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Research article

Evaluation of groups using cooperative game with fuzzy data envelopment analysis

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Abstract: Data Envelopment Analysis (DEA) is a prominent technique for evaluating the performance and ranking of a set of decision-making units (DMUs) that transform multiple inputs into multiple outputs. However, one of the challenges of the primary DEA models is facing imprecise data in real practical problems. To address this issue, fuzzy DEA have been proposed, which have been successfully applied in many real fields. On the other hand, in some real-world DEA applications, the primary objective of performance evaluation is the ranking of a group that consists of several DMUs that are typically under the control of a centralized management. In this paper, we try to use the theory of cooperative games and Shapley value method as a fair method to solve such games in order to rank groups in DEA. In this way, the resulting rank for groups is based on the average marginal shares of groups in different coalitions that are formed based on the theory of cooperative games. We applied the proposed method to rank groups of airlines considering fuzzy data. To the best of authors' knowledge, so far, no method has been presented in DEA literature for ranking groups in fuzzy environment and using game theory techniques.

Keywords: data envelopment analysis; ranking functions; fuzzy sets; cooperative game; shapley value
Mathematics Subject Classification: 91A12, 90C08, 90C70

1. Introduction

Data Envelopment Analysis (DEA) is an effective method that gives managers a tool to compare the performance of their firm against that of its competitors. Charnes, Cooper, and Rhodes [1] introduced this tool for the first time. The different DEA models can be used to analyze DMUs with many inputs and outputs, identifying efficient and inefficient units and trying to improve the latter.

Categories and groupings are used to reduce heterogeneity in evaluating units. The division of units based on the type of ownership such as private, government, semi-private, etc., division in terms of size and classification based on geographical regions, can provide the basis for creating different groups. For example, we can pay attention to regional and provincial divisions, which in regional divisions, several adjacent provinces are considered as a region and usually act under the supervision of a centralized decision maker. It can also be mentioned the competition of chain store agencies across a country that, in order to obtain a better result in performance, different regions operate under the supervision of a regional official and the existence of competition between different regions as well as between the units of provinces within each area is evident. Besides, every regional manager is trying to adopt a higher rank. This is true for all governmental and non-governmental organizations and banks that have different branches in different provinces. While there is competition between the branches within their province, the managers of the provinces are trying to improve the rank of the provincial group under their management as a group. Attempting to attract national and budget credits by the representatives of a province in competition with other provinces of a country in the first stage, and then trying to attract credits for the area represented by each representative within the province, at the political level is another example of the need to the ranking of groups.

Some authors have previously conducted studies on groups in DEA. In order to compare groups of DMUs where the internal inefficiency of the units may also be determined, Camanho and Dyson [2] employed DEA and the Malmquist productivity index. In another work, Cook and Zhu [3] proposed a goal programming model to get a set of common weights between groups, in such a way that these weights reduce the largest deviations between intergroup scores from their ideal level. The rank of the groups was not taken into account. Meta-frontier was used by O'Donnell et al. [4] to compare the technical effectiveness of businesses that can be classified into many groups. A DMU's distance from its own frontier and its own group frontier's distance from the meta-frontier were used to split the performance measured by the meta-frontier into two portions.

In the meta-frontier analysis, the focus is on obtaining individual efficiency, and the units are compared on only two levels (group boundary or inner layer, and meta boundary or outer layer). also, the performance score of each group is defined based on the performance of its members, and the ranking of the groups is not discussed. The score of a group is a function of the individual scores of its members relative to a general frontier.

That is, if the grouping is changed, again the new group efficiency score will be obtained only as a function of the same previously calculated efficiency scores of the individual efficiencies relative to the meta-frontier, and it seems that the new grouping is not much effective in the process of calculating the performance score. Bagherzadeh Valami [5] uses production technology to propose a model to evaluate the performance of a group in which the efficiency of each DMU is defined as the distance of that DMU to the group frontier. This model introduces the efficiency of a group as the geometric mean of the efficiency of all DMUs based on the production possibility set (PPS) constructed by the members of that group. The higher the geometric mean efficiency of all DMUs in a group, the higher the efficiency of that group. Bagherzadeh Valami's [5] method is highly dependent on the coordinate values of other units, and if there are units that are far away from the border of that group or are super-efficient, it is possible that the score of a group will be greatly affected, and due to the use of the geometric mean, this change will be more.

Since in some comparative performance evaluation frameworks, there are hidden or obvious competitions between units, it is possible to get help from game theory to make the evaluation of units more realistic. Also, in DEA, since the efficiency score of all efficient units is 1, it is not possible to distinguish between efficient units, so ranking units in competitive conditions using game theory seems important. In such a framework, the players of the game or the DMU can act together in competitive and non-competitive conditions. Cooperative games have an important contribution in such combined studies. Beside Nash bargaining method [6], as a cost-effective solution to the cooperative game, Shapley value has the largest contribution in the methods of obtaining the solution of cooperative games. Considering the competitive nature of the DEA performance evaluation process as well as some problems of cross efficient and super-efficient models, attempts were made to relate DEA models in game theory; Sugiyama et al. [7] and Wu et al. [8] used the Nash bargaining game to find a joint weight vector and an efficiency score. Li and Yang [9] prioritized the importance of input and output variables using Shapley value as a cooperative game solution. Wu et al. [10] using Shapley value, introduced a method to select the best partner. Wu et al. [11] used Nucleolus's solution and Shapley value in collaboration to determine the final weights in cross-efficiency. Nakabayashi and Tone [12] used the Shapley and Nucleolus solution for the fair allocation of a prize between the players of the game. Du et al. [13] used the bargaining game to evaluate the performance of two-stage networks. Asadi-Rahmati et al. [14] used a common framework to rank efficient units using Shapley value. Zhou et al. [15] used the bargaining game to analyze the efficiency of the two-stage centralized model. Li et al. [16] ranked efficient units using a cooperative game in a common framework.

Ignoring the uncertainties in measuring and collecting data, may have a negative effect on the evaluation results. One of the important ways of investigating the uncertainty in DEA models is to use a fuzzy approach based on fuzzy logic. The tolerance approach, the alpha-cut approach, the possibility approach, the ranking approach, the fuzzy arithmetic approach and multi-objective planning approach are the six main approaches for solving fuzzy DEA models [17]. On the other hand, traditional DEA models such as CCR and BCC do not deal with fuzzy data; they consider all inputs and outputs deterministic, while it is not always true in the real world especially when a set of DMUs have missing, random or sequential superiority data. The concept of decision making in fuzzy environment proposed by Belman and Zadeh [18]. Sengupta [19] was the first one who applied the fuzzy set theory to DEA and used the principles of that theory to introduce the fuzzy nature of the objective functions and limitations of the common models of DEA. Based on the fuzzy set theory and its combination with DEA, some fuzzy DEA models were presented. Various studies have been conducted in relation to Fuzzy FDEA, some of which we mention in recent years. Hatami-Marbini et al. [20] generalized the alpha-cutting approach to evaluate a fuzzy two-stage series system in which all the outputs of the first stage are the only inputs of second stage. Indeed, they developed that alpha cutting approach to evaluate a system in which a part of the fuzzy outputs of the first stage could be removed from the system, and the rest of them along with the other fuzzy inputs are the fuzzy inputs of the second stage. Hatami-Marbini et al. [17] used a completely fuzzy method to evaluate the performance of decision-making units by using a multi-objective lexicography method. Saini et al. [21] used the Pythagorean fuzzy entropy CCR model to rank decision-making units. Tabatabaei et al. [22] presented a two-stage network model of DEA to evaluate the performance of decision-making units with inaccurate data in the presence of undesirable outputs in a multi-objective linear programming structure. Izadikhah et al. [23] designed a fuzzy stochastic DEA model based on auxiliary variables in the SBM to evaluate the performance of health care efficiency. On this purpose, they evaluated the performance of eleven hospitals. In another study, Wang et al. [24] presented a fully fuzzy DEA model with large datasets for the evaluation of urban circular economy. The model took the uncertainties of circular economy indicators into account and introduced fuzzy trigonometric numbers.

