_\vartheta (\mathcal{N}_\theta)^\kappa \otimes (\mathcal{N}_\vartheta)^\varepsilon\right)\right)^{\frac{1}{\kappa+\varepsilon}}. \tag{3.7}$$

Similar to Theorem 4, Theorem 5 can be derived easily.

Theorem 5. *Let $\kappa, \varepsilon > 0, \mathcal{N}_\vartheta = ((\mathbf{s}_{\phi_\vartheta}, \Phi_\vartheta), (\mathbf{s}_{\psi_\vartheta}, \Psi_\vartheta), (\mathbf{s}_{\gamma_\vartheta}, \Upsilon_\vartheta))(\vartheta = 1, 2, \ldots, \mathfrak{N})$ be 2TLT-SFNs, $\xi = (\xi_1, \xi_2, \ldots, \xi_{\mathfrak{N}})^T$ is the vector of weights of $\mathcal{N}_\vartheta(\vartheta = 1, 2, \ldots, \mathfrak{N})$, where ξ_ϑ indicates the importance degree of $\mathcal{N}_\vartheta$, satisfying $\xi_\vartheta > 0(\vartheta = 1, 2, \ldots, \mathfrak{N})$ and $\sum_{\vartheta=1}^{\mathfrak{N}} \xi_\vartheta = 1$. Then the aggregated value by using the 2TLT-SFGWHM is actually a 2TLT-SFN, and we have*

$$\begin{aligned}
&\text{2TLT-SFGWHM}_{\xi}^{\kappa,\varepsilon}(\mathcal{N}_1, \mathcal{N}_2, \ldots, \mathcal{N}_{\mathfrak{N}}) \\
&= \left(\begin{array}{l}
\Delta\left(\Gamma\left(\sqrt[q]{1 - \prod_{\theta=1,\vartheta=\theta}^{\mathfrak{N}} \left(1 - \left(\frac{\Delta^{-1}(\mathbf{s}_{\phi_\theta}, \Phi_\theta)}{\Gamma}\right)^{q\kappa} \left(\frac{\Delta^{-1}(\mathbf{s}_{\phi_\vartheta}, \Phi_\vartheta)}{\Gamma}\right)^{q\varepsilon}\right)^{\xi_\theta \xi_\vartheta}}\right)^{\frac{1}{\kappa+\varepsilon}}\right), \\
\Delta\left(\Gamma \sqrt[q]{1 - \left(1 - \prod_{\theta=1,\vartheta=\theta}^{\mathfrak{N}} \left(1 - \left(1 - \left(\frac{\Delta^{-1}(\mathbf{s}_{\psi_\theta}, \Psi_\theta)}{\Gamma}\right)^q\right)^\kappa \left(1 - \left(\frac{\Delta^{-1}(\mathbf{s}_{\psi_\vartheta}, \Psi_\vartheta)}{\Gamma}\right)^q\right)^\varepsilon\right)^{\xi_\theta \xi_\vartheta}\right)^{\frac{1}{\kappa+\varepsilon}}}\right), \\
\Delta\left(\Gamma \sqrt[q]{1 - \left(1 - \prod_{\theta=1,\vartheta=\theta}^{\mathfrak{N}} \left(1 - \left(1 - \left(\frac{\Delta^{-1}(\mathbf{s}_{\gamma_\theta}, \Upsilon_\theta)}{\Gamma}\right)^q\right)^\kappa \left(1 - \left(\frac{\Delta^{-1}(\mathbf{s}_{\gamma_\vartheta}, \Upsilon_\vartheta)}{\Gamma}\right)^q\right)^\varepsilon\right)^{\xi_\theta \xi_\vartheta}\right)^{\frac{1}{\kappa+\varepsilon}}}\right)
\end{array}\right),
\end{aligned} \tag{3.8}$$

where $\kappa, \varepsilon \geq 0$.

The 2TLT-SFGWHM operator possesses the same properties as Theorem 2.

Subsequently, Yu [58] devised the geometric HM operator, which took into account both the HM and geometric mean operators. In contrast, the 2TLT-SFSs are effective tool for expressing concepts in realistic group decision-making situations, we generalize the geometric HM to the 2TLT-SFSs environment and propose the 2TLT-SFJHM operator for addressing the 2TLT-SFNs.

Definition 14. *Let $\mathcal{N}_\vartheta = ((\mathbf{s}_{\phi_\vartheta}, \Phi_\vartheta), (\mathbf{s}_{\psi_\vartheta}, \Psi_\vartheta), (\mathbf{s}_{\gamma_\vartheta}, \Upsilon_\vartheta))(\vartheta = 1, 2, \ldots, \mathfrak{N})$ be 2TLT-SFNs. The 2TLT-SFJHM is a mapping $P^{\mathfrak{N}} \to P$ such that*

$$\text{2TLT-SFJHM}^{\kappa,\varepsilon}(\mathcal{N}_1, \mathcal{N}_2, \ldots, \mathcal{N}_{\mathfrak{N}}) = \frac{1}{\kappa+\varepsilon}\left(\otimes_{\theta=1}^{\mathfrak{N}} \otimes_{\vartheta=\theta}^{\mathfrak{N}} \left(\kappa \mathcal{N}_\theta \oplus \varepsilon \mathcal{N}_\vartheta\right)\right)^{\frac{2}{\mathfrak{N}(\mathfrak{N}+1)}}, \tag{3.9}$$

where $\kappa, \varepsilon \geq 0$.

Utilizing the Definition 6, we can deduce the following theorem:

Theorem 6. *The aggregated value by using 2TLT-SFJHM operator is also 2TLT-SFN, and we have*

$$
\begin{aligned}
&2TLT\text{-}SFJHM^{\kappa,\varepsilon}(\mathcal{N}_1, \mathcal{N}_2, \ldots, \mathcal{N}_{\Re}) \\
&= \begin{pmatrix}
\Delta\left(\Gamma \sqrt[q]{1-\left(1-\prod_{\theta=1,\vartheta=\theta}^{\Re}\left(1-\left(1-\left(\frac{\Delta^{-1}(\mathbf{s}_{\phi_\theta},\Phi_\theta)}{\Gamma}\right)^q\right)^\kappa\left(1-\left(\frac{\Delta^{-1}(\mathbf{s}_{\phi_\vartheta},\Phi_\vartheta)}{\Gamma}\right)^q\right)^\varepsilon\right)^{\frac{2}{\Re(\Re+1)}}\right)^{\frac{1}{\kappa+\varepsilon}}}\right), \\
\Delta\left(\Gamma\left(\sqrt[q]{1-\prod_{\theta=1,\vartheta=\theta}^{\Re}\left(1-\left(\frac{\Delta^{-1}(\mathbf{s}_{\psi_\theta},\Psi_\theta)}{\Gamma}\right)^{q\kappa}\left(\frac{\Delta^{-1}(\mathbf{s}_{\psi_\vartheta},\Psi_\vartheta)}{\Gamma}\right)^{q\varepsilon}\right)^{\frac{2}{\Re(\Re+1)}}}\right)^{\frac{1}{\kappa+\varepsilon}}\right), \\
\Delta\left(\Gamma\left(\sqrt[q]{1-\prod_{\theta=1,\vartheta=\theta}^{\Re}\left(1-\left(\frac{\Delta^{-1}(\mathbf{s}_{\gamma_\theta},\Upsilon_\theta)}{\Gamma}\right)^{q\kappa}\left(\frac{\Delta^{-1}(\mathbf{s}_{\gamma_\vartheta},\Upsilon_\vartheta)}{\Gamma}\right)^{q\varepsilon}\right)^{\frac{2}{\Re(\Re+1)}}}\right)^{\frac{1}{\kappa+\varepsilon}}\right)
\end{pmatrix}, \quad (3.10)
\end{aligned}
$$

where $\kappa, \varepsilon \geq 0$.

Proof. According to Definition 6, we can derive

$$
\kappa\mathcal{N}_\theta = \left(\Delta\left(\Gamma\sqrt[q]{1-\left(1-\left(\frac{\Delta^{-1}(\mathbf{s}_{\phi_\theta},\Phi_\theta)}{\Gamma}\right)^q\right)^\kappa}\right), \Delta\left(\Gamma\left(\frac{\Delta^{-1}(\mathbf{s}_{\psi_\theta},\Psi_\theta)}{\Gamma}\right)^\kappa\right), \Delta\left(\Gamma\left(\frac{\Delta^{-1}(\mathbf{s}_{\gamma_\theta},\Upsilon_\theta)}{\Gamma}\right)^\kappa\right)\right),
$$

$$
\varepsilon\mathcal{N}_\vartheta = \left(\Delta\left(\Gamma\sqrt[q]{1-\left(1-\left(\frac{\Delta^{-1}(\mathbf{s}_{\phi_\vartheta},\Phi_\vartheta)}{\Gamma}\right)^q\right)^\varepsilon}\right), \Delta\left(\Gamma\left(\frac{\Delta^{-1}(\mathbf{s}_{\psi_\vartheta},\Psi_\vartheta)}{\Gamma}\right)^\varepsilon\right), \Delta\left(\Gamma\left(\frac{\Delta^{-1}(\mathbf{s}_{\gamma_\vartheta},\Upsilon_\vartheta)}{\Gamma}\right)^\varepsilon\right)\right).
$$

Thus

$$
(\kappa\mathcal{N}_\theta)\oplus(\varepsilon\mathcal{N}_\vartheta) = \begin{pmatrix}
\Delta\left(\Gamma\sqrt[q]{1-\left(1-\left(\frac{\Delta^{-1}(\mathbf{s}_{\phi_\theta},\Phi_\theta)}{\Gamma}\right)^q\right)^\kappa\left(1-\left(\frac{\Delta^{-1}(\mathbf{s}_{\phi_\vartheta},\Phi_\vartheta)}{\Gamma}\right)^q\right)^\varepsilon}\right), \\
\Delta\left(\Gamma\left(\frac{\Delta^{-1}(\mathbf{s}_{\psi_\theta},\Psi_\theta)}{\Gamma}\right)^\kappa\left(\frac{\Delta^{-1}(\mathbf{s}_{\psi_\vartheta},\Psi_\vartheta)}{\Gamma}\right)^\varepsilon\right), \Delta\left(\Gamma\left(\frac{\Delta^{-1}(\mathbf{s}_{\gamma_\theta},\Upsilon_\theta)}{\Gamma}\right)^\kappa\left(\frac{\Delta^{-1}(\mathbf{s}_{\gamma_\vartheta},\Upsilon_\vartheta)}{\Gamma}\right)^\varepsilon\right)
\end{pmatrix}.
$$

Therefore,

$$
\begin{aligned}
&\otimes_{\theta=1}^{\Re}\otimes_{\vartheta=\theta}^{\Re}((\kappa\mathcal{N}_\theta)\oplus(\varepsilon\mathcal{N}_\vartheta)) \\
&= \begin{pmatrix}
\Delta\left(\Gamma\sqrt[q]{\left(\prod_{\theta=1,\vartheta=\theta}^{\Re}\left(1-\left(1-\left(\frac{\Delta^{-1}(\mathbf{s}_{\phi_\theta},\Phi_\theta)}{\Gamma}\right)^q\right)^\kappa\left(1-\left(\frac{\Delta^{-1}(\mathbf{s}_{\phi_\vartheta},\Phi_\vartheta)}{\Gamma}\right)^q\right)^\varepsilon\right)\right)}\right), \\
\Delta\left(\Gamma\sqrt[q]{1-\prod_{\theta=1,\vartheta=\theta}^{\Re}\left(1-\left(\frac{\Delta^{-1}(\mathbf{s}_{\psi_\theta},\Psi_\theta)}{\Gamma}\right)^{q\kappa}\left(\frac{\Delta^{-1}(\mathbf{s}_{\psi_\vartheta},\Psi_\vartheta)}{\Gamma}\right)^{q\varepsilon}\right)}\right), \\
\Delta\left(\Gamma\sqrt[q]{1-\prod_{\theta=1,\vartheta=\theta}^{\Re}\left(1-\left(\frac{\Delta^{-1}(\mathbf{s}_{\gamma_\theta},\Upsilon_\theta)}{\Gamma}\right)^{q\kappa}\left(\frac{\Delta^{-1}(\mathbf{s}_{\gamma_\vartheta},\Upsilon_\vartheta)}{\Gamma}\right)^{q\varepsilon}\right)}\right)
\end{pmatrix}.
\end{aligned}
$$

