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Research article

T-spherical fuzzy information aggregation with multi-criteria decision-making

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Abstract: T-spherical fuzzy sets (T-SPFSs) have gained popularity because of their ability to account for uncertainty more effectively and spanning a larger domain. The sum of the t-*th* power of membership grades in T-SPFSs is close to a unit interval, allowing for greater uncertainty. As a result, this set outperforms traditional fuzzy structures. The "multi-criteria decision-making" (MCDM) approach is a widely used technique that requires the use of some aggregation tools, and various such aggregation operators (AOs) have been developed over the years to achieve this purpose. The purpose of this paper is to propose some new operational laws and AOs for use in a T-spherical fuzzy environment. In this regard, we presented some new neutral or fair operational rules that combine the concept of proportional distribution to provide a neutral or fair solution to the membership, abstinence, and non-membership of T-spherical fuzzy numbers (T-SPFNs). Based on the obtained operational rules, we presented the "T-spherical fuzzy fairly weighted average operator" and the "T-spherical fuzzy fairly ordered weighted averaging operator". Compared to earlier methodologies, the proposed AOs provide more generalised, reliable, and accurate information. In addition, under T-SPFSs, an MCDM approach is developed employing suggested AOs with several decision-makers (DMs) and partial weight details. Finally, to demonstrate the applicability of the innovative technique, we give an actual case study of "food waste treatment technology" (FWTT) selection under T-SPFSs scenarios. A comparison with an existing model has also been undertaken to confirm the validity and robustness of the acquired results.

Keywords: fairly operations; T-spherical fuzzy numbers; aggregation operators; optimization model
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1. Introduction

In practise, as a system becomes more sophisticated, it becomes more difficult for DMs to determine the best choice from a collection of feasible possibilities. It is challenging to convey how tough it is to achieve a single objective, yet it is not unattainable. Numerous organisations wrestled with the complexity of motivating employees, achievement, and worldview development. As a result, organizational choices, whether made by people or committees, include a number of concurrent objectives. Using optionally solved criteria, each DM should be confined to producing the optimal solution for each practical implication in actual issues. As a result, the decision maker is more concerned with developing more realistic and reliable methods for finding the best options.

MCDM is a widely used intellectual process. Its key feature is to help DMs choose the best possibility out of a limited number of choices by using their expert assessment. To solve these kinds of complications, Zadeh [1] was the first person to come up with the concept of "fuzzy set" (FS), which is a mathematical framework for expressing imprecision. Atanasov [2] took this unique initiative of FS and turned it into the theory of "intuitionistic fuzzy sets" (IFS). It is essential to remember that FS can be described in aspects of "membership functionalities" and IFS can be described in aspects of both "membership and non-membership functionalities". IFSs have been utilized a lot to solve big complications, but there are some situations that they can't handle. Assume that when it comes to voting, the human perspective contains more answers like "yes", "no", "don't know", and "refuse" that traditional FS and IFS can't fully represent. Cuong [3, 4] intoduced the idea of "picture fuzzy set" (PFS) to fix these issues.

Cuong and Hai [5] defined some basics for PFSs. Cuong [6] examined many technique for evaluating the distance between PFSs. Wei et al. introduced the "projection model" [8], "generalized dice similarity measures" [9] and "similarity measures" [10] for PFS. Singh [11] proposed some "correlation coefficients" and Son [12] defined "distributed picture fuzzy clustering method" for PFSs. Riaz et al. [13] revised several fundamental PFS operations. Phong et al. [7] investigated several PF-relation setups.

In recent eras, there has been a focus on the synthesis of knowledge and improved AOs. The effectiveness and limits of AOs are now a fundamental component of selection. It is evident that AO provides many principles for combining a finite set of fuzzy values into a single fuzzy value. Data analysis is crucial in the selection, business, medical, architecture, and surveillance domains. On PFSs, several AOs have been established in terms of their roles and operating regulations. Garg [14] advocated several averaging AOs for PFNs. Wei [15] and Jana et al. [15] introduced the notion of "Hamacher AOs", whereas Jana et al. [16] recommended "Dombi AOs" for PFNs. Tian et al. [17] gave the notion of "picture fuzzy power Choquet ordered geometric AOs" & "power shapley Choquet ordered AOs". Wang et al. [18] offered the selection of a hotel energy performance conversion project under PFSs. Wang et al. [19] proposed "Muirhead mean AOs" for PFNs.

Li et al. [20] introduced the unique concept of broadly applied, simplified neutrosophic Einstein AOs. Modified Einstein interacting geometric AOs for "q-rung orthopair fuzzy numbers" were provided by Farid & Riaz [23]. Many AOs for "linear diophantine fuzzy numbers" are given in [21,22]. Ashraf et al. suggested a distance metric for fuzzy collections of cubic picture fuzzy set [24,25]. Saha et al. [26, 27] provided the novel hybrid weighted AOs for some extensions of fuzzy. Wei and Zhang [28] selected important providers using single-valued neutrosophic Bonferroni power AOs. Some extraordinary work related to proposed work is given in [29–32]. Cao et al. [33] proposed decision-making method for power grid enterprizes. Liu et al. [34] introduced detection approach for thermal defects.

In everyday reality, we may meet many scenarios that cannot be addressed by PFS, including when the sum of PMD, N_uMD and N_gMD > 1. In just such a case, PFS has been unable to deliver an appropriate conclusion. In [35, 36] authors proposed the idea of "spherical fuzzy sets" (SFSs) independently in their articles. Mahmood et al. [37] defined T-SPFSs as an extension of SFSs. In T-SPFSs, the sum of the t-*th* power of membership grades is close to a unit interval, providing for higher uncertainty. As a result, this set outperforms conventional fuzzy structures. Munir et al. [38] developed the Einstein hybrid AOs for T-SPFSs with applications to MCDM. Zeng et al. [39] defined some Einstein interactive AOs for T-SPFSs with application to selection of photovoltaic cells. Liu et al. [40] proposed novel power Muirhead mean AOs for T-SPFSs. Kifayat-Ullah et al. [41] gave the notion of T-SPF Hamacher AOs with MCDM. Khan et al. [42] introduced the Schweizer-Sklar power Heronian mean AOs in the domain of T-SPFSs. More work related to MCDM is given in [43–45]. Some work related to AOs and graph structures can be seen in [46,47]. Feng et al. [48] proposed some novel score functions related to orthopair fuzzy set. Senapati and Yager proposed "Fermatean fuzzy set" as the extension of IFS [49]. Smarandache proposed a novel idea of "neutrosophic set" [50, 51].

In literature many AOs [37, 38, 40, 41] are there to solve MCDM problem in the framework of T-SPF, one can see the unbiased nature of aggregation process while dealing equal "membership degree" (MSD), "abstinence degree" (AD) and "non-membership degree" (N-MSD). For example, in the scenario where a DM directs a comparison work to all MSD, AD, and N-MSD, the values acquired by AOs which previously existent in the literature cannot be distinguished [37,38,40,41]. This indicates that the ultimate conclusion is unquestionably biassed. Consequently, several new procedures are necessary to handle MSD, AD, and N-MSD properly and to ensure the fair or neutral operation of T-SPFN. To attain actual satisfaction while assessing positive, abstinence, and negative membership, we provide two neutral or fair procedures based on the notion of proportionate distribution constraints for all functions.

The "Fermatean fuzzy Heronian mean" AOs and the "method based on the removal effects of criteria" (MEREC) approach were suggested by Rani et al. [52] with applications towards the selection of FWTT. Through the use of a spherical fuzzy technique, Buyuk and Temur [53] presented a new method for selecting treatment options for food waste. Rani et al. [54] was the one who came up with the idea of employing a single-valued neutrosophic framework to identify the best treatment procedure for multi-criteria food waste. Chen et al. [55] suggested a hybrid fuzzy evaluation approach for the purpose of safely evaluating food waste as a feed ingredient, with the method being based on entropy methods. The synthesis of the waste-water treatment process and the selection of suppliers were both introduced by Ho et al. [56] using fuzzy techniques.

This format is maintained for the rest of the article. In the second part, we will go through the fundamental notions of T-SPFS. In Section 3, we looked at a few different T-SPFN approaches. The appropriate AOs for T-SPFNs are provided in Section 4. In Section 5, an MCDM framework is shown for the suggested AOs, and in Section 6, a numerically comprehensive test case is provided. The most important conclusions from the research are discussed in the seventh part.

2. Preliminaries

Some key principles connected with T-SPFSs have just been addressed in this portion of the manuscript.

Definition 2.1. [37] A "T-spherical fuzzy set" (T-SPFS) in X is defined as

$$\chi = \{\langle \breve{\curlyvee}, {}^{\daleth}\mu_{\chi}(\breve{\curlyvee}), {}^{\daleth}\nu_{\chi}(\breve{\curlyvee}), {}^{\daleth}\tau_{\chi}(\breve{\curlyvee}) | \breve{\curlyvee} \in X\rangle\} \tag{2.1}$$

where ${}^{\daleth}\mu_{\chi}(\breve{\curlyvee}), {}^{\daleth}\nu_{\chi}(\breve{\curlyvee}), {}^{\daleth}\tau_{\chi}(\breve{\curlyvee}) \in [0,1]$, such that $0 \leq {}^{\daleth}\mu^{t}_{\chi}(\breve{\curlyvee}) + {}^{\daleth}\nu^{t}_{\chi}(\breve{\curlyvee}) + {}^{\daleth}\tau^{t}_{\chi}(\breve{\curlyvee}) \leq 1$ for all $\breve{\curlyvee} \in X$. ${}^{\daleth}\mu_{\chi}(\breve{\curlyvee}), {}^{\daleth}\nu_{\chi}(\breve{\curlyvee}), {}^{\daleth}\tau_{\chi}(\breve{\curlyvee})$ denote "membership degree (MSD), abstinence degree (AD) and non-membership degree (N-MSD)" respectively for some $\breve{\curlyvee} \in X$.

we denote this pair as $\sqsupset = ({}^{\daleth}\mu_{\sqsupset}, {}^{\daleth}\nu_{\sqsupset}, {}^{\daleth}\tau_{\sqsupset})$, throughout this article, and called as T-SPFN with the conditions ${}^{\daleth}\mu_{\sqsupset}, {}^{\daleth}\nu_{\sqsupset}, {}^{\daleth}\tau_{\sqsupset} \in [0,1]$ and ${}^{\daleth}\mu^{t}_{\sqsupset} + {}^{\daleth}\nu^{t}_{\sqsupset} + {}^{\daleth}\tau^{t}_{\sqsupset} \leq 1$.

Definition 2.2. [37] When implementing T-SPFNs to actual situations, it is crucial to categorize them. For this, "score function" (SF) corresponding to T-SPFN $\sqsupset = ({}^{\daleth}\mu_{\sqsupset}, {}^{\daleth}\nu_{\sqsupset}, {}^{\daleth}\tau_{\sqsupset})$ be defined as

$$S(\sqsupset) = {}^{\daleth}\mu^{t}_{\sqsupset} - {}^{\daleth}\tau^{t}_{\sqsupset}. \tag{2.2}$$

In many instances, however, the aforesaid function is inadequate of categorizing T-SPFNs under various settings, making it difficult to tell which is better. This is accomplished by defining an "accuracy function" H of $\sqsupset$ as

$$\breve{\Theta}^{\zeta}(\sqsupset) = {}^{\daleth}\mu^{t}_{\sqsupset} + {}^{\daleth}\nu^{t}_{\sqsupset} + {}^{\daleth}\tau^{t}_{\sqsupset}. \tag{2.3}$$

We shall provide operational principles for aggregating T-SPFNs.

Definition 2.3. [40] Let $\sqsupset_1 = \langle {}^{\daleth}\mu_1, {}^{\daleth}\nu_1, {}^{\daleth}\tau_1\rangle$ and $\sqsupset_2 = \langle {}^{\daleth}\mu_2, {}^{\daleth}\nu_2, {}^{\daleth}\tau_2\rangle$ be two T-SPFNs, then

$$\sqsupset_1^{c} = \left\langle {}^{\daleth}\tau_1, {}^{\daleth}\nu_1, {}^{\daleth}\mu_1 \right\rangle \tag{2.4}$$

$$\sqsupset_1 \vee \sqsupset_2 = \left\langle max\{{}^{\daleth}\mu_1, {}^{\daleth}\mu_2\}, min\{{}^{\daleth}\nu_1, {}^{\daleth}\nu_2\}, min\{{}^{\daleth}\tau_1, {}^{\daleth}\tau_2\} \right\rangle \tag{2.5}$$

$$\sqsupset_1 \wedge \sqsupset_2 = \left\langle min\{{}^{\daleth}\mu_1, {}^{\daleth}\mu_2\}, max\{{}^{\daleth}\nu_1, {}^{\daleth}\nu_2\}, max\{{}^{\daleth}\tau_1, {}^{\daleth}\tau_2\} \right\rangle \tag{2.6}$$

$$\sqsupset_1 \oplus \sqsupset_2 = \left\langle \sqrt[t]{{}^{\daleth}\mu_1^{t} + {}^{\daleth}\mu_2^{t} - {}^{\daleth}\mu_1^{t}\,{}^{\daleth}\mu_2^{t}}, {}^{\daleth}\nu_1\,{}^{\daleth}\nu_2, {}^{\daleth}\tau_1\,{}^{\daleth}\tau_2 \right\rangle \tag{2.7}$$

$$\sqsupset_1 \otimes \sqsupset_2 = \left\langle {}^{\daleth}\mu_1\,{}^{\daleth}\mu_2, \sqrt[t]{{}^{\daleth}\nu_1^{t} + {}^{\daleth}\nu_2^{t} - {}^{\daleth}\nu_1^{t}\,{}^{\daleth}\nu_2^{t}}, \sqrt[t]{{}^{\daleth}\tau_1^{t} + {}^{\daleth}\tau_2^{t} - {}^{\daleth}\tau_1^{t}\,{}^{\daleth}\tau_2^{t}} \right\rangle \tag{2.8}$$

$$\sigma\sqsupset_1 = \left\langle \sqrt[t]{1 - (1 - {}^{\daleth}\mu_1^{t})^{\sigma}}, {}^{\daleth}\nu_1^{\sigma}, {}^{\daleth}\tau_1^{\sigma} \right\rangle \tag{2.9}$$

$$\sqsupset_1^{\sigma} = \left\langle {}^{\daleth}\mu_1^{\sigma}, \sqrt[t]{1 - (1 - {}^{\daleth}\nu_1^{t})^{\sigma}}, \sqrt[t]{1 - (1 - {}^{\daleth}\tau_1^{t})^{\sigma}} \right\rangle. \tag{2.10}$$

Definition 2.4. Let $\beth_1 = \langle {}^{\gimel}\mu_1, {}^{\gimel}\nu_1, {}^{\gimel}\tau_1 \rangle$ and $\beth_2 = \langle {}^{\gimel}\mu_2, {}^{\gimel}\nu_2, {}^{\gimel}\tau_2 \rangle$ be two T-SPFNs and $\mathcal{Y}^{\gamma}, \mathcal{Y}^{\gamma}{}_1, \mathcal{Y}^{\gamma}{}_2 > 0$ be the real numbers, then we have

(1) $\beth_1 \oplus \beth_2 = \beth_2 \oplus \beth_1$

(2) $\beth_1 \otimes \beth_2 = \beth_2 \otimes \beth_1$

(3) $\mathcal{Y}^{\gamma} (\beth_1 \oplus \beth_2) = (\mathcal{Y}^{\gamma} \beth_1) \oplus (\mathcal{Y}^{\gamma} \beth_2)$

(4) $(\beth_1 \otimes \beth_2)^{\mathcal{Y}^{\gamma}} = \beth_1^{\mathcal{Y}^{\gamma}} \otimes \beth_2^{\mathcal{Y}^{\gamma}}$

(5) $(\mathcal{Y}^{\gamma}{}_1 + \mathcal{Y}^{\gamma}{}_2) \beth_1 = (\mathcal{Y}^{\gamma}{}_1 \beth_1) \oplus (\mathcal{Y}^{\gamma}{}_2 \beth_2)$

(6) $\beth_1^{\mathcal{Y}^{\gamma}{}_1 + \mathcal{Y}^{\gamma}{}_2} = \beth_1^{\mathcal{Y}^{\gamma}{}_1} \otimes \beth_2^{\mathcal{Y}^{\gamma}{}_2}$.

If ${}^{\gimel}\mu_{\beth_1} = {}^{\gimel}\nu_{\beth_1}$ and ${}^{\gimel}\mu_{\beth_2} = {}^{\gimel}\nu_{\beth_2}$ then from Definition 2.3 we get, ${}^{\gimel}\mu_{\beth_1 \oplus \beth_2} \neq {}^{\gimel}\nu_{\beth_1 \oplus \beth_2}, {}^{\gimel}\mu_{\beth_1 \otimes \beth_2} \neq {}^{\gimel}\nu_{\beth_1 \otimes \beth_2}, {}^{\gimel}\mu_{\mathcal{Y}^{\gamma}\beth_1} \neq {}^{\gimel}\nu_{\mathcal{Y}^{\gamma}\beth_1}, {}^{\gimel}\mu_{\beth_1^{\mathcal{Y}^{\gamma}}} \neq {}^{\gimel}\nu_{\beth_1^{\mathcal{Y}^{\gamma}}}$. Thus none of the operations $\beth_1 \oplus \beth_2, \beth_1 \otimes \beth_2, \mathcal{Y}^{\gamma}\beth_1, \beth_1^{\mathcal{Y}^{\gamma}}$ found to be neutral or fair indeed. Consequently, our focus must first be on developing fair operations amongst T-SPFNs.

3. Fairly operations on T-SPFNs

In this part, we construct several basic operations involving T-SPFNs and investigate their fundamental features.

Definition 3.1. Consider $\beth_1 = \left\langle {}^{\gimel}\mu_{\beth_1}, {}^{\gimel}\nu_{\beth_1}, {}^{\gimel}\tau_{\beth_1} \right\rangle$ and $\beth_2 = \left\langle {}^{\gimel}\mu_{\beth_2}, {}^{\gimel}\nu_{\beth_2}, {}^{\gimel}\tau_{\beth_2} \right\rangle$ be the two T-SPFNs and $\mathcal{Y}^{\gamma} > 0$. Then we define

$$\beth_1 \tilde{\oplus} \beth_2 = \begin{pmatrix} \sqrt[t]{\left(\frac{{}^{\gimel}\mu^t{}_{\beth_1} {}^{\gimel}\mu^t{}_{\beth_2}}{{}^{\gimel}\mu^t{}_{\beth_1} {}^{\gimel}\mu^t{}_{\beth_2} + {}^{\gimel}\nu^t{}_{\beth_1} {}^{\gimel}\nu^t{}_{\beth_2} + {}^{\gimel}\tau^t{}_{\beth_1} {}^{\gimel}\tau^t{}_{\beth_2}} \right) \times \left(1 - \left(1 - {}^{\gimel}\mu^t{}_{\beth_1} - {}^{\gimel}\nu^t{}_{\beth_1} - {}^{\gimel}\tau^t{}_{\beth_1} \right) \left(1 - {}^{\gimel}\mu^t{}_{\beth_2} - {}^{\gimel}\nu^t{}_{\beth_2} - {}^{\gimel}\tau^t{}_{\beth_2} \right) \right)}, \\ \sqrt[t]{\left(\frac{{}^{\gimel}\nu^t{}_{\beth_1} {}^{\gimel}\nu^t{}_{\beth_2}}{{}^{\gimel}\mu^t{}_{\beth_1} {}^{\gimel}\mu^t{}_{\beth_2} + {}^{\gimel}\nu^t{}_{\beth_1} {}^{\gimel}\nu^t{}_{\beth_2} + {}^{\gimel}\tau^t{}_{\beth_1} {}^{\gimel}\tau^t{}_{\beth_2}} \right) \times \left(1 - \left(1 - {}^{\gimel}\mu^t{}_{\beth_1} - {}^{\gimel}\nu^t{}_{\beth_1} - {}^{\gimel}\tau^t{}_{\beth_1} \right) \left(1 - {}^{\gimel}\mu^t{}_{\beth_2} - {}^{\gimel}\nu^t{}_{\beth_2} - {}^{\gimel}\tau^t{}_{\beth_2} \right) \right)}, \\ \sqrt[t]{\left(\frac{{}^{\gimel}\tau^t{}_{\beth_1} {}^{\gimel}\tau^t{}_{\beth_2}}{{}^{\gimel}\mu^t{}_{\beth_1} {}^{\gimel}\mu^t{}_{\beth_2} + {}^{\gimel}\nu^t{}_{\beth_1} {}^{\gimel}\nu^t{}_{\beth_2} + {}^{\gimel}\tau^t{}_{\beth_1} {}^{\gimel}\tau^t{}_{\beth_2}} \right) \times \left(1 - \left(1 - {}^{\gimel}\mu^t{}_{\beth_1} - {}^{\gimel}\nu^t{}_{\beth_1} - {}^{\gimel}\tau^t{}_{\beth_1} \right) \left(1 - {}^{\gimel}\mu^t{}_{\beth_2} - {}^{\gimel}\nu^t{}_{\beth_2} - {}^{\gimel}\tau^t{}_{\beth_2} \right) \right)} \end{pmatrix}$$

$$\mathcal{Y}^{\gamma} * \beth_1 = \begin{pmatrix} \sqrt[t]{\left(\frac{{}^{\gimel}\mu^{t\,\mathcal{Y}^{\gamma}}_{\beth_1}}{{}^{\gimel}\mu^{t\,\mathcal{Y}^{\gamma}}_{\beth_1} + {}^{\gimel}\nu^{t\,\mathcal{Y}^{\gamma}}_{\beth_1} + {}^{\gimel}\tau^{t\,\mathcal{Y}^{\gamma}}_{\beth_1}} \right) \times \left(1 - \left(1 - {}^{\gimel}\mu^t{}_{\beth_1} - {}^{\gimel}\nu^t{}_{\beth_1} - {}^{\gimel}\tau^t{}_{\beth_1} \right)^{\mathcal{Y}^{\gamma}} \right)}, \\ \sqrt[t]{\left(\frac{{}^{\gimel}\nu^{t\,\mathcal{Y}^{\gamma}}_{\beth_1}}{{}^{\gimel}\mu^{t\,\mathcal{Y}^{\gamma}}_{\beth_1} + {}^{\gimel}\nu^{t\,\mathcal{Y}^{\gamma}}_{\beth_1} + {}^{\gimel}\tau^{t\,\mathcal{Y}^{\gamma}}_{\beth_1}} \right) \times \left(1 - \left(1 - {}^{\gimel}\mu^t{}_{\beth_1} - {}^{\gimel}\nu^t{}_{\beth_1} - {}^{\gimel}\tau^t{}_{\beth_1} \right)^{\mathcal{Y}^{\gamma}} \right)}, \\ \sqrt[t]{\left(\frac{{}^{\gimel}\tau^{t\,\mathcal{Y}^{\gamma}}_{\beth_1}}{{}^{\gimel}\mu^{t\,\mathcal{Y}^{\gamma}}_{\beth_1} + {}^{\gimel}\nu^{t\,\mathcal{Y}^{\gamma}}_{\beth_1} + {}^{\gimel}\tau^{t\,\mathcal{Y}^{\gamma}}_{\beth_1}} \right) \times \left(1 - \left(1 - {}^{\gimel}\mu^t{}_{\beth_1} - {}^{\gimel}\nu^t{}_{\beth_1} - {}^{\gimel}\tau^t{}_{\beth_1} \right)^{\mathcal{Y}^{\gamma}} \right)} \end{pmatrix}.$$

It can be easily verified that $\beth_1 \tilde{\oplus} \beth_2, \mathcal{Y}^{\gamma} * \beth_1$ are the T-SPFNs.

