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Research article

Sensitive analysis of soliton solutions of nonlinear Landau-Ginzburg-Higgs equation with generalized projective Riccati method

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Abstract: The study aims to explore the nonlinear Landau-Ginzburg-Higgs equation, which describes nonlinear waves with long-range and weak scattering interactions between tropical tropospheres and mid-latitude, as well as the exchange of mid-latitude Rossby and equatorial waves. We use the recently enhanced rising procedure to extract the important, applicable and further general solitary wave solutions to the formerly stated nonlinear wave model via the complex travelling wave transformation. Exact travelling wave solutions obtained include a singular wave, a periodic wave, bright, dark and kink-type wave peakon solutions using the generalized projective Riccati equation. The obtained findings are represented as trigonometric and hyperbolic functions. Graphical comparisons are provided for Landau-Ginzburg-Higgs equation model solutions, which are presented diagrammatically by adjusting the values of the embedded parameters in the Wolfram Mathematica program. The propagating behaviours of the obtained results display in 3-D, 2-D and contour visualization to investigate the impact of different involved parameters. The velocity of soliton has a stimulating effect on getting the desired aspects according to requirement. The sensitivity analysis is demonstrated for the designed dynamical structural system's wave profiles, where the soliton wave velocity and wave number parameters regulate the water wave singularity. This study shows that the method utilized is effective and may be used to find appropriate closed-form solitary solitons to a variety of nonlinear evolution equations (NLEEs).

Keywords: generalized projective Riccati method; Landau-Ginzburg-Higgs equation; general solutions; sensitivity analysis
Mathematics Subject Classification: 35Q51, 35Q53

1. Introduction

The nonlinear partial differential equation is critical for investigating the characteristics of nonlinear physical situations. The Schrödinger type governing equation is a distinctive mechanism for appropriately understanding the complicated physical nonlinear model, with crucial applications in plasma, fibre optics, telecommunication engineering, mathematical physics and optics. Obtaining reliable analytical solutions for the Schrödinger equations is an essential research topic since exact solutions represent the physical features of nonlinear systems in applied mathematics [1–5]. Partial differential equations (PDEs) gained popularity and importance in applied and pure mathematics over the last decade. For mathematicians, computer technology has expanded the scope of applied sciences. Nonlinear models are becoming increasingly prevalent in mathematical physics and engineering sciences. Nonlinear PDEs have a wide range of practical applications, including mass and heat transportation, continuum mechanics, wave theory, hydrodynamics, chemical technology, plasma physics is rich in nonlinear systems and exhibits a wide range of phenomena associated with instabilities, coherent wave structures and turbulence [6, 7], complicated biological processes may be characterized by nonlinear ordinary differential equation systems [8, 9], population ecology [10], electromagnetic wave interaction in plasma [11], nonlinear may appear in quantum mechanics in several different ways [12–14] and so on.

In the geographical fields, environmental processes induced by energy transportation on a floating structure or a synthetic structure field, waves are the primary energy source.

The solitons discovery and the incredible diversity of its aesthetic features are all part of the mathematical explanation. The history of solitons begins with John Scott Russell's observation of the translation wave. Before the 1870s, when Russell's work was eventually proven, notable scientists and philosophers praised its scientific implications. Boussinesq's 1872 work practised extensively and predicted major concepts today employed by forward-thinking scientists and philosophers. Boussinesq expressed his opinion on the water wave equation. As a result, his estimate suggests that the movement may be a duplex. However, Boussinesq and Rayleigh's work still proves the essential issue of dispersion and non-linearity. It is still rendering to address Stokes and Airy's argument against making an equation of unidirectional motion that is now recognized by their names using bell-shaped and kink-shaped sech-solutions, simulating wave phenomena in plasmas, optical fibre, elastic media, chemical electrical circuits and other fields. The travelling wave solutions of the Korteweg de Vries equation and Boussinesq equation, which describe the water waves, are well-known examples. For more information, see [15–17].

Many schemes and approaches, such as the Kudryashov method [18], sine-Gordon expansion scheme [19], bilinear neural network technique [20] and extended equation method [21], have been developed to secure exact analytical solutions for partial nonlinear differential equations to find soliton solutions [22], F-expansion technique [23], unified auxiliary equation technique (m+$\frac{G}{G'}$) expansion strategy [24], Hirota bilinear technique [25], extended exponential function method [26], generalized exponential rational function method [27], power series method [28] and several others [29–32].

Diverse sets of researchers have investigated the NLEEs; Wang and Zhang [33] utilized the enhanced F-expansion approach and discovered just a few wave solutions, mainly general and periodic soliton solutions. They did not draw geometrical structures of these outcomes. Alquran et al. [34] found various explicit soliton solutions, such as periodic solutions, singular and kink soliton for a new two-mode version of the Burger–Huxley model. These findings have been represented using sinh and cosh functions. Akbar et al. [35] determine soliton solutions to the Boussinesq equation through the Kudryashov sine-Gordon methods. There are a few common strategies for finding accurate solutions to the integrable wave equations [36–40].

In the literature, the (LGH) model (1.1) was investigated through some methods, such as the technique of improved Bernoulli sub-equation function (IBSEF) [41], the sine–cosine and extended tanh function methods [42], power index method [43], inverse scattering transformation method [44], etc.

To our knowledge, the (LGH) model has not yet been investigated using the generalized projective Riccati equation method and sensitivity analysis is also ignored. As a result of the preceding investigations, the objective of the present article is to search and construct adequate, wide-ranging and further general soliton solutions linked with arbitrary parameters to the considered (LGH) model through the generalized projective Riccati equation method. Inserting specific values of arbitrary factors various wave solitons are created and these attained solitons are not established in the previous literature. The sensitive visualization of the model is presented and visualized on different initial conditions and attempts to fill this study gap.

