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Research article

Averaging aggregation operators under the environment of *q*-rung orthopair picture fuzzy soft sets and their applications in MADM problems

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Abstract: q-Rung orthopair fuzzy soft set handles the uncertainties and vagueness by membership and non-membership degree with attributes, here is no information about the neutral degree so to cover this gap and get a generalized structure, we present hybrid of picture fuzzy set and q-rung orthopair fuzzy soft set and initiate the notion of q-rung orthopair picture fuzzy soft set, which is characterized by positive, neutral and negative membership degree with attributes. The main contribution of this article is to investigate the basic operations and some averaging aggregation operators like q-rung orthopair picture fuzzy soft weighted averaging operator and q-rung orthopair picture fuzzy soft order weighted averaging operator under the environment of q-rung orthopair picture fuzzy soft set. Moreover, some fundamental properties and results of these aggregation operators are studied, and based on these proposed operators we presented a stepwise algorithm for MADM by taking the problem related to medical diagnosis under the environment of q-rung orthopair picture fuzzy soft set and finally, for the superiority we presented comparison analysis of proposed operators with existing operators.

Keywords: q-ROPF soft set; operational properties of $q\text{-}ROPFS_fSs$; $q\text{-}ROPFS_fWA$ operator; $q\text{-}ROPFS_fOWA$ operator and MADM problems
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List of abbreviations

MD	membership degree
NMD	non-membership degree
FS	Fuzzy set
IFS	Intuitionistic fuzzy set
P_yFS	Pythagorean fuzzy set
q-ROFS	q-rung orthopair fuzzy set
q-ROPFS$_f$Ss	q-rung orthopair picture fuzzy soft sets
PFS	Picture fuzyy set
AOs	Averaging operators
MADM	Multi attribute decision making
P_YFWA	Pythagorean fuzzy weighted averaging
P_YFWG	Pythagorean fuzzy weighted geometric
P_YFWPA operator	Pythagorean fuzzy weighted power averaging operator
P_YFWPG operator	Pythagorean fuzzy weighted power geometric operator
q-ROFWA operators	q-rung orthopair fuzzy weighted averaging operators
q-ROFWG operators	q-rung orthopair fuzzy weighted geometric operators
q-ROPFS$_f$WA operator	q-rung orthopair picture fuzzy weighted averaging operator
q-ROPFS$_f$OWA operator	q-rung orthopair picture fuzzy order weighted averaging operator
WV	Weighted vector

1. Introduction

In real-life situation, decision-making (DM) plays a vital role, for the selection of logical choice among several objects we use the process of MD. The foundation of a fuzzy set (FS) was laid by Zadeh [1] in 1965, characterized by member function belong to [0,1]. This concept was further extended by Zadeh [2] in 1975, and proposed an interval-valued fuzzy (IVF)set characterized by a lower fuzzy set and an upper fuzzy set. In the DM problem parameterized fuzzy operators were introduced by Song et al. [3]. In 1986, Atanassov [29] generalized the theory of FS and initiated intuitionistic fuzzy (IF)set by the affix of non-MF with the restriction $(MF) + (NMF) \leq 1$. Some aggregation operators like generalized AOs by Zhao et al. [4] and generalized geometric AOs by Tan et al. [5,6] under the environment of IFS. In IFS we study the MD and NMD, here we ignore the neutral degree, so to cover these gaps in 2014 Cuong [7] proposed the generalized structure of IFS and FS called picture fuzzy set, which is characterized by three membership degree positive, neutral and negative member degree with the restriction that the sum of this three-membership degree is less and equal than 1. In 1998, Smarandache [25] proposed a neutrosophic set, which is characterized by truth, indeterminacy, and falsehood membership degree, with the condition that the sum of truth, indeterminacy, and falsehood membership degree is less and equal to 3. In a neutrosophic set it is difficult to handle the voting problems when the expert's judgment is of a type like yes, abstinence, no, and rejection, as the sum of the three membership degrees is greater than 1, beside this it cannot be provided the information of voting of non-candidates of the above voting. In the other words, we say that picture fuzzy set is a special case of a neutrosophic set, because every picture fuzzy set can be neutrosophic set but the converse is not true. Some aggregation operators under the environment of picture fuzzy set are aggregation operators for PF set by Garg [8], PFAOs and their application in MADM by Wei [9], PF Einstein AOs by Khan et al. [10], and PF Dombi AOs by Jana et al. [11]. However, during the research, experts faced some issues when they have taken the value of MD is 0.8 and NMD is 0.6, then 0.8+0.6 ≰1, so here the condition of IFS failed. So, to cover this limitation in 2013 Yager [12] proposed the generalized structure of IFS, which is called the Pythagorean fuzzy set. A Pythagorean fuzzy set is characterized by MD and NMD, with the condition that $(MD)^2 + (NMD)^2 \leq 1$. Several aggregation operators under the environment of a Pythagorean fuzzy set such as $P_Y FWA$, $P_Y FWG$,$P_Y FWPA$and$P_Y FWPG$ operators proposed by Yager [13,14]. In 2021, Akrma et al. [38], handle MCGDM problem under the environment of complex Pythagorean fuzzy set by using CPF-VIKOR method. In 2021, Akram et al. [39], proposed two novel modified techniques, namely Pythagorean fuzzy hybrid order of preference by Similarity to an Ideal Solution (PFH-TOPSIS) method and Pythagorean fuzzy hybrid Elimination and Choice Translating Reality I (PPFH-ELECTRE I) method, in order to measure risk ranking in failure modes and effects analysis (FMEA). In 2021, Akram et al. [40], also proposed a novel multi-criteria optimization technique, namely, the complex Pythagorean fuzzy N-soft VIKOR (CPFNS-VIKOR) method that is highly proficient to express a great deal of linguistic imprecision and vagueness inherent in human assessments. In 2022, Akram et al. [41] proposed a new hybrid model with application under the environment of Complex fermatean fuzzy N-soft set to handle uncertainties. In 2016, Yager [20] made a new generalization of IFS and PFS, called q-rung orthopair fuzzy set. q-ROFS is described by MD and NMD with the restriction that $(MD)^q + (NMD)^q \leq 1 (q > 1)$. Different aggregation operators under the environment of q-ROFS are q-ROFWA operators by Liu and Wang [15], q-ROF Bonferroni mean weighted operator by y Liu and Liu [16], q-ROF power Maclaurin averaging operators by Liu et al. [17], q-ROF Dombi AOs by Jana et al. [18], q-ROF Neutrality AOs by Garg and Chen [19], MAGD with q-rung orthopair picture fuzzy information by Akram et al. [37]. In 2018, Joshi et al. [30] introduced the theory of interval-valued q-rung orthopair fuzzy soft set, which deals with the situation, of hesitation of assessment in the intervals. In such type of situation experts provide their grades in the closed subinterval of [0,1]. The concept of interval-valued q-rung orthopair fuzzy soft set was further modified in various structures see Hayat et al. [31], Yang et al. [32], and Hayat et al. [33]. In 1999, Molodtsov [21] proposed a new structure called soft set which deals with the attribute. The theory of soft set was further merged with different structures and developed a generalized concept like Maji [22,23,34–36]. In 2020, Hussain et al. [24] combined the structure of soft set and q-ROF set and proposed a new concept called q-rung orthopair fuzzy soft set, which is characterized by membership degree and non-membership degree with attributes, but here no information about the neutral degree, so to cover this gap we present a hybrid of picture fuzzy set and q-rung orthopair fuzzy soft set to get a generalized structure called q-rung orthopair picture fuzzy soft set, which deal the uncertainty problem with positive, neutral and negative membership degree by affix a parameterization tool.

The rest of this manuscript is as follows: Section 2, discusses some basic preliminaries. Section 3, presents a hybrid of picture fuzzy set and q-ROF soft set and develop a novel structure q-ROPF soft set, and also discuss their basic operations. In Section 4, we study some aggregation operators like q-ROPF soft weighted averaging operator and q-ROPF soft order weighted averaging operator and their related fundamental properties. In Section 5, we develop a step-wise algorithm for MADM problem. In Subsection 5.1, for application we consider a biological example of common disease "obstructive goiter". In Section 6, we present a comparison analysis of the proposed model with the existing model to show the superiority. In Section 7, provide a conclusion.

2. Preliminary

Definition 2.1. [29] An IFS $\widetilde{N}$ on a universe H is expressed by the two-mapping given as

$$\widetilde{N} = \{\langle \mu, \mathcal{L}_{\widetilde{N}}(\mu), G_{\widetilde{N}}(\mu)\rangle : \mu \epsilon H\}. \quad (1)$$

Where $\mathcal{L}_{\widetilde{N}}(\mu): H \to [0,1]$ and $G_{\widetilde{N}}(\mu): H \to [0,1]$ represent the MD and NMD, with the condition that $0 \le (\mathcal{L}_{\widetilde{N}}(\mu)) + (G_{\widetilde{N}}(\mu)) \le 1$.
And the score $\mathrm{S}(\widetilde{N})$ and accuracy $\mathrm{A}(\widetilde{N})$ function is represented as

$$\mathrm{S}(\widetilde{N}) = \mathcal{L}_{\widetilde{N}}(\mu) - G_{\widetilde{N}}(\mu), \mathrm{S}(\widetilde{N}) \in [-1,1].$$

$$(\widetilde{N}) = \mathcal{L}_{\widetilde{N}}(\mu) + G_{\widetilde{N}}(\mu), \mathrm{A}(\widetilde{N}) \in [0,1].$$

Definition 2.2. [12] By $P_Y FS$ $\widetilde{N}$ on a universe of discourse H is defined as

$$\widetilde{N} = \{\langle \mu, \mathcal{L}_{\widetilde{N}}(\mu), G_{\widetilde{N}}(\mu)\rangle : \mu \epsilon H\}. \quad (2)$$

Where $\mathcal{L}_{\widetilde{N}}(\mu): H \to [0,1]$ and $G_{\widetilde{N}}(\mu): H \to [0,1]$ represent the MD and NMD, with the condition that

$$0 \le (\mathcal{L}_{\widetilde{N}}(\mu))^2 + (G_{\widetilde{N}}(\mu))^2 \le 1$$

and the score and accuracy function of Pythagorean fuzzy set is represented as

$$\mathrm{S}(\widetilde{N}) = (\mathcal{L}_{\widetilde{N}}(\mu))^2 - (G_{\widetilde{N}}(\mu))^2, \mathrm{S}(\widetilde{N}) \in [-1,1].$$

$$\mathrm{A}(\widetilde{N}) = (\mathcal{L}_{\widetilde{N}}(\mu))^2 + (G_{\widetilde{N}}(\mu))^2, \mathrm{A}(\widetilde{N}) \in [0,1].$$

Definition 2.3. [20] A q-ROFS $\widetilde{N}$ on a universe of discourse H is defined as

$$\widetilde{N} = \{\langle \mu, \mathcal{L}_{\widetilde{N}}(\mu), G_{\widetilde{N}}(\mu)\rangle : \mu \epsilon H\}. \quad (3)$$

Where $\mathcal{L}_{\widetilde{N}}(\mu): H \to [0,1]$ and $G_{\widetilde{N}}(\mu): H \to [0,1]$ represent the MD and NMD, with the condition that

$$0 \le (\mathcal{L}_{\widetilde{N}}(\mu))^q + (G_{\widetilde{N}}(\mu))^q \le 1 \ (q \ge 1)$$

and the score and accuracy function of q-ROFS is represented as

$$\mathrm{S}(\widetilde{N}) = (\mathcal{L}_{\widetilde{N}}(\mu))^q - (G_{\widetilde{N}}(\mu))^q, \mathcal{S}(\widetilde{N}) \in [-1,1]$$

$$A(\widetilde{N}) = (\mathcal{L}_{\widetilde{N}}(\mu))^q + (G_{\widetilde{N}}(\mu))^q, A(\widetilde{N}) \in [0,1].$$

Definition 2.4. [21] Let an H be a fixed set and € represent the set of parameters and ₵⊆€. Then the pair (₣, ₵) is said to be soft set over H, where ₣ is a function define as ₣: ₵ → $P(H)$. $P(H)$ represent the power set of H.
Definition 2.5. [28] Let (H, ₵)be a soft universe and ₵⊆€. By Pythagorean fuzzy soft set we mean a pair $(\widetilde{N}, ₵)$ over H, where $\widetilde{N}$ is a function given by $\widetilde{N}$: ₵ → $PFS^{(H)}$ is defined as

$$\widetilde{N}_{\ddot{e}_j}(\mu_i) = \{\langle \mu_i, \mathcal{L}_j(\mu_i), G_j(\mu_i)\rangle : \mu_i \epsilon H\}. \quad (4)$$

Where $\mathcal{L}_j(\mu_i)$ represent the MD and $G_j(\mu_i)$ represent the NMD $\mu_i \epsilon H$ to a set $\widetilde{N}_{\ddot{e}_j}(\mu_i)$, with the condition that

$$0 \le (\mathcal{L}_j(\mu_i))^2 + (G_j(\mu_i))^2 \le 1,$$

which is simply denoted by $\widetilde{N}_{\ddot{e}_j}(\mu_i) = \langle \mu_i, \mathcal{L}_j(\mu_i), G_j(\mu_i)\rangle$.

Definition 2.6. [24] Let $(\mathrm{H}, \mathbb{C})$be a soft universe and $\mathbb{C}\subseteq€$. By q-rung orthopair fuzzy soft set we mean a pair $(\widetilde{N}, \mathbb{C})$ over H, where $\widetilde{N}$ is a function given by $\widetilde{N}: \mathbb{C} \to q - ROFS^{(H)}$ is defined as

$$\widetilde{N}_{\ddot{e}_j}(\mu_i)=\{\langle \mu_i, \mathcal{L}_j(\mu_i), G_j(\mu_i)\rangle : \mu_i \epsilon H\}. \tag{5}$$

Where $\mathcal{L}_j(\mu_i)$ represent the MD and $G_j(\mu_i)$ represent the NMD $\mu_i \epsilon H$ to a set $\widetilde{N}_{\ddot{e}_j}(\mu_i)$, with the condition that

$$0 \leq \left(\mathcal{L}_j(\mu_i)\right)^q + \left(G_j(\mu_i)\right)^q \leq 1 \ (q \geq 1),$$

which is simply denoted by $\widetilde{N}_{\ddot{e}_j}(\mu_i) = \langle \mu_i, \mathcal{L}_j(\mu_i), G_j(\mu_i)\rangle$ and the degree of indeterminacy of $q-ROPFS_f N$ is defined as $\pi_{\widetilde{N}_{\ddot{e}_{ij}}} = \sqrt[q]{1 - \left(\left(\mathcal{L}_j(\mu_i)\right)^q + \left(G_j(\mu_i)\right)^q\right)}$.

