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Research article

Robustness analysis of fuzzy BAM cellular neural network with time-varying delays and stochastic disturbances

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Abstract: Robustness analysis for the global exponential stability of fuzzy bidirectional associative memory cellular neural network (FBAMCNN) is explored in this paper. By applying Gronwall-Bellman lemma and other inequality techniques, the range limits of both time-varying delays and the intensity of noise that FBAMCNN can withstand to maintain globally exponentially stable is estimated. It means that if the intensities of interference are larger than the bounds we derived, then the perturbed system may lose global exponential stability. Several instances are given to support our main results.

Keywords: robustness analysis; fuzzy bidirectional memory cellular neural network; time-varying delays; stochastic disturbances
Mathematics Subject Classification: 93B35, 93D23

1. Introduction

Artificial neural networks (ANN) mimic biological neurons in design. Numerous extensions of ANN have been developed based on ANN and are in use today [1], such as cellular neural networks (CNN). CNN, a subclass of ANN that Chua and Yang initially presented in [2, 3], successfully addresses ANN's shortcomings by reducing the number of connections while maintaining the benefits of parallel processing. Additionally, neurons in a CNN can only communicate with neurons in the same region. As a result, CNN may simply be extended based on these characteristics without requiring structural changes.

As one of the most important extensions, the bidirectional associative memory cellular neural network (BAMCNN) model was first proposed by Kosko [4]. Two-layer neurons make up the pattern-matched hetero associative BAMCNN, which is a generalization of the single-layer auto-associative Hebbian network. Neurons in one layer are no interconnection, but they are interconnected with neurons in another layer fully. Based on these properties, BAMCNN has gained a lot of attentions since it was proposed [5–7].

In practical engineering applications, time delay and stochastic disturbances are inevitable due to the limited conversion speed of amplifier and the noise of signal transmission of electronic equipment, we can easily observe those two perturbations experimentally and numerically. Different time delays will lead to different response of the dynamical behaviours of the delayed systems. In our knowledge, the main types of time delay are constant delay [8], time varying delay [8], distributed delay [9], proportional delay [5] and so on [10]. Stochastic disturbances, where the structure of random disturbances is extremely fresh and complex, which is quite different from the traditional process. For BAMCNN affected by the above two disturbances, the research of dynamical behaviours of it gets more and more attention in decades [5, 11–13]. In [11], Park et al. estimate the covergence rate of delayed BAM neural network and criteria of exponential stability is studied. The problem of delay-dependent and independent state estimation of BAMCNN is explored in [5]. Wang et al. [12] discussed the stability of a delayed higher-order stochastic BAM neural network. In [13], novel criterion are given for stability of a type hybrid BAM neural network with stochastic noises by Li and Shen.

It is worth to point out that the literature above is mainly explore the stability of BAM neural networks disturbed by time delays or stochastic disturbances without fuzzy logic. However, neural networks with fuzzy logic can model these issues better than general neural network for those physical events for which mathematical models are hard to obtain, dynamic properties are challenging to master, or the changes are particularly large. Therefore, in 1996, Yang and Yang combine fuzzy logic with CNN for the first time and discuss its stability in detail [14, 15]. FCNN is a kind of CNN which include fuzzy logical in its structure, and also maintains the local connectivity of cells. With the use of fuzzy logic, we can integrate CNNs' low and high level information processing capabilities. And in [16], Yang et al. pointed out the differences between FCNN and CNN in mathematical morphological operations. Furthermore, with the gradual improvement of the theory of FCNN, various properties and applications of FCNN have been researched broadly [8, 17–21].

In addition, many researchers have extended FCNN to fuzzy BAMCNN (FBAMCNN) in recent decades. The stability of FBAMCNN with delays and stochastic disturbance has also been extensively studied by Lyapunov method, Razumikhin-type method etc, see [22–24]. In [22], the stability of delayed cohen-grossberg FBAMCNN with markovian switching is investigated. In [23, 24], Ali et al. derived novel criteria of robust stability and stability in mean-square of stochastic FBAMCNN respectively.

From the above discussions, we can see that all the above results mainly discuss the stability of FBAMCNN with perturbations, and do not explore the robustness of the stability of FBAMCNN with perturbations. The problem of the robustness of stability (RoS) was first proposed by Shen in [25], and has attracted wide attention in recent years [26, 27]. However, as far as we know, there are few researchers to study the RoS of FBAMCNN with perturbations.

Motivated by the above discussions, we explore the robustness of the exponential stability of FBAMCNN disturbed by time-varying delays and stochastic disturbances in this paper. In short, our work and contribution are listed below.

- The RoS of delayed BAMCNN (DFBAMCNN) and stochastic DFBAMCNN (SDBAMCNN) are investigated by utilizing Gronwall inequality as well as other inequality techniques, the upper limits of delays and noise intensities to sustain their original stability are estimated.
- Compared to [26], fuzzy logic are included in the systems we considered in this paper, hence, their complexity are improved to a certain extent. In addition, we extend the single layer neural

network to BAM neural network which also increases the difficulty of analysis. Furthermore, the FBAMCNN considered in this paper can handle imprecise or uncertain information better than BAMCNN. And, the results derived in this paper play an important role in the design and application of FBAMCNN.

Finally, there is a brief introduction to the main works of each section. We introduce the model of our system and the primaries we needed in the latter in Section 2. We explore the RoS of DFBAMCNN in Section 3, and we get the limit of time-varying delays. In Section 4, SDFBAMCNN is considered, and we get the max intensities of both time delay and stochastic noise. and Section 5 includes various numerical instances to test the usefulness of the results.

Notations: Denote $\mathbb{R} = (-\infty, +\infty)$, $\mathbb{R}^+ = [0, +\infty)$, $\mathbb{R}^m = \{\chi | \chi = \{\chi_1, \ldots, \chi_m\}, \chi_i \in \mathbb{R}, i = 1, 2, \ldots, m\}$. $\mathbb{N}^+ = \{1, 2, \cdots\}$. $|\cdot|$ represents the absolute value of real numbers and $\|\mathfrak{U}(t)\| = \sum_{i=1}^{m} |\mathfrak{U}_i(t)|$, where $\mathfrak{U}(t) = (\mathfrak{U}_1(t), \cdots, \mathfrak{U}_m(t))^T$. Complete filtered probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, P)$ embraces all P-null sets, where $\{\mathcal{F}_t\}_{t\geq 0}$ is a right continuous filtration that satisfies the usual conditions. Scalar Brownian movement $\mho(t)$ is defined at $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, P)$. Operator E is used to calculate mathematical expectations. Denote $L^2_{\mathcal{F}_0}([-\bar{\mathfrak{G}}, 0]; \mathbb{R}^n)$ as the family of all $\mathcal{F}_0$ measurable $C([-\bar{\mathfrak{G}}, 0]; \mathbb{R}^n)$ valued random variables $\hbar = \{\hbar(\theta) : -\bar{\mathfrak{G}} \leq \theta \leq 0\}$ such that $\sup_{-\bar{\mathfrak{G}} \leq \theta \leq 0} E\|\hbar(\theta)\|^2 < \infty$. $\lambda(t)$ and $\upsilon(t)$ are delay functions which satisfy $\lambda(t)'$, $\upsilon'(t) \leq \wp^* < 1$, and we assume that they have boundaries λ and υ respectively. $\bigwedge$ and $\bigvee$ denote AND and OR operations in fuzzy logic respectively.

2. Primaries

Consider the following FBAMCNN:

$$
\begin{cases}
\dot{\varpi}_k(t) = -c_k \varpi_k(t) + \bigwedge\limits_{r=1}^{p} \alpha_{kr} f_r(\zeta_r(t)) + \bigvee\limits_{r=1}^{p} \beta_{kr} f_r(\zeta_r(t)) + \bigwedge\limits_{r=1}^{p} F_{kr} u_r(t) + \bigvee\limits_{r=1}^{p} E_{kr} u_r(t) + I_k, \\
\dot{\zeta}_r(t) = -d_r \zeta_r(t) + \bigwedge\limits_{k=1}^{q} \omega_{rk} g_k(\varpi_k(t)) + \bigvee\limits_{k=1}^{q} \gamma_{rk} g_k(\varpi_k(t)) + \bigwedge\limits_{k=1}^{q} G_{rk} v_k(t) + \bigvee\limits_{k=1}^{q} H_{rk} v_k(t) + J_r
\end{cases}
\tag{2.1}
$$

with initial value $(\varpi_0, \zeta_0)^T$, where $k, r \in \mathbb{N}^+$ and $\varpi_0 = (\varpi_1(t_0), \ldots, \varpi_q(t_0))$, $\zeta_0 = (\zeta_1(t_0), \ldots, \zeta_p(t_0))$. With the networks and external inputs disconnecting, c_k and d_r are the rates that present as the kth and rth neuron reset their potential to the isolated resting state. α_{kr} and ω_{rk} are the elements of fuzzy feedback MIN template; β_{kr} and γ_{rk} are elements of fuzzy feedback MAX template; $\Gamma_r(\cdot)$ and $\Xi_k(\cdot)$ are the activation functions; F_{kr} and G_{rk}, E_{kr} and H_{rk} are elements of fuzzy feed-forward MIN template and fuzzy feed-forward MAX template respectively; ϖ_k and ζ_r are the kth and rth neuron respectively; $u_r(t)$ and $v_k(t)$ are the states of DFBAMCNN (2.1); Both of I_k and J_r are constant external inputs.