Theorem 3.2. *Consider* $\beth_1 =< {}^{\daleth}\mu_{\beth_1}, {}^{\daleth}\nu_{\beth_1}, {}^{\daleth}\tau_{\beth_1} >$ *and* $\beth_2 =< {}^{\daleth}\mu_{\beth_2}, {}^{\daleth}\nu_{\beth_2}, {}^{\daleth}\tau_{\beth_2} >$ *are the T-SPFNs. If* ${}^{\daleth}\mu_{\beth_1} = {}^{\daleth}\nu_{\beth_1}$, ${}^{\daleth}\mu_{\beth_2} = {}^{\daleth}\nu_{\beth_2}$ *and* ${}^{\daleth}\tau_{\beth_2} = {}^{\daleth}\tau_{\beth_2}$ *then we have*
(i) ${}^{\daleth}\mu_{\beth_1\tilde{\oplus}\beth_2} = {}^{\daleth}\nu_{\beth_1\tilde{\oplus}\beth_2} = {}^{\daleth}\tau_{\beth_1\tilde{\oplus}\beth_2}$.
(ii) ${}^{\daleth}\mu_{\mathcal{Y}^\gamma * \beth_1} = {}^{\daleth}\nu_{\mathcal{Y}^\gamma * \beth_1} = {}^{\daleth}\tau_{\mathcal{Y}^\gamma * \beth_1}$.

Proof. (i) As given ${}^{\daleth}\mu_{\aleph_1} = {}^{\daleth}\nu_{\aleph_1}$, ${}^{\daleth}\mu_{\aleph_2} = {}^{\daleth}\nu_{\aleph_2}$ and ${}^{\daleth}\tau_{\aleph_2} = {}^{\daleth}\tau_{\aleph_2}$

$$\begin{aligned}\frac{{}^{\daleth}\mu_{\aleph_1\tilde{\oplus}\aleph_2}}{{}^{\daleth}\nu_{\aleph_1\tilde{\oplus}\aleph_2}} &= \frac{\sqrt[t]{\left(\frac{{}^{\daleth}\mu^t{}_{\aleph_1}{}^{\daleth}\mu^t{}_{\aleph_2}}{{}^{\daleth}\mu^t{}_{\aleph_1}{}^{\daleth}\mu^t{}_{\aleph_2}+{}^{\daleth}\nu^t{}_{\aleph_1}{}^{\daleth}\nu^t{}_{\aleph_2}+{}^{\daleth}\tau^t{}_{\aleph_1}{}^{\daleth}\tau^t{}_{\aleph_2}}\right)\times\left(1-\left(1-{}^{\daleth}\mu^t{}_{\aleph_1}-{}^{\daleth}\nu^t{}_{\aleph_1}-{}^{\daleth}\tau^t{}_{\aleph_1}\right)\left(1-{}^{\daleth}\mu^t{}_{\aleph_2}-{}^{\daleth}\nu^t{}_{\aleph_2}-{}^{\daleth}\tau^t{}_{\aleph_2}\right)\right)}}{\sqrt[t]{\left(\frac{{}^{\daleth}\nu^t{}_{\aleph_1}{}^{\daleth}\nu^t{}_{\aleph_2}}{{}^{\daleth}\mu^t{}_{\aleph_1}{}^{\daleth}\mu^t{}_{\aleph_2}+{}^{\daleth}\nu^t{}_{\aleph_1}{}^{\daleth}\nu^t{}_{\aleph_2}+{}^{\daleth}\tau^t{}_{\aleph_1}{}^{\daleth}\tau^t{}_{\aleph_2}}\right)\times\left(1-\left(1-{}^{\daleth}\mu^t{}_{\aleph_1}-{}^{\daleth}\nu^t{}_{\aleph_1}-{}^{\daleth}\tau^t{}_{\aleph_1}\right)\left(1-{}^{\daleth}\mu^t{}_{\aleph_2}-{}^{\daleth}\nu^t{}_{\aleph_2}-{}^{\daleth}\tau^t{}_{\aleph_2}\right)\right)}} \\ &= 1\end{aligned}$$

and

$$\begin{aligned}\frac{{}^{\daleth}\nu_{\aleph_1\tilde{\oplus}\aleph_2}}{{}^{\daleth}\tau_{\aleph_1\tilde{\oplus}\aleph_2}} &= \frac{\sqrt[t]{\left(\frac{{}^{\daleth}\nu^t{}_{\aleph_1}{}^{\daleth}\nu^t{}_{\aleph_2}}{{}^{\daleth}\mu^t{}_{\aleph_1}{}^{\daleth}\mu^t{}_{\aleph_2}+{}^{\daleth}\nu^t{}_{\aleph_1}{}^{\daleth}\nu^t{}_{\aleph_2}+{}^{\daleth}\tau^t{}_{\aleph_1}{}^{\daleth}\tau^t{}_{\aleph_2}}\right)\times\left(1-\left(1-{}^{\daleth}\mu^t{}_{\aleph_1}-{}^{\daleth}\nu^t{}_{\aleph_1}-{}^{\daleth}\tau^t{}_{\aleph_1}\right)\left(1-{}^{\daleth}\mu^t{}_{\aleph_2}-{}^{\daleth}\nu^t{}_{\aleph_2}-{}^{\daleth}\tau^t{}_{\aleph_2}\right)\right)}}{\sqrt[t]{\left(\frac{{}^{\daleth}\tau^t{}_{\aleph_1}{}^{\daleth}\tau^t{}_{\aleph_2}}{{}^{\daleth}\mu^t{}_{\aleph_1}{}^{\daleth}\mu^t{}_{\aleph_2}+{}^{\daleth}\nu^t{}_{\aleph_1}{}^{\daleth}\nu^t{}_{\aleph_2}+{}^{\daleth}\tau^t{}_{\aleph_1}{}^{\daleth}\tau^t{}_{\aleph_2}}\right)\times\left(1-\left(1-{}^{\daleth}\mu^t{}_{\aleph_1}-{}^{\daleth}\nu^t{}_{\aleph_1}-{}^{\daleth}\tau^t{}_{\aleph_1}\right)\left(1-{}^{\daleth}\mu^t{}_{\aleph_2}-{}^{\daleth}\nu^t{}_{\aleph_2}-{}^{\daleth}\tau^t{}_{\aleph_2}\right)\right)}} \\ &= 1.\end{aligned}$$

Consequently, ${}^{\daleth}\mu_{\aleph_1\tilde{\oplus}\aleph_2} = {}^{\daleth}\nu_{\aleph_1\tilde{\oplus}\aleph_2} = {}^{\daleth}\tau_{\aleph_1\tilde{\oplus}\aleph_2}$. If ${}^{\daleth}\mu_{\aleph_1} = {}^{\daleth}\nu_{\aleph_1}$, ${}^{\daleth}\mu_{\aleph_2} = {}^{\daleth}\nu_{\aleph_2}$ and ${}^{\daleth}\tau_{\aleph_2} = {}^{\daleth}\tau_{\aleph_2}$.
(ii)

$$\begin{aligned}\frac{{}^{\daleth}\mu_{\mathcal{Y}^\gamma * \aleph_1}}{{}^{\daleth}\nu_{\mathcal{Y}^\gamma * \aleph_1}} &= \frac{\sqrt[t]{\left(\frac{{}^{\daleth}\mu^t{}_{\aleph_1}^{\mathcal{Y}^\gamma}}{{}^{\daleth}\mu^t{}_{\aleph_1}^{\mathcal{Y}^\gamma}+{}^{\daleth}\nu^t{}_{\aleph_1}^{\mathcal{Y}^\gamma}+{}^{\daleth}\tau^t{}_{\aleph_1}^{\mathcal{Y}^\gamma}}\right)\times\left(1-\left(1-{}^{\daleth}\mu^t{}_{\aleph_1}-{}^{\daleth}\nu^t{}_{\aleph_1}-{}^{\daleth}\tau^t{}_{\aleph_1}\right)^{\mathcal{Y}^\gamma}\right)}}{\sqrt[t]{\left(\frac{{}^{\daleth}\nu^t{}_{\aleph_1}^{\mathcal{Y}^\gamma}}{{}^{\daleth}\mu^t{}_{\aleph_1}^{\mathcal{Y}^\gamma}+{}^{\daleth}\nu^t{}_{\aleph_1}^{\mathcal{Y}^\gamma}+{}^{\daleth}\tau^t{}_{\aleph_1}^{\mathcal{Y}^\gamma}}\right)\times\left(1-\left(1-{}^{\daleth}\mu^t{}_{\aleph_1}-{}^{\daleth}\nu^t{}_{\aleph_1}-{}^{\daleth}\tau^t{}_{\aleph_1}\right)^{\mathcal{Y}^\gamma}\right)}} \\ &= 1\end{aligned}$$

and

$$\begin{aligned}\frac{{}^{\daleth}\nu_{\mathcal{Y}^\gamma * \aleph_1}}{{}^{\daleth}\tau_{\mathcal{Y}^\gamma * \aleph_1}} &= \frac{\sqrt[t]{\left(\frac{{}^{\daleth}\nu^t{}_{\aleph_1}^{\mathcal{Y}^\gamma}}{{}^{\daleth}\mu^t{}_{\aleph_1}^{\mathcal{Y}^\gamma}+{}^{\daleth}\nu^t{}_{\aleph_1}^{\mathcal{Y}^\gamma}+{}^{\daleth}\tau^t{}_{\aleph_1}^{\mathcal{Y}^\gamma}}\right)\times\left(1-\left(1-{}^{\daleth}\mu^t{}_{\aleph_1}-{}^{\daleth}\nu^t{}_{\aleph_1}-{}^{\daleth}\tau^t{}_{\aleph_1}\right)^{\mathcal{Y}^\gamma}\right)}}{\sqrt[t]{\left(\frac{{}^{\daleth}\tau^t{}_{\aleph_1}^{\mathcal{Y}^\gamma}}{{}^{\daleth}\mu^t{}_{\aleph_1}^{\mathcal{Y}^\gamma}+{}^{\daleth}\nu^t{}_{\aleph_1}^{\mathcal{Y}^\gamma}+{}^{\daleth}\tau^t{}_{\aleph_1}^{\mathcal{Y}^\gamma}}\right)\times\left(1-\left(1-{}^{\daleth}\mu^t{}_{\aleph_1}-{}^{\daleth}\nu^t{}_{\aleph_1}-{}^{\daleth}\tau^t{}_{\aleph_1}\right)^{\mathcal{Y}^\gamma}\right)}} \\ &= 1.\end{aligned}$$

Consequently, ${}^{\daleth}\mu_{\mathcal{Y}^\gamma * \aleph_1} = {}^{\daleth}\nu_{\mathcal{Y}^\gamma * \aleph_1} = {}^{\daleth}\tau_{\mathcal{Y}^\gamma * \aleph_1}$. If ${}^{\daleth}\mu_{\aleph_1} = {}^{\daleth}\nu_{\aleph_1}$, ${}^{\daleth}\mu_{\aleph_2} = {}^{\daleth}\nu_{\aleph_2}$ and ${}^{\daleth}\tau_{\aleph_2} = {}^{\daleth}\tau_{\aleph_2}$. □

This theorem reveled that the operations $\beth_1\tilde{\oplus}\beth_2, \mathcal{Y}^\gamma * \beth_1$ show the fairly or neutral nature to the DMs, when MSD, AD and N-MSD are equal initially. This is why we call the operations $\tilde{\bigoplus}$, * fairly operations.

Theorem 3.3. *Consider* $\beth_1 =< {}^{\gimel}\mu_{\beth_1}, {}^{\gimel}\nu_{\beth_1}, {}^{\gimel}\tau_{\beth_1} >$ *and* $\beth_2 =< {}^{\gimel}\mu_{\beth_2}, {}^{\gimel}\nu_{\beth_2}, {}^{\gimel}\tau_{\beth_2} >$ *are the T-SPFNs and* $\mho, \mho_1$ *and* $\mho_2$ *are any three real numbers, then we have*
(i) $\beth_1 \tilde{\oplus} \beth_2 = \beth_2 \tilde{\oplus} \beth_1$
(ii) $\mho * (\beth_1 \tilde{\oplus} \beth_2) = (\mho * \beth_1) \tilde{\oplus} (\mho * \beth_2)$
(iii) $(\mho_1 + \mho_2) * \beth_1 = (\mho_1 * \beth_1) \tilde{\oplus} (\mho_2 * \beth_1)$.

Proof. (i) This one is trivial.
(ii) $\mho * (\aleph_1 \tilde{\oplus} \aleph_2)$

$$= \left(\begin{array}{c}
\sqrt[t]{\dfrac{\left(\frac{\gimel\mu^t_{\aleph_1}\gimel\mu^t_{\aleph_2}}{\gimel\mu^t_{\aleph_1}\gimel\mu^t_{\aleph_2}+\gimel\nu^t_{\aleph_1}\gimel\nu^t_{\aleph_2}+\gimel\tau^t_{\aleph_1}\gimel\tau^t_{\aleph_2}}\right)^{\mho}}{\left(\frac{\gimel\mu^t_{\aleph_1}\gimel\mu^t_{\aleph_2}}{\gimel\mu^t_{\aleph_1}\gimel\mu^t_{\aleph_2}+\gimel\nu^t_{\aleph_1}\gimel\nu^t_{\aleph_2}+\gimel\tau^t_{\aleph_1}\gimel\tau^t_{\aleph_2}}\right)^{\mho}+\left(\frac{\gimel\nu^t_{\aleph_1}\gimel\nu^t_{\aleph_2}}{\gimel\mu^t_{\aleph_1}\gimel\mu^t_{\aleph_2}+\gimel\nu^t_{\aleph_1}\gimel\nu^t_{\aleph_2}+\gimel\tau^t_{\aleph_1}\gimel\tau^t_{\aleph_2}}\right)^{\mho}+\left(\frac{\gimel\tau^t_{\aleph_1}\gimel\tau^t_{\aleph_2}}{\gimel\mu^t_{\aleph_1}\gimel\mu^t_{\aleph_2}+\gimel\nu^t_{\aleph_1}\gimel\nu^t_{\aleph_2}+\gimel\tau^t_{\aleph_1}\gimel\tau^t_{\aleph_2}}\right)^{\mho}} \times \left(1-\left(1-\left(1-\left(1-\gimel\mu^t_{\aleph_1}-\gimel\nu^t_{\aleph_1}-\gimel\tau^t_{\aleph_1}\right)\left(1-\gimel\mu^t_{\aleph_2}-\gimel\nu^t_{\aleph_2}-\gimel\tau^t_{\aleph_2}\right)\right)\right)^{\mho}\right)}, \\
\sqrt[t]{\dfrac{\left(\frac{\gimel\nu^t_{\aleph_1}\gimel\nu^t_{\aleph_2}}{\gimel\mu^t_{\aleph_1}\gimel\mu^t_{\aleph_2}+\gimel\nu^t_{\aleph_1}\gimel\nu^t_{\aleph_2}+\gimel\tau^t_{\aleph_1}\gimel\tau^t_{\aleph_2}}\right)^{\mho}}{\left(\frac{\gimel\mu^t_{\aleph_1}\gimel\mu^t_{\aleph_2}}{\gimel\mu^t_{\aleph_1}\gimel\mu^t_{\aleph_2}+\gimel\nu^t_{\aleph_1}\gimel\nu^t_{\aleph_2}+\gimel\tau^t_{\aleph_1}\gimel\tau^t_{\aleph_2}}\right)^{\mho}+\left(\frac{\gimel\nu^t_{\aleph_1}\gimel\nu^t_{\aleph_2}}{\gimel\mu^t_{\aleph_1}\gimel\mu^t_{\aleph_2}+\gimel\nu^t_{\aleph_1}\gimel\nu^t_{\aleph_2}+\gimel\tau^t_{\aleph_1}\gimel\tau^t_{\aleph_2}}\right)^{\mho}+\left(\frac{\gimel\tau^t_{\aleph_1}\gimel\tau^t_{\aleph_2}}{\gimel\mu^t_{\aleph_1}\gimel\mu^t_{\aleph_2}+\gimel\nu^t_{\aleph_1}\gimel\nu^t_{\aleph_2}+\gimel\tau^t_{\aleph_1}\gimel\tau^t_{\aleph_2}}\right)^{\mho}} \times \left(1-\left(1-\left(1-\left(1-\gimel\mu^t_{\aleph_1}-\gimel\nu^t_{\aleph_1}-\gimel\tau^t_{\aleph_1}\right)\left(1-\gimel\mu^t_{\aleph_2}-\gimel\nu^t_{\aleph_2}-\gimel\tau^t_{\aleph_2}\right)\right)\right)^{\mho}\right)}, \\
\sqrt[t]{\dfrac{\left(\frac{\gimel\tau^t_{\aleph_1}\gimel\tau^t_{\aleph_2}}{\gimel\mu^t_{\aleph_1}\gimel\mu^t_{\aleph_2}+\gimel\nu^t_{\aleph_1}\gimel\nu^t_{\aleph_2}+\gimel\tau^t_{\aleph_1}\gimel\tau^t_{\aleph_2}}\right)^{\mho}}{\left(\frac{\gimel\mu^t_{\aleph_1}\gimel\mu^t_{\aleph_2}}{\gimel\mu^t_{\aleph_1}\gimel\mu^t_{\aleph_2}+\gimel\nu^t_{\aleph_1}\gimel\nu^t_{\aleph_2}+\gimel\tau^t_{\aleph_1}\gimel\tau^t_{\aleph_2}}\right)^{\mho}+\left(\frac{\gimel\nu^t_{\aleph_1}\gimel\nu^t_{\aleph_2}}{\gimel\mu^t_{\aleph_1}\gimel\mu^t_{\aleph_2}+\gimel\nu^t_{\aleph_1}\gimel\nu^t_{\aleph_2}+\gimel\tau^t_{\aleph_1}\gimel\tau^t_{\aleph_2}}\right)^{\mho}+\left(\frac{\gimel\tau^t_{\aleph_1}\gimel\tau^t_{\aleph_2}}{\gimel\mu^t_{\aleph_1}\gimel\mu^t_{\aleph_2}+\gimel\nu^t_{\aleph_1}\gimel\nu^t_{\aleph_2}+\gimel\tau^t_{\aleph_1}\gimel\tau^t_{\aleph_2}}\right)^{\mho}} \times \left(1-\left(1-\left(1-\left(1-\gimel\mu^t_{\aleph_1}-\gimel\nu^t_{\aleph_1}-\gimel\tau^t_{\aleph_1}\right)\left(1-\gimel\mu^t_{\aleph_2}-\gimel\nu^t_{\aleph_2}-\gimel\tau^t_{\aleph_2}\right)\right)\right)^{\mho}\right)}
\end{array} \right)$$

$$= \left(\begin{array}{c}
\sqrt[t]{\frac{\gimel\mu^{t\mho}_{\aleph_1}\gimel\mu^{t\mho}_{\aleph_2}}{\gimel\mu^{t\mho}_{\aleph_1}\gimel\mu^{t\mho}_{\aleph_2}+\gimel\nu^{t\mho}_{\aleph_1}\gimel\nu^{t\mho}_{\aleph_2}+\gimel\tau^{t\mho}_{\aleph_1}\gimel\tau^{t\mho}_{\aleph_2}}\left(1-\left(1-\gimel\mu^t_{\aleph_1}-\gimel\nu^t_{\aleph_1}-\gimel\tau^t_{\aleph_1}\right)^{\mho}\left(1-\gimel\mu^t_{\aleph_2}-\gimel\nu^t_{\aleph_2}-\gimel\tau^t_{\aleph_2}\right)^{\mho}\right)}, \\
\sqrt[t]{\frac{\gimel\nu^{t\mho}_{\aleph_1}\gimel\nu^{t\mho}_{\aleph_2}}{\gimel\mu^{t\mho}_{\aleph_1}\gimel\mu^{t\mho}_{\aleph_2}+\gimel\nu^{t\mho}_{\aleph_1}\gimel\nu^{t\mho}_{\aleph_2}+\gimel\tau^{t\mho}_{\aleph_1}\gimel\tau^{t\mho}_{\aleph_2}}\left(1-\left(1-\gimel\mu^t_{\aleph_1}-\gimel\nu^t_{\aleph_1}-\gimel\tau^t_{\aleph_1}\right)^{\mho}\left(1-\gimel\mu^t_{\aleph_2}-\gimel\nu^t_{\aleph_2}-\gimel\tau^t_{\aleph_2}\right)^{\mho}\right)}, \\
\sqrt[t]{\frac{\gimel\tau^{t\mho}_{\aleph_1}\gimel\tau^{t\mho}_{\aleph_2}}{\gimel\mu^{t\mho}_{\aleph_1}\gimel\mu^{t\mho}_{\aleph_2}+\gimel\nu^{t\mho}_{\aleph_1}\gimel\nu^{t\mho}_{\aleph_2}+\gimel\tau^{t\mho}_{\aleph_1}\gimel\tau^{t\mho}_{\aleph_2}}\left(1-\left(1-\gimel\mu^t_{\aleph_1}-\gimel\nu^t_{\aleph_1}-\gimel\tau^t_{\aleph_1}\right)^{\mho}\left(1-\gimel\mu^t_{\aleph_2}-\gimel\nu^t_{\aleph_2}-\gimel\tau^t_{\aleph_2}\right)^{\mho}\right)},
\end{array} \right)$$