This paper uses the generalized projective Riccati equation approach to find atypical stable soliton solutions to the Landau-Ginzburg-Higgs (LGH) equation. This study establishes the framework for the exact solutions to the Landau-Ginzburg-Higgs equation using the generalized projective Riccati equation method. The present methodology has some advantages over the previously studied techniques in the form of more generalized solutions and its performance is more efficient and effective. The solitonic patterns of the Landau-Ginzburg-Higgs equation have been successfully illustrated, with exact solutions offered by the travelling wave solutions obtained, including periodic wave, singular wave, bright, dark and kink-type wave peakon solutions by using generalized projective Riccati equation (GPRE). The obtained findings are represented as trigonometric and hyperbolic functions. The survey results are compared to the highly recognized results and the originality of the obtained results is presented. Graphical comparisons are provided for Landau-Ginzburg-Higgs equation model solutions, which are presented diagrammatically by adjusting the values of the embedded parameters in `Mathematica`. The results are visually displayed in 3-D, contour and 2-D for accuracy. We also demonstrated sensitivity analysis for the redesigned dynamical structural system's wave profiles, where the soliton wave velocity and wave number parameters regulate the water wave singularity. We are confident that our research will assist physicists in predicting new notions in mathematical physics.
The Landau-Ginzburg-Higgs equation [45] is stated as,

$$\frac{\partial^2 \mathbb{Z}}{\partial t^2} - \frac{\partial^2 \mathbb{Z}}{\partial x^2} - g^2 \mathbb{Z} + h^2 \mathbb{Z}^3 = 0, \tag{1.1}$$

where $\mathbb{Z}(x,t)$ denote ion-cyclotron wave for electrostatic potential, t and x implies for the nonlinear temporal and spatial coordinates where h and g are non-zero parameters. Lev Devidovich Landau and Vitaly Lazarevich Ginzburg developed the NLEE (1) to explain superconductivity and drift cyclotron waves for coherent ion-cyclotron waves in radially in-homogeneous plasma. Landau-Ginzburg-Higgs equation plays a significant role in various scientific and engineering fields, such as, optical fibres, solid-state physics, fluid mechanics, plasma physics, chemical kinematics, chemical physics geochemistry, etc. Nonlinear wave phenomena of diffusion, reaction, dispersion, dissipation, and convection are very important in nonlinear wave equations, It is a mathematical physical theory used to describe superconductivity. In its initial form, it was postulated as a phenomenological model which could describe type-I superconductors without examining their microscopic properties. One GL-type superconductor is the famous YBCO, and generally all Cuprates. The theory can also be given a general geometric setting, placing it in the context of Riemannian geometry, where in many cases exact solutions can be given. This general setting then extends to quantum field theory and string theory, again owing to its solvability and close relation to other, similar systems. Several approaches have been used to assess distinctive soliton solutions to integrable NLEE (1.1).

In Section 2, we constructed exact solutions using the generalized projective Riccati equation method. The application of the generalized projective Riccati equation method is shown in Section 3. Furthermore, for different values of wave velocity and wave number, we saw diverse wave textures in the 3-D, contour and 2-D graphical depictions of the solutions. We discussed the visual representation of the study findings in Section 4. Section 5 presents a sensitivity assessment for wave velocity profiles graphically with the analysis and discussions of the results. The study's conclusion is given in section 6.

2. Structures of exact solutions

2.1. Generalized projective Riccati equation

In order to find the solution to Eq (1.1) the methodology can be constructed as follows,

Step 1. Assume a general NLEE of the type:

$$Y(\Omega,\ \Omega_t,\ \Omega_x,\ \Omega_{xt},\ \Omega_{xx},\cdots) = 0, \tag{2.1}$$

where Ω is a polynomial in $\Omega(x,t)$ and its partial derivative in which non-linear term and highest order derivative are involved. The Eq (2.1) can be converted into the ordinary differential equation with the help of transformation [46, 47].

We use the following transformation,

$$\Omega(x,t) = \eta(\xi),\ \xi = x - c\,t, \tag{2.2}$$

$$Q(\eta,\eta',\eta'',...) = 0, \tag{2.3}$$

where c, Q and $\eta^{'} = \frac{d\eta}{d\xi}$ are the velocity polynomial of $\eta(\xi)$ respectively.

Step 2. Assume the solution of Eq (2.3) can be written as,

$$\eta(\xi) = A_0 + \sum_{i=1}^{N} \sigma^{i-1}(\xi)\left[A_i\sigma(\xi) + B_i\tau(\xi)\right], \tag{2.4}$$

where A_0 , A_i and B_i, $(i = 1,...,l)$, are arbitrary constants to be determined. The functions $\sigma(\xi)$ and $\tau(\xi)$ satisfies the ODEs,

$$\begin{aligned} &\tau^{'}(\xi) = \kappa + \epsilon\,\tau^2(\xi) - \mu\,\tau\,(\xi), \qquad \epsilon = \pm 1,\\ &\sigma^{'}(\xi) = \epsilon\,\tau(\xi)\,\sigma(\xi), \end{aligned} \tag{2.5}$$

such that,

$$\tau^2(\xi) = -\epsilon\left[\kappa - 2\,\epsilon^2\mu\,\sigma(\xi) + \frac{\mu^2\epsilon^2(\sigma(\xi))^2}{\kappa} - \frac{\epsilon^2(\sigma(\xi))^2}{\kappa}\right], \tag{2.6}$$

where κ and μ are non zero constants.

If $\kappa = \mu = 0$ in Eq (2.5),

$$\eta(\xi) = \sum_{i=1}^{N} A_i\tau^i(\xi), \tag{2.7}$$

where $\tau^i(\xi)$ satisfies the nonlinear ODE,

$$\tau^{'}(\xi) = \tau^2(\xi). \tag{2.8}$$

Step 3. The positive integer number N in Eq (2.4) must be determined by using the homogeneous balance between the highest-order derivatives and the highest nonlinear terms in Eq (2.3).
Step 4. Substitute (2.4) along with (2.5) and (2.6) into (2.3) and collecting all terms of the same order of $\sigma^j(\xi)$ and $\tau^i(\xi)$, $(j = 0, 1.., i = 0, 1..)$. Setting each coefficient to zero results in a set of algebraic equations that can be solved to determine the values of desired parameters.
Step 5. Proposed solutions of Eq (2.5) is as follows [48]:
(Family 1) if $\epsilon = -1$, $\kappa \neq 0$,

$$\sigma_1(x,t) = \frac{\kappa\,\text{sech}(\sqrt{\kappa}\,\xi)}{\mu\,\text{sech}(\sqrt{\kappa}\,\xi) + 1}, \qquad \tau_1(x,t) = \frac{\sqrt{\kappa}\,\tanh(\sqrt{\kappa}\,\xi)}{\mu\,\text{sech}(\sqrt{\kappa}\,\xi) + 1}, \tag{2.9}$$

$$\sigma_2(x,t) = \frac{\kappa\,\text{csch}(\sqrt{\kappa}\,\xi)}{\mu\,\text{csch}(\sqrt{\kappa}\,\xi) + 1}, \qquad \tau_2(x,t) = \frac{\sqrt{\kappa}\,\coth(\sqrt{\kappa}\,\xi)}{\mu\,\text{csch}(\sqrt{\kappa}\,\xi) + 1}, \tag{2.10}$$