In the studies conducted in fuzzy DEA, the theory of cooperative games and Shapley value have not been used to rank groups with fuzzy data, which we discuss in this research. By examining the literature of the group, game theory and imprecise data in DEA, we find some research gaps. The existing methods for ranking groups have some shortcomings. They have not addressed the effect of one group on the boundaries of another PPS or the boundary of the global set. This is despite the fact that some existing methods in DEA use such a concept to determine the rank. Super efficiency methods are included in this category. Also, the combined literature of game theory and DEA has poverty in the ranking of groups. On the other hand, this gap can be seen in the ranking of groups with imprecise data in DEA texts covering imprecise data. Therefore, in this paper, in order to fill these research gaps, we are going to present a method for ranking groups with imprecise data using the concepts of cooperative game theory.

In our method, different sets (coalition) including different groups are made and the effect of each group in that set is measured, considering the largest possible set including all groups as the largest possible coalition, causes reaching to the meta-frontier. Therefore, the score obtained from the meta-frontier method for the members of each group can be considered as one of the $2^n - 1$ possible modes of calculating the performance score of a group. Since the effect of a group is calculated for the sets of production possibilities resulting from different frontiers, and because the Shapley value, which is the average of the marginal effects of each group in various coalitions, was used in the calculation of the ranking score of each group, therefore the value is not dependent on a specific value or frontier, and therefore the results are more reliable. Also, the proposed method is a method for ranking groups, and the ranking of a group will be considered as an entity that competes with other groups. In the proposed method, the effect of each group in other coalitions is evident.

The rest of this article is organized as follows. The second part deals with the prerequisites. The third part presents the proposed method. In the fourth part, a numerical example is provided to show the applicability of the model. Finally, the conclusion of the study and some suggestions for future research are given in the fifth part.

2. Preliminaries

2.1. Fuzzy set theory

Fuzzy numbers make up a special type of fuzzy set that is convex and normal with a unique support. A wide range of fuzzy numbers have been presented and used with different names and properties, but an important principle in the application of a fuzzy theory is its computational efficiency.

Definition 2.1. *Certain members in a fuzzy set whose membership degree is at least α $(\alpha \geq 0)$ form a set called α-cut $\tilde{A}$ defined as follows:*

$$A_\alpha = \{x \in X | \mu_{\tilde{A}}(x) \geq \alpha\}.$$

Definition 2.2. *Fuzzy set $\tilde{A}$ defined in terms of the universal set of real numbers is said to be fuzzy if its membership function has the following characteristics:*

a) $\tilde{A}$ is convex, i.e., $\forall x_1, x_2 \in R$, $\forall\ \lambda \in [0,1]$, $\mu_{\tilde{A}} \lambda(x_1) + (1 - \lambda x_2) \geq\ min\{\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)\}$

b) $\tilde{A}$ is normal, i.e., $\exists x \in R$, $\mu_{\tilde{A}}(x) = 1$
c) $\mu_{\tilde{A}}$ is continuous piecewise.

2.2. Triangular fuzzy numbers

Doing calculations with fuzzy numbers is very complicated because of their special structure. To facilitate the handling of these numbers, a specific set of them is used in calculations. Such special numbers are bell, triangular, trapezoidal, L-R triangular, L-R trapezoidal, and so on. In this regard, there are several methods to obtain a fuzzy membership function. The present study benefits from triangular fuzzy numbers due to their flexibility, ease of mathematical calculations, ease of interpretation and linear membership functions. A triangular fuzzy number can be represented by an ordered triplet (l, m, u), as in Figure 1, where l and u are the lower and upper bounds, respectively, and m is the middle value, and x is an element between l and u.

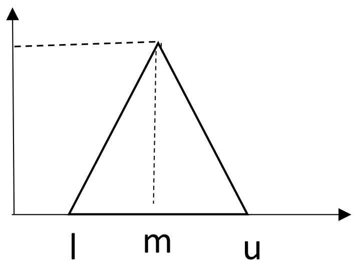

Figure 1. Triangular numbers.

Fuzzy number $\tilde{A}$=$\{a^L, a^M, a^U\}$ is called a triangular fuzzy number whenever its membership function $\mu_{\tilde{A}}(x)$:$R \to [0, 1]$ is as follows:

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x-a^L}{a^M-a^L} & a^L \le x \le a^M \\ \frac{a^U-x}{a^U-a^M} & a^M \le x \le a^U \\ 0 & otherwise, \end{cases}$$

where a^L and a^U represent the lower and upper bounds, and a^M is the mean value of the fuzzy number $\tilde{A}$.

Definition 2.3. *The fuzzy number $\tilde{A} = (a^L, a^M, a^U)$ is called non-negative if and only if $a^L \ge 0$.*

Definition 2.4. *The two triangular fuzzy numbers $\tilde{A} = (a^L, a^M, a^U)$ and $\tilde{B} = (b^L, b^M, b^U)$ are equal if a^L=b^L and a^M=b^M and a^U=b^U.*

Definition 2.5. *Suppose $\tilde{A} = (a^L, a^M, a^U)$ and $\tilde{B} = (b^L, b^M, b^U)$ are two non-negative triangular fuzzy numbers and $k \in R$. The operators on $\tilde{A}$ and $\tilde{B}$ are defined as follows:*

$$\begin{array}{ll} If\ k \ge 0 \to K\tilde{A}=(ka^L, ka^M, ka^U), & a^L \ge 0, b^L \ge 0, \\ If\ k \le 0 \to K\tilde{A} = (ka^U, k\,a^M, ka^L), & a^L \ge 0, b^L \ge 0, \\ \tilde{A}+\tilde{B} = (a^L+b^L, a^M+b^M, a^U+b^U), & a^L \ge 0, b^L \ge 0, \\ \tilde{A}\times\tilde{B} \cong (a^L\times b^L, a^M\times b^M, a^U\times b^U), & a^L \ge 0, b^L \ge 0, \\ \tilde{A} - \tilde{B} = (a^L-b^U, a^M-b^M, a^U-b^L), & a^L \ge 0, b^L \ge 0, \end{array}$$

$$\frac{\tilde{A}}{\tilde{B}} \cong \left(\frac{a^L}{b^U}, \frac{a^M}{b^M}, \frac{a^U}{b^L}\right), \qquad a^L \geq 0, b^L \geq 0.$$

The sum of the and subtraction of two triangular fuzzy numbers is a triangular fuzzy number, but their multiplication and division is not necessarily a triangular fuzzy number. For this reason, we assume that their multiplication and division are almost a triangular fuzzy number.