Furthermore,

$$
\left(\otimes_{\theta=1}^{\Re}\otimes_{\vartheta=\theta}^{\Re}((\kappa\mathcal{N}_\theta)\oplus(\varepsilon\mathcal{N}_\vartheta))\right)^{\frac{2}{\Re(\Re+1)}}
$$

$$= \begin{pmatrix} \Delta\left(\Gamma\left(\sqrt[q]{\prod_{\theta=1,\vartheta=\theta}^{\Re}\left(1-\left(1-\left(\frac{\Delta^{-1}(s_{\phi_\theta},\Phi_\theta)}{\Gamma}\right)^q\right)^\kappa\left(1-\left(\frac{\Delta^{-1}(s_{\phi_\vartheta},\Phi_\vartheta)}{\Gamma}\right)^q\right)^\varepsilon\right)}\right)^{\frac{2}{\Re(\Re+1)}}\right), \\ \Delta\left(\Gamma\sqrt[q]{1-\prod_{\theta=1,\vartheta=\theta}^{\Re}\left(1-\left(\frac{\Delta^{-1}(s_{\psi_\theta},\Psi_\theta)}{\Gamma}\right)^{q\kappa}\left(\frac{\Delta^{-1}(s_{\psi_\vartheta},\Psi_\vartheta)}{\Gamma}\right)^{q\varepsilon}\right)^{\frac{2}{\Re(\Re+1)}}}\right), \\ \Delta\left(\Gamma\sqrt[q]{1-\prod_{\theta=1,\vartheta=\theta}^{\Re}\left(1-\left(\frac{\Delta^{-1}(s_{\gamma_\theta},\Upsilon_\theta)}{\Gamma}\right)^{q\kappa}\left(\frac{\Delta^{-1}(s_{\gamma_\vartheta},\Upsilon_\vartheta)}{\Gamma}\right)^{q\varepsilon}\right)^{\frac{2}{\Re(\Re+1)}}}\right) \end{pmatrix}.$$

Therefore,

$$2TLT\text{-}SFJHM^{\kappa,\varepsilon}(\mathcal{N}_1,\mathcal{N}_2,\ldots,\mathcal{N}_\Re)$$

$$= \begin{pmatrix} \Delta\left(\Gamma\sqrt[q]{1-\left(1-\prod_{\theta=1,\vartheta=\theta}^{\Re}\left(1-\left(1-\left(\frac{\Delta^{-1}(s_{\phi_\theta},\Phi_\theta)}{\Gamma}\right)^q\right)^\kappa\left(1-\left(\frac{\Delta^{-1}(s_{\phi_\vartheta},\Phi_\vartheta)}{\Gamma}\right)^q\right)^\varepsilon\right)^{\frac{2}{\Re(\Re+1)}}\right)^{\frac{1}{\kappa+\varepsilon}}}\right), \\ \Delta\left(\Gamma\left(\sqrt[q]{1-\prod_{\theta=1,\vartheta=\theta}^{\Re}\left(1-\left(\frac{\Delta^{-1}(s_{\psi_\theta},\Psi_\theta)}{\Gamma}\right)^{q\kappa}\left(\frac{\Delta^{-1}(s_{\psi_\vartheta},\Psi_\vartheta)}{\Gamma}\right)^{q\varepsilon}\right)^{\frac{2}{\Re(\Re+1)}}}\right)^{\frac{1}{\kappa+\varepsilon}}\right), \\ \Delta\left(\Gamma\left(\sqrt[q]{1-\prod_{\theta=1,\vartheta=\theta}^{\Re}\left(1-\left(\frac{\Delta^{-1}(s_{\gamma_\theta},\Upsilon_\theta)}{\Gamma}\right)^{q\kappa}\left(\frac{\Delta^{-1}(s_{\gamma_\vartheta},\Upsilon_\vartheta)}{\Gamma}\right)^{q\varepsilon}\right)^{\frac{2}{\Re(\Re+1)}}}\right)^{\frac{1}{\kappa+\varepsilon}}\right) \end{pmatrix}.$$

□

The 2TLT-SFJHM operator possesses the same properties as Theorem 2.

It can be seen in Definition 14 that the 2TLT-SFJHM aggregation operator doesn't show the weighting values of attributes. Attribute weights, expert weights, and attribute evaluation values are all crucial components in solving MAGDM problems. We utilize the proposed AOs to solve MAGDM problems in which attribute weights are known, and the evaluation values are represented by 2TLT-SFSs. To get around the restriction of 2TLT-SFJHM, we propose the 2TLT-SFWJHM, in this subsection.

Definition 15. *Let* $\kappa,\varepsilon > 0, \mathcal{N}_\vartheta = ((s_{\phi_\vartheta},\Phi_\vartheta),(s_{\psi_\vartheta},\Psi_\vartheta),(s_{\gamma_\vartheta},\Upsilon_\vartheta))(\vartheta = 1,2,\ldots,\Re)$ *be 2TLT-SFNs,* $\xi = (\xi_1,\xi_2,\ldots,\xi_\Re)^T$ *is the vector of weights of* $\mathcal{N}_\vartheta(\vartheta = 1,2,\ldots,\Re)$*, where* ξ_ϑ *indicates the importance degree of* $\mathcal{N}_\vartheta$*, satisfying* $\xi_\vartheta > 0(\vartheta = 1,2,\ldots,\Re)$ *and* $\sum_{\vartheta=1}^{\Re}\xi_\vartheta = 1$*. If*

$$2TLT\text{-}SFWJHM_\xi^{\kappa,\varepsilon}(\mathcal{N}_1,\mathcal{N}_2,\ldots,\mathcal{N}_\Re) = \frac{1}{\kappa+\varepsilon}\left(\otimes_{\theta=1}^{\Re}\otimes_{\vartheta=\theta}^{\Re}\left(\kappa\mathcal{N}_\theta\oplus\varepsilon\mathcal{N}_\vartheta\right)^{\xi_\theta\xi_\vartheta}\right), \tag{3.11}$$

where $\kappa,\varepsilon \geq 0$.

Theorem 7. *Let* $\kappa,\varepsilon > 0, \mathcal{N}_\vartheta = ((s_{\phi_\vartheta},\Phi_\vartheta),(s_{\psi_\vartheta},\Psi_\vartheta),(s_{\gamma_\vartheta},\Upsilon_\vartheta))(\vartheta = 1,2,\ldots,\Re)$ *be 2TLT-SFNs,* $\xi = (\xi_1,\xi_2,\ldots,\xi_\Re)^T$ *is the vector of weights of* $\mathcal{N}_\vartheta(\vartheta = 1,2,\ldots,\Re)$*, where* ξ_ϑ *indicates the importance degree of* $\mathcal{N}_\vartheta$*, satisfying* $\xi_\vartheta > 0(\vartheta = 1,2,\ldots,\Re)$ *and* $\sum_{\vartheta=1}^{\Re}\xi_\vartheta = 1$*. Then the aggregated value by the 2TLT-SFWJHM is actually a 2TLT-SFN, and we have*

$$2TLT\text{-}SFWJHM_\xi^{\kappa,\varepsilon}(\mathcal{N}_1,\mathcal{N}_2,\ldots,\mathcal{N}_\Re)$$

$$= \left(\begin{array}{l} \Delta\left(\Gamma\sqrt[q]{1-\left(1-\prod_{\theta=1,\vartheta=\theta}^{\mathfrak{N}}\left(1-\left(1-\left(\frac{\Delta^{-1}(\mathbf{s}_{\phi_\theta},\Phi_\theta)}{\Gamma}\right)^q\right)^\kappa\left(1-\left(\frac{\Delta^{-1}(\mathbf{s}_{\phi_\vartheta},\Phi_\vartheta)}{\Gamma}\right)^q\right)^\varepsilon\right)^{\xi_\theta\xi_\vartheta}\right)^{\frac{1}{\kappa+\varepsilon}}}\right), \\ \Delta\left(\Gamma\left(\sqrt[q]{1-\prod_{\theta=1,\vartheta=\theta}^{\mathfrak{N}}\left(1-\left(\frac{\Delta^{-1}(\mathbf{s}_{\psi_\theta},\Psi_\theta)}{\Gamma}\right)^{q\kappa}\left(\frac{\Delta^{-1}(\mathbf{s}_{\psi_\vartheta},\Psi_\vartheta)}{\Gamma}\right)^{q\varepsilon}\right)^{\xi_\theta\xi_\vartheta}}\right)^{\frac{1}{\kappa+\varepsilon}}\right), \\ \Delta\left(\Gamma\left(\sqrt[q]{1-\prod_{\theta=1,\vartheta=\theta}^{\mathfrak{N}}\left(1-\left(\frac{\Delta^{-1}(\mathbf{s}_{\gamma_\theta},\Upsilon_\theta)}{\Gamma}\right)^{q\kappa}\left(\frac{\Delta^{-1}(\mathbf{s}_{\gamma_\vartheta},\Upsilon_\vartheta)}{\Gamma}\right)^{q\varepsilon}\right)^{\xi_\theta\xi_\vartheta}}\right)^{\frac{1}{\kappa+\varepsilon}}\right) \end{array} \right), \quad (3.12)$$

where $\kappa, \varepsilon \geq 0$.

The 2TLT-SFWJHM operator possesses the same properties as Theorem 2.

4. The 2TLT-SF-MABAC method for MAGDM

To cope with group decision-making problems a novel MABAC proves to be an effective tool. So, it is crucial to develop a new 2TLT-SF-MABAC method by extending the MABAC method into 2TLT-SFNs in order to deal with the linguistic assessment information. Therefore, this section develops the 2TLT-SF-MABAC model based on 2TLT-SFGWHM and 2TLT-SFWJHM operators by considering the flexibility of 2TLT-SFNs.

Specifically, suppose that there are $\mathfrak{M}$ alternatives $F = \{F_1, F_2, \ldots, F_{\mathfrak{M}}\}$, $\mathfrak{N}$ attributes $\aleph = \{\aleph_1, \aleph_2, \ldots, \aleph_{\mathfrak{N}}\}$, and λ experts $\mathcal{D} = \{\mathcal{D}_1, \mathcal{D}_2, \ldots, \mathcal{D}_\lambda\}$, and let $\xi = (\xi_1, \xi_2, \ldots, \xi_{\mathfrak{N}})^T$ and $\omega = (\omega_1, \omega_2, \ldots, \omega_\lambda)^T$ be the vector of weights of the $\aleph_\vartheta$ and vector of weights of the DMs fulfilling $\xi_\vartheta \in [0,1]$, $\omega_\flat \in [0,1]$, $\sum_{\vartheta=1}^{\mathfrak{N}} \xi_\vartheta = 1$, and $\sum_{\flat=1}^{\lambda} \omega_\flat = 1$.

The next phases outline the strategy for constructing the 2TLT-SF-MABAC methodology.

Step 1. Establish the 2TLT-SF evaluation matrix $R = [F^\lambda_{\theta\vartheta}]_{\mathfrak{M}\times\mathfrak{N}} = ((\mathbf{s}_{\phi_{\theta\vartheta}}, \Phi)^\lambda, (\mathbf{s}_{\psi_{\theta\vartheta}}, \Psi)^\lambda, (\mathbf{s}_{\gamma_{\theta\vartheta}}, \Upsilon)^\lambda)$ as:

$$\begin{aligned} R &= [F^\lambda_{\theta\vartheta}]_{\mathfrak{M}\times\mathfrak{N}} \quad (4.1) \\ &= \begin{bmatrix} ((\mathbf{s}_{\phi_{11}},\Phi)^\lambda,(\mathbf{s}_{\psi_{11}},\Psi)^\lambda,(\mathbf{s}_{\gamma_{11}},\Upsilon)^\lambda) & ((\mathbf{s}_{\phi_{12}},\Phi)^\lambda,(\mathbf{s}_{\psi_{12}},\Psi)^\lambda,(\mathbf{s}_{\gamma_{12}},\Upsilon)^\lambda) & \ldots & ((\mathbf{s}_{\phi_{1\mathfrak{N}}},\Phi)^\lambda,(\mathbf{s}_{\psi_{1\mathfrak{N}}},\Psi)^\lambda,(\mathbf{s}_{\gamma_{1\mathfrak{N}}},\Upsilon)^\lambda) \\ ((\mathbf{s}_{\phi_{21}},\Phi)^\lambda,(\mathbf{s}_{\psi_{21}},\Psi)^\lambda,(\mathbf{s}_{\gamma_{21}},\Upsilon)^\lambda) & ((\mathbf{s}_{\phi_{22}},\Phi)^\lambda,(\mathbf{s}_{\psi_{22}},\Psi)^\lambda,(\mathbf{s}_{\gamma_{22}},\Upsilon)^\lambda) & \ldots & ((\mathbf{s}_{\phi_{2\mathfrak{N}}},\Phi)^\lambda,(\mathbf{s}_{\psi_{2\mathfrak{N}}},\Psi)^\lambda,(\mathbf{s}_{\gamma_{2\mathfrak{N}}},\Upsilon)^\lambda) \\ \vdots & \vdots & \ddots & \vdots \\ ((\mathbf{s}_{\phi_{\mathfrak{M}1}},\Phi)^\lambda,(\mathbf{s}_{\psi_{\mathfrak{M}1}},\Psi)^\lambda,(\mathbf{s}_{\gamma_{\mathfrak{M}1}},\Upsilon)^\lambda) & ((\mathbf{s}_{\phi_{\mathfrak{M}2}},\Phi)^\lambda,(\mathbf{s}_{\psi_{\mathfrak{M}2}},\Psi)^\lambda,(\mathbf{s}_{\gamma_{\mathfrak{M}2}},\Upsilon)^\lambda) & \ldots & ((\mathbf{s}_{\phi_{\mathfrak{M}\mathfrak{N}}},\Phi)^\lambda,(\mathbf{s}_{\psi_{\mathfrak{M}\mathfrak{N}}},\Psi)^\lambda,(\mathbf{s}_{\gamma_{\mathfrak{M}\mathfrak{N}}},\Upsilon)^\lambda) \end{bmatrix} \end{aligned}$$

where $F^\lambda_{\theta\vartheta} = ((\mathbf{s}_{\phi_{\theta\vartheta}}, \Phi)^\lambda, (\mathbf{s}_{\psi_{\theta\vartheta}}, \Psi)^\lambda, (\mathbf{s}_{\gamma_{\theta\vartheta}}, \Upsilon)^\lambda)(\theta = 1, 2, \ldots, \mathfrak{M}, \vartheta = 1, 2, \ldots, \mathfrak{N}$, and $\lambda = 1, 2, \ldots, \flat)$ describes the 2TLT-SF information of alternatives F_θ on attributes $\aleph_\vartheta$ by decision experts $\mathcal{D}_\lambda$.

