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Research article

A predictor-corrector interior-point algorithm for $P_*(\kappa)$-weighted linear complementarity problems

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Abstract: In this paper, we present a predictor-corrector interior-point algorithm for $P_*(\kappa)$-weighted linear complementarity problems. Based on the kernel function $\varphi(t) = \sqrt{t}$, the search direction of the algorithm is obtained. By choosing appropriate parameters, we prove that the algorithm is feasible and convergent. It is shown that the proposed algorithm has polynomial iteration complexity. Numerical results illustrate the effectiveness of the algorithm.

Keywords: predictor-corrector interior-point algorithm; $P_*(\kappa)$-weighted linear complementarity problem; search direction; proximity measure; polynomial complexity
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1. Introduction

The weighted linear complementarity problem (WLCP) is to find a pair of vectors belonging to the intersection of a manifold with a cone, such that their product equals a given weight vector. Many equilibrium problems can be modeled as a nonlinear complementarity problem (CP) [1] or a WLCP. The latter may lead to some highly efficient algorithms for solving the corresponding equilibrium problems [2]. It is shown that the Fisher market equilibrium problem [3] can be reformulated as a WLCP. What is more, when the weight vector is a zero vector, a WLCP reduces to a linear complementarity problem (LCP). It should be mentioned that the analysis of WLCP becomes more difficult than the theory of LCP. Now, we summarize some versions of WLCP. In 2016, Potra [4] introduced the sufficient WLCP, generalized the characterization of the sufficient LCP to the sufficient WLCP and then presented a corrector-predictor interior point algorithm (IPA) for its numerical solution. Zhang [5] gave a smoothing Newton algorithm [6] for solving monotone WLCP and proved its global and local convergence. In [7], a variant nonmonotone smoothing algorithm was proposed by Tang for solving monotone WLCP. He and Tang [8] introduced a Levenberg-Marquardt method for WLCP. Recently, Chi, Gowda and Tao [9] presented some existence and uniqueness results for weighted horizontal linear complementarity problem (WHLCP) in the setting of Euclidean Jordan algebras.

IPA can be extended for solving WLCP. Since Karmarkar [10] presented the well-known IPA, which becomes one of the most effective algorithms for optimization. Asadi et al. [11] proposed a full-Newton step IPA for monotone weighted linear complementarity problems (MWLCP) and proved that this algorithm has a quadratic rate of convergence to the target point on the central path. In 2021, Chi and Wang [12] presented a full-Newton step infeasible interior-point method (IIPM) for a special WLCP [13] over the nonnegative orthant.

The IPA based on kernel functions is a popular algorithm in optimization. In this IPA, kernel functions are used to define the search directions and measure the distance to the central path. Darvay presented a new technique for finding search directions for LP problems [14], namely the algebraic equivalent transformation (AET). The most frequently used function in AET technology is the identity map. The idea of this method is to apply a continuously differentiable φ on the centering equation of the central path problem. Darvay [14] used the square root function for constructing search directions. Based on a search direction generated by considering the function $\varphi(t) = \frac{\sqrt{t}}{2(1+\sqrt{t})}$ in the AET, Kheirfam and Haghighi [15] defined IPA for $P_*(\kappa)$-LCPs. In 2020, Darvay et al. [16] used the function $\varphi(t) = t - \sqrt{t}$ for solving $P_*(\kappa)$-LCPs in the AET method. Recently, he considered the kernel function $\varphi(t) = t^2$ in the new AET $v^2 = v$ and proposed a predictor-corrector (PC) IPA [17] for $P_*(\kappa)$-LCP. Based on a simple locally-kernel function, Zhang et al. [18] proposed a full-Newton step infeasible IPA for monotone linear complementarity problems (MLCP), which obtains the same search directions as [14]. In 2012, Mansouri and Pirhaji [19] considered a continuously differentiable kernel function $\varphi(t) = \sqrt{t}$ based on Darvay's technique [14] for linear optimization (LO), and they proposed an IPA for MLCP.

Motivated by the aforementioned work, in this paper we consider a PC IPA for $P_*(\kappa)$-WLCP. We use AET technology for the system of central path based on the function $\varphi(t) = \sqrt{t}$. By applying Newton's method to the transformed system, the search directions are obtained. We prove the global convergence of the algorithm and derive the iteration bound. Our algorithm has the following properties: (1) Our algorithm is well-defined and a solution of $P_*(\kappa)$-WLCP can be obtained from a sequence of feasible point of the problem. (2) No line-searches are needed at each iteration. (3) It is shown that our algorithm is convergent.

The remainder of this paper is organized as follows. In Sect. 2, we introduce the search directions of the PC IPA for $P_*(\kappa)$-WLCP. The convergence analysis and the iteration bound of the algorithm are shown in Sect. 3. In Sect. 4, we present some numerical results. Finally, some conclusions are given in Sect. 5.

The symbols used throughout the paper are as follows. $\mathbb{R}^n_+$ ($\mathbb{R}^n_{++}$) represents the non-negative (positive) orthant on $\mathbb{R}^n$. The vector of all ones is denoted by e. As usual, $\|\cdot\|$ and $\|\cdot\|_\infty$ denote the Euclidean and the infinity norms for vectors, respectively. For two given vectors $x, y \in \mathbb{R}^n$, the inner product is defined as $x^T y = \sum_{i=1}^{n} x_i y_i$. We shall also use the notation $xy = (x_i y_i)_{1 \le i \le n}$. Similarly, the coordinate-wise division of vectors x, y is defined as $x/y = (x_i/y_i)_{1 \le i \le n}$, where y_i $(1 \le i \le n)$ is non-zero. $\min v = \min\{v_i : i = 1, 2, ..., n\}$ $(\max v = \max\{v_i : i = 1, 2, ..., n\})$ is the smallest (maximum) element of a vector v.

2. A PC IPA for $P_*(\kappa)$-WLCP

In this section, we give the central path and search directions for $P_*(\kappa)$-WLCP. We consider the AET $v^2 = e$ for defining search directions. We define the proximity measure in order to measure the distance from the iteration point to the central path.

