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Research article

Modelling exchange rate volatility under jump process and application analysis

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Abstract: Exchange rate is an important part of financial markets. Our analysis finds that the fluctuations of exchange rates have several obvious features, such as spikes, thick tails, fluctuation aggregations and asymmetry. Based on this, we build novel GARCH class model by introducing a jumping process to describe the dynamics of their fluctuations. Our empirical results show that the models with jump factors can better characterize the agglomeration and thick tail characteristics of these return fluctuations than the models without jump factors. In particular, the model with double exponential jumps can fully handle and capture the fluctuation characteristics of the returns. Our findings will be useful for individuals and governments to predict exchange rate fluctuations, provide reference for the effective management of exchange rate risk in China, and further improve the financial risk management mechanism.

Keywords: exchange rate; jump process; fluctuations; GARCH class models
Mathematics Subject Classification: 62P20, 62P25, 91B05

1. Introduction

Exchange rate (also known as the foreign exchange rate), the exchange rate between two currencies, can be regarded as the value of one country's currency against the value of another country's currency. It is both the most important adjustment mechanism for foreign exchange and an important indicator reflecting the degrees of economic stability of a country. Therefore, exchange rate risk cannot be ignored [1]. The exchange rate changes can lead to bias in the underlying elasticities [2]. And Exchange rates also are connected to fundamentals irrespective of the forecast horizons [3]. In addition, the change of exchange rate and other economic variables, such as the stock price, oil price, energy prices, monetary aggregate, and other import and export products, may influence each other. For example, Yang [4] investigated the short-run and long-run dynamics between exchange rates and stock prices. Chen et al. [5] found that the relationships between oil and exchange rates are not nonlinear. Malik and Umar [6] found that the connectedness between oil price shocks and exchange rates has increased since financial crisis. Gokmenoglu et al. [7] studied the relationship between exchange rate and stock market returns for selected emerging countries by using the quantile-on-quantile approach. There are some other related studies [8–15].

Since the implementation of the managed floating exchange rate on July 21, 2005 in China, the role of exchange rates in financial markets has also gradually increased. In October 2016, the CNY was added to the Special Drawing Rights (SDR) currency basket. In the future, the expected change in the CNY exchange rate will become more frequent, and the expectations of market participants will also have more pronounced effects on the CNY exchange rate movement. Therefore, the effective management of CNY exchange rate to avoid domestic market fluctuations are important issues facing China's macroeconomic control in the future, and it has important practical significance. The exchange rate of major currency pairs in the foreign exchange market are volatile all the time. However, the fluctuation patterns in exchange rates are ever-changing. In fact, similar to other commodities, they are ultimately determined by the relationship between supply and demand. On the one hand, it will be constrained by the commodity structure, openness level, and degrees of linkage with the international financial market. It will also be affected by changes in various economic factors, such as the CNY internationalization, foreign exchange reserves, consumer price index, net exports and cost of carry. On the other hand, the exchange rate will affect the domestic economy and international trade, and even the entire international economy. Thus, the exchange rate is not an isolated factor, and often interacts with other economic variables. The movement of exchange rate is related to the economic development trend, and has a significant and profound impact on the economy. For example, Table 1 simply reports the pairwise correlations between USD-CNY exchange rate of Chinese yuan (CNY) against the U.S. dollar (USD) and the following five relevant influence factors: the CNY internationalization (RGI), foreign exchange reserves (FER), consumer price index (CPI), net exports (NE) and cost of carry (CC). Some findings are shown below. There are significant correlations between the USD-CNY exchange rate and the following three influence factors, the CNY internationalization (RGI), the foreign exchange reserve (FER) and the net exports (NE). In particular, the USD-CNY exchange rate is positively correlated with the CNY internationalization and the net exports at the 10% and 5% levels, respectively. And it is negative correlation between the exchange rate and the foreign exchange reserve at the 1% level. The possible reasons are as follows. First, the higher the CNY internationalization, the more CNY will be circulated in the international financial market. When China adopts a tightening monetary policy and raises interest rate to control inflation, the CNY circulating internationally will speculatively enter the Chinese market. Then the CNY supply increases, the CNY depreciates, the exchange rate increases, and vice versa. Second, when the net exports increase, the foreign exchange supply increases, and the CNY demand decreases. Then the CNY depreciates, and the exchange rate rises. And vice versa. Finally, in order to cope with the risks of exchange rate fluctuations, the government must hold a large amount of foreign exchange reserves. The increase in foreign exchange reserves will cause the rapidly increasing pressure of appreciation of the CNY, then the exchange rate will decline. And vice versa. In addition, there are negative correlations between the exchange rate and the consumer price index (CPI) and first difference of cost of carry (CC). This could be explained that the greater the consumer price index and the cost of carry, the higher the CNY demand. And then the CNY appreciates, the exchange rate falls. Vice versa. The above results demonstrate that the CNY exchange rate is an important lever for economic adjustment in China. It has important relationship with other economic factors, affects the internal and external economies of the country and even the international economy, and plays an important role in macroeconomic regulations and economic stability.

Table 1. The correlation analysis.

Variables	USD-CNY	RGI	FER	CPI	NE
RGI	0.260 (0.051)				
FER	−0.512 (0.000)	0.147 (0.276)			
CPI	−0.048 (0.724)	0.334 (0.011)	0.383 (0.003)		
NE	0.322 (0.015)	−0.153 (0.255)	−0.496 (0.000)	−0.272 (0.041)	
CC	−0.063 (0.642)	0.043 (0.754)	0.018 (0.894)	0.172 (0.200)	−0.110 (0.416)

Notes: The values in parentheses are *p*-values. The sample is the monthly data from September 2012 to June 2017. These data were obtained from the Wind Finance Database.

The exchange rate has always been the focus of macroeconomic regulations and policy control of government. Its level can reflect the basic conditions of a country's macroeconomic operations. It has extremely important regulatory effects on macroeconomic operations and microeconomic activities. At present, the following questions, whether the CNY will appreciate or not, and the magnitude of appreciation or depreciation, have become the focal point worldwide. This is a good illustration of the important role of the exchange rate in the country and even in the world economy.

With the increase of degrees of exchange rate marketization, the flexibility of exchange rate fluctuations has also enhanced significantly. The characteristics of exchange rate fluctuations have always been concerned by academic researchers and policy makers alike. The asymmetry is the main feature of exchange rate fluctuations. In the existing literature, researches have focused on the GARCH models. Gong [16] empirically studied the fluctuations of USD-CNY exchange rate based on the GARCH model. He found that the exchange rate fluctuation has a positive asymmetry. When analyzing the volatility of exchange rates or to forecast the trend of exchange rates, most of scholars consider using the exchange rate of Chinese yuan against the U.S. dollar, and have obtained many interesting results. But considering that the structural adjustment of China's foreign exchange reserves and the gradual diversification of foreign trade after the exchange rate reform, the trade relationship between China and European countries has become closer. Therefore, the Euro is also becoming more and more important. In view of this, this article will take into account the exchange rate of Chinese yuan against U.S. dollar (USD-CNY) and the exchange rate of Chinese yuan against Euro (EUR-CNY). In addition, several CNY exchange rate indices are also introduced. Because the exchange rate index can be used to comprehensively calculate the change of the weighted average exchange rate of a country's currency against a basket of foreign currencies, which can fully reflect the value change of the country's currency.