(Family 2) if $\epsilon = 1$, $\kappa \neq 0$,

$$\sigma_3(x,t) = \frac{\kappa\,\sec(\sqrt{\kappa}\,\xi)}{\mu\,\sec(\sqrt{\kappa}\,\xi) + 1}, \qquad \tau_3(x,t) = \frac{\sqrt{\kappa}\,\tan(\sqrt{\kappa}\,\xi)}{\mu\,\sec(\sqrt{\kappa}\,\xi) + 1}, \tag{2.11}$$

(2.12)

$$\sigma_4(x,t) = \frac{\kappa\,\csc(\sqrt{\kappa}\,\xi)}{\mu\,\csc(\sqrt{\kappa}\,\xi) + 1}, \qquad \tau_4(x,t) = \frac{\sqrt{\kappa}\,\cot(\sqrt{\kappa}\,\xi)}{\mu\,\csc(\sqrt{\kappa}\,\xi) + 1}, \tag{2.13}$$

(Family 3) if $\mu = \kappa = 0$,

$$\sigma_5(x,t) = \frac{c_1}{\xi}, \qquad \tau_5(x,t) = \frac{1}{\epsilon\,\xi}, \tag{2.14}$$

where c_1 is a constant parameter.
Step 6. Substituting the values of obtained desired parameters as well as the solutions (2.9–2.14) into Eq (2.4), we obtain the exact solution of (2.1).

3. Application of generalized projective Riccati equation method

In order to find, the analytical exact solution of the Landau-Ginzburg-Higgs system, the next wave transformation apply on the system (1.1),

$$\mathbb{Z}(x,t) = \mathbb{Z}(\xi),\ \xi = (\lambda\, x - c\, t), \tag{3.1}$$

where, c is the wave velocity of the travelling wave and λ denotes the wave number, for Eq (1.1) we complete the structure of an ODE as follows,

$$(c^2 - \lambda^2)\,\frac{d^2\,\mathbb{Z}}{d\,\xi^2} - g^2\,\mathbb{Z} + h^2\,\mathbb{Z}^3 = 0. \tag{3.2}$$

According to the homogeneous balancing principle of Eq (3.2) gives $N = 1$. Thus, the general solution of the examined model is based on a generalized projective Riccati equation method which is given below,

$$\mathbb{Z}(\xi) = b_0 + b_1\,\sigma(\xi) + b_2\,\Lambda(\xi)\,, \tag{3.3}$$

system of Eq (3.3) is substitute in Eq (3.2). We get an algebraic system by equating the coefficients of distinct powers of $\sigma(\xi)$ and $\tau(\xi)$,

$$\begin{aligned}
\sigma(\xi)^0\tau(\xi)^0 &= -\,3\,\kappa h^2\epsilon\, b_0 {b_2}^2 + h^2 {b_0}^3 - g^2 b_0 = 0,\\
\sigma(\xi)^0\tau(\xi)^1 &= -\,\kappa h^2\epsilon\, {b_2}^3 + 3\,h^2 {b_0}^2 b_2 - g^2 b_2 = 0,\\
\sigma(\xi)^1\tau(\xi)^0 &= -\,2\,\kappa c^2\epsilon^3 b_1 - 3\,\kappa h^2\epsilon\, b_1 {b_2}^2 + 2\,\kappa\lambda^2\epsilon^3 b_1 + 6\,h^2\mu\,\epsilon\, b_0 {b_2}^2 + \kappa c^2\epsilon\, b_1\\
&\quad -\kappa\lambda^2\epsilon\, b_1 + 3\,h^2 {b_0}^2 b_1 - g^2 b_1 = 0,\\
\sigma(\xi)^1\tau(\xi)^a n1 &= 2\,c^2\mu\,\epsilon^3 b_2 + 2\,h^2\mu\,\epsilon\, {b_2}^3 - 2\,\lambda^2\mu\,\epsilon^3 b_2 - c^2\mu\,\epsilon\, b_2 + 6\,h^2 b_0 b_1 b_2 + \lambda^2\mu\,\epsilon\, b_2 = 0,\\
\sigma(\xi)^2\tau(\xi)^0 &= 3\,h^2 b_0 {b_1}^2 + 4\,b_1\epsilon^3\mu\, c^2 - 4\,b_1\epsilon^3\mu\,\lambda^2 - b_1\epsilon\,\mu\, c^2 + b_1\epsilon\,\mu\,\lambda^2 + \frac{3\,h^2 b_0 {b_2}^2\epsilon}{\kappa}\\
&\quad + 6\,h^2 b_1 {b_2}^2\mu\,\epsilon - \frac{3\,h^2 b_0 {b_2}^2\mu^2\epsilon}{\kappa} = 0,\\
\sigma(\xi)^2\tau(\xi)^1 &= 3\,h^2 {b_1}^2 b_2 + \frac{2\,b_2\epsilon^3 c^2}{\kappa} - \frac{2\,b_2\epsilon^3\lambda^2}{\kappa} + \frac{h^2 {b_2}^3\epsilon}{\kappa} - \frac{2\,b_2\mu^2\epsilon^3 c^2}{\kappa} + \frac{2\,b_2\mu^2\epsilon^3\lambda^2}{\kappa} - \frac{h^2 {b_2}^3\mu^2\epsilon}{\kappa} = 0,\\
\sigma(\xi)^3 &= \frac{2\,b_1\epsilon^3 c^2}{\kappa} + h^2 {b_1}^3 - \frac{2\,b_1\epsilon^3\lambda^2}{\kappa} + \frac{3\,h^2 b_1 {b_2}^2\epsilon}{\kappa} - \frac{2\,b_1\epsilon^3\mu^2 c^2}{\kappa} + \frac{2\,b_1\epsilon^3\mu^2\lambda^2}{\kappa} - \frac{3\,h^2 b_1 {b_2}^2\mu^2\epsilon}{\kappa} = 0.
\end{aligned} \tag{3.4}$$