2.3. De-fuzzification of fuzzy numbers

In order to convert a fuzzy number into an exact value, there are various methods, such as the cutting method, the center of gravity method, the maximum membership function method, the ranking function method, etc. Since we will use the ranking function method in the following, we will describe it. Using ranking functions, fuzzy numbers are easily compared with each other and fuzzy inequalities are converted into absolute inequalities.

Ranking function is a function like $R : F(\mathbb{R}) \to \mathbb{R}$ that maps every fuzzy number to a real value. For both fuzzy numbers $\tilde{A}$ and $\tilde{B}$ we define:

$$\begin{aligned} &\tilde{A} \geq \tilde{B} && \textbf{iff } \mathbf{R}\left(\tilde{A}\right) \geq \mathbf{R}\left(\tilde{B}\right), \\ &\tilde{A} > \tilde{B} && \textbf{iff } \mathbf{R}\left(\tilde{A}\right) > \mathbf{R}\left(\tilde{B}\right), \\ &\tilde{A} \cong \tilde{B} && \textbf{iff } \mathbf{R}\left(\tilde{A}\right) = \mathbf{R}\left(\tilde{B}\right). \end{aligned} \tag{2.1}$$

Here we assume that R is an arbitrary linear ranking function. In this case, for every fuzzy number $\tilde{A}$ and $\tilde{B}$ in $F(\mathbb{R})$ and every fixed value $k \in \mathbb{R}$, we will have:

$$R(k\tilde{A} + \tilde{B}) = kR(\tilde{A}) + R(\tilde{B}). \tag{2.2}$$

Using the ranking function provided by Fortemps and Roubens [25] for a triangular fuzzy number $\widetilde{A} = (a^L, a^M, a^U)$, the desired ranking function is defined as follows:

$$\boldsymbol{R}\left(\tilde{A}\right) = \frac{1}{2} \int (\inf \widetilde{A}_\alpha + \text{ Sup}\, \tilde{A}_\alpha) d_\alpha \tag{2.3}$$

The above relationship can be considered as follows:

$$R\left(\tilde{A}\right) = \frac{1}{4}\left(a^L + 2a^M + a^U\right). \tag{2.4}$$

Therefore, for both fuzzy numbers, we will have $\widetilde{A} = (a^L, a^M, a^U)$ and $\widetilde{B} = (b^L, b^M, b^U)$

$$\begin{aligned} &\tilde{A} \geq \tilde{B} && \textbf{iff } \left(a^L + 2a^M + a^U\right) \geq \left(b^L + 2b^M + b^U\right), \\ &\tilde{A} > \tilde{B} && \textbf{iff } \left(a^L + 2a^M + a^U\right) > \left(b^L + 2b^M + b^U\right), \\ &\tilde{A} \cong \widetilde{B} && \textbf{iff } \left(a^L + 2a^M + a^U\right) = \left(b^L + 2b^M + b^U\right). \end{aligned} \tag{2.5}$$

2.4. A non-radial model to evaluate the performance of DMUs with fuzzy data

Barzegarinezhad et al. [26], proposed a model to evaluate DMUs with fuzzy data as follow:

Let set of n DMUs, DMU_j, $j \in \{1,\ldots,n\}$ with m triangular fuzzy input ($x_{ij}, i \in \{1,\ldots,m\}$) that produce s triangular fuzzy output (y_{rj} , $r \in \{1,\ldots,s\}$) which is indicated by $\widetilde{x}_{ij}=\left(x_{ij}^L, x_{ij}^M, x_{ij}^U\right)$ and $\widetilde{y}_{rj}=\left(y_{rj}^L, y_{rj}^M, r_{ij}^U\right)$ where $x_{ij}^L> 0$ and $y_{rj}^L> 0$. Suppose we want to improve the efficiency of DMU**p**, we do this by decreasing all inputs and increasing all outputs. In other words, we want:

$$\begin{cases} \widetilde{\theta}\, \tilde{x}_{ip} \;\le \tilde{x}_{ip} & i = 1,\ldots,m \\ \widetilde{\varphi}_{r\tilde{y}_{rp}} \;\ge \widetilde{y}_{rp} & r = 1,\ldots,s \end{cases} \tag{2.6}$$

where ($\widetilde{\theta}_i = \theta_i^L, \theta_i^M\ , \theta_i^U$) and ($\widetilde{\varphi}_r\ = \varphi_r^L, \varphi_r^M, \varphi_r^U$) and also $\begin{cases} \widetilde{\theta}_i\, \tilde{x}_{ip} \\ \widetilde{\varphi}_r\, \tilde{y}_{rp} \end{cases}$ belonging to the PPS

$$T_c^* = \left\{ (\widetilde{x}, \tilde{y})\,,\ \widetilde{x} \ge \sum_{j=1}^{n} \widetilde{\lambda}_j \tilde{x}_j,\ \widetilde{y} \le \sum_{j=1}^{n} \widetilde{\lambda}_j \tilde{y}_j, \widetilde{\lambda}_j \ge 0 \right\}.$$

According to Definition 2.5, the fuzzy inequalities (2.6) can be written as follows:

$$\begin{aligned} &\left(\theta_i^L x_{ip}^L, \theta_i^M x_{ip}^M, \theta_i^U\ \ x_{ip}^U\right) \le \left(x_{ip}^L, x_{ip}^M, x_{ip}^U\right), && i = 1,\ldots,m, \\ &\left(\varphi_r^L y_{rp}^L, \varphi_r^M y_{rp}^M, \varphi_r^U y_{rp}^U\right) \ge \left(y_{rp}^L, y_{rp}^M, y_{rp}^U\right), && r = 1,\ldots,s. \end{aligned} \tag{2.7}$$

By using the presented ranking function, the above fuzzy inequalities can be easily converted into the following absolute inequalities:

$$\begin{aligned} &\theta_i^L x_{ip}^L + 2\theta_i^M x_{ip}^M + \theta_i^U\ \ x_{ip}^U \le\ x_{ip}^L + 2x_{ip}^M + x_{ip}^U && i= 1,\ldots,m \\ &\varphi_r^L y_{rp}^L + 2\ \varphi_r^M y_{rp}^M + \varphi_r^U y_{rp}^U \ge y_{rp}^L + 2y_{rp}^M + y_{rp}^U, && r = 1,\ldots,s. \end{aligned} \tag{2.8}$$

On the other hand, under the following conditions, the relations (2.8) are always established.