Compared with the reference to a single currency, referring to a basket of currencies can better reflect the comprehensive competitiveness of goods and services. More importantly, it can better reflect the role of exchange rate to adjust import and export, international investment, and balance of payments.

First, this article will analyze the importance of exchange rate in the financial market based on the recently developed econometrics models. Then, we will build novel GARCH class model by introducing a jumping process to describe the dynamics of their fluctuations, such as spikes, thick tails, fluctuation aggregations and asymmetry. In particular, Kou [17] adopted the double exponential jump diffusion processes. His results indicated that double exponential distribution can well describe the phenomenon of financial asset fluctuations and showed to be promising in modeling fluctuations in the exchange rate. Therefore, this article will further introduce the double exponential jumps into the GARCH class models to describe the fluctuations of exchange rates and exchange rate indices. Our attempt is new in the existing literature. Our findings will be helpful for individuals and governments to effectively manage the exchange rate risk. In our study, the maximum likelihood estimation method is used to estimate the model parameters. To assess the model forecast performance, we use two statistical error loss functions, the *MAE* and *RMSE* loss functions, and the modified Diebold–Mariano (MDM) statistical test [18]. Our study can provide suggestions for investors to understand fluctuations in foreign exchange market and for their investment decision-making and provide references for policy makers.

In a word, this paper built novel GARCH class models by introducing a jumping process. Then we present the empirical results based on the daily observations in exchange rates (including USD-CNY and EUR-CNY) from 1 December 2015 to 29 December 2022, and the weekly observations in CNY exchange rate indices (including CNYX-CFETS, CNYX-SDR and CNYX-BIS) from 11 December 2015 to 23 December 2022. The sample data is split into two small samples, in-sample data (the daily observations from 1 December 2015 to 9 July 2018, the weekly observations 11 December 2015 to 29 June 2018) for estimating the parameters and out-of-sample data (the daily observations from 10 July 2018 to 29 December 2022, the weekly observations 6 July 2018 to 23 December 2022) to analyze the predicted performance of models. Our empirical results show that the fluctuations of exchange rates and exchange rate indices have features such as spikes, heavy tails and asymmetry. Models with jumps have better fittings than models without jumps, and can better characterize the agglomeration and heavy tail characteristics of the return fluctuations. Our findings will be useful for individuals and governments to predict exchange rate fluctuations.

The rest of the paper is organized as follows: Section 2 is a review of the related literature. Section 3 explains the methodology. Section 4 presents the estimation results and the empirical findings are discussed. Finally, Section 5 summarizes and concludes.

2. Literature review

In recent years, the formation mechanism of central parity of the Chinese yuan (Renminbi, or CNY) to the U.S. dollar has been improved; the normalized intervention in the foreign exchange market has been gradually withdrawn; and the role of the market in the formation of the USD-CNY (the Chinese yuan per U.S. dollar) exchange rate has been continuously strengthened. In July 2005, China began to implement a managed floating exchange rate to start and promote the reform of exchange rate. Since then, the CNY exchange rate has been rising steadily. The adjustment of China's exchange rate policy and the changing trend of CNY exchange rate have currently become hot issues for the public. Now, as the flexibility of CNY exchange rate increases, once the revaluation of CNY's appreciation is completely reversed, the asset price increase under the background of appreciation expectations may disappear instantly, causing a serious blow to the country's economic and financial systems.

The development of financial market requires financial theories to continuously innovate and provide guidance. Similarly, the continuous development of financial market promotes the development and innovation of financial theories as well. Topics related to exchange rates have been extensively studied by many scholars [19–25]. GARCH-based processes are the most popular models to predict volatility of financial data because of their ability to capture clustering and persistence in time series volatilities (see Engle [26]; Bollerslev [27]). Therefore, several papers have focused on modeling and predicting asset volatilities based on such models [28–32]. In particular, Bentes [28] showed that GARCH forecasted volatility outperforms implied volatility in forecasting out-of-sample realized volatility. Abounoori et al. [29] used a set of standard GARCH and Markov Regime-Switching GARCH models to forecast Tehran Stock Exchange (TSE) volatility and compared their prediction ability. These researches indicated the versatility and usefulness of GARCH-based processes. However, further research showed that the phenomenon of fluctuations in financial asset prices is characterized by peaks, thick tails and fluctuations in aggregation. It is well known that GARCH processes are parametric models that assume a linear correlation structure in data. In addition, they are restricted to stationary and normal distribution of variables and errors. However, these assumptions are not realistic in real life situations. GARCH models may not capture nonlinear patterns in data, and linear approximation approach of those complex time series may not be satisfactory.

In particular, scholars empirically showed that price fluctuations in financial market can be divided into small-scale fluctuations and jump fluctuations. They pointed out that financial asset price-fitting models must distinguish between these two types of fluctuations. Liu and Luger [33] proposed a new and flexible GARCH-type model for autoregressive conditional higher moments, such as volatility, skewness and kurtosis. In addition, some scholars began to introduce jump factors into GARCH models, thus proposing a traditional GARCH-JUMP model with Poisson jumps. Duan et al. [34] provided a risk-neutralization of a discrete-time model with jumps, but they did not allow for time-varying jump intensity. As a more general case, Christoffersen et al. [35] investigated the GARCH framework allowing for dynamic jumps in returns as well as in volatilities. Byun et al. [36] developed a GARCH option valuation model using the variance-dependent pricing kernel of [37] under a framework allowing for dynamic volatility and dynamic jump intensity. Qiao et al. [38] studied VIX forecasting based on discrete time GARCH-type model with observable dynamic jump intensity by incorporating high frequency information (DJI-GARCH model). In addition, other scholars also contributed in this research area [39]. Hu et al. [39] examined the empirical effects of hyperbolic jump diffusion models. Their results showed that hyperbolic jump diffusion models can capture the following characteristics: thick and heavy tail of asset returns, and asymmetry and volatility smiles in option pricing. At the same time, the hyperbolic jump diffusion models are better than GARCH and traditional GARCH-JUMP models. Barunik et al. [40] proposed a realized Jump-GARCH model which can be able to account for impact of jumps. And their results showed that disentangling jump variation from the integrated variation is important for forecasting performance.

With the development of global economic integration, international trade has been increasing significantly. The impact of exchange rate on the economy is extremely complex. It will not only affect the domestic economy, but also affect the international economy. Therefore, it is necessary to further explore the fluctuation of exchange rate. Note that earlier studies have shown that the CNY exchange rate fluctuations are characterized by asymmetry. The asymmetry of financial assets, also known as the leverage effect, refers to the fact that the decline in financial asset prices has a greater impact on the fluctuation of asset prices than the price increase of the same magnitude. That is to say, the price fluctuations caused by bad news is greater than the price fluctuations caused by good news. Therefore, the asymmetry of exchange rate fluctuations is still a hot topic for scholars. In our study, the asymmetry will be explored as well. Our research can provide a valuable reference for investors and policy makers.