and

$$
\begin{aligned}
&(\mho * \boldsymbol{\aleph}_1)\,\tilde{\oplus}\,(\mho * \boldsymbol{\aleph}_2)\\
&= \left(\begin{array}{l}
\sqrt[t]{\left(\frac{\gimel\mu^{t\mho}_{\aleph_1}}{\gimel\mu^{t\mho}_{\aleph_1}+\gimel\nu^{t\mho}_{\aleph_1}+\gimel\tau^{t\mho}_{\aleph_1}}\right)\times\left(1-\left(1-\gimel\mu^{t}_{\aleph_1}-\gimel\nu^{t}_{\aleph_1}-\gimel\tau^{t}_{\aleph_1}\right)^{\mho}\right)},\\
\sqrt[t]{\left(\frac{\gimel\nu^{t\mho}_{\aleph_1}}{\gimel\mu^{t\mho}_{\aleph_1}+\gimel\nu^{t\mho}_{\aleph_1}+\gimel\tau^{t\mho}_{\aleph_1}}\right)\times\left(1-\left(1-\gimel\mu^{t}_{\aleph_1}-\gimel\nu^{t}_{\aleph_1}-\gimel\tau^{t}_{\aleph_1}\right)^{\mho}\right)},\\
\sqrt[t]{\left(\frac{\gimel\tau^{t\mho}_{\aleph_1}}{\gimel\mu^{t\mho}_{\aleph_1}+\gimel\nu^{t\mho}_{\aleph_1}+\gimel\tau^{t\mho}_{\aleph_1}}\right)\times\left(1-\left(1-\gimel\mu^{t}_{\aleph_1}-\gimel\nu^{t}_{\aleph_1}-\gimel\tau^{t}_{\aleph_1}\right)^{\mho}\right)}\tilde{\oplus}\\
\sqrt[t]{\left(\frac{\gimel\mu^{t\mho}_{\aleph_2}}{\gimel\mu^{t\mho}_{\aleph_2}+\gimel\nu^{t\mho}_{\aleph_2}+\gimel\tau^{t\mho}_{\aleph_2}}\right)\times\left(1-\left(1-\gimel\mu^{t}_{\aleph_2}-\gimel\nu^{t}_{\aleph_2}-\gimel\tau^{t}_{\aleph_2}\right)^{\mho}\right)},\\
\sqrt[t]{\left(\frac{\gimel\nu^{t\mho}_{\aleph_2}}{\gimel\mu^{t\mho}_{\aleph_2}+\gimel\nu^{t\mho}_{\aleph_2}+\gimel\tau^{t\mho}_{\aleph_2}}\right)\times\left(1-\left(1-\gimel\mu^{t}_{\aleph_2}-\gimel\nu^{t}_{\aleph_2}-\gimel\tau^{t}_{\aleph_2}\right)^{\mho}\right)},\\
\sqrt[t]{\left(\frac{\gimel\tau^{t\mho}_{\aleph_2}}{\gimel\mu^{t\mho}_{\aleph_2}+\gimel\nu^{t\mho}_{\aleph_2}+\gimel\tau^{t\mho}_{\aleph_2}}\right)\times\left(1-\left(1-\gimel\mu^{t}_{\aleph_2}-\gimel\nu^{t}_{\aleph_2}-\gimel\tau^{t}_{\aleph_2}\right)^{\mho}\right)}
\end{array}\right)
\end{aligned}
$$

$$
= \left(\begin{array}{l}
\sqrt[t]{\begin{array}{r}\frac{\frac{\gimel\mu^{t\mho}_{\aleph_1}}{\gimel\mu^{t\mho}_{\aleph_1}+\gimel\nu^{t\mho}_{\aleph_1}+\gimel\tau^{t\mho}_{\aleph_1}}\times\frac{\gimel\mu^{t\mho}_{\aleph_2}}{\gimel\mu^{t\mho}_{\aleph_2}+\gimel\nu^{t\mho}_{\aleph_2}+\gimel\tau^{t\mho}_{\aleph_2}}}{\frac{\gimel\mu^{t\mho}_{\aleph_1}}{\gimel\mu^{t\mho}_{\aleph_1}+\gimel\nu^{t\mho}_{\aleph_1}+\gimel\tau^{t\mho}_{\aleph_1}}\times\frac{\gimel\mu^{t\mho}_{\aleph_2}}{\gimel\mu^{t\mho}_{\aleph_2}+\gimel\nu^{t\mho}_{\aleph_2}+\gimel\tau^{t\mho}_{\aleph_2}}+\frac{\gimel\nu^{t\mho}_{\aleph_1}}{\gimel\mu^{t\mho}_{\aleph_1}+\gimel\nu^{t\mho}_{\aleph_1}+\gimel\tau^{t\mho}_{\aleph_1}}\times\frac{\gimel\nu^{t\mho}_{\aleph_2}}{\gimel\mu^{t\mho}_{\aleph_2}+\gimel\nu^{t\mho}_{\aleph_2}+\gimel\tau^{t\mho}_{\aleph_2}}+\frac{\gimel\tau^{t\mho}_{\aleph_1}}{\gimel\mu^{t\mho}_{\aleph_1}+\gimel\nu^{t\mho}_{\aleph_1}+\gimel\tau^{t\mho}_{\aleph_1}}\times\frac{\gimel\tau^{t\mho}_{\aleph_2}}{\gimel\mu^{t\mho}_{\aleph_2}+\gimel\nu^{t\mho}_{\aleph_2}+\gimel\tau^{t\mho}_{\aleph_2}}}\\
\times\left(1-\left(1-\left(1-\left(1-\gimel\mu^{t}_{\aleph_1}-\gimel\nu^{t}_{\aleph_1}-\gimel\tau^{t}_{\aleph_1}\right)^{\mho}\right)\right)\times\left(1-\left(1-\left(1-\gimel\mu^{t}_{\aleph_2}-\gimel\nu^{t}_{\aleph_2}-\gimel\tau^{t}_{\aleph_2}\right)^{\mho}\right)\right)\right)\end{array}},\\
\sqrt[t]{\begin{array}{r}\frac{\frac{\gimel\nu^{t\mho}_{\aleph_1}}{\gimel\mu^{t\mho}_{\aleph_1}+\gimel\nu^{t\mho}_{\aleph_1}+\gimel\tau^{t\mho}_{\aleph_1}}\times\frac{\gimel\nu^{t\mho}_{\aleph_2}}{\gimel\mu^{t\mho}_{\aleph_2}+\gimel\nu^{t\mho}_{\aleph_2}+\gimel\tau^{t\mho}_{\aleph_2}}}{\frac{\gimel\mu^{t\mho}_{\aleph_1}}{\gimel\mu^{t\mho}_{\aleph_1}+\gimel\nu^{t\mho}_{\aleph_1}+\gimel\tau^{t\mho}_{\aleph_1}}\times\frac{\gimel\mu^{t\mho}_{\aleph_2}}{\gimel\mu^{t\mho}_{\aleph_2}+\gimel\nu^{t\mho}_{\aleph_2}+\gimel\tau^{t\mho}_{\aleph_2}}+\frac{\gimel\nu^{t\mho}_{\aleph_1}}{\gimel\mu^{t\mho}_{\aleph_1}+\gimel\nu^{t\mho}_{\aleph_1}+\gimel\tau^{t\mho}_{\aleph_1}}\times\frac{\gimel\nu^{t\mho}_{\aleph_2}}{\gimel\mu^{t\mho}_{\aleph_2}+\gimel\nu^{t\mho}_{\aleph_2}+\gimel\tau^{t\mho}_{\aleph_2}}+\frac{\gimel\tau^{t\mho}_{\aleph_1}}{\gimel\mu^{t\mho}_{\aleph_1}+\gimel\nu^{t\mho}_{\aleph_1}+\gimel\tau^{t\mho}_{\aleph_1}}\times\frac{\gimel\tau^{t\mho}_{\aleph_2}}{\gimel\mu^{t\mho}_{\aleph_2}+\gimel\nu^{t\mho}_{\aleph_2}+\gimel\tau^{t\mho}_{\aleph_2}}}\\
\times\left(1-\left(1-\left(1-\left(1-\gimel\mu^{t}_{\aleph_1}-\gimel\nu^{t}_{\aleph_1}-\gimel\tau^{t}_{\aleph_1}\right)^{\mho}\right)\right)\times\left(1-\left(1-\left(1-\gimel\mu^{t}_{\aleph_2}-\gimel\nu^{t}_{\aleph_2}-\gimel\tau^{t}_{\aleph_2}\right)^{\mho}\right)\right)\right)\end{array}},\\
\sqrt[t]{\begin{array}{r}\frac{\frac{\gimel\tau^{t\mho}_{\aleph_1}}{\gimel\mu^{t\mho}_{\aleph_1}+\gimel\nu^{t\mho}_{\aleph_1}+\gimel\tau^{t\mho}_{\aleph_1}}\times\frac{\gimel\tau^{t\mho}_{\aleph_2}}{\gimel\mu^{t\mho}_{\aleph_2}+\gimel\nu^{t\mho}_{\aleph_2}+\gimel\tau^{t\mho}_{\aleph_2}}}{\frac{\gimel\mu^{t\mho}_{\aleph_1}}{\gimel\mu^{t\mho}_{\aleph_1}+\gimel\nu^{t\mho}_{\aleph_1}+\gimel\tau^{t\mho}_{\aleph_1}}\times\frac{\gimel\mu^{t\mho}_{\aleph_2}}{\gimel\mu^{t\mho}_{\aleph_2}+\gimel\nu^{t\mho}_{\aleph_2}+\gimel\tau^{t\mho}_{\aleph_2}}+\frac{\gimel\nu^{t\mho}_{\aleph_1}}{\gimel\mu^{t\mho}_{\aleph_1}+\gimel\nu^{t\mho}_{\aleph_1}+\gimel\tau^{t\mho}_{\aleph_1}}\times\frac{\gimel\nu^{t\mho}_{\aleph_2}}{\gimel\mu^{t\mho}_{\aleph_2}+\gimel\nu^{t\mho}_{\aleph_2}+\gimel\tau^{t\mho}_{\aleph_2}}+\frac{\gimel\tau^{t\mho}_{\aleph_1}}{\gimel\mu^{t\mho}_{\aleph_1}+\gimel\nu^{t\mho}_{\aleph_1}+\gimel\tau^{t\mho}_{\aleph_1}}\times\frac{\gimel\tau^{t\mho}_{\aleph_2}}{\gimel\mu^{t\mho}_{\aleph_2}+\gimel\nu^{t\mho}_{\aleph_2}+\gimel\tau^{t\mho}_{\aleph_2}}}\\
\times\left(1-\left(1-\left(1-\left(1-\gimel\mu^{t}_{\aleph_1}-\gimel\nu^{t}_{\aleph_1}-\gimel\tau^{t}_{\aleph_1}\right)^{\mho}\right)\right)\times\left(1-\left(1-\left(1-\gimel\mu^{t}_{\aleph_2}-\gimel\nu^{t}_{\aleph_2}-\gimel\tau^{t}_{\aleph_2}\right)^{\mho}\right)\right)\right)\end{array}}
\end{array}\right)
$$

$$
= \begin{pmatrix}
\sqrt[t]{\frac{\gimel\mu^{t\mho}_{\aleph_1}\gimel\mu^{t\mho}_{\aleph_2}}{\gimel\mu^{t\mho}_{\aleph_1}\gimel\mu^{t\mho}_{\aleph_2}+\gimel\nu^{t\mho}_{\aleph_1}\gimel\nu^{t\mho}_{\aleph_2}+\gimel\tau^{t\mho}_{\aleph_1}\gimel\tau^{t\mho}_{\aleph_2}}\left(1-\left(1-\gimel\mu^{t}_{\aleph_1}-\gimel\nu^{t}_{\aleph_1}-\gimel\tau^{t}_{\aleph_1}\right)^{\mho}\left(1-\gimel\mu^{t}_{\aleph_2}-\gimel\nu^{t}_{\aleph_2}-\gimel\tau^{t}_{\aleph_2}\right)^{\mho}\right)}, \\
\sqrt[t]{\frac{\gimel\nu^{t\mho}_{\aleph_1}\gimel\nu^{t\mho}_{\aleph_2}}{\gimel\mu^{t\mho}_{\aleph_1}\gimel\mu^{t\mho}_{\aleph_2}+\gimel\nu^{t\mho}_{\aleph_1}\gimel\nu^{t\mho}_{\aleph_2}+\gimel\tau^{t\mho}_{\aleph_1}\gimel\tau^{t\mho}_{\aleph_2}}\left(1-\left(1-\gimel\mu^{t}_{\aleph_1}-\gimel\nu^{t}_{\aleph_1}-\gimel\tau^{t}_{\aleph_1}\right)^{\mho}\left(1-\gimel\mu^{t}_{\aleph_2}-\gimel\nu^{t}_{\aleph_2}-\gimel\tau^{t}_{\aleph_2}\right)^{\mho}\right)}, \\
\sqrt[t]{\frac{\gimel\tau^{t\mho}_{\aleph_1}\gimel\tau^{t\mho}_{\aleph_2}}{\gimel\mu^{t\mho}_{\aleph_1}\gimel\mu^{t\mho}_{\aleph_2}+\gimel\nu^{t\mho}_{\aleph_1}\gimel\nu^{t\mho}_{\aleph_2}+\gimel\tau^{t\mho}_{\aleph_1}\gimel\tau^{t\mho}_{\aleph_2}}\left(1-\left(1-\gimel\mu^{t}_{\aleph_1}-\gimel\nu^{t}_{\aleph_1}-\gimel\tau^{t}_{\aleph_1}\right)^{\mho}\left(1-\gimel\mu^{t}_{\aleph_2}-\gimel\nu^{t}_{\aleph_2}-\gimel\tau^{t}_{\aleph_2}\right)^{\mho}\right)}
\end{pmatrix}.
$$

Hence, $\mho * (\aleph_1 \tilde{\oplus} \aleph_2) = (\mho * \aleph_1)\,\tilde{\oplus}\,(\mho * \aleph_2)$.
(iii) $(\mho_1 * \aleph_1)\,\tilde{\oplus}\,(\mho_2 * \aleph_1)$

$$
= \begin{pmatrix}
\sqrt[t]{\left(\frac{\gimel\mu^{t\mho_1}_{\aleph_1}}{\gimel\mu^{t\mho_1}_{\aleph_1}+\gimel\nu^{t\mho}_{\aleph_1}+\gimel\tau^{t\mho}_{\aleph_1}}\right)\times\left(1-\left(1-\gimel\mu^{t}_{\aleph_1}-\gimel\nu^{t}_{\aleph_1}-\gimel\tau^{t}_{\aleph_1}\right)^{\mho_1}\right)}, \\
\sqrt[t]{\left(\frac{\gimel\nu^{t\mho_1}_{\aleph_1}}{\gimel\mu^{t\mho_1}_{\aleph_1}+\gimel\nu^{t\mho}_{\aleph_1}+\gimel\tau^{t\mho}_{\aleph_1}}\right)\times\left(1-\left(1-\gimel\mu^{t}_{\aleph_1}-\gimel\nu^{t}_{\aleph_1}-\gimel\tau^{t}_{\aleph_1}\right)^{\mho_1}\right)}, \\
\sqrt[t]{\left(\frac{\gimel\tau^{t\mho_1}_{\aleph_1}}{\gimel\mu^{t\mho_1}_{\aleph_1}+\gimel\nu^{t\mho}_{\aleph_1}+\gimel\tau^{t\mho}_{\aleph_1}}\right)\times\left(1-\left(1-\gimel\mu^{t}_{\aleph_1}-\gimel\nu^{t}_{\aleph_1}-\gimel\tau^{t}_{\aleph_1}\right)^{\mho_1}\right)\Bigg\rangle}\tilde{\oplus} \\
\sqrt[t]{\Bigg\langle\left(\frac{\gimel\mu^{t\mho_2}_{\aleph_1}}{\gimel\mu^{t\mho_2}_{\aleph_1}+\gimel\nu^{t\mho}_{\aleph_1}+\gimel\tau^{t\mho}_{\aleph_1}}\right)\times\left(1-\left(1-\gimel\mu^{t}_{\aleph_1}-\gimel\nu^{t}_{\aleph_1}-\gimel\tau^{t}_{\aleph_1}\right)^{\mho_2}\right)}, \\
\sqrt[t]{\left(\frac{\gimel\nu^{t\mho_2}_{\aleph_1}}{\gimel\mu^{t\mho_2}_{\aleph_1}+\gimel\nu^{t\mho}_{\aleph_1}+\gimel\tau^{t\mho}_{\aleph_1}}\right)\times\left(1-\left(1-\gimel\mu^{t}_{\aleph_1}-\gimel\nu^{t}_{\aleph_1}-\gimel\tau^{t}_{\aleph_1}\right)^{\mho_2}\right)}, \\
\sqrt[t]{\left(\frac{\gimel\tau^{t\mho_2}_{\aleph_1}}{\gimel\mu^{t\mho_2}_{\aleph_1}+\gimel\nu^{t\mho}_{\aleph_1}+\gimel\tau^{t\mho}_{\aleph_1}}\right)\times\left(1-\left(1-\gimel\mu^{t}_{\aleph_1}-\gimel\nu^{t}_{\aleph_1}-\gimel\tau^{t}_{\aleph_1}\right)^{\mho_2}\right)}
\end{pmatrix}
$$

$$
= \begin{pmatrix}
\sqrt[t]{\left(\frac{\gimel\mu^{t\mho_1+\mho_2}_{\aleph_1}}{\gimel\mu^{t\mho_1+\mho_2}_{\aleph_1}+\gimel\nu^{t\mho}_{\aleph_1}+\gimel\tau^{t\mho}_{\aleph_1}}\right)\times\left(1-\left(1-\gimel\mu^{t}_{\aleph_1}-\gimel\nu^{t}_{\aleph_1}-\gimel\tau^{t}_{\aleph_1}\right)^{\mho_1+\mho_2}\right)}, \\
\sqrt[t]{\left(\frac{\gimel\nu^{t\mho_1+\mho_2}_{\aleph_1}}{\gimel\mu^{t\mho_1+\mho_2}_{\aleph_1}+\gimel\nu^{t\mho}_{\aleph_1}+\gimel\tau^{t\mho}_{\aleph_1}}\right)\times\left(1-\left(1-\gimel\mu^{t}_{\aleph_1}-\gimel\nu^{t}_{\aleph_1}-\gimel\tau^{t}_{\aleph_1}\right)^{\mho_1+\mho_2}\right)}, \\
\sqrt[t]{\left(\frac{\gimel\tau^{t\mho_1+\mho_2}_{\aleph_1}}{\gimel\mu^{t\mho_1+\mho_2}_{\aleph_1}+\gimel\nu^{t\mho}_{\aleph_1}+\gimel\tau^{t\mho}_{\aleph_1}}\right)\times\left(1-\left(1-\gimel\mu^{t}_{\aleph_1}-\gimel\nu^{t}_{\aleph_1}-\gimel\tau^{t}_{\aleph_1}\right)^{\mho_1+\mho_2}\right)}
\end{pmatrix}
$$

and

$(\mho_1 + \mho_2) * \aleph_1$

$$= \begin{pmatrix} \sqrt[t]{\left(\frac{{}^{\gimel}\mu'^{\mho_1+\mho_2}_{\aleph_1}}{{}^{\gimel}\mu'^{\mho_1+\mho_2}_{\aleph_1}+{}^{\gimel}\nu'^{\mho}_{\aleph_1}+{}^{\gimel}\tau'^{\mho}_{\aleph_1}}\right) \times \left(1-\left(1-{}^{\gimel}\mu^{t}_{\aleph_1}-{}^{\gimel}\nu^{t}_{\aleph_1}-{}^{\gimel}\tau^{t}_{\aleph_1}\right)^{\mho_1+\mho_2}\right)}, \\ \sqrt[t]{\left(\frac{{}^{\gimel}\nu'^{\mho_1+\mho_2}_{\aleph_1}}{{}^{\gimel}\mu'^{\mho_1+\mho_2}_{\aleph_1}+{}^{\gimel}\nu'^{\mho}_{\aleph_1}+{}^{\gimel}\tau'^{\mho}_{\aleph_1}}\right) \times \left(1-\left(1-{}^{\gimel}\mu^{t}_{\aleph_1}-{}^{\gimel}\nu^{t}_{\aleph_1}-{}^{\gimel}\tau^{t}_{\aleph_1}\right)^{\mho_1+\mho_2}\right)}, \\ \sqrt[t]{\left(\frac{{}^{\gimel}\tau'^{\mho_1+\mho_2}_{\aleph_1}}{{}^{\gimel}\mu'^{\mho_1+\mho_2}_{\aleph_1}+{}^{\gimel}\nu'^{\mho}_{\aleph_1}+{}^{\gimel}\tau'^{\mho}_{\aleph_1}}\right) \times \left(1-\left(1-{}^{\gimel}\mu^{t}_{\aleph_1}-{}^{\gimel}\nu^{t}_{\aleph_1}-{}^{\gimel}\tau^{t}_{\aleph_1}\right)^{\mho_1+\mho_2}\right)} \end{pmatrix}.$$

Hence, $(\mho_1 + \mho_2) * \aleph_1 = (\mho_1 * \aleph_1) \tilde{\oplus} (\mho_2 * \aleph_1)$. □

4. Fairly AOs for T-SPFNs

This section addresses the evolution of fair AOs for T-SPFNs and their respective properties.