$$\begin{aligned} &\theta_i^L \le 1, \theta_i^M\ \le 1, \theta_i^U \le 1, && i= 1,\ldots,m \\ &\varphi_r^L \ge 1, \varphi_r^M \ge 1, \varphi_r^U \ge 1, && r= 1,\ldots,s. \end{aligned} \tag{2.9}$$

Now, in order to check the performance of the DMUs, according to the mentioned contents and considering the R ranking function, they proposed the following model:

$$\begin{aligned} \min\ \ & \{\theta_1^L,\ldots,\theta_m^L,\ \ \theta_1^M,\ldots,\theta_m^M, \theta_1^U\ ,\ \ldots,\theta_m^U\} \\ \max\ \ & \{\varphi_1^L,\ldots,\varphi_s^L,\ \ \varphi_1^M,\ldots,\varphi_s^M,\ \varphi_1^U\ ,\ \ldots,\varphi_s^U\} \\ \text{s.t.}\ \ & \\ & \begin{pmatrix} \left(\theta_i^L x_{ip}^L, \theta_i^M x_{ip}^M, \theta_i^U\ \ x_{ip}^U\right) \\ \left(\varphi_r^L y_{rp}^L, \varphi_r^M y_{rp}^M, \varphi_r^U y_{rp}^U\right) \end{pmatrix} \in T_c^* \\ & \theta_i^L \le 1, \theta_i^M\ \le 1, \theta_i^U \le 1, \qquad i= 1,\ldots,m \\ & \varphi_r^L \ge 1, \varphi_r^M \ge 1, \varphi_r^U \ge 1, \qquad r= 1,\ldots,s. \end{aligned} \tag{2.10}$$

The model (2.10) is a completely fuzzy model. According to the definition of T_c^*, the model (2.10) becomes the following model:

$$\begin{array}{ll}
\min & \{\theta_1^L,\ldots,\theta_m^L,\theta_1^M,\ldots,\theta_m^M,\theta_1^U,\ldots,\theta_m^U\} \\
\max & \{\varphi_1^L,\ldots,\varphi_s^L,\varphi_1^M,\ldots,\varphi_s^M,\varphi_1^U,\ldots,\varphi_s^U\} \\
\text{s.t.} & \\
& \sum_{j=1}^{n}(\lambda_j^L x_{ij}^L,\lambda_j^M x_{ij}^M,\lambda_j^U x_{ij}^U) \le \theta_i^L x_{ip}^L,\theta_i^M x_{ip}^M,\theta_i^U x_{ip}^U, \quad i=1,\ldots m, \\
& \sum_{j=1}^{n}(\lambda_j^L y_{rj}^L,\lambda_j^M y_{rj}^M,\lambda_j^U y_{rj}^U) \ge \varphi_r^L y_{rp}^L,\varphi_r^M y_{rp}^M,\varphi_r^U y_{rp}^U, \quad r=,\ldots,s, \\
& \theta_I^L \le 1,\theta_I^M \le 1,\theta_I^U \le 1, \quad i=1,\ldots,m, \\
& \varphi_r^L \ge 1,\varphi_r^M \ge 1,\varphi_r^U \ge 1, \quad r=1,\ldots,s, \\
& \lambda_j^M-\lambda_j^L \ge 0,\ \lambda_j^U-\lambda_j^M \ge 0,\lambda_j^L \ge 0, \quad j=1,\ldots,n.
\end{array} \tag{2.11}$$

By using the ranking function R, the multi-objective programming problem (2.11) can be written as follows:

$$\begin{array}{ll}
\min & \tau=\dfrac{\frac{1}{4m}\sum_{i=1}^{m}(\theta_i^L+2\theta_i^M+\theta_i^U)}{\frac{1}{4s}\sum_{r=1}^{s}(\varphi_r^L+2\varphi_r^M+\varphi_r^U)} \\
\text{s.t.} & \\
& \sum_{j=1}^{n}(\lambda_j^L x_{ij}^L+2\lambda_j^M x_{ij}^M+\lambda_j^U x_{ij}^U) \le \theta_i^L x_{ip}^L+2\theta_i^M x_{ip}^M+\theta_i^U x_{ip}^U, \quad i=1,\ldots,m, \\
& \sum_{j=1}^{n}(\lambda_j^L y_{rj}^L+2\lambda_j^M y_{rj}^M+\lambda_j^U y_{rj}^U) \ge \varphi_r^L y_{rp}^L+2\varphi_r^M y_{rp}^M+\varphi_r^U y_{rp}^U, \quad r=1,\ldots,s, \\
& \theta_I^L \le 1,\theta_I^M \le 1,\theta_I^U \le 1, \quad i=1,\ldots,m \\
& \varphi_r^L \ge 1,\varphi_r^M \ge 1,\varphi_r^U \ge 1, \quad r=1,\ldots,s, \\
& \varepsilon \le t \le 1, \\
& \lambda_j^M-\lambda_j^L \ge 0,\ \lambda_j^U-\lambda_j^M \ge 0,\lambda_j^L \ge 0, \quad j=1,\ldots,n,
\end{array} \tag{2.12}$$

Using Charnes and Cooper transformations [27], the model (2.12) becomes the following linear model:

$$\frac{1}{4s}\sum_{i=1}^{m}(\varphi_r^l+2\varphi_r^m+\varphi_r^u)=\frac{1}{t}. \tag{2.13}$$

Suppose:

$$t\theta_i^L=\Theta_i^L,\quad t\theta_i^U=\Theta_i^U,\quad t\theta_i^M=\Theta_i^M,\quad t\varphi_r^L=\Phi_r^L,\quad t\varphi_r^M=\Phi_r^M,\quad t\varphi_i^U=\Phi_r^U,\quad \mu_j^L=t\lambda_j^L,\quad \mu_j^M=t\lambda_j^M,\quad \mu_j^U=t\lambda_j^U$$

Then we will have:

$$\begin{array}{ll}
\tau^*=\min & \tau=\frac{1}{4m}\sum_{i=1}^{m}(\Theta_i^L+2\,\Theta_i^M+\Theta_i^U) \\
\text{s.t.} & \\
& \frac{1}{4s}\sum_{r=1}^{s}(\Phi_r^L+2\,\Phi_r^M+\Phi_r^U)=1 \\
& \sum_{j=1}^{n}(\mu_j^L x_{ij}^L+2\mu_j^M x_{ij}^M+\mu_j^U x_{ij}^U) \le \Theta_i^L x_{ip}^L+2\,\Theta_i^M x_{ip}^M+\Theta_i^U x_{ip}^U, \quad i=1,\ldots m, \\
& \sum_{j=1}^{n}(\mu_j^L y_{rj}^L+2\mu_j^M y_{rj}^M+\mu_j^U y_{rj}^U) \ge \Phi_r^L y_{rP}^L+2\Phi_r^M y_{rp}^M+\Phi_r^U y_{rp}^U, \quad r=1,\ldots,s, \\
& 0 \le \Theta_i^L \le \Theta_i^M \le \Theta_i^U \le t \quad i=1,\ldots,m, \\
& \Phi_r^L \ge \Phi_r^M \ge \Phi_r^U \ge t, \quad r=1,\ldots,s, \\
& \varepsilon \le t \le 1, \\
& \mu_j^M-\mu_j^L \ge 0,\ \mu_j^U-\mu_j^M \ge 0,\ \mu_j^L \ge 0, \quad j=1,\ldots,n.
\end{array} \tag{2.14}$$

2.5. Game theory

Game theory is an interdisciplinary approach that examines the behavior between two or more actors or a group by using its own characteristics. The idea of game theory is to find optimal strategies, increase productivity or reduce costs, for actors. To choose the optimal strategy, some assumptions are presented as follows:

- Each actor is able to perform two or more actions or a set of actions.
- Any action of the actors can lead to the desired final state.
- Each actor's decision is determined based on the rules of the game and other actors.