Assume that $(\varpi(t), \zeta(t))^T$ is one of solutions of FBAMCNN (2.1), and assume $(\varpi^*, \zeta^*)^T$ is the equilibrium point (Ep) of FBAMCNN (2.1), where $\varpi(t) = (\varpi_1(t), \varpi_2(t), \ldots, \varpi_q(t))$, $\zeta(t) = (\zeta_1(t), \zeta_2(t), \ldots, \zeta_p(t))$, $\varpi^* = (\varpi_1^*, \varpi_2^*, \ldots, \varpi_q^*)$, $\zeta^* = (\zeta_1^*, \zeta_2^*, \ldots, \zeta_p^*)$. Let $\Lambda_k(t) = \varpi_k(t) - \varpi_k^*$, $\Upsilon_r(t) = \zeta_r(t) - \zeta_r^*$ and $\Gamma_r(\Upsilon_r(t)) = f_r(\Upsilon_r(t) + \zeta_r^*) - f_r(\zeta_r^*)$, $\Xi_k(\Lambda_k(t)) = g_k(\Lambda_k(t) + \varpi_k^*) - g_k(\varpi_k^*)$. Then

FBAMCNN (2.1) is equivalent with

$$\begin{cases} \dot{\Lambda}_k(t) = -c_k\Lambda_k(t) + \bigwedge_{r=1}^{p} \alpha_{kr}\Gamma_r(\Upsilon_r(t)) + \bigvee_{r=1}^{p} \beta_{kr}\Gamma_r(\Upsilon_r(t)), \\ \dot{\Upsilon}_r(t) = -d_r\Upsilon_r(t) + \bigwedge_{k=1}^{q} \omega_{rk}\Xi_k(\Lambda_k(t)) + \bigvee_{k=1}^{q} \gamma_{rk}\Xi_k(\Lambda_k(t)), \end{cases} \tag{2.2}$$

where initial value of FBAMCNN (2.2) is $(\Lambda_0, \Upsilon_0)^T = (\varpi_0, \zeta_0)^T - (\varpi^*, \zeta^*)^T$. After transform we can observe that the origin is Ep of FBAMCNN (2.2). Hence, the properties of origin point of FBAMCNN (2.2) is the same as the properties of the Ep of FBAMCNN (2.1). Then we give an assumption and a definition we need.

Assumption H(1): There exist $l_r > 0$ and $l_k > 0$ such that

$$|\Gamma_r(\varrho) - \Gamma_r(\varsigma)| \le l_r|\varrho - \varsigma|,$$
$$|\Xi_k(\varrho) - \Xi_k(\varsigma)| \le l_k|\varrho - \varsigma|,$$

and $\Gamma_r(0) = 0$, $\Xi_k(0) = 0$.

Next, the definition of globally exponential stability (GES) of FBAMCNN (2.2) is given below.

Definition 1. *[28] FBAMCNN* (2.2) *is GES if*

$$\|\Phi(t)\| \le \mathcal{L}\|\Theta\|e^{-\mathcal{K}(t-t_0)}, \quad \forall t \in \mathbb{R}^+$$

holds, where $\mathcal{K}$ and $\mathcal{L}$ are two positive constants. $\Phi(t) = (\Lambda_1(t), \cdots, \Lambda_q(t), \Upsilon_1(t), \cdots, \Upsilon_p(t))^T$ is the state of FBAMCNN (2.2)*, and $\Theta = (\phi_1(0), \ldots, \phi_q(0), \varphi_1(0), \ldots, \varphi_p(0))^T$ is the initial value of FBAMCNN* (2.2)*.*

Unless otherwise stated, FBAMCNN (2.1) is GES throughout this article.

Lemma 1. *[14] For FBAMCNN* (2.2)*, we have*

$$|\bigwedge_{r=1}^{p} \alpha_{kr}\Gamma_r(u) - \bigwedge_{r=1}^{p} \alpha_{kr}\Gamma_r(v)| \le \sum_{r=1}^{p} |\alpha_{kr}||\Gamma_r(u) - \Gamma_r(v)|,$$
$$|\bigvee_{r=1}^{p} \beta_{kr}\Gamma_r(u) - \bigvee_{r=1}^{p} \beta_{kr}\Gamma_r(v)| \le \sum_{r=1}^{p} |\beta_{kr}||\Gamma_r(u) - \Gamma_r(v)|,$$
$$|\bigwedge_{k=1}^{q} \omega_{rk}\Xi_k(u) - \bigwedge_{k=1}^{q} \omega_{rk}\Xi_k(v)| \le \sum_{k=1}^{q} |\omega_{rk}||\Xi_k(u) - \Xi_k(v)|,$$
$$|\bigvee_{k=1}^{q} \gamma_{rk}\Xi_k(u) - \bigvee_{k=1}^{q} \gamma_{rk}\Xi_k(v)| \le \sum_{k=1}^{q} |\gamma_{rk}||\Xi_k(u) - \Xi_k(v)|,$$

where u and v are states of (2.2).

3. Robustness of stability of DFBAMCNN

The following is the model of DFBAMCNN we considered in this part.

$$\begin{cases} \dot{\eta}_k(t) = -c_k\eta_k(t) + \bigwedge_{r=1}^{p} \alpha_{kr}\mathit{\Gamma}_r(\vartheta_r(t-\lambda(t))) + \bigvee_{r=1}^{p} \beta_{kr}\mathit{\Gamma}_r(\vartheta_r(t-\lambda(t))), \\ \dot{\vartheta}_r(t) = -d_r\vartheta_r(t) + \bigwedge_{k=1}^{q} \omega_{rk}\mathit{\Xi}_k(\eta_k(t-\upsilon(t))) + \bigvee_{k=1}^{q} \gamma_{rk}\mathit{\Xi}_k(\eta_k(t-\upsilon(t))), \end{cases} \tag{3.1}$$

with initial conditions $(\eta_k(t_0), \vartheta_r(t_0))^T = (\phi_k(t_0), \varphi_r(t_0))^T$, $k, r \in \mathbb{N}^+$ where $\phi \in C([-\bar{\mathfrak{G}}, 0]; \mathbb{R}^q)$, $\varphi \in C([-\bar{\mathfrak{G}}, 0]; \mathbb{R}^p)$. If $\lambda(t) = \upsilon(t) = 0$, the DFBAMCNN (3.1) degenerate to FBAMCNN (2.2).

Lemma 2. *[9] The DFBAMCNN* (3.1) *can be seen as a special instance of the result in [9], thus, the solution* $(\mathit{\Lambda}(t), \mathit{\Upsilon}(t))^T$ *is unique.*

Then, we explore the robustness of DFBAMCNN (3.1).

Theorem 1. *Let H(1) holds, DFBAMCNN* (3.1) *is said to be GES if* $\mathfrak{G} < \min\{\Delta/2, \bar{\mathfrak{G}}\}$, *where* $\bar{\mathfrak{G}}$ *is the positive root of the following equation*

$$(\mathfrak{D} + \mathfrak{C}\mathcal{L}/\mathcal{K})\exp(2\mathfrak{E}\Delta) + \mathcal{L}\exp[-\mathcal{K}(\Delta - \mathfrak{G})] = 1, \tag{3.2}$$

where

$$\Delta > \ln\mathcal{L}/\mathcal{K}, \mathfrak{D} = M_5[\mathfrak{G} + 2\mathfrak{G}(1-\wp^*)^{-1} + M_5\mathfrak{G}^2(1-\wp^*)^{-1}], \mathfrak{C} = M_4\mathfrak{G} + M_5^2\mathfrak{G}(1-\wp^*)^{-1},$$

$$\mathfrak{E} = \max\{m_1 + m_4 + \mathfrak{C}, m_2 + m_3 + \mathfrak{C}\}, m_1 = \max_{1\le k\le q}|c_k|, \quad m_2 = \max_{1\le r\le p} l_r \sum_{k=1}^{q}(|\alpha_{kr}| + |\beta_{kr}|),$$

$$m_3 = \max_{1\le r\le p}|d_r|, \quad m_4 = \max_{1\le k\le q} l_k \sum_{r=1}^{p}(|\omega_{rk}| + |\gamma_{rk}|), M_4 = \max\{m_1m_4, m_2m_3\}, M_5 = \max\{m_2, m_4\}.$$

Proof. By (2.2), (3.1), we have

$$\begin{aligned} |\mathit{\Lambda}_k(t) - \eta_k(t)| \le& \int_{t_0}^{t}\Big[|c_k||\mathit{\Lambda}_k(s) - \eta_k(s)| + |\bigwedge_{r=1}^{p}\alpha_{kr}\mathit{\Gamma}_r(\mathit{\Upsilon}_r(s)) - \bigwedge_{r=1}^{p}\alpha_{kr}\mathit{\Gamma}_r(\vartheta_r(s-\lambda(s)))| \\ &+ |\bigvee_{r=1}^{p}\beta_{kr}\mathit{\Gamma}_r(\mathit{\Upsilon}_r(s)) - \bigvee_{r=1}^{p}\beta_{kr}\mathit{\Gamma}_r(\vartheta_r(s-\lambda(s)))|\Big]ds \\ \le& \int_{t_0}^{t}\Big[|c_k||\mathit{\Lambda}_k(s) - \eta_k(s)| + \sum_{r=1}^{p}|\alpha_{kr}|l_r|\mathit{\Upsilon}_r(s) - \vartheta_r(s-\lambda(s))| \\ &+ \sum_{r=1}^{p}|\beta_{kr}|l_r|\mathit{\Upsilon}_r(s) - \vartheta_r(s-\lambda(s))|\Big]ds. \end{aligned} \tag{3.3}$$