Definition 2.7. [7] A picture fuzzy set $\widetilde{N}$ on a fixed set H is displayed as

$$\widetilde{N} = \{\langle \mu, \mathcal{L}_{\widetilde{N}}(\mu), G_{\widetilde{N}}(\mu), Ł_{\widetilde{N}}(\mu)\rangle : \mu \epsilon H\}. \tag{6}$$

Where $\mathcal{L}_{\widetilde{N}}(\mu), G_{\widetilde{N}}(\mu), Ł_{\widetilde{N}}(\mu): H \to [0,1]$ represent the positive MD, neutral MD and negative MD, with the condition that, with the condition that

$$0 \leq \left(\mathcal{L}_{\widetilde{N}}(\mu)\right) + \left(G_{\widetilde{N}}(\mu) + \left(Ł_{\widetilde{N}}(\mu)\right)\right) \leq 1.$$

3. q-Rung orthopair picture fuzzy soft (q-$ROPFS_fS$) set

Definition 3.1. Let $(\mathrm{H}, \mathbb{C})$be a soft universe and $\mathbb{C}\subseteq€$. By q-rung orthopair picture fuzzy soft set we mean a pair $(\widetilde{N}, \mathbb{C})$ over H, where $\widetilde{N}$ is a function given by $\widetilde{N}: \mathbb{C} \to q - ROPFS^{(H)}$ is defined as

$$\widetilde{N}_{\ddot{e}_j}(\mu_i)=\{\langle \mu_i, \mathcal{L}_j(\mu_i), G_j(\mu_i), Ł_j(\mu_i) \rangle : \mu_i \epsilon H \text{ and } q \geq 1\} \tag{7}$$

where $\mathcal{L}_j(\mu_i)$ represent the positive MD and $G_j(\mu_i)$ represent the neutral MD and $Ł_j(\mu_i)$ denoted negative MD of $\mu_i \epsilon H$ to a set $\widetilde{N}_{\ddot{e}_j}(\mu_i)$, with the condition that

$$0 \leq \left(\mathcal{L}_j(\mu_i)\right)^q + \left(G_j(\mu_i)\right)^q + \left(Ł_j(\mu_i)\right)^q \leq 1 \ (q \geq 1)$$

which is simply denoted by $\widetilde{N}_{\ddot{e}_j}(\mu_i)=\langle \mu_i, \mathcal{L}_j(\mu_i), G_j(\mu_i), Ł_j(\mu_i)\rangle_q$ and the degree of indeterminacy of q-$ROPFS_f N$ is defined as

$$\pi_{\widetilde{N}_{\ddot{e}_{ij}}} = \sqrt[q]{1 - \left(\left(\mathcal{L}_j(\mu_i)\right)^q + \left(G_j(\mu_i)\right)^q + \left(Ł_j(\mu_i)\right)^q\right)}.$$

Basic operations on q-ROPF soft set

Let $\widetilde{N} = (\mathcal{L}, G, Ł)$ be any tree q-$ROPFS_f Ns$ and $\widetilde{N}_{\ddot{e}_{1j}} = (\mathcal{L}_{1j}, G_{1j}, Ł_{1j})$ (j=1,2) and λ, λ1, λ2>0. Then the operations of q-$ROPFS_f Ns$ are define as

(1) $\widetilde{N}_{\ddot{e}_{11}} \cup \widetilde{N}_{\ddot{e}_{11}} = (\max(\mathcal{L}_{11}, \mathcal{L}_{12}), \min(G_{11}, G_{12}), \min(Ł_{11}, Ł_{12}))$;
(2) $\widetilde{N}_{\ddot{e}_{11}} \cap \widetilde{N}_{\ddot{e}_{11}} = (\min(\mathcal{L}_{11}, \mathcal{L}_{12}), \min(G_{11}, G_{12}), \max(Ł_{11}, Ł_{12}))$;
(3) $\widetilde{N}^C = (Ł, G, \mathcal{L})$;

(4) $\widetilde{N}_{\ddot{e}_{11}} \leq \widetilde{N}_{\ddot{e}_{11}}$ if and only if $(\mathcal{L}_{11} \leq \mathcal{L}_{12}, G_{11} \leq G_{12}, Ł_{11} \geq Ł_{12})$;

(5) $\widetilde{N}_{\ddot{e}_{11}} \oplus \widetilde{N}_{\ddot{e}_{11}} = \left(\sqrt[q]{(\mathcal{L}_{11})^q + (\mathcal{L}_{12})^q - (\mathcal{L}_{11})^q(\mathcal{L}_{12})^q}, G_{11}G_{12}, Ł_{11}Ł_{12}\right)$;

(6) $\widetilde{N}_{\ddot{e}_{11}} \otimes \widetilde{N}_{\ddot{e}_{11}} = \left(\mathcal{L}_{11}\mathcal{L}_{12}, \sqrt[q]{(G_{11})^q + (G_{12})^q - (G_{11})^q(G_{12})^q}, \sqrt[q]{(Ł_{11})^q + (Ł_{12})^q - (Ł_{11})^q(Ł_{12})^q}\right)$;

(7) $\lambda\widetilde{N} = \left(\sqrt[q]{1-(1-\mathcal{L}^q)^\lambda}, G^\lambda, Ł^\lambda\right)$;

(8) $\widetilde{N}^\lambda = \left(\mathcal{L}^\lambda, \sqrt[q]{1-(1-G^q)^\lambda}, \sqrt[q]{1-(1-Ł^q)^\lambda}\right)$.

Definition 3.2. A score function of $q-ROPFS_fN$ $\widetilde{N}_{\ddot{e}_{ij}} = \left(\mathcal{L}_{ij}, G_{ij}, Ł_{ij}\right)$ can be define as

$$\left(\widetilde{N}_{\ddot{e}_{ij}}\right) = \mathcal{L}_{ij}^q - G_{ij}^q - Ł_{ij}^q + \left(\frac{e^{\mathcal{L}_{ij}^q - G_{ij}^q - Ł_{ij}^q}}{e^{\mathcal{L}_{ij}^q - G_{ij}^q - Ł_{ij}^q} + 1} - \frac{1}{2}\right)\pi^q_{\widetilde{N}_{\ddot{e}_{ij}}} \tag{8}$$

where $q \geq 1$ and $S\left(\widetilde{N}_{\ddot{e}_{ij}}\right) \in [-1,1]$.

Example 3.1. Assume that a person wants to select a car out of five possible alternatives in a market that is $\bar{U} = \{t_1, t_2, t_3, t_4, t_5\}$ with the parameters € = $\{e_1, e_2, e_3, e_4\}$.

e_1 = Price

e_2 = Comfort

e_3 = Fuel Efficiency

e_4 = Looks.

Let $\widetilde{N}_{\ddot{e}_{11}} = (\mathcal{L}_{11}, G_{11}, Ł_{11})$ and $\widetilde{N}_{\ddot{e}_{12}} = (\mathcal{L}_{12}, G_{12}, Ł_{12})$ be two $q-ROPFS_fNs$. Then

(i) $S\left(\widetilde{N}_{\ddot{e}_{11}}\right) > S\left(\widetilde{N}_{\ddot{e}_{12}}\right)$, $\widetilde{N}_{\ddot{e}_{11}} \geq \widetilde{N}_{\ddot{e}_{12}}$

(ii) $S\left(\widetilde{N}_{\ddot{e}_{11}}\right) < S\left(\widetilde{N}_{\ddot{e}_{12}}\right)$, $\widetilde{N}_{\ddot{e}_{11}} \leq \widetilde{N}_{\ddot{e}_{12}}$

(iii) $S\left(\widetilde{N}_{\ddot{e}_{11}}\right) = S\left(\widetilde{N}_{\ddot{e}_{12}}\right)$, then

(a) $\pi_{\widetilde{N}_{\ddot{e}_{11}}} > \pi_{\widetilde{N}_{\ddot{e}_{11}}}$, then $\widetilde{N}_{\ddot{e}_{11}} < \widetilde{N}_{\ddot{e}_{12}}$

(b) $\pi^q_{\widetilde{N}_{\ddot{e}_{11}}} > \pi^q_{\widetilde{N}_{\ddot{e}_{12}}}$, then $\widetilde{N}_{\ddot{e}_{11}} = \widetilde{N}_{\ddot{e}_{12}}$.

From "Table 1" we show the result in the form of $q-ROPFS_fNs$, by evaluated the alternative with rating values.

Table 1. Tabular representation of $q-ROPFS_fS$ $(\mathcal{L}, G, Ł)$ for $q \geq 3$.

$\bar{U}$	e_1	e_2	e_3	e_4
t_1	(0.6, 0.2, 0.3)	(0.5, 0.1, 0.4)	(0.3, 0.1, 0.5)	(0.5, 0.4, 0.1)
t_2	(0.5, 0.2, 0.1)	(0.3, 0.1, 0.2)	(0.4, 0.2, 0.3)	(0.4, 0.3, 0.2)
t_3	(0.6, 0.2, 0.1)	(0.4, 0.2, 0.3)	(0.3, 0.2, 0.5)	(0.4, 0.5, 0.1)
t_4	(0.4, 0.1, 0.3)	(0.6, 0.1, 0.4)	(0.3, 0.1, 0.4)	(0.4, 0.2, 0.3)
t_5	(0.5, 0.3, 0.2)	(0.4, 0.1, 0.5)	(0.3, 0.1, 0.5)	(0.6, 0.1, 0.4)

Theorem 3.1. Let $\widetilde{N}_{\ddot{e}_{ij}} = \left(\mathcal{L}_{ij}, G_{ij}, Ł_{ij}\right)$ and $\widetilde{N} = (\mathcal{L}, G, Ł)$ be any two $q-ROPFS_fNs$ and $\lambda, \lambda_1, \lambda_2 > 0$, having the properties.

(i) $\widetilde{N}_{\ddot{e}_{11}} \oplus \widetilde{N}_{\ddot{e}_{12}} = \widetilde{N}_{\ddot{e}_{12}} \oplus \widetilde{N}_{\ddot{e}_{11}}$

(ii) $\widetilde{N}_{\ddot{e}_{11}} \otimes \widetilde{N}_{\ddot{e}_{12}} = \widetilde{N}_{\ddot{e}_{12}} \otimes \widetilde{N}_{\ddot{e}_{11}}$

(iii) $\lambda\left(\widetilde{N}_{\ddot{e}_{11}} \oplus \widetilde{N}_{\ddot{e}_{12}}\right) = \lambda\widetilde{N}_{\ddot{e}_{11}} \oplus \lambda\widetilde{N}_{\ddot{e}_{12}}$

(iv) $(\lambda_1 \oplus \lambda_2)\widetilde{N} = \lambda_1\widetilde{N} \oplus \lambda_2\widetilde{N}$

(v) $\widetilde{N}^{(\lambda_1 \oplus \lambda_2)} = {}^{\lambda_1}\otimes \widetilde{N}^{\lambda_2}$

(vi)$\widetilde{N}_{ë_{11}}^{\lambda} \otimes \widetilde{N}_{ë_{12}}^{\lambda} = \left(\widetilde{N}_{ë_{11}} \oplus \widetilde{N}_{ë_{12}}\right)^{\lambda}$.

Proof. Straightforward.

4. Average aggregation operator under the environment of *q*-ROPF soft set

In this section, we discuss some aggregation operators like $q-ROPFS_fWA$ and $q-ROPFS_fOWA$ operators and their related results.

4.1. *q*-ROPF soft weighted average ($q-ROPFS_fWA$) operators

Definition 4.1. Assume that $\widetilde{N}_{ë_{ij}} = (\mathcal{L}_{ij}, G_{ij}, Ł_{ij})$ for ($i = 1,2, \dots$ n and j $= 1,2, \dots$ m) be collection of $q = ROPFS_fNs$ and weight vector ω= {ω1, ω2, ... ,ωn} with the condition $\sum_{i=1}^{n} \omega_i = 1$ and υ= {υ1, υ2, ..., υm} with the condition that $\sum_{i=1}^{n} \upsilon_i = 1$ for alternatives x_i and parameters e_j, respectively. The mapping $q-ROPFS_fWA: Đ^n \to Đ$ is said to be $q-ROPFS_fWA$ operator. (Đ is the collection of $q-ROPFS_fNs$).

$$q-ROPFS_fWA\left(\widetilde{N}_{ë_{11}}, \widetilde{N}_{ë_{12}}, \dots, \widetilde{N}_{ë_{nm}}\right) = \oplus_{j=1}^{m} \upsilon_i \left(\oplus_{i=1}^{n} \omega_i \widetilde{N}_{ë_{ij}}\right). \tag{9}$$

Theorem 4.1. Consider the collection of $q-ROPFS_fNs$ $\widetilde{N}_{ë_{ij}} = (\mathcal{L}_{ij}, G_{ij}, Ł_{ij})$ then the aggregation result for $q-ROPFS_fWA$ operator is expressed:

$$q-ROPFS_fWA\left(\widetilde{N}_{ë_{11}}, \widetilde{N}_{ë_{12}}, \dots, \widetilde{N}_{ë_{nm}}\right) = \oplus_{j=1}^{m} \upsilon_i \left(\oplus_{i=1}^{n} \omega_i \widetilde{N}_{ë_{ij}}\right)$$

$$= \left(\sqrt[q]{1 - \prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(1 - \mathcal{L}_{ij}^{q}\right)^{\omega_i}\right)^{\upsilon_i}}, \prod_{j=1}^{m}\left(\prod_{i=1}^{n} G_{ij}^{\omega_i}\right)^{\upsilon_i}, \prod_{j=1}^{m}\left(\prod_{i=1}^{n} Ł_{ij}^{\omega_i}\right)^{\upsilon_i}\right)$$

$$q-ROPFS_fWA\left(\widetilde{N}_{ë_{11}}, \widetilde{N}_{ë_{12}}, \dots, \widetilde{N}_{ë_{nm}}\right) = \begin{pmatrix} \sqrt[q]{1 - \prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(1 - \mathcal{L}_{ij}^{q}\right)^{\omega_i}\right)^{\upsilon_i}}, \\ \prod_{j=1}^{m}\left(\prod_{i=1}^{n} G_{ij}^{\omega_i}\right)^{\upsilon_i}, \\ \prod_{j=1}^{m}\left(\prod_{i=1}^{n} Ł_{ij}^{\omega_i}\right)^{\upsilon_i} \end{pmatrix} \tag{10}$$

Ω = {ω1, ω2, ..., ωn} with the condition $\sum_{i=1}^{n} \omega_i = 1$ and υ= {υ1, υ2, ..., υm} with the condition that $\sum_{i=1}^{n} \upsilon_i = 1$ for alternatives x_i and parameters e_j, respectively.