The aforementioned system (3.4) is solved with the help of modern software `Mathematica` and get the values of desired parameters,

Case-1:

$$c = \pm\lambda,\ h = h,\ g = 0, \lambda = \lambda,\ b_0 = b_0,\ b_1 = b_1,\ b_2 = b_2. \tag{3.5}$$

We obtain the general solutions by substituting Eq (3.5) into Eq (3.3),

For case 1,

(Family 1): if $\epsilon = -1$, $\kappa \neq 0$,

$$\mathbb{Z}_1(x,t) = b_0 + \frac{b_1\,\kappa\ \mathrm{sech}(\sqrt{\kappa}\,\xi)}{\mu\ \mathrm{sech}(\sqrt{\kappa}\,\xi) + 1} + \frac{b_2\ \sqrt{\kappa}\ \tanh(\sqrt{\kappa}\,\xi)}{\mu\ \mathrm{sech}(\sqrt{\kappa}\,\xi) + 1}. \tag{3.6}$$

$$\mathbb{Z}_2(x,t) = b_0 + \frac{b_1\,\kappa\ \text{csch}(\sqrt{\kappa}\,\xi)}{\mu\ \text{csch}(\sqrt{\kappa}\,\xi)+1} + \frac{b_2\,\sqrt{\kappa}\ \text{coth}(\sqrt{\kappa}\,\xi)}{\mu\ \text{csch}(\sqrt{\kappa}\,\xi)+1}. \tag{3.7}$$

(Family 2): if $\epsilon = 1, \kappa \neq 0$,

$$\mathbb{Z}_3(x,t) = b_0 + \frac{b_1\,\kappa\ \sec(\sqrt{\kappa}\,\xi)}{\mu\ \sec(\sqrt{\kappa}\,\xi)+1} + \frac{b_2\,\sqrt{\kappa}\ \tan(\sqrt{\kappa}\,\xi)}{\mu\ \sec(\sqrt{\kappa}\,\xi)+1}. \tag{3.8}$$

$$\mathbb{Z}_4(x,t) = b_0 + \frac{b_1\,\kappa\ \csc(\sqrt{\kappa}\,\xi)}{\mu\ \csc(\sqrt{\kappa}\,\xi)+1} + \frac{b_2\,\sqrt{\kappa}\ \cot(\sqrt{\kappa}\,\xi)}{\mu\ \csc(\sqrt{\kappa}\,\xi)+1}. \tag{3.9}$$

(Family 3): if $\mu = \kappa = 0$

$$\mathbb{Z}_5(x,t) = b_0 + \frac{b_1\,c_1}{\xi} + \frac{b_2}{\epsilon\,\xi} \tag{3.10}$$

4. Graphical representation

4.1. Graphical discussion

This section is dedicated to see the physical aspects of the wave pattern of considered dynamical systems and presented graphically. we showed 3-D, contour and 2-D graphs of the calculated solutions to the soliton velocity number. The graphs representing the solutions to the succeeding nonlinear evolution equations of the Landau-Ginzburg-Higgs equation are now shown. Current modern programming software is utilised to plot the graph for better presentation, corresponding numerical values for the parameters can be used based on their physical ranges.

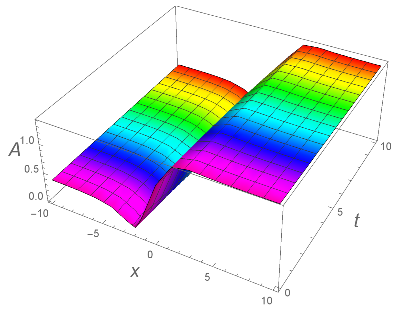
In Figures 1 and 2, we explained the graphical representation for $\mathbb{Z}_1(x,t)$ of the obtained solutions for soliton velocity and wave number for different values of wave speed and wave number.

In Figure 1, we depicted graphs for soliton velocity at the parametric values $\kappa = 0.9$, $b_0 = 0.5$, $b_1 = 0.75$, $b_2 = 0.90$, $\lambda = 0.8$, $\mu = 0.5$, in the form of 3-D, 2-D and contour form respectively. At $c = 0.09$ flat kink-shaped solitonic behaviour is observed in a 3-D profile and for more visualisation contour was plotted and found the singular soliton and 2-D shows the bright periodic soliton and for fixed speed and different values of time soliton propagate translatory.

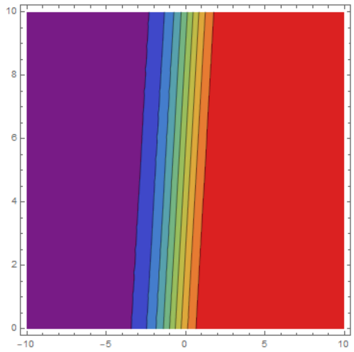
A similar pattern is observed in Figure 1d, 1e, 1f, 1g, 1h and 1b respectively, for different values of wave speed.

In Figure 2, we depicted graphs for wave numbers at the parametric values $\kappa = 0.9$, $b_0 = 0.5$, $b_1 = 0.75$, $b_2 = 0.90$, $c = 0.9$, $\mu = 0.5$, of 3-D, contour and 2-D form respectively. At $\lambda = 0.2$ Flat kink-shaped solitonic behaviour is observed in a 3-D profile and contour plotted and identified the single soliton for better visualisation and 2-D displays the brilliant periodic soliton and for fixed speed and varied values of time soliton propagate translatory.

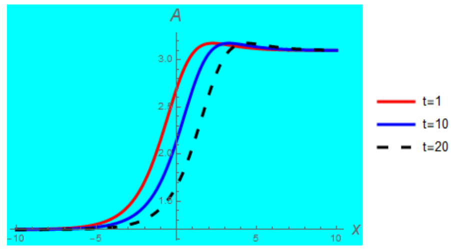
A similar pattern is observed in Figure 2d, 2e, 2f, 2g, 2h and 2b respectively, for different values of wave number.