2.1. The central path and search directions for $P_*(\kappa)$-WLCP

For a given matrix $M \in \mathbb{R}^{n \times n}$ and a vector $q \in \mathbb{R}^n$, the $P_*(\kappa)$-WLCP in $\mathbb{R}^n$ consists in finding a pair vectors $(x, s) \in \mathbb{R}^n \times \mathbb{R}^n$ such that

$$\begin{bmatrix} -Mx + s \\ xs \end{bmatrix} = \begin{bmatrix} q \\ \omega \end{bmatrix}, \quad x, s \ge 0, \tag{2.1}$$

where $\omega \in \mathbb{R}^n_{++}$ is a given weight vector. For a nonnegative number κ, we call that M is a $P_*(\kappa)$-matrix [20] if

$$(1 + 4\kappa) \sum_{i \in I_+} x_i (Mx)_i + \sum_{i \in I_-} x_i (Mx)_i \ge 0, \ \forall x \in \mathbb{R}^n,$$

where $I_+(x) = \{i : x_i (Mx)_i > 0\}$, $I_-(x) = \{i : x_i (Mx)_i < 0\}$. The handicap of the matrix M is defined as: $\hat{\kappa}(M) := \min\{\kappa : \kappa \ge 0, M \text{ is } P_*(\kappa) - \text{matrix}\}$. When $\kappa = 0$, a $P_*(0)$-WLCP reduces to a MWLCP.

Let $\mathcal{F} = \{(x, s) | -Mx + s = q, \ x \ge 0, \ s \ge 0\}$ denote the set of all feasible solutions of $P_*(\kappa)$-WLCP. Define the strictly feasible region of $P_*(\kappa)$-WLCP (2.1) as

$$\mathcal{F}^0 = \{(x, s) \in \mathcal{F} | x > 0, \ s > 0\}.$$

It is proved in [4] that if WLCP is not only monotone but also strictly feasible, then it has a solution. Choosing a strictly feasible initial point $\left(x^0, s^0\right) \in \mathcal{F}^0$ such that $x^0 s^0 \ge \omega$, we define

$$\omega(t) = t x^0 s^0 + (1 - t)\omega, \tag{2.2}$$

where $t \in (0, 1]$. The central path [14] of $P_*(\kappa)$-WLCP (2.1) is formed by the unique solution of the system:

$$\begin{bmatrix} -Mx + s \\ xs \end{bmatrix} = \begin{bmatrix} q \\ \omega(t) \end{bmatrix}, \quad x, s \ge 0. \tag{2.3}$$

System (2.3) has a solution if the interior-point condition (IPC) holds [21], i.e., there exists a strictly feasible solution $\left(x^0, s^0\right) \in \mathcal{F}^0$. Furthermore, if t tends to zero, system (2.3) reduces to (2.1). Then we obtain a solution of $P_*(\kappa)$-WLCP (2.1).

Now, we present the search directions for $P_*(\kappa)$-WLCP. Define the notations

$$v = \sqrt{\frac{xs}{\omega(t)}}, \ \ d_x = \frac{v\Delta x}{x}, \ \ d_s = \frac{v\Delta s}{s}, \ \ d = \sqrt{\frac{x}{s}}. \tag{2.4}$$

Let us consider the continuously differentiable function $\varphi : \mathbb{R}^n \to \mathbb{R}^n$ and suppose that the inverse function φ^{-1} exists. We use the AET:

$$xs = \omega(t) \Leftrightarrow \frac{xs}{\omega(t)} = e \Leftrightarrow v^2 = e.$$

Therefore, system (2.3) can be written as

$$\begin{bmatrix} -Mx + s \\ \varphi\left(\dfrac{xs}{\omega(t)}\right) \end{bmatrix} = \begin{bmatrix} q \\ \varphi(e) \end{bmatrix}, \quad x, s \geq 0. \tag{2.5}$$

For $t \in (0, 1]$, consider the following function

$$f(x, s) = \begin{bmatrix} -Mx + s - q \\ \varphi\left(\dfrac{xs}{\omega(t)}\right) - \varphi(e) \end{bmatrix}.$$

We can see that system (2.5) is equivalent to $f(x, s) = 0$. Using Newton's method, we obtain

$$J_f(x, s)\begin{bmatrix} \Delta x \\ \Delta s \end{bmatrix} = -f(x, s),$$

where $J_f(x, s)$ denotes the Jacobian matrix of f. After some simple calculations, we have the Newton's system:

$$\begin{bmatrix} -M\Delta x + \Delta s \\ \dfrac{s}{\omega(t)}\varphi'\left(\dfrac{xs}{\omega(t)}\right)\Delta x + \dfrac{x}{\omega(t)}\varphi'\left(\dfrac{xs}{\omega(t)}\right)\Delta s \end{bmatrix} = \begin{bmatrix} 0 \\ \varphi(e) - \varphi\left(\dfrac{xs}{\omega(t)}\right) \end{bmatrix}, \quad x, s \geq 0, \tag{2.6}$$

where Δx and Δs are search directions. Substituting relation (2.4) into system (2.6), we get

$$\begin{bmatrix} -\overline{M}d_x + d_s \\ d_x + d_s \end{bmatrix} = \begin{bmatrix} 0 \\ p_v \end{bmatrix}, \tag{2.7}$$

where $\overline{M} = \sqrt{W(t)}^{-1}DMD\sqrt{W(t)}$, $D = diag(d)$, $W(t) = diag(\omega(t))$ and $p_v = \dfrac{\varphi(e) - \varphi(v^2)}{v\varphi'(v^2)}$. By choosing function $\varphi(t)$ appropriately, the system (2.7) can be used to define a class of search directions. For example:

- $\varphi(t) = t$ gives the classical search directions [21] $p_v = v^{-1} - v$.
- $\varphi(t) = \frac{\sqrt{t}}{2(1+\sqrt{t})}$ gives the search directions $p_v = e - v^2$ which used by Kheirfam in [15].
- $\varphi(t) = t - \sqrt{t}$ gives the search directions $p_v = \frac{2(v-v^2)}{2v-e}$ which used by Darvay in [16].

In this paper, we consider the function $\varphi(t) = \sqrt{t}$ [14]. Thus, from system (2.7), we have $p_v = 2(e - v)$. We define the distance from the current iteration point (x, s) to the central path as a proximity measure

$$\delta(v) = \delta(x, s; \omega(t)) = \frac{\|p_v\|}{2} = \|e - v\|. \tag{2.8}$$

For $(x, s) \in \mathcal{F}^0$, we have

$$\delta(x, s; \omega(t)) = 0 \Leftrightarrow e = v \Leftrightarrow xs = \omega(t).$$

Hence, $\delta(v)$ can be considered as a measure from point (x, s) to the central path. Lemma 1 gives a bound for the component of v, which will be used in the proof of the feasibility.