3. Methodology

In this section, we will describe the generalized ARCH (GARCH) model of Bollerslev [41] and the threshold GARCH (GJR-GARCH) model of Glosten et al. [42], and further modify the above models considering jump factors, i.e., the Poisson jumps and the double exponential jumps (see Kou [17]), totaling five models, which are utilized to forecast exchange rate returns in this paper.

3.1. GARCH(1,1) model

Define $y_t = 100 \times \ln(S_t / S_{t-1})$ as the exchange rate or exchange rate index, expressed in the rate of returns, where S_t is the values of relevant variables (including the USD-CNY, EUR-CNY, CNYX-CFETS, CNYX-SDR, and CNYX-BIS in this article).

First, the GARCH(1,1) model is expressed as follows:

$$\begin{cases} y_t = c + e_t,\, e_t = z_t \sigma_t,\, z_t \sim i.i.d.\, N(0,1), \\ \sigma_t^2 = \omega + \alpha e_{t-1}^2 + \beta \sigma_{t-1}^2, \end{cases} \tag{3.1}$$

where e_t is the current error, c and σ_t^2 are the conditional mean and variance of returns, respectively, $i.i.d.$ denotes that the standardized errors z_t are independent and identically distributed, and the variance parameters ω, α and β are the parameters to be estimated and obey the constraints $\omega,\ \alpha,\ \beta > 0$. In the GARCH model, the variance of the random error term is affected not only by the previous random error term but also by the variance of the previous period. Therefore, this model is more suitable to the in-depth study and analysis of the volatility and correlation of the financial time series.

3.2. GARCH(1,1) model with Poisson jumps

The GARCH(1,1) model with Poisson jumps is an extension of the GARCH(1,1) model and it allows the jump variable J_t, an indicator of jumps which accounts for the spikes. The complete GARCH(1,1) model with Poisson jumps can be represented as follows:

$$\begin{cases} y_t = c + e_t + \sum_{j=0}^{J_t} V_j, \\ e_t \mid \psi^{t-1} = z_t \sigma_t,\, z_t \sim i.i.d.\, N(0,1), \\ J_t \sim \dfrac{e^{-\lambda} \lambda^j}{j!},\ V_j \sim N(\theta, \delta^2), \\ \sigma_t^2 = \omega + \alpha e_{t-1}^2 + \beta \sigma_{t-1}^2, \end{cases} \tag{3.2}$$

where the definitions of y_t, e_t, c, ω, α, β, σ_t^2, z_t and $i.i.d$ are the same as in Eq (3.1). J_t is assumed to be a Poisson random variable with parameter λ. It is assumed that each Poisson event causes a discrete jump of size $\exp(V_j),\ j=1,2,\ldots,J_t$. Hence, jumps are assumed to be independently lognormally distributed random variables, which are independent of e_t, while V_j is assumed to be an *i.i.d.* normally distributed random variable with mean θ and variance δ^2.

3.3. GJR-GARCH(1,1) model with Poisson jumps

The standard GARCH model is symmetrical about the processing of good news and bad news. That is, they have the same effect on fluctuations. However, the impact of good news and bad news on fluctuations may be asymmetric in financial market. In general, when bad news hits the financial market, asset prices tend to enter a turbulent stage and their fluctuation increases. However, when there is good news, the fluctuation tends to be smaller and the market will enter a period of stability. In other words, there may be asymmetry in the financial market. Thus, GJR-GARCH model introduces an asymmetric term, and the treatment of good news and bad news is asymmetric. Therefore, this paper will apply the GJR-GARCH model to study fluctuations of exchange rates and exchange rate indices.

The GJR-GARCH(1,1) model with Poisson jumps can be represented as follows:

$$\begin{cases} y_t = c + e_t + \sum_{j=0}^{J_t} V_j, \\ e_t \mid \psi^{t-1} = z_t\sigma_t,\ z_t \sim i.i.d.\ N(0,1), \\ J_t \sim \dfrac{e^{-\lambda}\lambda^j}{j!},\ V_j \sim N(\theta,\delta^2), \\ \sigma_t^2 = \omega + (\alpha + \gamma I_{t-1}^-)e_{t-1}^2 + \beta\sigma_{t-1}^2, \end{cases} \tag{3.3}$$

where the definitions of y_t, e_t, c, σ_t^2, J_t, z_t and *i.i.d.* are the same as in Eq (3.2). The parameters ω, α, β and γ obey the constraints $\omega,\ \alpha,\ \beta > 0$ and $\alpha + \beta + 0.5\gamma < 1$. I_{t-1}^- is an indicator dummy that takes the value 1 if $e_{t-1} < 0$ (bad news) and 0 otherwise (good news), and parameter γ is used to capture the leverage effect of volatility. If $\gamma > 0$, the price fluctuations from a bad news are larger than those from the same degrees of good news. And there is a positive asymmetry. If $\gamma < 0$, in contrast to the former case, there is a negative asymmetry. If $\gamma = 0$, there is no asymmetry.

3.4. GARCH(1,1) model with double exponential jumps

Kou [17] proposed the double exponential jump diffusion processes which can well describe the phenomenon of financial asset fluctuations. Therefore, this article will further combine the double exponential jump process with the GARCH models to explore the fluctuations of exchange rate in China. The GARCH(1,1) model with double exponential jumps is an extension of the GARCH(1,1) model and allows the jump variable J_t, an indicator of jumps which accounts for the spikes. The complete GARCH(1,1) model with double exponential jumps can be represented as follows:

$$\begin{cases} y_t = c + e_t + \sum_{j=0}^{J_t} V_j, \\ e_t \mid \psi^{t-1} = z_t\sigma_t,\ z_t \sim i.i.d.\ N(0,1), \\ f_{V_j}(v) = \frac{1}{2\eta} e^{\frac{-|v-k|}{\eta}},\ 0 < \eta < 1 \\ \sigma_t^2 = \omega + \alpha e_{t-1}^2 + \beta\sigma_{t-1}^2, \\ p(J_t = l) = \frac{e^{-\lambda}\lambda^k}{l!}, \end{cases} \tag{3.4}$$

where the definitions of y_t, e_t, c, λ, ω, α, β, σ_t^2, J_t, z_t and $i.i.d.$ are the same as in Eq (3.2). V_j follows an asymmetric double exponential distribution. k is the mean of v, $p(v-k=\xi) = p(v-k=-\xi) = 0.5$. This model assumes that the jumps of exchange rate are mainly the responses to major external information in the financial market, and the favorable information and interest spread information arrive at the Poisson process mean. Therefore, the whole jump process in the model also takes place in the Poisson process. At the same time, the model also points out that the jump rate of the yield series obeys the double exponential distribution, which is more conducive in describing the leverage effect in the fluctuation process of the exchange rate series.