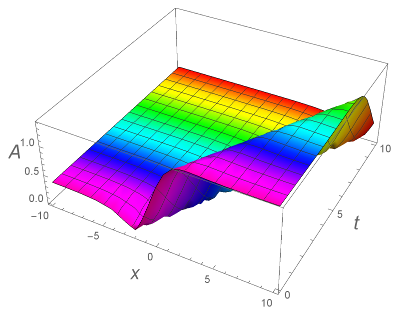
(a) 3-D soliton wave velocity profile at $c = 0.09$



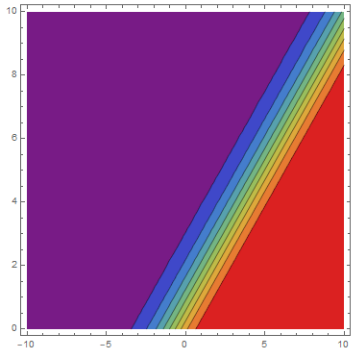
(b) Contour soliton wave velocity profile at $c = 0.09$



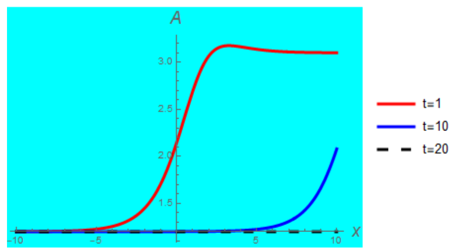
(c) 2-D soliton wave velocity profile at $c = 0.09$



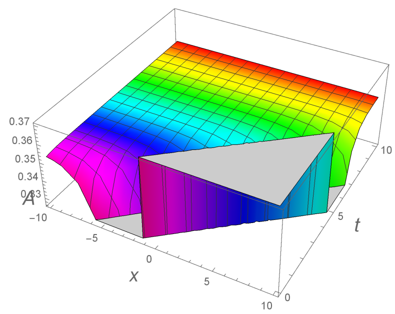
(d) 3-D soliton wave velocity profile at $c = 0.9$



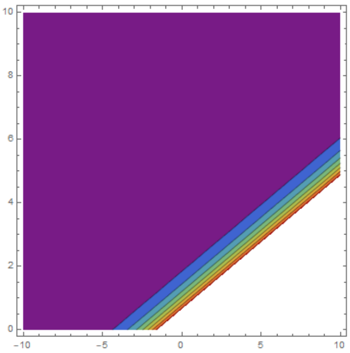
(e) Contour soliton wave velocity profile at $c = 0.9$



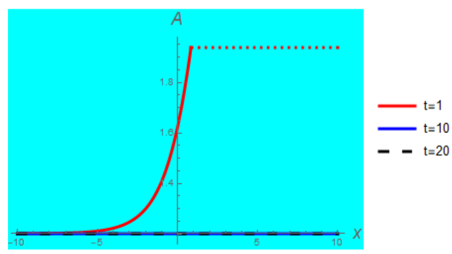
(f) 2-D soliton wave velocity profile at $c = 0.9$



(g) 3-D soliton wave velocity profile at $c = 1.9$

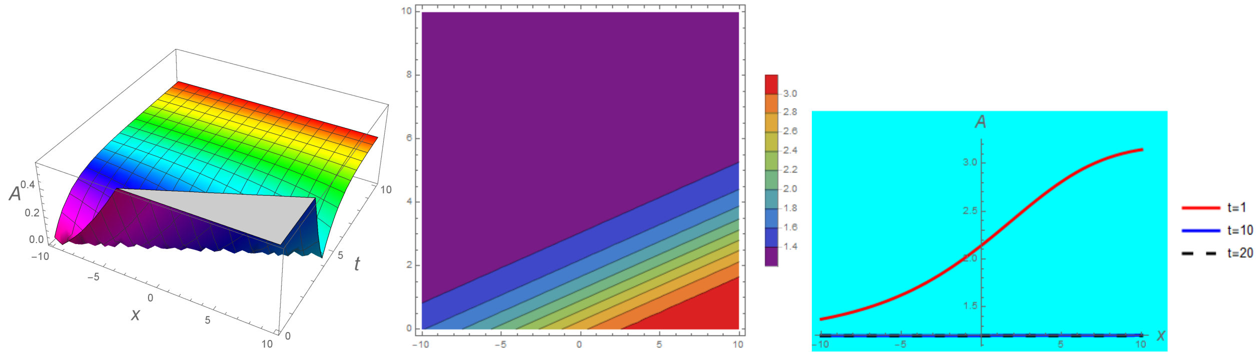


(h) Contour soliton wave velocity profile at $c = 1.9$



(i) 2-D soliton wave velocity profile at $c = 1.9$

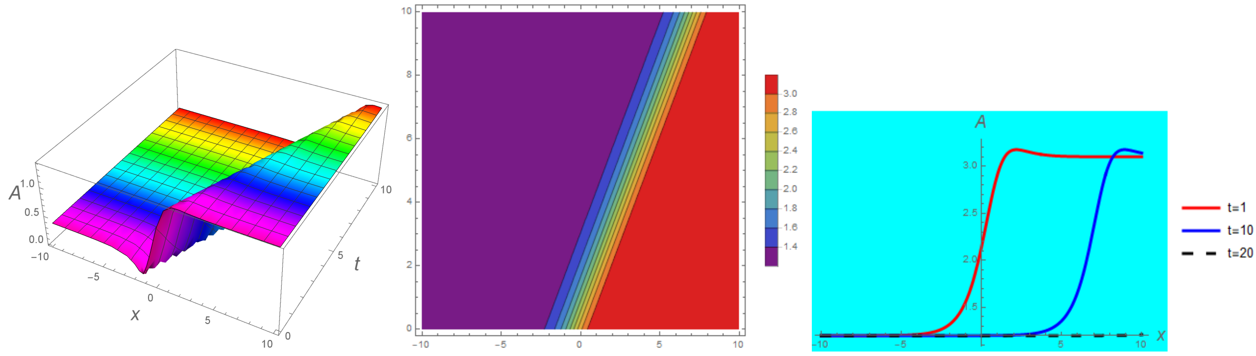
Figure 1. 3-D, contour and 2-D graphical illustration for $\mathbb{Z}_1(x, t)$.