Similarly,

$$|\Upsilon_r(t)-\vartheta_r(t)| \le \int_{t_0}^{t}\Big[|d_r||\Upsilon_r(s)-\vartheta_r(s)| + \sum_{k=1}^{q}|\omega_{rk}|l_k|\Lambda_k(s)-\eta_k(s-\upsilon(s))|$$
$$+\sum_{k=1}^{q}|\gamma_{rk}|l_k|\Lambda_k(s)-\eta_k(s-\upsilon(s))|\Big]ds. \tag{3.4}$$

Then from (3.3), (3.4),

$$\begin{aligned}
&|\Lambda_k(t)-\eta_k(t)| + |\Upsilon_r(t)-\vartheta_r(t)| \\
\le& \int_{t_0}^{t}\Big[|c_k||\Lambda_k(s)-\eta_k(s)| + \sum_{r=1}^{p}|\alpha_{kr}|l_r|\Upsilon_r(s)-\vartheta_r(s-\lambda(s))| + \sum_{r=1}^{p}|\beta_{kr}|l_r|\Upsilon_r(s) \\
&-\vartheta_r(s-\lambda(s))|\Big]ds + \int_{t_0}^{t}\Big[|d_r||\Upsilon_r(s)-\vartheta_r(s)| + \sum_{k=1}^{q}|\omega_{rk}|l_k|\Lambda_k(s)-\eta_k(s-\upsilon(s))| \\
&+\sum_{k=1}^{q}|\gamma_{rk}|l_k|\Lambda_k(s)-\eta_k(s-\upsilon(s))|\Big]ds.
\end{aligned} \tag{3.5}$$

Furthermore,

$$\begin{aligned}
&\sum_{k=1}^{q}|\Lambda_k(t)-\eta_k(t)| + \sum_{r=1}^{p}|\Upsilon_r(t)-\vartheta_r(t)| \\
\le& \int_{t_0}^{t}\Big[\sum_{k=1}^{q}|c_k||\Lambda_k(s)-\eta_k(s)| + \sum_{k=1}^{q}\sum_{r=1}^{p}|\alpha_{kr}|l_r|\Upsilon_r(s)-\vartheta_r(s-\lambda(s))| + \sum_{k=1}^{q}\sum_{r=1}^{p}|\beta_{kr}|l_r|\Upsilon_r(s)-\vartheta_r(s-\lambda(s))|\Big]ds \\
&+\int_{t_0}^{t}\Big[\sum_{r=1}^{p}|d_r||\Upsilon_r(s)-\vartheta_r(s)| + \sum_{r=1}^{p}\sum_{k=1}^{q}|\omega_{rk}|l_k|\Lambda_k(s)-\eta_k(s-\upsilon(s))| + \sum_{r=1}^{p}\sum_{k=1}^{q}|\gamma_{rk}|l_k|\Lambda_k(s)-\eta_k(s-\upsilon(s))|\Big]ds \\
\le& \int_{t_0}^{t}\Big[\sum_{k=1}^{q}|c_k||\Lambda_k(s)-\eta_k(s)| + \sum_{r=1}^{p}\sum_{k=1}^{q}|\alpha_{kr}|l_r|\Upsilon_r(s)-\vartheta_r(s-\lambda(s))| + \sum_{r=1}^{p}\sum_{k=1}^{q}|\beta_{kr}|l_r|\Upsilon_r(s)-\vartheta_r(s-\lambda(s))|\Big]ds \\
&+\int_{t_0}^{t}\Big[\sum_{r=1}^{p}|d_r||\Upsilon_r(s)-\vartheta_r(s)| + \sum_{k=1}^{q}\sum_{r=1}^{p}|\omega_{rk}|l_k|\Lambda_k(s)-\eta_k(s-\upsilon(s))| + \sum_{k=1}^{q}\sum_{r=1}^{p}|\gamma_{rk}|l_k|\Lambda_k(s)-\eta_k(s-\upsilon(s))|\Big]ds.
\end{aligned} \tag{3.6}$$

Let $m_1=\max\limits_{1\le k\le q}|c_k|$, $m_2=\max\limits_{1\le r\le p} l_r\sum\limits_{k=1}^{q}(|\alpha_{kr}|+|\beta_{kr}|)$, $m_3=\max\limits_{1\le r\le p}|d_r|$, $m_4=\max\limits_{1\le k\le q} l_k\sum\limits_{r=1}^{p}(|\omega_{rk}|+|\gamma_{rk}|)$. Then,

$$\begin{aligned}
&\|\Lambda(t)-\eta(t)\| + \|\Upsilon(t)-\vartheta(t)\| \\
\le& \int_{t_0}^{t}\Big[m_1\|\Lambda(s)-\eta(s)\| + m_3\|\Upsilon(s)-\vartheta(s)\| + m_2\|\Upsilon(s)-\vartheta(s-\lambda(s))\| + m_4\|\Lambda(s)-\eta(s-\upsilon(s))\|\Big]ds \\
\le& \int_{t_0}^{t}\Big[(m_1+m_4)\|\Lambda(s)-\eta(s)\| + (m_3+m_2)\|\Upsilon(s)-\vartheta(s)\| + m_2\|\vartheta(s)-\vartheta(s-\lambda(s))\|
\end{aligned}$$

$$+ m_4\|\eta(s) - \eta(s - \upsilon(s))\|\Big]ds. \tag{3.7}$$

Since

$$\begin{aligned}
&\int_{t_0}^{t} m_2\|\vartheta(s) - \vartheta(s - \lambda(s))\|ds \\
=&\int_{t_0}^{t_0+\lambda} m_2\|\vartheta(s) - \vartheta(s - \lambda(s))\|ds + \int_{t_0+\lambda}^{t} m_2\|\vartheta(s) - \vartheta(s - \lambda(s))\|ds,
\end{aligned}$$

and

$$\begin{aligned}
&\int_{t_0}^{t} m_4\|\eta(s) - \eta(s - \upsilon(s))\|ds \\
=&\int_{t_0}^{t_0+\upsilon} m_4\|\eta(s) - \eta(s - \upsilon(s))\|ds + \int_{t_0+\upsilon}^{t} m_4\|\eta(s) - \eta(s - \upsilon(s))\|ds.
\end{aligned}$$

Then, when $t_0 + \lambda \leq t$,

$$\begin{aligned}
&\int_{t_0+\lambda}^{t} m_2\|\vartheta(s) - \vartheta(s - \lambda(s))\|ds \\
\leq& m_2 \int_{t_0+\lambda}^{t} \int_{s-\lambda}^{s} \Big[m_3\|\vartheta(r)\| + m_4\|x(r - \upsilon(r))\|\Big]drds \\
\leq& m_2 \int_{t_0}^{t} dr \int_{\max\{t_0+\lambda, r\}}^{\min\{r+\lambda, t\}} \Big[m_3\|\vartheta(r)\| + m_4\|x(r - \upsilon(r))\|\Big]ds \\
\leq& m_2 m_3 \lambda \int_{t_0}^{t} \|\vartheta(r)\|dr + m_2 m_4 \upsilon(1 - \wp^*)^{-1} \int_{t_0}^{t} \|\eta(u)\|du + m_2 m_4 \upsilon^2(1 - \wp^*)^{-1}\Big(\sup_{t_0-\upsilon\leq s\leq t_0} \|\eta(s)\|\Big).
\end{aligned} \tag{3.8}$$

Likewise, when $t_0 + \upsilon \leq t$,

$$\begin{aligned}
&\int_{t_0+\upsilon}^{t} m_4\|\eta(s) - \eta(s - \upsilon(s))\|ds \\
\leq& m_4 m_1 \upsilon \int_{t_0}^{t} \|\eta(r)\|dr + m_4 m_2 \lambda(1 - \wp^*)^{-1} \int_{t_0}^{t} \|\vartheta(u)\|du + m_4 m_2 \lambda^2(1 - \wp^*)^{-1}\Big(\sup_{t_0-\lambda\leq s\leq t_0} \|\vartheta(s)\|\Big).
\end{aligned} \tag{3.9}$$

We denote $\mathfrak{G} = \max\{\upsilon, \lambda\}$. Thus, for all $t_0 + \mathfrak{G} \leq t$,

$$\begin{aligned}
&\int_{t_0}^{t} m_2\|\vartheta(s) - \vartheta(s - \lambda(s))\|ds \\
\leq& m_2[\lambda + 2\lambda(1 - \wp^*)^{-1}]\Big(\sup_{t_0-\lambda\leq s\leq t_0+\lambda} \|\vartheta(s)\|\Big) + m_2 m_3 \lambda \int_{t_0}^{t} \|\vartheta(r)\|dr \\
&+ m_2 m_4 \upsilon(1 - \wp^*)^{-1} \int_{t_0}^{t} \|\eta(u)\|du + m_2 m_4 \upsilon^2(1 - \wp^*)^{-1}\Big(\sup_{t_0-\upsilon\leq s\leq t_0} \|\eta(s)\|\Big),
\end{aligned} \tag{3.10}$$

and

$$\int_{t_0}^{t} m_4\|\eta(s) - \eta(s - \upsilon(s))\|ds$$

$$\leq m_4[\upsilon + 2\upsilon(1-\wp^*)^{-1}]\Big(\sup_{t_0-\upsilon\leq s\leq t_0+\upsilon}\|\eta(s)\|\Big) + m_4 m_1 \upsilon\int_{t_0}^{t}\|\eta(r)\|dr$$
$$+ m_4 m_2 \lambda(1-\wp^*)^{-1}\int_{t_0}^{t}\|\vartheta(u)\|du + m_4 m_2\lambda^2(1-\wp^*)^{-1}\Big(\sup_{t_0-\lambda\leq s\leq t_0}\|\vartheta(s)\|\Big). \tag{3.11}$$