4.1. T-SPFFWA operator

Definition 4.1. Let $\beth_h = \langle {}^{\gimel}\mu_h, {}^{\gimel}\nu_h, {}^{\gimel}\tau_h \rangle$ be the jumble of T-SPFNs, and T-SPFFWA: $\mathscr{R}^n \to \mathscr{R}$, be the mapping. If

$$\text{T-SPFFWA}(\beth_1, \beth_2, \ldots \beth_e) = (\daleth_1 * \beth_1 \tilde{\oplus} \daleth_2 * \beth_2 \tilde{\oplus} \ldots, \tilde{\oplus} \daleth_e * \beth_e) \tag{4.1}$$

then the mapping T-SPFFWA is called "T-spherical fuzzy fairly weighted averaging (T-SPFFWA) operator", here $\daleth_i$ is the "weight vector" (WV) of $\beth_i$ with $\daleth_i > 0$ and $\sum_{i=1}^{e} \daleth_i = 1$.

We may also investigate T-SPFFWA using proposed operational rules, as demonstrated in the theorem that follows.

Theorem 4.2. Let $\beth_h = \langle {}^{\gimel}\mu_h, {}^{\gimel}\nu_h, {}^{\gimel}\tau_h \rangle$ be the jumble of T-SPFNs, we can also find T-SPFFWA by

$\text{T-SPFFWA}(\beth_1, \beth_2, \ldots, \beth_e)$

$$= \begin{pmatrix} \sqrt[t]{\frac{\prod_{i=1}^{e}({}^{\gimel}\mu^{t}_{i})^{\daleth_i}}{\prod_{i=1}^{e}({}^{\gimel}\mu^{t}_{i})^{\daleth_i}+\prod_{i=1}^{e}({}^{\gimel}\nu^{t}_{i})^{\daleth_i}+\prod_{i=1}^{e}({}^{\gimel}\tau^{t}_{i})^{\daleth_i}} \times \left(1-\prod_{i=1}^{e}\left(1-{}^{\gimel}\mu^{t}_{i}-{}^{\gimel}\nu^{t}_{i}-{}^{\gimel}\tau^{t}_{i}\right)^{\daleth_i}\right)}, \\ \sqrt[t]{\frac{\prod_{i=1}^{e}({}^{\gimel}\nu^{t}_{i})^{\daleth_i}}{\prod_{i=1}^{e}({}^{\gimel}\mu^{t}_{i})^{\daleth_i}+\prod_{i=1}^{e}({}^{\gimel}\nu^{t}_{i})^{\daleth_i}+\prod_{i=1}^{e}({}^{\gimel}\tau^{t}_{i})^{\daleth_i}} \times \left(1-\prod_{i=1}^{e}\left(1-{}^{\gimel}\mu^{t}_{i}-{}^{\gimel}\nu^{t}_{i}-{}^{\gimel}\tau^{t}_{i}\right)^{\daleth_i}\right)}, \\ \sqrt[t]{\frac{\prod_{i=1}^{e}({}^{\gimel}\tau^{t}_{i})^{\daleth_i}}{\prod_{i=1}^{e}({}^{\gimel}\mu^{t}_{i})^{\daleth_i}+\prod_{i=1}^{e}({}^{\gimel}\nu^{t}_{i})^{\daleth_i}+\prod_{i=1}^{e}({}^{\gimel}\tau^{t}_{i})^{\daleth_i}} \times \left(1-\prod_{i=1}^{e}\left(1-{}^{\gimel}\mu^{t}_{i}-{}^{\gimel}\nu^{t}_{i}-{}^{\gimel}\tau^{t}_{i}\right)^{\daleth_i}\right)} \end{pmatrix}$$

where $\daleth_i$ is the WV of $\beth_i$ with $\daleth_i > 0$ and $\sum_{i=1}^{e} \daleth_i = 1$.

Proof. We will start this prove using mathematical induction.
For $e = 1$, we have $\aleph_1 = \langle {}^{\gimel}\mu_1, {}^{\gimel}\nu_1, {}^{\gimel}\tau_1 \rangle$ and $\daleth = 1$.

$$\text{T-SPFFWA}(\aleph_1) = \daleth_1 * \aleph_1 = \begin{pmatrix} \sqrt[t]{\frac{(\gimel\mu^t{}_1)^{\daleth_1}}{(\gimel\mu^t{}_1)^{\daleth_1}+(\gimel\nu^t{}_1)^{\daleth}_1+(\gimel\tau^t{}_1)^{\daleth_1}} \times \left(1-\left(1-\gimel\mu^t{}_1-\gimel\nu^t{}_1-\gimel\tau^t{}_1\right)^{\daleth_1}\right)}, \\ \sqrt[t]{\frac{(\gimel\nu^t{}_1)^{\daleth_1}}{(\gimel\mu^t{}_1)^{\daleth_1}+(\gimel\nu^t{}_1)^{\daleth_1}+(\gimel\tau^t{}_1)^{\daleth_1}} \times \left(1-\left(1-\gimel\mu^t{}_1-\gimel\nu^t{}_1-\gimel\tau^t{}_1\right)^{\daleth_1}\right)}, \\ \sqrt[t]{\frac{(\gimel\tau^t{}_1)^{\daleth_1}}{(\gimel\mu^t{}_1)^{\daleth_1}+(\gimel\nu^t{}_1)^{\daleth_1}+(\gimel\tau^t{}_1)^{\daleth_1}} \times \left(1-\left(1-\gimel\mu^t{}_1-\gimel\nu^t{}_1-\gimel\tau^t{}_1\right)^{\daleth_1}\right)} \end{pmatrix}.$$

Theorem is valid for $e = 1$, now we consider this is valid for $e = g$, i.e.,

$$\text{T-SPFFWA}(\aleph_1, \aleph_2, \ldots, \aleph_g) = \begin{pmatrix} \sqrt[t]{\frac{\prod_{i=1}^{g}(\gimel\mu^t{}_i)^{\daleth_i}}{\prod_{i=1}^{g}(\gimel\mu^t{}_i)^{\daleth_i}+\prod_{i=1}^{g}(\gimel\nu^t{}_i)^{\daleth_i}+\prod_{i=1}^{g}(\gimel\tau^t{}_i)^{\daleth_i}} \times \left(1-\prod_{i=1}^{g}\left(1-\gimel\mu^t{}_i-\gimel\nu^t{}_i-\gimel\tau^t{}_i\right)^{\daleth_i}\right)}, \\ \sqrt[t]{\frac{\prod_{i=1}^{g}(\gimel\nu^t{}_i)^{\daleth_i}}{\prod_{i=1}^{g}(\gimel\mu^t{}_i)^{\daleth_i}+\prod_{i=1}^{g}(\gimel\nu^t{}_i)^{\daleth_i}+\prod_{i=1}^{g}(\gimel\tau^t{}_i)^{\daleth_i}} \times \left(1-\prod_{i=1}^{g}\left(1-\gimel\mu^t{}_i-\gimel\nu^t{}_i-\gimel\tau^t{}_i\right)^{\daleth_i}\right)}, \\ \sqrt[t]{\frac{\prod_{i=1}^{g}(\gimel\tau^t{}_i)^{\daleth_i}}{\prod_{i=1}^{g}(\gimel\mu^t{}_i)^{\daleth_i}+\prod_{i=1}^{g}(\gimel\nu^t{}_i)^{\daleth_i}+\prod_{i=1}^{g}(\gimel\tau^t{}_i)^{\daleth_i}} \times \left(1-\prod_{i=1}^{g}\left(1-\gimel\mu^t{}_i-\gimel\nu^t{}_i-\gimel\tau^t{}_i\right)^{\daleth_i}\right)} \end{pmatrix}.$$

We will prove for $e = g + 1$.
$\text{T-SPFFWA}(\aleph_1, \aleph_2, \ldots, \aleph_{g+1}) = \text{T-SPFFWA}(\aleph_1, \aleph_2, \ldots, \aleph_g)\tilde{\oplus}(\daleth_{g+1} * \aleph_{g+1})$

$$= \begin{pmatrix} \sqrt[t]{\frac{\prod_{i=1}^{g}(\gimel\mu^t{}_i)^{\daleth_i}}{\prod_{i=1}^{g}(\gimel\mu^t{}_i)^{\daleth_i}+\prod_{i=1}^{g}(\gimel\nu^t{}_i)^{\daleth_i}+\prod_{i=1}^{g}(\gimel\tau^t{}_i)^{\daleth_i}} \times \left(1-\prod_{i=1}^{g}\left(1-\gimel\mu^t{}_i-\gimel\nu^t{}_i-\gimel\tau^t{}_i\right)^{\daleth_i}\right)}, \\ \sqrt[t]{\frac{\prod_{i=1}^{g}(\gimel\nu^t{}_i)^{\daleth_i}}{\prod_{i=1}^{g}(\gimel\mu^t{}_i)^{\daleth_i}+\prod_{i=1}^{g}(\gimel\nu^t{}_i)^{\daleth_i}+\prod_{i=1}^{g}(\gimel\tau^t{}_i)^{\daleth_i}} \times \left(1-\prod_{i=1}^{g}\left(1-\gimel\mu^t{}_i-\gimel\nu^t{}_i-\gimel\tau^t{}_i\right)^{\daleth_i}\right)}, \\ \sqrt[t]{\frac{\prod_{i=1}^{g}(\gimel\tau^t{}_i)^{\daleth_i}}{\prod_{i=1}^{g}(\gimel\mu^t{}_i)^{\daleth_i}+\prod_{i=1}^{g}(\gimel\nu^t{}_i)^{\daleth_i}+\prod_{i=1}^{g}(\gimel\tau^t{}_i)^{\daleth_i}} \times \left(1-\prod_{i=1}^{g}\left(1-\gimel\mu^t{}_i-\gimel\nu^t{}_i-\gimel\tau^t{}_i\right)^{\daleth_i}\right)} \end{pmatrix} \tilde{\oplus}$$

$$\begin{pmatrix} \sqrt[t]{\left(\frac{\gimel\mu^{t\,\daleth_{g+1}}_{\aleph_{g+1}}}{\gimel\mu^{t\,\daleth_{g+1}}_{\aleph_{g+1}}+\gimel\nu^{t\,\daleth_{g+1}}_{\aleph_{g+1}}+\gimel\tau^{t\,\daleth_{g+1}}_{\aleph_{g+1}}}\right) \times \left(1-\left(1-\gimel\mu^t{}_{\aleph_{g+1}}-\gimel\nu^t{}_{\aleph_{g+1}}-\gimel\tau^t{}_{\aleph_{g+1}}\right)^{\daleth_{g+1}}\right)}, \\ \sqrt[t]{\left(\frac{\gimel\nu^{t\,\daleth_{g+1}}_{\aleph_{g+1}}}{\gimel\mu^{t\,\daleth_{g+1}}_{\aleph_{g+1}}+\gimel\nu^{t\,\daleth_{g+1}}_{\aleph_{g+1}}+\gimel\tau^{t\,\daleth_{g+1}}_{\aleph_{g+1}}}\right) \times \left(1-\left(1-\gimel\mu^t{}_{\aleph_{g+1}}-\gimel\nu^t{}_{\aleph_{g+1}}-\gimel\tau^t{}_{\aleph_{g+1}}\right)^{\daleth_{g+1}}\right)}, \\ \sqrt[t]{\left(\frac{\gimel\tau^{t\,\daleth_{g+1}}_{\aleph_{g+1}}}{\gimel\mu^{t\,\daleth_{g+1}}_{\aleph_{g+1}}+\gimel\nu^{t\,\daleth_{g+1}}_{\aleph_{g+1}}+\gimel\tau^{t\,\daleth_{g+1}}_{\aleph_{g+1}}}\right) \times \left(1-\left(1-\gimel\mu^t{}_{\aleph_{g+1}}-\gimel\nu^t{}_{\aleph_{g+1}}-\gimel\tau^t{}_{\aleph_{g+1}}\right)^{\daleth_{g+1}}\right)} \end{pmatrix}$$

$$= \left(\begin{array}{l} \sqrt[t]{\frac{\prod_{i=1}^{g}\left(\gimel\mu^{t}{}_{i}\right)^{\daleth_i} \times \left(\gimel\mu^{t}{}_{g+1}\right)^{\daleth_{g+1}}}{\prod_{i=1}^{g}\left(\gimel\mu^{t}{}_{i}\right)^{\daleth_i} \times \left(\gimel\mu^{t}{}_{g+1}\right)^{\daleth_{g+1}} + \prod_{i=1}^{g}\left(\gimel\nu^{t}{}_{i}\right)^{\daleth_i} \times \left(\gimel\nu^{t}{}_{g+1}\right)^{\daleth_{g+1}} + \prod_{i=1}^{g}\left(\gimel\tau^{t}{}_{i}\right)^{\daleth_i}\left(\gimel\tau^{t}{}_{g+1}\right)^{\daleth_{g+1}}} \times \left(1 - \prod_{i=1}^{g}\left(1 - \gimel\mu^{t}{}_{i} - \gimel\nu^{t}{}_{i} - \gimel\tau^{t}{}_{i}\right)^{\daleth_i} \times \left(1 - \gimel\mu^{t}{}_{g+1} - \gimel\nu^{t}{}_{g+1} - \gimel\tau^{t}{}_{g+1}\right)^{\daleth_{g+1}}\right)}, \\ \sqrt[t]{\frac{\prod_{i=1}^{g}\left(\gimel\nu^{t}{}_{i}\right)^{\daleth_i} \times \left(\gimel\nu^{t}{}_{g+1}\right)^{\daleth_{g+1}}}{\prod_{i=1}^{g}\left(\gimel\mu^{t}{}_{i}\right)^{\daleth_i} \times \left(\gimel\mu^{t}{}_{g+1}\right)^{\daleth_{g+1}} + \prod_{i=1}^{g}\left(\gimel\nu^{t}{}_{i}\right)^{\daleth_i} \times \left(\gimel\nu^{t}{}_{g+1}\right)^{\daleth_{g+1}} + \prod_{i=1}^{g}\left(\gimel\tau^{t}{}_{i}\right)^{\daleth_i}\left(\gimel\tau^{t}{}_{g+1}\right)^{\daleth_{g+1}}} \times \left(1 - \prod_{i=1}^{g}\left(1 - \gimel\mu^{t}{}_{i} - \gimel\nu^{t}{}_{i} - \gimel\tau^{t}{}_{i}\right)^{\daleth_i} \times \left(1 - \gimel\mu^{t}{}_{g+1} - \gimel\nu^{t}{}_{g+1} - \gimel\tau^{t}{}_{g+1}\right)^{\daleth_{g+1}}\right)}, \\ \sqrt[t]{\frac{\prod_{i=1}^{g}\left(\gimel\tau^{t}{}_{i}\right)^{\daleth_i} \times \left(\gimel\tau^{t}{}_{g+1}\right)^{\daleth_{g+1}}}{\prod_{i=1}^{g}\left(\gimel\mu^{t}{}_{i}\right)^{\daleth_i} \times \left(\gimel\mu^{t}{}_{g+1}\right)^{\daleth_{g+1}} + \prod_{i=1}^{g}\left(\gimel\nu^{t}{}_{i}\right)^{\daleth_i} \times \left(\gimel\nu^{t}{}_{g+1}\right)^{\daleth_{g+1}} + \prod_{i=1}^{g}\left(\gimel\tau^{t}{}_{i}\right)^{\daleth_i}\left(\gimel\tau^{t}{}_{g+1}\right)^{\daleth_{g+1}}} \times \left(1 - \prod_{i=1}^{g}\left(1 - \gimel\mu^{t}{}_{i} - \gimel\nu^{t}{}_{i} - \gimel\tau^{t}{}_{i}\right)^{\daleth_i} \times \left(1 - \gimel\mu^{t}{}_{g+1} - \gimel\nu^{t}{}_{g+1} - \gimel\tau^{t}{}_{g+1}\right)^{\daleth_{g+1}}\right)} \end{array} \right)$$

$$= \left(\begin{array}{l} \sqrt[t]{\frac{\prod_{i=1}^{g+1}\left(\gimel\mu^{t}{}_{i}\right)^{\daleth_i}}{\prod_{i=1}^{g+1}\left(\gimel\mu^{t}{}_{i}\right)^{\daleth_i} + \prod_{i=1}^{g+1}\left(\gimel\nu^{t}{}_{i}\right)^{\daleth_i} + \prod_{i=1}^{g+1}\left(\gimel\tau^{t}{}_{i}\right)^{\daleth_i}} \times \left(1 - \prod_{i=1}^{g+1}\left(1 - \gimel\mu^{t}{}_{i} - \gimel\nu^{t}{}_{i} - \gimel\tau^{t}{}_{i}\right)^{\daleth_i}\right)}, \\ \sqrt[t]{\frac{\prod_{i=1}^{g+1}\left(\gimel\nu^{t}{}_{i}\right)^{\daleth_i}}{\prod_{i=1}^{g+1}\left(\gimel\mu^{t}{}_{i}\right)^{\daleth_i} + \prod_{i=1}^{g+1}\left(\gimel\nu^{t}{}_{i}\right)^{\daleth_i} + \prod_{i=1}^{g+1}\left(\gimel\tau^{t}{}_{i}\right)^{\daleth_i}} \times \left(1 - \prod_{i=1}^{g+1}\left(1 - \gimel\mu^{t}{}_{i} - \gimel\nu^{t}{}_{i} - \gimel\tau^{t}{}_{i}\right)^{\daleth_i}\right)}, \\ \sqrt[t]{\frac{\prod_{i=1}^{g+1}\left(\gimel\tau^{t}{}_{i}\right)^{\daleth_i}}{\prod_{i=1}^{g+1}\left(\gimel\mu^{t}{}_{i}\right)^{\daleth_i} + \prod_{i=1}^{g+1}\left(\gimel\nu^{t}{}_{i}\right)^{\daleth_i} + \prod_{i=1}^{g+1}\left(\gimel\tau^{t}{}_{i}\right)^{\daleth_i}} \times \left(1 - \prod_{i=1}^{g+1}\left(1 - \gimel\mu^{t}{}_{i} - \gimel\nu^{t}{}_{i} - \gimel\tau^{t}{}_{i}\right)^{\daleth_i}\right)} \end{array} \right).$$

Therefore, the result also stands for $e = g + 1$. As a result, according to the principle of induction on 'e', the conclusion holds for any and all e. □

Example 4.3. Consider $\gimel_1 = \langle 0.415, 0.245, 0.145\rangle$, $\gimel_2 = \langle 0.339, 0.334, 0.119\rangle$ and $\gimel_3 = \langle 0.474, 0.144, 0.109\rangle$ are three T-SPFNs with WV $\daleth = (0.399, 0.276, 0.325)$, we take $t = 3$, then

$$\sqrt[t]{\frac{\prod_{i=1}^{3}\left(\gimel\mu^{t}{}_{i}\right)^{\daleth_i}}{\prod_{i=1}^{3}\left(\gimel\mu^{t}{}_{i}\right)^{\daleth_i} + \prod_{i=1}^{3}\left(\gimel\nu^{t}{}_{i}\right)^{\daleth_i} + \prod_{i=1}^{3}\left(\gimel\tau^{t}{}_{i}\right)^{\daleth_i}} \times \left(1 - \prod_{i=1}^{3}\left(1 - \gimel\mu^{t}{}_{i} - \gimel\nu^{t}{}_{i} - \gimel\tau^{t}{}_{i}\right)^{\daleth_i}\right)} = 0.426140$$

$$\sqrt[t]{\frac{\prod_{i=1}^{3}\left({}^{\gimel}\nu^{t}{}_{i}\right)^{\daleth_i}}{\prod_{i=1}^{3}\left({}^{\gimel}\mu^{t}{}_{i}\right)^{\daleth_i}+\prod_{i=1}^{3}\left({}^{\gimel}\nu^{t}{}_{i}\right)^{\daleth_i}+\prod_{i=1}^{3}\left({}^{\gimel}\tau^{t}{}_{i}\right)^{\daleth_i}}\times\left(1-\prod_{i=1}^{3}\left(1-{}^{\gimel}\mu^{t}{}_{i}-{}^{\gimel}\nu^{t}{}_{i}-{}^{\gimel}\tau^{t}{}_{i}\right)^{\daleth_i}\right)} = 0.235385$$

$$\sqrt[t]{\frac{\prod_{i=1}^{3}\left({}^{\gimel}\tau^{t}{}_{i}\right)^{\daleth_i}}{\prod_{i=1}^{3}\left({}^{\gimel}\mu^{t}{}_{i}\right)^{\daleth_i}+\prod_{i=1}^{3}\left({}^{\gimel}\nu^{t}{}_{i}\right)^{\daleth_i}+\prod_{i=1}^{3}\left({}^{\gimel}\tau^{t}{}_{i}\right)^{\daleth_i}}\times\left(1-\prod_{i=1}^{3}\left(1-{}^{\gimel}\mu^{t}{}_{i}-{}^{\gimel}\nu^{t}{}_{i}-{}^{\gimel}\tau^{t}{}_{i}\right)^{\daleth_i}\right)} = 0.132880.$$

T-SPFFWA$(\beth_1, \beth_2, \beth_3)$

$$= \left(\begin{array}{c} \sqrt[t]{\frac{\prod_{i=1}^{3}\left({}^{\gimel}\mu^{t}{}_{i}\right)^{\daleth_i}}{\prod_{i=1}^{3}\left({}^{\gimel}\mu^{t}{}_{i}\right)^{\daleth_i}+\prod_{i=1}^{3}\left({}^{\gimel}\nu^{t}{}_{i}\right)^{\daleth_i}+\prod_{i=1}^{3}\left({}^{\gimel}\tau^{t}{}_{i}\right)^{\daleth_i}}\times\left(1-\prod_{i=1}^{3}\left(1-{}^{\gimel}\mu^{t}{}_{i}-{}^{\gimel}\nu^{t}{}_{i}-{}^{\gimel}\tau^{t}{}_{i}\right)^{\daleth_i}\right)}, \\ \sqrt[t]{\frac{\prod_{i=1}^{3}\left({}^{\gimel}\nu^{t}{}_{i}\right)^{\daleth_i}}{\prod_{i=1}^{3}\left({}^{\gimel}\mu^{t}{}_{i}\right)^{\daleth_i}+\prod_{i=1}^{3}\left({}^{\gimel}\nu^{t}{}_{i}\right)^{\daleth_i}+\prod_{i=1}^{3}\left({}^{\gimel}\tau^{t}{}_{i}\right)^{\daleth_i}}\times\left(1-\prod_{i=1}^{3}\left(1-{}^{\gimel}\mu^{t}{}_{i}-{}^{\gimel}\nu^{t}{}_{i}-{}^{\gimel}\tau^{t}{}_{i}\right)^{\daleth_i}\right)}, \\ \sqrt[t]{\frac{\prod_{i=1}^{3}\left({}^{\gimel}\tau^{t}{}_{i}\right)^{\daleth_i}}{\prod_{i=1}^{3}\left({}^{\gimel}\mu^{t}{}_{i}\right)^{\daleth_i}+\prod_{i=1}^{3}\left({}^{\gimel}\nu^{t}{}_{i}\right)^{\daleth_i}+\prod_{i=1}^{3}\left({}^{\gimel}\tau^{t}{}_{i}\right)^{\daleth_i}}\times\left(1-\prod_{i=1}^{3}\left(1-{}^{\gimel}\mu^{t}{}_{i}-{}^{\gimel}\nu^{t}{}_{i}-{}^{\gimel}\tau^{t}{}_{i}\right)^{\daleth_i}\right)} \end{array}\right)$$

$$= (0.426140, 0.235385, 0.132880).$$

The proposed AO meets a few special requirements, which are presented in the context of the theorems underneath.