Proof. To solve this result we use the mathematical induction. We have,

$$\widetilde{N}_{ë_{11}} \oplus \widetilde{N}_{ë_{12}} = \left(\sqrt[q]{(\mathcal{L}_{11})^q + (\mathcal{L}_{12})^q - (\mathcal{L}_{11})^q(\mathcal{L}_{12})^q}, G_{11}G_{12}, Ł_{11}Ł_{12}\right)$$

$$\lambda\left(\widetilde{N} = \left(\sqrt[q]{1 - [1 - \mathcal{L}^q]^{\lambda}}, G^{\lambda}, Ł^{\lambda}\right)\right)$$

for λ ≥ 1. For n = 2 and m = 2, Eq (10) is true.

$$q-ROPFS_fWA\left(\tilde{N}_{ë_{11}},\tilde{N}_{ë_{12}}\right)=\oplus_{j=1}^{2}\upsilon_i\left(\oplus_{i=1}^{2}\omega_i\tilde{N}_{ë_{ij}}\right)=\upsilon_1\left(\oplus_{i=1}^{2}\omega_i\tilde{N}_{ë_{11}}\right)\oplus\upsilon_2\left(\oplus_{i=1}^{2}\omega_i\tilde{N}_{ë_{12}}\right)$$

$$=\upsilon_1\left(\omega_1\tilde{N}_{ë_{11}}\oplus\omega_2\tilde{N}_{ë_{21}}\right)\oplus\upsilon_2\left(\omega_1\tilde{N}_{ë_{12}}\oplus\omega_2\tilde{N}_{ë_{22}}\right)$$

$$=\upsilon_1\left\{\begin{matrix}\left(\sqrt[q]{1-\left(1-\mathcal{L}_{11}^{q}\right)^{\omega_1}},G_{11}^{\omega_1},Ł_{11}^{\omega_1}\right)\oplus\\ \left(\sqrt[q]{1-\left(1-\mathcal{L}_{21}^{q}\right)^{\omega_2}},G_{21}^{\omega_2},Ł_{21}^{\omega_2}\right)\end{matrix}\right\}\oplus\upsilon_2\left\{\begin{matrix}\left(\sqrt[q]{1-\left(1-\mathcal{L}_{12}^{q}\right)^{\omega_1}},G_{12}^{\omega_1},Ł_{12}^{\omega_1}\right)\oplus\\ \left(\sqrt[q]{1-\left(1-\mathcal{L}_{22}^{q}\right)^{\omega_2}},G_{22}^{\omega_2},Ł_{22}^{\omega_2}\right)\end{matrix}\right\}$$

$$=\upsilon_1\begin{pmatrix}\sqrt[q]{1-\Pi_{i=1}^{2}\left(1-\mathcal{L}_{i1}^{q}\right)^{\omega_i}},\\ \Pi_{i=1}^{2}G_{i1}^{\omega_i},\Pi_{i=1}^{2}Ł_{i1}^{\omega_i}\end{pmatrix}\oplus\upsilon_2\begin{pmatrix}\sqrt[q]{1-\Pi_{i=1}^{2}\left(1-\mathcal{L}_{i2}^{q}\right)^{\omega_i}},\\ \Pi_{i=1}^{2}G_{i2}^{\omega_i},\Pi_{i=1}^{2}Ł_{i2}^{\omega_i}\end{pmatrix}$$

$$=\upsilon_1\begin{pmatrix}\sqrt[q]{1-\left(\Pi_{i=1}^{2}\left(1-\mathcal{L}_{i1}^{q}\right)^{\omega_i}\right)^{\upsilon_1}},\\ \left(\Pi_{i=1}^{2}G_{i1}^{\omega_i}\right)^{\upsilon_1},\left(\Pi_{i=1}^{2}Ł_{i1}^{\omega_i}\right)^{\upsilon_1}\end{pmatrix}\oplus\upsilon_2\begin{pmatrix}\sqrt[q]{1-\left(\Pi_{i=1}^{2}\left(1-\mathcal{L}_{i2}^{q}\right)^{\omega_i}\right)^{\upsilon_2}},\\ \left(\Pi_{i=1}^{2}G_{i2}^{\omega_i}\right)^{\upsilon_2},\left(\Pi_{i=1}^{2}Ł_{i2}^{\omega_i}\right)^{\upsilon_2}\end{pmatrix}$$

$$=\left(\sqrt[q]{1-\prod_{j=1}^{2}\left(\prod_{i=1}^{2}\left(1-\mathcal{L}_{ij}^{q}\right)^{\omega_i}\right)^{\upsilon_i}},\Pi_{j=1}^{2}\left(\Pi_{i=1}^{2}G_{ij}^{\omega_i}\right)^{\upsilon_i},\Pi_{j=1}^{2}\left(\Pi_{i=1}^{2}Ł_{ij}^{\omega_i}\right)^{\upsilon_i}\right).$$

Next, we will check for n=κ_1 and m=κ_2.

$$q-ROPFS_fWA\left(\tilde{N}_{ë_{11}},\tilde{N}_{ë_{12}},\ldots,\tilde{N}_{ë_{\kappa_1\kappa_2}}\right)=\oplus_{j=1}^{\kappa_2}\upsilon_i\left(\oplus_{i=1}^{\kappa_1}\omega_i\tilde{N}_{ë_{ij}}\right)$$

$$=\left(\sqrt[q]{1-\prod_{j=1}^{\kappa_2}\left(\prod_{i=1}^{\kappa_1}\left(1-\mathcal{L}_{ij}^{q}\right)^{\omega_i}\right)^{\upsilon_i}},\Pi_{j=1}^{\kappa_2}\left(\Pi_{i=1}^{\kappa_1}G_{ij}^{\omega_i}\right)^{\upsilon_i},\Pi_{j=1}^{\kappa_2}\left(\Pi_{i=1}^{\kappa_1}Ł_{ij}^{\omega_i}\right)^{\upsilon_i}\right).$$

And further for n=κ_1+1 and m=κ_2+1.

$$q-ROPFS_fWA\left(\tilde{N}_{ë_{11}},\tilde{N}_{ë_{12}},\ldots,\tilde{N}_{ë_{\kappa_1+1\,\kappa_2+1}}\right)=\left\{\oplus_{j=1}^{\kappa_2}\upsilon_i\left(\oplus_{i=1}^{\kappa_1}\omega_i\tilde{N}_{ë_{ij}}\right)\right\}\oplus$$
$$\upsilon_{(\kappa_1+1)}\left(\omega_{\kappa_2+1}\tilde{N}_{ë_{(\kappa_1+1)(\kappa_2+1)}}\right)$$

$$=\begin{pmatrix}\sqrt[q]{1-\prod_{j=1}^{\kappa_2}\left(\prod_{i=1}^{\kappa_1}\left(1-\mathcal{L}_{ij}^{q}\right)^{\omega_i}\right)^{\upsilon_i}},\\ \Pi_{j=1}^{\kappa_2}\left(\Pi_{i=1}^{\kappa_1}G_{ij}^{\omega_i}\right)^{\upsilon_i},\Pi_{j=1}^{\kappa_2}\left(\Pi_{i=1}^{\kappa_1}Ł_{ij}^{\omega_i}\right)^{\upsilon_i}\end{pmatrix}\oplus\upsilon_{(\kappa_1+1)}\left(\omega_{\kappa_2+1}\tilde{N}_{ë_{(\kappa_1+1)(\kappa_2+1)}}\right)$$

$$=\left(\sqrt[q]{1-\prod_{j=1}^{(\kappa_2+1)}\left(\prod_{i=1}^{(\kappa_1+1)}\left(1-\mathcal{L}_{ij}^{q}\right)^{\omega_i}\right)^{\upsilon_i}},\Pi_{j=1}^{(\kappa_2+1)}\left(\Pi_{i=1}^{(\kappa_1+1)}G_{ij}^{\omega_i}\right)^{\upsilon_i},\Pi_{j=1}^{(\kappa_2+1)}\left(\Pi_{i=1}^{(\kappa_1+1)}Ł_{ij}^{\omega_i}\right)^{\upsilon_i}\right).$$

Hence by the induction process we prove that Eq (10) is true for all m, n≥1 and also Eq (10) is true for $n=\kappa_1+1$ and $m=\kappa_2+1$. Moreover, to obtained the aggregated result from $q-ROPFS_fWA$ operator is again $q-ROPFS_fNs$. For, any $\tilde{N}_{ë_{ij}}=\left(\mathcal{L}_{ij},G_{ij},Ł_{ij}\right)$ be collection of $q-ROPFS_fNs$ and weight vector ω = {$\omega_1, \omega_2, \ldots, \omega_n$} with the condition $\sum_{i=1}^{n}\omega_i=1$ and υ = {$\upsilon_1, \upsilon_2, \ldots, \upsilon_m$} with the condition that $\sum_{i=1}^{n}\upsilon_i=1$ for alternatives x_i and parameters e_j, respectively. So,

$$0\le\mathcal{L}_{ij}\le1$$

$$\Rightarrow0\le1-\mathcal{L}_{ij}\le1$$

$\Rightarrow 0 \leq \left(1-\mathcal{L}_{ij}^{q}\right)^{\omega_i} \leq 1$

$\Rightarrow 0 \leq \Pi_{i=1}^{n}\left(1-\mathcal{L}_{ij}^{q}\right)^{\omega_i} \leq 1$

$\Rightarrow 0 \leq \Pi_{j=1}^{m}\left(\Pi_{i=1}^{n}\left(1-\mathcal{L}_{ij}^{q}\right)^{\omega_i}\right)^{\upsilon_i} \leq 1$

$\Rightarrow 0 \leq \sqrt[q]{\Pi_{j=1}^{m}\left(\Pi_{i=1}^{n}\left(1-\mathcal{L}_{ij}^{q}\right)^{\omega_i}\right)^{\upsilon_i}} \leq 1.$

Now, for $0 \leq \mathrm{G}_{ij} \leq 1 \Rightarrow 0 \leq \Pi_{i=1}^{n} \mathrm{G}_{ij}^{\omega_i} \leq 1 \Rightarrow 0 \leq \Pi_{j=1}^{m}\left(\Pi_{i=1}^{n} \mathrm{G}_{ij}^{\omega_i}\right)^{\upsilon_i} \leq 1$ and

$0 \leq Ł_{ij} \leq 1 \Rightarrow 0 \leq \Pi_{i=1}^{n} Ł_{ij}^{\omega_i} \leq 1 \Rightarrow 0 \leq \Pi_{j=1}^{m}\left(\Pi_{i=1}^{n} Ł_{ij}^{\omega_i}\right)^{\upsilon_i} \leq 1.$

As, $0 \leq \mathcal{L}_{ij}^{q} + \mathrm{G}_{ij}^{q} + Ł_{ij}^{q} \leq 1 \Rightarrow \mathrm{G}_{ij}^{q} + Ł_{ij}^{q} \leq 1 - \mathcal{L}_{ij}^{q}$

$\Rightarrow \Pi_{i=1}^{n}\left(\mathrm{G}_{ij}^{q}\right)^{\omega_i} + \Pi_{i=1}^{n}\left(Ł_{ij}^{q}\right)^{\omega_i} \leq \Pi_{i=1}^{n}\left(1-\mathcal{L}_{ij}^{q}\right)^{\omega_i}$

$$\Rightarrow \Pi_{j=1}^{m}\left(\Pi_{i=1}^{n}\left(\mathrm{G}_{ij}^{q}\right)^{\omega_i}\right)^{\upsilon_i} + \Pi_{j=1}^{m}\left(\Pi_{i=1}^{n}\left(Ł_{ij}^{q}\right)^{\omega_i}\right)^{\upsilon_i} \leq \Pi_{j=1}^{m}\left(\Pi_{i=1}^{n}\left(1-\mathcal{L}_{ij}^{q}\right)^{\omega_i}\right)^{\upsilon_i}. \tag{11}$$

Now, we have

$$0 \leq \left\{\sqrt[q]{1-\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(1-\mathcal{L}_{ij}^{q}\right)^{\omega_i}\right)^{\upsilon_i}}\right\}^{q} + \left\{\Pi_{j=1}^{m}\left(\Pi_{i=1}^{n}\mathrm{G}_{ij}^{\omega_i}\right)^{\upsilon_i}\right\}^{q} + \left\{\Pi_{j=1}^{m}\left(\Pi_{i=1}^{n}Ł_{ij}^{\omega_i}\right)^{\upsilon_i}\right\}^{q}.$$

By Eq (11), $0 \leq 1-\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(1-\mathcal{L}_{ij}^{q}\right)^{\omega_i}\right)^{\upsilon_i} + \Pi_{j=1}^{m}\left(\Pi_{i=1}^{n}\mathrm{G}_{ij}^{\omega_i}\right)^{\upsilon_i} + \Pi_{j=1}^{m}\left(\Pi_{i=1}^{n}Ł_{ij}^{\omega_i}\right)^{\upsilon_i} = 1.$

Therefore,

$$0 \leq \left\{\sqrt[q]{1-\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(1-\mathcal{L}_{ij}^{q}\right)^{\omega_i}\right)^{\upsilon_i}}\right\}^{q} + \left\{\Pi_{j=1}^{m}\left(\Pi_{i=1}^{n}\mathrm{G}_{ij}^{\omega_i}\right)^{\upsilon_i}\right\}^{q} + \left\{\Pi_{j=1}^{m}\left(\Pi_{i=1}^{n}Ł_{ij}^{\omega_i}\right)^{\upsilon_i}\right\}^{q} \leq 1.$$

Hence, we proved the required result.

Example 4.1. Assume that a person wants to purchase a new laptop in the domain set

$\overline{U}$= {t_1, t_2, t_3, t_4, t_5}

t_1=HP Pavilion,

t_2=Dell Inspiron,

t_3=Apple iBook,

t_4=Toshiba,

t_5=Lenovo

and with the parameters €= $\{e_1, e_2, e_3, e_4\}$

e_1=Battery life,

e_2=Memory and storage,

e_3=Carrying weight,

e_4=Warranty.

Suppose that the weight vectors ω= {0.15,0.16,0.20,0.25,0.24} and υ= {0.5,0.17,0.13,0.20} for the expert x_i and parameters e_j, respectively. From "Table 2" we decision maker show the result in the form of $q-ROPFS_f Ns$, by evaluated each laptop to their corresponding parameters.

Table 2. Tabular representation of $q-ROPFS_fS$ $\widetilde{\mathrm{N}}_{\ddot{\mathrm{e}}_{ij}}=(\mathcal{L}_{ij}, \mathrm{G}_{ij}, Ł_{ij})$ for q ≥ 3.

$\bar{U}$	e_1	e_2	e_3	e_4
t_1 = HP Pavilion	$(0.66, 0.4, 0.2)$	$(0.6, 0.5, 0.3)$	$(0.45.0.25, 0.1)$	$(0.73, 0.22, 0.1)$
t_2 = Dell Inspiron	$(0.76, 0.2, 0.1)$	$(0.55, 0.5, 0.2)$	$(0.77, 0.4, 0.3)$	$(0.87, 0.32, 0.15)$
t_3 = Apple iBook	$(0.71, 0.3, 0.1)$	$(0.9, 0.5, 0.3)$	$(0.66, 0.54, 0.2)$	$(0.7, 0.5, 0.1)$
t_4 = Toshiba	$(0.8, 0.4, 0.3)$	$(0.65, 0.35, 0.15)$	$(0.83, 0.2, 0.1)$	$(0.8, 0.3, 0.2)$
t_5 = Lenovo	$(0.62, 0.5, 0.2)$	$(0.8, 0.33, 0.1)$	$(0.58, 0.3, 0.1)$	$(0.84, 0.35, 0.1)$

By Eq (10) we have

$$q-ROPFS_fWA(\widetilde{\mathrm{N}}_{\ddot{\mathrm{e}}_{11}}, \widetilde{\mathrm{N}}_{\ddot{\mathrm{e}}_{12}}, \ldots, \widetilde{\mathrm{N}}_{\ddot{\mathrm{e}}_{54}})=\begin{pmatrix} \sqrt[q]{1-\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(1-\mathcal{L}_{ij}^{q}\right)^{\omega_i}\right)^{\upsilon_i}}, \\ \prod_{j=1}^{m}\left(\prod_{i=1}^{n}\mathrm{G}_{ij}^{\omega_i}\right)^{\upsilon_i}, \\ \prod_{j=1}^{m}\left(\prod_{i=1}^{n}Ł_{ij}^{\omega_i}\right)^{\upsilon_i} \end{pmatrix}$$

$$=\left(\begin{array}{c} \sqrt[3]{\begin{array}{c} 1-\{(1-0.66^3)^{0.15}(1-0.76^3)^{0.16}(1-0.71^3)^{0.20}(1-0.8^3)^{0.25}(1-0.62^3)^{0.24}\}^{0.5} \\ 1-\{(1-0.6^3)^{0.15}(1-0.55^3)^{0.16}(1-0.09^3)^{0.20}(1-0.65^3)^{0.25}(1-0.8^3)^{0.24}\}^{0.17} \\ 1-\{(1-0.45^3)^{0.15}(1-0.77^3)^{0.16}(1-0.66^3)^{0.20}(1-0.83^3)^{0.25}(1-0.58^3)^{0.24}\}^{0.13}, \\ 1-\{(1-0.73^3)^{0.15}(1-0.87^3)^{0.16}(1-0.7^3)^{0.20}(1-0.8^3)^{0.25}(1-0.84^3)^{0.24}\}^{0.20} \end{array}} \\ \begin{pmatrix} \{(1-0.4^3)^{0.15}(1-0.2^3)^{0.16}(1-0.3^3)^{0.20}(1-0.4^3)^{0.25}(1-0.5^3)^{0.24}\}^{0.5} \\ \{(1-0.5^3)^{0.15}(1-0.5^3)^{0.16}(1-0.5^3)^{0.20}(1-0.35^3)^{0.25}(1-0.33^3)^{0.24}\}^{0.17} \\ \{(1-0.25^3)^{0.15}(1-0.4^3)^{0.16}(1-0.54^3)^{0.20}(1-0.2^3)^{0.25}(1-0.3^3)^{0.24}\}^{0.13} \\ \{(1-0.22^3)^{0.15}(1-0.32^3)^{0.16}(1-0.5^3)^{0.20}(1-0.3^3)^{0.25}(1-0.35^3)^{0.24}\}^{0.20} \end{pmatrix}, \\ \begin{pmatrix} \{(1-0.2)^{0.15}(1-0.1^3)^{0.16}(1-0.1^3)^{0.20}(1-0.3^3)^{0.25}(1-0.2^3)^{0.24}\}^{0.5} \\ \{(1-0.3^3)^{0.15}(1-0.2^3)^{0.16}(1-0.3^3)^{0.20}(1-0.15^3)^{0.25}(1-0.1^3)^{0.24}\}^{0.17} \\ \{(1-0.1^3)^{0.15}(1-0.3^3)^{0.16}(1-0.2^3)^{0.20}(1-0.1^3)^{0.25}(1-0.1^3)^{0.24}\}^{0.13} \\ \{(1-0.1^3)^{0.15}(1-0.15^3)^{0.16}(1-0.1^3)^{0.20}(1-0.2^3)^{0.25}(1-0.1^3)^{0.24}\}^{0.20} \end{pmatrix} \end{array}\right)$$

= $(0.7471, 0.3543, 0.1588)$.