Let $M_1 = \max\{m_1 m_4 \upsilon + m_2 m_4 \upsilon(1-\wp^*)^{-1}, m_2 m_3\lambda + m_2 m_4 \lambda(1-\wp^*)^{-1}\}$, $M_2 = \max\{m_2[\lambda + 2\lambda(1-\wp^*)^{-1}] + m_2 m_4\lambda^2(1-\wp^*)^{-1}, m_2 m_4 \upsilon^2(1-\wp^*)^{-1} + m_4[\upsilon + 2\upsilon(1-\wp^*)^{-1}]\}$. Thus, by Definition 1,

$$\begin{aligned}
&\|\varLambda(t)-\eta(t)\| + \|\varUpsilon(t)-\vartheta(t)\|\\
\leq &\int_{t_0}^{t}\Big[(m_1+m_4+M_1)\|\varLambda(s)-\eta(s)\| + (m_3+m_2+M_1)\|\varUpsilon(s)-\vartheta(s)\|\Big]ds\\
&+ M_1\int_{t_0}^{t}\Big[\|\varLambda(s)\| + \|\varUpsilon(s)\|\Big]ds + M_2\sup_{t_0-\mathfrak{G}\leq s\leq t_0+\mathfrak{G}}(\|\eta(s)\| + \|\vartheta(s)\|)\\
\leq &\int_{t_0}^{t}\Big[(m_1+m_4+M_1)\|\varLambda(s)-\eta(s)\| + (m_3+m_2+M_1)\|\varUpsilon(s)-\vartheta(s)\|\Big]ds\\
&+ (M_2 + M_1\mathcal{L}/\mathcal{K})\sup_{t_0-\mathfrak{G}\leq s\leq t_0+\mathfrak{G}}(\|\eta(s)\| + \|\vartheta(s)\|).
\end{aligned} \tag{3.12}$$

For simplicity, we denote $M_3 = \max\{m_1+m_4+M_1, m_2+m_3+M_1\}$, $\mathfrak{A} = \|\varLambda(t)-\eta(t)\| + \|\varUpsilon(t)-\vartheta(t)\|$, $\mathfrak{B} = \|\eta(s)\| + \|\vartheta(s)\|$, then, when $t \leq t_0 + 2\Delta$, by applying Gronwall-Bellman lemma,

$$\begin{aligned}
\mathfrak{A} \leq &(M_2 + M_1\mathcal{L}/\mathcal{K})\exp(M_3(t-t_0))\sup_{t_0-\mathfrak{G}\leq s\leq t_0+\mathfrak{G}}\mathfrak{B}\\
\leq &(M_2 + M_1\mathcal{L}/\mathcal{K})\exp(2M_3\Delta)\sup_{t_0-\mathfrak{G}\leq s\leq t_0+\mathfrak{G}}\mathfrak{B}.
\end{aligned} \tag{3.13}$$

Thus, when $t_0 + \mathfrak{G} \leq t \leq t_0 + 2\Delta$,

$$\begin{aligned}
\|\eta(t)\| + \|\vartheta(t)\| \leq &\|\varLambda(t)-\eta(t)\| + \|\varUpsilon(t)-\vartheta(t)\| + \|\varLambda(t)\| + \|\varUpsilon(t)\|\\
\leq &(M_2 + M_1\mathcal{L}/\mathcal{K})\exp(2M_3\Delta)\sup_{t_0-\mathfrak{G}\leq s\leq t_0+\mathfrak{G}}\mathfrak{B} + \mathcal{L}\|\Theta\|\exp\{-\mathcal{K}(t-t_0)\}.
\end{aligned} \tag{3.14}$$

Noting that $\mathfrak{G} < \Delta/2$, therefore,

$$\begin{aligned}
\|\eta(t)\| + \|\vartheta(t)\| \leq &(M_2 + M_1\mathcal{L}/\mathcal{K})\exp(2M_3\Delta)\sup_{t_0-\mathfrak{G}\leq s\leq t_0-\mathfrak{G}+\Delta}\mathfrak{B} + \mathcal{L}\|\Theta\|\exp[-\mathcal{K}(\Delta-\mathfrak{G})]\\
\leq &\Big\{(M_2 + M_1\mathcal{L}/\mathcal{K})\exp(2M_3\Delta) + \mathcal{L}\exp[-\mathcal{K}(\Delta-\mathfrak{G})]\Big\}\sup_{t_0-\mathfrak{G}\leq s\leq t_0-\mathfrak{G}+\Delta}\mathfrak{B}
\end{aligned} \tag{3.15}$$

for $t_0 - \mathfrak{G} + \Delta \leq t \leq t_0 - \mathfrak{G} + 2\Delta$ holds.

Since $M_1 = \max\{m_1 m_4 \upsilon + m_2 m_4 \upsilon(1-\wp^*)^{-1}, m_2 m_3\lambda + m_2 m_4\lambda(1-\wp^*)^{-1}\}$, $M_2 = \max\{m_2[\lambda + 2\lambda(1-\wp^*)^{-1}] + m_2 m_4 \lambda^2(1-\wp^*)^{-1}, m_2 m_4\upsilon^2(1-\wp^*)^{-1} + m_4[\upsilon + 2\upsilon(1-\wp^*)^{-1}]\}$, we denote $M_4 = \max\{m_1 m_4, m_2 m_3\}$, $M_5 = \max\{m_2, m_4\}$ thus, $M_1 \leq M_4\mathfrak{G} + M_5^2\mathfrak{G}(1-\wp^*)^{-1} = \mathfrak{C}$, $M_2 \leq M_5[\mathfrak{G} + 2\mathfrak{G}(1-\wp^*)^{-1} + M_5\mathfrak{G}^2(1-\wp^*)^{-1}] = \mathfrak{D}$. Thus, $M_3 \leq \max\{m_1 + m_4 + \mathfrak{C}, m_2 + m_3 + \mathfrak{C}\} = \mathfrak{E}$ by (3.13)–(3.15),

$$\|\eta(t)\| + \|\vartheta(t)\| \leq \Big\{(\mathfrak{D} + \mathfrak{C}\mathcal{L}/\mathcal{K})\exp(2\mathfrak{E}\Delta) + \mathcal{L}\exp[-\mathcal{K}(\Delta-\mathfrak{G})]\Big\}\sup_{t_0-\mathfrak{G}\leq s\leq t_0-\mathfrak{G}+\Delta}\mathfrak{B}. \tag{3.16}$$

Let $\mathfrak{F}(\mathfrak{G}) = (\mathfrak{D} + \mathfrak{C}\mathcal{L}/\mathcal{K})\exp(2\mathfrak{E}\Delta) + \mathcal{L}\exp[-\mathcal{K}(\Delta - \mathfrak{G})]$, therefore, $\mathfrak{F}(\mathfrak{G})$ is strictly increasing for $\mathfrak{G}$. Thus, there must exists $\bar{\mathfrak{G}} > 0$, such that $\mathfrak{F}(\mathfrak{G}) < 1$, $\forall \mathfrak{G} \in (0, \bar{\mathfrak{G}})$.

Select $\mathcal{F} = -\ln \mathfrak{F}/\Delta$, so $\mathcal{F} > 0$, when $\mathfrak{G} \in (0, \bar{\mathfrak{G}})$, from (3.16), we have

$$\sup_{t_0-\mathfrak{G}+\Delta\le s\le t_0-\mathfrak{G}+2\Delta} \mathfrak{B} \le \exp(-\mathcal{F}\Delta)\Big(\sup_{t_0-\mathfrak{G}\le s\le t_0-\mathfrak{G}+\Delta} \mathfrak{B}\Big). \tag{3.17}$$

Thus, by mathematical induction and the existence and uniqueness of (3.9), an integer $\mathfrak{Z} \in \mathbb{N}^+$ exists such that when $t \ge t_0 + (\mathfrak{Z} - 1)\Delta$,

$$\begin{aligned}\sup_{t_0-\mathfrak{G}+\mathfrak{Z}\Delta\le s\le t_0+-\mathfrak{G}+(\mathfrak{Z}+1)\Delta} \mathfrak{B} \le & \sup_{t_0-\mathfrak{G}+(\mathfrak{Z}-1)\Delta+\Delta\le s\le t_0-\mathfrak{G}+\mathfrak{Z}\Delta+\Delta} \mathfrak{B}\\ &\le \exp(-\mathcal{F}\Delta) \sup_{t_0-\mathfrak{G}+(\mathfrak{Z}-1)\Delta\le s\le t_0-\mathfrak{G}+\mathfrak{Z}\Delta} \mathfrak{B}\\ &\quad \dots\\ &\le \exp(-\mathfrak{Z}\mathcal{F}\Delta) \sup_{t_0-\mathfrak{G}\le s\le t_0-\mathfrak{G}+\Delta} \mathfrak{B}\\ &= \mathfrak{Y}\exp(-\mathfrak{Z}\mathcal{F}\Delta),\end{aligned} \tag{3.18}$$

where $\mathfrak{Y} = \sup_{t_0-\mathfrak{G}\le t\le t_0-\mathfrak{G}+\Delta} \mathfrak{B}$. So, $\forall t > t_0 + \Delta$, there is an arbitrary integer $\mathfrak{Z}$ such that

$$\mathfrak{B} \le \mathfrak{Y}\exp(\mathcal{F}\Delta)\exp(-\mathcal{F}(t - t_0)), \quad t_0 + \mathfrak{Z}\Delta \le t \le t_0 + (\mathfrak{Z} + 1)\Delta \tag{3.19}$$

holds.