3.5. GJR-GARCH(1,1) model with double exponential jumps

The GJR-GARCH(1,1) model with double exponential jumps can be represented as follows:

$$\begin{cases} y_t = c + e_t + \sum_{j=0}^{J_t} V_j, \\ e_t \mid \psi^{t-1} = z_t\sigma_t, z_t \sim i.i.d.\ N(0,1), \\ f_{V_j}(v) = \frac{1}{2\eta} e^{\frac{-|v-k|}{\eta}},\ 0 < \eta < 1 \\ \sigma_t^2 = \omega + (\alpha + \gamma I_{t-1}^-) e_{t-1}^2 + \beta\sigma_{t-1}^2, \\ p(J_t = l) = \frac{e^{-\lambda}\lambda^k}{l!}, \end{cases} \tag{3.5}$$

where the definitions of y_t, e_t, c, λ, ω, α, β, γ, σ_t^2, J_t, z_t, I_{t-1}^- and $i.i.d.$ are the same as in Eq (3.3). The definitions of V_j, k and v are the same as in Eq (3.4).

4. Empirical analysis

In this section, we perform empirical analysis applying these models discussed in Section 3 to the following five variables, the USD-CNY exchange rate, the EUR-CNY exchange rate, the CNY exchange rate index referring to the basket currencies of China Foreign Exchange Trading System, the CNY exchange rate index referring to the basket currencies of Special Drawing Right and the CNY exchange rate index referring to the basket currencies of Bank for International Settlements. The notations about models and variables are shown in Table 2.

Table 2. Model and variable notations.

Model		Variable	
M1	GARCH(1,1) model	USD-CNY	the exchange rate of the Chinese yuan per U.S. dollar
M2	GARCH(1,1) model with Poisson jumps	EUR-CNY	the exchange rate of the Chinese yuan per Euro
M3	GJR-GARCH(1,1) model with Poisson jumps	CNYX-CFETS	the CNY exchange rate index referring to the basket currencies of China Foreign Exchange Trading System
M4	GARCH(1,1) model with double exponential jumps	CNYX-SDR	the CNY exchange rate index referring to the basket currencies of Special Drawing Right
M5	GJR-GARCH(1,1) model with double exponential jumps	CNYX-BIS	the CNY exchange rate index referring to the basket currencies of Bank for International Settlements

4.1. Data and descriptive statistics

We consider the daily observations in exchange rates (including USD-CNY and EUR-CNY) from 1 December 2015 to 29 December 2022, and the weekly observations in CNY exchange rate indices (including CNYX-CFETS, CNYX-SDR and CNYX-BIS) from 11 December 2015 to 23 December 2022. These data were obtained from the China Wind Finance Database. After this, the sample is split into two small samples, in-sample data (the daily observations from 1 December 2015 to 9 July 2018, the weekly observations 11 December 2015 to 29 June 2018) and out-of-sample data (the daily observations from 10 July 2018 to 29 December 2022, the weekly observations 6 July 2018 to 23 December 2022). We estimate the parameters of the model based on in-sample data first. Then, we will explore the forecast performance of each model during out-of-sample period. Now, we convert into continuous composite percentage returns using the following equation:

$$y_t = 100 \times (\ln(S_t) - \ln(S_{t-1})), \tag{3.6}$$

where S_t is any of the above five variables at time *t*.

Figure 1 shows the graphical representation of all variables in their raw data and log-returns. From these diagrams, we can find that during the sample periods, the log-returns of exchange rates and CNY exchange rate indices show obvious fluctuations and aggregation. At the same time, there are obvious characteristics of fluctuant jumps, especially the USD-CNY exchange rate and EUR-CNY exchange rate. These characteristics are consistent with the previous theoretical elaboration and the conclusions of many existing empirical studies. Further, we elaborate the fluctuation characteristics of the sequence by comparing and analyzing the basic statistics of return series. The summary statistics of daily or weekly returns are shown in the following Tables 3 and 4.

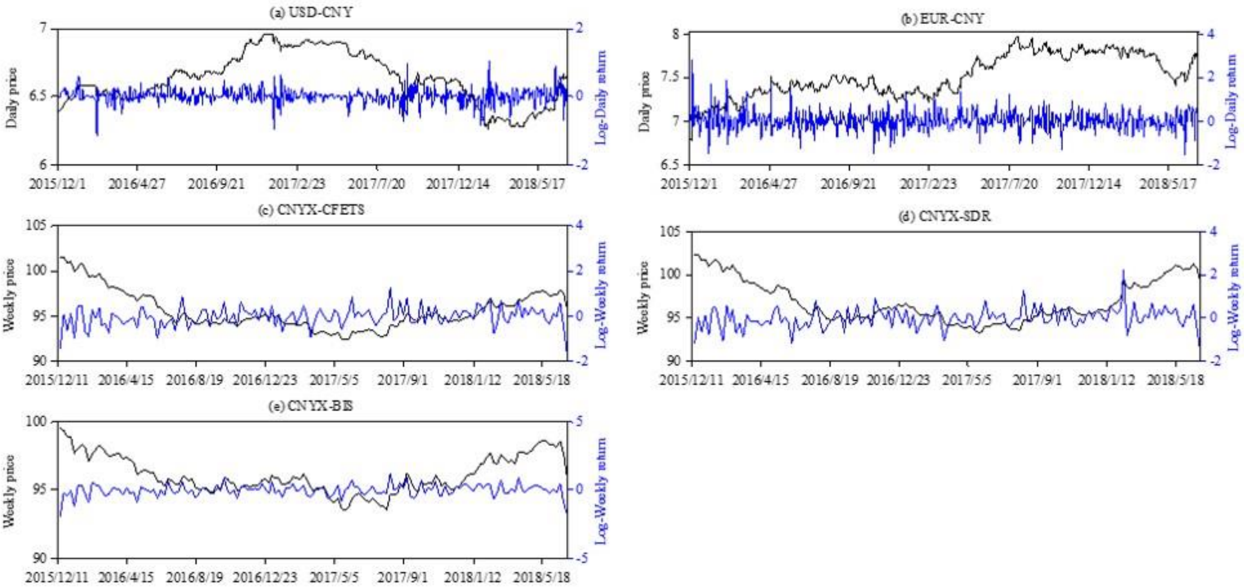

Figure 1. Time-series plots of all variables (original data and returns).

Table 3. Summary statistics of all variables (in returns).

	USD-CNY	EUR-CNY	CNYX-CFETS	CNYX-SDR	CNYX-BIS
Mean	0.005	0.022	−0.050	−0.030	−0.039
Maximum	1.055	2.842	1.252	2.229	1.190
Minimum	−1.150	−1.512	−1.782	−1.643	−2.029
Standard deviation	0.219	0.445	0.441	0.488	0.447
Skewness	−0.285	0.578	−0.460	0.340	−0.922
Kurtosis	7.091	6.920	4.590	6.062	6.558
Jarque-Bera test	452.147***	442.605***	20.679***	60.238***	98.351***

Notes: (1) The Jarque-Bera statistics are based on Jarque and Bera [43] and are asymptotically chi-squared-distributed with two degrees of freedom. (2) *** denotes significantly at the 1% level.