Theorem 4.2. Assume that $\widetilde{\mathrm{N}}_{\ddot{\mathrm{e}}_{ij}}=(\mathcal{L}_{ij}, \mathrm{G}_{ij}, Ł_{ij})$ be the collection of $q-ROPFS_fNs$ and weight vector ω = {ω_1, ω_2, ..., ω_n} with the condition $\sum_{i=1}^{n}\omega_i = 1$ and υ = {υ_1, υ_2, ..., υ_m} with the condition that $\sum_{i=1}^{n}\upsilon_i = 1$ for alternatives x_i and parameters e_j, respectively. Then the $q-ROPFS_fWA$ operator hold the following properties:

(1) **Idempotency**: If $\widetilde{\mathrm{N}}_{\ddot{\mathrm{e}}_{ij}}=\mathfrak{R}_{\ddot{\mathrm{e}}}$, where $\mathfrak{R}_{\ddot{\mathrm{e}}}=(Ꞇ, ə, ɖ)$ and for all $(i=1,2,\ldots,n \text{ and } j=1,2,\ldots,m)$, then $q-ROPFS_fWA(\widetilde{\mathrm{N}}_{\ddot{\mathrm{e}}_{11}}, \widetilde{\mathrm{N}}_{\ddot{\mathrm{e}}_{12}}, \ldots, \widetilde{\mathrm{N}}_{\ddot{\mathrm{e}}_{nm}})=\mathfrak{R}_{\ddot{\mathrm{e}}}$.

(2) **Boundedness:** If

$$\widetilde{\mathrm{N}}_{\ddot{\mathrm{e}}_{ij}}^{-}=\left\{\min_j \min_i(\mathcal{L}_{ij}), \min_j \min_i(\mathrm{G}_{ij}), \max_j \max_i(Ł_{ij})\right\}$$

and

$$\widetilde{\mathrm{N}}^{+}_{\ddot{e}_{ij}}=\left\{\max_{j}\max_{i}\left(\mathcal{L}_{ij}\right),\ \min_{j}\min_{i}\left(\mathrm{G}_{ij}\right),\ \min_{j}\min_{i}\left(\text{Ł}_{ij}\right)\right\},$$

then

$$\widetilde{\mathrm{N}}^{-}_{\ddot{e}_{ij}} \leq q-ROPFS_{f}WA\left(\widetilde{\mathrm{N}}_{\ddot{e}_{11}},\widetilde{\mathrm{N}}_{\ddot{e}_{12}},\ldots,\widetilde{\mathrm{N}}_{\ddot{e}_{nm}}\right) \leq \widetilde{\mathrm{N}}^{+}_{\ddot{e}_{ij}}.$$

(3) **Monotonicity:** If $\mathfrak{R}_{\ddot{e}_{ij}}=\left(\text{Ꞇ}_{ij},\text{Ə}_{ij},\ \text{Ɥ}_{ij}\right)$ be the collection of $q-ROPFS_{f}Ns$ such that $\mathcal{L}_{ij} \leq \text{Ꞇ}_{ij},\ \mathrm{G}_{ij} \leq \text{Ə}_{ij}, \text{Ł}_{ij} \geq \text{Ɥ}_{ij}$ then

$$q-ROPFS_{f}WA\left(\widetilde{\mathrm{N}}_{\ddot{e}_{11}},\widetilde{\mathrm{N}}_{\ddot{e}_{12}},\ldots,\widetilde{\mathrm{N}}_{\ddot{e}_{nm}}\right) \leq q-ROPFS_{f}WA\left(\mathfrak{R}_{\ddot{e}_{11}},\mathfrak{R}_{\ddot{e}_{12}},\ldots,\mathfrak{R}_{\ddot{e}_{nm}}\right).$$

(4) **Shift Invariance:** If $\mathfrak{R}_{\ddot{e}}=\left(\text{Ꞇ}_{ij},\text{Ə}_{ij},\ \text{Ɥ}_{ij}\right)$ is $q-ROPFS_{f}Ns$, then

$$q-ROPFS_{f}WA\left(\widetilde{\mathrm{N}}_{\ddot{e}_{11}}\oplus\mathfrak{R}_{\ddot{e}},\widetilde{\mathrm{N}}_{\ddot{e}_{12}}\oplus\mathfrak{R}_{\ddot{e}},\ldots,\widetilde{\mathrm{N}}_{\ddot{e}_{nm}}\oplus\mathfrak{R}_{\ddot{e}}\right)=$$

$$q-ROPFS_{f}WA\left(\widetilde{\mathrm{N}}_{\ddot{e}_{11}},\widetilde{\mathrm{N}}_{\ddot{e}_{12}},\ldots,\widetilde{\mathrm{N}}_{\ddot{e}_{nm}}\right)\oplus\mathfrak{R}_{\ddot{e}}.$$

(5) **Homogeneity:** If $\lambda \geq 0$, then

$$q-ROPFS_{f}WA\left(\lambda\widetilde{\mathrm{N}}_{\ddot{e}_{11}},\lambda\widetilde{\mathrm{N}}_{\ddot{e}_{12}},\ldots,\lambda\widetilde{\mathrm{N}}_{\ddot{e}_{nm}}\right)=\lambda\ q-ROPFS_{f}WA\left(\widetilde{\mathrm{N}}_{\ddot{e}_{11}},\widetilde{\mathrm{N}}_{\ddot{e}_{12}},\ldots,\widetilde{\mathrm{N}}_{\ddot{e}_{nm}}\right).$$

Proof. We know that $\widetilde{\mathrm{N}}_{\ddot{e}_{11}}=\mathfrak{R}_{\ddot{e}}=(\text{Ꞇ},\text{Ə},\text{Ɥ})$, then we have

$$q-ROPFS_{f}WA\left(\widetilde{\mathrm{N}}_{\ddot{e}_{11}},\widetilde{\mathrm{N}}_{\ddot{e}_{12}},\ldots,\widetilde{\mathrm{N}}_{\ddot{e}_{nm}}\right)=\begin{pmatrix}\sqrt[q]{1-\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(1-\mathcal{L}_{ij}^{q}\right)^{\omega_i}\right)^{v_i}},\\ \prod_{j=1}^{m}\left(\prod_{i=1}^{n}\mathrm{G}_{ij}^{\omega_i}\right)^{v_i},\\ \prod_{j=1}^{m}\left(\prod_{i=1}^{n}\text{Ł}_{ij}^{\omega_i}\right)^{v_i}\end{pmatrix}$$

$$=\begin{pmatrix}\sqrt[q]{1-\prod_{j=1}^{m}(\prod_{i=1}^{n}(1-\text{Ꞇ}^{q})^{\omega_i})^{v_i}},\\ \prod_{j=1}^{m}(\prod_{i=1}^{n}\text{Ə}^{q})^{v_i},\\ \prod_{j=1}^{m}(\prod_{i=1}^{n}\text{Ɥ}^{q})^{v_i}\end{pmatrix}$$

$$=\left(\sqrt[q]{1-(1-\text{Ꞇ}^{q})},\text{Ə}^{q},\text{Ɥ}^{q}\right).$$

Hence, $q-ROPFS_{f}WA\left(\widetilde{\mathrm{N}}_{\ddot{e}_{11}},\widetilde{\mathrm{N}}_{\ddot{e}_{12}},\ldots,\widetilde{\mathrm{N}}_{\ddot{e}_{nm}}\right)=\mathfrak{R}_{\ddot{e}}$.

(2) **Boundedness:** We know that

$$\widetilde{\mathrm{N}}^{-}_{\ddot{e}_{ij}}=\left\{\min_{j}\min_{i}\left(\mathcal{L}_{ij}\right),\ \min_{j}\min_{i}\left(\mathrm{G}_{ij}\right),\ \max_{j}\max_{i}\left(\text{Ł}_{ij}\right)\right\}$$

and

$$\widetilde{\mathrm{N}}^{+}_{\ddot{e}_{ij}}=\left\{\max_{j}\max_{i}\left(\mathcal{L}_{ij}\right),\ \min_{j}\min_{i}\left(\mathrm{G}_{ij}\right),\ \min_{j}\min_{i}\left(\text{Ł}_{ij}\right)\right\}.$$

To show that,

$$\widetilde{\mathrm{N}}^{-}_{\ddot{e}_{ij}} \leq q-ROPFS_{f}WA\left(\widetilde{\mathrm{N}}_{\ddot{e}_{11}},\widetilde{\mathrm{N}}_{\ddot{e}_{12}},\ldots,\widetilde{\mathrm{N}}_{\ddot{e}_{nm}}\right) \leq \widetilde{\mathrm{N}}^{+}_{\ddot{e}_{ij}}.$$

$$\Rightarrow \min_j \min_i \{\mathcal{L}_{\mathrm{ij}}\} \leq \mathcal{L}_{\mathrm{ij}} \leq \max_j \max_i \{\mathcal{L}_{\mathrm{ij}}\}$$

$$\Leftrightarrow 1 - \max_j \max_i \{\mathcal{L}_{ij}^q\} \leq 1 - \mathcal{L}_{ij}^q \leq 1 - \min_j \min_i \{\mathcal{L}_{ij}^q\}$$

$$\Leftrightarrow \Pi_{j=1}^m \left(\Pi_{i=1}^n \left(1 - \max_j \max_i \{\mathcal{L}_{ij}^q\}\right)^{\omega_i}\right)^{v_i} \leq \Pi_{j=1}^m \left(\Pi_{i=1}^n \left(1 - \mathcal{L}_{ij}^q\right)^{\omega_i}\right)^{v_i} \leq \Pi_{j=1}^m \left(\Pi_{i=1}^n \left(1 - \min_j \min_i \{\mathcal{L}_{ij}^q\}\right)^{\omega_i}\right)^{v_i}$$

$$\Leftrightarrow \left(\left(1 - \max_j \max_i \{\mathcal{L}_{ij}^q\}\right)^{\sum_{i=1}^n \omega_i}\right)^{\sum_{i=1}^m v_i} \leq \Pi_{j=1}^m \left(\Pi_{i=1}^n \left(1 - \mathcal{L}_{ij}^q\right)^{\omega_i}\right)^{v_i} \leq \left(\left(1 - \min_j \min_i \{\mathcal{L}_{ij}^q\}\right)^{\sum_{i=1}^n \omega_i}\right)^{\sum_{i=1}^m v_i}$$

$$\Leftrightarrow \left(1 - \max_j \max_i \{\mathcal{L}_{ij}^q\}\right) \leq \Pi_{j=1}^m \left(\Pi_{i=1}^n \left(1 - \mathcal{L}_{ij}^q\right)^{\omega_i}\right)^{v_i} \leq \left(1 - \min_j \min_i \{\mathcal{L}_{ij}^q\}\right)$$

$$\Leftrightarrow 1 - \left(1 - \max_j \max_i \{\mathcal{L}_{ij}^q\}\right) \leq 1 - \Pi_{j=1}^m \left(\Pi_{i=1}^n \left(1 - \mathcal{L}_{ij}^q\right)^{\omega_i}\right)^{v_i} \leq 1 - \left(1 - \min_j \min_i \{\mathcal{L}_{ij}^q\}\right)$$

$$\Leftrightarrow \min_j \min_i \{\mathcal{L}_{\mathrm{ij}}\} \leq \sqrt[q]{1 - \Pi_{j=1}^m \left(\Pi_{i=1}^n \left(1 - \mathcal{L}_{ij}^q\right)^{\omega_i}\right)^{v_i}} \leq \max_j \max_i \{\mathcal{L}_{\mathrm{ij}}\}. \quad (12)$$