Obviously, this condition also holds for $t_0 - \mathfrak{G} \le t \le t_0 - \mathfrak{G} + \Delta$. Thus, FBAMCNN (3.1) is GES. □

Remark 1. Since inequality techniques are mainly used in this paper, the results obtained by Theorem 1 is sufficient condition for DFBAMCNN to maintain its exponential stability. That is to say, when two different time-varying delays are larger than the derived value, DFBAMCNN will lose its original stability, but the reverse is not necessarily true.

4. Robustness of stability of SDFBAMCNN

Firstly, we give the model of SDFBAMCNN.

$$\begin{cases} d\eta_k(t) = \Big[-c_k\eta_k(t) + \bigwedge_{r=1}^{p} \alpha_{kr}\Gamma_r(\vartheta_r(t - \lambda(t))) + \bigvee_{r=1}^{p} \beta_{kr}\Gamma_r(\vartheta_r(t - \lambda(t)))\Big]dt + \mathcal{Q}_k\eta_k(t)d\mho(t),\\ d\vartheta_r(t) = \Big[-d_r\vartheta_r(t) + \bigwedge_{k=1}^{q} \omega_{rk}\Xi_k(\eta_k(t - \upsilon(t))) + \bigvee_{k=1}^{q} \gamma_{rk}\Xi_k(\eta_k(t - \upsilon(t)))\Big]dt + \mathcal{R}_r\vartheta_r(t)d\mho(t),\end{cases} \tag{4.1}$$

where $\eta_k(t) = \phi_k(t)$, $\vartheta_r(t) = \varphi_r(t)$, $t \in [-\bar{\mathfrak{G}}, 0]$. $\mathcal{Q}_k$, $\mathcal{R}_r$ are constant noise intensities.

The definitions of mean square exponential stability (MSES) and almost surely globally exponential stability (ASGES) of SDFBAMCNN (4.1) are as follows.

Definition 2. *[13] SDFBAMCNN* (4.1) *is said to be MSES, if $\mathcal{A} > 0$, $\mathcal{B} > 0$, for any $t_0 \in \mathbb{R}^+$, $\phi \in L^2_{\mathcal{F}_0}([-\bar{\mathfrak{G}}, 0]; \mathbb{R}^q)$ and $\varphi \in L^2_{\mathcal{F}_0}([-\bar{\mathfrak{G}}, 0]; \mathbb{R}^p)$ such that*

$$E(\mathcal{U}(t))^2 \le \mathcal{A}E(\mathcal{U}_0)^2 e^{-\mathcal{B}(t-t_0)}, \tag{4.2}$$

or

$$\limsup_{t\to\infty}(\ln(E(\mathcal{U}(t;t_0,(\phi,\varphi)^T))^2/t)) < 0. \tag{4.3}$$

where $\mathcal{U}(t) = \|\eta(t)\| + \|\vartheta(t)\|$, $\mathcal{U}_0 = \|\phi\| + \|\varphi\|$, *and* $\phi = \eta(t_0)$, $\varphi = \vartheta(t_0)$.

Definition 3. *[13] SDFBAMCNN* (4.1) *is said to be ASGES, if for any* $t_0 \in \mathbb{R}^+$, $\phi \in L^2_{\mathcal{F}_0}([-\bar{\mathfrak{G}}, 0]; \mathbb{R}^q)$ *and* $\varphi \in L^2_{\mathcal{F}_0}([-\bar{\mathfrak{G}}, 0]; \mathbb{R}^p)$*, such that*

$$\mathcal{U}(t) \le \mathcal{A}\mathcal{U}_0 e^{-\mathcal{B}(t-t_0)},$$

or

$$\limsup_{t\to\infty}(\ln(E(\mathcal{U}(t;t_0,(\phi,\varphi)^T)))/t) < 0$$

almost surely.

From the Definitions 2 and 3 above, it means that MSES can not imply ASGES, but the reverse is not true. If assumption H(1) holds, MSES implies ASGES [29].

Theorem 2. *Let H(1) holds, and* m_1*-*m_4 *are defined in Theorem 1, the SDFBAMCNN* (4.1) *is said to be MSES if there exist* $|\mathcal{G}| \le \bar{\mathcal{G}}/\sqrt{2}$, $\mathfrak{G} \le \min\{\Delta/2, \bar{\mathfrak{G}}\}$*, where* $\bar{\mathcal{G}}$ *and* $\bar{\mathfrak{G}}$ *are the unique roots of the following two equations respectively.*

$$16\mathcal{G}^2\mathcal{A}/\mathcal{B} + 2\mathcal{A}\exp\{-\mathcal{B}\Delta\} = 1,$$

and

$$2[(2N_1 + 4\bar{\mathcal{G}}^2)\mathcal{A}/\mathcal{B} + N_2]\exp\{16\mathfrak{G}(\widetilde{N}_3 + 4\bar{\mathcal{G}}^2)\Delta\} + 2\mathcal{A}\exp\{-\mathcal{B}(\Delta - \mathfrak{G})\} = 1,$$

where

$$\begin{aligned}
&\Delta > \ln\mathcal{A}/\mathcal{B}, \widetilde{N}_3 = \max\{(m_1+m_4)^2 + 2N_1, (m_2+m_3)^2 + 2N_1\}, \bar{N}_3 = \max\{(m_1+m_4)^2, (m_2+m_3)^2\},\\
&N_1 = \max\{\acute{\mathfrak{z}}, \grave{\mathfrak{z}}\}, N_2 = \max\{\hat{\mathfrak{z}}, \check{\mathfrak{z}}\}, \acute{\mathfrak{z}} = 2\mathfrak{G}^2 m_1^2 m_4^2 + 2\mathfrak{G}^2 m_4^2 m_2^2 (1-\wp^*)^{-1},\\
&\grave{\mathfrak{z}} = 2\mathfrak{G}^2 m_3^2 m_2^2 + 2\mathfrak{G}^2 m_2^2 m_4^2 (1-\wp^*)^{-1}, \hat{\mathfrak{z}} = 2\mathfrak{G}^3 m_2^2 m_4^2 (1-\wp^*)^{-1} + 2\mathfrak{G} m_4^2[1 + 2(1-\wp^*)^{-1}],\\
&\check{\mathfrak{z}} = 2\mathfrak{G}^3 m_4^2 m_2^2 (1-\wp^*)^{-1} + 2\mathfrak{G} m_2^2[1 + 2(1-\wp^*)^{-1}].
\end{aligned}$$

Proof. By (2.2), (4.1), let $\mathcal{G} = \max\{\mathcal{Q}, \mathcal{R}\}$, from (3.7), similarly,

$$\begin{aligned}
&\|\Lambda(t) - \eta(t)\| + \|\Upsilon(t) - \vartheta(t)\|\\
\le &\int_{t_0}^{t}\Big[(m_1+m_4)\|\Lambda(s) - \eta(s)\| + (m_3+m_2)\|\Upsilon(s) - \vartheta(s)\|\\
&+ m_2\|\vartheta(s) - \vartheta(s-\lambda(s))\| + m_4\|\eta(s) - \eta(s-\upsilon(s))\|\Big]ds + \mathcal{G}\int_{t_0}^{t}\mathfrak{B}d\mho(s).
\end{aligned} \tag{4.4}$$

When $t_0 \le t + \mathfrak{G}$, let $\mathfrak{R} = \|\Lambda(t) - \eta(t)\| + \|\Upsilon(t) - \vartheta(t)\|$, we have

$$E(\mathfrak{R})^2 \le 2(t-t_0)\int_{t_0}^{t}\Big\{E\Big[(m_1+m_4)\|\Lambda(s) - \eta(s)\| + (m_3+m_2)\|\Upsilon(s) - \vartheta(s)\|$$

$$
\begin{aligned}
&+ m_2\|\vartheta(s) - \vartheta(s-\lambda(s))\| + m_4\|\eta(s) - \eta(s-\upsilon(s))\|\Big]^2\Big\}ds + 2\mathcal{G}^2 E\int_{t_0}^{t}\mathfrak{B}^2 ds \\
\leq &8(t-t_0)\int_{t_0}^{t}\Big[(m_1+m_4)^2 E\|\Lambda(s)-\eta(s)\|^2 + (m_3+m_2)^2 E\|\Upsilon(s)-\vartheta(s)\|^2 \\
&+ m_2^2 E\|\vartheta(s)-\vartheta(s-\lambda(s))\|^2 + m_4^2 E\|\eta(s)-\eta(s-\upsilon(s))\|^2\Big]ds + 2\mathcal{G}^2\int_{t_0}^{t}E(\mathfrak{B})^2 ds \\
\leq &8\mathfrak{G}\int_{t_0}^{t}\Big\{\Big|(m_1+m_4)^2 E\|\Lambda(s)-\eta(s)\|^2 + (m_3+m_2)^2 E\|\Upsilon(s)-\vartheta(s)\|^2 \\
&+ m_2^2 E\|\vartheta(s)-\vartheta(s-\lambda(s))\|^2 + m_4^2 E\|\eta(s)-\eta(s-\upsilon(s))\|^2\Big|\Big\}ds + 2\mathcal{G}^2\int_{t_0}^{t}E(\mathfrak{B})^2 ds. \qquad (4.5)
\end{aligned}
$$