Step 2. According to the 2TLT-SFGWHM or 2TLT-SFWJHM operators, we utilize overall $F^\lambda_{\theta\vartheta}$ to $F_{\theta\vartheta}$, the fused 2TLT-SFNs matrix $r = [F_{\theta\vartheta}]_{\mathfrak{M}\times\mathfrak{N}}$ constructed as:

$$\begin{aligned} r &= [F_{\theta\vartheta}]_{\mathfrak{M}\times\mathfrak{N}} \quad (4.2) \\ &= \begin{bmatrix} ((\mathbf{s}_{\phi_{11}},\Phi),(\mathbf{s}_{\psi_{11}},\Psi),(\mathbf{s}_{\gamma_{11}},\Upsilon)) & ((\mathbf{s}_{\phi_{12}},\Phi),(\mathbf{s}_{\psi_{12}},\Psi),(\mathbf{s}_{\gamma_{12}},\Upsilon)) & \ldots & ((\mathbf{s}_{\phi_{1\mathfrak{N}}},\Phi),(\mathbf{s}_{\psi_{1\mathfrak{N}}},\Psi),(\mathbf{s}_{\gamma_{1\mathfrak{N}}},\Upsilon)) \\ ((\mathbf{s}_{\phi_{21}},\Phi),(\mathbf{s}_{\psi_{21}},\Psi),(\mathbf{s}_{\gamma_{21}},\Upsilon)) & ((\mathbf{s}_{\phi_{22}},\Phi),(\mathbf{s}_{\psi_{22}},\Psi),(\mathbf{s}_{\gamma_{22}},\Upsilon)) & \ldots & ((\mathbf{s}_{\phi_{2\mathfrak{N}}},\Phi),(\mathbf{s}_{\psi_{2\mathfrak{N}}},\Psi),(\mathbf{s}_{\gamma_{2\mathfrak{N}}},\Upsilon)) \\ \vdots & \vdots & \ddots & \vdots \\ ((\mathbf{s}_{\phi_{\mathfrak{M}1}},\Phi),(\mathbf{s}_{\psi_{\mathfrak{M}1}},\Psi),(\mathbf{s}_{\gamma_{\mathfrak{M}1}},\Upsilon)) & ((\mathbf{s}_{\phi_{\mathfrak{M}2}},\Phi),(\mathbf{s}_{\psi_{\mathfrak{M}2}},\Psi),(\mathbf{s}_{\gamma_{\mathfrak{M}2}},\Upsilon)) & \ldots & ((\mathbf{s}_{\phi_{\mathfrak{M}\mathfrak{N}}},\Phi),(\mathbf{s}_{\psi_{\mathfrak{M}\mathfrak{N}}},\Psi),(\mathbf{s}_{\gamma_{\mathfrak{M}\mathfrak{N}}},\Upsilon)) \end{bmatrix} \end{aligned}$$

Step 3. Normalize the aggregated matrix $r = [F_{\theta\vartheta}]_{\mathfrak{M}\times\mathfrak{N}}$ utilizing a computation based on each attribute:
For benefit attributes:

$$N_{\theta\vartheta} = F_{\theta\vartheta} = ((\mathbf{s}_{\phi_{\theta\vartheta}}, \Phi_{\theta\vartheta}), (\mathbf{s}_{\psi_{\theta\vartheta}}, \Psi_{\theta\vartheta}), (\mathbf{s}_{\gamma_{\theta\vartheta}}, \Upsilon_{\theta\vartheta})),\ \theta = 1, 2, \ldots, \mathfrak{M},\ \vartheta = 1, 2, \ldots, \mathfrak{N} \tag{4.3}$$

For cost attributes:

$$N_{\theta\vartheta} = F^c_{\theta\vartheta} = ((\mathbf{s}_{\gamma_{\theta\vartheta}}, \Upsilon_{\theta\vartheta}), (\mathbf{s}_{\psi_{\theta\vartheta}}, \Psi_{\theta\vartheta}), (\mathbf{s}_{\phi_{\theta\vartheta}}, \Phi_{\theta\vartheta})),\ \theta = 1, 2, \ldots, \mathfrak{M},\ \vartheta = 1, 2, \ldots, \mathfrak{N} \tag{4.4}$$

Step 4. By utilizing the normalized matrix $N_{\theta\vartheta} = ((\mathbf{s}_{\phi_{\theta\vartheta}}, \Phi_{\theta\vartheta}), (\mathbf{s}_{\psi_{\theta\vartheta}}, \Psi_{\theta\vartheta}), (\mathbf{s}_{\gamma_{\theta\vartheta}}, \Upsilon_{\theta\vartheta}))(\theta = 1, 2, \ldots, \mathfrak{M}, \vartheta = 1, 2, \ldots, \mathfrak{N})$ and attribute's weights $\xi_\vartheta(\vartheta = 1, 2, \ldots, \mathfrak{N})$, the 2TL$T$-SF weighted normalized matrix $WN_{\theta\vartheta} = ((\mathbf{s}'_{\phi_{\theta\vartheta}}, \Phi'_{\theta\vartheta}), (\mathbf{s}'_{\psi_{\theta\vartheta}}, \Psi'_{\theta\vartheta}), (\mathbf{s}'_{\gamma_{\theta\vartheta}}, \Upsilon'_{\theta\vartheta}))(\theta = 1, 2, \ldots, \mathfrak{M}, \vartheta = 1, 2, \ldots, \mathfrak{N})$ utilizing the 2TLT-SFGWHM operator can be constructed as:

$$WN_{\theta\vartheta} = \xi_\vartheta \otimes N_{\theta\vartheta} = \left(\Delta\left(\Gamma \sqrt[q]{1 - \left(1 - \left(\frac{\Delta^{-1}(\mathbf{s}_\phi, \Phi)}{\Gamma}\right)^q\right)^{\xi_\vartheta}}\right), \Delta\left(\Gamma\left(\frac{\Delta^{-1}(\mathbf{s}_\psi, \Psi)}{\Gamma}\right)^{\xi_\vartheta}\right),\ \Delta\left(\Gamma\left(\frac{\Delta^{-1}(\mathbf{s}_\gamma, \Upsilon)}{\Gamma}\right)^{\xi_\vartheta}\right) \right). \tag{4.5}$$

By utilizing the normalized matrix $N_{\theta\vartheta} = ((\mathbf{s}_{\phi_{\theta\vartheta}}, \Phi_{\theta\vartheta}), (\mathbf{s}_{\psi_{\theta\vartheta}}, \Psi_{\theta\vartheta}), (\mathbf{s}_{\gamma_{\theta\vartheta}}, \Upsilon_{\theta\vartheta}))(\theta = 1, 2, \ldots, \mathfrak{M}, \vartheta = 1, 2, \ldots, \mathfrak{N})$ and attribute's weights $\xi_\vartheta(\vartheta = 1, 2, \ldots, \mathfrak{N})$, the 2TL$T$-SF weighted normalized matrix $WN_{\theta\vartheta} = ((\mathbf{s}'_{\phi_{\theta\vartheta}}, \Phi'_{\theta\vartheta}), (\mathbf{s}'_{\psi_{\theta\vartheta}}, \Psi'_{\theta\vartheta}), (\mathbf{s}'_{\gamma_{\theta\vartheta}}, \Upsilon'_{\theta\vartheta}))(\theta = 1, 2, \ldots, \mathfrak{M}, \vartheta = 1, 2, \ldots, \mathfrak{N})$ utilizing the 2TLT-SFWJHM operator can be constructed as:

$$WN_{\theta\vartheta} = \xi_\vartheta \otimes N_{\theta\vartheta} = \left(\Delta\left(\Gamma\left(\frac{\Delta^{-1}(\mathbf{s}_\phi, \Phi)}{\Gamma}\right)^{\xi_\vartheta}\right), \Delta\left(\Gamma \sqrt[q]{1 - \left(1 - \left(\frac{\Delta^{-1}(\mathbf{s}_\psi, \Psi)}{\Gamma}\right)^q\right)^{\xi_\vartheta}}\right), \Delta\left(\Gamma \sqrt[q]{1 - \left(1 - \left(\frac{\Delta^{-1}(\mathbf{s}_\gamma, \Upsilon)}{\Gamma}\right)^q\right)^{\xi_\vartheta}}\right) \right) \tag{4.6}$$

Step 5. Determine the BAA matrix $G = [g_\vartheta]_{1\times\mathfrak{N}}$. The element g_ϑ utilizing the 2TLT-SFGWHM operator can be computed as:

$$g_\vartheta = \left(\prod_{\theta=1}^{\mathfrak{M}} WN_{\theta\vartheta}\right)^{1/\mathfrak{M}} = \left(\begin{array}{l} \Delta\left(\Gamma\left(\prod_{\theta=1}^{\mathfrak{M}} \frac{\Delta^{-1}(\mathbf{s}'_\phi, \Phi')}{\Gamma}\right)^{\frac{1}{\mathfrak{M}}}\right), \Delta\left(\Gamma \sqrt[q]{1 - \prod_{\theta=1}^{\mathfrak{M}} \left(1 - \left(\frac{\Delta^{-1}(\mathbf{s}'_\psi, \Psi')}{\Gamma}\right)^q\right)^{\frac{1}{\mathfrak{M}}}}\right), \\ \Delta\left(\Gamma \sqrt[q]{1 - \prod_{\theta=1}^{\mathfrak{M}} \left(1 - \left(\frac{\Delta^{-1}(\mathbf{s}'_\gamma, \Upsilon')}{\Gamma}\right)^q\right)^{\frac{1}{\mathfrak{M}}}}\right) \end{array} \right). \tag{4.7}$$

Determine the BAA matrix $G = [g_\vartheta]_{1\times\mathfrak{N}}$. The element g_ϑ utilizing the 2TLT-SFWJHM operator can be computed as:

$$g_\vartheta = \left(\prod_{\theta=1}^{\mathfrak{M}} WN_{\theta\vartheta}\right)^{1/\mathfrak{M}} = \left(\begin{array}{l} \Delta\left(\Gamma \sqrt[q]{1 - \prod_{\theta=1}^{\mathfrak{M}} \left(1 - \left(\frac{\Delta^{-1}(\mathbf{s}'_\phi, \Phi')}{\Gamma}\right)^q\right)^{\frac{1}{\mathfrak{M}}}}\right), \Delta\left(\Gamma\left(\prod_{\theta=1}^{\mathfrak{M}} \frac{\Delta^{-1}(\mathbf{s}'_\psi, \Psi')}{\Gamma}\right)^{\frac{1}{\mathfrak{M}}}\right), \\ \Delta\left(\Gamma\left(\prod_{\theta=1}^{\mathfrak{M}} \frac{\Delta^{-1}(\mathbf{s}'_\gamma, \Upsilon')}{\Gamma}\right)^{\frac{1}{\mathfrak{M}}}\right) \end{array} \right). \tag{4.8}$$

Step 6. The distance matrix $D = [d_{\theta\vartheta}]_{\mathfrak{M}\times\mathfrak{N}}$ can be computed as follows:

$$d_{\theta\vartheta} = \begin{cases} d(WN_{\theta\vartheta}, g_{\vartheta}), & if \quad WN_{\theta\vartheta} > g_{\vartheta} \\ 0, & if \quad WN_{\theta\vartheta} = g_{\vartheta} \\ -d(WN_{\theta\vartheta}, g_{\vartheta}), & if \quad WN_{\theta\vartheta} < g_{\vartheta} \end{cases} \tag{4.9}$$

where $d_{\theta\vartheta}$ denotes the distance between alternatives $F_{\theta}(\theta = 1, 2, \ldots, \mathfrak{M})$ and BAA, and $d(WN_{\theta\vartheta}, g_{\vartheta})$ represents the distance between $WN_{\theta\vartheta}$ and g_{ϑ}.

Step 7. Add the values of each alternatives's $d_{\theta\vartheta}$ as:

$$S_{\theta} = \sum_{\vartheta=1}^{\mathfrak{N}} d_{\theta\vartheta} \tag{4.10}$$

The order of all alternatives can be determined based on the comprehensive assessment result S_{θ}; obviously, the larger the comprehensive assessment result S_{θ}, the better the decision.

The flowchart of proposed approach is geometrically represented in Figure 1.