Next, we have

$$\Leftrightarrow \min_j \min_i \{\mathrm{G}_{\mathrm{ij}}\} \leq \mathrm{G}_{\mathrm{ij}} \leq \max_j \max_i \{\mathrm{G}_{\mathrm{ij}}\}$$

$$\Leftrightarrow \Pi_{j=1}^m \left(\Pi_{i=1}^n \left(\min_j \min_i \{\mathrm{G}_{\mathrm{ij}}\}\right)^{\omega_i}\right)^{v_i} \leq \Pi_{j=1}^m \left(\Pi_{i=1}^n \left(\mathrm{G}_{\mathrm{ij}}\right)^{\omega_i}\right)^{v_i} \leq \Pi_{j=1}^m \left(\Pi_{i=1}^n \left(\max_j \max_i \{\mathrm{G}_{\mathrm{ij}}\}\right)^{\omega_i}\right)^{v_i}$$

$$\Leftrightarrow \left(\left(\min_j \min_i \{\mathrm{G}_{\mathrm{ij}}\}\right)^{\sum_{i=1}^n \omega_i}\right)^{\sum_{i=1}^m v_i} \leq \Pi_{j=1}^m \left(\Pi_{i=1}^n \left(\mathrm{G}_{\mathrm{ij}}\right)^{\omega_i}\right)^{v_i} \leq \left(\left(\max_j \max_i \{\mathrm{G}_{\mathrm{ij}}\}\right)^{\sum_{i=1}^n \omega_i}\right)^{\sum_{i=1}^m v_i}$$

$$\Leftrightarrow \min_j \min_i \{\mathrm{G}_{\mathrm{ij}}\} \leq \Pi_{j=1}^m \left(\Pi_{i=1}^n \left(1 - \mathcal{L}_{ij}^q\right)^{\omega_i}\right)^{v_i} \leq \max_j \max_i \{\mathrm{G}_{\mathrm{ij}}\} \quad (13)$$

and

$$\Leftrightarrow \min_j \min_i \{\text{Ł}_{\mathrm{ij}}\} \leq \text{Ł}_{\mathrm{ij}} \leq \max_j \max_i \{\text{Ł}_{\mathrm{ij}}\}$$

$$\Leftrightarrow \Pi_{j=1}^m \left(\Pi_{i=1}^n \left(\min_j \min_i \{\text{Ł}_{\mathrm{ij}}\}\right)^{\omega_i}\right)^{v_i} \leq \Pi_{j=1}^m \left(\Pi_{i=1}^n \left(\text{Ł}_{\mathrm{ij}}\right)^{\omega_i}\right)^{v_i} \leq \Pi_{j=1}^m \left(\Pi_{i=1}^n \left(\max_j \max_i \{\text{Ł}_{\mathrm{ij}}\}\right)^{\omega_i}\right)^{v_i}$$

$$\Leftrightarrow \left(\left(\min_j \min_i \{\text{Ł}_{\mathrm{ij}}\}\right)^{\sum_{i=1}^n \omega_i}\right)^{\sum_{i=1}^m v_i} \leq \Pi_{j=1}^m \left(\Pi_{i=1}^n \left(\text{Ł}_{\mathrm{ij}}\right)^{\omega_i}\right)^{v_i} \leq \left(\left(\max_j \max_i \{\text{Ł}_{\mathrm{ij}}\}\right)^{\sum_{i=1}^n \omega_i}\right)^{\sum_{i=1}^m v_i}$$

$$\Leftrightarrow \min_j \min_i \{\text{Ł}_{\mathrm{ij}}\} \leq \Pi_{j=1}^m \left(\Pi_{i=1}^n \left(\text{Ł}_{\mathrm{ij}}\right)^{\omega_i}\right)^{v_i} \leq \max_j \max_i \{\text{Ł}_{\mathrm{ij}}\} \quad (14)$$

Therefore, from Eqs (12)–(14), we have

$$\Leftrightarrow \min_j \min_i \{\mathcal{L}_{ij}\} \leq \sqrt[q]{1-\Pi_{j=1}^{m}\left(\Pi_{i=1}^{n}\left(1-\mathcal{L}_{ij}^{q}\right)^{\omega_i}\right)^{\upsilon_i}} \leq \max_j \max_i \{\mathcal{L}_{ij}\}$$

$$\Leftrightarrow \min_j \min_i \{\mathrm{G}_{ij}\} \leq \Pi_{j=1}^{m}\left(\Pi_{i=1}^{n}\left(1-\mathcal{L}_{ij}^{q}\right)^{\omega_i}\right)^{\upsilon_i} \leq \max_j \max_i \{\mathrm{G}_{ij}\}$$

$$\Leftrightarrow \min_j \min_i \{\text{Ł}_{ij}\} \leq \Pi_{j=1}^{m}\left(\Pi_{i=1}^{n}\left(\text{Ł}_{ij}\right)^{\omega_i}\right)^{\upsilon_i} \leq \max_j \max_i \{\text{Ł}_{ij}\}.$$

Let $\delta = q-ROPFS_f WA\left(\tilde{\mathrm{N}}_{\ddot{e}_{11}}, \tilde{\mathrm{N}}_{\ddot{e}_{12}}, \ldots, \tilde{\mathrm{N}}_{\ddot{e}_{nm}}\right) = (\mathcal{L}_\delta, \mathrm{G}_\delta, \text{Ł}_\delta)$, then by score function

$$S(\delta)=\mathcal{L}_\delta^q - G_\delta^q - \text{Ł}_\delta^q + \left(\frac{e^{\mathcal{L}_\delta^q - G_\delta^q - \text{Ł}_\delta^q}}{e^{\mathcal{L}_\delta^q - G_\delta^q - \text{Ł}_\delta^q}+1} - \frac{1}{2}\right)\pi_\delta^q$$

$$\leq \left(\max_j \max_i \{\mathcal{L}_{ij}\}\right)^q \leq \left(\min_j \min_i \{G_{ij}\}\right)^q \leq \left(\min_j \min_i \{\text{Ł}_{ij}\}\right)^q +$$

$$\left(\frac{e^{\left(\max_j \max_i \{\mathcal{L}_{ij}\}\right)^q - \left(\min_j \min_i \{G_{ij}\}\right)^q - \left(\min_j \min_i \{\text{Ł}_{ij}\}\right)^q}}{e^{\left(\max_j \max_i \{\mathcal{L}_{ij}\}\right)^q - \left(\min_j \min_i \{G_{ij}\}\right)^q - \left(\min_j \min_i \{\text{Ł}_{ij}\}\right)^q}+1} - \frac{1}{2}\right)\pi_{\tilde{\mathrm{N}}_{\ddot{e}_{ij}}^{+}}^{q}$$

$$= S\left(\tilde{\mathrm{N}}_{\ddot{e}_{ij}}^{+}\right)$$

$$\Rightarrow S(\delta) \leq S\left(\tilde{\mathrm{N}}_{\ddot{e}_{ij}}^{+}\right)$$

and

$$\Leftrightarrow S(\delta)=\mathcal{L}_\delta^q - G_\delta^q - \text{Ł}_\delta^q + \left(\frac{e^{\mathcal{L}_\delta^q - G_\delta^q - \text{Ł}_\delta^q}}{e^{\mathcal{L}_\delta^q - G_\delta^q - \text{Ł}_\delta^q}+1} - \frac{1}{2}\right)\pi_\delta^q$$

$$\geq \left(\min_j \min_i \{\mathcal{L}_{ij}\}\right)^q \leq \left(\min_j \min_i \{G_{ij}\}\right)^q \leq \left(\max_j \max_i \{\text{Ł}_{ij}\}\right)^q +$$

$$\left(\frac{e^{\left(\min_j \min_i \{\mathcal{L}_{ij}\}\right)^q - \left(\min_j \min_i \{G_{ij}\}\right)^q - \left(\max_j \max_i \{\text{Ł}_{ij}\}\right)^q}}{e^{\left(\min_j \min_i \{\mathcal{L}_{ij}\}\right)^q - \left(\min_j \min_i \{G_{ij}\}\right)^q - \left(\max_j \max_i \{\text{Ł}_{ij}\}\right)^q}+1} - \frac{1}{2}\right)\pi_{\tilde{\mathrm{N}}_{\ddot{e}_{ij}}^{-}}^{q}$$

$$= S\left(\tilde{\mathrm{N}}_{\ddot{e}_{ij}}^{-}\right)$$

$$\Rightarrow S(\delta) \geq S\left(\tilde{\mathrm{N}}_{\ddot{e}_{ij}}^{-}\right).$$

Now we have the following cases:

(i) If $S(\delta) \leq S\left(\tilde{\mathrm{N}}_{\ddot{e}_{ij}}^{+}\right)$ and $S(\delta) \geq S\left(\tilde{\mathrm{N}}_{\ddot{e}_{ij}}^{-}\right)$, by the comparison of these two $q-ROPFS_f Ns$, we get

$$\tilde{\mathrm{N}}_{\ddot{e}_{ij}}^{-} < q-ROPFS_f WA\left(\tilde{\mathrm{N}}_{\ddot{e}_{11}}, \tilde{\mathrm{N}}_{\ddot{e}_{12}}, \ldots, \tilde{\mathrm{N}}_{\ddot{e}_{nm}}\right) < \tilde{\mathrm{N}}_{\ddot{e}_{ij}}^{+}.$$

(ii) If $S(\delta) = S\left(\tilde{\mathrm{N}}_{\ddot{e}_{ij}}^{+}\right)$, then

$$\mathcal{L}_{\delta}^{q}-G_{\delta}^{q}-Ł_{\delta}^{q}+\left(\frac{e^{\mathcal{L}_{\delta}^{q}-G_{\delta}^{q}-Ł_{\delta}^{q}}}{e^{\mathcal{L}_{\delta}^{q}-G_{\delta}^{q}-Ł_{\delta}^{q}}+1}-\frac{1}{2}\right)\pi_{\delta}^{q}$$

$$=\left(\max_{j}\max_{i}\{\mathcal{L}_{ij}\}\right)^{q}\leq\left(\min_{j}\min_{i}\{G_{ij}\}\right)^{q}\leq\left(\min_{j}\min_{i}\{Ł_{ij}\}\right)^{q}+$$

$$\left(\frac{e^{\left(\max_{j}\max_{i}\{\mathcal{L}_{ij}\}\right)^{q}-\left(\min_{j}\min_{i}\{G_{ij}\}\right)^{q}-\left(\min_{j}\min_{i}\{Ł_{ij}\}\right)^{q}}}{e^{\left(\max_{j}\max_{i}\{\mathcal{L}_{ij}\}\right)^{q}-\left(\min_{j}\min_{i}\{G_{ij}\}\right)^{q}-\left(\min_{j}\min_{i}\{Ł_{ij}\}\right)^{q}}+1}-\frac{1}{2}\right)\pi_{\tilde{N}_{ë_{ij}}^{+}}^{q}$$

then by above inequalities, we get

$\Longrightarrow \mathcal{L}_{\delta}=\max_{j}\max_{i}\{\mathcal{L}_{ij}\},\ G_{\delta}=\min_{j}\min_{i}(G_{ij}),\ Ł_{\delta}=\max_{j}\max_{i}(Ł_{ij})$

$\Longrightarrow \pi_{\delta}^{q}=\pi_{\tilde{N}_{ë_{ij}}^{-}}^{q}$

$\Longrightarrow q-ROPFS_{f}WA(\tilde{N}_{ë_{11}},\tilde{N}_{ë_{12}},\ldots,\tilde{N}_{ë_{nm}})=\tilde{N}_{ë_{ij}}^{-}$.

Hence

$$\tilde{N}_{ë_{ij}}^{-}\leq q-ROPFS_{f}WA(\tilde{N}_{ë_{11}},\tilde{N}_{ë_{12}},\ldots,\tilde{N}_{ë_{nm}})\leq\tilde{N}_{ë_{ij}}^{+}.$$

(3) **Monotonicity:** Since $\mathcal{L}_{ij}\leq Ꞇ_{ij}, G_{ij}\leq Ə_{ij}$ and $Ł_{ij}\geq Ⅎ_{ij}$, then

$\Rightarrow \mathcal{L}_{ij}\leq Ꞇ_{ij}\Rightarrow 1-Ꞇ_{ij}\leq 1-\mathcal{L}_{ij}\Rightarrow 1-Ꞇ_{ij}^{q}\leq 1-\mathcal{L}_{ij}^{q}$

$\Rightarrow \Pi_{j=1}^{m}\left(\Pi_{i=1}^{n}\left(1-Ꞇ_{ij}^{q}\right)^{\omega_i}\right)^{\upsilon_i}\leq\Pi_{j=1}^{m}\left(\Pi_{i=1}^{n}\left(1-\mathcal{L}_{ij}^{q}\right)^{\omega_i}\right)^{\upsilon_i}$

$\Rightarrow 1-\Pi_{j=1}^{m}\left(\Pi_{i=1}^{n}\left(1-\mathcal{L}_{ij}^{q}\right)^{\omega_i}\right)^{\upsilon_i}\leq 1-\Pi_{j=1}^{m}\left(\Pi_{i=1}^{n}\left(1-Ꞇ_{ij}^{q}\right)^{\omega_i}\right)^{\upsilon_i}$

$\Rightarrow \sqrt[q]{1-\Pi_{j=1}^{m}\left(\Pi_{i=1}^{n}\left(1-\mathcal{L}_{ij}^{q}\right)^{\omega_i}\right)^{\upsilon_i}}\leq\sqrt[q]{1-\Pi_{j=1}^{m}\left(\Pi_{i=1}^{n}\left(1-Ꞇ_{ij}^{q}\right)^{\omega_i}\right)^{\upsilon_i}}$

next $G_{ij}\leq Ə_{ij}$

$\Rightarrow \Pi_{i=1}^{n}(G_{ij})^{\omega_i}\leq\Pi_{i=1}^{n}(Ə_{ij})^{\omega_i}$

$\Rightarrow \Pi_{j=1}^{m}\left(\Pi_{i=1}^{n}(G_{ij})^{\omega_i}\right)^{\upsilon_i}\leq\Pi_{j=1}^{m}\left(\Pi_{i=1}^{n}(Ə_{ij})^{\omega_i}\right)^{\upsilon_i}$

and $Ł_{ij}\geq Ⅎ_{ij}$

$\Rightarrow \Pi_{i=1}^{n}(Ł_{ij})^{\omega_i}\geq\Pi_{i=1}^{n}(Ⅎ_{ij})^{\omega_i}$

$\Rightarrow \Pi_{j=1}^{m}\left(\Pi_{i=1}^{n}(Ł_{ij})^{\omega_i}\right)^{\upsilon_i}\leq\Pi_{j=1}^{m}\left(\Pi_{i=1}^{n}(Ⅎ_{ij})^{\omega_i}\right)^{\upsilon_i}$.

Suppose that $\delta_{\tilde{N}}=q-ROPFS_{f}WA(\tilde{N}_{ë_{11}},\tilde{N}_{ë_{12}},\ldots,\tilde{N}_{ë_{nm}})=(\mathcal{L}_{\delta_{\tilde{N}}},G_{\delta_{\tilde{N}}},Ł_{\delta_{\tilde{N}}})$ and

$$\delta_{\mathfrak{R}}=q-ROPFS_{f}WA(\mathfrak{R}_{ë_{11}},\mathfrak{R}_{ë_{12}},\ldots,\mathfrak{R}_{ë_{nm}})=(Ꞇ_{\delta_{\mathfrak{R}}},Ə_{\delta_{\mathfrak{R}}},Ⅎ_{\delta_{\mathfrak{R}}}).$$

Now, from the above equation, we have

$$\mathcal{L}_{ij} \le \mathrm{C}_{ij}, \mathrm{G}_{ij} \le \mathrm{Ə}_{ij} \text{ and } \mathrm{Ł}_{ij} \ge \mathrm{Ƚ}_{ij}$$

then by the score function we have $S(\delta_{\widetilde{N}}) \le S(\delta_{\Re})$.
Now, we have the following cases:

(I) By the comparison of two q-ROPF soft numbers, if $S(\delta_{\widetilde{N}}) < S(\delta_{\Re})$, then

$$q-ROPFS_f WA(\widetilde{N}_{ë_{11}}, \widetilde{N}_{ë_{12}}, \ldots, \widetilde{N}_{ë_{nm}}) < q-ROPFS_f WA(\Re_{ë_{11}}, \Re_{ë_{12}}, \ldots, \Re_{ë_{nm}}).$$

(II) If $S(\delta_{\widetilde{N}}) = S(\delta_{\Re})$, where

$$S(\delta_{\widetilde{N}}) = \mathcal{L}^q_{\delta_{\widetilde{N}}} - G^q_{\delta_{\widetilde{N}}} - Ł^q_{\delta_{\widetilde{N}}} + \left(\frac{e^{\mathcal{L}^q_{\delta_{\widetilde{N}}} - G^q_{\delta_{\widetilde{N}}} - Ł^q_{\delta_{\widetilde{N}}}}}{e^{\mathcal{L}^q_{\delta_{\widetilde{N}}} - G^q_{\delta_{\widetilde{N}}} - Ł^q_{\delta_{\widetilde{N}}+1}}} - \frac{1}{2} \right) \pi^q_{\delta_{\widetilde{N}}}$$

$$S(\delta_{\Re}) = \mathcal{L}^q_{\delta_{\Re}} - G^q_{\delta_{\Re}} - Ł^q_{\delta_{\Re}} + \left(\frac{e^{\mathcal{L}^q_{\delta_{\widetilde{N}}} - G^q_{\delta_{\widetilde{N}}} - Ł^q_{\delta_{\widetilde{N}}}}}{e^{\mathcal{L}^q_{\delta_{\Re}} - G^q_{\delta_{\Re}} - Ł^q_{\delta_{\Re}+1}}} - \frac{1}{2} \right) \pi^q_{\delta_{\widetilde{\Re}}}.$$

We have, $\mathcal{L}_{\delta_{\widetilde{N}}} = \mathrm{C}_{\delta_{\Re}}, \mathrm{G}_{\delta_{\widetilde{N}}} = \mathrm{Ə}_{\delta_{\Re}}$ and $\mathrm{Ł}_{\delta_{\widetilde{N}}} = \mathrm{Ƚ}_{\delta_{\Re}}$. Hence

$\Rightarrow \pi^q_{\delta_{\widetilde{N}}} = \pi^q_{\delta_{\Re}}$

$\Rightarrow (\mathcal{L}_{\delta_{\widetilde{N}}}, \mathrm{G}_{\delta_{\widetilde{N}}}, \mathrm{Ł}_{\delta_{\widetilde{N}}}) = (\mathrm{C}_{\delta_{\Re}}, \mathrm{Ə}_{\delta_{\Re}}, \mathrm{Ƚ}_{\delta_{\Re}})$.