Theorem 4.4. Let $\beth_i = \langle {}^{\gimel}\mu_i, {}^{\gimel}\nu_i, {}^{\gimel}\tau_i \rangle$ be the jumble of T-SPFNs and $\beth_\diamond = \langle {}^{\gimel}\mu_\diamond, {}^{\gimel}\nu_\diamond, {}^{\gimel}\tau_\diamond \rangle$ be the T-SPFNs such that, $\beth_i = \beth_\diamond \forall i$. Then

$$\text{T-SPFFWA}(\beth_1, \beth_2, \ldots, \beth_e) = \beth_\diamond. \tag{4.2}$$

Proof. Given that $\aleph_i = \aleph_\diamond \forall i$, by this, ${}^{\gimel}\mu_i = {}^{\gimel}\mu_\diamond$, ${}^{\gimel}\nu_i = {}^{\gimel}\nu_\diamond$ and ${}^{\gimel}\tau_i = {}^{\gimel}\tau_\diamond \forall i$.

T-SPFFWA$(\aleph_1, \aleph_2, \ldots, \aleph_e)$

$$= \left(\begin{array}{c} \sqrt[t]{\frac{\prod_{i=1}^{e}\left({}^{\gimel}\mu^{t}{}_{i}\right)^{\daleth_i}}{\prod_{i=1}^{e}\left({}^{\gimel}\mu^{t}{}_{i}\right)^{\daleth_i}+\prod_{i=1}^{e}\left({}^{\gimel}\nu^{t}{}_{i}\right)^{\daleth_i}+\prod_{i=1}^{e}\left({}^{\gimel}\tau^{t}{}_{i}\right)^{\daleth_i}}\times\left(1-\prod_{i=1}^{e}\left(1-{}^{\gimel}\mu^{t}{}_{i}-{}^{\gimel}\nu^{t}{}_{i}-{}^{\gimel}\tau^{t}{}_{i}\right)^{\daleth_i}\right)}, \\ \sqrt[t]{\frac{\prod_{i=1}^{e}\left({}^{\gimel}\nu^{t}{}_{i}\right)^{\daleth_i}}{\prod_{i=1}^{e}\left({}^{\gimel}\mu^{t}{}_{i}\right)^{\daleth_i}+\prod_{i=1}^{e}\left({}^{\gimel}\nu^{t}{}_{i}\right)^{\daleth_i}+\prod_{i=1}^{e}\left({}^{\gimel}\tau^{t}{}_{i}\right)^{\daleth_i}}\times\left(1-\prod_{i=1}^{e}\left(1-{}^{\gimel}\mu^{t}{}_{i}-{}^{\gimel}\nu^{t}{}_{i}-{}^{\gimel}\tau^{t}{}_{i}\right)^{\daleth_i}\right)}, \\ \sqrt[t]{\frac{\prod_{i=1}^{e}\left({}^{\gimel}\tau^{t}{}_{i}\right)^{\daleth_i}}{\prod_{i=1}^{e}\left({}^{\gimel}\mu^{t}{}_{i}\right)^{\daleth_i}+\prod_{i=1}^{e}\left({}^{\gimel}\nu^{t}{}_{i}\right)^{\daleth_i}+\prod_{i=1}^{e}\left({}^{\gimel}\tau^{t}{}_{i}\right)^{\daleth_i}}\times\left(1-\prod_{i=1}^{e}\left(1-{}^{\gimel}\mu^{t}{}_{i}-{}^{\gimel}\nu^{t}{}_{i}-{}^{\gimel}\tau^{t}{}_{i}\right)^{\daleth_i}\right)} \end{array}\right)$$

$$=\left(\begin{array}{l}\sqrt[t]{\frac{\prod_{i=1}^{e}({}^{\gimel}\mu^{t}{}_{\diamond})^{\daleth_i}}{\prod_{i=1}^{e}({}^{\gimel}\mu^{t}{}_{\diamond})^{\daleth_i}+\prod_{i=1}^{e}({}^{\gimel}\nu^{t}{}_{\diamond})^{\daleth_i}+\prod_{i=1}^{e}({}^{\gimel}\tau^{t}{}_{\diamond})^{\daleth_i}}\times\left(1-\prod_{i=1}^{e}\left(1-{}^{\gimel}\mu^{t}{}_{\diamond}-{}^{\gimel}\nu^{t}{}_{\diamond}-{}^{\gimel}\tau^{t}{}_{\diamond}\right)^{\daleth_i}\right)},\\ \sqrt[t]{\frac{\prod_{i=1}^{e}({}^{\gimel}\nu^{t}{}_{\diamond})^{\daleth_i}}{\prod_{i=1}^{e}({}^{\gimel}\mu^{t}{}_{\diamond})^{\daleth_i}+\prod_{i=1}^{e}({}^{\gimel}\nu^{t}{}_{\diamond})^{\daleth_i}+\prod_{i=1}^{e}({}^{\gimel}\tau^{t}{}_{\diamond})^{\daleth_i}}\times\left(1-\prod_{i=1}^{e}\left(1-{}^{\gimel}\mu^{t}{}_{\diamond}-{}^{\gimel}\nu^{t}{}_{\diamond}-{}^{\gimel}\tau^{t}{}_{\diamond}\right)^{\daleth_i}\right)},\\ \sqrt[t]{\frac{\prod_{i=1}^{e}({}^{\gimel}\tau^{t}{}_{\diamond})^{\daleth_i}}{\prod_{i=1}^{e}({}^{\gimel}\mu^{t}{}_{\diamond})^{\daleth_i}+\prod_{i=1}^{e}({}^{\gimel}\nu^{t}{}_{\diamond})^{\daleth_i}+\prod_{i=1}^{e}({}^{\gimel}\tau^{t}{}_{\diamond})^{\daleth_i}}\times\left(1-\prod_{i=1}^{e}\left(1-{}^{\gimel}\mu^{t}{}_{\diamond}-{}^{\gimel}\nu^{t}{}_{\diamond}-{}^{\gimel}\tau^{t}{}_{\diamond}\right)^{\daleth_i}\right)}\end{array}\right)$$

$$=\left(\begin{array}{l}\sqrt[t]{\frac{({}^{\gimel}\mu^{t}{}_{\diamond})^{\sum_{i=1}^{e}\daleth_i}}{({}^{\gimel}\mu^{t}{}_{\diamond})^{\sum_{i=1}^{e}\daleth_i}+({}^{\gimel}\nu^{t}{}_{\diamond})^{\sum_{i=1}^{e}\daleth_i}+({}^{\gimel}\tau^{t}{}_{\diamond})^{\sum_{i=1}^{e}\daleth_i}}\times\left(1-\left(1-{}^{\gimel}\mu^{t}{}_{\diamond}-{}^{\gimel}\nu^{t}{}_{\diamond}-{}^{\gimel}\tau^{t}{}_{\diamond}\right)^{\sum_{i=1}^{e}\daleth_i}\right)},\\ \sqrt[t]{\frac{({}^{\gimel}\nu^{t}{}_{\diamond})^{\sum_{i=1}^{e}\daleth_i}}{({}^{\gimel}\mu^{t}{}_{\diamond})^{\sum_{i=1}^{e}\daleth_i}+({}^{\gimel}\nu^{t}{}_{\diamond})^{\sum_{i=1}^{e}\daleth_i}+({}^{\gimel}\tau^{t}{}_{\diamond})^{\sum_{i=1}^{e}\daleth_i}}\times\left(1-\left(1-{}^{\gimel}\mu^{t}{}_{\diamond}-{}^{\gimel}\nu^{t}{}_{\diamond}-{}^{\gimel}\tau^{t}{}_{\diamond}\right)^{\sum_{i=1}^{e}\daleth_i}\right)},\\ \sqrt[t]{\frac{({}^{\gimel}\tau^{t}{}_{\diamond})^{\sum_{i=1}^{e}\daleth_i}}{({}^{\gimel}\mu^{t}{}_{\diamond})^{\sum_{i=1}^{e}\daleth_i}+({}^{\gimel}\nu^{t}{}_{\diamond})^{\sum_{i=1}^{e}\daleth_i}+({}^{\gimel}\tau^{t}{}_{\diamond})^{\sum_{i=1}^{e}\daleth_i}}\times\left(1-\left(1-{}^{\gimel}\mu^{t}{}_{\diamond}-{}^{\gimel}\nu^{t}{}_{\diamond}-{}^{\gimel}\tau^{t}{}_{\diamond}\right)^{\sum_{i=1}^{e}\daleth_i}\right)}\end{array}\right)$$

$$=\left(\begin{array}{l}\sqrt[t]{\frac{({}^{\gimel}\mu^{t}{}_{\diamond})}{({}^{\gimel}\mu^{t}{}_{\diamond})+({}^{\gimel}\nu^{t}{}_{\diamond})+({}^{\gimel}\tau^{t}{}_{\diamond})}\times\left(1-\left(1-{}^{\gimel}\mu^{t}{}_{\diamond}-{}^{\gimel}\nu^{t}{}_{\diamond}-{}^{\gimel}\tau^{t}{}_{\diamond}\right)\right)},\sqrt[t]{\frac{({}^{\gimel}\nu^{t}{}_{\diamond})}{({}^{\gimel}\mu^{t}{}_{\diamond})+({}^{\gimel}\nu^{t}{}_{\diamond})+({}^{\gimel}\tau^{t}{}_{\diamond})}\times\left(1-\left(1-{}^{\gimel}\mu^{t}{}_{\diamond}-{}^{\gimel}\nu^{t}{}_{\diamond}-{}^{\gimel}\tau^{t}{}_{\diamond}\right)\right)},\\ \sqrt[t]{\frac{({}^{\gimel}\tau^{t}{}_{\diamond})}{({}^{\gimel}\mu^{t}{}_{\diamond})+({}^{\gimel}\nu^{t}{}_{\diamond})+({}^{\gimel}\tau^{t}{}_{\diamond})}\times\left(1-\left(1-{}^{\gimel}\mu^{t}{}_{\diamond}-{}^{\gimel}\nu^{t}{}_{\diamond}-{}^{\gimel}\tau^{t}{}_{\diamond}\right)\right)}\end{array}\right)$$

$$=\langle{}^{\gimel}\mu_{\diamond},{}^{\gimel}\nu_{\diamond},{}^{\gimel}\tau_{\diamond}\rangle=\aleph_{\diamond}.$$

□

Theorem 4.5. Assume that $\beth_i=\langle{}^{\gimel}\mu_i,{}^{\gimel}\nu_i,{}^{\gimel}\tau_i\rangle$ and $\beth_{i^*}=\langle{}^{\gimel}\mu_{i^*},{}^{\gimel}\nu_{i^*},{}^{\gimel}\tau_{i^*}\rangle$ are the families of T-SPFNs, and also consider

$$\text{T-SPFFWA}(\beth_1,\beth_2,\dots\beth_e)\;=\;\beth\;=\;\langle{}^{\gimel}\mu,{}^{\gimel}\nu,{}^{\gimel}\tau\rangle$$

and

$$\text{T-SPFFWA}(\beth_{1^*},\beth_{2^*},\dots\beth_{e^*})\;=\;\beth_*\;=\;\langle{}^{\gimel}\mu_*,{}^{\gimel}\nu_*,{}^{\gimel}\tau_*\rangle.$$

Then,

$${}^{\gimel}\mu^{t}+{}^{\gimel}\nu^{t}+{}^{\gimel}\tau^{t}\le{}^{\gimel}\mu^{t}{}_{*}+{}^{\gimel}\nu^{t}{}_{*}+{}^{\gimel}\tau^{t}{}_{*},\quad\text{if}\quad{}^{\gimel}\mu^{t}{}_{i}+{}^{\gimel}\nu^{t}{}_{i}+{}^{\gimel}\tau^{t}{}_{i}\le{}^{\gimel}\mu^{t}{}_{i^*}+{}^{\gimel}\nu^{t}{}_{i^*}+{}^{\gimel}\tau^{t}{}_{i^*}$$

Proof. By applying Theorem 4.2 on the both collection of T-SPFNs namely, $\aleph_i=\langle{}^{\gimel}\mu^{t}{}_{i},{}^{\gimel}\nu^{t}{}_{i},{}^{\gimel}\tau^{t}{}_{i}\rangle$ and $\aleph_{i^*}=\langle{}^{\gimel}\mu^{t}{}_{i^*},{}^{\gimel}\nu^{t}{}_{i^*},{}^{\gimel}\tau^{t}{}_{i^*}\rangle$, we get

$${}^{\gimel}\mu^{t}\;=\;\frac{\prod_{i=1}^{e}\left({}^{\gimel}\mu^{t}{}_{i}\right)^{\daleth_i}}{\prod_{i=1}^{e}\left({}^{\gimel}\mu^{t}{}_{i}\right)^{\daleth_i}+\prod_{i=1}^{e}\left({}^{\gimel}\nu^{t}{}_{i}\right)^{\daleth_i}+\prod_{i=1}^{e}\left({}^{\gimel}\tau^{t}{}_{i}\right)^{\daleth_i}}\times\left(1-\prod_{i=1}^{e}\left(1-{}^{\gimel}\mu^{t}{}_{i}-{}^{\gimel}\nu^{t}{}_{i}-{}^{\gimel}\tau^{t}{}_{i}\right)^{\daleth_i}\right)$$

$$
\begin{aligned}
{}^{\gimel}\nu^{t} &= \frac{\prod_{i=1}^{e}\left({}^{\gimel}\nu^{t}{}_{i}\right)^{\daleth_i}}{\prod_{i=1}^{e}\left({}^{\gimel}\mu^{t}{}_{i}\right)^{\daleth_i}+\prod_{i=1}^{e}\left({}^{\gimel}\nu^{t}{}_{i}\right)^{\daleth_i}+\prod_{i=1}^{e}\left({}^{\gimel}\tau^{t}{}_{i}\right)^{\daleth_i}}\times\left(1-\prod_{i=1}^{e}\left(1-{}^{\gimel}\mu^{t}{}_{i}-{}^{\gimel}\nu^{t}{}_{i}-{}^{\gimel}\tau^{t}{}_{i}\right)^{\daleth_i}\right)\\
{}^{\gimel}\tau^{t} &= \frac{\prod_{i=1}^{e}\left({}^{\gimel}\tau^{t}{}_{i}\right)^{\daleth_i}}{\prod_{i=1}^{e}\left({}^{\gimel}\mu^{t}{}_{i}\right)^{\daleth_i}+\prod_{i=1}^{e}\left({}^{\gimel}\nu^{t}{}_{i}\right)^{\daleth_i}+\prod_{i=1}^{e}\left({}^{\gimel}\tau^{t}{}_{i}\right)^{\daleth_i}}\times\left(1-\prod_{i=1}^{e}\left(1-{}^{\gimel}\mu^{t}{}_{i}-{}^{\gimel}\nu^{t}{}_{i}-{}^{\gimel}\tau^{t}{}_{i}\right)^{\daleth_i}\right)
\end{aligned}
$$

and

$$
\begin{aligned}
{}^{\gimel}\mu^{t}{}_{*} &= \frac{\prod_{i=1}^{e}\left({}^{\gimel}\mu^{t}{}_{i^*}\right)^{\daleth_i}}{\prod_{i=1}^{e}\left({}^{\gimel}\mu^{t}{}_{i^*}\right)^{\daleth_i}+\prod_{i=1}^{e}\left({}^{\gimel}\nu^{t}{}_{i^*}\right)^{\daleth_i}+\prod_{i=1}^{e}\left({}^{\gimel}\tau^{t}{}_{i^*}\right)^{\daleth_i}}\times\left(1-\prod_{i=1}^{e}\left(1-{}^{\gimel}\mu^{t}{}_{i^*}-{}^{\gimel}\nu^{t}{}_{i^*}-{}^{\gimel}\tau^{t}{}_{i^*}\right)^{\daleth_i}\right)\\
{}^{\gimel}\nu^{t}{}_{*} &= \frac{\prod_{i=1}^{e}\left({}^{\gimel}\nu^{t}{}_{i^*}\right)^{\daleth_i}}{\prod_{i=1}^{e}\left({}^{\gimel}\mu^{t}{}_{i^*}\right)^{\daleth_i}+\prod_{i=1}^{e}\left({}^{\gimel}\nu^{t}{}_{i^*}\right)^{\daleth_i}+\prod_{i=1}^{e}\left({}^{\gimel}\tau^{t}{}_{i^*}\right)^{\daleth_i}}\times\left(1-\prod_{i=1}^{e}\left(1-{}^{\gimel}\mu^{t}{}_{i^*}-{}^{\gimel}\nu^{t}{}_{i^*}-{}^{\gimel}\tau^{t}{}_{i^*}\right)^{\daleth_i}\right)\\
{}^{\gimel}\tau^{t}{}_{*} &= \frac{\prod_{i=1}^{e}\left({}^{\gimel}\tau^{t}{}_{i^*}\right)^{\daleth_i}}{\prod_{i=1}^{e}\left({}^{\gimel}\mu^{t}{}_{i^*}\right)^{\daleth_i}+\prod_{i=1}^{e}\left({}^{\gimel}\nu^{t}{}_{i^*}\right)^{\daleth_i}+\prod_{i=1}^{e}\left({}^{\gimel}\tau^{t}{}_{i^*}\right)^{\daleth_i}}\times\left(1-\prod_{i=1}^{e}\left(1-{}^{\gimel}\mu^{t}{}_{i^*}-{}^{\gimel}\nu^{t}{}_{i^*}-{}^{\gimel}\tau^{t}{}_{i^*}\right)^{\daleth_i}\right).
\end{aligned}
$$

By this, if ${}^{\gimel}\mu^{t}{}_{i}+{}^{\gimel}\nu^{t}{}_{i}+{}^{\gimel}\tau^{t}{}_{i}\leq{}^{\gimel}\mu^{t}{}_{i^*}+{}^{\gimel}\nu^{t}{}_{i^*}+{}^{\gimel}\tau^{t}{}_{i^*}$ then we have,

$$
{}^{\gimel}\mu^{t}+{}^{\gimel}\nu^{t}+{}^{\gimel}\tau^{t}=1-\prod_{i=1}^{e}\left(1-\left\{{}^{\gimel}\mu^{t}{}_{i}+{}^{\gimel}\nu^{t}{}_{i}+{}^{\gimel}\tau^{t}{}_{i}\right\}\right)^{\daleth_i}\leq 1-\prod_{i=1}^{e}\left(1-\left\{{}^{\gimel}\mu^{t}{}_{i^*}+{}^{\gimel}\nu^{t}{}_{i^*}+{}^{\gimel}\tau^{t}{}_{i^*}\right\}\right)^{\daleth_i}\leq{}^{\gimel}\mu^{t}{}_{*}+{}^{\gimel}\nu^{t}{}_{*}+{}^{\gimel}\tau^{t}{}_{*}.
$$

□

Theorem 4.6. Let $\beth_i=\langle{}^{\gimel}\mu_i,{}^{\gimel}\nu_i,{}^{\gimel}\tau_i\rangle$ be the jumble of T-SPFNs. Then for T-SPFFWA$(\beth_1,\beth_2,\ldots,\beth_e)=\langle{}^{\gimel}\mu_x,{}^{\gimel}\nu_x,{}^{\gimel}\tau_x\rangle$, we have

$$
\min_i\left\{{}^{\gimel}\mu^{t}{}_{i}+{}^{\gimel}\nu^{t}+{}^{\gimel}\tau^{t}{}_{i}\right\}\leq{}^{\gimel}\mu^{t}{}_{x}+{}^{\gimel}\nu^{t}{}_{x}+{}^{\gimel}\tau^{t}{}_{x}\leq\max_i\left\{{}^{\gimel}\mu^{t}{}_{i}+{}^{\gimel}\nu^{t}{}_{i}+{}^{\gimel}\tau^{t}{}_{i}\right\}.
$$