For $t \leq t_0 + \mathfrak{G}$, we can obtain

$$
\begin{aligned}
&\int_{t_0}^{t}E\|\vartheta(s)-\vartheta(s-\lambda(s))\|^2 ds \\
=&\int_{t_0}^{t_0+\mathfrak{G}}E\|\vartheta(s)-\vartheta(s-\lambda(s))\|^2 ds + \int_{t_0+\mathfrak{G}}^{t}E\|\vartheta(s)-\vartheta(s-\lambda(s))\|^2 ds.
\end{aligned}
$$

and

$$
\begin{aligned}
&\int_{t_0}^{t}E\|\eta(s)-\eta(s-\upsilon(s))\|^2 ds \\
=&\int_{t_0}^{t_0+\mathfrak{G}}E\|\eta(s)-\eta(s-\upsilon(s))\|^2 ds + \int_{t_0+\mathfrak{G}}^{t}E\|\eta(s)-\eta(s-\upsilon(s))\|^2 ds.
\end{aligned}
$$

Thus, similar to (3.8),

$$
\begin{aligned}
\int_{t_0+\mathfrak{G}}^{t}E\|\vartheta(s)-\vartheta(s-\lambda(s))\|^2 ds \leq &2\mathfrak{G}^2 m_3^2\int_{t_0}^{t}E\|\vartheta(s)\|^2 ds + 2\mathfrak{G}^2 m_4^2(1-\wp^*)^{-1}\int_{t_0}^{t}E\|\eta(s)\|^2 ds \\
&+ 2\mathfrak{G}^3 m_4^2(1-\wp^*)^{-1}\sup_{t_0-\mathfrak{G}\leq s\leq t_0}E\|\eta(s)\|^2, \qquad (4.6)
\end{aligned}
$$

and

$$
\begin{aligned}
\int_{t_0+\mathfrak{G}}^{t}E\|\eta(s)-\eta(s-\upsilon(s))\|^2 ds \leq &2\mathfrak{G}^2 m_1^2\int_{t_0}^{t}E\|\eta(s)\|^2 ds + 2\mathfrak{G}^2 m_2^2(1-\wp^*)^{-1}\int_{t_0}^{t}E\|\vartheta(s)\|^2 ds \\
&+ 2\mathfrak{G}^3 m_2^2(1-\wp^*)^{-1}\sup_{t_0-\mathfrak{G}\leq s\leq t_0}E\|\vartheta(s)\|^2. \qquad (4.7)
\end{aligned}
$$

Thus,

$$
\begin{aligned}
&\int_{t_0}^{t} E\|\eta(s)-\eta(s-\upsilon(s))\|^2 ds \\
\leq& 2\mathfrak{G}^2 m_1^2 \int_{t_0}^{t} E\|\eta(s)\|^2 ds + 2\mathfrak{G}^2 m_2^2 (1-\wp^*)^{-1} \int_{t_0}^{t} E\|\vartheta(s)\|^2 ds \\
&+ 2\mathfrak{G}^3 m_2^2 (1-\wp^*)^{-1} \sup_{t_0-\mathfrak{G}\leq s\leq t_0} E\|\vartheta(s)\|^2 + 2\mathfrak{G}[1+2(1-\wp^*)^{-1}] \sup_{t_0-\mathfrak{G}\leq s\leq t_0+\mathfrak{G}} E\|\eta(s)\|^2, \qquad (4.8)
\end{aligned}
$$

and

$$
\begin{aligned}
&\int_{t_0}^{t} E\|\vartheta(s)-\vartheta(s-\lambda(s))\|^2 ds \\
\leq& 2\mathfrak{G}^2 m_3^2 \int_{t_0}^{t} E\|\vartheta(s)\|^2 ds + 2\mathfrak{G}^2 m_4^2 (1-\wp^*)^{-1} \int_{t_0}^{t} E\|\eta(s)\|^2 ds \\
&+ 2\mathfrak{G}^3 m_4^2 (1-\wp^*)^{-1} \sup_{t_0-\mathfrak{G}\leq s\leq t_0} E\|\eta(s)\|^2 + 2\mathfrak{G}[1+2(1-\wp^*)^{-1}] \sup_{t_0-\mathfrak{G}\leq s\leq t_0+\mathfrak{G}} E\|\vartheta(s)\|^2. \qquad (4.9)
\end{aligned}
$$

Let

$$N_1 = \max\{\acute{\mathfrak{z}}, \grave{\mathfrak{z}}\},$$

and

$$N_2 = \max\{\hat{\mathfrak{z}}, \check{\mathfrak{z}}\},$$

where $\acute{\mathfrak{z}} = 2\mathfrak{G}^2 m_1^2 m_4^2 + 2\mathfrak{G}^2 m_4^2 m_2^2 (1-\wp^*)^{-1}$, $\grave{\mathfrak{z}} = 2\mathfrak{G}^2 m_3^2 m_2^2 + 2\mathfrak{G}^2 m_2^2 m_4^2 (1-\wp^*)^{-1}$, $\hat{\mathfrak{z}} = 2\mathfrak{G}^3 m_2^2 m_4^2 (1-\wp^*)^{-1} + 2\mathfrak{G} m_4^2 [1+2(1-\wp^*)^{-1}]$, $\check{\mathfrak{z}} = 2\mathfrak{G}^3 m_4^2 m_2^2 (1-\wp^*)^{-1} + 2\mathfrak{G} m_2^2 [1+2(1-\wp^*)^{-1}]$.

Then from (4.8), (4.9), we can get

$$
\begin{aligned}
&\int_{t_0}^{t} \Big[m_2^2 E\|\vartheta(s)-\vartheta(s-\lambda(s))\|^2 + m_4^2 E\|\eta(s)-\eta(s-\upsilon(s))\|^2 \Big] ds \\
&\leq N_1 \int_{t_0}^{t} \Big[E\|\eta(s)\|^2 + E\|\vartheta(s)\|^2 \Big] ds + N_2 \sup_{t_0-\mathfrak{G}\leq s\leq t_0+\mathfrak{G}} (E\|\eta(s)\|^2 + E\|\vartheta(s)\|^2). \qquad (4.10)
\end{aligned}
$$

From (4.5), we obtain

$$
\begin{aligned}
E(\mathfrak{N})^2 \leq& 8\mathfrak{G} \int_{t_0}^{t} \Big\{ [(m_1+m_4)^2 + 2N_1] E\|\varLambda(s)-\eta(s)\|^2 \\
&+ [(m_3+m_2)^2 + 2N_1] E\|\varUpsilon(s)-\vartheta(s)\|^2 \Big\} ds + 2N_1 \int_{t_0}^{t} \Big[\|\varLambda(s)\|^2 + \|\varUpsilon(s)\|^2 \Big] ds \\
&+ N_2 \sup_{t_0-\mathfrak{G}\leq s\leq t_0+\mathfrak{G}} (E\|\eta(s)\|^2 + E\|\vartheta(s)\|^2) + 4\mathcal{G}^2 \int_{t_0}^{t} \Big[E\|\eta(s)\|^2 + E\|\vartheta(s)\|^2 \Big] ds \\
\leq& 8\mathfrak{G} \int_{t_0}^{t} \Big\{ [(m_1+m_4)^2 + 8\mathcal{G}^2 + 2N_1] E\|\varLambda(s)-\eta(s)\|^2 \\
&+ [(m_3+m_2)^2 + 8\mathcal{G}^2 + 2N_1] E\|\varUpsilon(s)-\vartheta(s)\|^2 \Big\} ds + (2N_1 + 8\mathcal{G}^2) \int_{t_0}^{t} \Big[\|\varLambda(s)\|^2 + \|\varUpsilon(s)\|^2 \Big] ds
\end{aligned}
$$

$$+ N_2 \sup_{t_0-\mathfrak{G}\le s\le t_0+\mathfrak{G}} (E\|\eta(s)\|^2 + E\|\vartheta(s)\|^2). \tag{4.11}$$

Let $N_3 = \max\{(m_1+m_4)^2 + 2N_1 + 8\mathcal{G}^2, (m_3+m_2)^2 + 2N_1 + 8\mathcal{G}^2\}$, thus,

$$\begin{aligned} E(\mathfrak{R})^2 \le & 8\mathfrak{G}N_3 \int_{t_0}^{t} \Big[E\|\Lambda(s)-\eta(s)\|^2 + E\|\Upsilon(s)-\vartheta(s)\|^2\Big]ds \\ & + (2N_1 + 8\mathcal{G}^2)\int_{t_0}^{t}\Big[\|\Lambda(s)\|^2 + \|\Upsilon(s)\|^2\Big]ds + N_2 \sup_{t_0-\mathfrak{G}\le s\le t_0+\mathfrak{G}} (E\|\eta(s)\|^2 + E\|\vartheta(s)\|^2). \end{aligned} \tag{4.12}$$

Then,

$$\begin{aligned} E(\mathfrak{R})^2 \le & 8\mathfrak{G}N_3 \int_{t_0}^{t} E(\mathfrak{R})^2 ds + (2N_1 + 8\mathcal{G}^2)\int_{t_0}^{t} (\|\Lambda(s)\| + \|\Upsilon(s)\|)^2 ds \\ & + N_2 \sup_{t_0-\mathfrak{G}\le s\le t_0+\mathfrak{G}} E(\|\eta(s)\| + \|\vartheta(s)\|)^2 \\ \le & 8\mathfrak{G}N_3 \int_{t_0}^{t} E(\mathfrak{R})^2 ds + (2N_1 + 8\mathcal{G}^2)\mathcal{A}/\mathcal{B}E(\mathfrak{R}_0)^2 \\ & + N_2 \sup_{t_0-\mathfrak{G}\le s\le t_0+\mathfrak{G}} E(\|\eta(s)\| + \|\vartheta(s)\|)^2 \\ \le & 8\mathfrak{G}N_3 \int_{t_0}^{t} E(\mathfrak{R})^2 ds + [(2N_1 + 8\mathcal{G}^2)\mathcal{A}/\mathcal{B} + N_2] \sup_{t_0-\mathfrak{G}\le s\le t_0+\mathfrak{G}} E(\mathfrak{B})^2. \end{aligned} \tag{4.13}$$