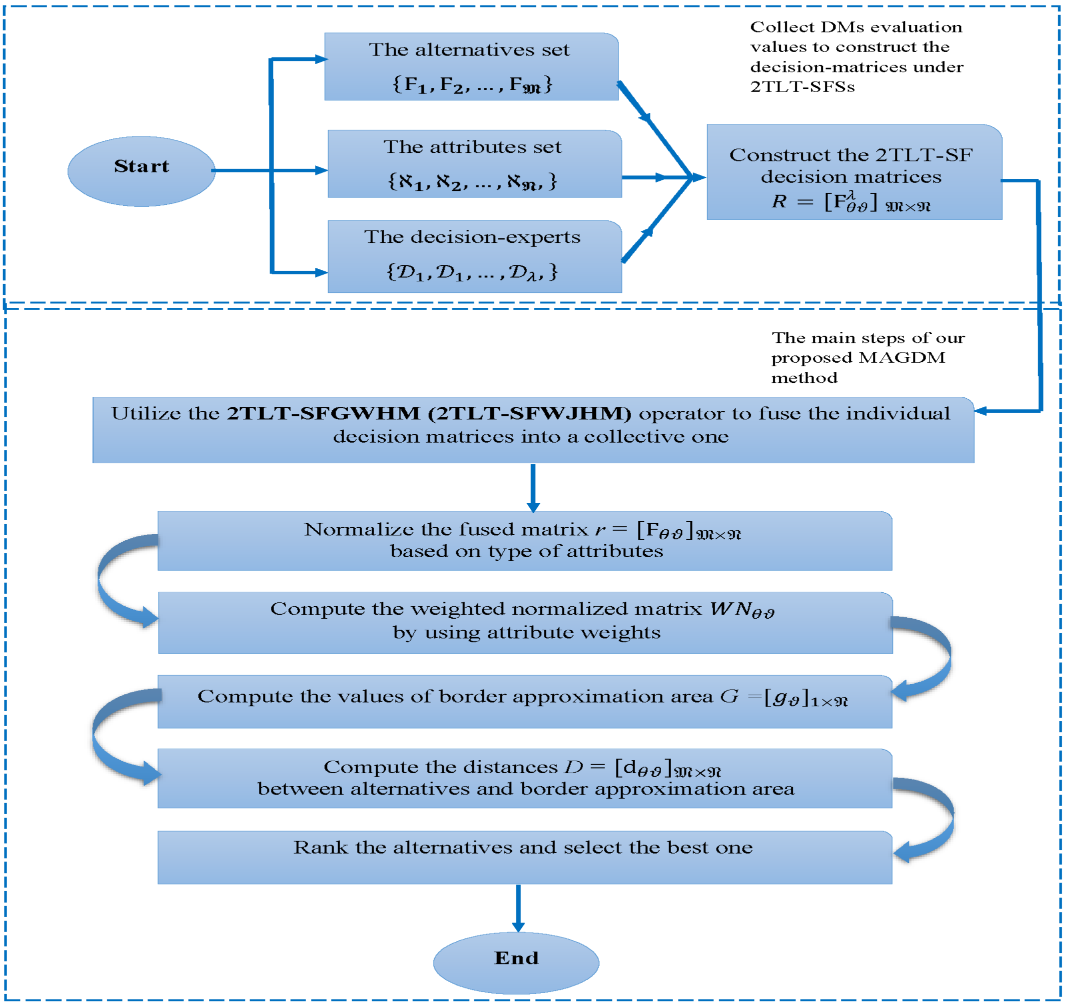

Figure 1. The geometrical representation of constructed approach.

5. Numerical example and discussion

5.1. An application to alternative fuel selection

In this subsection, we demonstrate an actual decision-making application to select the best alternative fuel for diminishing the impact of greenhouse gas emission by using the 2TLT-SF-MABAC model.

As we know that major sources of fuel, such as diesel and petroleum are diminishing. Since these are non-renewable, the demand for them is increasing. Many oil wells are drying up. We are forced to look for alternative fuels to deal with such a situation. The use of alternative fuels reduces the number of harmful emissions such as carbon dioxide, carbon monoxide, and sulfur dioxide that affect the Earth protective ozone layer [43]. The costs of alternative fuels, moreover, are less. Therefore, to select the best alternative fuels, the data is collected from four DMs $\mathcal{D} = \{\mathcal{D}_1, \mathcal{D}_2, \mathcal{D}_3, \mathcal{D}_4\}$(professors and scholars with many years of experience in control the impact of greenhouse gas emission project and FS theory), with vector of weights $\omega = (0.2, 0.4, 0.3, 0.1)^T$. To determine the criteria, five different fuels are chosen in this alternative fuel selection as follows:

(1) Bio-diesel (F_1);

(2) Ethanol (F_2);

(3) Propane (F_3);

(4) Natural gas (F_4);

(5) Hydrogen (F_5).

The evaluation of five alternative fuels include the four attributes as:

(1) CO_2 emission level ($\aleph_1$);

(2) Fuel cost ($\aleph_2$);

(3) Environmental safety ($\aleph_3$);

(4) Technical cost ($\aleph_4$).

Figure 2 is the geometrical structure of selected case study.

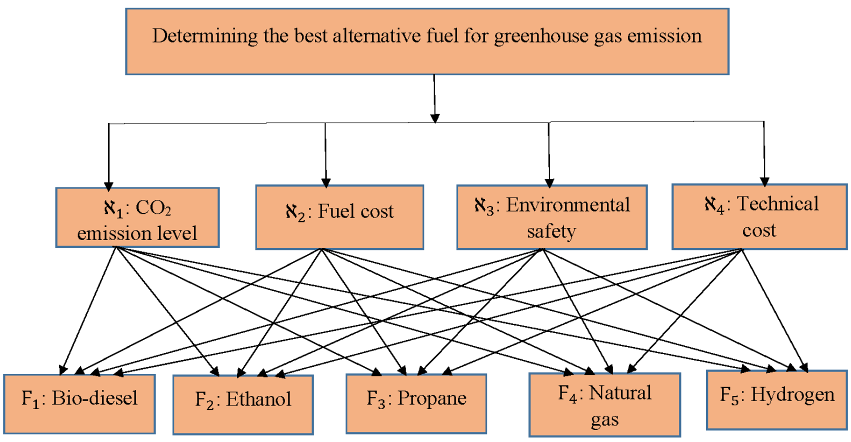

Figure 2. Hierarchical structure of selected case study.

All attributes are of benefit type. The decision experts subjectively assign the essential weights to the attributes as $\xi = (0.17, 0.31, 0.27, 0.25)^T$. The 2TL$T$-SFNs to assess the ability of each fuel to control greenhouse gas emission for each attribute are given in Table 2.

Table 2. Linguistic variables and their 2TLT-SFNs.

Linguistic variables	2TLT-SFNs
Extremely High Importance (EHI)	$((\mathbf{s}_8, 0), (\mathbf{s}_0, 0), (\mathbf{s}_0, 0))$
Absolutely High Importance (AHI)	$((\mathbf{s}_7, 0), (\mathbf{s}_1, 0), (\mathbf{s}_1, 0))$
More Importance (MI)	$((\mathbf{s}_6, 0), (\mathbf{s}_2, 0), (\mathbf{s}_2, 0))$
Medium High Importance (MHI)	$((\mathbf{s}_5, 0), (\mathbf{s}_3, 0), (\mathbf{s}_3, 0))$
Uniformly Importance (UI)	$((\mathbf{s}_4, 0), (\mathbf{s}_4, 0), (\mathbf{s}_4, 0))$
Medium Less Importance (MLI)	$((\mathbf{s}_3, 0), (\mathbf{s}_3, 0), (\mathbf{s}_5, 0))$
Less Importance (LI)	$((\mathbf{s}_2, 0), (\mathbf{s}_2, 0), (\mathbf{s}_6, 0))$
Absolutely Less Importance (ALI)	$((\mathbf{s}_1, 0), (\mathbf{s}_1, 0), (\mathbf{s}_7, 0))$
Extremely Less Importance (ELI)	$((\mathbf{s}_0, 0), (\mathbf{s}_0, 0), (\mathbf{s}_8, 0))$

The 2TLT-SF evaluation matrix describing the assessments of four decision experts

$$R = [F^{\lambda}_{\theta\vartheta}]_{\mathfrak{M}\times\mathfrak{N}} = ((\mathbf{s}_{\phi_{\theta\vartheta}}, \Phi_{\theta\vartheta})^{\lambda}, (\mathbf{s}_{\psi_{\theta\vartheta}}, \Psi_{\theta\vartheta})^{\lambda}, (\mathbf{s}_{\gamma_{\theta\vartheta}}, \Upsilon_{\theta\vartheta}))^{\lambda}(\theta = 1, 2, \ldots, \mathfrak{M}, \vartheta = 1, 2, \ldots, \mathfrak{N})$$

is established in Table 3 in accordance with the linguistic values indicated in Table 2.

Table 3. Linguistic assessing matrix.

Decision makers	Alternatives	$\aleph_1$	$\aleph_2$	$\aleph_3$	$\aleph_4$
	F_1	LI	MLI	MHI	UI
	F_2	UI	AHI	MI	MHI
$\mathcal{D}_1$	F_3	AHI	UI	AHI	MI
	F_4	MI	LI	ALI	AHI
	F_5	AHI	MLI	LI	ALI
	F_1	ALI	LI	MLI	UI
	F_2	MHI	UI	LI	MLI
$\mathcal{D}_2$	F_3	MLI	UI	ALI	LI
	F_4	LI	MLI	UI	ALI
	F_5	MHI	AHI	LI	AHI
	F_1	MHI	LI	MHI	ALI
	F_2	MHI	ALI	LI	UI
$\mathcal{D}_3$	F_3	AHI	MHI	UI	LI
	F_4	LI	UI	MHI	ALI
	F_5	UI	AHI	MHI	AHI
	F_1	MI	ALI	MI	ALI
	F_2	MI	ALI	MHI	MHI
$\mathcal{D}_4$	F_3	MHI	MI	ALI	MI
	F_4	AHI	ALI	MI	MHI
	F_5	ALI	MI	LI	MHI

Conversion of the linguistic evaluation matrix given in Table 3 into 2TLT-SF decision matrix shown in Table 4.

Table 4. The evaluation matrix with 2TLT-SFNs.