According to the two assumptions of rationality and maximization of income, actors work to maximize their efficiency and productivity.

The main cases of using game theory are in the fields where the environment has conflicting conditions and one person's strategies have an impact on the players of other parties who have conflicting interests. When decision makers have conflicting interests, game theory is applicable. A game includes a set of players, sets of moves or strategies, and a specific outcome for each combination of strategies. Consider a competitive situation with a set of players; Players can compete in two ways:

(1) Non-cooperative game: players act their game individually and their achievement is personal. In this game, each player chooses her strategy, which makes her achieve the most.
(2) Cooperative game: The players are expected to form a coalition to boost their achievement. Cooperative games are usually characterized by the players in the game.

2.6. Cooperative game

In this game, it is expected that players will form alliances to increase their achievements. A cooperative game with players and a characteristic function $\langle N, C\langle S\rangle\rangle$ are represented. Let S be a subset of players who cooperate and form a coalition to increase their success. The value of the characteristic function m of the coalition $C(S)$, which is the achievement of the players of the coalition S, is the achievement that the members of the coalition S are confident that they will get from cooperating with each other and forming a coalition. For both subsets of players A and B that have no players in common, the characteristic function is a superset function; That is, it applies in the following condition:

$$C(A \cup B) \geq C(A) + C(B).$$

What the coalition players get should be divided fairly among them. Let $X = \{x_1, x_2, \ldots, x_n\}$ be the player's prize vector; That is, x_i is the prize of the i-th player.

The reward vector must have the following two conditions:

a) Group rationality $\quad C(N) = \sum_{i=1}^{n} x_1$.
b) Personal rationality $\quad x_i \geq C(\{I\})$.

The relationship A shows that the amount that each rational reward vector should assign to all players is equal to the amount that can be obtained by the super coalition including all players. Relationship B, that is, the i-th player prize must be at least as large as the value it can obtain on its own.

There are several ways to find the values of the bonus vector; Such as kernel method, constant set, nucleolus kernel and Shapley value [28], among which the solution of Shapley value is more understandable and more widely used for dividing the prize of the game. Also, the prize vector obtained from the Shapley solution is unique and fairly distributed among the players. In the Shapley value solution, the prize amount of the i-th player is calculated as follows:

$$x_i = \sum_{\forall S \subseteq N, i \notin S} \frac{(s-1)!(n-s)!}{n!}(C(S) - C(S \cup \{i\})).$$

3. Evaluation of groups using cooperative game with fuzzy data

Suppose that there is n DMUs, where each DMUj, $j \in 1, \ldots, n$ consumes m input $(x_{ij},\ i \in \{1, \ldots, m\})$ to produce s output $(y_{rj},\ r \in \{1, \ldots, s\})$ Consider what they do. Furthermore, suppose that these DMUs are divided into q group $A_1, A_2, \ldots, A_q$. The efficiency of DMU$_o$ with respect to the group frontier A_t is defined as follows with the condition that the PPS is made by the DMUs belonging to the group A_t $(t \in \{1, 2, \ldots, q\})$:

$$\begin{array}{lll}
\text{Min} & R_o^{A_t} = \dfrac{\frac{1}{4m}\sum_{i=1}^{m}(\theta_i^L + 2\theta_i^M + \theta_i^U)}{\frac{1}{4s}\sum_{r=1}^{s}(\varphi_r^L + 2\varphi_r^M + \varphi_r^U)} & \\
\text{s.t.} & & \\
& \sum_{j\in At}^{n}(\lambda_j^L x_{ij}^L + 2\lambda_j^M x_{ij}^M + \lambda_j^U x_{ij}^U) \le \theta_i^L x_{ip}^L + 2\theta_i^M x_{ip}^M + \theta_i^U x_{ip}^U, & i = 1, \ldots m, \\
& \sum_{j\in At}^{n}(\lambda_j^L y_{rj}^L + 2\lambda_j^M y_{rj}^M + \lambda_j^U y_{rj}^U) \ge \varphi_r^L y_{rp}^L + 2\varphi_r^M y_{rp}^M + \varphi_r^U y_{rp}^U, & r = 1, \ldots, s, \\
& \theta_i^L \le 1, \theta_i^M \le 1, \theta_i^U \le 1, & i = 1, \ldots, m, \\
& \varphi_r^L \ge 1, \varphi_r^M \ge 1, \varphi_r^U \ge 1, & r = 1, \ldots, s, \\
& \varepsilon \le t \le 1, & \\
& \lambda_j^M - \lambda_j^L \ge 0,\ \lambda_j^U - \lambda_j^M \ge 0, \lambda_j^L \ge 0, & j = 1, \ldots, n.
\end{array} \tag{3.1}$$

Using the transformations of Charnes and Cooper [27], the model becomes the following linear model:

$$\begin{array}{lll}
R_o^{A_t} = \min & \frac{1}{4m}\sum_{i=1}^{m}(\Theta_i^L + 2\,\Theta_i^M + \Theta_i^U) & \\
\text{s.t.} & & \\
& \sum_{j\in At}^{n}(\mu_j^L x_{ij}^L + 2\mu_j^M x_{ij}^M + \mu_j^U x_{ij}^U) \le \Theta_i^L x_{ip}^L + 2\,\Theta_i^M x_{ip}^M + \Theta_i^U x_{ip}^U, & i = 1, \ldots m, \\
& \sum_{j\in At}^{n}(\mu_j^L y_{rj}^L + 2\mu_j^M y_{rj}^M + \mu_j^U y_{rj}^U) \ge \Phi_r^L y_{rP}^L + 2\Phi_r^M y_{rp}^M + \Phi_r^U y_{rp}^U, & r = 1, \ldots, s, \\
& 0 \le \Theta_i^L \le \Theta_i^M \le \Theta_i^U \le t, & i = 1, \ldots, m \\
& \Phi_r^L \ge \Phi_r^M \ge \Phi_r^U \ge t, & r = 1, \ldots, s, \\
& \varepsilon \le t \le 1, & \\
& \mu_j^M - \mu_j^L \ge 0,\ \mu_j^U - \mu_j^M \ge 0, \mu_j^L \ge 0, & j = 1, \ldots, n.
\end{array} \tag{3.2}$$

The above model is always feasible and $\mathbf{0} \le R_o^{A_t *} \le \mathbf{1}$.