(a) 3-D soliton wave number profile at $\lambda = 0.2$

(b) Contour soliton wave number profile at $\lambda = 0.2$

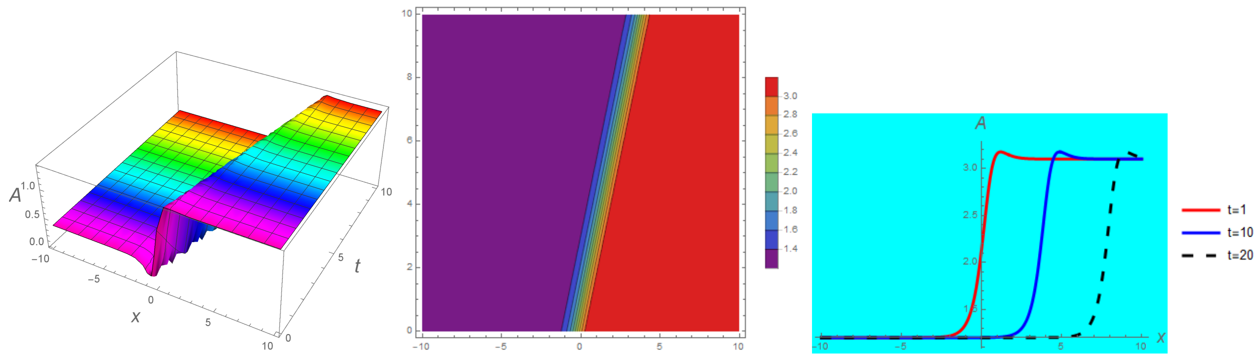
(c) 2-D soliton wave number profile at $\lambda = 0.2$



(d) 3-D soliton wave number profile at $\lambda = 1.2$

(e) Contour soliton wave number profile at $\lambda = 1.2$

(f) 2-D soliton wave number profile at $\lambda = 1.2$

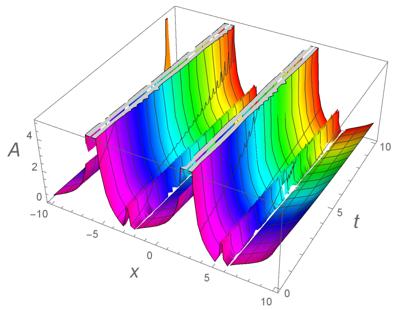


(g) 3-D soliton wave number profile at $\lambda = 2.2$

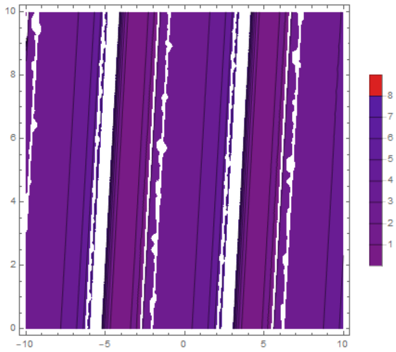
(h) Contour soliton wave number profile at $\lambda = 2.2$

(i) 2-D soliton wave number profile at $\lambda = 2.2$

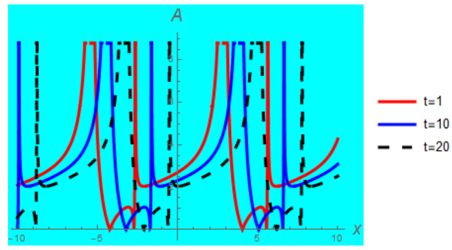
Figure 2. 3-D, contour and 2-D graphical illustration for $\mathbb{Z}_1(x, t)$.



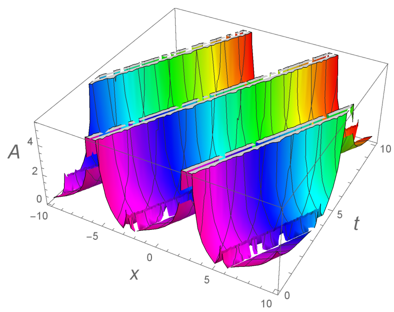
(a) 3-D soliton wave velocity profile at $c = 0.09$



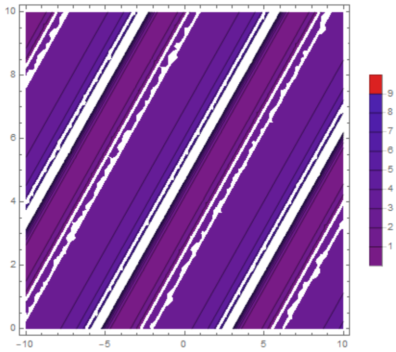
(b) Contour soliton wave velocity profile at $c = 0.09$



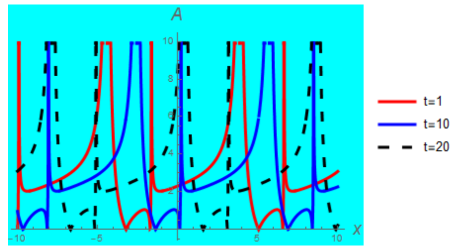
(c) 2-D soliton wave velocity profile at $c = 0.09$



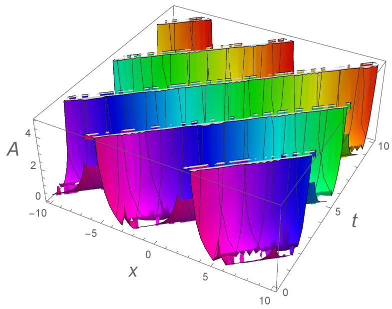
(d) 3-D soliton wave velocity profile at $c = 0.9$



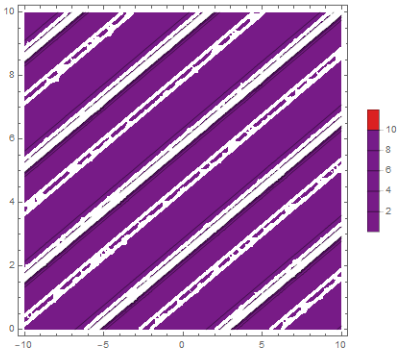
(e) Contour soliton wave velocity profile at $c = 0.9$



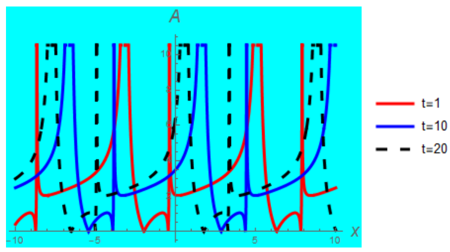
(f) 2-D soliton wave velocity profile at $c = 0.9$