Proved that $q-ROPFS_f WA(\widetilde{N}_{ë_{11}}, \widetilde{N}_{ë_{12}}, \ldots, \widetilde{N}_{ë_{nm}}) < q-ROPFS_f WA(\Re_{ë_{11}}, \Re_{ë_{12}}, \ldots, \Re_{ë_{nm}})$.

(4) **Shift Invariance:** Since $\Re_{ë} = (\mathrm{C}, \mathrm{Ə}, \mathrm{Ƚ})$ and $\widetilde{N}_{ë_{ij}} = (\mathcal{L}_{ë_{ij}}, \mathrm{G}_{ë_{ij}}, \mathrm{Ł}_{ë_{ij}})$ are the q-ROPF soft numbers, so $\widetilde{N}_{ë_{ij}} \oplus \Re_{ë} = \left(\sqrt[q]{(1 - \mathcal{L}^q_{ij})(1 - \mathrm{C}^q)}, \mathrm{G}^q_{ij}\mathrm{Ə}, \mathrm{Ł}^q_{ij}\mathrm{Ƚ} \right)$. Therefore,

$$q-ROPFS_f WA(\widetilde{N}_{ë_{11}} \oplus \Re_{ë}, \widetilde{N}_{ë_{12} \oplus \Re_{ë}}, \ldots, \widetilde{N}_{ë_{nm}} \oplus \Re_{ë}) = \oplus_{j=1}^m \upsilon_i \left(\oplus_{i=1}^n \omega_i \left(\widetilde{N}_{ë_{ij}} \oplus \Re_{ë} \right) \right)$$

$$= \left(\sqrt[q]{1 - \Pi_{j=1}^m \left(\Pi_{i=1}^n \left(1 - \mathcal{L}^q_{ij}\right)^{\omega_i} (1 - \mathrm{C}^q)^{\omega_i} \right)^{\upsilon_i}}, \Pi_{j=1}^m \left(\Pi_{i=1}^n \mathrm{G}_{ij}^{\omega_i} \mathrm{Ə}^{\omega_i} \right)^{\upsilon_i}, \Pi_{j=1}^m \left(\Pi_{i=1}^n \mathrm{Ł}_{ij}^{\omega_i} \mathrm{Ƚ}^{\omega_i} \right)^{\upsilon_i} \right)$$

$$= \left(\sqrt[q]{1 - (1 - \mathrm{C}^q) \Pi_{j=1}^m \left(\Pi_{i=1}^n \left(1 - \mathcal{L}^q_{ij}\right)^{\omega_i} \right)^{\upsilon_i}}, \mathrm{Ə}\Pi_{j=1}^m \left(\Pi_{i=1}^n \mathrm{G}_{ij}^{\omega_i} \right)^{\upsilon_i}, \mathrm{Ƚ}\Pi_{j=1}^m \left(\Pi_{i=1}^n \mathrm{Ł}_{ij}^{\omega_i} \right)^{\upsilon_i} \right)$$

$$= \left(\sqrt[q]{1 - \Pi_{j=1}^m \left(\Pi_{i=1}^n \left(1 - \mathcal{L}^q_{ij}\right)^{\omega_i} \right)^{\upsilon_i}}, \Pi_{j=1}^m \left(\Pi_{i=1}^n \mathrm{G}_{ij}^{\omega_i} \right)^{\upsilon_i}, \Pi_{j=1}^m \left(\Pi_{i=1}^n \mathrm{Ł}_{ij}^{\omega_i} \right)^{\upsilon_i} \right) \oplus (\mathrm{C}, \mathrm{Ə}, \mathrm{Ƚ})$$

$$q-ROPFS_f WA(\widetilde{N}_{ë_{11}}, \widetilde{N}_{ë_{12}}, \ldots, \widetilde{N}_{ë_{nm}}) \oplus \Re_{ë}.$$

(5) **Homogeneity:** Let $\widetilde{N}_{ë_{ij}} = (\mathcal{L}_{ë_{ij}}, \mathrm{G}_{ë_{ij}}, \mathrm{Ł}_{ë_{ij}})$ be a $q-ROPFS_f Ns$ and $\lambda \ge 0$, be any real number, then

$$\lambda \widetilde{N} = \left(\sqrt[q]{1 - \left(1 - \mathcal{L}^q_{ij}\right)^{\lambda}}, \mathrm{G}^q_{ij}, \mathrm{Ł}^q_{ij} \right)$$

$$q-ROPFS_fWA\left(\lambda\widetilde{N}_{ë_{11}},\lambda\widetilde{N}_{ë_{12}},\ldots,\widetilde{\lambda N}_{ë_{nm}}\right)=\begin{pmatrix}\sqrt[q]{1-\left(\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(1-\mathcal{L}_{ij}^{q}\right)^{\omega_i}\right)^{\upsilon_i}\right)^{\lambda}}, \\ \left(\prod_{j=1}^{m}\left(\prod_{i=1}^{n}G_{ij}^{\omega_i}\right)^{\upsilon_i}\right)^{\lambda}, \\ \left(\prod_{j=1}^{m}\left(\prod_{i=1}^{n}Ł_{ij}^{\omega_i}\right)^{\upsilon_i}\right)^{\lambda}\end{pmatrix}$$

$$=\lambda\ q-ROPFS_fWA\left(\widetilde{N}_{ë_{11}},\widetilde{N}_{ë_{12}},\ldots,\widetilde{N}_{ë_{nm}}\right).$$

Hence, the property proved.

4.2. q-ROPF soft ordered weighted average ($q-ROPFS_fOWA$) operator

Definition 4.2. Assume that $\widetilde{N}_{ë_{ij}}=\left(\mathcal{L}_{ij},G_{ij},Ł_{ij}\right)$ for ($i=1,2,\ldots n$ and $j=1,2,\ldots m$) be collection of $q=ROPFS_fNs$ and weight vector ω= {ω_1, ω_2, ... ,ω_n} with the condition $\sum_{i=1}^{n}\omega_i=1$ and υ= {υ_1, υ_2, ..., υ_m} with the condition that $\sum_{i=1}^{n}\upsilon_i=1$ for alternatives x_i and parameters e_j, respectively. The mapping $q-ROPFS_fOWA: Đ^n\rightarrow Đ$ is said to be $q-ROPFS_fOWA$ operator. (Đ is the collection of $q-ROPFS_fNs$).

$$q-ROPFS_fOWA\left(\widetilde{N}_{ë_{11}},\widetilde{N}_{ë_{12}},\ldots,\widetilde{N}_{ë_{nm}}\right)=\oplus_{j=1}^{m}\upsilon_i\left(\oplus_{i=1}^{n}\omega_i\widetilde{N}_{\sigma ë_{ij}}\right). \quad (15)$$

Theorem 4.3. Consider the collection of $q-ROPFS_fNs$ $\widetilde{N}_{ë_{ij}}=\left(\mathcal{L}_{ij},G_{ij},Ł_{ij}\right)$ then the aggregation result for $q-ROPFS_fOWA$ operator is expressed:

$$q-ROPFS_fWA\left(\widetilde{N}_{ë_{11}},\widetilde{N}_{ë_{12}},\ldots,\widetilde{N}_{ë_{nm}}\right)=\oplus_{j=1}^{m}\upsilon_i\left(\oplus_{i=1}^{n}\omega_i\widetilde{N}_{\sigma ë_{ij}}\right)$$

$$=\left(\sqrt[q]{1-\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(1-\mathcal{L}_{\sigma ij}^{q}\right)^{\omega_i}\right)^{\upsilon_i}},\prod_{j=1}^{m}\left(\prod_{i=1}^{n}G_{\sigma ij}^{\omega_i}\right)^{\upsilon_i},\prod_{j=1}^{m}\left(\prod_{i=1}^{n}Ł_{\sigma ij}^{\omega_i}\right)^{\upsilon_i}\right)$$

$$q-ROPFS_fWA\left(\widetilde{N}_{ë_{11}},\widetilde{N}_{ë_{12}},\ldots,\widetilde{N}_{ë_{nm}}\right)=\begin{pmatrix}\sqrt[q]{1-\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(1-\mathcal{L}_{\sigma ij}^{q}\right)^{\omega_i}\right)^{\upsilon_i}}, \\ \prod_{j=1}^{m}\left(\prod_{i=1}^{n}G_{\sigma ij}^{\omega_i}\right)^{\upsilon_i}, \\ \prod_{j=1}^{m}\left(\prod_{i=1}^{n}Ł_{\sigma ij}^{\omega_i}\right)^{\upsilon_i}\end{pmatrix} \quad (16)$$

ω= {ω_1, ω_2, ..., ω_n} with the condition $\sum_{i=1}^{n}\omega_i=1$ and υ= {υ_1, υ_2, ..., υ_m} with the condition that $\sum_{i=1}^{n}\upsilon_i=1$ for alternatives x_i and parameters e_j, respectively.

Proof. Proof is similar to the theory of "$q-ROPFS_fWA$" operator.

Example 4.2. From "Table 2" of Example 4.1, we take the collections $q-ROPFS_fNs$ $\widetilde{N}_{ë_{ij}}=\left(\mathcal{L}_{ij},G_{ij},Ł_{ij}\right)$) by using the score function, the we obtain the tabular representation of $\widetilde{N}_{ë_{ij}}=\left(\mathcal{L}_{\sigma ij},G_{\sigma ij},Ł_{\sigma ij}\right)$ is presented in "Table 3".

Table 3. Tabular representation of $q-ROPFS_f S$ $\tilde{N}_{\ddot{e}_{ij}}=(\mathcal{L}_{\sigma ij}, G_{\sigma ij}, \text{Ł}_{\sigma ij})$ for $q \geq 3$.

$\bar{U}$	e_1	e_2	e_3	e_4
t_1	$(0.8, 0.4, 0.3)$	$(0.9, 0.5, 0.3)$	$(0.83, 0.2, 0.1)$	$(0.87, 0.32, 0.15)$
t_2	$(0.76, 0.2, 0.1)$	$(0.8, 0.33, 0.1)$	$(0.77, 0.4, 0.3)$	$(0.84, 0.35, 0.1)$
t_3	$(0.71, 0.3, 0.1)$	$(0.65, 0.35, 0.15)$	$(0.66, 0.54, 0.2)$	$(0.8, 0.3, 0.2)$
t_4	$(0.66, 0.4, 0.2)$	$(0.6, 0.5, 0.3)$	$(0.58, 0.3, 0.1)$	$(0.73, 0.22, 0.1)$
t_5	$(0.62, 0.5, 0.2)$	$(0.55, 0.5, 0.2)$	$(0.45.0.25, 0.1)$	$(0.7, 0.5, 0.1)$

By Eq (16) we have

$$q-ROPFS_f WA(\tilde{N}_{\ddot{e}_{11}}, \tilde{N}_{\ddot{e}_{12}}, \ldots, \tilde{N}_{\ddot{e}_{nm}}) = \begin{pmatrix} \sqrt[q]{1-\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(1-\mathcal{L}_{\sigma ij}^{q}\right)^{\omega_i}\right)^{\upsilon_i}}, \\ \prod_{j=1}^{m}\left(\prod_{i=1}^{n} G_{\sigma ij}^{\omega_i}\right)^{\upsilon_i}, \\ \prod_{j=1}^{m}\left(\prod_{i=1}^{n} \text{Ł}_{\sigma ij}^{\omega_i}\right)^{\upsilon_i} \end{pmatrix}$$

$$= \begin{pmatrix} \sqrt[3]{\begin{matrix} 1-\{(1-0.8^3)^{0.15}(1-0.76^3)^{0.16}(1-0.71^3)^{0.20}(1-0.66^3)^{0.25}(1-0.62^3)^{0.24}\}^{0.5} \\ 1-\{(1-0.9^3)^{0.15}(1-0.8^3)^{0.16}(1-0.65^3)^{0.20}(1-0.6^3)^{0.25}(1-0.55^3)^{0.24}\}^{0.17} \\ 1-\{(1-0.83^3)^{0.15}(1-0.77^3)^{0.16}(1-0.66^3)^{0.20}(1-0.58^3)^{0.25}(1-0.45^3)^{0.24}\}^{0.13}, \\ 1-\{(1-0.87^3)^{0.15}(1-0.84^3)^{0.16}(1-0.8^3)^{0.20}(1-0.73^3)^{0.25}(1-0.7^3)^{0.24}\}^{0.20} \end{matrix}} \\ \begin{pmatrix} \{(1-0.4^3)^{0.15}(1-0.2^3)^{0.16}(1-0.3^3)^{0.20}(1-0.4^3)^{0.25}(1-0.5^3)^{0.24}\}^{0.5} \\ \{(1-0.5^3)^{0.15}(1-0.33^3)^{0.16}(1-0.35^3)^{0.20}(1-0.5^3)^{0.25}(1-0.5^3)^{0.24}\}^{0.17} \\ \{(1-0.2^3)^{0.15}(1-0.4^3)^{0.16}(1-0.54^3)^{0.20}(1-0.3^3)^{0.25}(1-0.25^3)^{0.24}\}^{0.13} \\ \{(1-0.32^3)^{0.15}(1-0.35^3)^{0.16}(1-0.3^3)^{0.20}(1-0.22^3)^{0.25}(1-0.5^3)^{0.24}\}^{0.20} \end{pmatrix}, \\ \begin{pmatrix} \{(1-0.3^3)^{0.15}(1-0.1^3)^{0.16}(1-0.1^3)^{0.20}(1-0.2^3)^{0.25}(1-0.2^3)^{0.24}\}^{0.5} \\ \{(1-0.3^3)^{0.15}(1-0.1^3)^{0.16}(1-0.15^3)^{0.20}(1-0.3^3)^{0.25}(1-0.2^3)^{0.24}\}^{0.17} \\ \{(1-0.1^3)^{0.15}(1-0.3^3)^{0.16}(1-0.2^3)^{0.20}(1-0.1^3)^{0.25}(1-0.1^3)^{0.24}\}^{0.13} \\ \{(1-0.15^3)^{0.15}(1-0.1^3)^{0.16}(1-0.2^3)^{0.20}(1-0.1^3)^{0.25}(1-0.1^3)^{0.24}\}^{0.20} \end{pmatrix} \end{pmatrix}$$

$= (0.7264, 0.3568, 0.1568)$.