Decision makers	Alternatives	$\aleph_1$	$\aleph_2$	$\aleph_3$	$\aleph_4$
	F_1	$((\mathbf{s}_2,0),(\mathbf{s}_2,0),(\mathbf{s}_6,0))$	$((\mathbf{s}_3,0),(\mathbf{s}_3,0),(\mathbf{s}_5,0))$	$((\mathbf{s}_5,0),(\mathbf{s}_3,0),(\mathbf{s}_3,0))$	$((\mathbf{s}_4,0),(\mathbf{s}_4,0),(\mathbf{s}_4,0))$
	F_2	$((\mathbf{s}_4,0),(\mathbf{s}_4,0),(\mathbf{s}_4,0))$	$((\mathbf{s}_7,0),(\mathbf{s}_1,0),(\mathbf{s}_1,0))$	$((\mathbf{s}_6,0),(\mathbf{s}_2,0),(\mathbf{s}_2,0))$	$((\mathbf{s}_5,0),(\mathbf{s}_3,0),(\mathbf{s}_3,0))$
$\mathcal{D}_1$	F_3	$((\mathbf{s}_7,0),(\mathbf{s}_1,0),(\mathbf{s}_1,0))$	$((\mathbf{s}_4,0),(\mathbf{s}_4,0),(\mathbf{s}_4,0))$	$((\mathbf{s}_7,0),(\mathbf{s}_1,0),(\mathbf{s}_1,0))$	$((\mathbf{s}_6,0),(\mathbf{s}_2,0),(\mathbf{s}_2,0))$
	F_4	$((\mathbf{s}_6,0),(\mathbf{s}_2,0),(\mathbf{s}_2,0))$	$((\mathbf{s}_2,0),(\mathbf{s}_2,0),(\mathbf{s}_6,0))$	$((\mathbf{s}_1,0),(\mathbf{s}_1,0),(\mathbf{s}_7,0))$	$((\mathbf{s}_7,0),(\mathbf{s}_1,0),(\mathbf{s}_1,0))$
	F_5	$((\mathbf{s}_7,0),(\mathbf{s}_1,0),(\mathbf{s}_1,0))$	$((\mathbf{s}_3,0),(\mathbf{s}_3,0),(\mathbf{s}_5,0))$	$((\mathbf{s}_2,0),(\mathbf{s}_2,0),(\mathbf{s}_6,0))$	$((\mathbf{s}_1,0),(\mathbf{s}_1,0),(\mathbf{s}_7,0))$
	F_1	$((\mathbf{s}_1,0),(\mathbf{s}_1,0),(\mathbf{s}_7,0))$	$((\mathbf{s}_2,0),(\mathbf{s}_2,0),(\mathbf{s}_6,0))$	$((\mathbf{s}_3,0),(\mathbf{s}_3,0),(\mathbf{s}_5,0))$	$((\mathbf{s}_4,0),(\mathbf{s}_4,0),(\mathbf{s}_4,0))$
	F_2	$((\mathbf{s}_5,0),(\mathbf{s}_3,0),(\mathbf{s}_3,0))$	$((\mathbf{s}_4,0),(\mathbf{s}_4,0),(\mathbf{s}_4,0))$	$((\mathbf{s}_2,0),(\mathbf{s}_2,0),(\mathbf{s}_6,0))$	$((\mathbf{s}_3,0),(\mathbf{s}_3,0),(\mathbf{s}_5,0))$
$\mathcal{D}_2$	F_3	$((\mathbf{s}_3,0),(\mathbf{s}_3,0),(\mathbf{s}_5,0))$	$((\mathbf{s}_4,0),(\mathbf{s}_4,0),(\mathbf{s}_4,0))$	$((\mathbf{s}_1,0),(\mathbf{s}_1,0),(\mathbf{s}_7,0))$	$((\mathbf{s}_2,0),(\mathbf{s}_2,0),(\mathbf{s}_6,0))$
	F_4	$((\mathbf{s}_2,0),(\mathbf{s}_2,0),(\mathbf{s}_6,0))$	$((\mathbf{s}_3,0),(\mathbf{s}_3,0),(\mathbf{s}_5,0))$	$((\mathbf{s}_4,0),(\mathbf{s}_4,0),(\mathbf{s}_4,0))$	$((\mathbf{s}_1,0),(\mathbf{s}_1,0),(\mathbf{s}_7,0))$
	F_5	$((\mathbf{s}_5,0),(\mathbf{s}_3,0),(\mathbf{s}_3,0))$	$((\mathbf{s}_7,0),(\mathbf{s}_1,0),(\mathbf{s}_1,0))$	$((\mathbf{s}_2,0),(\mathbf{s}_2,0),(\mathbf{s}_6,0))$	$((\mathbf{s}_7,0),(\mathbf{s}_1,0),(\mathbf{s}_1,0))$
	F_1	$((\mathbf{s}_5,0),(\mathbf{s}_3,0),(\mathbf{s}_3,0))$	$((\mathbf{s}_2,0),(\mathbf{s}_2,0),(\mathbf{s}_6,0))$	$((\mathbf{s}_5,0),(\mathbf{s}_3,0),(\mathbf{s}_3,0))$	$((\mathbf{s}_1,0),(\mathbf{s}_1,0),(\mathbf{s}_7,0))$
	F_2	$((\mathbf{s}_5,0),(\mathbf{s}_3,0),(\mathbf{s}_3,0))$	$((\mathbf{s}_1,0),(\mathbf{s}_1,0),(\mathbf{s}_7,0))$	$((\mathbf{s}_2,0),(\mathbf{s}_2,0),(\mathbf{s}_6,0))$	$((\mathbf{s}_4,0),(\mathbf{s}_4,0),(\mathbf{s}_4,0))$
$\mathcal{D}_3$	F_3	$((\mathbf{s}_7,0),(\mathbf{s}_1,0),(\mathbf{s}_1,0))$	$((\mathbf{s}_5,0),(\mathbf{s}_3,0),(\mathbf{s}_3,0))$	$((\mathbf{s}_4,0),(\mathbf{s}_4,0),(\mathbf{s}_4,0))$	$((\mathbf{s}_2,0),(\mathbf{s}_2,0),(\mathbf{s}_6,0))$
	F_4	$((\mathbf{s}_2,0),(\mathbf{s}_2,0),(\mathbf{s}_6,0))$	$((\mathbf{s}_4,0),(\mathbf{s}_4,0),(\mathbf{s}_4,0))$	$((\mathbf{s}_5,0),(\mathbf{s}_3,0),(\mathbf{s}_3,0))$	$((\mathbf{s}_1,0),(\mathbf{s}_1,0),(\mathbf{s}_7,0))$
	F_5	$((\mathbf{s}_4,0),(\mathbf{s}_4,0),(\mathbf{s}_4,0))$	$((\mathbf{s}_7,0),(\mathbf{s}_1,0),(\mathbf{s}_1,0))$	$((\mathbf{s}_5,0),(\mathbf{s}_3,0),(\mathbf{s}_3,0))$	$((\mathbf{s}_7,0),(\mathbf{s}_1,0),(\mathbf{s}_1,0))$
	F_1	$((\mathbf{s}_6,0),(\mathbf{s}_2,0),(\mathbf{s}_2,0))$	$((\mathbf{s}_1,0),(\mathbf{s}_1,0),(\mathbf{s}_7,0))$	$((\mathbf{s}_6,0),(\mathbf{s}_2,0),(\mathbf{s}_2,0))$	$((\mathbf{s}_1,0),(\mathbf{s}_1,0),(\mathbf{s}_7,0))$
	F_2	$((\mathbf{s}_6,0),(\mathbf{s}_2,0),(\mathbf{s}_2,0))$	$((\mathbf{s}_1,0),(\mathbf{s}_1,0),(\mathbf{s}_7,0))$	$((\mathbf{s}_5,0),(\mathbf{s}_3,0),(\mathbf{s}_3,0))$	$((\mathbf{s}_5,0),(\mathbf{s}_3,0),(\mathbf{s}_3,0))$
$\mathcal{D}_4$	F_3	$((\mathbf{s}_5,0),(\mathbf{s}_3,0),(\mathbf{s}_3,0))$	$((\mathbf{s}_6,0),(\mathbf{s}_2,0),(\mathbf{s}_2,0))$	$((\mathbf{s}_1,0),(\mathbf{s}_1,0),(\mathbf{s}_7,0))$	$((\mathbf{s}_6,0),(\mathbf{s}_2,0),(\mathbf{s}_2,0))$
	F_4	$((\mathbf{s}_7,0),(\mathbf{s}_1,0),(\mathbf{s}_1,0))$	$((\mathbf{s}_1,0),(\mathbf{s}_1,0),(\mathbf{s}_7,0))$	$((\mathbf{s}_6,0),(\mathbf{s}_2,0),(\mathbf{s}_2,0))$	$((\mathbf{s}_5,0),(\mathbf{s}_3,0),(\mathbf{s}_3,0))$
	F_5	$((\mathbf{s}_1,0),(\mathbf{s}_1,0),(\mathbf{s}_7,0))$	$((\mathbf{s}_6,0),(\mathbf{s}_2,0),(\mathbf{s}_2,0))$	$((\mathbf{s}_2,0),(\mathbf{s}_2,0),(\mathbf{s}_6,0))$	$((\mathbf{s}_5,0),(\mathbf{s}_3,0),(\mathbf{s}_3,0))$

5.2. Decision-making criteria utilizing the 2TLT-SFGWHM operator

In this subsection, the evaluation procedure for the selection of the best alternative is described by using the 2TLT-SF-MABAC method based on the 2TLT-SFGWHM operator.

Step 1. We constructed the 2TLT-SF evaluation matrix $R = [F^{\lambda}_{\theta\vartheta}]_{5\times4}((\mathbf{s}_{\phi_{\theta\vartheta}}, \Phi)^{\lambda}, (\mathbf{s}_{\psi_{\theta\vartheta}}, \Psi)^{\lambda}, (\mathbf{s}_{\gamma_{\theta\vartheta}}, \Upsilon)^{\lambda})_{5\times4}(\theta = 1,2,3,4,5, \vartheta = 1,2,3,4$, and $\lambda = 1,2,3,4)$, which is describing the assessments of four DMs as computed in Tables 2–4.

Step 2. According to the 2TLT-SFGWHM aggregation operator, we utilize overall $F^{\lambda}_{\theta\vartheta}$ to $F_{\theta\vartheta}$, the fused 2TLT-SFNs matrix $r = [F_{\theta\vartheta}]_{\mathfrak{M}\times\mathfrak{N}}$ shown in Table 5. (Suppose $q = 4$, $\Gamma = 8$, $\kappa = 2$, $\varepsilon = 3$, and $\omega = (0.2, 0.4, 0.3, 0.1)^T$)

Step 3. According to the normalized matrix $N_{\theta\vartheta} = ((\mathbf{s}_{\phi_{\theta\vartheta}}, \Phi_{\theta\vartheta}), (\mathbf{s}_{\psi_{\theta\vartheta}}, \Psi_{\theta\vartheta})(\mathbf{s}_{\gamma_{\theta\vartheta}}, \Upsilon_{\theta\vartheta}))(\theta = 1,2,\ldots 5, \vartheta = 1,2,3,4)$ and attribute weights $\xi = (0.17, 0.31, 0.27, 0.25)^T$, the results of 2TL$T$-SF weighted normalized matrix $WN_{\theta\vartheta} = ((\mathbf{s}'_{\phi_{\theta\vartheta}}, \Phi'_{\theta\vartheta}), (\mathbf{s}'_{\psi_{\theta\vartheta}}, \Psi'_{\theta\vartheta}), (\mathbf{s}'_{\gamma_{\theta\vartheta}}, \Upsilon'_{\theta\vartheta}))(\theta = 1,2,\ldots 5, \vartheta = 1,2,3,4)$ are recorded in Table 6.

Step 4. Determine the scores of 2TLT-SF elements of weighted normalized matrix $WN_{\theta\vartheta}$ (see Table 7).

Step 5. Compute the BAA matrix $G = [g_{\vartheta}]_{1\times\mathfrak{N}}$. The results of BAA matrix $G = [g_{\vartheta}]_{1\times\mathfrak{N}}$ are shown in Table 8.

Step 6. Compute the normalized Hamming distance $D = [d_{\theta\vartheta}]_{\mathfrak{M}\times\mathfrak{N}}$ utilizing Eq (2.4), between alternatives and BAA (see Table 9).

Step 7. Sum the values of each alternatives $d_{\theta\vartheta}$.

$$S_1 = \sum_{\vartheta=1}^{\mathfrak{N}} d_{1\vartheta} = d_{11} + d_{12} + d_{13} + d_{14} = -0.0527 + (-0.0854) + 0.0444 + (-0.0593) = -0.1529$$

$$S_2 = \sum_{\vartheta=1}^{\mathfrak{N}} d_{2\vartheta} = d_{21} + d_{22} + d_{23} + d_{24} = 0.0504 + 0.0389 + (-0.0305) + (-0.0567) = 0.0022$$

$$S_3 = \sum_{\vartheta=1}^{\mathfrak{N}} d_{3\vartheta} = d_{31} + d_{32} + d_{33} + d_{34} = 0.0597 + 0.0744 + 0.0466 + (-0.0240) = 0.1566$$

$$S_4 = \sum_{\vartheta=1}^{\mathfrak{N}} d_{4\vartheta} = d_{41} + d_{42} + d_{43} + d_{44} = -0.0426 + (-0.0509) + 0.0424 + (-0.0777) = -0.1287$$

$$S_5 = \sum_{\vartheta=1}^{\mathfrak{N}} d_{5\vartheta} = d_{51} + d_{52} + d_{53} + d_{54} = 0.0353 + 0.1924 + (-0.0285) + 0.1630 = 0.3622.$$

So, by utilizing the result of S_θ, the alternatives $F_\theta(\theta = 1, 2, \ldots, 5)$ can be ranked, easily. The best alternative has the highest value of S_θ. The ranking of alternatives is as follows:

$$F_5 > F_3 > F_2 > F_4 > F_1.$$

Therefore, F_5 is the best alternative fuel.

Table 5. Collective 2TLT-SF matrix according to the 2TLT-SFGWHM operator.

	$\aleph_1$	$\aleph_2$
F_1	$((\mathbf{s}_5, -0.0741), (\mathbf{s}_3, -0.2659), (\mathbf{s}_5, 0.2839))$	$((\mathbf{s}_3, -0.4422), (\mathbf{s}_3, -0.0642), (\mathbf{s}_6, 0.1556))$
F_2	$((\mathbf{s}_5, 0.0906), (\mathbf{s}_4, -0.1869), (\mathbf{s}_4, -0.1869))$	$((\mathbf{s}_6, -0.0297), (\mathbf{s}_3, -0.0044), (\mathbf{s}_5, -0.0985))$
F_3	$((\mathbf{s}_6, 0.4532), (\mathbf{s}_3, -0.1301), (\mathbf{s}_3, 0.4688))$	$((\mathbf{s}_5, -0.0362), (\mathbf{s}_4, 0.1751), (\mathbf{s}_4, 0.1751))$
F_4	$((\mathbf{s}_6, -0.2726), (\mathbf{s}_3, -0.2323), (\mathbf{s}_5, 0.1826))$	$((\mathbf{s}_4, -0.4301), (\mathbf{s}_4, -0.2450), (\mathbf{s}_5, 0.3820))$
F_5	$((\mathbf{s}_6, -0.0185), (\mathbf{s}_4, -0.4067), (\mathbf{s}_4, 0.0018))$	$((\mathbf{s}_7, -0.3221), (\mathbf{s}_2, 0.2739), (\mathbf{s}_3, -0.4955))$

	$\aleph_3$	$\aleph_4$
F_1	$((\mathbf{s}_5, 0.0069), (\mathbf{s}_4, -0.3367), (\mathbf{s}_4, 0.3754))$	$((\mathbf{s}_4, -0.2467), (\mathbf{s}_4, -0.4693), (\mathbf{s}_5, 0.3930))$
F_2	$((\mathbf{s}_5, 0.1239), (\mathbf{s}_3, -0.0651), (\mathbf{s}_5, 0.3664))$	$((\mathbf{s}_4, 0.4101), (\mathbf{s}_4, 0.0152), (\mathbf{s}_5, -0.2794))$
F_3	$((\mathbf{s}_6, -0.0299), (\mathbf{s}_3, -0.3536), (\mathbf{s}_5, 0.1030))$	$((\mathbf{s}_5, 0.2534), (\mathbf{s}_3, -0.1589), (\mathbf{s}_5, 0.2737))$
F_4	$((\mathbf{s}_5, -0.0484), (\mathbf{s}_4, -0.2365), (\mathbf{s}_5, -0.4636))$	$((\mathbf{s}_6, -0.0278), (\mathbf{s}_2, -0.0126), (\mathbf{s}_5, 0.3974))$
F_5	$((\mathbf{s}_4, 0.4331), (\mathbf{s}_3, 0.1454), (\mathbf{s}_5, 0.3792))$	$((\mathbf{s}_7, -0.3309), (\mathbf{s}_2, -0.0126), (\mathbf{s}_3, -0.2815))$

Table 6. The weighted normalized matrix $WN_{\theta\vartheta}(\theta = 1, 2, \ldots 5, \vartheta = 1, 2, 3, 4)$ with 2TLT-SFNs.