(g) 3-D soliton wave velocity profile at $c = 1.9$

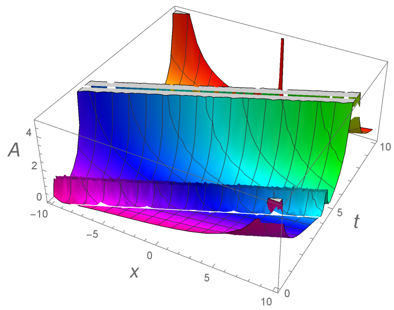


(h) Contour soliton wave velocity profile at $c = 1.9$

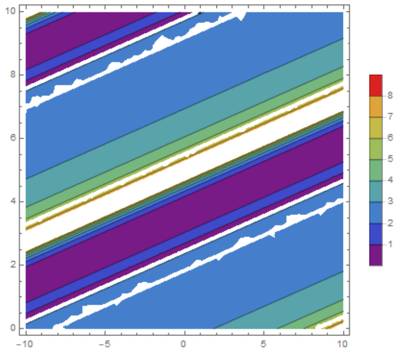


(i) 2-D soliton wave velocity profile at $c = 1.9$

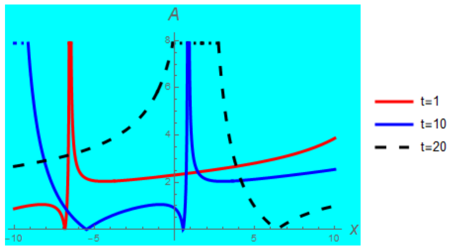
Figure 3. 3-D, contour and 2-D graphical illustration for $\mathbb{Z}_3(x, t)$.



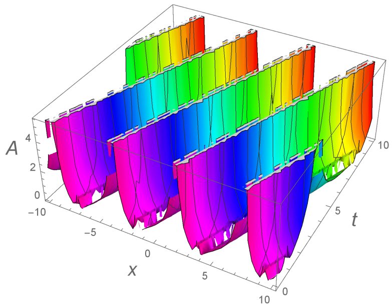
(a) 3-D soliton wave number profile at $\lambda = 0.2$



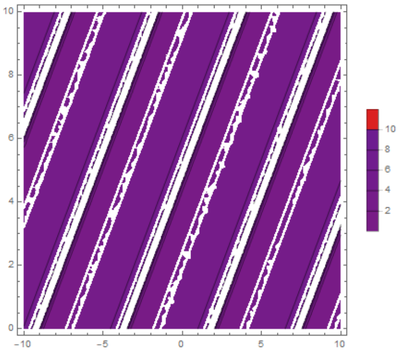
(b) Contour soliton wave number profile at $\lambda = 0.2$



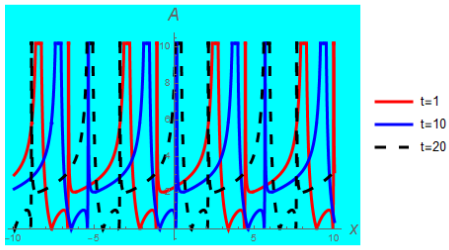
(c) 2-D soliton wave number profile at $\lambda = 0.2$



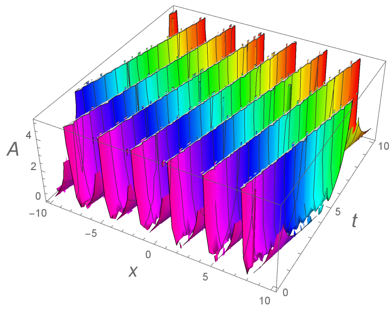
(d) 3-D soliton wave number profile at $\lambda = 1.2$



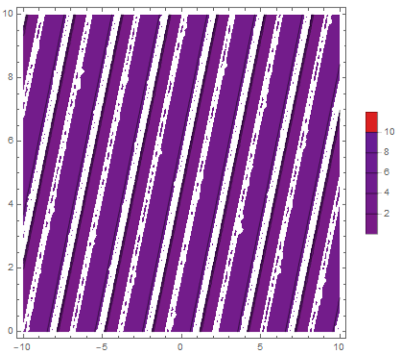
(e) Contour soliton wave number profile at $\lambda = 1.2$



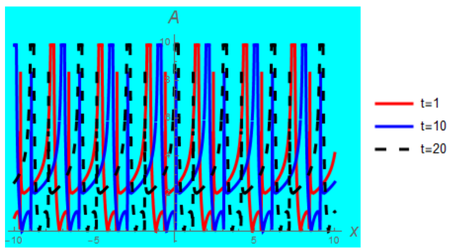
(f) 2-D soliton wave number profile at $\lambda = 1.2$



(g) 3-D soliton wave number profile at $\lambda = 2.2$



(h) Contour soliton wave number profile at $\lambda = 2.2$



(i) 2-D soliton wave number profile at $\lambda = 2.2$

Figure 4. 3-D, contour and 2-D graphical illustration for $\mathbb{Z}_3(x,t)$.

In Figures 3 and 4, we explained the graphical representation for $\mathbb{Z}_3(x,t)$ of the obtained solutions for soliton velocity and wave number for different values of wave speed and wave number.

In Figure 3, we depicted graphs for soliton velocity at the parametric values $\kappa = 0.9$, $b_0 = 0.5$, $b_1 = 0.75$, $b_2 = 0.90$, $\lambda = 0.8$, $\mu = 0.5$, in form of 3-D, contour and 2-D form respectively. At $c = 0.09$ continuous periodic solitonic behaviour is observed in a 3-D profile and for more visualisation contour plotted and found the singular soliton and 2-D shows the bright continuous periodic soliton and for fixed speed and different values of time soliton propagate translatory. The Same behaviour is observed for different values of wave velocity.

In Figure 4, we depicted graphs for wave number at the parametric values $\kappa = 0.9$, $b_0 = 0.5$, $b_1 = 0.75$, $b_2 = 0.90$, $c = 0.9$ and $\mu = 0.5$, in form of 3-D, contour and 2-D form respectively. At $\lambda = 0.2$, singular periodic solitonic behaviour is observed in a 3-D, singular soliton in contour and 2-D shows the bright continuous periodic soliton. For $\lambda = 1.2$ and $\lambda = 2.2$, we observed continuous periodic solitonic behaviour in a 3-D profile and for more visualisation contour plotted and found the singular soliton and 2-D shows the bright continuous periodic soliton.

As a result, these physical descriptions of our novel results may be useful for nonlinear wave problems in applied sciences for further research.