Proof. We start with

$$
\begin{aligned}
\min_i\left\{{}^{\gimel}\mu^{t}{}_{i}+{}^{\gimel}\nu^{t}{}_{i}+{}^{\gimel}\tau^{t}{}_{i}\right\} &= 1-\left(1-\min_i\left\{{}^{\gimel}\mu^{t}{}_{i}+{}^{\gimel}\nu^{t}{}_{i}+{}^{\gimel}\tau^{t}{}_{i}\right\}\right)\\
&= 1-\left(1-\min_i\left\{{}^{\gimel}\mu^{t}{}_{i}+{}^{\gimel}\nu^{t}{}_{i}+{}^{\gimel}\tau^{t}{}_{i}\right\}\right)^{\sum_{i=1}^{e}\daleth_i}\\
&= 1-\prod_{i=1}^{e}\left(1-\min_i\left\{{}^{\gimel}\mu^{t}{}_{i}+{}^{\gimel}\nu^{t}{}_{i}+{}^{\gimel}\tau^{t}{}_{i}\right\}\right)^{\daleth_i}\\
&\leq 1-\prod_{i=1}^{e}\left(1-\left\{{}^{\gimel}\mu^{t}{}_{i}+{}^{\gimel}\nu^{t}{}_{i}+{}^{\gimel}\tau^{t}{}_{i}\right\}\right)^{\daleth_i}\\
&\leq 1-\prod_{i=1}^{e}\left(1-\max_i\left\{{}^{\gimel}\mu^{t}{}_{i}+{}^{\gimel}\nu^{t}{}_{i}+{}^{\gimel}\tau^{t}{}_{i}\right\}\right)^{\daleth_i}
\end{aligned}
$$

$$= 1 - \left(1 - \max_i \left\{\gimel\mu^t_i + \gimel\nu^t_i + \gimel\tau^t_i\right\}\right)^{\sum_{i=1}^{e} \daleth_i}$$
$$= \max_i \left\{\gimel\mu^t_i + \gimel\nu^t_i + \gimel\tau^t_i\right\}.$$

By Theorem 4.2, we get

$$\gimel\mu_x = \sqrt[t]{\frac{\prod_{i=1}^{e}\left(\gimel\mu^t_i\right)^{\daleth_i}}{\prod_{i=1}^{e}\left(\gimel\mu^t_i\right)^{\daleth_i} + \prod_{i=1}^{e}\left(\gimel\nu^t_i\right)^{\daleth_i} + \prod_{i=1}^{e}\left(\gimel\tau^t_i\right)^{\daleth_i}} \times \left(1 - \prod_{i=1}^{e}\left(1 - \gimel\mu^t_i - \gimel\nu^t_i - \gimel\tau^t_i\right)^{\daleth_i}\right)}$$

$$\gimel\nu_x = \sqrt[t]{\frac{\prod_{i=1}^{e}\left(\gimel\nu^t_i\right)^{\daleth_i}}{\prod_{i=1}^{e}\left(\gimel\mu^t_i\right)^{\daleth_i} + \prod_{i=1}^{e}\left(\gimel\nu^t_i\right)^{\daleth_i} + \prod_{i=1}^{e}\left(\gimel\tau^t_i\right)^{\daleth_i}} \times \left(1 - \prod_{i=1}^{e}\left(1 - \gimel\mu^t_i - \gimel\nu^t_i - \gimel\tau^t_i\right)^{\daleth_i}\right)}$$

$$\gimel\tau_x = \sqrt[t]{\frac{\prod_{i=1}^{e}\left(\gimel\tau^t_i\right)^{\daleth_i}}{\prod_{i=1}^{e}\left(\gimel\mu^t_i\right)^{\daleth_i} + \prod_{i=1}^{e}\left(\gimel\nu^t_i\right)^{\daleth_i} + \prod_{i=1}^{e}\left(\gimel\tau^t_i\right)^{\daleth_i}} \times \left(1 - \prod_{i=1}^{e}\left(1 - \gimel\mu^t_i - \gimel\nu^t_i - \gimel\tau^t_i\right)^{\daleth_i}\right)}.$$

From this, we get

$$\gimel\mu^t_x + \gimel\nu^t_x + \gimel\tau^t_x = \left(1 - \prod_{i=1}^{e}\left(1 - \gimel\mu^t_i - \gimel\nu^t_i - \gimel\tau^t_i\right)^{\daleth_i}\right).$$

Consequently,

$$\min_i \left\{\gimel\mu^t_i + \gimel\nu^t + \gimel\tau^t_i\right\} \leq \gimel\mu^t_x + \gimel\nu^t_x + \gimel\tau^t_x \leq \max_i \left\{\gimel\mu^t_i + \gimel\nu^t_i + \gimel\tau^t_i\right\} .$$

□

4.2. T-SPFFOWA operator

Definition 4.7. Let $\sqsupset_h = \langle \gimel\mu_h, \gimel\nu_h, \gimel\tau_h \rangle$ be the jumble of T-SPFNs, and T-SPFFOWA: $\mathscr{R}^n \rightarrow \mathscr{R}$, be the mapping. If

$$\text{T-SPFFOWA}(\sqsupset_1, \sqsupset_2, \ldots \sqsupset_e) = \left(\daleth_1 * \sqsupset_{\xi(1)} \tilde{\oplus} \daleth_2 * \sqsupset_{\xi(2)} \tilde{\oplus} \ldots, \tilde{\oplus} \daleth_e * \sqsupset_{\xi(e)}\right) \tag{4.3}$$

then the mapping T-SPFFOWA is called "T-spherical fuzzy fairly ordered weighted averaging (T-SPFFOWA) operator", here $\daleth_i$ is the WV of $\sqsupset_i$ with $\daleth_i > 0$ and $\sum_{i=1}^{e} \daleth_i = 1$.
$\xi : 1, 2, 3, ..., n \rightarrow 1, 2, 3, ..., n$ is a permutation map s.t. $\sqsupset_{\xi(i-1)} \geq \sqsupset_{\xi(i)}$.

We may also investigate T-SPFFOWA using proposed operational rules, as demonstrated in the theorem that follows.

Theorem 4.8. Let $\sqsupset_h = \langle \gimel\mu_h, \gimel\nu_h, \gimel\tau_h \rangle$ be the jumble of T-SPFNs, we can also find T-SPFFOWA by

$$\text{T-SPFFOWA}(\beth_1, \beth_2, \ldots, \beth_e)$$

$$= \left(\begin{array}{c} \sqrt[t]{\frac{\prod_{i=1}^{e}\left({}^{\gimel}\mu^{t}{}_{\xi_{(i)}}\right)^{\daleth_i}}{\prod_{i=1}^{e}\left({}^{\gimel}\mu^{t}{}_{\xi_{(i)}}\right)^{\daleth_{\xi(i)}}+\prod_{i=1}^{e}\left({}^{\gimel}\nu^{t}{}_{\xi_{(i)}}\right)^{\daleth_i}+\prod_{i=1}^{e}\left({}^{\gimel}\tau^{t}{}_{\xi_{(i)}}\right)^{\daleth_i}} \times \left(1-\prod_{i=1}^{e}\left(1-{}^{\gimel}\mu^{t}{}_{\xi_{(i)}}-{}^{\gimel}\nu^{t}{}_{\xi_{(i)}}-{}^{\gimel}\tau^{t}{}_{\xi_{(i)}}\right)^{\daleth_i}\right)}, \\ \sqrt[t]{\frac{\prod_{i=1}^{e}\left({}^{\gimel}\nu^{t}{}_{\xi_{(i)}}\right)^{\daleth_i}}{\prod_{i=1}^{e}\left({}^{\gimel}\mu^{t}{}_{\xi_{(i)}}\right)^{\daleth_i}+\prod_{i=1}^{e}\left({}^{\gimel}\nu^{t}{}_{\xi_{(i)}}\right)^{\daleth_i}+\prod_{i=1}^{e}\left({}^{\gimel}\tau^{t}{}_{\xi_{(i)}}\right)^{\daleth_i}} \times \left(1-\prod_{i=1}^{e}\left(1-{}^{\gimel}\mu^{t}{}_{\xi_{(i)}}-{}^{\gimel}\nu^{t}{}_{\xi_{(i)}}-{}^{\gimel}\tau^{t}{}_{\xi_{(i)}}\right)^{\daleth_i}\right)}, \\ \sqrt[t]{\frac{\prod_{i=1}^{e}\left({}^{\gimel}\tau^{t}{}_{\xi_{(i)}}\right)^{\daleth_i}}{\prod_{i=1}^{e}\left({}^{\gimel}\mu^{t}{}_{\xi_{(i)}}\right)^{\daleth_i}+\prod_{i=1}^{e}\left({}^{\gimel}\nu^{t}{}_{\xi_{(i)}}\right)^{\daleth_i}+\prod_{i=1}^{e}\left({}^{\gimel}\tau^{t}{}_{\xi_{(i)}}\right)^{\daleth_i}} \times \left(1-\prod_{i=1}^{e}\left(1-{}^{\gimel}\mu^{t}{}_{\xi_{(i)}}-{}^{\gimel}\nu^{t}{}_{\xi_{(i)}}-{}^{\gimel}\tau^{t}{}_{\xi_{(i)}}\right)^{\daleth_i}\right)} \end{array} \right)$$

where $\daleth_i$ is the WV of $\beth_i$ with $\daleth_i > 0$ and $\sum_{i=1}^{e} \daleth_i = 1$.

Theorem 4.9. *Let $\beth_i = \langle {}^{\gimel}\mu_i, {}^{\gimel}\nu_i, {}^{\gimel}\tau_i \rangle$ be the jumble of T-SPFNs and $\beth_\diamond = \langle {}^{\gimel}\mu_\diamond, {}^{\gimel}\nu_\diamond, {}^{\gimel}\tau_\diamond \rangle$ be the T-SPFNs such that, $\beth_i = \beth_\diamond \forall i$. Then*

$$\textit{T-SPFFOWA}(\beth_1, \beth_2, \ldots, \beth_e) = \beth_\diamond. \tag{4.4}$$

Theorem 4.10. Let $\beth_i = \langle {}^{\gimel}\mu_i, {}^{\gimel}\nu_i, {}^{\gimel}\tau_i \rangle$ be the jumble of T-SPFNs. Then for T-SPFFOWA$(\beth_1, \beth_2, \ldots, \beth_e) = \langle {}^{\gimel}\mu_x, {}^{\gimel}\nu_x, {}^{\gimel}\tau_x \rangle$, we have

$$\min_{\xi_{(i)}} \left\{ {}^{\gimel}\mu^{t}{}_{\xi_{(i)}} + {}^{\gimel}\nu^{t}{}_{\xi_{(i)}} + {}^{\gimel}\tau^{t}{}_{\xi_{(i)}} \right\} \leq {}^{\gimel}\mu^{t}{}_{x} + {}^{\gimel}\nu^{t}{}_{x} + {}^{\gimel}\tau^{t}{}_{x} \leq \max_{\xi_{(i)}} \left\{ {}^{\gimel}\mu^{t}{}_{\xi_{(i)}} + {}^{\gimel}\nu^{t}{}_{\xi_{(i)}} + {}^{\gimel}\tau^{t}{}_{\xi_{(i)}} \right\}.$$

Theorem 4.11. Assume that $\beth_i = \langle {}^{\gimel}\mu_i, {}^{\gimel}\nu_i, {}^{\gimel}\tau_i \rangle$ and $\beth_{i^*} = \langle {}^{\gimel}\mu_{i^*}, {}^{\gimel}\nu_{i^*}, {}^{\gimel}\tau_{i^*} \rangle$ are the families of T-SPFNs, and also consider

$$\text{T-SPFFOWA}(\beth_1, \beth_2, \ldots \beth_e) = \beth = \langle {}^{\gimel}\mu, {}^{\gimel}\nu, {}^{\gimel}\tau \rangle$$

and

$$\text{T-SPFFOWA}(\beth_{1^*}, \beth_{2^*}, \ldots \beth_{e^*}) = \beth_* = \langle {}^{\gimel}\mu_*, {}^{\gimel}\nu_*, {}^{\gimel}\tau_* \rangle.$$

Then,

$${}^{\gimel}\mu^{t} + {}^{\gimel}\nu^{t} + {}^{\gimel}\tau^{t} \leq {}^{\gimel}\mu^{t}{}_{*} + {}^{\gimel}\nu^{t}{}_{*} + {}^{\gimel}\tau^{t}{}_{*}, \quad \text{if} \quad {}^{\gimel}\mu^{t}{}_{\xi_{(i)}} + {}^{\gimel}\nu^{t}{}_{\xi_{(i)}} + {}^{\gimel}\tau^{t}{}_{\xi_{(i)}} \leq {}^{\gimel}\mu^{t}{}_{\xi^*_{(i)}} + {}^{\gimel}\nu^{t}{}_{\xi^*_{(i)}} + {}^{\gimel}\tau^{t}{}_{\xi^*_{(i)}}.$$

5. Decision-making approach

We explore an MCDM issue by comparing each of the n different options to a set of m distinct qualities. In this circumstance, it is necessary to have a group of p specialists whose ratings must be larger than zero, but whose total is one.

As one can remember, the alternative $Ш^{\varsigma}{}_j$ $(j = 1, 2, \ldots, n)$ should be provided by a group of experts $\mathscr{P}^{Q}{}_k$ $(k = 1, 2, \ldots, p)$. The characteristics $\mathscr{C}_i^{\daleth}$ $(i = 1, 2, \ldots, m)$ were likewise chosen by experts after deliberation; hence, the evaluation result is supplied in terms of T-SPFNs, $\alpha^{\varphi p}_{ji} = \left\langle \xi^{p}_{ji}, \rho^{p}_{ji}, \kappa^{p}_{ji} \right\rangle$. Assume further that $\mathscr{W}_t$ is the WV for the feature $\mathscr{C}_i^{\daleth}$ meeting the criteria, $\mathscr{W}_t \geq 0$ and $\sum_{t=1}^{m} \mathscr{W}_t = 1$. The proposed operator is used to create an MCDM for the T-SPF data, which includes the following steps:

Algorithm

Step 1:

Determine the ratings of DMs based on the DMs' importance as "linguistics terms" (LTs) as indicated in T-SPFNs. The LTs listed in Table 1. Assume $\aleph_k = \left\langle {}^{\gimel}\mu_k, {}^{\gimel}\nu_k, {}^{\gimel}\tau_k \right\rangle$ is the T-SPFN for the importance of k-th DM. Then the weight ζ_k of k-th DM can be calculated as follows:

$$\zeta_k = \frac{\aleph_k}{\sum_{k=1}^{p} \aleph_k}, k = 1, 2, 3, \ldots, p \tag{5.1}$$

where $\aleph_k = {}^{\gimel}\mu^{t}{}_{k} + \left(1 - {}^{\gimel}\mu^{t}{}_{k} - {}^{\gimel}\nu^{t}{}_{k} - {}^{\gimel}\tau^{t}{}_{k}\right)\left(\frac{{}^{\gimel}\nu^{t}{}_{k}}{{}^{\gimel}\mu^{t}{}_{k}+{}^{\gimel}\nu^{t}{}_{k}+{}^{\gimel}\tau^{t}k}\right)\left(\frac{{}^{\gimel}\tau^{t}{}_{k}}{{}^{\gimel}\mu^{t}{}_{k}+{}^{\gimel}\nu^{t}{}_{k}+{}^{\gimel}\tau^{t}k}\right)$ and clearly $\sum_{k=1}^{p} \zeta_k = 1$.

Table 1. Linguistic terms for DMs.

LTs	T-SPFNs
Very suitable	0.800, 0.100, 0.050
Suitable	0.650, 0.150, 0.100
Medium suitable	0.550, 0.200, 0.150
Un-suitable	0.300, 0.300, 0.200
Very un-suitable	0.100, 0.400, 0.400

Step 2:

Acquire a decision matrix $\mathscr{P}^{Q}{}_{(p)} = (\mathfrak{Y}_{ji}^{(p)})_{n\times m}$ in the form of T-SPFNs from the DMs.

$$\begin{array}{ccc}
 & & \begin{array}{cccc} \mathscr{C}_1^{\daleth} & \mathscr{C}_2^{\daleth} & & \mathscr{C}_m^{\daleth} \end{array} \\
\begin{array}{c} \mathscr{P}^{Q}{}_1 \\ \\ \\ \\ \\ \mathscr{P}^{Q}{}_2 \\ \\ \\ \\ \\ \mathscr{P}^{Q}{}_p \\ \\ \\ \\ \end{array} &
\begin{array}{c} \text{Ш}^{\varsigma}{}_1 \\ \text{Ш}^{\varsigma}{}_2 \\ \\ \text{Ш}^{\varsigma}{}_n \\ \\ \text{Ш}^{\varsigma}{}_1 \\ \text{Ш}^{\varsigma}{}_2 \\ \\ \text{Ш}^{\varsigma}{}_n \\ \\ \text{Ш}^{\varsigma}{}_1 \\ \text{Ш}^{\varsigma}{}_2 \\ \\ \text{Ш}^{\varsigma}{}_n \end{array} &
\begin{bmatrix}
({}^{\gimel}\mu_{11}^{1}, {}^{\gimel}\nu_{11}^{1}, {}^{\gimel}\tau_{11}^{1}) & ({}^{\gimel}\mu_{12}^{1}, {}^{\gimel}\nu_{12}^{1}, {}^{\gimel}\tau_{12}^{1}) & \cdots\cdots & (\langle{}^{\gimel}\mu_{1m}^{1}, {}^{\gimel}\nu_{1m}^{1}, {}^{\gimel}\tau_{1m}^{1}) \\
({}^{\gimel}\mu_{21}^{1}, {}^{\gimel}\nu_{21}^{1}, {}^{\gimel}\tau_{21}^{1}) & ({}^{\gimel}\mu_{22}^{1}, {}^{\gimel}\nu_{22}^{1}, {}^{\gimel}\tau_{22}^{1}) & \cdots\cdots & ({}^{\gimel}\mu_{2m}^{1}, {}^{\gimel}\nu_{2m}^{1}, {}^{\gimel}\tau_{2m}^{1}) \\
\vdots & \vdots & \ddots\ddots & \vdots \\
({}^{\gimel}\mu_{n1}^{1}, {}^{\gimel}\nu_{n1}^{1}, {}^{\gimel}\tau_{n1}^{1}) & ({}^{\gimel}\mu_{n2}^{1}, {}^{\gimel}\nu_{n2}^{1}, {}^{\gimel}\tau_{n2}^{1}) & \cdots\cdots & ({}^{\gimel}\mu_{nm}^{1}, {}^{\gimel}\nu_{nm}^{1}, {}^{\gimel}\tau_{nm}^{1}) \\
 & & & \\
({}^{\gimel}\mu_{11}^{2}, {}^{\gimel}\nu_{11}^{2}, {}^{\gimel}\tau_{11}^{2}) & ({}^{\gimel}\mu_{12}^{2}, {}^{\gimel}\nu_{12}^{2}, {}^{\gimel}\tau_{12}^{2}) & \cdots\cdots & ({}^{\gimel}\mu_{1m}^{2}, {}^{\gimel}\nu_{1m}^{2}, {}^{\gimel}\tau_{1m}^{2}) \\
({}^{\gimel}\mu_{21}^{2}, {}^{\gimel}\nu_{21}^{2}, {}^{\gimel}\tau_{21}^{2}) & ({}^{\gimel}\mu_{22}^{2}, {}^{\gimel}\nu_{22}^{2}, {}^{\gimel}\tau_{22}^{2}) & \cdots\cdots & ({}^{\gimel}\mu_{2m}^{2}, {}^{\gimel}\nu_{2m}^{2}, {}^{\gimel}\tau_{2m}^{2}) \\
\vdots & \vdots & \ddots\ddots & \vdots \\
({}^{\gimel}\mu_{n1}^{2}, {}^{\gimel}\nu_{n1}^{2}, {}^{\gimel}\tau_{n1}^{2}) & ({}^{\gimel}\mu_{n2}^{2}, {}^{\gimel}\nu_{n2}^{2}, {}^{\gimel}\tau_{n2}^{2}) & \cdots\cdots & ({}^{\gimel}\mu_{nm}^{2}, {}^{\gimel}\nu_{nm}^{2}, {}^{\gimel}\tau_{nm}^{2}) \\
 & & & \\
({}^{\gimel}\mu_{11}^{p}, {}^{\gimel}\nu_{11}^{p}, {}^{\gimel}\tau_{11}^{p}) & ({}^{\gimel}\mu_{12}^{p}, {}^{\gimel}\nu_{12}^{p}, {}^{\gimel}\tau_{12}^{p}) & \cdots\cdots & ({}^{\gimel}\mu_{1m}^{p}, {}^{\gimel}\nu_{1m}^{p}, {}^{\gimel}\tau_{1m}^{p}) \\
({}^{\gimel}\mu_{21}^{p}, {}^{\gimel}\nu_{21}^{p}, {}^{\gimel}\tau_{21}^{p}) & ({}^{\gimel}\mu_{22}^{p}, {}^{\gimel}\nu_{22}^{p}, {}^{\gimel}\tau_{22}^{p}) & \cdots\cdots & ({}^{\gimel}\mu_{2m}^{p}, {}^{\gimel}\nu_{2m}^{p}, {}^{\gimel}\tau_{2m}^{p}) \\
\vdots & \vdots & \ddots\ddots & \vdots \\
({}^{\gimel}\mu_{n1}^{p}, {}^{\gimel}\nu_{n1}^{p}, {}^{\gimel}\tau_{n1}^{p}) & ({}^{\gimel}\mu_{n2}^{p}, {}^{\gimel}\nu_{n2}^{p}, {}^{\gimel}\tau_{n2}^{p}) & \cdots\cdots & ({}^{\gimel}\mu_{nm}^{p}, {}^{\gimel}\nu_{nm}^{p}, {}^{\gimel}\tau_{nm}^{p})
\end{bmatrix}
\end{array}$$

Step 3:

Establish a consolidated T-SPF judgement matrix. It is necessary to note that, while developing the aggregated T-SPF decision matrix, all individual perspectives must be summed and included to generate a collective viewpoint. The proposed AO will provide the things to this end:

Consider $H = \left(H_{ji}\right)_{n\times m}$ is the aggregated T-SPF decision matrix, where

$$H_{ji} = T - SPFFWA\ \left(\mathfrak{Y}_{ji}^{(1)}, \mathfrak{Y}_{ji}^{(2)}, \ldots, \mathfrak{Y}_{ji}^{(p)}\right)$$

or

$$H_{ji} = T - SPFFOWA\ \left(\mathfrak{Y}_{ji}^{(1)}, \mathfrak{Y}_{ji}^{(2)}, \ldots, \mathfrak{Y}_{ji}^{(p)}\right).$$

For convenience, we take H_{ji} as $H_{ji} = \left\langle {}^{\daleth}\mu_{ji}, {}^{\daleth}\nu_{ji}, {}^{\daleth}\tau_{ji}\right\rangle$

Step 4:

If required, normalise the T-SPFNs by changing each cost kind attribute (${}^{\daleth}\tau_c$) to benefit kind attribute (${}^{\daleth}\tau_b$) with the help of formula given by:

$$(\aleph_{ji}^{N})_{n\times m} = \begin{cases} (\tilde{\eth}_{ji})^c; & i \in {}^{\daleth}\tau_c \\ \tilde{\eth}_{ji}; & i \in {}^{\daleth}\tau_b, \end{cases} \tag{5.2}$$

where $(\tilde{\eth}_{ji})^c$ show the compliment of $(\tilde{\eth}_{ji})$. The normalised decision matrix will be $\Gamma_N = \left(\aleph_{ji}^{N}\right)_{n\times m} = \left({}^{\daleth}\breve{\mu}_{ji}, {}^{\daleth}\breve{\nu}_{ji}, {}^{\daleth}\breve{\tau}_{ji}\right)_{n\times m}$.