Table 3 summarizes the sample statistics of all variables in returns. According to Table 3, the average returns of all variables are quite low compared with the standard deviation, indicating high volatility. On the one hand, we can find that the skewness of each variable is not equal to zero, and the Kurtosis of each index is greater than 3, indicating heavy tails and leptokurtic characteristic, respectively. On the other hand, the Jarque-Bera normality test statistics are all significant at the 1% level, indicating that the hypothesis of normality is rejected. Thus, they are not normally distributed. According to Table 4, the Kolmogorov-Smirnov (K-S) Z values of the log-daily returns of USD-CNY exchange rate is 2.485 that is more than the absolute value of the most extreme difference of 0.099, and the approximate probability value is 0.000 that is significantly less than the 1% level. Thus, the log-daily returns of USD-CNY exchange rate does not follow normal distribution. Similar results can be obtained for the log-daily returns of EUR-CNY exchange rate or other log-weekly returns of the exchange rate indices. In addition, Table 5 reports the Augmented Dickey–Fuller (ADF) test results. The results show that return series of all variables are stationary. It means that the GARCH model can be applied to these return series.

Table 4. Kolmogorov-Smirnov test of all variables (in returns).

		USD-CNY	EUR-CNY	CNYX-CFETS	CNYX-SDR	CNYX-BIS
Number of observations		636	636	147	147	147
The most extreme difference	Absolute value	0.099	0.057	0.050	0.069	0.099
	Positive	0.083	0.057	0.041	0.055	0.055
	Negative	−0.099	−0.046	−0.050	−0.069	−0.099
Kolmogorov-Smirnov Z		2.485	1.443	0.061	0.835	1.206
Asymptotic significance (two-sided test)		0.000	0.031	0.851	0.489	0.109

Notes: (1) The null hypothesis of the Kolmogorov-Smirnov (K-S) Z statistics is that the test distribution is assumed to obey normal distribution. (2) The "Absolute value" is the maximum value between the "Positive value" and the absolute value of the "Negative value".

Table 5. The Augmented Dickey-Fuller (ADF) test statistic of all variables (in returns).

Variables	ADF value	P-value	Test result
USD-CNY	−23.662	0.000	stationary
EUR-CNY	−27.564	0.000	stationary
CNYX-CFETS	−11.732	0.000	stationary
CNYX-SDR	−12.053	0.000	Stationary
CNYX-BIS	−11.909	0.000	stationary

In summary, we find that there are obvious volatility clustering and non-normal characteristics with high peaks and heavy tails in the log-returns. Further, there may be jump volatility phenomena. Therefore, in the empirical estimation in the following subsection, we introduce the GARCH model, the GARCH models with jump factors and the GJR-GARCH models with jump factors as the models to compare with each other.

4.2. Parameter estimation

Danielsson [44] used the maximum likelihood estimation method to estimate the stochastic volatility in asset prices. In this study, we will also use the maximum likelihood estimation method to estimate parameters of the GARCH(1,1) model, GARCH(1,1) model with Poisson jumps, GJR-GARCH(1,1) model with Poisson jumps, GARCH(1,1) model with double exponential jumps, and GJR-GARCH(1,1) model with double exponential jumps. The parameter estimation results are reported in Table 6. We have the following findings. (1) The coefficients $\hat{\omega}$, $\hat{\alpha}$ and $\hat{\beta}$ of the models are all positive, and most of the coefficients are significant at the 1% level. (2) The constraint of $\hat{\alpha}+\hat{\beta}+0.5\hat{\gamma}<1$ is obeyed for the GJR-GARCH(1,1) model with Poisson jumps and the GJR-GARCH(1,1) model with double exponential jumps, thus ensuring that the conditions for stationary covariance hold. (3) For the GJR-GARCH(1,1) with Poisson jumps, $\hat{\gamma}$ of the USD-CNY, EUR-CNY and CNYX-BIS are 0.854, 0.007 and 0.332 respectively, which are significantly greater than 0 at the 1%, 10% and 1% levels respectively. So the fluctuations of USD-CNY, EUR-CNY and CNYX-BIS from a bad news are larger than those from the same degrees of good news. However, $\hat{\gamma}$ of the

CNYX-CFETS and CNYX-SDR are −0.057 and −0.068 respectively, which are both significantly less than 0 at the 1% level. Therefore, the fluctuations of CNYX-CFETS and CNYX-SDR from a good news are larger than those from the same degrees of bad news. For the GJR-GARCH(1,1) with double exponential jumps, similar result is obtained. (4) Except the GARCH(1,1) model with Poisson jumps on the EUR-CNY and the GJR-GARCH(1,1) model with double exponential jumps on CNYX-BIS, the values of jump parameter $\hat{\lambda}$ are all significant at least 5% significant level. This can be explained that the flow of information that arrives in the market can be described part of the time by the heteroscedastic diffusion process, but is often subject to jump risks. (5) In GARCH(1,1) model with double exponential jumps and GJR-GARCH(1,1) model with double exponential jumps, the values of parameter $\hat{k}$ are significantly greater than 0 at least 10% level in most cases. This shows that the critical points of the response of exchange rates and CNY exchange rate indices to good news and bad news are significantly not equal to 0 and is positive. This means that the market will see some good news with little profit as the start of bad news, so as to respond equivalently to the arrival of bad news, which implies the vigilance of market participants for bad news.

Table 6. The parameter estimation results of each model.

Parameter	USD-CNY	EUR-CNY	CNYX-CFETS	CNYX-SDR	CNYX-BIS
Panel A: GARCH(1,1) model					
$\hat{c}$	0.013	0.019	−0.020	−0.016	−0.014
	(0.008)	(0.017)	(0.036)	(0.041)	(0.036)
$\hat{\omega}$	0.014***	0.006***	0.019***	0.029	0.046**
	(0.002)	(0.000)	(0.003)	(0.026)	(0.020)
$\hat{\alpha}$	0.307***	0.048***	0.186***	0.020	0.032
	(0.056)	(0.006)	(0.030)	(0.014)	(0.038)
$\hat{\beta}$	0.449***	0.913***	0.755***	0.888***	0.693***
	(0.071)	(0.005)	(0.042)	(0.111)	(0.121)
LR	102.197	−352.447	−73.669	−100.916	−81.112
Panel B: GARCH(1,1) model with Poisson jumps					
$\hat{c}$	0.019***	0.022	−0.029	0.022	0.022
	(0.007)	(0.017)	(0.019)	(0.017)	(0.017)
$\hat{\omega}$	0.040***	0.005***	0.022***	0.005***	0.005***
	(0.003)	(0.000)	(0.001)	(0.000)	(0.000)
$\hat{\alpha}$	0.594***	0.039***	0.163***	0.039***	0.039***
	(0.061)	(0.004)	(0.011)	(0.004)	(0.004)
$\hat{\beta}$	0.006	0.905***	0.815***	0.915***	0.825***
	(0.045)	(0.003)	(0.012)	(0.003)	(0.003)
$\hat{\theta}$	0.014***	0.647**	−1.476***	2.221	−1.814
	(0.003)	(0.069)	(0.056)	(0.109)	(0.130)
$\hat{\delta}^2$	0.040**	1.075***	0.001***	0.001**	0.001***
	(0.504)	(0.003)	(0.002)	(0.005)	(0.006)
$\hat{\lambda}$	1.138**	0.024	0.014***	0.007**	0.015*
	(0.035)	(0.085)	(0.073)	(0.081)	(0.203)
LR	15.558	−394.543	−82.628	−115.315	−91.768