	$\aleph_1$	$\aleph_2$
F_1	$((\mathbf{s}_3, 0.2136), (\mathbf{s}_7, -0.3344), (\mathbf{s}_7, 0.4552))$	$((\mathbf{s}_2, -0.0896), (\mathbf{s}_6, -0.1368), (\mathbf{s}_7, 0.3752))$
F_2	$((\mathbf{s}_3, 0.3288), (\mathbf{s}_7, 0.0528), (\mathbf{s}_7, 0.0528))$	$((\mathbf{s}_5, -0.4056), (\mathbf{s}_6, -0.1000), (\mathbf{s}_7, -0.1272))$
F_3	$((\mathbf{s}_4, 0.3736), (\mathbf{s}_7, -0.2800), (\mathbf{s}_7, -0.0592))$	$((\mathbf{s}_4, -0.2456), (\mathbf{s}_7, -0.4608), (\mathbf{s}_7, -0.4608))$
F_4	$((\mathbf{s}_4, -0.2080), (\mathbf{s}_7, -0.3208), (\mathbf{s}_7, 0.4304))$	$((\mathbf{s}_3, -0.3272), (\mathbf{s}_6, 0.3280), (\mathbf{s}_7, 0.0752))$
F_5	$((\mathbf{s}_4, -0.0128), (\mathbf{s}_7, -0.0176), (\mathbf{s}_7, 0.1112))$	$((\mathbf{s}_5, 0.2544), (\mathbf{s}_5, 0.4168), (\mathbf{s}_6, -0.4184))$
	$\aleph_3$	$\aleph_4$
F_1	$((\mathbf{s}_4, -0.3360), (\mathbf{s}_6, 0.4792), (\mathbf{s}_7, -0.2032))$	$((\mathbf{s}_3, -0.3336), (\mathbf{s}_7, -0.4800), (\mathbf{s}_7, 0.2488))$
F_2	$((\mathbf{s}_4, -0.2448), (\mathbf{s}_6, 0.1024), (\mathbf{s}_7, 0.1824))$	$((\mathbf{s}_3, 0.1472), (\mathbf{s}_7, -0.2664), (\mathbf{s}_7, 0.0120))$
F_3	$((\mathbf{s}_4, 0.4464), (\mathbf{s}_6, -0.0656), (\mathbf{s}_7, 0.0856))$	$((\mathbf{s}_4, -0.2144), (\mathbf{s}_6, 0.1752), (\mathbf{s}_7, 0.2088))$
F_4	$((\mathbf{s}_4, -0.3792), (\mathbf{s}_7, -0.4736), (\mathbf{s}_7, -0.1360))$	$((\mathbf{s}_4, 0.3664), (\mathbf{s}_6, -0.3520), (\mathbf{s}_7, 0.2504))$
F_5	$((\mathbf{s}_3, 0.2240), (\mathbf{s}_6, 0.2176), (\mathbf{s}_7, 0.1872))$	$((\mathbf{s}_5, -0.0048), (\mathbf{s}_6, -0.3520), (\mathbf{s}_6, 0.1080))$

Table 7. Score functions of weighted normalized matrix.

	$\aleph_1$	$\aleph_2$	$\aleph_3$	$\aleph_4$
F_1	0.1359	0.1404	0.2614	0.1691
F_2	0.2129	0.2820	0.1994	0.2169
F_3	0.2614	0.3010	0.2400	0.1955
F_4	0.1531	0.2004	0.2500	0.2070
F_5	0.2187	0.4746	0.1875	0.4061

Table 8. The G matrix with 2TLT-SFNs.

$g's$	The 2TLT-SFNs for $g's$	Score
g_1	$((\mathbf{s}_4, -0.2852), (\mathbf{s}_7, -0.1645), (\mathbf{s}_7, 0.2317))$	0.1894
g_2	$((\mathbf{s}_3, 0.4125), (\mathbf{s}_6, 0.0713), (\mathbf{s}_7, -0.1439))$	0.2468
g_3	$((\mathbf{s}_4, -0.2783), (\mathbf{s}_6, 0.2735), (\mathbf{s}_7, 0.0395))$	0.2237
g_4	$((\mathbf{s}_4, -0.3006), (\mathbf{s}_6, 0.2280), (\mathbf{s}_7, 0.0517))$	0.2210

Table 9. The distance matrix with 2TLT-SFNs.

	$\aleph_1$	$\aleph_2$	$\aleph_3$	$\aleph_4$
F_1	$d_{11} = -0.0527$	$d_{12} = -0.0854$	$d_{13} = 0.0444$	$d_{14} = -0.0593$
F_2	$d_{21} = 0.0504$	$d_{22} = 0.0389$	$d_{23} = -0.0305$	$d_{24} = -0.0567$
F_3	$d_{31} = 0.0597$	$d_{32} = 0.0744$	$d_{33} = 0.0466$	$d_{34} = -0.0240$
F_4	$d_{41} = -0.0426$	$d_{42} = -0.0509$	$d_{43} = 0.0424$	$d_{44} = -0.0777$
F_5	$d_{51} = 0.0353$	$d_{52} = 0.1924$	$d_{53} = -0.0285$	$d_{54} = 0.1630$

5.3. Decision-making criteria utilizing the 2TLT-SFWJHM operator

In this subsection, the evaluation procedure for the selection of the best alternative is described by using the 2TLT-SF-MABAC method based on the 2TLT-SFWJHM operator.

Step 1. We constructed the 2TLT-SF evaluation matrix $R = [F^{\lambda}_{\theta\vartheta}]_{5\times4}((\mathbf{s}_{\phi_{\theta\vartheta}}, \Phi)^{\lambda}, (\mathbf{s}_{\psi_{\theta\vartheta}}, \Psi)^{\lambda}, (\mathbf{s}_{\gamma_{\theta\vartheta}}, \Upsilon)^{\lambda})_{5\times4}(\theta = 1,2,3,4,5, \vartheta = 1,2,3,4$, and $\lambda = 1,2,3,4)$, which is describing the assessments of four DMs as computed in Tables 2–4.

Step 2. According to the 2TLT-SFWJHM aggregation operator, we utilize overall $F^{\lambda}_{\theta\vartheta}$ to $F_{\theta\vartheta}$, the fused 2TLT-SFNs matrix $r = [F_{\theta\vartheta}]_{\mathfrak{M}\times\mathfrak{N}}$ shown in Table 10. (Suppose $q = 4$, $\Gamma = 8$, $\kappa = 2$, $\varepsilon = 3$, and $\omega = (0.2, 0.4, 0.3, 0.1)^T$)

Step 3. According to the normalized matrix $N_{\theta\vartheta} = ((\mathbf{s}_{\phi_{\theta\vartheta}}, \Phi_{\theta\vartheta}), (\mathbf{s}_{\psi_{\theta\vartheta}}, \Psi_{\theta\vartheta})(\mathbf{s}_{\gamma_{\theta\vartheta}}, \Upsilon_{\theta\vartheta}))(\theta = 1,2,\ldots 5, \vartheta = 1,2,3,4)$ and attribute weights $\xi = (0.17, 0.31, 0.27, 0.25)^T$, the results of 2TL$T$-SF weighted normalized matrix $WN_{\theta\vartheta} = ((\mathbf{s}'_{\phi_{\theta\vartheta}}, \Phi'_{\theta\vartheta}), (\mathbf{s}'_{\psi_{\theta\vartheta}}, \Psi'_{\theta\vartheta}), (\mathbf{s}'_{\gamma_{\theta\vartheta}}, \Upsilon'_{\theta\vartheta}))(\theta = 1,2,\ldots 5, \vartheta = 1,2,3,4)$ are recorded in Table 11.

Step 4. Determine the scores of 2TLT-SF elements of weighted normalized matrix $WN_{\theta\vartheta}$ (see Table 12).

Step 5. Compute the the BAA matrix $G = [g_{\vartheta}]_{1\times\mathfrak{N}}$. The results of BAA matrix $G = [g_{\vartheta}]_{1\times\mathfrak{N}}$ are shown in Table 13.

Step 6. Compute the normalized Hamming distance $D = [d_{\theta\vartheta}]_{\mathfrak{M}\times\mathfrak{N}}$ utilizing Eq (2.4), between alternatives and BAA (see Table 14).

Step 7. Sum the values of each alternatives $d_{\theta\vartheta}$.

$$\mathrm{S}_1 = \sum_{\vartheta=1}^{\mathfrak{N}} d_{\theta\vartheta} = d_{11} + d_{12} + d_{13} + d_{14} = -0.0635 + (-0.0904) + 0.0462 + (-0.0406) = -0.1483$$

$$\mathrm{S}_2 = \sum_{\vartheta=1}^{\mathfrak{N}} d_{2\vartheta} = d_{21} + d_{22} + d_{23} + d_{24} = 0.0321 + (-0.0632) + (-0.0196) + 0.0332 = -0.0175$$

$$\mathrm{S}_3 = \sum_{\vartheta=1}^{\mathfrak{N}} d_{3\vartheta} = d_{31} + d_{32} + d_{33} + d_{34} = 0.0233 + 0.0360 + (-0.0493) + (-0.0218) = -0.0118$$

$$\mathrm{S}_4 = \sum_{\vartheta=1}^{\mathfrak{N}} d_{4\vartheta} = d_{41} + d_{42} + d_{43} + d_{44} = -0.0402 + (-0.0495) + 0.0311 + (-0.0788) = -0.1374$$

$$\mathrm{S}_5 = \sum_{\vartheta=1}^{\mathfrak{N}} d_{5\vartheta} = d_{51} + d_{52} + d_{53} + d_{54} = 0.0218 + 0.0870 + (-0.0191) + 0.0623 = 0.1520.$$

So, by utilizing the result of S_θ, the alternatives $F_\theta(\theta = 1,2,\ldots,5)$ can be ranked, easily. The best alternative has the highest value of S_θ. The ranking of alternatives is as follows:

$$F_5 > F_3 > F_2 > F_4 > F_1.$$

Therefore, F_5 is the best alternative fuel.

Table 10. Collective 2TLT-SF matrix according to the 2TLT-SFWJHM operator.

	$\aleph_1$	$\aleph_2$
F_1	$((\mathbf{s}_3, 0.4127), (\mathbf{s}_3, -0.3365), (\mathbf{s}_6, 0.4421))$	$((\mathbf{s}_3, -0.0642), (\mathbf{s}_3, -0.4422), (\mathbf{s}_6, 0.0308))$
F_2	$((\mathbf{s}_5, 0.3485), (\mathbf{s}_3, 0.4278), (\mathbf{s}_3, 0.4278))$	$((\mathbf{s}_4, -0.1710), (\mathbf{s}_4, -0.3502), (\mathbf{s}_6, 0.3352))$
F_3	$((\mathbf{s}_5, 0.2422), (\mathbf{s}_3, -0.2252), (\mathbf{s}_5, -0.4376))$	$((\mathbf{s}_5, -0.0087), (\mathbf{s}_4, -0.2427), (\mathbf{s}_4, -0.2427))$
F_4	$((\mathbf{s}_4, -0.2916), (\mathbf{s}_2, -0.0589), (\mathbf{s}_6, -0.2907))$	$((\mathbf{s}_4, -0.2450), (\mathbf{s}_4, -0.4301), (\mathbf{s}_6, -0.2193))$
F_5	$((\mathbf{s}_5, 0.1032), (\mathbf{s}_4, -0.4306), (\mathbf{s}_6, -0.4196))$	$((\mathbf{s}_6, 0.1617), (\mathbf{s}_3, -0.4455), (\mathbf{s}_4, 0.2567))$

	$\aleph_3$	$\aleph_4$
F_1	$((\mathbf{s}_5, -0.2295), (\mathbf{s}_3, -0.0882), (\mathbf{s}_5, -0.4355))$	$((\mathbf{s}_4, -0.4693), (\mathbf{s}_4, -0.2467), (\mathbf{s}_6, 0.3373))$
F_2	$((\mathbf{s}_4, -0.3635), (\mathbf{s}_2, 0.4206), (\mathbf{s}_6, -0.2907))$	$((\mathbf{s}_4, 0.4566), (\mathbf{s}_4, -0.4178), (\mathbf{s}_5, -0.4229))$
F_3	$((\mathbf{s}_3, 0.4495), (\mathbf{s}_4, -0.4537), (\mathbf{s}_6, 0.4844))$	$((\mathbf{s}_4, -0.3176), (\mathbf{s}_2, -0.0426), (\mathbf{s}_6, -0.2907))$
F_4	$((\mathbf{s}_5, -0.4307), (\mathbf{s}_4, -0.3456), (\mathbf{s}_6, -0.0297))$	$((\mathbf{s}_3, -0.1729), (\mathbf{s}_2, 0.3831), (\mathbf{s}_7, -0.3320))$
F_5	$((\mathbf{s}_4, -0.3695), (\mathbf{s}_3, -0.3298), (\mathbf{s}_6, -0.3064))$	$((\mathbf{s}_6, -0.4591), (\mathbf{s}_2, 0.3831), (\mathbf{s}_6, -0.0304))$

Table 11. The weighted normalized matrix $WN_{\theta\vartheta}(\theta = 1, 2, \ldots 5, \vartheta = 1, 2, 3, 4)$ with 2TLT-SFNs.