Lemma 1. ([22]) For any $v \in \mathbb{R}^n$, we have

$$1 - \delta(v) \leq v_i \leq 1 + \delta(v), \quad i = 1, \cdots, n.$$

2.2. The corrector step and predictor step

In this subsection, we give the search directions for $P_*(\kappa)$-WLCP. We compute $(\Delta x, \Delta s)$ from (2.7) by using (2.4). The corrector step is $x^+ = x + \Delta x$, $s^+ = s + \Delta s$. To simplify the analysis, we define

$$q_v = d_x - d_s.$$

Then

$$d_x = \frac{p_v + q_v}{2}, \;\; d_s = \frac{p_v - q_v}{2}, \;\; d_x d_s = \frac{p_v^2 - q_v^2}{4}. \tag{2.9}$$

Define

$$v_+ = \sqrt{\frac{x^+ s^+}{\omega(t)}}, \; d^+ = \sqrt{\frac{x^+}{s^+}}, \; D^+ = diag(d^+), \; \overline{M}_+ = \sqrt{W(t)}^{-1} D^+ M D^+ \sqrt{W(t)}.$$

The predictor system is

$$\begin{bmatrix} -\overline{M}_+ d_x^p + d_s^p \\ d_x^p + d_s^p \end{bmatrix} = \begin{bmatrix} 0 \\ -2v_+ \end{bmatrix}. \tag{2.10}$$

Then we obtain the search directions $(\Delta^p x, \Delta^p s)$ from

$$\Delta^p x = \frac{x^+}{v_+} d_x^p, \; \Delta^p s = \frac{s^+}{v_+} d_s^p. \tag{2.11}$$

After a predictor step, the new iterate is

$$(x^p, \; s^p) = (x^+, \; s^+) + \theta t (\Delta^p x, \; \Delta^p s), \tag{2.12}$$

where θ is a update parameter.

Now, we give a PC IPA for $P_*(\kappa)$-WLCP in details. For a given weight vector $\omega > 0$, we choose a strictly feasible initial point (x^0, s^0) such that $x^0 s^0 \geq \omega$. The update parameter is $\theta \in \left(0, \frac{1}{2}\right)$. Our PC IPA takes one corrector step and one predictor step in a main iteration. The corrector step stays in the neighbourhood of the central path. In the corrector step, we take a full-Newton step. We apply Newton's method to system (2.7) and then obtain a search direction $(\Delta x, \Delta s)$ of the corrector step for $P_*(\kappa)$-WLCP from (2.4). The goal of the predictor step is to reach optimality. We can calculate the predictor search directions $(\Delta^p x, \Delta^p s)$ from (2.10) and (2.11). In order to determine the new search directions, the presented PC IPA applies the kernel function $\varphi(t) = \sqrt{t}$ to the equation $v^2 = e$. The stopping criterion is $\|x^k s^k - \omega\| \leq \varepsilon$, where $\varepsilon > 0$ is an accuracy parameter. The framework of our algorithm is described as *Algorithm* 1.

3. Analysis of PC IPA for $P_*(\kappa)$-WLCP

In this section, we analyze that *Algorithm* 1 is well-defined. Then we establish an upper bound for the value of proximity measure after a full-Newton step. We show that *Algorithm* 1 can solve $P_*(\kappa)$-WLCP in polynomial complexity.

Algorithm 1. A PC IPA for $P_*(\kappa)$-WLCP

Input
Accuracy parameter $\varepsilon > 0$;
Update parameter $\theta \in \left(0, \frac{1}{2}\right)$;
Let $(x^0, s^0) \in \mathcal{F}^0$, where $\delta(x^0, s^0; \omega(t_0)) \leq \frac{t_0}{2(1+4\kappa')}$ with $t_0 = 1$;
Set $k := 0$;
begin
$x := x^0, \ s := s^0; \ t := t_0$;
  while $\|x^k s^k - \omega\| > \varepsilon$ **do**
  begin
    (corrector step)
    obtain $(\Delta x, \Delta s)$ by solving the system (2.7) and using (2.4);
    let $x^+ := x + \Delta x$, $s^+ := s + \Delta s$;
    (predictor step)
    obtain $(\Delta^p x, \Delta^p s)$ by solving the system (2.10) and using (2.11);
    let $x^p := x^+ + \theta t \Delta^p x$, $s^p := s^+ + \theta t \Delta^p s$;
    set $\omega^p(t) := (1 - 2\theta t)\omega(t)$, $t_p := (1 - 2\theta)t$;
    $x^k := x^p$, $s^k := s^p$; $\omega(t) = \omega^p(t)$; $t := t_p$; $k := k + 1$;
  end
end

3.1. Feasibility of algorithm

In the following, Lemma 2 gives the estimates of $d_x^T d_s$ and $\|d_x d_s\|_\infty$ which are important for analyzing the feasibility of *Algorithm* 1. After a corrector step, we define the new point as $x^+ = x + \triangle x, \ \ s^+ = s + \Delta s$.

Lemma 2. $(c.f.$ [20, 23]) Let $\delta = \delta(x, s; \omega(t))$ and $x^0 s^0 \geq \omega$. Then the following inequality holds:

$$\|q_v\|^2 \leq 4(1 + 4\kappa')\delta^2, \ -4\kappa'\delta^2 \leq d_x^T d_s \leq \delta^2, \ \|d_x d_s\|_\infty \leq (1 + 4\kappa')\delta^2,$$

where $\kappa' = \dfrac{(1 + 4\kappa)\max(x^0 s^0) - \min \omega}{4 \min \omega}$ and κ' is the handicap of the matrix $\overline{M}$.

Lemma 3. Let $x^0 s^0 \geq \omega$. The iterate (x^+, s^+) is positive if $\delta < \dfrac{1}{\sqrt{2 + 4\kappa'}}$.