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Parameter	USD-CNY	EUR-CNY	CNYX-CFETS	CNYX-SDR	CNYX-BIS
Panel C: GJR-GARCH(1,1) model with Poisson jumps					
$\hat{c}$	0.008	0.030*	−0.042***	−0.051	−0.009
	(0.008)	(0.016)	(0.012)	(0.041)	(0.031)
$\hat{\omega}$	0.039***	0.005***	0.017***	0.012***	0.078***
	(0.002)	(0.000)	(0.001)	(0.002)	(0.012)
$\hat{\alpha}$	0.277***	0.041***	0.116***	0.054***	0.118***
	(0.046)	(0.000)	(0.022)	(0.008)	(0.005)
$\hat{\beta}$	0.003	0.906***	0.731***	0.817***	0.516***
	(0.030)	(0.000)	(0.009)	(0.007)	(0.067)
$\hat{\gamma}$	0.854***	0.007*	−0.057***	−0.068***	0.332***
	(0.148)	(0.004)	(0.018)	(0.013)	(0.044)
$\hat{\theta}$	−0.115*	1.698	−1.476**	1.698	1.698*
	(0.610)	(0.396)	(0.034)	(0.396)	(0.396)
$\hat{\delta}^2$	0.302**	0.161*	0.001***	0.001***	0.001***
	(0.089)	(0.702)	(0.002)	(0.000)	(0.000)
$\hat{\lambda}$	0.039**	0.010***	0.013***	0.007***	0.015*
	(0.070)	(0.005)	(0.002)	(0.000)	(0.014)
LR	24.536	−392.384	−77.749	−99.022	−86.438
Panel D: GARCH(1,1) model with double exponential jumps					
$\hat{c}$	0.010	0.010	−0.014**	0.010	0.010
	(0.007)	(0.016)	(0.007)	(0.016)	(0.016)
$\hat{\omega}$	0.008***	0.006***	0.019***	0.006***	0.006***
	(0.001)	(0.001)	(0.007)	(0.001)	(0.001)
$\hat{\alpha}$	0.230***	0.021***	0.160***	0.021***	0.021***
	(0.057)	(0.006)	(0.028)	(0.006)	(0.006)
$\hat{\beta}$	0.602***	0.982***	0.726***	0.962***	0.932***
	(0.074)	(0.007)	(0.050)	(0.007)	(0.007)
$\hat{\lambda}$	0.988**	0.987***	1.245**	1.209***	1.275***
	(0.402)	(0.110)	(0.290)	(0.900)	(0.098)
$\hat{k}$	1.087	1.094*	0.998**	1.010**	1.209*
	(0.900)	(0.870)	(0.041)	(0.067)	(0.076)
$\hat{\eta}$	0.930*	0.930***	0.967**	0.189**	0.978**
	(0.093)	(0.011)	(0.083)	(0.059)	(0.042)
LR	143.809	−349.121	−377.987	−95.965	−76.745
Panel E: GJR-GARCH(1,1) model with double exponential jumps					
$\hat{c}$	0.009	0.006	−0.033	−0.073***	−0.009
	(0.007)	(0.017)	(0.028)	(0.018)	(0.033)
$\hat{\omega}$	0.008***	0.005***	0.015***	0.010***	0.069***
	(0.001)	(0.001)	(0.005)	(0.003)	(0.023)
$\hat{\alpha}$	0.183***	0.042***	0.090*	0.043*	0.116***
	(0.058)	(0.002)	(0.053)	(0.025)	(0.030)

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Parameter	USD-CNY	EUR-CNY	CNYX-CFETS	CNYX-SDR	CNYX-BIS
$\hat{\beta}$	0.560^{***}	0.604^{***}	0.015^{***}	0.427^{***}	0.579^{***}
	(0.070)	(0.003)	(0.052)	(0.008)	(0.141)
$\hat{\gamma}$	0.174^{**}	0.014	-0.063^{*}	-0.090^{***}	0.182^{***}
	(0.082)	(0.009)	(0.033)	(0.021)	(0.062)
$\hat{\lambda}$	1.106^{***}	1.074^{**}	1.339^{**}	1.291^{*}	1.373
	(0.073)	(0.064)	(0.005)	(0.103)	(0.714)
$\hat{k}$	0.983^{**}	0.981^{***}	0.954^{***}	0.968^{***}	0.951^{***}
	(0.017)	(0.008)	(0.007)	(0.014)	(0.003)
$\hat{\eta}$	0.945^{***}	0.967	0.939^{*}	0.964	0.877^{***}
	(0.032)	(0.105)	(0.807)	(0.102)	(0.011)
LR	145.689	−347.038	−76.801	−89.770	−74.607

Notes: (1) LL indicates the log-likelihood value. (2) ***, ** and * denote significantly at the 1%, 5% and 10% levels, respectively.

Next, we compare the forecast performance of the above models. We draw the return charts of each model on basis of the estimate results in Figure 2. In addition, in order to assess the model performance, we will first calculate two model evaluation error loss functions, the mean absolute error (*MAE*) and the root mean square error (*RMSE*).

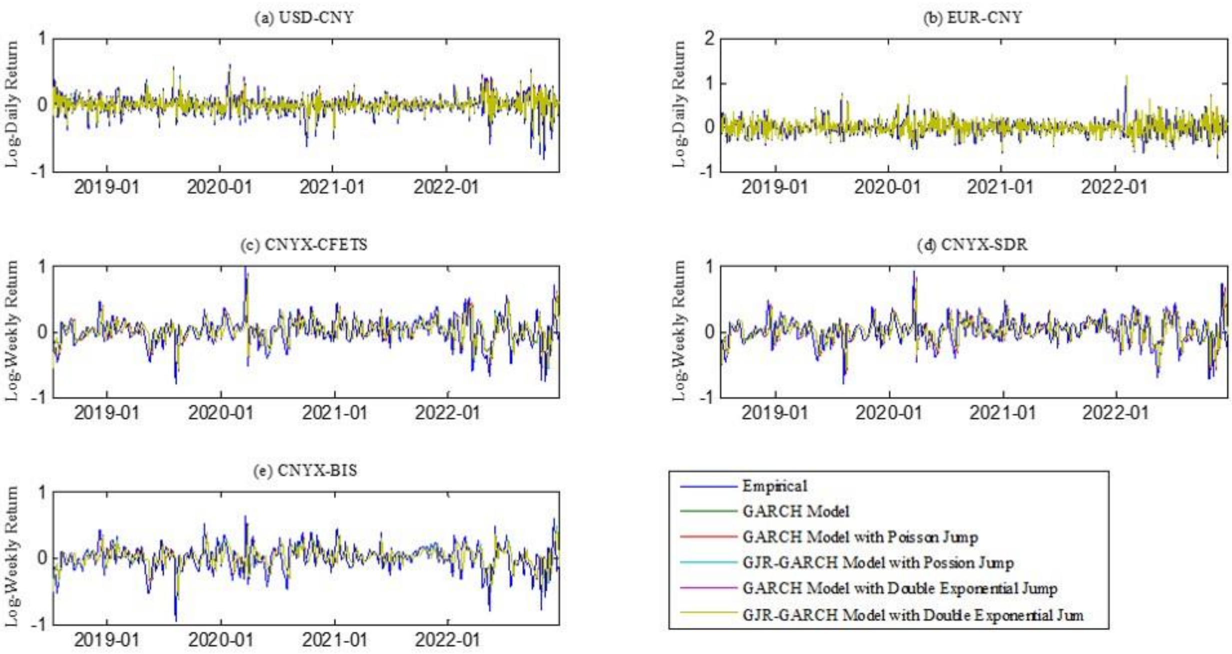

Figure 2. The forecast performance of all models.