	$\aleph_1$	$\aleph_2$
F_1	$((\mathbf{s}_7, -0.0786), (\mathbf{s}_2, -0.2877), (\mathbf{s}_4, 0.3646))$	$((\mathbf{s}_6, -0.1368), (\mathbf{s}_2, -0.0899), (\mathbf{s}_5, -0.3530))$
F_2	$((\mathbf{s}_7, 0.4708), (\mathbf{s}_2, 0.2090), (\mathbf{s}_2, 0.2090))$	$((\mathbf{s}_6, 0.3662), (\mathbf{s}_3, -0.2664), (\mathbf{s}_5, -0.0762))$
F_3	$((\mathbf{s}_7, 0.4453), (\mathbf{s}_2, -0.2153), (\mathbf{s}_3, -0.0366))$	$((\mathbf{s}_7, -0.0884), (\mathbf{s}_3, -0.1841), (\mathbf{s}_3, -0.1841))$
F_4	$((\mathbf{s}_7, 0.0200), (\mathbf{s}_1, 0.2467), (\mathbf{s}_4, -0.2212))$	$((\mathbf{s}_6, 0.3280), (\mathbf{s}_3, -0.3272), (\mathbf{s}_4, 0.4293))$
F_5	$((\mathbf{s}_7, 0.4114), (\mathbf{s}_2, 0.3017), (\mathbf{s}_4, -0.3179))$	$((\mathbf{s}_7, 0.3780), (\mathbf{s}_2, -0.0923), (\mathbf{s}_3, 0.1991))$

	$\aleph_3$	$\aleph_4$
F_1	$((\mathbf{s}_7, -0.0423), (\mathbf{s}_2, 0.1025), (\mathbf{s}_3, 0.3239))$	$((\mathbf{s}_7, -0.4796), (\mathbf{s}_3, -0.3335), (\mathbf{s}_5, -0.3147))$
F_2	$((\mathbf{s}_6, 0.4662), (\mathbf{s}_2, -0.2536), (\mathbf{s}_4, 0.2264))$	$((\mathbf{s}_7, -0.0885), (\mathbf{s}_3, -0.4571), (\mathbf{s}_3, 0.2704))$
F_3	$((\mathbf{s}_6, 0.3746), (\mathbf{s}_3, -0.4344), (\mathbf{s}_5, -0.0939))$	$((\mathbf{s}_7, -0.4105), (\mathbf{s}_1, 0.3847), (\mathbf{s}_4, 0.1489))$
F_4	$((\mathbf{s}_7, -0.1226), (\mathbf{s}_3 - 0.3551), (\mathbf{s}_4, 0.4461))$	$((\mathbf{s}_6, 0.1682), (\mathbf{s}_2, -0.3136), (\mathbf{s}_5, -0.0057))$
F_5	$((\mathbf{s}_6, 0.4631), (\mathbf{s}_2, -0.0729), (\mathbf{s}_4, 0.2132))$	$((\mathbf{s}_7, 0.2981), (\mathbf{s}_2, -0.3136), (\mathbf{s}_4, 0.3647))$

Table 12. Score functions of weighted normalized matrix.

	$\aleph_1$	$\aleph_2$	$\aleph_3$	$\aleph_4$
F_1	0.7358	0.5873	0.7712	0.6618
F_2	0.8774	0.6288	0.6745	0.7646
F_3	0.8657	0.7709	0.6308	0.6940
F_4	0.7716	0.6488	0.7254	0.6008
F_5	0.8459	0.8489	0.6745	0.8020

Table 13. The G matrix with 2TLT-SFNs.

$g's$	The 2TLT-SFNs for $g's$	Score
g_1	$((\mathbf{s}_7, 0.2937), (\mathbf{s}_2, -0.1911), (\mathbf{s}_3, 0.3104))$	0.8308
g_2	$((\mathbf{s}_7, -0.2784), (\mathbf{s}_2, 0.3713), (\mathbf{s}_4, -0.0908))$	0.7207
g_3	$((\mathbf{s}_7, -0.3422), (\mathbf{s}_2, 0.1691), (\mathbf{s}_4, 0.1898))$	0.7022
g_4	$((\mathbf{s}_7, -0.2195), (\mathbf{s}_2, -0.0711), (\mathbf{s}_4, 0.2495))$	0.7182

Table 14. The distance matrix with 2TLT-SFNs.

	$\aleph_1$	$\aleph_2$	$\aleph_3$	$\aleph_4$
F_1	$d_{11} = -0.0635$	$d_{12} = -0.0904$	$d_{13} = 0.0462$	$d_{14} = -0.0406$
F_2	$d_{21} = 0.0321$	$d_{22} = -0.0632$	$d_{23} = -0.0196$	$d_{24} = 0.0332$
F_3	$d_{31} = 0.0233$	$d_{32} = 0.0360$	$d_{33} = -0.0493$	$d_{34} = -0.0218$
F_4	$d_{41} = -0.0402$	$d_{42} = -0.0495$	$d_{43} = 0.0311$	$d_{44} = -0.0788$
F_5	$d_{51} = 0.0218$	$d_{52} = 0.0870$	$d_{53} = -0.0191$	$d_{54} = 0.0623$

5.4. The effects of parameters on the outcomes

The influence of parameters on the score functions and ranking outcomes is detailed in the following subsection. The parameters κ, ε, and q play a significant role in the evaluation of alternatives, and the variation of these parameters also affects the ranking of outcomes. The variation of parameters κ, ε, and q enable DMs to extend their decision assessment space based on the 2TLT-SFGWHM and the 2TLT-SFWJHM operators as well as the influence of parameters on the ranking of outcomes is analyzed to check the validity and effectiveness of the proposed approach. We deal with the variation of these three parameters to determine how they affect the outcomes: (1) Let κ=3, $\varepsilon \in [1, 10]$, and q=4 the influence of ε on the ranking outcomes is investigated. (2) Let $\kappa \in [1, 10]$, ε=3, and q=4 the influence of κ on the ranking results is studied. (3) We measure the overall evaluation scores by assigning κ and ε fixed values such as κ=ε=3, where $q \in [1, 10]$. We also calculate the alternative ranking scores. Hence, we solve the concerned problem for a variety of parameter values and ranked the outcomes in Tables 15 and 16. Further, details can be found in Figures 3–8. The ranking results show that F_5 is the best alternative fuel for controlling the impact of greenhouse gas emissions.

Table 15. Parameter analysis according to the 2TLT-SFGWHM operator ($q = 4$).

Parameters	Score Function $\mathcal{S}(d_\theta)(\theta = 1,2,3,4,5)$	Ranking Results
$\kappa = \varepsilon = 1$	$\mathcal{S}(F_1) = -0.1389$ $\mathcal{S}(F_2) = 0.1108$ $\mathcal{S}(F_3) = 0.0707$ $\mathcal{S}(F_4) = -0.1281$ $\mathcal{S}(F_5) = 0.3727$	$F_5 > F_2 > F_3 > F_4 > F_1$
$\kappa = \varepsilon = 2$	$\mathcal{S}(F_1) = -0.1505$ $\mathcal{S}(F_2) = -0.0001$ $\mathcal{S}(F_3) = 0.1558$ $\mathcal{S}(F_4) = -0.1283$ $\mathcal{S}(F_5) = 0.3658$	$F_5 > F_3 > F_2 > F_4 > F_1$
$\kappa = \varepsilon = 3$	$\mathcal{S}(F_1) = -0.1569$ $\mathcal{S}(F_2) = 0.0105$ $\mathcal{S}(F_3) = 0.1542$ $\mathcal{S}(F_4) = -0.1306$ $\mathcal{S}(F_5) = 0.3593$	$F_5 > F_3 > F_2 > F_4 > F_1$
$\kappa = 1, \varepsilon = 2$	$\mathcal{S}(F_1) = -0.1434$ $\mathcal{S}(F_2) = 0.1150$ $\mathcal{S}(F_3) = 0.0706$ $\mathcal{S}(F_4) = -0.1256$ $\mathcal{S}(F_5) = 0.3690$	$F_5 > F_2 > F_3 > F_4 > F_1$
$\kappa = 2, \varepsilon = 1$	$\mathcal{S}(F_1) = -0.1485$ $\mathcal{S}(F_2) = 0.1104$ $\mathcal{S}(F_3) = 0.1539$ $\mathcal{S}(F_4) = -0.1299$ $\mathcal{S}(F_5) = 0.3681$	$F_5 > F_3 > F_2 > F_4 > F_1$
$\kappa = 2, \varepsilon = 3$	$\mathcal{S}(F_1) = -0.1529$ $\mathcal{S}(F_2) = 0.0022$ $\mathcal{S}(F_3) = 0.1566$ $\mathcal{S}(F_4) = -0.1287$ $\mathcal{S}(F_5) = 0.3622$	$F_5 > F_3 > F_2 > F_4 > F_1$
$\kappa = 3, \varepsilon = 2$	$\mathcal{S}(F_1) = -0.1550$ $\mathcal{S}(F_2) = 0.0088$ $\mathcal{S}(F_3) = 0.1534$ $\mathcal{S}(F_4) = -0.1303$ $\mathcal{S}(F_5) = 0.3622$	$F_5 > F_3 > F_2 > F_4 > F_1$
$\kappa = \varepsilon = 3/2$	$\mathcal{S}(F_1) = -0.1462$ $\mathcal{S}(F_2) = 0.1135$ $\mathcal{S}(F_3) = 0.1569$ $\mathcal{S}(F_4) = -0.1280$ $\mathcal{S}(F_5) = 0.3692$	$F_5 > F_3 > F_2 > F_4 > F_1$

Table 16. Parameter analysis according to the 2TLT-SFWJHM operator ($q = 4$).

Parameters	Score Function $\mathcal{S}(d_\theta)(\theta = 1,2,3,4,5)$	Ranking Results
$\kappa = \varepsilon = 1$	$\mathcal{S}(F_1) = -0.1545$ $\mathcal{S}(F_2) = -0.0287$ $\mathcal{S}(F_3) = -0.0152$ $\mathcal{S}(F_4) = -0.1473$ $\mathcal{S}(F_5) = 0.1559$	$F_5 > F_3 > F_2 > F_4 > F_1$
$\kappa = \varepsilon = 2$	$\mathcal{S}(F_1) = -0.1472$ $\mathcal{S}(F_2) = -0.0198$ $\mathcal{S}(F_3) = -0.0128$ $\mathcal{S}(F_4) = -0.1420$ $\mathcal{S}(F_5) = 0.1503$	$F_5 > F_3 > F_2 > F_4 > F_1$
$\kappa = \varepsilon = 3$	$\mathcal{S}(F_1) = -0.1417$ $\mathcal{S}(F_2) = -0.0118$ $\mathcal{S}(F_3) = -0.0121$ $\mathcal{S}(F_4) = -0.1345$ $\mathcal{S}(F_5) = 0.1514$	$F_5 > F_2 > F_3 > F_4 > F_1$
$\kappa = 1, \varepsilon = 2$	$\mathcal{S}(F_1) = -0.1583$ $\mathcal{S}(F_2) = -0.0293$ $\mathcal{S}(F_3) = -0.0125$ $\mathcal{S}(F_4) = -0.1462$ $\mathcal{S}(F_5) = 0.1544$	$F_5 > F_3 > F_2 > F_4 > F_1$
$\kappa = 2, \varepsilon = 1$	$\mathcal{S}(F_1) = -0.1431$ $\mathcal{S}(F_2) = -0.0178$ $\mathcal{S}(F_3) = -0.0126$ $\mathcal{S}(F_4) = -0.1463$ $\mathcal{S}(F_5) = 0.1497$	$F_5 > F_3 > F_2 > F_1 > F_4$
$\kappa = 2, \varepsilon = 3$	$\mathcal{S}(F_1) = -0.1482$ $\mathcal{S}(F_2) = -0.0175$ $\mathcal{S}(F_3) = -0.0119$ $\mathcal{S}(F_4) = -0.1374$ $\mathcal{S}(F_5) = 0.1520$	$F_5 > F_3 > F_2 > F_4 > F_1$
$\kappa = 3, \varepsilon = 2$	$\mathcal{S}(F_1) = -0.1402$ $\mathcal{S}(F_2) = -0.0132$ $\mathcal{S}(F_3) = -0.0128$ $\mathcal{S}(F_4) = -0.1383$ $\mathcal{S}(F_5) = 0.1503$	$F_5 > F_3 > F_2 > F_4 > F_1$
$\kappa = \varepsilon = 3/2$	$\mathcal{S}(F_1) = -0.1507$ $\mathcal{S}(F_2) = -0.0242$ $\mathcal{S}(F_3) = 0.1569$ $\mathcal{S}(F_4) = -0.1466$ $\mathcal{S}(F_5) = 0.1522$	$F_3 > F_5 > F_2 > F_4 > F_1$

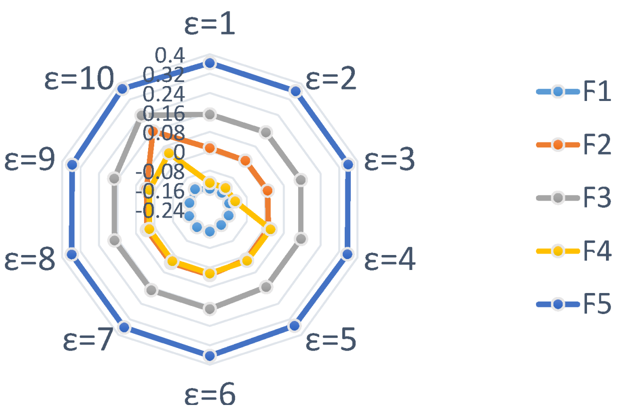

Figure 3. Scores of five alternatives when $\kappa = 3$, $\varepsilon \in [1, 10]$ based on 2TLT-SFGWHM operator ($q = 4$).