Proof. For any $\alpha \in [0, 1]$, let $x(\alpha) = x + \alpha \Delta x$, $s(\alpha) = s + \alpha \Delta s$. By (2.4), we get

$$x(\alpha) = \sqrt{\omega(t)}\sqrt{\frac{x}{s}}(v + \alpha d_x), \quad s(\alpha) = \sqrt{\omega(t)}\sqrt{\frac{s}{x}}(v + \alpha d_s).$$

From the second equation in (2.7), it follows that

$$\begin{aligned} x(\alpha)s(\alpha) &= \omega(t)(v + \alpha d_x)(v + \alpha d_s) \\ &= \omega(t)\left[(1 - \alpha)v^2 + \alpha\left(v^2 + v p_v\right) + \alpha^2 d_x d_s\right]. \end{aligned} \tag{3.1}$$

Since $p_v = 2(e - v)$, we have for any $\alpha \in [0, 1]$

$$
\begin{aligned}
x(\alpha)s(\alpha) &= \omega(t)\left[(1-\alpha)v^2 + \alpha\left(-v^2 + 2v\right) + \alpha^2 d_x d_s\right] \\
&\geq \omega(t)\left[(1-\alpha)v^2 + \alpha^2\left(-v^2 + 2v + d_x d_s\right)\right]. \qquad (3.2)
\end{aligned}
$$

By Lemma 1, we have $-v^2 + 2v \geq e - \delta^2 e$. Now we obtain from (3.2) and Lemma 2

$$
\begin{aligned}
x(\alpha)s(\alpha) &\geq \omega(t)\left[(1-\alpha)v^2 + \alpha^2\left(e - \delta^2 e + d_x d_s\right)\right] \\
&\geq \omega(t)\left[(1-\alpha)v^2 + \alpha^2\left(1 - \delta^2 - \|d_x d_s\|_\infty\right)e\right] \\
&\geq \omega(t)\left[(1-\alpha)v^2 + \alpha^2\left(1 - \delta^2 - (1+4\kappa')\delta^2\right)e\right] \\
&= \omega(t)\left[(1-\alpha)v^2 + \alpha^2\left(1 - (2+4\kappa')\delta^2\right)e\right].
\end{aligned}
$$

Thus, for $0 \leq \alpha \leq 1$, none of the entries of $x(\alpha)$ and $s(\alpha)$ vanishes if $\delta < \frac{1}{\sqrt{2+4\kappa'}}$. Since $x(\alpha)$ and $s(\alpha)$ are both linear functions of α and $x(0) = x^0 > 0,\ s(0) = s^0 > 0$, this implies that $x(\alpha) > 0,\ s(\alpha) > 0$. Hence, $x^+ = x(1) > 0$ and $s^+ = s(1) > 0$. The proof is complete.

Lemma 3 shows that after the predictor step, both x^+ and s^+ are strictly feasible. The next lemma gives an upper bound on $\left\|e - v_+^2\right\|$.

Lemma 4. Let $v_+ = \sqrt{\frac{x^+ s^+}{\omega(t)}}$ and $x^0 s^0 \geq \omega$. If $\delta = \delta(x, s; \omega(t)) < \frac{1}{\sqrt{2+4\kappa'}}$, then

$$\left\|e - v_+^2\right\| \leq (1+4\kappa')\delta^2.$$

Proof. By (2.7), (2.9) and (3.1), we obtain

$$
\begin{aligned}
e - v_+^2 &= e - v^2 - v p_v - \frac{p_v^2 - q_v^2}{4} \\
&= e - \left(v + \frac{p_v}{2}\right)^2 + \frac{q_v^2}{4} \\
&= \frac{q_v^2}{4}. \qquad (3.3)
\end{aligned}
$$

It follows from (3.3) and Lemma 2 that

$$\left\|e - v_+^2\right\| = \frac{\left\|q_v^2\right\|}{4} \leq \frac{\|q_v\|^2}{4} \leq (1+4\kappa')\delta^2. \qquad (3.4)$$

3.2. Convergence of algorithm

In this subsection, we investigate the upper bound of the proximity measure after a corrector step and a predictor step. Then we show the convergence of *Algorithm* 1. First, we give an upper bound for $\delta(v_+)$ after a full-Newton step when $\omega(t)$ is fixed.

Lemma 5. Let $\delta(v_+) = \delta(x^+, s^+; \omega(t))$ and $x^0 s^0 \geq \omega$. If $\delta = \delta(x, s; \omega(t)) < \frac{1}{\sqrt{2+4\kappa'}}$, then

$$\delta(v_+) \leq \frac{(1+4\kappa')\delta^2}{1 + \sqrt{1 - (1+4\kappa')\delta^2}}.$$

Proof. We have by (2.8)

$$\delta(v_+) = \|e - v_+\| = \left\|\frac{e - v_+^2}{e + v_+}\right\| \leq \left\|\frac{e}{e + v_+}\right\|_\infty \left\|e - v_+^2\right\|. \tag{3.5}$$

Then, we will provide an upper bound for $\left\|\frac{e}{e + v_+}\right\|_\infty$. From (3.3) and (3.4), we obtain

$$\min v_+^2 = \min\left(e - \frac{q_v^2}{4}\right) \geq 1 - \left\|\frac{q_v^2}{4}\right\|_\infty \geq 1 - \frac{\|q_v\|^2}{4} \geq 1 - (1 + 4\kappa')\delta^2. \tag{3.6}$$

It follows from (3.6) that

$$\min v_+ \geq \sqrt{1 - (1 + 4\kappa')\delta^2},$$

which implies

$$\left\|\frac{e}{e + v_+}\right\|_\infty \leq \frac{1}{1 + \min v_+} \leq \frac{1}{1 + \sqrt{1 - (1 + 4\kappa')\delta^2}}. \tag{3.7}$$

According to (3.5), (3.7) and Lemma 4, we derive that

$$\delta(v_+) \leq \frac{(1 + 4\kappa')\delta^2}{1 + \sqrt{1 - (1 + 4\kappa')\delta^2}}.$$

This completes the proof of the lemma.

Let $v^p = \sqrt{\frac{x^p s^p}{\omega^p(t)}}$ with $\omega^p(t) = (1 - 2\theta t)\omega(t)$. In the following lemma, we give an upper bound of proximity measure after a predictor step with $\omega(t)$ updated.