Theorem 4.4. Consider the collection of $q-ROPFS_f Ns$ $\tilde{N}_{\ddot{e}_{ij}}=(\mathcal{L}_{ij}, G_{ij}, \text{Ł}_{ij})$ of weight vector $\omega = \{\omega_1, \omega_2, \ldots, \omega_n\}$ with the condition $\sum_{i=1}^{n}\omega_i = 1$ and $\upsilon = \{\upsilon_1, \upsilon_2, \ldots, \upsilon_m\}$ with the condition that $\sum_{i=1}^{n}\upsilon_i = 1$ for alternatives x_i and parameters e_j, respectively. Then the $q-ROPFS_f OWA$ operator hold the following properties:

(1) **Idempotency:** If $\tilde{N}_{\ddot{e}_{ij}} = \mathfrak{R}_{\ddot{e}}$, where $\mathfrak{R}_{\ddot{e}} = (\text{Ꞓ}, \text{Ə}, \text{Ɉ})$ and for all $(i = 1,2,\ldots,n \text{ and } j = 1,2,\ldots,m)$, then $q-ROPFS_f OWA(\tilde{N}_{\ddot{e}_{11}}, \tilde{N}_{\ddot{e}_{12}}, \ldots, \tilde{N}_{\ddot{e}_{nm}}) = \mathfrak{R}_{\ddot{e}}$.

(2) **Boundedness:** If

$$\tilde{N}_{\ddot{e}_{ij}}^{-} = \left\{\min_j \min_i (\mathcal{L}_{ij}), \min_j \min_i (G_{ij}), \max_j \max_i (\text{Ł}_{ij})\right\}$$

and

$$\widetilde{\text{N}}^{+}_{\text{ë}_{\text{ij}}}=\left\{\max_{j}\max_{i}\left(\mathcal{L}_{\text{ij}}\right),\ \min_{j}\min_{i}\left(\text{G}_{\text{ij}}\right),\ \min_{j}\min_{i}\left(\text{Ł}_{\text{ij}}\right)\right\},$$

then

$$\widetilde{\text{N}}^{-}_{\text{ë}_{\text{ij}}} \leq q-ROPFS_{f}OWA\left(\widetilde{\text{N}}_{\text{ë}_{11}},\widetilde{\text{N}}_{\text{ë}_{12}},\ldots,\widetilde{\text{N}}_{\text{ë}_{\text{nm}}}\right) \leq \widetilde{\text{N}}^{+}_{\text{ë}_{\text{ij}}}.$$

(3) **Monotonicity:** If $\Re_{\text{ë}_{\text{ij}}}=\left(\text{ɕ}_{\text{ij}},\text{Ə}_{\text{ij}},\ \text{Ⅎ}_{\text{ij}}\right)$ be the collection of $q-ROPFS_{f}Ns$ such that $\mathcal{L}_{\text{ij}} \leq \text{ɕ}_{\text{ij}},\ \text{G}_{\text{ij}} \leq \text{Ə}_{\text{ij}}, \text{Ł}_{\text{ij}} \geq\ \text{Ⅎ}_{\text{ij}}$ then

$$q-ROPFS_{f}OWA\left(\widetilde{\text{N}}_{\text{ë}_{11}},\widetilde{\text{N}}_{\text{ë}_{12}},\ldots,\widetilde{\text{N}}_{\text{ë}_{\text{nm}}}\right) \leq q-ROPFS_{f}OWA\left(\Re_{\text{ë}_{11}},\Re_{\text{ë}_{12}},\ldots,\Re_{\text{ë}_{\text{nm}}}\right).$$

(4) **Shift Invariance:** If $\Re_{\text{ë}} = \left(\text{ɕ}_{\text{ij}},\text{Ə}_{\text{ij}},\ \text{Ⅎ}_{\text{ij}}\right)$ is $q-ROPFS_{f}Ns$, then

$$q-ROPFS_{f}OWA\left(\widetilde{\text{N}}_{\text{ë}_{11}} \oplus \Re_{\text{ë}},\widetilde{\text{N}}_{\text{ë}_{12}} \oplus \Re_{\text{ë}},\ldots,\widetilde{\text{N}}_{\text{ë}_{\text{nm}}} \oplus \Re_{\text{ë}}\right) =$$

$$q-ROPFS_{f}OWA\left(\widetilde{\text{N}}_{\text{ë}_{11}},\widetilde{\text{N}}_{\text{ë}_{12}},\ldots,\widetilde{\text{N}}_{\text{ë}_{\text{nm}}}\right) \oplus \Re_{\text{ë}}.$$

(5) **Homogeneity:** If $\lambda \geq 0$, then

$$q-ROPFS_{f}OWA\left(\lambda\widetilde{\text{N}}_{\text{ë}_{11}},\widetilde{\lambda\text{N}}_{\text{ë}_{12}},\ldots,\lambda\widetilde{\text{N}}_{\text{ë}_{\text{nm}}}\right) = \lambda\ q-ROPFS_{f}OWA\left(\widetilde{\text{N}}_{\text{ë}_{11}},\widetilde{\text{N}}_{\text{ë}_{12}},\ldots,\widetilde{\text{N}}_{\text{ë}_{\text{nm}}}\right).$$

Proof. Straight forward.

5. MADM under *q*-ROPF soft information

In real life situation DM play very important role, and it is a pre-plan process of selecting the best choice out of many alternatives. Let Ą= {ą₁, ą₂, ...ąₗ} be the set of alternative and corresponding set parameter Ĉ= {c₁, c₂….., cₘ}. The team of n senior expert Đ₁, Đ₂, ..., Đn} evaluate to each alternative $ą_s$ to their corresponding parameters c_j. The group of senior experts provide their evaluation in terms of $\widetilde{\text{N}}_{\text{ë}_{\text{ij}}}=\left(\mathcal{L}_{\text{ij}},\text{G}_{\text{ij}},\text{Ł}_{\text{ij}}\right)$ with the weight vector $\omega= (\omega_1,\omega2,\ldots,\omega n)^{\text{T}}$ with the condition $\sum_{i=1}^{n}\omega_i = 1$ and $\upsilon = (\upsilon1,\upsilon2,\ldots,\upsilon m)^{\text{T}}$ with the condition that $\sum_{i=1}^{n}\upsilon_i = 1$ for alternatives x_i and parameters e_j, respectively. Where the collective information of senior expert are described by the decision matrix $\text{M}=\left[\widetilde{\text{N}}_{\text{ë}_{\text{ij}}}\right]_{\text{m}\times\text{n}}$ and the aggregated *q*-ROPF soft number $\mho_{\text{S}}$ for alternative (S=1, 2,….,l) is given as $\mho_{\text{S}}=(\mathcal{L}_{\text{S}},\text{G}_{\text{S}},\text{Ł}_{\text{S}})$. Finally, we apply the score function on each aggregated *q*-ROPF soft number $\mho_{\text{S}}=(\mathcal{L}_{\text{S}},\text{G}_{\text{S}},\text{Ł}_{\text{S}})$ for the alternative and rank them in a specific ordered to get the best option. Steps involve in algorithm for solving MADM applications.

Algorithm:

Step 1. Construct a decision matrix $\text{M}=\left[\widetilde{\text{N}}_{\text{ë}_{\text{ij}}}\right]_{\text{m}\times\text{n}}$:

$$M = \begin{bmatrix} (\mathcal{L}_{11},\text{G}_{11},\text{Ł}_{11}) & (\mathcal{L}_{12},\text{G}_{12},\text{Ł}_{12}) & \cdots & (\mathcal{L}_{1m},\text{G}_{1m},\text{Ł}_{1m}) \\ (\mathcal{L}_{21},\text{G}_{21},\text{Ł}_{21}) & (\mathcal{L}_{22},\text{G}_{22},\text{Ł}_{22}) & \cdots & (\mathcal{L}_{2m},\text{G}_{2m},\text{Ł}_{2m}) \\ \vdots & \vdots & \ddots & \vdots \\ (\mathcal{L}_{1m},\text{G}_{1m},\text{Ł}_{1m}) & (\mathcal{L}_{2m},\text{G}_{2m},\text{Ł}_{2m}) & \cdots & (\mathcal{L}_{nm},\text{G}_{nm},\text{Ł}_{nm}) \end{bmatrix}$$

Step 2. Normalization of *q*-ROPF soft decision matrix $\text{M}=\left[\widetilde{\text{N}}_{\text{ë}_{\text{ij}}}\right]_{\text{m}\times\text{n}}$

$$P_{ij} = \begin{cases} \text{for cost type parameter we use } \widetilde{N}^{C}_{\ddot{e}_{ij}} \\ \text{for benefit type parameter we use } \widetilde{N}_{\ddot{e}_{ij}} \end{cases}$$

where $\widetilde{N}^{C}_{\ddot{e}_{ij}}\}$ reprsent the complement of $\widetilde{N}_{\ddot{e}_{ij}}$.

Step 3. To aggregate the -ROPF soft number $\widetilde{N}_{\ddot{e}_{ij}}=(\mathcal{L}_{ij}, G_{ij}, Ł_{ij})$ for each alternative.

Step 4. To calculate the score value.

Step 5. At the end arrange the score value to choose the best option.

5.1. Application of the proposed model to MCDM

For a decision-making problem, we consider a numerical example. Let consider a $Đ_1$, $Đ_2$, $Đ_3$, $Đ_4$, $Đ_5$ which represents the set of senior expert doctors having $\omega=(0.16,0.26,0.15,0.20,0.23)^T$ represent the weight-vector which evaluate a common disease "obstructive goiter" of four different patients (alternatives) z_1, z_2, z_3 and z_4 based on the following signs and symptoms may include:

$$\hat{C} = \begin{Bmatrix} c_1 = \text{Difficulty swallowing} \\ c_2 = \text{Difficulty breathing with exertion} \\ c_3 = \text{Cough} \\ c_4 = \text{Hoarseness} \\ c_5 = \text{Snoring} \end{Bmatrix}$$

which represent the set of parameters having weight vectors $\upsilon=(0.28,0.20,0.1,0.15,0.27)^T$. To diagnose the illness patients, we construct a step-wise algorithm.

By *q*-ROPF soft weight averaging operator:

Step 1. Construct a decision matrix $M=\left[\widetilde{N}_{\ddot{e}_{ij}}\right]_{m\times n}$ expressed in *q*-ROPF soft numbers, which are given in Tables 4–7, respectively

Table 4. *q*-ROPF soft matrix for patient z_1.

	c_1	c_2	c_3	c_4	c_5
$Đ_1$	$(0.71,0.25,0.1)$	$(0.77,0.2,0.15)$	$(0.88,0.22,0.11)$	$(0.81,0.18,0.11)$	$(0.79,0.2,0.1)$
$Đ_2$	$(0.8,0.22,0.11)$	$(0.85,0.12,0.11)$	$(0.7,0.3,0.15)$	$(0.75,0.15,0.1)$	$(0.74,0.4,0.14)$
$Đ_3$	$(0.77,0.2,0.1)$	$(0.75,0.25,0.15)$	$(0.84,0.12,0.11)$	$(0.86,0.2,0.1)$	$(0.86,0.2,0.1)$
$Đ_4$	$(0.78,0.18,0.1)$	$(0.7,0.18,0.11)$	$(0.75,0.25,0.1)$	$(0.7,0.25,0.15)$	$(0.65,0.16,0.11)$
$Đ_5$	$(0.7,0.35,0.25)$	$(0.8,0.19,0.1)$	$(0.74,0.2,0.1)$	$(0.6,0.3,0.2)$	$(0.5,0.3,0.1)$

Table 5. *q*-ROPF soft matrix for patient z_2.

	c_1	c_2	c_3	c_4	c_5
$Đ_1$	$(0.6,0.3,0.15)$	$(0.7,0.22,0.11)$	$(0.66,0.25,0.1)$	$(0.8,0.2,0.1)$	$(0.63,0.3,0.1)$
$Đ_2$	$(0.5,0.25,0.1)$	$(0.8,0.25,0.13)$	$(0.75,0.16,0.12)$	$(0.66,0.25,0.18)$	$(0.71,0.17,0.2)$
$Đ_3$	$(0.64,0.25,0.1)$	$(0.5,0.2,0.1)$	$(0.85,0.24,0.1)$	$(0.7,0.3,0.2)$	$(0.6,0.26,0.15)$
$Đ_4$	$(0.66,0.4,0.25)$	$(0.6,0.3,0.18)$	$(0.76,0.2,0.1)$	$(0.68,0.25,0.15)$	$(0.55,0.25,0.15)$
$Đ_5$	$(0.7,0.5,0.2)$	$(0.75,0.2,0.18)$	$(0.67,0.25,0.15)$	$(0.6,0.3,0.2)$	$(0.7,0.3,0.1)$

Table 6. q-ROPF soft matrix for patient z_3.

	c_1	c_2	c_3	c_4	c_5
Đ$_1$	$(0.7, 0.25, 0.15)$	$(0.55, 0.33, 0.11)$	$(0.76, 0.2, 0.1)$	$(0.8, 0.19, 0.1)$	$(0.65, 0.22, 0.1)$
Đ$_2$	$(0.65, 0.22, 0.11)$	$(0.8, 0.3, 0.1)$	$(0.8, 0.2, 0.18)$	$(0.5, 0.15, 0.1)$	$(0.9, 0.1, 0.1)$
Đ$_3$	$(0.87, 0.23, 0.1)$	$(0.6, 0.25, 0.15)$	$(0.7, 0.2, 0.1)$	$(0.76, 0.21, 0.11)$	$(0.76, 0.23, 0.11)$
Đ$_4$	$(0.8, 0.3, 0.2)$	$(0.78, 0.13, 0.1)$	$(0.75, 0.25, 0.15)$	$(0.4, 0.2, 0.1)$	$(0.66, 0.3, 0.2)$
Đ$_5$	$(0.75, 0.4, 0.2)$	$(0.65, 0.3, 0.1)$	$(0.6, 0.1, 0.1)$	$(0.6, 0.23, 0.1)$	$(0.66, 0.3, 0.2)$

Table 7. q-ROPF soft matrix for patient z_4.

	c_1	c_2	c_3	c_4	c_5
Đ$_1$	$(0.76, 0.3, 0.1)$	$(0.85, 0.22, 0.11)$	$(0.84, 0.2, 0.1)$	$(0.78, 0.3, 0.1)$	$(0.65, 0.26, 0.1)$
Đ$_2$	$(0.82, 0.14, 0.11)$	$(0.78, 0.18, 0.1)$	$(0.6, 0.12, 0.11)$	$(0.73, 0.17, 0.11)$	$(0.9, 0.3, 0.1)$
Đ$_3$	$(0.72, 0.22, 0.1)$	$(0.83, 0.25, 0.1)$	$(0.84, 0.13, 0.11)$	$(0.72, 0.22, 0.11)$	$(0.77, 0.2, 0.1)$
Đ$_4$	$(0.6, 0.27, 0.16)$	$(0.6, 0.3, 0.2)$	$(0.7, 0.3, 0.2)$	$(0.83, 0.13, 0.11)$	$(0.6, 0.25, 0.15)$
Đ$_5$	$(0.66, 0.23, 0.11)$	$(0.80, 0.22, 0.12)$	$(0.77, 0.25, 0.15)$	$(0.70, 0.17, 0.12)$	$(0.5, 0.3, 0.2)$

Step 2. Normalization is not necessary because all the parameters are similar.