Therefore, $R_o^{A_t}$ efficient, assume that the PPS is constructed by DMUs belonging to the group A_t. If $R_o^{A_t} = 1$, DMU$_o$ is on the frontier of the PPS made by DMUs belonging to the group A_t, and is efficient compared to that group. If $R_o^{A_t} < 1$ it is, DMU$_o$ it is included in the set of production possibilities made by DMUs belonging to the group A_t and DMU$_o$ it is inefficient compared to the group A_t. If $R_o^{A_t} > 1$, DMU$_o$ it is outside the PPS made by DMUs belonging to the group A_t and DMU$_o$ is super-efficient compared to the group A_t. It may DMU$_o$ be efficient with the frontier of one group, but it may not be efficient with the frontier of another group or the frontier of the PPS made by all groups. Although the values of θ_i and φ_r may not be unique in the model (; but the value $R_o^{A_t}$ is always unique.

3.1. Group efficiency

The efficiency of group $A_t$is defined as the sum of the of the efficiency of all DMUs relative to the frontier of that group:

$$E(A_t) = \sum_{j=1}^{n} R_j^{A_t}.$$

Since the groups compete to obtain the best performance, we consider them as players in a cooperative game so that we can compare them.

Let G be the set of g group and the coalition S is a subset of s group. The characteristic function of the coalition, S, is defined $C(S) : 2^g \to \boldsymbol{R}$ as the sum of the of the efficiency of all DMUs; Assume that the PPS is made by all DMUs belonging to s of coalition group S:

$$C(S) = \sum_{j=1}^{n} R_j^S.$$

Since the value of R_j^s all of them is unique, their sum of the, that is, the value of the characteristic function, is also unique. If $A_k \notin s$, then the characteristic function $C(S \cup A_K)$ is defined as the sum of the of the efficiency of all DMUs, assume that the set of production possibilities is made by all DMUs s of the coalition member group S and the group A_k:

$$C(S \cup A_k) = \sum_{j=1}^{n} R_j^{S\cup\{A_k\}}.$$

With these definitions, the marginal effect of the group A_k in the coalition S, which is the amount of change created in the sum of the of the efficiency of DMUs by adding the group A_k to the coalition S, is defined as follows:

$$\mathrm{ME}^S(A_k) = C(S) - C(S \cup A_k).$$

The characteristic function $C(S)$ is the gain that members of coalition S expect to get from cooperating with each other. What is gained by the players participating in the alliance must be divided fairly among them. There are different solutions to divide this achievement, among which the Shapley value interpretation is easier and more understandable. The Shapley value to obtain the solution of this cooperative game, that is, the share of players from participating in coalition S is as follows:

$$\begin{aligned} \varphi_{A_k}(C) &= \sum_{\substack{S\subseteq\{A_1,A_2,\ldots,A_q\} \\ A_k\notin S}} \frac{(s)!(q-s-1)!}{q!}(C(S) - C(S \cup \{A_k\}) \\ &= \sum_{\substack{S\subseteq\{A_1,A_2,\ldots,A_q\} \\ A_k\notin S}} \frac{(s)!(q-s-1)!}{q!}(\mathrm{ME}^S(A_k)) \end{aligned} \tag{3.3}$$

where q is the total number of groups and s is the number of groups participating in coalition S. $\varphi_{A_k}(C)$ is the amount that the group A_k expect to gain by participating in coalition S in the cooperative game. The higher the Shapley value of a group, the better the group is ranked. Due to the uniqueness of the Shapley value, the ranking of groups with this method is unique. In addition, due to the use of cooperative play, this ranking is fair and acceptable to all groups. In the next section, using a real example, the applicability of the presented method is shown and the results are compared with the meta-frontier method.

4. Numerical examples

The role and importance of the aviation sector as one of the important pillars of the infrastructure sectors of any country in the mobility and dynamism and growth of society using new technologies in providing desirable services to users is not hidden from anyone. In this regard, the performance of airlines as part of this structure, which is responsible for moving passengers and cargo, is important. In this paper, to check the applicability of the proposed method in ranking groups, we have tried to evaluate Iranian airlines.

Iran is a country with an area of 1648195 $(km)^2$ located in the Middle East in Asia. The Iranian Aviation Organization dates back to 1946. According to the latest aviation statistics book published annually by the Iran Civil Aviation Organization (`https://www.cao.ir/statistical-yearbook`), there are currently 17 airlines operating in Iran. Mahmoudi et al. [29] presented a classification of the inputs and outputs into 6 categories used in the evaluation of the aviation system in DEA framework. As mentioned in [30], the financial indicators that fall into the categories of capital inputs and financial outputs do not exist in the statistical yearbooks of the Iranian Aviation Organization (`https://www.cao.ir/statistical-yearbook`). Also, since we did not find information about the categories of Environmental and energy inputs and Environmental and safety outputs by the airlines under study in these books, we considered the indicators related to the other categories. Among the various indicators, previous studies [30–32] have considered 'The number of employees', 'Available seat-kilometer', 'Available ton-kilometer' and 'Fleet seat' as inputs, and 'Number of flights', 'Passenger-kilometer performed' and 'Ton-kilometer performed' as outputs. Because the number of airlines is not much compared to the number of indicators, we inevitably reduced the other two indicators. For this purpose, among the inputs, we kept two indicators (Available seat-kilometer and Available ton-kilometer) that were conceptually more related to the outputs (Number of flights, Passenger-kilometer performed and Ton-kilometer performed) and eliminated the other two inputs. See Table 1, for a briefly explanation of the variables. Table 2, shows the data of 17 airlines according to the mentioned indicators. On the other hand, in order to avoid the arbitrary classification of companies to create different groups for the purpose of ranking in this paper, we applied the K-means clustering method as one of the most common clustering methods used in the field of data mining and machine learning. Previously, this method and other clustering methods have been used in applied DEA studies. Classification of the units can be seen in the second column of Table 2.

Table 1. Variables of inputs and outputs.

		Variable	Units	Descriptions
Inputs	I1	Available seat-kilometer	Thousands seat-km	Sum of the products obtained by multiplying the number of passenger seats available for sale on each flight by the flight distance.
	I2	Available ton-kilometer	Thousands ton-km	Sum of the products obtained by multiplying the number of tons available for the carriage of revenue load on each flight by the flight distance.
Outputs	O1	Number of flights	Flight	Number of performed flights
	O2	Passenger-kilometer performed	Thousands	Sum of products obtained by multiplying the number of revenue passengers carried on each flight by the flight distance
	O3	Ton-kilometer performed	Thousands ton-km	Multiplication of carried weight (ton) in every origin and destination of flight by the distance between the same origin and destination.

Table 2. Fuzzy input and output values of DMUs.