Lemma 6. ([24]) Let $x^0 s^0 \geq \omega$. One has

$$\left\|d_x^p d_s^p\right\| \leq 2(1 + 2\kappa')\,\|v_+\|^2.$$

Lemma 7. Let $\delta < \frac{1}{\sqrt{2 + 4\kappa'}}$ and $x^0 s^0 \geq \omega$, then

$$\left\|d_x^p d_s^p\right\| \leq 2n(1 + 2\kappa')\left[1 + (1 + 4\kappa')\delta^2\right].$$

Proof. From (3.2) with $\alpha = 1$, we have

$$\|v_+\|^2 = \sum_{i=1}^{n}\left(-v_i^2 + 2v_i + d_{x_i} d_{s_i}\right).$$

Let $g(\lambda) = -\lambda^2 + 2\lambda$, then $g'(\lambda) = -2\lambda + 2$. When $\delta < \frac{1}{\sqrt{2 + 4\kappa'}}$, it follows from Lemma 1 that $0 < v_i < 2$ and $g(\lambda) \leq g(1) = 1$. By Lemma 2, we get

$$\|v_+\|^2 \leq n + \sum_{i=1}^{n}\left|d_{x_i} d_{s_i}\right| \leq n + n\,\|d_x d_s\|_\infty \leq n\left[1 + (1 + 4\kappa')\delta^2\right].$$

Using Lemma 6, we obtain the desired result.

Lemma 8. Let $x^+ > 0,\ s^+ > 0$, $x^0 s^0 \geq \omega$ and $\theta < \dfrac{1}{2\sqrt{2n(1+2\kappa')}}$ with $n \geq 2$. If $\delta < \dfrac{1}{\sqrt{3(1+4\kappa')}}$, then $x^p > 0,\ s^p > 0$.

Proof. Let us define

$$x^p(\alpha) = x^+ + \alpha\theta t \Delta^p x,\ s^p(\alpha) = s^+ + \alpha\theta t \Delta^p s,$$

for $0 \leq \alpha \leq 1$. We have from (2.12)

$$x^p(\alpha) = \frac{x^+}{v_+}(v_+ + \alpha\theta t d_x^p),\ s^p(\alpha) = \frac{s^+}{v_+}(v_+ + \alpha\theta t d_s^p).$$

Therefore, using system (2.10) we obtain

$$\begin{aligned} x^p(\alpha)s^p(\alpha) &= \omega(t)\left[v_+^2 + \alpha\theta t v_+(d_x^p + d_s^p) + \alpha^2\theta^2 t^2 d_x^p d_s^p\right] \\ &= \omega(t)\left[(1-2\alpha\theta t)v_+^2 + \alpha^2\theta^2 t^2 d_x^p d_s^p\right], \end{aligned} \tag{3.8}$$

which implies that

$$\begin{aligned} \min \frac{x^p(\alpha)s^p(\alpha)}{\omega(t)(1-2\alpha\theta t)} &\geq \min v_+^2 - \frac{\alpha^2\theta^2 t^2}{1-2\alpha\theta t}\left\|d_x^p d_s^p\right\|_\infty \\ &\geq \min v_+^2 - \frac{\theta^2 t^2}{1-2\theta t}\left\|d_x^p d_s^p\right\|. \end{aligned} \tag{3.9}$$

Combining (3.6), (3.9) and Lemma 7 yields that

$$\begin{aligned} \min \frac{x^p(\alpha)s^p(\alpha)}{\omega(t)(1-2\alpha\theta t)} &\geq 1 - (1+4\kappa')\delta^2 - \frac{2n(1+2\kappa')\theta^2 t^2}{1-2\theta t}\left[1 + (1+4\kappa')\delta^2\right] \\ &= h(\delta, \theta, n). \end{aligned} \tag{3.10}$$

Now we give the strict positivity of $h(\delta,\theta,n)$. Let $n \geq 2$, $\theta < \dfrac{1}{2\sqrt{2n(1+2\kappa')}}$ and $\delta < \dfrac{1}{\sqrt{3(1+4\kappa')}}$, we have

$$\begin{aligned} h(\delta,\theta,n) &\geq 1 - \frac{1}{3} - \frac{2n(1+2\kappa')\theta^2 t^2}{1-2\theta t}\left(1 + \frac{1}{3}\right) \\ &\geq \frac{2}{3} - \frac{\sqrt{2n(1+2\kappa')}}{3\left(\sqrt{2n(1+2\kappa')} - 1\right)} \\ &> 0. \end{aligned}$$

The second inequality follows from the fact that $f(\gamma) = \dfrac{2\gamma^2}{1-2\gamma}$ is increasing for $0 < \gamma < \dfrac{1}{2}$. Then $h(\delta,\theta,n) > 0$ with $\kappa' \geq 0$ and $n \geq 2$. This implies that $x^p(\alpha)s^p(\alpha) > 0$ for $0 \leq \alpha \leq 1$ and $0 < \theta < \dfrac{1}{2\sqrt{2n(1+2\kappa')}}$. Hence $x^p(\alpha) > 0$ and $s^p(\alpha) > 0$ for all $0 \leq \alpha \leq 1$. Using $x^p(0) = x^+ > 0$ and $s^p(0) = s^+ > 0$, we have $x^p(1) = x^p > 0$ and $s^p(1) = s^p > 0$.

In the next lemma, we investigate the value of the proximity measure after a predictor step.

Lemma 9. Let $x^0 s^0 \geq \omega$, $\delta < \frac{1}{\sqrt{3(1+4\kappa')}}$ and $\omega^p(t) = (1-2\theta t)\omega(t)$ with $\theta < \frac{1}{2\sqrt{2n(1+2\kappa')}}$, then

$$\delta(v^p) := \delta(x^p, s^p; \omega^p(t)) \leq 1 - \sqrt{h(\delta, \theta, n)},$$

where $h(\delta, \theta, n) = 1 - (1+4\kappa')\delta^2 - \frac{2n(1+2\kappa')\theta^2 t^2}{1-2\theta t}\left[1 + (1+4\kappa')\delta^2\right]$.

Proof. By the definition of $\delta(v)$, we get

$$\delta(v^p) = \|e - v^p\| \leq \frac{\left\|e - (v^p)^2\right\|}{1 + \min v^p}. \tag{3.11}$$

We first estimate an lower bound on the component of v^p. By (3.10), we obtain

$$\min(v^p)^2 = \min \frac{x^p s^p}{(1-2\theta t)\omega(t)} \geq h(\delta, \theta, n),$$

which implies

$$\min v^p \geq \sqrt{h(\delta, \theta, n)}. \tag{3.12}$$

Using (3.8), we have

$$\left\|e - (v^p)^2\right\| = \left\|e - v_+^2 - \frac{\theta^2 t^2}{1-2\theta t} d_x^p d_s^p\right\| \leq \left\|e - v_+^2\right\| + \frac{\theta^2 t^2}{1-2\theta t}\left\|d_x^p d_s^p\right\|.$$

It follows from Lemma 4, Lemma 7 and (3.10) that

$$\left\|e - (v^p)^2\right\| \leq (1+4\kappa')\delta^2 + \frac{2n(1+2\kappa')\theta^2 t^2\left[1 + (1+4\kappa')\delta^2\right]}{1-2\theta t} = 1 - h(\delta, \theta, t). \tag{3.13}$$

Combining (3.11)–(3.13) yields that

$$\delta(v^p) \leq \frac{1 - h(\delta, \theta, t)}{1 + \sqrt{h(\delta, \theta, t)}} = 1 - \sqrt{h(\delta, \theta, t)}.$$

Lemma 10. Let $x^0 s^0 \geq \omega$, $t_p = (1-2\theta)t$ and $\theta \leq \frac{2-t}{8(1+4\kappa')\sqrt{n}}$ with $n \geq 2$. If $\delta(v) \leq \frac{t}{2(1+4\kappa')}$, then $\delta(v^p) \leq \frac{t_p}{2(1+4\kappa')}$.