Step 5:

Construct the accuracy matrix, by utilizing the accuracy function of T-SPFNs as $\Psi = \left(\breve{\Theta}^{\zeta}\left(\aleph_{ji}^{N}\right)\right)_{n\times m}$.

$$\begin{array}{c} \\ Ш^{\varsigma}{}_1 \\ Ш^{\varsigma}{}_2 \\ Ш^{\varsigma}{}_3 \\ \vdots \\ Ш^{\varsigma}{}_m \end{array} \begin{array}{c} \begin{array}{ccccc} \mathscr{C}_1^{\daleth} & \mathscr{C}_2^{\daleth} & \mathscr{C}_3^{\daleth} & \cdots & \mathscr{C}_m^{\daleth} \end{array} \\ \begin{bmatrix} \breve{\Theta}^{\zeta}\left(\aleph_{11}^{N}\right) & \breve{\Theta}^{\zeta}\left(\aleph_{12}^{N}\right) & \breve{\Theta}^{\zeta}\left(\aleph_{13}^{N}\right) & \cdots & \breve{\Theta}^{\zeta}\left(\aleph_{1m}^{N}\right) \\ \breve{\Theta}^{\zeta}\left(\aleph_{21}^{N}\right) & \breve{\Theta}^{\zeta}\left(\aleph_{22}^{N}\right) & \breve{\Theta}^{\zeta}\left(\aleph_{23}^{N}\right) & \cdots & \breve{\Theta}^{\zeta}\left(\aleph_{2m}^{N}\right) \\ \breve{\Theta}^{\zeta}\left(\aleph_{31}^{N}\right) & \breve{\Theta}^{\zeta}\left(\aleph_{32}^{N}\right) & \breve{\Theta}^{\zeta}\left(\aleph_{33}^{N}\right) & \cdots & \breve{\Theta}^{\zeta}\left(\aleph_{3m}^{N}\right) \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \left(\aleph_{n1}^{N}\right) & \breve{\Theta}^{\zeta}\left(\aleph_{n2}^{N}\right) & \breve{\Theta}^{\zeta}\left(\aleph_{n3}^{N}\right) & \cdots & \breve{\Theta}^{\zeta}\left(\aleph_{nm}^{N}\right) \end{bmatrix} \end{array}.$$

Step 6:

Just on basis of this scoring matrix Ψ, a "weighted sum of the scores of each alternate $Ш^{\varsigma}{}_j$" is calculated by

$$¥(Ш^{\varsigma}{}_j) = \sum_{i=1}^{m} \mathscr{W}_i^{\gamma}\ \breve{\Theta}^{\zeta}\left(\aleph_{ji}^{N}\right), \quad (j = 1, 2, \ldots, n),$$

where, $\mathscr{W}_1^{\gamma}, \mathscr{W}_2^{\gamma}, \ldots \mathscr{W}_m^{\gamma}$ be the WV of the given criterion.

Assume that the weights are indeterminate and that $\widehat{\coprod}$ represents a subset of them. We evaluate these indeterminate weights using the underlying computational model:

$$Max\ \ g = \sum_{i=1}^{m} ¥(Ш^{\varsigma}{}_{j})$$

under the conditions $\sum_{i=1}^{m} \mathscr{W}_i^{\gamma} = 1$. By using this methodology, the WV is normalised. Using a linear programming model, we calculate the weights of criteria subject to certain limitations.

Step 7:

Using the normalised decision matrix Γ_N and the WV $\mathscr{W}^{\gamma}$, analyze the consolidated weighted T-SPF decision matrix. We used the suggested AOs listed below.

$$T - SPFFWA(\aleph_{j1}^{N}, \aleph_{j2}^{N}, \ldots, \aleph_{jm}^{N})$$

$$= \left(\begin{array}{c} \frac{\prod_{j=1}^{m}\left(\breve{\mu}^{t}{}_{j\xi_{(i)}}\right)^{\mathscr{W}^{\gamma}{}_{i}}}{\prod_{j=1}^{m}\left(\breve{\mu}^{t}{}_{j\xi_{(i)}}\right)^{\mathscr{W}^{\gamma}{}_{i}}+\prod_{j=1}^{m}\left(\breve{\nu}^{t}{}_{j\xi_{(i)}}\right)^{\mathscr{W}^{\gamma}{}_{i}}+\prod_{j=1}^{m}\left(\breve{\tau}^{t}{}_{j\xi_{(i)}}\right)^{\mathscr{W}_{i}^{\gamma}}} \times \left(1 - \prod_{j=1}^{m}\left(1 - \breve{\mu}^{t}{}_{j\xi_{(i)}} - \breve{\nu}^{t}{}_{j\xi_{(i)}} - \breve{\tau}^{t}{}_{j\xi_{(i)}}\right)^{\mathscr{W}_{i}^{\gamma}}\right), \\ \frac{\prod_{j=1}^{m}\left(\breve{\nu}^{t}{}_{j\xi_{(i)}}\right)^{\mathscr{W}_{i}^{\gamma}}}{\prod_{j=1}^{m}\left(\breve{\mu}^{t}{}_{j\xi_{(i)}}\right)^{\mathscr{W}_{i}^{\gamma}}+\prod_{j=1}^{m}\left(\breve{\nu}^{t}{}_{j\xi_{(i)}}\right)^{\mathscr{W}_{i}^{\gamma}}+\prod_{j=1}^{m}\left(\breve{\tau}^{t}{}_{j\xi_{(i)}}\right)^{\mathscr{W}_{i}^{\gamma}}} \times \left(1 - \prod_{j=1}^{m}\left(1 - \breve{\mu}^{t}{}_{j\xi_{(i)}} - \breve{\nu}^{t}{}_{j\xi_{(i)}} - \breve{\tau}^{t}{}_{j\xi_{(i)}}\right)^{\mathscr{W}_{i}^{\gamma}}\right), \\ \frac{\prod_{j=1}^{m}\left(\breve{\tau}^{t}{}_{j\xi_{(i)}}\right)^{\mathscr{W}_{i}^{\gamma}}}{\prod_{j=1}^{m}\left(\breve{\mu}^{t}{}_{j\xi_{(i)}}\right)^{\mathscr{W}_{i}^{\gamma}}+\prod_{j=1}^{m}\left(\breve{\nu}^{t}{}_{j\xi_{(i)}}\right)^{\mathscr{W}_{i}^{\gamma}}+\prod_{j=1}^{m}\left(\breve{\tau}^{t}{}_{j\xi_{(i)}}\right)^{\mathscr{W}_{i}^{\gamma}}} \times \left(1 - \prod_{j=1}^{m}\left(1 - \breve{\mu}^{t}{}_{j\xi_{(i)}} - \breve{\nu}^{t}{}_{j\xi_{(i)}} - \breve{\tau}^{t}{}_{j\xi_{(i)}}\right)^{\mathscr{W}_{i}^{\gamma}}\right) \end{array} \right)$$

or

$$T - SPFFOWA(\aleph_{j1}^{N}, \aleph_{j2}^{N}, \ldots, \aleph_{jm}^{N})$$

$$= \left(\begin{array}{c} \frac{\prod_{j=1}^{m}\left(\breve{\mu}^{t}{}_{j\xi_{(i)}}\right)^{\mathscr{W}^{\gamma}{}_{i}}}{\prod_{j=1}^{m}\left(\breve{\mu}^{t}{}_{j\xi_{(i)}}\right)^{\mathscr{W}^{\gamma}{}_{i}}+\prod_{j=1}^{m}\left(\breve{\nu}^{t}{}_{j\xi_{(i)}}\right)^{\mathscr{W}^{\gamma}{}_{i}}+\prod_{j=1}^{m}\left(\breve{\tau}^{t}{}_{j\xi_{(i)}}\right)^{\mathscr{W}_{i}^{\gamma}}} \times \left(1 - \prod_{j=1}^{m}\left(1 - \breve{\mu}^{t}{}_{j\xi_{(i)}} - \breve{\nu}^{t}{}_{j\xi_{(i)}} - \breve{\tau}^{t}{}_{j\xi_{(i)}}\right)^{\mathscr{W}_{i}^{\gamma}}\right), \\ \frac{\prod_{j=1}^{m}\left(\breve{\nu}^{t}{}_{j\xi_{(i)}}\right)^{\mathscr{W}_{i}^{\gamma}}}{\prod_{j=1}^{m}\left(\breve{\mu}^{t}{}_{j\xi_{(i)}}\right)^{\mathscr{W}_{i}^{\gamma}}+\prod_{j=1}^{m}\left(\breve{\nu}^{t}{}_{j\xi_{(i)}}\right)^{\mathscr{W}_{i}^{\gamma}}+\prod_{j=1}^{m}\left(\breve{\tau}^{t}{}_{j\xi_{(i)}}\right)^{\mathscr{W}_{i}^{\gamma}}} \times \left(1 - \prod_{j=1}^{m}\left(1 - \breve{\mu}^{t}{}_{j\xi_{(i)}} - \breve{\nu}^{t}{}_{j\xi_{(i)}} - \breve{\tau}^{t}{}_{j\xi_{(i)}}\right)^{\mathscr{W}_{i}^{\gamma}}\right), \\ \frac{\prod_{j=1}^{m}\left(\breve{\tau}^{t}{}_{j\xi_{(i)}}\right)^{\mathscr{W}_{i}^{\gamma}}}{\prod_{j=1}^{m}\left(\breve{\mu}^{t}{}_{j\xi_{(i)}}\right)^{\mathscr{W}_{i}^{\gamma}}+\prod_{j=1}^{m}\left(\breve{\nu}^{t}{}_{j\xi_{(i)}}\right)^{\mathscr{W}_{i}^{\gamma}}+\prod_{j=1}^{m}\left(\breve{\tau}^{t}{}_{j\xi_{(i)}}\right)^{\mathscr{W}_{i}^{\gamma}}} \times \left(1 - \prod_{j=1}^{m}\left(1 - \breve{\mu}^{t}{}_{j\xi_{(i)}} - \breve{\nu}^{t}{}_{j\xi_{(i)}} - \breve{\tau}^{t}{}_{j\xi_{(i)}}\right)^{\mathscr{W}_{i}^{\gamma}}\right) \end{array} \right).$$

Step 8:

Using T-SPF, compute the score value of the whole weighted consolidated result. Rank each option depending on the SF, and afterwards select the most desired one (s).

Demonstration of the suggested method is shown in Figure 1.

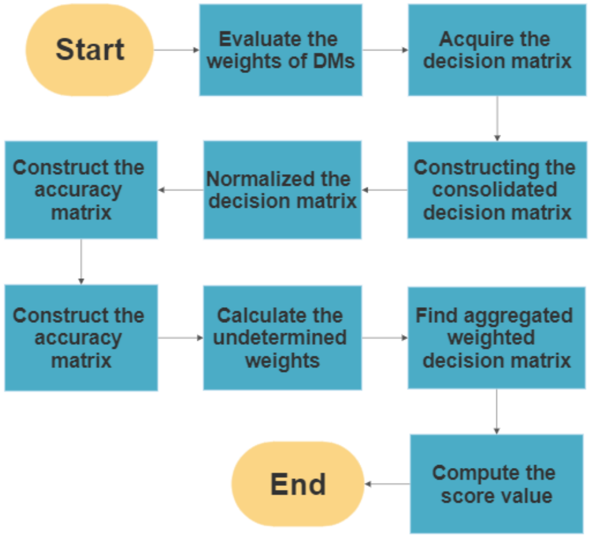

Figure 1. Pictorial view of proposed algorithm.

6. An application to food waste treatment technology selection

Food waste treatment and management has become a major concern due to the considerable ecological, sociological, and monetary consequences. Because of the correlation of multiple qualitative and quantitative qualities, selecting the best acceptable food waste treatment procedure from a group of alternatives might be considered an MCDM challenge. The colossal sum of "food waste" (FW) is produced each year around the world, and the sum of the FW era has expanded over time [58]. For occasion, within the Joined together States, FW accounted for 21.1% of the disposed of metropolitan squander stream in 2012 which is identical to 31.4 million tons. In China, 90 million tons of FW was produced in 2010 which made up almost 51% of the "municipal solid waste" (MSW) generation [59]. It is evaluated that over one-fourth of nourishment delivered around the world annually is squandered amid production, handling, dispersion, utilization, and transfer. In developed nations such as Japan, the United Kingdom, South Korea and Australia, the sums of FW produced per year were 9.9, 7.0, 5.7 and 4.4 million tons respectively on normal [60].

It is calculated that 1.6 billion tons of "primary product equivalents" are lost due to food waste throughout the world every year. The edible portion of this represents 1.3 billion tones of foodstuff that is lost or wasted every year. It is projected that 3.3 billion tones of CO_2 equivalence of greenhouse gases are emitted into the sky each year as a result of food being wasted. The entire amount of water utilized each year in the production of food that is either lost over time (250 km3) is similar to the yearly flow of the Volga River in Russia or 3 times the volume of Lake Geneva. This is a significant issue that has to be addressed. In a similar vein, each year, 1.4 billion hectares of land, which accounts for 28 percent of the total agricultural acreage on the planet, is used to produce food that is either not consumed or is wasted. Agriculture is the primary cause of the bulk of the dangers that face endangered plant and animal species that are monitored by the International Union for the Conservation of Nature (IUCN). Composting only accounts for a small fraction of the total amount of wasted food; the vast majority of it is transferred to landfills, where it contributes significantly to the volume of municipal solid waste. One of the most significant contributors to greenhouse gas emissions from the waste industry is the methane that is released from landfills. Composting at home has the ability to divert up to 150 kg per household per year of food waste away from the collecting authority in the community. FW at the retail and consumer level is often greater in areas with middle- and high-incomes, but food losses during agricultural production are more common in developing nations. To exclude fish and seafood from the calculation, the yearly cost of the direct economic effects of food waste is estimated to be 750 billion dollars. Total FW per year and estimated FW per capita for different countries given in Figures 2 and 3 respectively [57].

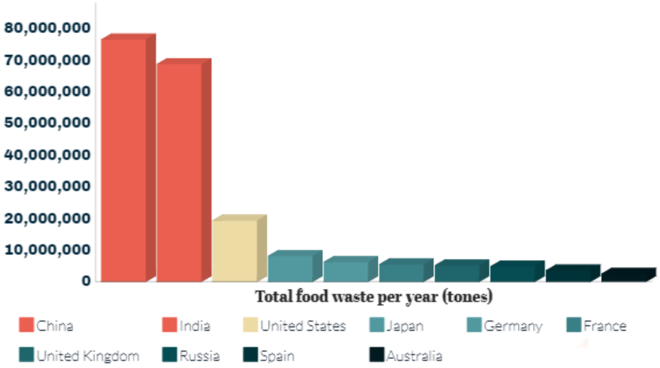

Figure 2. Total food waste in year 2021 for different countries.

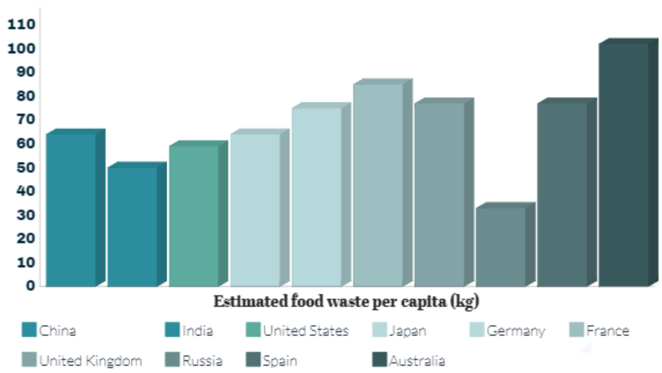

Figure 3. Total food waste in year 2021 per capita for different countries.

FW presents transfer challenges basically due to its tall dampness substance, oil substance, and heterogeneous nature. The choice of waste management technology could be a complex issue from two major perspectives. Firstly, it requires thought of data and investigation from a few distinctive disciplines, to be specific financial matters, supply administration, designing, material science, and chemistry. Besides, determination requires communication and understanding of diverse viewpoints from inside an organization from providers to handlers, and security officers to administration [61]. A recent review of MCDM within the improvement of industrial waste administration approaches universally appears how the goals of these frameworks have become progressively complex over later decades. Babalola conducted a ponder to assess diverse treatment choices for large volumes of nourishment and natural squander and their reasonableness for Japan utilizing MCDM [62]. Additionally in 2016, Mir et al. utilized MCDM to compare and rank 11 MSW treatment strategy scenarios for Malaysia ecologically and financially [63]. The choice investigation, in this manner, has been broadly supported and executed as a implies to supply a straightforward examination and a way to consider and reconcile diverse points of view within the determination of waste administration. Partner engagement utilizing MCDM to back metropolitan solid waste administration has been detailed to facilitate communication, assess clashes and bolster the positioning of diverse techniques.

FW demands several treatment techniques. Burning, anaerobic assimilation, landfilling, pyrolysis, ethanol ageing, and gasification are a few innovations for FW treatment (FWT) that involve diverse financial guesses and have diverse social and environmental repercussions. Various writers have conducted a small number of investigations in this area. In general, FWTT determination necessitates a full analysis of arrangement planning, country's GDP, garbage collection, transparency index, availability of suitable technology, people's awareness of supportability, etc., in which several viable technological options are studied. Consequently, it can be acknowledged that determining an optimal FWTT may involve a complex decision-making process governed by a variety of variables. MCDM methods are applied well for this purpose.

In this section, we actualize the presented decision-making strategy on the choice of alluring FWTT among a set of choices, which uncovers the viability and practicality of the developed strategy. After a preparatory investigation, a gather of specialists ($\mathscr{P}^{Q}{}_1$, $\mathscr{P}^{Q}{}_2$, $\mathscr{P}^{Q}{}_3$) has chosen, five technologies as the candidate of FWTT, which are anaerobic absorption $\amalg^{\varsigma}{}_1$, incineration $\amalg^{\varsigma}{}_2$, composting $\amalg^{\varsigma}{}_3$, landfill $\amalg^{\varsigma}{}_4$, and gasification $\amalg^{\varsigma}{}_5$. DMs evaluating the five technologies under the criterion given in Table 2. Here we take $t = 3$.

Table 2. Criterion for land selection.

	Criterion
$\mathscr{C}_1^{\urcorner}$	Environmental capacity involvement with the developed scenario
$\mathscr{C}_2^{\urcorner}$	Time and space are required for residential waste management
$\mathscr{C}_3^{\urcorner}$	number of eco-friendly purchases
$\mathscr{C}_3^{\urcorner}$	Number of participants in promoting the model

6.1. Decision-making process

Step 1:

LTs for each DM given in Table 3. By the LTs find the DMs weights by the Eq (5.1), Then the DMs weight are $\zeta_1 = 0.3990$, $\zeta_2 = 0.2760$ and $\zeta_3 = 0.3250$.

Table 3. Linguistic terms for DMs.

DM	Linguistic terms
$\mathscr{P}^{Q}{}_1$	Very suitable
$\mathscr{P}^{Q}{}_2$	Medium suitable
$\mathscr{P}^{Q}{}_3$	Suitable

Step 2:

Obtain the decision matrix $\mathscr{P}^{Q}{}_{(p)} = (\mathfrak{Y}^{(p)}_{ji})_{n\times m}$ in the format of T-SPFNs from DMs. The judgement values, given by three DMs, are given in Tables 4–6.

Table 4. Assessment matrix acquired from $\mathscr{P}^{Q}{}_1$.