Denote $\mathfrak{S} = \sup_{t_0-\mathfrak{G}\le s\le t_0+\mathfrak{G}} E(\mathfrak{B})^2$, applying Gronwall-Bellman lemma,

$$E(\mathfrak{R})^2 \le [(2N_1 + 8\mathcal{G}^2)\mathcal{A}/\mathcal{B} + N_2]\mathfrak{S}\exp\{8\mathfrak{G}N_3(t-t_0)\}. \tag{4.14}$$

Therefore, when $t_0 + \mathfrak{G} \le t \le \Delta$,

$$E(\mathfrak{R})^2 \le [(2N_1 + 8\mathcal{G}^2)\mathcal{A}/\mathcal{B} + N_2]\mathfrak{S}\exp\{16\mathfrak{G}N_3\Delta\}. \tag{4.15}$$

Noting that $\mathfrak{G} \le \min\{\bar{\mathfrak{G}}, \Delta/2\}$, for $t_0 - \mathfrak{G} + \Delta \le t \le t_0 - \mathfrak{G} + 2\Delta$

$$\begin{aligned} E(\mathfrak{B})^2 \le & 2E(\mathfrak{R}(t))^2 + 2E(\|\Lambda(x)\| + \|\Upsilon(x)\|)^2 \\ \le & 2[(2N_1 + 8\mathcal{G}^2)\mathcal{A}/\mathcal{B} + N_2]\mathfrak{S}\exp\{16\mathfrak{G}N_3\Delta\} + 2\mathcal{A}\mathfrak{S}\exp(-\mathcal{B}(\Delta-\mathfrak{G})) \\ \le & \Big\{2[(2N_1 + 8\mathcal{G}^2)\mathcal{A}/\mathcal{B} + N_2]\exp\{16\mathfrak{G}N_3\Delta\} + 2\mathcal{A}\exp\{-\mathcal{B}(\Delta-\mathfrak{G})\}\Big\}\mathfrak{S}. \end{aligned} \tag{4.16}$$

Select $\mathcal{H}(\mathfrak{G},\mathcal{G}) = [(2N_1 + 8\mathcal{G}^2)\mathcal{A}/\mathcal{B} + N_2]\exp\{16\mathfrak{G}N_3\Delta\} + 2\mathcal{A}\exp\{-\mathcal{B}(\Delta-\mathfrak{G})\}$, noting that $\Delta > \ln\mathcal{A}/\mathcal{B}$, therefore, $\mathcal{H}(0,0) < 1$. Since $\mathcal{H}(\mathfrak{G},\mathcal{G})$ is strictly increasing for $\mathcal{G}$, thus, there must be a $\bar{\mathcal{G}}$, such that $\mathcal{H}(0,\mathcal{G}) < 1$, when $|\mathcal{G}| < \bar{\mathcal{G}}$. Furthermore, for $\mathfrak{G}$, $\mathcal{H}(\mathcal{G},\mathfrak{G})$ is also strictly increases, thus, $\exists\bar{\mathfrak{G}} > 0$ such that $\mathcal{H}(\mathcal{G},\mathfrak{G}) < 1$ when $|\mathcal{G}| \le \bar{\mathcal{G}}/\sqrt{2}$ and $\mathfrak{G} < \min\{\bar{\mathfrak{G}}, \Delta/2\}$. We skip the second part of the proof here since it is the same as the discussion in Theorem 1.

Therefore, if $|\mathcal{G}| \le \bar{\mathcal{G}}/\sqrt{2}$ and $\mathfrak{G} < \min\{\bar{\mathfrak{G}}, \Delta/2\}$ hold, the system (4.1) is MSES, furthermore, the system is also ASGES. □

Remark 2. The bounds of disturbances we derived in Theorem 2 is not a simple superposition of the results of Theorem 1. In the derivation process, we can see that $\mathcal{G}$ and $\mathfrak{G}$ in Theorem 2 are mutually restricted, and only one parameter satisfies the upper bounds we deduced is not enough to make SDFBAMCNN keep its original stability.

Remark 3. A brief comparison of our study and some of current literature is provided in Table 1. The elements of the comparison are BAM, fuzzy logic (F-L), CNN, time delays (TDs), stochastic disturbances (SDs), asymptotically stable (AS), GES, MSES, ASGES, robustness of stability (RoS). Furthermore, since there is fuzzy logic in the BAM neural network, its complexity is improved. In addition, BAMCNN with fuzzy logic can better simulate the human-like derivation style and better deal with fuzzy problems. By the way, the study of robustness in this paper offers a theoretical foundation for the construction of FBAMCNN.

Table 1. The differences between our study and current literature.

	BAM	F-L	CNN	TDs	SDs	AS	GES	MSES	ASGES	RoS
Nagamani et al. (2021) [5]	✓	-	✓	✓	-	✓	-	-	-	-
Ali et al. (2017) [23]	✓	✓	-	✓	✓	✓	-	-	-	-
Si et al. (2021) [26]	-	-	-	-	✓	-	✓	✓	✓	✓
Fang et al. (2023) [27]	-	✓	✓	-	✓	-	✓	✓	✓	✓
Oliveira (2022) [30]	✓	-	-	✓	-	✓	✓	-	-	-
This paper	✓	✓	✓	✓	✓	-	✓	✓	✓	✓

5. Examples

Example 1. Let $p = q = 2$ and

$$C = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}, \quad D = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}, \quad \alpha = \begin{bmatrix} -0.01 & 0.01 \\ 0.01 & -0.01 \end{bmatrix},$$
$$\beta = \begin{bmatrix} -0.03 & 0.03 \\ 0.03 & -0.03 \end{bmatrix}, \quad \omega = \begin{bmatrix} -0.04 & 0.04 \\ 0.04 & -0.04 \end{bmatrix}, \quad \gamma = \begin{bmatrix} -0.02 & 0.02 \\ 0.02 & -0.02 \end{bmatrix},$$

and $\mathit{\Gamma}_r(x) = \frac{1}{2}(|x+1| - |x-1|)$, $\mathit{\Xi}_k(x) = \tanh x$.

Thus, the following is the form of the DFBAMCNN we considered.

$$\begin{cases} \dot{\eta}_1(t) = -2\eta_1(t) + \bigwedge\limits_{r=1}^{2} \alpha_{1r}\mathit{\Gamma}_r(\vartheta_r(t-\upsilon(t))) + \bigvee\limits_{r=1}^{2} \beta_{1r}\mathit{\Gamma}_r(\vartheta_r(t-\upsilon(t))), \\ \dot{\eta}_2(t) = -2\eta_2(t) + \bigwedge\limits_{r=1}^{2} \alpha_{2r}\mathit{\Gamma}_r(\vartheta_r(t-\upsilon(t))) + \bigvee\limits_{r=1}^{2} \beta_{2r}\mathit{\Gamma}_r(\vartheta_r(t-\upsilon(t))), \\ \dot{\vartheta}_1(t) = -2\vartheta_1(t) + \bigwedge\limits_{k=1}^{2} \omega_{1k}\mathit{\Xi}_k(\eta_k(t-\lambda(t))) + \bigvee\limits_{k=1}^{q} \gamma_{1k}\mathit{\Xi}_k(\eta_k(t-\lambda(t))), \\ \dot{\vartheta}_2(t) = -2\vartheta_2(t) + \bigwedge\limits_{k=1}^{2} \omega_{2k}\mathit{\Xi}_k(\eta_k(t-\lambda(t))) + \bigvee\limits_{k=1}^{2} \gamma_{2k}\mathit{\Xi}_k(\eta_k(t-\lambda(t))). \end{cases} \tag{5.1}$$

Then from the principle of comparison, we know that the system without time delay

$$\begin{cases} \dot{\eta}_1(t) = -2\eta_1(t) + \bigwedge_{r=1}^{2} \alpha_{1r}\Gamma_r(\vartheta_r(t)) + \bigvee_{r=1}^{2} \beta_{1r}\Gamma_r(\vartheta_r(t)), \\ \dot{\eta}_2(t) = -2\eta_2(t) + \bigwedge_{r=1}^{2} \alpha_{2r}\Gamma_r(\vartheta_r(t)) + \bigvee_{r=1}^{2} \beta_{2r}\Gamma_r(\vartheta_r(t)), \\ \dot{\vartheta}_1(t) = -2\vartheta_1(t) + \bigwedge_{k=1}^{2} \omega_{1k}\Xi_k(\eta_k(t)) + \bigvee_{k=1}^{2} \gamma_{1k}\Xi_k(\eta_k(t)), \\ \dot{\vartheta}_2(t) = -2\vartheta_2(t) + \bigwedge_{k=1}^{2} \omega_{2k}\Xi_k(\eta_k(t)) + \bigvee_{k=1}^{2} \gamma_{2k}\Xi_k(\eta_k(t)) \end{cases} \tag{5.2}$$

is GES with $\mathcal{L} = 1, \mathcal{K} = 0.8$.