Proof. From Lemma 9, it follows that $\delta(v^p) \leq \frac{t_p}{2(1+4\kappa')}$ holds, if

$$1 - \sqrt{h(\delta, \theta, t)} \leq \frac{(1-2\theta)t}{2(1+4\kappa')}.$$

Then, we have

$$\frac{(1-2\theta)^2 t^2}{4(1+4\kappa')^2} - \frac{(1-2\theta)t}{1+4\kappa'} + (1+4\kappa')\delta^2 + \frac{2n(1+2\kappa')\theta^2 t^2}{1-2\theta t}\left[1 + (1+4\kappa')\delta^2\right] \leq 0. \tag{3.14}$$

Since $\delta(v) \le \frac{t}{2(1+4\kappa')}$ and $\theta_{\max} = \frac{1}{4(1+4\kappa')\sqrt{n}} \le \frac{1}{4\sqrt{2}}$, we obtain from (3.14) that

$$\begin{aligned}
&(1-2\theta)^3 t - 4(1-2\theta)^2 + (1-2\theta)t + 8\theta^2 tn(1+2\kappa')(1+4\kappa')\left[1 + \frac{t^2}{4(1+4\kappa')}\right] \\
&\le (1-2\theta)^3 - 4(1-2\theta)^2 + (1-2\theta) + 2\theta^2 n\left[4(1+2\kappa')(1+4\kappa') + (1+2\kappa')\right] \\
&\le -8\theta^3 - 4\theta^2 + 8\theta - 2 + 10\theta^2 n(1+4\kappa')^2 \\
&\le -8\left(\frac{1}{4\sqrt{2}}\right)^3 - 4\left(\frac{1}{4\sqrt{2}}\right)^2 + 8\left(\frac{1}{4\sqrt{2}}\right) - 2 + 10\theta_{\max}^2 n(1+4\kappa')^2 \\
&\le 1.2451 - 2 + 0.625 \\
&= -0.1299.
\end{aligned}$$

Here the third inequality follows from the fact that $-8\theta^3 - 4\theta^2 + 8\theta$ is increasing for $0 < \theta \le \frac{1}{4\sqrt{2}}$. Then (3.14) holds, which implies the result.

3.3. Complexity bound

In this subsection, we will give an upper bound of iterations for *Algorithm* 1. We first examine the value of Gap=$\|xs - \omega\|$.

Lemma 11. Let $x^0 s^0 \ge \omega$. If $\delta(v) \le \frac{t}{2(1+4\kappa')}$ and $\theta \le \frac{2-t}{8(1+4\kappa')\sqrt{n}}$ with $n \ge 2$, then

$$\|x^p s^p - \omega\| \le \left[\frac{17}{16(1+4\kappa')}\max\left(x^0 s^0\right) + \left\|x^0 s^0 - \omega\right\|\right] t,$$

where $t \in (0,1]$.

Proof. From (2.2), (3.13) and $x^0 s^0 \ge \omega$, we have

$$\begin{aligned}
\|x^p s^p - \omega\| &\le \|x^p s^p - \omega^p(t)\| + \|\omega^p(t) - \omega(t)\| + \|\omega(t) - \omega\| \\
&\le \left\|e - (v^p)^2\right\| \|\omega^p(t)\|_\infty + 2\theta t\|\omega(t)\| + \left\|x^0 s^0 - \omega\right\| t \\
&\le \left\{(1+4\kappa')\delta^2 + \frac{2n(1+2\kappa')\theta^2 t^2\left[1 + (1+4\kappa')\delta^2\right]}{1-2\theta t} + 2\theta\sqrt{n}t\right\}\max\left(x^0 s^0\right) + \left\|x^0 s^0 - \omega\right\| t.
\end{aligned}$$

Suppose that $\delta(v) \le \frac{t}{2(1+4\kappa')}$. Due to the fact that $\theta_{\max} = \frac{1}{4(1+4\kappa')\sqrt{n}} \le \frac{1}{4\sqrt{2}}$ and $t \in (0,1]$, we obtain

$$\begin{aligned}
\|x^p s^p - \omega\| &\le \left\{\frac{t}{4(1+4\kappa')} + \frac{2n(1+2\kappa')\theta^2 t}{1-2\theta t}\left[1 + \frac{t^2}{4(1+4\kappa')}\right] + 2\theta\sqrt{n}\right\}\max\left(x^0 s^0\right) t + \left\|x^0 s^0 - \omega\right\| t \\
&\le \left\{\frac{1}{4(1+4\kappa')} + \frac{2n(1+2\kappa')}{16n(1+4\kappa')^2}\cdot\frac{1}{1-\frac{1}{2\sqrt{2}}}\cdot\left[1 + \frac{1}{4(1+4\kappa')}\right] + \frac{1}{2(1+4\kappa')}\right\}\max\left(x^0 s^0\right) t \\
&\quad + \left\|x^0 s^0 - \omega\right\| t
\end{aligned}$$

$$
\begin{aligned}
&< \left[\frac{1}{4(1+4\kappa')} + \frac{1}{8(1+4\kappa')}\cdot 2\cdot\frac{5}{4} + \frac{1}{2(1+4\kappa')}\right]\max\left(x^0s^0\right)t + \left\|x^0s^0-\omega\right\|t \\
&\le \frac{17}{16(1+4\kappa')}\max\left(x^0s^0\right) + \left\|x^0s^0-\omega\right\|.
\end{aligned}
$$

Here the third inequality holds because $\frac{1}{1-\frac{1}{2\sqrt{2}}} < \frac{1}{1-\frac{1}{2}} = 2$ and $1+\frac{1}{4(1+4\kappa')} \le \frac{5}{4}$. This completes the proof of the lemma.