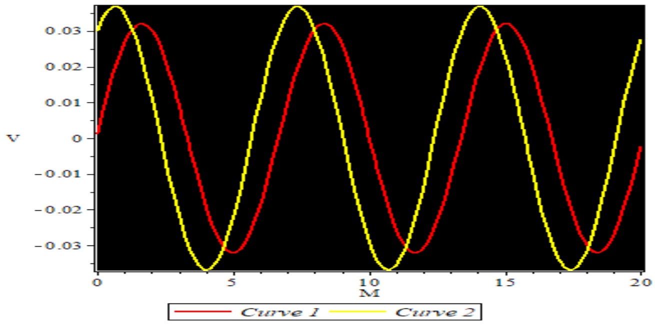
5. The sensitivity assessment

In order to display the sensitivity of the Landau-Ginzburg-Higgs equation, the dynamic planer system can be contributed by using the Galilean transformation process. Thus the Galilean transformation yields the dynamic system of Eq (3.2) as follows:

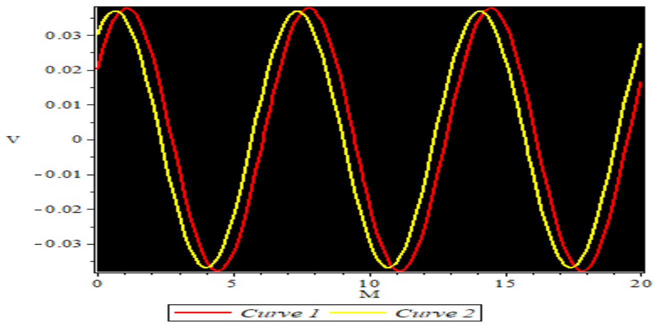
$$\begin{aligned} \frac{d\mathbb{Z}}{d\xi} &= S, \\ \frac{dS}{d\xi} &= \frac{g^2\mathbb{Z}}{(c^2-\lambda^2)} - \frac{h^2\mathbb{Z}^3}{(c^2-\lambda^2)}. \end{aligned} \tag{5.1}$$

This section discusses the sensitivity behaviour of the complex structure solution with parameters $c = 1.2$, $g = 1.5$, $\lambda = 2$, $h = 0.9$, taking into account the two separate initial conditions with the red and yellow curve drawn. In Figure 5, we investigate sensitive phenomena of the perturbed system. The sensitivity analysis is a process that assesses how sensitive our system is. The system's sensitivity will be poor if only a minor adjustment is made to the initial conditions. However, the system will be extremely sensitive if the system suffers a considerable shift due to minor changes in starting circumstances. We will look at how the frequency term affects the model under consideration. For this, we will establish the physical characteristics of the investigated model and discuss the influence of the perturbation's force as well as frequency. Therefore in the segment, we want to know the sensitivity for the solution of the perturbed dynamical structural system by using distinctive initial conditions.

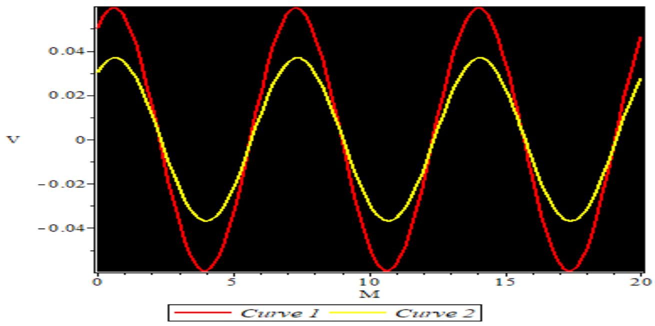
In Figure 5a, it can be seen that a slight change in the initial state affects the solutions that contribute to the disorderly behaviour of the curve same experiment is replicated in Figure 5b, 5c and 5d, by retaining the values of the parameters and greater change in the initial conditions and the same effects are found, as a result, the system is sensitive in this situation. The change in amplitude and frequency of the wave velocity in sensitivity graphs shows that the physical explanation for the system's sensitivity.



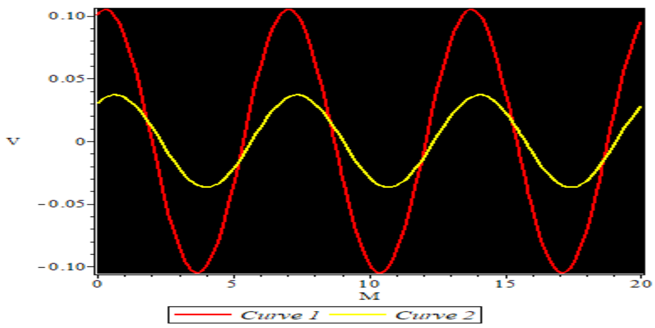
(a) Sensitive analysis for the curve 1 and curve 2 at $(0.001, 0.03)$ and $(0.03, 0.02)$ respectively



(b) Sensitive analysis for the curve 1 and curve 2 at $(0.02, 0.03)$ and $(0.03, 0.02)$ respectively



(c) Sensitive analysis for the curve the 1 and curve 2 at $(0.05, 0.03)$ and $(0.03, 0.02)$ respectively



(d) Sensitive analysis for the curve 1 and curve 2, at $(0.1, 0.03)$ and $(0.03, 0.02)$ respectively

Figure 5. Sensitivity of assessment at different initial conditions.

6. Conclusions

This study has successfully established and examined the exact solutions and more general solitary wave solitons of the remarkable Landau-Ginzburg-higgs model utilizing the generalized projective Riccati method approach. As a result of this,

- 3-D, 2-D and contour presentation of a periodic, singular, bright, dark, kink-type and bell-shaped wave peakon solutions are derived.
- Singularity of water waves can be reduced for larger values of wave velocity and wave number.
- Sensitive analysis presentation of the obtained system is displayed with the appropriate values of involved parameters.
- The solutions might be functional to better realize the mechanics of complicated nonlinear physical phenomena and have potential uses in dispersive wave systems to study resonant nonlinear relations.
- The generalized projective Riccati method performance is dependable and effective and it provides additional solutions. The applied methodology will be an advantage in future studies to develop new solutions for different nonlinear wave equations.

Researchers and professionals may apply these results to new nonlinear equations and complex nonlinear systems more quickly and efficiently.

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Conflict of interest

The authors state that they do not have any competing interests.

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