Units	Group	Number of seats supplied (thousands)	Ton kilometers supplied (thousands)	Number of flights	Passenger kilometers transported (thousands)	Ton kilometers transported (thousands)
Iran Air	A	1410597,1567331,1724064	163488.6,181654,199819.4	25008,27787,30565	880677.9,978531,1076384.1	90221.4,100246,110270.6
Iranairtour	A	1288778,1431976,1575173	121286.7,134763,148239.3	9888.3,10987,12085	949612.5,1055125,1160637	83573.1,92859,102144.9
Aseman	A	1583775.9,1759751,1935726.1	163822.5,182025,200227.5	9403,10448,11492	769634.1,855149,940663.9	70037.1,77819,85.600.9
Zagros	A	239011.2,265568,292124.8	20601.9,22891,25180.1	1959,2177,2394	161822.7,179803,197783.3	13434.3,14927,16419.7
Mahan	A	407883.6,453204,498524.4	40614.3,45127,49639.7	6211,6902,7592	284774,316416,348057	25126.2,27918,30709.8
Pouyaair	B	1458639,1620710,1782781	489161.7,543513,597864.3	11327,12586,13844	919920.6,1022134,1124347	88545.6,98348,108222.4
Saha	B	23776.2,26418,29059.8	58126.5,64585,71043.5	3194,3549,3903	16369.2,18188,20006.8	333938.7,371043,408147.3
Meraj	B	558436.5,620485,682533.5	58126.5,64585,71043.5	5090,5656,6221	372951.9,414391,455830.1	32959.8,36622,40284.2
Fly persia	B	1076069.7,1195633,1315196.3	113073.3,125637,138200.7	8577,9530,10483	697069.8,774522,851974.2	61938.9,68821,75703.1
Taban	C	52424.1,58249,64037.9	27902.7,31003,34103.3	1091.7,1213,1334.3	15917.4,17686,19454.6	9838.8,10932,12025.2
Sepahran	C	136478.7,151643,166807.3	14650.2,16278,17905.8	1556,1729,1901	94287.6,104764,115240.4	7020,7800,8580
Kish Air	C	721219.5,801355,881490.5	88598.7,98443,108287.3	6891,7657,8422	606511.8,673902,741292.2	57498.3,63887,70275.7
Karoon	C	1053271.8,1170302,1287332.2	106762.5,118625,130487.5	9216,10241,11265	648248.4,720276,792303.6	58041,64490,70939
Varesh	C	953395.2,1059328,1165260.8	81690.3,90767,99843.7	8802,9780,1075	642154.5,713505,784855.5	57006,63340,69674
Ata	D	1203020.1,1336689,1470357.9	117845.1,130939,144032.9	16740,18601,20461	837736.2,930818,1023899.8	74043.9,82271,90498.1
Gheshm	D	446939.1,496599,546258.9	47838.6,53154,58469.4	3313,3682,4050	291708,324120,356532	39928.5,44365,48801.45
Kaspian	D	1037706.3,1153007,1268307.7	103478.4,114976,126437.6	2778,3087,3395	924264,1026960,1129656	84106.8,93452,102797.2

Table 3 shows the efficiency values of all decision-making units in all possible coalitions.

Table 3. The efficiency of the coalition of groups.

Units \ Coalition	A	B	C	D	A,B	A,C	A,D	B,C	B,D	C,D	A,B,C	A,B,D	A,C,D	B,C,D	A,B,C,D
Iran Air	1.00	0.46	0.31	17.0	1.41	1.00	1.00	0.94	0.60	1.50	1.00	1.00	1.00	0.94	1
Iranairtour	1.00	1.00	0.26	1.07	1.00	0.89	1.00	0.85	0.87	0.89	0.85	1.00	0.89	0.85	0.8506
Aseman	0.82	1.47	0.28	2.04	1.00	0.79	0.82	0.53	1.49	0.79	0.53	0.54	0.79	0.53	0.525
Zagros	0.25	0.10	0.06	0.33	0.56	0.24	0.25	0.09	0.10	0.24	0.09	0.10	0.24	0.09	0.088
Mahan	0.40	0.05	0.08	0.53	0.10	0.37	0.40	0.04	0.04	0.37	0.04	0.04	0.37	0.04	0.036
Pouyaair	0.50	0.10	0.22	0.93	0.04	0.53	1.03	0.09	0.10	0.54	0.09	0.10	0.53	0.09	0.0932
Saha	0.00	1.00	0.43	1.47	0.10	1.72	1.33	0.81	0.88	1.72	0.81	0.84	1.72	0.81	0.8058
Meraj	2.71	0.86	0.37	0.84	1.00	12.0	1.01	0.80	0.83	8.24	0.80	0.83	12.00	0.80	0.8009
Fly persia	0.77	0.56	0.31	0.86	0.84	0.75	0.77	0.50	0.52	0.75	0.50	0.51	0.75	0.50	0.5019
Taban	0.46	0.26	0.08	0.95	0.55	1.32	0.79	0.23	0.24	1.57	0.23	0.24	1.32	0.23	0.2255
Sepahran	1.03	0.12	0.08	1.02	0.25	0.95	18.0	0.84	4.29	0.95	0.84	4.29	0.95	0.84	0.8441
Kish Air	33.0	0.77	0.27	2.03	0.42	1.00	33.0	1.00	0.70	1.00	1.00	3.68	1.00	1.00	1
Karoon	0.89	3.33	0.34	2.53	0.76	0.87	0.89	0.66	7.37	0.87	0.66	0.68	0.87	0.66	0.6634
Varesh	0.83	0.83	0.30	0.93	0.70	0.80	0.83	0.67	0.69	0.80	0.67	0.69	0.80	0.67	0.6705
Ata	0.82	0.74	0.26	1.30	0.74	0.80	0.82	0.55	0.57	0.80	0.55	0.57	0.80	0.55	0.553
Gheshm	0.65	0.49	0.17	0.81	0.60	0.62	0.65	0.43	0.45	0.62	0.43	0.45	0.62	0.43	0.4295
Kaspian	1.70	1.07	0.27	1.00	0.47	1.0	1.25	1.00	1.07	1.00	1.00	1.26	1.00	1.00	1

In Table 4, using the results obtained from Table 3, we obtain the sum efficiency of DMUs in all possible coalitions.

Table 4. The sum of the efficiency of all decision-making units.

Possible coalitions of groups	Sum of the efficiency
$\{A\}$	46.8356
$\{B\}$	13.2082
$\{C\}$	4.076
$\{D\}$	35.6426
$\{A, B\}$	10.5481 25.6494
$\{A, D\}$	63.8616
$\{B, C\}$	10.031
$\{B, D\}$	20.804
$\{C, D\}$	22.643
$\{A, B, C\}$	10.0901
$\{A, B, D\}$	16.814
$\{A, C, D\}$	25.6467
$\{B, C, D\}$	10.0283
$\{A, B, C, D\}$	10.0874

The presented method is used to calculate the marginal effects of four groups in different coalitions and is presented in Table 5. The second, third, fourth and fifth columns of the Table 5 respectively show the marginal effect of groups A, B, C, and D in different coalitions. As an explanation, the marginal effect of adding group C to the coalition $\{A\}, \{B\}, \{C\}$ which is given in the second row and the fourth column and the third row and the fourth column and the fourth row and the fourth column of Table 5, It is obtained as follows:

$$\mathrm{ME}^{(A)}\,(C){=}C\,(A)-C\,(A\cup C) = 46.8356 - 25.6494 = 21.1862$$
$$\mathrm{ME}^{(B)}\,(C){=}C\,(B)-C\,(B\cup C) = 13.2082{-}10.031{=}\ 3.1772$$
$$\mathrm{ME}^{(C)}\,(C){=}C\,(C)-C\,(C\cup C) = 4.076{-}4.076{=}\ 0.$$

Table 5. Marginal effect of groups in different coalitions.