Step 3. To aggregate the $q-ROPFS_fWA$ operator for each alternative, so we get

$$Ʊ_1 = (0.7590, 0.2206, 0.1198)$$

$$Ʊ_2 = (0.6772, 0.2621, 0.1429)$$

$$Ʊ_3 = (0.7361, 0.2172, 0.1210)$$

$$Ʊ_4 = (0.7557, 0.2220, 0.1204).$$

Step 4. To calculate the score value.

$$S(Ʊ_1) = 0.5120$$

$$S(Ʊ_2) = 0.3539$$

$$S(Ʊ_3) = 0.4681$$

$$S(Ʊ_4) = 0.5052.$$

Step 5. At the end arrange the score value to choose the best option.

$$S(Ʊ_1) > S(Ʊ_4) > S(Ʊ_3) > S(Ʊ_2).$$

Figure 1 shows the ranking order of alternatives of $q-ROPFS_fWA$ operator.

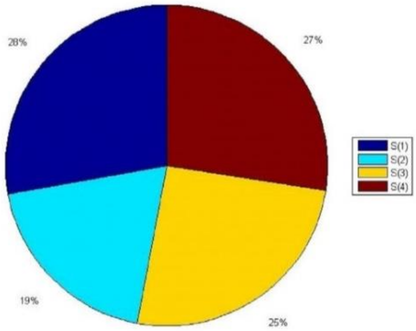

Figure 1. The ranking order of alternatives of $q-ROPFS_fWA$ operator.

So, from the above analysis, it is observed that patient z_1 is more illness.
By q-ROPF soft ordered weight averaging operator:
Step 1. Construct a decision matrix $M=\left[\tilde{N}_{\ddot{e}_{ij}}\right]_{m\times n}$ expressed in q-ROPF soft numbers, which are given in Table 8–11, respectively.

Table 8. q-ROPF soft matrix for patient z_1.

	c_1	c_2	c_3	c_4	c_5
Đ$_1$	$(0.8,0.22,0.11)$	$(0.85,0.12,0.11)$	$(0.88,0.22,0.11)$	$(0.86,0.2,0.1)$	$(0.86,0.2,0.1)$
Đ$_2$	$(0.78,0.18,0.1)$	$(0.8,0.19,0.1)$	$(0.84,0.12,0.11)$	$(0.81,0.18,0.11)$	$(0.79,0.2,0.1)$
Đ$_3$	$(0.77,0.2,0.1)$	$(0.77,0.2,0.15)$	$(0.75,0.25,0.1)$	$(0.75,0.15,0.1)$	$(0.74,0.4,0.14)$
Đ$_4$	$(0.71,0.25,0.1)$	$(0.75,0.25,0.15)$	$(0.74,0.2,0.1)$	$(0.7,0.25,0.15)$	$(0.65,0.16,0.11)$
Đ$_5$	$(0.7,0.35,0.25)$	$(0.7,0.18,0.11)$	$(0.7,0.3,0.15)$	$(0.6,0.3,0.2)$	$(0.5,0.3,0.1)$

Table 9. q-ROPF soft matrix for patient z_2.

	c_1	c_2	c_3	c_4	c_5
Đ$_1$	$(0.8,0.22,0.11)$	$(0.85,0.12,0.11)$	$(0.88,0.22,0.11)$	$(0.86,0.2,0.1)$	$(0.86,0.2,0.1)$
Đ$_2$	$(0.78,0.18,0.1)$	$(0.8,0.19,0.1)$	$(0.84,0.12,0.11)$	$(0.81,0.18,0.11)$	$(0.79,0.2,0.1)$
Đ$_3$	$(0.77,0.2,0.1)$	$(0.77,0.2,0.15)$	$(0.75,0.25,0.1)$	$(0.75,0.15,0.1)$	$(0.74,0.4,0.14)$
Đ$_4$	$(0.71,0.25,0.1)$	$(0.75,0.25,0.15)$	$(0.74,0.2,0.1)$	$(0.7,0.25,0.15)$	$(0.65,0.16,0.11)$
Đ$_5$	$(0.7,0.35,0.25)$	$(0.7,0.18,0.11)$	$(0.7,0.3,0.15)$	$(0.6,0.3,0.2)$	$(0.5,0.3,0.1)$

Table 10. q-ROPF soft matrix for patient z_3.

	c_1	c_2	c_3	c_4	c_5
Đ$_1$	$(0.87,0.23,0.1)$	$(0.8,0.3,0.1)$	$(0.8,0.2,0.18)$	$(0.8,0.19,0.1)$	$(0.9,0.1,0.1)$
Đ$_2$	$(0.8,0.3,0.2)$	$(0.78,0.13,0.1)$	$(0.76,0.2,0.1)$	$(0.76,0.21,0.11)$	$(0.76,0.23,0.11)$
Đ$_3$	$(0.75,0.4,0.2)$	$(0.65,0.3,0.1)$	$(0.75,0.25,0.15)$	$(0.6,0.23,0.1)$	$(0.66,0.3,0.2)$
Đ$_4$	$(0.7,0.25,0.15)$	$(0.6,0.25,0.15)$	$(0.7,0.2,0.1)$	$(0.5,0.15,0.1)$	$(0.65,0.22,0.1)$
Đ$_5$	$(0.65,0.22,0.11)$	$(0.55,0.33,0.11)$	$(0.6,0.1,0.1)$	$(0.4,0.2,0.1)$	$(0.55,0.15,0.1)$

Table 11. q-ROPF soft matrix for patient z_4.

	c_1	c_2	c_3	c_4	c_5
$Đ_1$	$(0.82, 0.14, 0.11)$	$(0.85, 0.22, 0.11)$	$(0.84, 0.2, 0.1)$	$(0.83, 0.13, 0.11)$	$(0.9, 0.3, 0.1)$
$Đ_2$	$(0.76, 0.3, 0.1)$	$(0.83, 0.25, 0.1)$	$(0.80, 0.13, 0.11)$	$(0.78, 0.3, 0.1)$	$(0.77, 0.2, 0.1)$
$Đ_3$	$(0.72, 0.22, 0.1)$	$(0.80, 0.22, 0.12)$	$(0.77, 0.25, 0.15)$	$(0.73, 0.17, 0.11)$	$(0.65, 0.26, 0.1)$
$Đ_4$	$(0.66, 0.23, 0.11)$	$(0.78, 0.18, 0.1)$	$(0.7, 0.3, 0.2)$	$(0.72, 0.22, 0.11)$	$(0.6, 0.25, 0.15)$
$Đ_5$	$(0.6, 0.27, 0.16)$	$(0.6, 0.3, 0.2)$	$(0.6, 0.12, 0.11)$	$(0.70, 0.17, 0.12)$	$(0.5, 0.3, 0.2)$

Step 2. Normalization is not necessary because all the parameters are similar.
Step 3. To aggregate the $q-ROPFS_f OWA$ operator for each alternative, so we get

$$Ʊ_1 = (0.7597, 0.2172, 0.1186)$$
$$Ʊ_2 = (0.6698, 0.2648, 0.1421)$$
$$Ʊ_3 = (0.7295, 0.2176, 0.1201)$$
$$Ʊ_4 = (0.7473, 0.2277, 0.1201).$$

Step 4. To calculate the score value.

$$S(Ʊ_1) = 0.5140$$
$$S(Ʊ_2) = 0.3413$$
$$S(Ʊ_3) = 0.4557$$
$$S(Ʊ_4) = 0.4878.$$

Step 5. At the end arrange the score value to choose the best option.

$$S(Ʊ_1) > S(Ʊ_4) > S(Ʊ_3) > S(Ʊ_2).$$

Figure 2 shows the ranking order of alternatives of $q-ROPFS_f OWA$ operator.

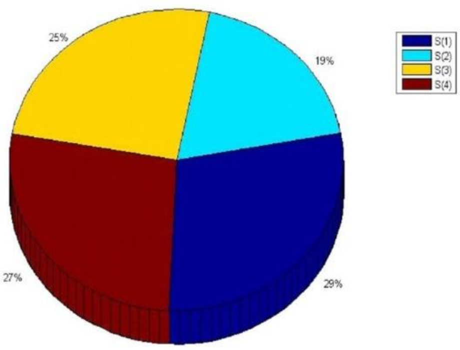

Figure 2. The ranking order of alternatives of $q-ROPFS_f OWA$ operator.

So, from the above analysis, it is observed that patient z_1 is more illness.

6. Comparative analysis

In this section we compare the result of our proposed model with the existing methods based on different operators, to show superiority and influence. In the existing method of various operators (see [15,24,26–28]) we handle the DM problem with the help of membership and non-membership degree with attributes, but it cannot handle the situation when the expert's judgment is of a type like yes, abstinence, no and rejection, because there is no information about the neutral degree. So, the logic behind our proposed model is that they have the capability to handle the situations with more generality than the existing concepts, with positive, neutral, and negative degrees $(0 \le (\mu)^q + (\eta)^q + (\nu)^q \le 1)$ with parameterization tools, which is more generalized than the previous concept. Based on q-ROPF soft weighted averaging operator and q-ROPF soft order weighted averaging operator we construct a decision matrix $M = \left[\widetilde{N}_{\ddot{e}_{ij}}\right]_{m\times n}$ expressed in q-ROPF soft numbers, which are given in Table 4–11 than aggregated this decision matrix with weight vector $\omega = (0.16, 0.26, 0.15, 0.20, 0.23)^T$ and their corresponding results for each candidate are given in Table 12 and also Figure 3 shows the graphical representation of proposed operators with existing operators.

Table 12. Comparison analysis with existing operators.

Methods	z_1	z_2	z_3	z_4	Ranking
IFWA [26]	0.220169	0.211734	0.332146	0.270078	$Ʊ_3>Ʊ_4>Ʊ_1>Ʊ_2$
IFOWA [26]	0.230663	0.217267	0.329021	0.254617	$Ʊ_3>Ʊ_4>Ʊ_1>Ʊ_2$
IFHA [26]	0.232465	0.225013	0.32471	0.240629	$Ʊ_3>Ʊ_4>Ʊ_1>Ʊ_2$
IFS_fWA [27]	0.516859	0.548324	0.604673	0.590214	$Ʊ_3>Ʊ_4>Ʊ_2>Ʊ_1$
PFS_fWA [28]	0.522097	0.565965	0.621904	0.590214	$Ʊ_3>Ʊ_4>Ʊ_2>Ʊ_1$
PFS_fOWA [28]	0.532526	0.575719	0.621094	0.593809	$Ʊ_3>Ʊ_4>Ʊ_2>Ʊ_1$
PFS_fHA [28]	−0.39452	−0.37378	−0.34634	−0.33975	$Ʊ_3>Ʊ_4>Ʊ_2>Ʊ_1$
q-ROFWA [15]	0.81579	0.79845	0.147617	0.099586	$Ʊ_3>Ʊ_4>Ʊ_1>Ʊ_2$
$q-$ROFS_fWA [24]	0.414877	0.46537	0.522354	0.484856	$Ʊ_3>Ʊ_4>Ʊ_2>Ʊ_1$
$q-$ROFS_fOWA [24]	0.426939	0.475573	0.521928	0.483572	$Ʊ_3>Ʊ_4>Ʊ_2>Ʊ_1$
$q-$ROFS_fHA [24]	−0.29764	−0.27858	−0.2507	−0.5753	$Ʊ_3>Ʊ_4>Ʊ_2>Ʊ_1$
$q-$ROPFS_fWA	0.5120	0.3539	0.4681	0.5052	$Ʊ_1>Ʊ_4>Ʊ_3>Ʊ_2$
$q-$ROPFS_fOWA	0.5140	0.3413	0.4557	0.4878	$Ʊ_1>Ʊ_4>Ʊ_3>Ʊ_2$

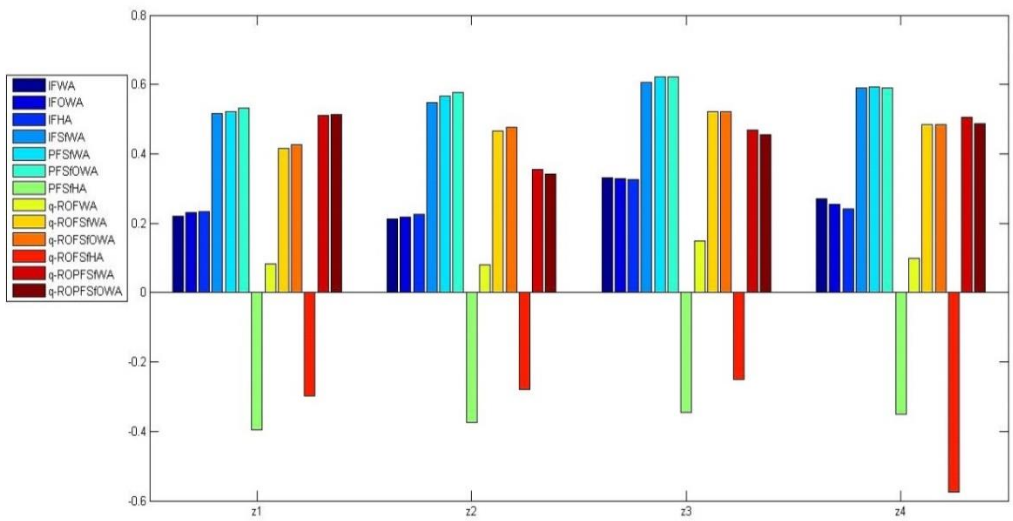

Figure 3. Graphical representation of comparison analysis.

7. Conclusions

In this article we present the hybrid of picture fuzzy set and q-rung orthopair fuzzy soft set, to get the generalized structure of q-rung orthopair fuzzy soft set called q-rung orthopair picture fuzzy soft set $q-ROPFS_fS$, which is characterized by positive, neutral and negative membership degree by affixing a parameterization tool to solve the uncertainties. The notion of $q-ROPFS_fS$ S covers the gap of neutral degree, in the existing concept of q-rung orthopair fuzzy soft set. The main contribution of this article is to investigate the basic operations and aggregation operators like q-ROPF soft weighted averaging operator and q-ROPF soft ordered weighted averaging operator under the environment of q-rung orthopair picture fuzzy soft set. Moreover, some fundamental properties like idempotency, boundedness, monotonicity, shift invariance, and homogeneity based on these operators are studied. Under the environment of q-ROPF soft set, we consider a biological problem (medical problem) and construct a stepwise algorithm for decision-making problem. Finally, we make a comparison analysis to compare the result of our proposed model with the existing methods, for show superiority and influence. The advantage of our proposed model is that they can handle the situations with more generality than an existing concept (q-rung orthopair fuzzy soft set), i.e., the existing concept, we deal the real-life problems with membership degree and non-membership degree ($0\leq(\mu)^q + (\eta)^q\leq 1$) with attributes but in the proposed method we handle the situations with positive, neutral and negative degree ($0\leq(\mu)^q + (\eta)^q + (\nu)^q \leq 1$) with parameterization tools, which is more generalized than the previous concept. This proposed work will be extended in various directions such as q-rung orthopair interval-valued picture fuzzy soft set, q-rung orthopair bi-polar picture fuzzy soft set, q-rung orthopair m-polar picture fuzzy soft set, q-rung orthopair cubic picture fuzzy soft set and q-rung orthopair neutrosophic fuzzy soft set, etc.

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Conflict of interest

The authors disclose no conflict of interests in publishing this paper.

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