The *MAE* is written as

$$MAE = \frac{1}{N}\sum_{i=1}^{N}|e_i| = \frac{1}{N}\sum_{i=1}^{N}\left|\hat{A}_i - A_i\right|, \tag{3.7}$$

and the *RMSE* is expressed as

$$RMSE = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(e_i)^2} = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(\hat{A}_i - A_i\right)^2}\ , \tag{3.8}$$

where e_i denotes the estimation error; *N* is the number of observations; A_i and $\hat{A}_i$ represent the actual market value and predicted value of each model, respectively.

Table 7 presents the return describing forecast performance of each model base on the *MAE* and *RMSE* loss functions of each model. First, for both the *MAE* and *RMSE*, the forecast performance of GJR-GARCH(1,1) model with Poisson jumps is the best for the USD-CNY and EUR-CNY. Second, for the CNYX-CFETS, the CNYX-SDR, and the CNYX-BIS, the *MAE* and *RMSE* of GARCH(1,1) model are the smallest, but the numerical difference between the GARCH(1,1) model and other models is very small.

Table 7. The *MAE* and *RMSE* of all return models.

	USD-CNY	EUR-CNY	CNYX-CFETS	CNYX-SDR	CNYX-BIS	USD-CNY	EUR-CNY	CNYX-CFETS	CNYX-SDR	CNYX-BIS
	MAE					*RMSE*				
M1	0.11578	0.17912	**0.20371**	**0.19608**	**0.19541**	0.16625	0.23958	**0.27397**	**0.26198**	**0.25781**
M2	0.11443	0.17926	0.20916	0.20597	0.19876	0.16434	0.23979	0.28170	0.27425	0.26189
M3	**0.11382**	**0.17905**	0.20727	0.20181	0.19804	**0.16346**	**0.23951**	0.27896	0.26910	0.26102
M4	0.11386	0.17932	0.20703	0.20301	0.19707	0.16351	0.23983	0.27865	0.27058	0.25985
M5	0.11414	0.17908	0.20611	0.20003	0.19664	0.16392	0.23953	0.27730	0.26690	0.25932

Notes: (1) M1, M2, M3, M4 and M5 are explained in Table 2. (2) Boldface numbers denote the lowest values of loss function when the predictive accuracies of all five models are compared with each other.

Further, we use the MDM test (see Choudhry and Wu [18]) to investigate whether any two sets of forecast errors, for example, $e_{1,i}$ and $e_{2,i}$, have equal mean values. Based on the *MAE* criteria, the MDM test statistic is given by

$$S^* = \sqrt{\frac{N+1-2h+N^{-1}h(h-1)}{N}}S, \tag{3.9}$$

where

$$S=\frac{\bar{d}}{\sqrt{Var(\bar{d})}},\ \bar{d} = \frac{1}{N}\sum_{i=1}^{N}d_i,\ d_i = |e_{1,i}| - |e_{2,i}|, Var(\bar{d}) \approx \frac{1}{N}\left[\eta_0 + 2\sum_{j=1}^{h-1}\eta_j\right].$$

In the above formula, η_j is the jth auto-covariance of d_i, N is the size of sample period, h denotes the forecast horizon, and the MDM test statistic follows the Student's *t*-distribution with $N-1$ degrees of freedom. Each MDM test generates two statistics, S_1 and S_2, based on two hypothesizes:

(a) H_0^1: there is no statistical difference between two sets of forecast errors ($e_{1,i}$ and $e_{2,i}$).

H_1^1: the first set of forecast errors is significantly smaller than the second.

(b) H_0^2: there is no statistical difference between two sets of forecast errors ($e_{1,i}$ and $e_{2,i}$).

H_1^2 : the second set of forecast errors is significantly smaller than the first.

In this study, the significance levels for S_1 and S_2 statistics are set at the 10% level. The adjusted statistics provide three possible answers to the superiority between two rival models: if S_1 is significant, the first forecasting model outperforms the second; if S_2 is significant, the second forecasting model outperforms the first; and if both S_1 and S_2 are not significant, then the two models produce equally accurate forecasts. Table 8 reports the results of the MDM test (Choudhry and Wu [18]), using loss functions *MAE* as the error criterion. There are ten MDM tests (T1–T10) in Table 8.

Table 8. The MDM test results based on *MAE* loss function.

	USD-CNY	EUR-CNY	SUM1	CNYX-CFETS	CNYX-SDR	CNYX-BIS	SUM2	SUM
T1(M1 vs. M2)	(0,1,0)	(1,0,0)	(**1**,**1**,0)	(1,0,0)	(1,0,0)	(1,0,0)	(**3**,0,0)	(**4**,1,0)
T2(M1 vs. M3)	(0,1,0)	(0,1,0)	(0,**2**,0)	(1,0,0)	(1,0,0)	(1,0,0)	(**3**,0,0)	(**3**,2,0)
T3(M1 vs. M4)	(0,1,0)	(1,0,0)	(**1**,**1**,0)	(1,0,0)	(1,0,0)	(1,0,0)	(**3**,0,0)	(**4**,1,0)
T4(M1 vs. M5)	(0,1,0)	(0,1,0)	(0,**2**,0)	(1,0,0)	(1,0,0)	(1,0,0)	(**3**,0,0)	(**3**,2,0)
T5(M2 vs. M3)	(0,1,0)	(0,1,0)	(0,**2**,0)	(0,1,0)	(0,1,0)	(0,1,0)	(0,**3**,0)	(0,**5**,0)
T6(M2 vs. M4)	(0,1,0)	(1,0,0)	(**1**,**1**,0)	(0,1,0)	(0,1,0)	(0,1,0)	(0,**3**,0)	(1,**4**,0)
T7(M2 vs. M5)	(0,0,1)	(0,1,0)	(0,**1**,**1**)	(0,1,0)	(0,1,0)	(0,1,0)	(0,**3**,0)	(0,**4**,1)
T8(M3 vs. M4)	(0,0,1)	(1,0,0)	(**1**,0,**1**)	(0,0,1)	(1,0,0)	(0,1,0)	(**1**,**1**,**1**)	(**2**,1,**2**)
T9(M3 vs. M5)	(1,0,0)	(1,0,0)	(**2**,0,0)	(0,1,0)	(0,1,0)	(0,1,0)	(0,**3**,0)	(2,**3**,0)
T10(M4 vs. M5)	(1,0,0)	(0,1,0)	(**1**,**1**,0)	(0,1,0)	(0,1,0)	(0,1,0)	(0,**3**,0)	(1,**4**,0)