Lemma 12 provides an upper bound for the number of iterations generated by *Algorithm* 1.

Theorem 12. Let $(x^0, s^0) \in \mathcal{F}^0$, $x^0s^0 \ge \omega$ and $\theta \le \frac{2-t}{8(1+4\kappa')\sqrt{n}}$ with $n \ge 2$. Then *Algorithm* 1 provides an ε-optimal solution of the $P_*(\kappa)$-WLCP (2.1) after at most

$$
O\left((1+4\kappa')\sqrt{n}\log\frac{\frac{17}{16(1+4\kappa')}\max\left(x^0s^0\right)+\left\|x^0s^0-\omega\right\|}{\varepsilon}\right)
$$

iterations.

Proof. Let $t_0 = 1$ and $t_+ = (1-2\theta)t$. From Lemma 11, we obtain

$$
\begin{aligned}
\left\|x^ks^k-\omega\right\| &\le \left[\frac{17}{16(1+4\kappa')}\max\left(x^0s^0\right)+\left\|x^0s^0-\omega\right\|\right]t_{k-1} \\
&\le \left[\frac{17}{16(1+4\kappa')}\max\left(x^0s^0\right)+\left\|x^0s^0-\omega\right\|\right](1-2\theta_{\min})^{k-1}.
\end{aligned}
$$

The inequality $\left\|x^ks^k-\omega\right\| \le \varepsilon$ holds if

$$
\left[\frac{17}{16(1+4\kappa')}\max\left(x^0s^0\right)+\left\|x^0s^0-\omega\right\|\right](1-2\theta_{\min})^{k-1} \le \varepsilon. \tag{3.15}
$$

Taking logarithms of both sides and using the inequality $\log(1-\xi) \le -\xi$, (3.15) holds if

$$
k \ge \frac{1}{2\theta_{\min}}\log\frac{\frac{17}{16(1+4\kappa')}\max\left(x^0s^0\right)+\left\|x^0s^0-\omega\right\|}{\varepsilon}+1.
$$

Then *Algorithm* 1 finds an ε-optimal solution in at most

$$
\left\lfloor\frac{1}{2\theta_{\min}}\log\frac{\frac{17}{16(1+4\kappa')}\max\left(x^0s^0\right)+\left\|x^0s^0-\omega\right\|}{\varepsilon}\right\rfloor+1
$$

iterations. Since $t \in (0,1]$ and $\theta_{\min} = \frac{1}{8(1+4\kappa')\sqrt{n}}$, the proof is straightforward.

4. Numerical examples

In this section, we present some numerical results of $P_*(\kappa)$-WLCPs to show the effectiveness of *Algorithm* 1. All the experiments were performed on a personal computer with Intel(R) Core(TM) i5-10210U CPU @2.11 GHz 8.00GB memory. The operating system was Windows 10 and the implementations were done in MATLAB (R2018a). In the implementation of *Algorithm* 1, let $x^0 s^0 \geq \omega$, accuracy parameter $\varepsilon = 10^{-5}$ and Gap= $\|xs - \omega\|$.

Example 1. [25] Consider the $P_*(\kappa)$-WLCP (2.1) in $\mathbb{R}^{4\times 4}$, where

$$M = \begin{bmatrix} 2 & 1 & 1 & 1 \\ 1 & 2 & 0 & 1 \\ 1 & 0 & 1 & 2 \\ -1 & -1 & -2 & 0 \end{bmatrix}, \quad q = [-4 \ \ -3 \ \ -3 \ \ 5]^T, \quad \omega = rand(4,1).$$

The strictly feasible initial point for *Algorithm* 1 is $x^0 = s^0 = [1 \ \ 1 \ \ 1 \ \ 1]^T$. We set the update parameter $\theta = 0.1$. The unique solution of Example 1 is

$$x^* = [1.3091 \ \ 0.6449 \ \ 0.8426 \ \ 1.3953],$$

$$s^* = [0.5903 \ \ 1.0682 \ \ 0.1496 \ \ 0.6438],$$

which takes 0.0261 seconds and 52 iterations.

Example 2. [26] Let us consider the $P_*(\kappa)$-WLCP (2.1) with

$$M = \begin{bmatrix} 1 & 2 & 2 & \cdots & 2 \\ 2 & 5 & 6 & \cdots & 6 \\ 2 & 6 & 9 & \cdots & 10 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 2 & 6 & 10 & \cdots & 4n-3 \end{bmatrix}, \quad q = [-1 \ \ -1 \ \ -1 \ \ \cdots \ \ -1]^T, \quad \omega = rand(n,1).$$

We choose the initial point $x^0 = e, \ s^0 = Mx^0 + q$. The value of update parameter is $\theta = 0.05$. The running time and iterations of *Algorithm* 1 for solving Example 2 are denoted by 'CPU' and 'Iter'. Gap and $\delta(v)$ are the values of $\|xs - \omega\|$ and $\|e - v\|$, respectively. The numerical results with different n are summarized in Table 1.

Table 1. Numerical results for Example 2.

n	CPU	Iter	Gap	$\delta(v)$
10	0.0207	114	9.8003e-6	1.4574e-12
100	0.5537	126	9.7951e-6	5.0575e-11
300	9.1477	133	9.9024e-6	6.0770e-12
600	36.4226	136	9.8653e-6	5.7824e-10
900	102.6694	138	9.5108e-6	7.1522e-10
1200	214.4561	149	9.8031e-6	1.6408e-09

Example 3. [27] Consider the $P_*(\kappa)$-WLCP (2.1), where the matrix $M \in \mathbb{R}^{n\times n}$ and the weight vector $\omega \in \mathbb{R}^n$ are given by

$$M = \begin{bmatrix} 6 & -4 & 2 & \cdots & 0 \\ -4 & 6 & -4 & \cdots & 0 \\ 2 & -4 & 6 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 6 \end{bmatrix}, \quad \omega = rand(n, 1).$$

In this example, we take $x^0 = s^0 = e$ and $q = -Mx^0 + s^0$. Set the update parameters as $\theta \in \{0.05,\ 0.10,\ 0.15,\ 0.20,\ 0.25\}$ and dimensions of the example as $n \in \{100,\ 300,\ 500,\ 700,\ 900\}$. The numerical results of Example 3 for different θ and n are shown in Table 2.