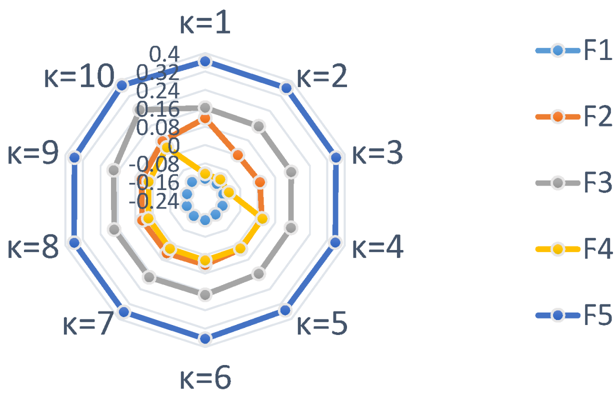

Figure 4. Scores of five alternatives when $\varepsilon = 3$, $\kappa \in [1, 10]$ based on 2TLT-SFGWHM operator ($q = 4$).

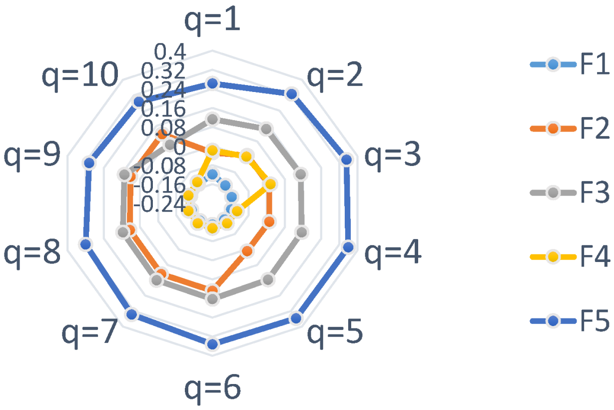

Figure 5. Scores of five alternatives when $\kappa = \varepsilon = 3$, $q \in [1, 10]$ based on 2TLT-SFGWHM operator.

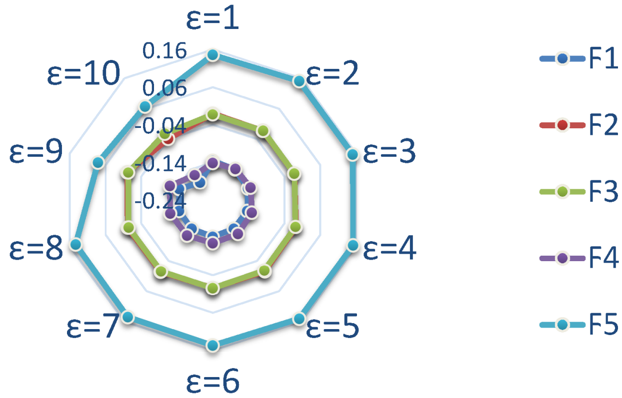

Figure 6. Scores of five alternatives when $\kappa = 3$, $\varepsilon \in [1, 10]$ based on 2TLT-SFWJHM operator ($q = 4$).

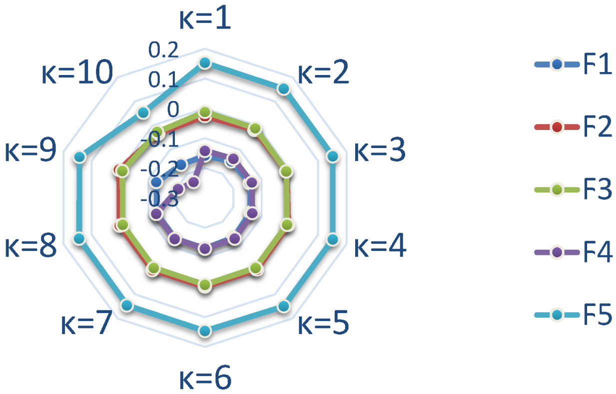

Figure 7. Scores of five alternatives when $\varepsilon = 3$, $\kappa \in [1, 10]$ based on 2TLT-SFWJHM operator ($q = 4$).

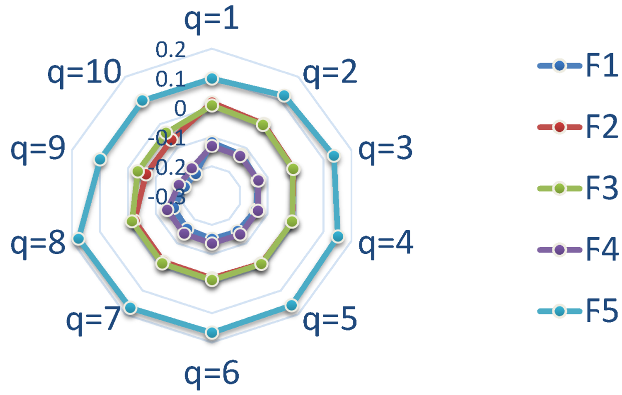

Figure 8. Scores of five alternatives when $\kappa = \varepsilon = 3$, $q \in [1, 10]$ based on 2TLT-SFWJHM operator.

5.5. Comparative analysis with existing MAGDM methods

In this subsection, we use certain validated approaches to cope with the proposed MAGDM problem and analyze the outcomes with our framework to check its feasibility and effectiveness. We carefully compute the evaluation outcomes for the selection of an optimal alternative fuel by using these strategies. Tables 17 and 18 summarizes the output of the comparisons among the developed MABAC method and existing EDAS and CODAS methods.

Table 17. Comparative analysis according to the 2TLT-SFGWHM operator.

Alternatives	MABAC	Ranking	EDAS [28]	Ranking	CODAS [29]	Ranking
F_1	-0.1529	V	0.0224	V	-2.6522	V
F_2	0.0022	III	0.3901	III	-0.4684	III
F_3	0.1566	II	0.5628	II	0.6624	II
F_4	-0.1287	IV	0.2824	IV	-0.5218	IV
F_5	0.3622	I	0.9172	I	2.9800	I

Table 18. Comparative analysis according to the 2TLT-SFWJHM operator.

Alternatives	MABAC	Ranking	EDAS [28]	Ranking	CODAS [29]	Ranking
F_1	-0.1483	V	0.1769	V	-1.6316	V
F_2	-0.0175	III	0.5603	III	0.3430	II
F_3	-0.0118	II	0.6007	II	0.2473	III
F_4	-0.1374	IV	0.1067	I	-1.5629	IV
F_5	0.1520	I	1.0000	IV	2.6043	I

Due to the fundamental behavior of the multiple aggregation methods, there are some variations in the ranking order of alternatives. But, the most acceptable alternative as shown in Tables 17 and 18 is the same in both the existing methodologies and the proposed methodology. Hence, from the comparison outcomes with EDAS and CODAS methods, we can conclude that F_5 is the best alternative fuel for control of impact of greenhouse gas emissions.

The proposed MABAC method is more applicable because it does not only determine the relationships between the given arguments but it also enables the DMs to show their fuzzy evaluation information more effectively. Moreover, the presented technique allows DMs to select their risk preferences based on the variation of parameters. The 2TLT-SF-MABAC method can also provide robust and flexible information integration, enabling risk MAGDM problems more feasible. The existing operators and methods cannot control the certainty degrees. Our proposed model can effectively redistribute the MD, AD, and NMD in 2TLT-SFNs by different rules, allowing us to extract more detailed and objective data from the original 2TLT-SFS. In a summary, the strategy we propose is broad and applicable to solve MAGDM problems with 2TLT-SFNs. Further detail about the comparison outcomes plotted in Figures 9 and 10.

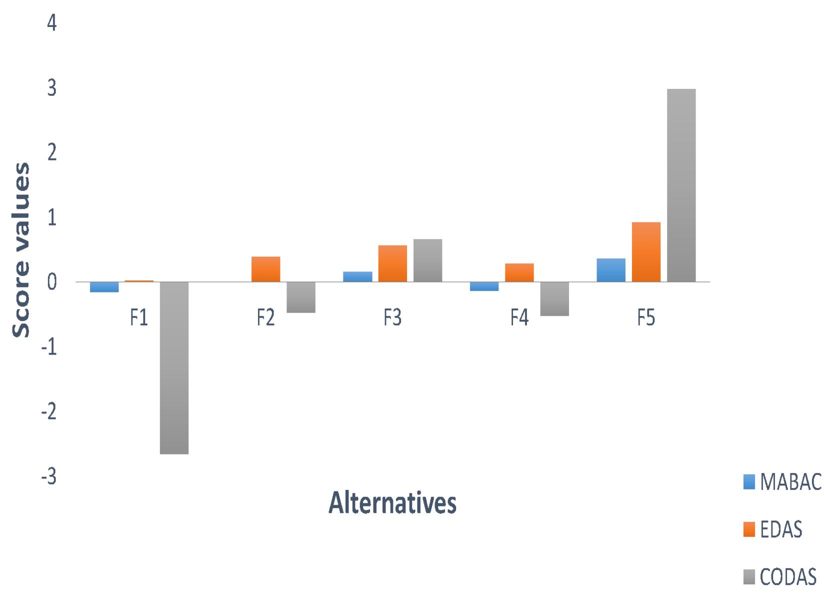

Figure 9. Comparative outcomes with 2TLT-SFGWHM operator.

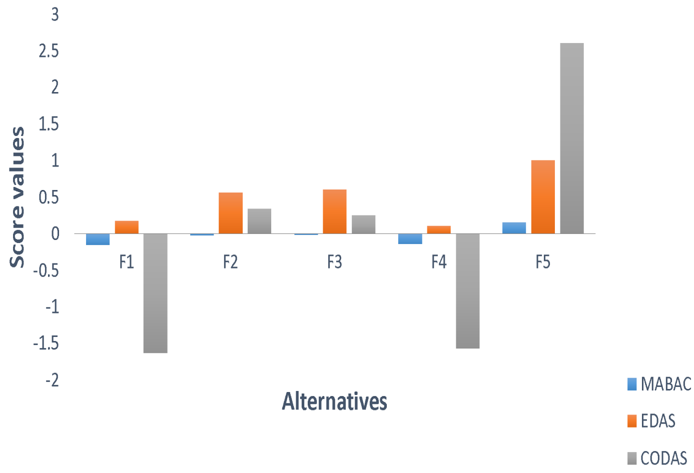

Figure 10. Comparative outcomes with 2TLT-SFWJHM operator.

6. Conclusions

The T-SFS increases the range for assigning MD, AD and NMD, and the 2TL terms can best describe the information given by DMs in natural language. In this manuscript, to evaluate linguistic problems the combining concept of T-SFS and 2TL terms called the 2TLT-SFS has been utilized. The 2TLT-SFNs can be converted into 2TL spherical fuzzy numbers and 2TL picture fuzzy numbers when $q = 2$ and $q = 1$, respectively. Therefore, the 2TLT-SFS has a good ability to other fuzzy numbers which represent linguistic information. The generalized HM operator can solve real world MAGDM problems by taking into account the correlation of attribute values which are obvious in the modern environment. So, we have proposed a family of 2TLT-SF generalized and geometric HM AOs, such as the 2TLT-SFGHM, the 2TLT-SFGWHM, the 2TLT-SFJHM, and the 2TLT-SFWJHM operators for aggregating 2TLT-SFNs. In this article, by considering the stability, feasibility, and simplicity of the MABAC method in the calculation process, a new tool called the 2TLT-SF-MABAC method based on the 2TLT-SFGWHM and the 2TLT-SFWJHM has been proposed, which is useful, powerful, and flexible to express DMs' complicated and uncertain decision information in the MAGDM environment. The proposed method can flexibly assign different weights to the attributes according to specific problems to obtain results that are consistent with actual conditions. Numerical illustration has demonstrated that our proposed method is more flexible than other existing methods. However, our proposed approach is still more crucial to deal with realistic MAGDM problems than some others owing to its advantages and superiorities. In the future, our work will be extended w.r.t two aspects in fuzzy circumstances. First, we will try to expand our proposed method to solve different classical MAGDM problems; supplier selection, medical diagnosis, etc. Second, we will investigate more AOs for 2TLT-SFS and study their applications in an MAGDM environment.

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Conflict of interest

The authors declare no conflict of interest.

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