Possible coalitions of groups	A	B	C	D
$\{A\}$	0	36.2875	21.1862	17.026
$\{B\}$	2.6601	0	3.1772	7.5958
$\{C\}$	21.5734	5.955	0	18.567
$\{D\}$	28.219	14.8386	12.9996	0
$\{A,B\}$	0	0	0.458	6.2659
$\{A,C\}$	0	15.5593	0	0.0027
$\{A,D\}$	0	47.0476	38.2149	0
$\{B,C\}$	0.0591	0	0	0.0027
$\{B,D\}$	3.99	0	10.7757	0
$\{C,D\}$	3.0037	12.6147	0	0
$\{A,B,C\}$	0	0	0	0.0027
$\{A,B,D\}$	0	0	6.7266	0
$\{A,C,D\}$	0	15.5593	0	0
$\{B,C,D\}$	0.0591	0	0	0

We have discussed the marginal contribution of player k in coalition S as $c(S) - c(S - \{A_k\})$, The results of which are shown in Table 5. It is reasonable to evaluate k's contribution to the entire game as the average of marginal contribution to coalitions which include k. With regard to this scheme, the Shapley value is the representative imputation of cooperative game.

In our DEA game, it goes as follows. We introduce "ordering" in forming a coalition. For example, suppose that A is the first comer to the coalition and B follows him, and further C and D join the coalition in this order. We denote this ordered coalition by $A \leftarrow B \leftarrow C \leftarrow D$. In this coalition D's marginal contribution is evaluated as:

$$c\{\{A,B,C,D\}) - c\{\{A,B,C\}) = 10.901 - 10.0847 = 0.0027$$

C has marginal contribution (independent of D) as taken from Tables 5.

$$c\{\{A,B,C\}) - c\{A,B\}) = 10.901 - 10.5481 = 0.458$$
$$c\{A,B\} - c\{A\} = 46.8356 - 10.5481 = 36.2875.$$

The first row of Table 6 displays this decomposition. Similarly, we count all orderings (permutations) of players and evaluate their marginal contribution to the coalition. See Table 6. We assume that all permutations occur with equal probability. Thus, we have what is referred to as the "Shapley value" of each player as the average of the marginal contributions exhibited in the second column of Table 7, obtained by using Eq (3.3).

Table 6. All states of different permutations of groups.

	Permutations	A	B	C	D
1	$A \leftarrow B \leftarrow C \leftarrow D$	0	36.2875	0.458	0.0027
2	$A \leftarrow B \leftarrow D \leftarrow C$	0	36.2875	6.7266	6.2659
3	$A \leftarrow C \leftarrow B \leftarrow D$	0	15.5593	21.1862	7.5958
4	$A \leftarrow C \leftarrow D \leftarrow B$	0	15.5593	21.1862	0.0027
5	$A \leftarrow D \leftarrow B \leftarrow C$	0	47.0476	3.1772	17.026
6	$A \leftarrow D \leftarrow C \leftarrow B$	0	0.0027	38.2149	17.026
7	$B \leftarrow A \leftarrow C \leftarrow D$	2.6601	0	0.458	7.5958
8	$B \leftarrow A \leftarrow D \leftarrow C$	2.6601	0	6.7266	6.2659
9	$B \leftarrow C \leftarrow A \leftarrow D$	0.0591	0	3.1772	17.026
10	$B \leftarrow C \leftarrow D \leftarrow A$	0.0591	0	6.7266	0.0027
11	$B \leftarrow D \leftarrow A \leftarrow C$	3.99	0	21.1862	7.5958
12	$B \leftarrow D \leftarrow C \leftarrow A$	0.0591	0	10.7757	7.5958
13	$C \leftarrow A \leftarrow B \leftarrow D$	21.5734	36.2875	0	7.5958
14	$C \leftarrow A \leftarrow D \leftarrow B$	21.5734		0	17.026
15	$C \leftarrow B \leftarrow A \leftarrow D$	5.955	5.955	0	17.026
16	$C \leftarrow B \leftarrow D \leftarrow A$	0.0591	5.955	0	0.0027
17	$C \leftarrow D \leftarrow A \leftarrow B$	3.0037	15.5593	0	18.567
18	$C \leftarrow D \leftarrow B \leftarrow A$	0.0591	12.6147	0	18.567
19	$D \leftarrow A \leftarrow B \leftarrow C$	28.219	36.2875	3.1772	0
20	$D \leftarrow A \leftarrow C \leftarrow B$	28.219	15.5593	21.1862	0
21	$D \leftarrow B \leftarrow A \leftarrow C$	3.99	14.8386	6.7266	0
22	$D \leftarrow B \leftarrow C \leftarrow A$	0.0591	14.8386	3.1772	0
23	$D \leftarrow C \leftarrow A \leftarrow B$	3.0037	36.2875	12.9996	0
24	$D \leftarrow C \leftarrow B \leftarrow A$	18.567	12.6147	12.9996	0

In Table 7, the obtained Shapley values is given in the second column and the efficiency compared to meta-frontier is shown in the fourth column, also, the rank obtained by the Shepley value method of the groups is given in the third column and the rank obtained by the Meta-frontier method is given in the fifth column of the table.

Table 7. Shapley value and ranking of groups.

Group	Shapley value	Ranking by Shapley value	Efficiency evaluated by meta-frontier	Ranking by meta-frontier
A	5.990375	4	0.49992	4
B	15.54529	1	0.55045	3
C	8.344408	2	0.6807	1
D	7.1994	3	0.660833	2

5. Conclusions

Applying a fuzzy approach based on fuzzy logic is an important way for investigating uncertainty in DEA. In this study, to evaluate the groups with fuzzy data, they have been investigated from the point of view of cooperative game. Each group is considered as a player and a subset of groups as a coalition. By defining a characteristic function for coalition S as the sum of the of efficiency of all DMUs when the PPS is made of DMUs of member groups of coalition S, the marginal effect of groups in different coalitions has been obtained. Then, using the marginal effect values, the Shapley value ranks the groups as cooperative game solutions. The higher the Shapley value of a group, the better that group performs. In the presented method, by evaluating the groups, a fair evaluation has been done that is accepted by all the groups. In addition, the marginal effect of a mixed group of possible coalitions has been applied to evaluate that group, which makes the evaluation of that group more accurate; Because this ranking is done by the Shapley value, it is unique.

For future studies, other methods of obtaining cooperative game solutions, noncooperative methods such as Nash bargaining, can be used to evaluate groups. In a situation where the number of groups is large, implementing this method is time-consuming; which can use advanced algorithms such as genetic algorithm.

Conflict of interest

The authors declare no conflict of interest.

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