	$\mathscr{C}_1^{\urcorner}$	$\mathscr{C}_2^{\urcorner}$	$\mathscr{C}_3^{\urcorner}$	$\mathscr{C}_4^{\urcorner}$
$Ш^{\varsigma}{}_1$	(0.425, 0.255, 0.155)	(0.352, 0.256, 0.106)	(0.359, 0.215, 0.178)	(0.313, 0.243, 0.212)
$Ш^{\varsigma}{}_2$	(0.356, 0.154, 0.221)	(0.332, 0.312, 0.211)	(0.290, 0.312, 0.145)	(0.434, 0.255, 0.157)
$Ш^{\varsigma}{}_3$	(0.203, 0.178, 0.248)	(0.154, 0.287, 0.246)	(0.257, 0.235, 0.153)	(0.275, 0.385, 0.213)
$Ш^{\varsigma}{}_4$	(0.345, 0.120, 0.158)	(0.483, 0.268, 0.121)	(0.472, 0.181, 0.231)	(0.335, 0.265, 0.235)
$Ш^{\varsigma}{}_5$	(0.165, 0.370, 0.135)	(0.480, 0.260, 0.195)	(0.280, 0.315, 0.320)	(0.115, 0.175, 0.335)

Table 5. Assessment matrix acquired from $\mathscr{P}^{Q}{}_2$.

	$\mathscr{C}_1^{\urcorner}$	$\mathscr{C}_2^{\urcorner}$	$\mathscr{C}_3^{\urcorner}$	$\mathscr{C}_4^{\urcorner}$
$Ш^{\varsigma}{}_1$	(0.340, 0.335, 0.120)	(0.560, 0.125, 0.130)	(0.345, 0.140, 0.115)	(0.515, 0.310, 0.111)
$Ш^{\varsigma}{}_2$	(0.515, 0.235, 0.134)	(0.165, 0.355, 0.220)	(0.370, 0.425, 0.120)	(0.452, 0.111, 0.123)
$Ш^{\varsigma}{}_3$	(0.240, 0.121, 0.315)	(0.175, 0.470, 0.265)	(0.255, 0.375, 0.240)	(0.253, 0.135, 0.145)
$Ш^{\varsigma}{}_4$	(0.135, 0.240, 0.265)	(0.280, 0.330, 0.145)	(0.275, 0.255, 0.340)	(0.253, 0.235, 0.365)
$Ш^{\varsigma}{}_5$	(0.365, 0.201, 0.235)	(0.281, 0.240, 0.355)	(0.280, 0.135, 0.330)	(0.154, 0.253, 0.335)

Table 6. Assessment matrix acquired from $\mathscr{P}^{Q}{}_3$.

	$\mathscr{C}_1^{\urcorner}$	$\mathscr{C}_2^{\urcorner}$	$\mathscr{C}_3^{\urcorner}$	$\mathscr{C}_4^{\urcorner}$
$Ш^{\varsigma}{}_1$	(0.475, 0.145, 0.110)	(0.345, 0.252, 0.203)	(0.555, 0.303, 0.115)	(0.425, 0.135, 0.111)
$Ш^{\varsigma}{}_2$	(0.335, 0.245, 0.145)	(0.145, 0.225, 0.335)	(0.230, 0.153, 0.420)	(0.235, 0.435, 0.245)
$Ш^{\varsigma}{}_3$	(0.370, 0.265, 0.275)	(0.135, 0.210, 0.545)	(0.235, 0.152, 0.220)	(0.265, 0.225, 0.433)
$Ш^{\varsigma}{}_4$	(0.220, 0.465, 0.135)	(0.250, 0.415, 0.245)	(0.415, 0.152, 0.140)	(0.265, 0.445, 0.135)
$Ш^{\varsigma}{}_5$	(0.143, 0.154, 0.465)	(0.270, 0.458, 0.165)	(0.120, 0.415, 0.260)	(0.215, 0.115, 0.435)

Step 3:

To construct the aggregated T-SPF decision matrix, all individual opinions must be totalled up and integrated to form a group opinion.

$H = \left(\tilde{\eth}_{ji}\right)_{5\times 4}$ be the aggregated T-SPF decision matrix, where

$\tilde{\eth}_{ji} = T - SPFFWA \left(\mathfrak{Y}_{ji}^{(1)}, \mathfrak{Y}_{ji}^{(2)}, \mathfrak{Y}_{ji}^{(3)}\right) = \left(\zeta_1 * \mathfrak{Y}_{ji}^{(1)} \tilde{\oplus} \zeta_2 * \mathfrak{Y}_{ji}^{(2)} \tilde{\oplus} \zeta_3 * \mathfrak{Y}_{ji}^{(3)}\right)$. Aggregated T-SPF decision matrix given in Table 7.

Table 7. Aggregated T-SPF decision matrix.

	$\mathscr{C}_1^{\urcorner}$	$\mathscr{C}_2^{\urcorner}$
$Ш^{\varsigma}{}_1$	(0.285906, 0.114804, 0.050480)	(0.272807, 0.114013, 0.056356)
$Ш^{\varsigma}{}_2$	(0.254533, 0.094056, 0.078675)	(0.117772, 0.178783, 0.139180)
$Ш^{\varsigma}{}_3$	(0.142084, 0.088203, 0.156125)	(0.077527, 0.205063, 0.236169)
$Ш^{\varsigma}{}_4$	(0.180479, 0.147796, 0.104477)	(0.240093, 0.212379, 0.072306)
$Ш^{\varsigma}{}_5$	(0.124448, 0.185574, 0.161027)	(0.253588, 0.199940, 0.113938)

	$\mathscr{C}_3^{\urcorner}$	$\mathscr{C}_4^{\urcorner}$
$Ш^{\varsigma}{}_1$	(0.294449, 0.114730, 0.0600663)	(0.274437, 0.145051, 0.0669727)
$Ш^{\varsigma}{}_2$	(0.198061, 0.180828, 0.110345)	(0.268425, 0.153467, 0.0866869)
$Ш^{\varsigma}{}_3$	(0.139668, 0.122039, 0.0922995)	(0.174100, 0.165420, 0.151573)
$Ш^{\varsigma}{}_4$	(0.195787, 0.090117, 0.113357)	(0.182522, 0.204743, 0.117831)
$Ш^{\varsigma}{}_5$	(0.116394, 0.175757, 0.193227)	(0.060994, 0.072372, 0.232129)

Step 4:
Here is no cost type attribute so, the normalized decision matrix will be $\Gamma_N = \left(\aleph_{ji}^{N}\right)_{n\times m}$ $= \left(\breve{\mu}_{ji}, \breve{\nu}_{ji}, \breve{\tau}_{ji}\right)_{5\times 4}$, given in Table 8.

Table 8. Normalized T-SPF decision matrix.

	$\mathscr{C}_1^{\urcorner}$	$\mathscr{C}_2^{\urcorner}$
Ш^{ς}_1	(0.285906, 0.114804, 0.050480)	(0.272807, 0.114013, 0.056356)
Ш^{ς}_2	(0.254533, 0.094056, 0.078675)	(0.117772, 0.178783, 0.139180)
Ш^{ς}_3	(0.142084, 0.088203, 0.156125)	(0.077527, 0.205063, 0.236169)
Ш^{ς}_4	(0.180479, 0.147796, 0.104477)	(0.240093, 0.212379, 0.072306)
Ш^{ς}_5	(0.124448, 0.185574, 0.161027)	(0.253588, 0.199940, 0.113938)
	$\mathscr{C}_3^{\urcorner}$	$\mathscr{C}_4^{\urcorner}$
Ш^{ς}_1	(0.294449, 0.114730, 0.0600663)	(0.274437, 0.145051, 0.0669727)
Ш^{ς}_2	(0.198061, 0.180828, 0.110345)	(0.268425, 0.153467, 0.0866869)
Ш^{ς}_3	(0.139668, 0.122039, 0.0922995)	(0.174100, 0.165420, 0.151573)
Ш^{ς}_4	(0.195787, 0.090117, 0.113357)	(0.182522, 0.204743, 0.117831)
Ш^{ς}_5	(0.116394, 0.175757, 0.193227)	(0.060994, 0.072372, 0.232129)

Step 5:
Construct the score matrix, by utilizing the SF of T-SPFNs as $\Psi = \left(\breve{\Theta}^{\zeta}\left(\aleph_{ji}^{N}\right)\right)_{5\times 4}$.

$$
\begin{array}{c|cccc}
 & \mathscr{C}_1^{\urcorner} & \mathscr{C}_2^{\urcorner} & \mathscr{C}_3^{\urcorner} & \mathscr{C}_4^{\urcorner} \\
\text{Ш}^{\varsigma}_1 & 0.0250124 & 0.0219643 & 0.0272556 & 0.0240217 \\
\text{Ш}^{\varsigma}_2 & 0.0178093 & 0.0100441 & 0.0150260 & 0.0236064 \\
\text{Ш}^{\varsigma}_3 & 0.0073595 & 0.0222615 & 0.0053284 & 0.0132859 \\
\text{Ш}^{\varsigma}_4 & 0.0102475 & 0.0237974 & 0.0096936 & 0.0162993 \\
\text{Ш}^{\varsigma}_5 & 0.0124935 & 0.0257794 & 0.0142206 & 0.0131140
\end{array}
$$

Step 6:
Consider that the DMs provide the following partial weight details about the attribute weights:
$\Psi = 0.10 \leq \mathscr{W}_1^{\gamma} \leq 0.35, 0.20 \leq \mathscr{W}_2^{\gamma} \leq 0.53, 0.10 \leq \mathscr{W}_3^{\gamma} \leq 0.70, 0.15 \leq \mathscr{W}_4^{\gamma} \leq 0.65$
Relying on this data, the following optimization framework can be developed:

$$
\begin{aligned}
Max\ \ g \ = \ & 0.0250124\mathscr{W}_1^{\gamma} + 0.0178093\mathscr{W}_1^{\gamma} + 0.0073595\mathscr{W}_1^{\gamma} + 0.0102475\mathscr{W}_1^{\gamma} + 0.0124935\mathscr{W}_1^{\gamma} \\
& 0.0219643\mathscr{W}_2^{\gamma} + 0.0100441\mathscr{W}_2^{\gamma} + 0.0222615\mathscr{W}_2^{\gamma} + 0.0237974\mathscr{W}_2^{\gamma} + 0.0257794\mathscr{W}_2^{\gamma} \\
& 0.0272556\mathscr{W}_3^{\gamma} + 0.0150260\mathscr{W}_3^{\gamma} + 0.0053284\mathscr{W}_3^{\gamma} + 0.0096936\mathscr{W}_3^{\gamma} + 0.0142206\mathscr{W}_3^{\gamma} \\
& 0.0240217\mathscr{W}_4^{\gamma} + 0.0236064\mathscr{W}_4^{\gamma} + 0.0132859\mathscr{W}_4^{\gamma} + 0.0162993\mathscr{W}_4^{\gamma} + 0.0131140\mathscr{W}_4^{\gamma},
\end{aligned}
$$

such that,
$0.10 \leq \mathscr{W}_1^{\gamma} \leq 0.35,\quad 0.20 \leq \mathscr{W}_2^{\gamma} \leq 0.53,\quad 0.10 \leq \mathscr{W}_3^{\gamma} \leq 0.70,\quad 0.15 \leq \mathscr{W}_4^{\gamma} \leq 0.65,$
$\mathscr{W}_1^{\gamma} + \mathscr{W}_2^{\gamma} + \mathscr{W}_3^{\gamma} + \mathscr{W}_4^{\gamma} = 1,\quad \mathscr{W}_1^{\gamma}, \mathscr{W}_2^{\gamma}, \mathscr{W}_3^{\gamma}, \mathscr{W}_4^{\gamma} \geq 0.$
By solving this model we get, $\mathscr{W}_1^{\gamma} = 0.1, \mathscr{W}_2^{\gamma} = 0.53, \mathscr{W}_3^{\gamma} = 0.1, \mathscr{W}_4^{\gamma} = 0.27.$
Step 7:
Evaluate the aggregated weighted T-SPF decision matrix by using proposed AOs given by Table 9.

Table 9. Aggregated weighted T-SPF decision matrix.

$\text{Ш}^{\varsigma}{}_1$	(0.277325, 0.122136, 0.058918)
$\text{Ш}^{\varsigma}{}_2$	(0.189635, 0.182495, 0.128065)
$\text{Ш}^{\varsigma}{}_3$	(0.120535, 0.187258, 0.202940)
$\text{Ш}^{\varsigma}{}_4$	(0.221568, 0.194255, 0.093429)
$\text{Ш}^{\varsigma}{}_5$	(0.186953, 0.187209, 0.189453)

Step 8:
Compute the score values of all alternatives.

$$\begin{aligned} S\left(\text{Ш}^{\varsigma}{}_1\right) &= 0.0211243 \\ S\left(\text{Ш}^{\varsigma}{}_2\right) &= 0.00471918 \\ S\left(\text{Ш}^{\varsigma}{}_3\right) &= -0.0066068 \\ S\left(\text{Ш}^{\varsigma}{}_4\right) &= 0.01006180 \\ S\left(\text{Ш}^{\varsigma}{}_5\right) &= -0.0002658. \end{aligned}$$

At the end, the final ranking will be

$$\text{Ш}^{\varsigma}{}_1 \succ \text{Ш}^{\varsigma}{}_2 \succ \text{Ш}^{\varsigma}{}_4 \succ \text{Ш}^{\varsigma}{}_3 \succ \text{Ш}^{\varsigma}{}_5.$$

Thus, we conclude that the optimal FWTT option is incineration $\text{Ш}^{\varsigma}{}_1$.

6.2. Comparison with existing AOs

To check the validity and authenticity of the proposed AOs in this article, we solve our given problem employing fundamental averaging AOs suggested by [37]. We take the “normalized T-SPF decision matrix” from **Step 4** and the WV obtained from the partial weight details from **Step 6** as $\mathscr{W}_1^{\gamma} = 0.32, \mathscr{W}_2^{\gamma} = 0.20, \mathscr{W}_3^{\gamma} = 0.33, \mathscr{W}_4^{\gamma} = 0.15$. We apply “T-spherical fuzzy weighted average (T-SPFWA) operator”, on the T-SPFNs given in Table 10 and get the following results given in Table 11.

Table 10. T-SPF decision matrix.

	$\mathscr{C}_1^{\urcorner}$	$\mathscr{C}_2^{\urcorner}$
$Ц^{\varsigma}_1$	(0.431824, 0.235033, 0.135906)	(0.411300, 0.229910, 0.143730)
$Ц^{\varsigma}_2$	(0.395850, 0.203841, 0.180950)	(0.234851, 0.310206, 0.262513)
$Ц^{\varsigma}_3$	(0.267909, 0.194961, 0.285270)	(0.171292, 0.327612, 0.359957)
$Ц^{\varsigma}_4$	(0.295482, 0.258636, 0.205240)	(0.374920, 0.345483, 0.168449)
$Ц^{\varsigma}_5$	(0.229988, 0.300186, 0.273095)	(0.386231, 0.329630, 0.226572)
	$\mathscr{C}_3^{\urcorner}$	$\mathscr{C}_4^{\urcorner}$
$Ц^{\varsigma}_1$	(0.435387, 0.232267, 0.150877)	(0.415799, 0.271811, 0.162375)
$Ц^{\varsigma}_2$	(0.325725, 0.306545, 0.220540)	(0.403580, 0.278008, 0.189969)
$Ц^{\varsigma}_3$	(0.263540, 0.240870, 0.199947)	(0.299333, 0.289299, 0.272920)
$Ц^{\varsigma}_4$	(0.329464, 0.196406, 0.228867)	(0.313588, 0.338549, 0.234237)
$Ц^{\varsigma}_5$	(0.233374, 0.307167, 0.327199)	(0.154416, 0.173071, 0.376405)

Table 11. Aggregated weighted T-SPF decision matrix by using T-SPFWA operator.

$Ц^{\varsigma}_1$	(0.277183, 0.121832, 0.124118)
$Ц^{\varsigma}_2$	(0.178633, 0.161073, 0.167273)
$Ц^{\varsigma}_3$	(0.119849, 0.168856, 0.182187)
$Ц^{\varsigma}_4$	(0.215161, 0.186141, 0.198276)
$Ц^{\varsigma}_5$	(0.151333, 0.148903, 0.176793)

Compute the score values of all alternatives.

$$S\left(Ц^{\varsigma}_1\right) = 0.0193839$$

$$S\left(Ц^{\varsigma}_2\right) = 0.0010198$$

$$S\left(Ц^{\varsigma}_3\right) = -0.004325$$

$$S\left(Ц^{\varsigma}_4\right) = 0.0021658$$

$$S\left(Ц^{\varsigma}_5\right) = -0.002060.$$

At the end, the final ranking will be

$$Ц^{\varsigma}_1 \succ Ц^{\varsigma}_4 \succ Ц^{\varsigma}_2 \succ Ц^{\varsigma}_3 \succ Ц^{\varsigma}_5.$$

We achieve the same optimum solution employing the T-SPFWA operator, illustrating the resilience of our proposed AOs.

By analyzing the inputs with specific pre-existing AOs and obtaining an equal optimal solution, we equalize our outcomes. This proves the robustness and reliability of the suggested AOs. Due to its unbiased or neutral operation for T-SPFNs, the offered technique is more practicable and preferable to some previous AOs. The comparison of proposed AOs with some existing operators are given in Table 12.

Table 12. Comparison of proposed AOs with some exiting operators.

Authors	AOs	Ranking of alternatives	The optimal alternative
Mahmood *et al.* [37]	TSFWG	$\amalg^{\varsigma}_1 > \amalg^{\varsigma}_2 > \amalg^{\varsigma}_4 > \amalg^{\varsigma}_3 > \amalg^{\varsigma}_5$	$\amalg^{\varsigma}_1$
Munir *et al.* [38]	T-SFEWA	$\amalg^{\varsigma}_1 > \amalg^{\varsigma}_3 > \amalg^{\varsigma}_2 > \amalg^{\varsigma}_4 > \amalg^{\varsigma}_5$	$\amalg^{\varsigma}_1$
	T-SFEOWA	$\amalg^{\varsigma}_1 > \amalg^{\varsigma}_4 > \amalg^{\varsigma}_3 > \amalg^{\varsigma}_2 > \amalg^{\varsigma}_5$	$\amalg^{\varsigma}_1$
Zeng *et al.* [39]	T-SFEWIG	$\amalg^{\varsigma}_1 > \amalg^{\varsigma}_2 > \amalg^{\varsigma}_4 > \amalg^{\varsigma}_3 > \amalg^{\varsigma}_5$	$\amalg^{\varsigma}_1$
	T-SFEOWIG	$\amalg^{\varsigma}_1 > \amalg^{\varsigma}_4 > \amalg^{\varsigma}_2 > \amalg^{\varsigma}_3 > \amalg^{\varsigma}_5$	$\amalg^{\varsigma}_1$
Liu *et al.* [40]	SPFPMM	$\amalg^{\varsigma}_1 > \amalg^{\varsigma}_3 > \amalg^{\varsigma}_2 > \amalg^{\varsigma}_4 > \amalg^{\varsigma}_5$	$\amalg^{\varsigma}_1$
	SPFPDMM	$\amalg^{\varsigma}_1 > \amalg^{\varsigma}_5 > \amalg^{\varsigma}_2 > \amalg^{\varsigma}_3 > \amalg^{\varsigma}_4$	$\amalg^{\varsigma}_1$
Kifayat-Ullah *et al.* [41]	TSFHWA	$\amalg^{\varsigma}_1 > \amalg^{\varsigma}_4 > \amalg^{\varsigma}_2 > \amalg^{\varsigma}_5 > \amalg^{\varsigma}_3$	$\amalg^{\varsigma}_1$
	TSFOHWA	$\amalg^{\varsigma}_1 > \amalg^{\varsigma}_4 > \amalg^{\varsigma}_2 > \amalg^{\varsigma}_3 > \amalg^{\varsigma}_5$	$\amalg^{\varsigma}_1$
Khan *et al.* [42]	T-SPHFSSPHEM	$\amalg^{\varsigma}_1 > \amalg^{\varsigma}_5 > \amalg^{\varsigma}_2 > \amalg^{\varsigma}_3 > \amalg^{\varsigma}_4$	$\amalg^{\varsigma}_1$
	T-SPHFSSPGHEM	$\amalg^{\varsigma}_1 > \amalg^{\varsigma}_2 > \amalg^{\varsigma}_3 > \amalg^{\varsigma}_4 > \amalg^{\varsigma}_5$	$\amalg^{\varsigma}_1$
Proposed	T-SPFFWA	$\amalg^{\varsigma}_1 > \amalg^{\varsigma}_4 > \amalg^{\varsigma}_2 > \amalg^{\varsigma}_3 > \amalg^{\varsigma}_5$	$\amalg^{\varsigma}_1$

7. Conclusions

Present research indicates that when a DM provides an equal number of membership, abstinence and non-membership evaluations for an object, the resulting aggregate assessments are uneven. In this scenario, we proposed some novel fairly or neutrality operations based on T-SPFS and proportional distribution rules for membership, abstinence and non-membership functions, with an emphasis on correctness and applicability throughout decision making impacted by the DM's attitude. We contributed some "T-spherical fuzzy fairly weighted averaging (T-SPFFWA) operator" and "T-spherical fuzzy fairly ordered weighted averaging (T-SPFFOWA) operator" for the T-SPFN information, inspired by fairly operations. We discussed the prospective AOs' characteristics in considerable detail. The primary benefit of the suggested operators is that they facilitate not just interaction amongst pairs of diverse T-SPFNs, but also the exploration of DM attitude characteristics by enabling for a categorical treatment of the T-SPFS degrees. The developed strategy was compared with extant strategies to confirm its legitimacy. The results gotten by the proposed strategy confirm that it encompasses a better than average viability and soundness and is reliable with extant models. There is no interaction between the membership, abstention, and non-membership degrees as we discuss the limitations of proposed AOs. On this side of the proposed AOs, the hybrid structure of interactive AOs is implemented. This model was expanded to incorporate the global dynamics of a three-species spatial food chain model [64] and a multi-source fluid queue-based stochastic model [65].

Conflict of interest

The authors declare that they have no conflict of interest.

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