By calculating, we get that $m_1 = 2$, $m_2 = 0.8$, $m_3 = 2$, $m_4 = 1.2$, $M_4 = 2.4$, $M_5 = 1.2$. We select $\wp^* = 0$, $\Delta = 0.2$, therefore we can obtain $\bar{\mathfrak{G}} = 0.0047$ from the following transcendental equation

$$[1.2(3\bar{\mathfrak{G}} + 1.2\bar{\mathfrak{G}}^2) + 2.4\bar{\mathfrak{G}} + 1.8\bar{\mathfrak{G}}^2]\exp\{0.4(2.8 + 2.4\bar{\mathfrak{G}} + 1.44\bar{\mathfrak{G}}^2)\} + \exp(-0.8(0.2 - \bar{\mathfrak{G}})) = 1. \tag{5.3}$$

Therefore, according Theorem 1, when $\mathfrak{G} < \min\{\Delta/2, \bar{\mathfrak{G}}\}$, system (5.1) is GES. In Figure 1 , we take $\mathfrak{G} = 0.001 < 0.0047$, hence, the states in Figure 1 is GES.

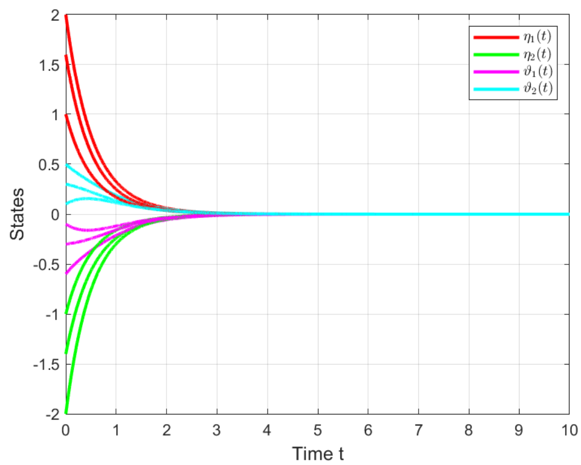

Figure 1. The state of DFBAMCNN (5.1) with $\mathfrak{G} = 0.001$.

Example 2. Let $p = q = 2$, and the SDFBAMCNN model that we taken into account is as follows:

$$\begin{cases} d\eta_1(t) = \Big[-2\eta_1(t) + \bigwedge_{r=1}^{2} \alpha_{1r}\Gamma_r(\vartheta_r(t-\lambda(t))) + \bigvee_{r=1}^{2} \beta_{1r}\Gamma_r(\vartheta_r(t-\lambda(t)))\Big]dt + \mathcal{Q}\eta_1(t)d\mho(t), \\ d\eta_2(t) = \Big[-2\eta_2(t) + \bigwedge_{r=1}^{2} \alpha_{2r}\Gamma_r(\vartheta_r(t-\lambda(t))) + \bigvee_{r=1}^{2} \beta_{2r}\Gamma_r(\vartheta_r(t-\lambda(t)))\Big]dt + \mathcal{Q}\eta_2(t)d\mho(t), \\ d\vartheta_1(t) = \Big[-2\vartheta_1(t) + \bigwedge_{k=1}^{2} \omega_{1k}\Xi_k(\eta_k(t-\upsilon(t))) + \bigvee_{k=1}^{2} \gamma_{1k}\Xi_k(\eta_k(t-\upsilon(t)))\Big]dt + \mathcal{R}\vartheta_1(t)d\mho(t), \\ d\vartheta_2(t) = \Big[-2\vartheta_2(t) + \bigwedge_{k=1}^{2} \omega_{2k}\Xi_k(\eta_k(t-\upsilon(t))) + \bigvee_{k=1}^{2} \gamma_{2k}\Xi_k(\eta_k(t-\upsilon(t)))\Big]dt + \mathcal{R}\vartheta_2(t)d\mho(t), \end{cases} \tag{5.4}$$

where the parameters of (5.4) are as follows:

$$C = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}, \quad D = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}, \quad \alpha = \begin{bmatrix} -0.2 & 0.2 \\ 0.2 & -0.2 \end{bmatrix},$$
$$\beta = \begin{bmatrix} -0.3 & 0.3 \\ 0.3 & -0.3 \end{bmatrix}, \quad \omega = \begin{bmatrix} -0.3 & 0.3 \\ 0.3 & -0.3 \end{bmatrix}, \quad \gamma = \begin{bmatrix} -0.1 & 0.1 \\ 0.1 & -0.1 \end{bmatrix},$$

and select $\Gamma_r(x) = \frac{1}{2}(|x+1| - |x-1|)$, $\Xi_k(x) = \tanh x$.

By computing the parameters above, we can easily get $m_1 = 1$, $m_2 = 0.1$, $m_3 = 1$, $m_4 = 0.08$, $\bar{N}_3 = 1.21$. On the other hand, by principle of comparison, we can get the system without time-varying delays and disturbances is GES with $\mathcal{A} = 1$ and $\mathcal{B} = 0.9$. We select $\wp^* = 0$ and $\Delta = 0.9$. Then, from Theorem 2, we can get the following two equations:

$$17.7778\mathcal{G}^2 + 2\exp(-0.81) = 1, \tag{5.5}$$

and

$$\begin{aligned} 2[1.1111(0.04256\mathfrak{G}^2 + 4\bar{\mathcal{G}}^2) + 0.00128\mathfrak{G}^3 + 0.06\mathfrak{G}]\exp\{14.4\mathfrak{G}(1.21 + 4\bar{\mathcal{G}}^2)\} \\ + 2\exp\{-0.9(0.9 - \mathfrak{G})\} = 1. \end{aligned} \tag{5.6}$$

Thus, we can easily get $\bar{\mathcal{G}} = 0.0788$ and $\bar{\mathfrak{G}} = 0.0256$. From Theorem 2, the perturbed FBAMCNN (5.4) is said to be MSES if times delays $\upsilon(t)$, $\lambda(t)$ and noise intensities $\mathcal{Q}$, $\mathcal{R}$ are lower than the bounds we derived above, that is $|\mathcal{G}| \leq \bar{\mathcal{G}}$ and $\mathfrak{G} \leq \min\{\bar{\mathfrak{G}}, \Delta/2\}$.

Figure 2 shows the states of SDFBAMCNN (5.4) with different initial value, where $\mathfrak{G} = 0.0133$ and $\mathcal{G} = 0.02$. Since the max delay and the max intensity of noises are lower than the limits we derived, thus, SDFBAMCNN is MSES and ASGES.

Figures 3 and 4 depict the impacts of too large time delay and noise intensity. In Figure 3, $\mathfrak{G} = 0.01$, $\mathcal{G} = 0.08$, we can easily observe that $\mathcal{G}$ is larger than $\bar{\mathcal{G}}$, thus it is unstable. From Figure 4, it is clearly that $\mathfrak{G}$ and $\mathcal{G}$ are both larger than the bounds of theoretical results, therefore it is also unstable.

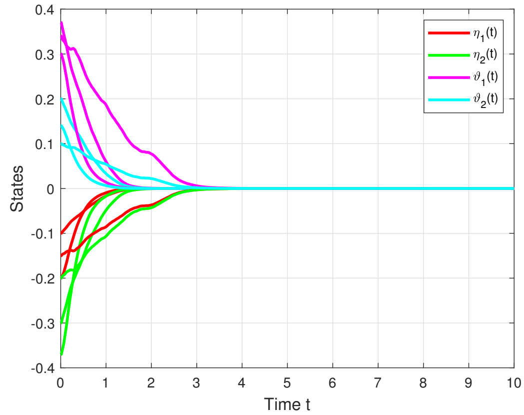

Figure 2. States of SDFBAMCNN (5.4) with $\mathfrak{G} = 0.0133$ and $\mathcal{G} = 0.02$.

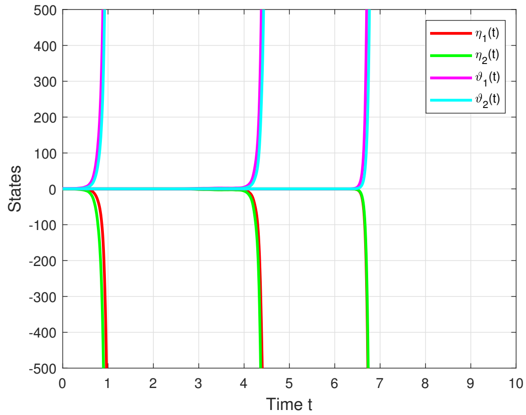

Figure 3. The state of SDFBAMCNN (5.4) with $\mathfrak{G} = 0.01$ and $\mathcal{G} = 0.08$.

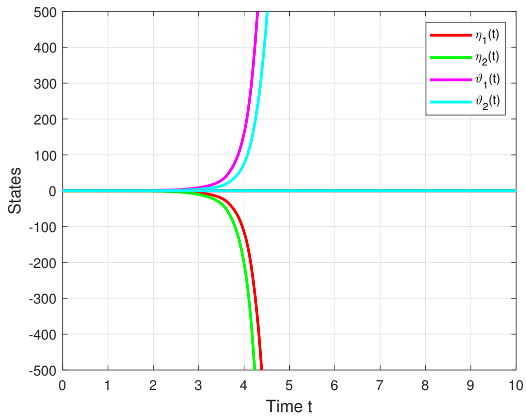

Figure 4. The state of SDFBAMCNN (5.4) with $\mathfrak{G} = 0.04$ and $\mathcal{G} = 0.04$.

6. Conclusions

This study examines the robustness of FBAMCNN when it is affected by time-varying delays and stochastic disturbances. Maximum duration of delays and the upper boundaries of noises must be identified in order for a perturbed FBAMCNN to remain GES. We may calculate these upper bounds of the interference by using inequality techniques. The results we derived provide a solid foundation for FBAMCNN applications and designs. Future study may focus on enhancing the upper limits and considering employing classical approaches to optimize the computation process, such as the LMI method and the Lyapunov function method. Furthermore, more sophisticated structural disturbances, such as Markov jump, impulses, state-dependent delays, and so on, can be taken into account.

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Conflict of interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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