Table 2. Numerical results for Example 3 with different θ and n.

n	$\theta = 0.05$		$\theta = 0.10$		$\theta = 0.15$		$\theta = 0.20$		$\theta = 0.25$	
	CPU	Iter	CPU	Iter	CPU	Iter	CPU	Iter	CPU	Iter
100	0.6147	129	0.2651	62	0.1889	36	0.0984	28	0.0912	18
300	8.0243	134	3.4047	64	2.9213	37	1.5469	29	1.3415	19
500	27.8059	135	12.8321	65	7.3513	39	5.5678	30	4.5624	21
700	54.3689	137	25.7613	66	14.7698	40	11.0765	32	8.8229	22
900	101.4634	138	50.2722	68	28.1039	42	21.8132	34	16.3462	23

It is obvious from Table 1 that the running time and the number of iterations are depend on n. The running time and the number of iterations decrease as n reduces. Furthermore, the number of iterations grows slowly as n increases. It can be seen from Table 2 that the larger θ gives the less running time. Obviously, the minimum value of θ leads to the largest number of iterations.

Example 4. [27] We randomly generate five $P_*(\kappa)$-WLCPs (2.1) with $M = A^TA$, where $A \in \mathbb{R}^{n\times n}$, $rank(A) = n$ and $n \in \{20, 100, 200, 400, 600\}$. The initial point is $x^0 = s^0 = e$. Let the weight vector $\omega = \frac{x^0 s^0}{2}$ and update parameter $\theta = 0.1$. In Table 3, we compare the result of *Algorithm* 1 with the algorithm in MLCP [19]. Moreover, we denote by *Algorithm* 2 the algorithm introduced by Mansouri and Pirhaji [19].

Table 3. Numerical results for Example 4 by *Algorithm* 1 and *Algorithm* 2.

	Algorithm 1				*Algorithm* 2			
n	CPU	Iter	Gap	$\delta(v)$	CPU	Iter	Gap	$\delta(v)$
20	0.0152	57	9.0261e-06	1.7586e-12	0.0246	60	8.4706e-06	4.9652e-13
100	0.1531	60	9.6533e-06	4.6658e-13	0.3127	63	8.4829e-06	1.0427e-12
200	0.8112	62	9.9522e-06	2.2747e-13	1.3889	65	9.4029e-06	5.5448e-13
400	7.1878	63	9.2343e-06	1.8058e-13	7.2245	66	9.6939e-06	2.7296e-12
600	12.5847	70	9.1608e-06	5.6522e-12	21.7197	81	9.6168e-06	1.7469e-12

Figures 1 and 2 show the Gap of Example 1 and Example 2 are convergent in the iterative process. Besides, based on the value of $\delta(v)$ in Figures 1 and 2, $\delta(v)$ reduces to 0 as t tends to zero. The values of Gap in Figures 3 and 4 are provided to demonstrate the convergence of Example 3 and Example 4 with different n. Thus, *Algorithm* 1 could efficiently solve $P_*(\kappa)$-WLCP.

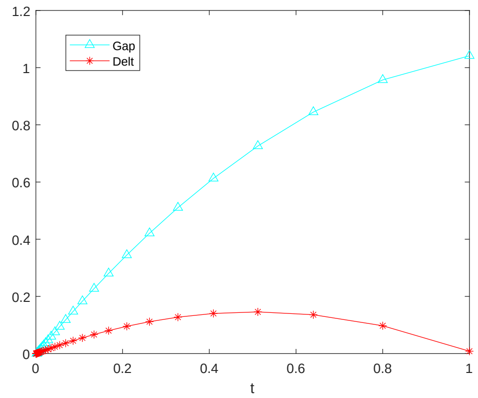

Figure 1. The results for Example 1.

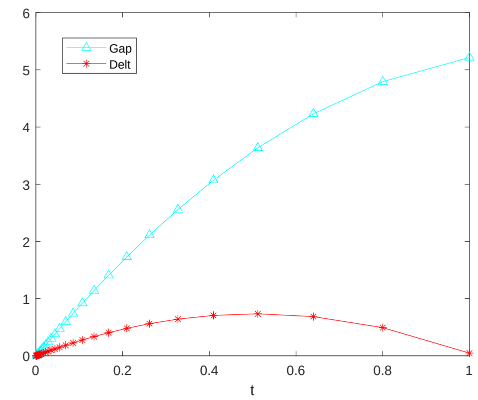

Figure 2. The results for Example 2.

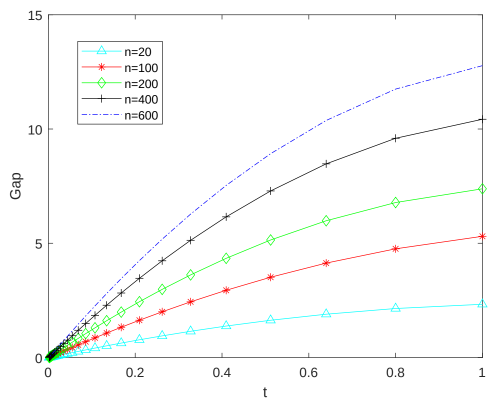

Figure 3. The value of Gap for Example 3.

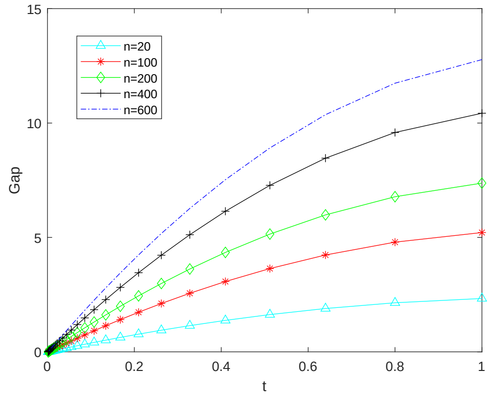

Figure 4. The value of Gap for Example 4.

5. Conclusions

We propose a PC IPA for $P_*(\kappa)$-WLCP based on the kernel function $\phi(t) = \sqrt{t}$. Applying this function to the central path, new search directions for $P_*(\kappa)$-WLCP are obtained. The analysis of $P_*(\kappa)$-WLCP is more complicated than $P_*(\kappa)$-LCP because of the nonzero weight vector. We prove the feasibility and convergence of *Algorithm* 1. Numerical results indicate the efficiency of our algorithm.

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Conflict of interest

All authors declare no conflicts of interest in this paper.

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