Notes: (1) M1, M2, M3, M4 and M5 are explained in Table 2. (2) There are ten groups (T1−T10) of models that are used to perform the MDM tests based on *MAE* loss function. For each group, the models inside the bracket, respectively, are the first and the second models in the corresponding MDM test. (3) According to Choudhry and Wu [18], each MDM test generates two statistics, S_1 and S_2. The null hypothesis of both S_1 and S_2 is that no statistical difference exists between two sets of forecast errors, whereas the alternative hypothesis of S_1 (or, respectively, S_2) is that the first (or, respectively, second) set of forecast errors is significantly smaller than the second (or, respectively, first). (4) If S_1 is significant, then the first forecasting model in the bracket beside the corresponding test outperforms the second model in that bracket. If S_2 is significant, then the second forecasting model in the bracket beside the corresponding test outperforms the first model in that bracket. If neither of S_1 and S_2 is significant, then the two models produce equally accurate forecasts. (5) The first (or, respectively, second) number in parenthesis of each cell takes the value of 1 if statistic S_1 (or, respectively, S_2) is significant at 10% level and 0 otherwise whereas the third number takes the value of 1 if neither of statistic S_1 and S_2 is significant at 10% level. (6) The first (or, respectively, second) number in parenthesis of column 'SUM' denotes the total number of all variables that the first (or, respectively, second) model outperforms the second (or, respectively, first) whereas the third number represents the total number of all variables that the two models produce equally accurate forecasts. The first (or, respectively, second) number in parenthesis of column 'SUM1' denotes the total number of two exchange rates that the first (or, respectively, second) model outperforms the second (or, respectively, first) whereas the third number represents the total number of two exchange rates that the two models produce equally accurate forecasts. The first (or, respectively, second) number in parenthesis of column 'SUM2' denotes the total number of three exchange rate indices that the first (or, respectively, second) model outperforms the second (or, respectively, first) whereas the third number represents the total number of three exchange rate indices that the two models produce equally accurate forecasts. (7) The bold font denotes the largest value among three numbers inside the parenthesis for each cell.

From three aspects, a discussion of the results in Table 8 is as follows. First, according to the column 'SUM1' about exchange rates, the USD-CNY and the EUR-CNY, the following conclusions can be obtained. (1) The first number inside the parenthesis is not greater than the second number and the third number at the same time for the T1–T4 tests. This indicates the superiority of the models with jump factors. (2) The T5 test in Table 8 shows that the GJR-GARCH(1,1) model with Poisson jumps (M3) is obviously outperform the GARCH(1,1) model with Poisson jumps (M2). It implies that the exchange rate return series of USD-CNY and EUR-CNY may have asymmetry. (3) The GARCH(1,1) model with Poisson jumps (M2) is not superior to the GARCH(1,1) model with double exponential jumps (M4) (see T6 in Table 8). It reflects the rote of the double exponential distribution. (4) The GJR-GARCH(1,1) model with double exponential jumps (M5) is obviously superior to the GARCH(1,1) model with Poisson jumps (M2) (see T7 in Table 8). It is consistent with the above conclusions (2) and (3). (4) The GJR-GARCH(1,1) model with double exponential jumps (M5) is the same as the GJR-GARCH(1,1) model with double exponential jumps (M5) (see T10 in Table 8). Compared with the above conclusion (3), we can find that when describing the return change of exchange rate, the double exponential jump factor is more important to the individual asymmetry and Poisson jump factor. In summary, the GARCH(1,1) model with double exponential jumps (M4) is the best of the five models to describe the return series of exchange rate.

Second, according to column 'SUM2' about the three exchange rate indices, the CNYX-CFETS, the CNYX-SDR and the CNYX-BIS, we obtain the following results. (1) The second number inside the parenthesis is bigger than the first number and the third number at the same time for the T5–T7 tests. This implies the return series of exchange rate indices are not symmetrical. (2) T8 in Table 8 shows that the GJR-GARCH(1,1) model with Poisson jumps (M3) and the GARCH(1,1) model with double exponential jumps (M4) produce equally accurate forecasts. (3) The GJR-GARCH(1,1) model with double exponential jumps (M5) is obviously superior to the GJR-GARCH(1,1) model with Poisson jumps (M3) (see T9 in Table 8) and the GARCH(1,1) model with double exponential jumps (M4) (see T10 in Table 8). It indicates that the GJR-GARCH(1,1) model with double exponential jumps (M5) can be applied to the return series of exchange rate indices. (4) However, according to T1–T4, the GARCH(1,1) model (M1) obviously outperform the following four models, the GARCH(1,1) model with Poisson jumps (M2), the GJR-GARCH(1,1) model with Poisson jumps (M3), the GARCH(1,1) model with double exponential jumps (M4) and the GJR-GARCH(1,1) model with double exponential jumps (M5). It implies that return series of exchange rate indices have not obvious jump characteristics. The above results comprehensively indicate that the GARCH(1,1) model (M1) is the best choice for the return series of exchange rate indices.

Third, according to column 'SUM' about all variables in Table 8, these tests indicate the following. (1) According to the T5–T7 tests, the GARCH(1,1) model with Poisson jumps (M2) is obviously worse than the following three models, the GJR-GARCH(1,1) model with Poisson jumps (M3), the GARCH(1,1) model with double exponential jumps (M4) and the GJR-GARCH(1,1) model with double exponential jumps (M5). It implies that return series of exchange rate indices have obvious Poisson jump characteristics. (2) The GJR-GARCH(1,1) model with double exponential jumps (M5) is superior to the GJR-GARCH(1,1) model with Poisson jumps (M3) (see T9 in Table 8) and the GARCH(1,1) model with double exponential jumps (M4) (see T10 in Table 8). These results are consistent with the results derived from the return *MAE* in Table 7. This further confirms the superiority of the double exponential distribution. These results are consistent with the results derived from the return *MAE* in Table 7. In summary, the GJR-GARCH(1,1) model with double exponential jumps (M5) is the best model, which indicates that there is characteristics of asymmetry and double exponential jumps in return series.

In addition, Table 9 presents the MDM test results based on the *RMSE* loss function as the error criterion. On the one hand, for the results of column 'SUM1' about exchange rates and column 'SUM2' about exchange rate indices, similar results to Table 8 are obtained. When analyzing the return series of CNY exchange rates and CNY exchange rate indices, the corresponding best choices are the GJR-GARCH(1,1) model with double exponential jumps (M5). On the other hand, the results of column 'SUM' about all variables show that the GARCH(1,1) model with double exponential jumps (M4) outperforms the GARCH(1,1) model (M1) (see T13 in Table 9), and the GJR-GARCH(1,1) model with double exponential jumps (M5) is obviously superior to the GJR-GARCH(1,1) model with Poisson jumps (M3) (see T19 in Table 9). This indicates that the double exponential distribution is more conducive in describing the leverage effect in the fluctuation process of return series.

Table 9. The MDM test results based on *RMSE* loss function.

	USD-CNY	EUR-CNY	SUM1	CNYX-CFETS	CNYX-SDR	CNYX-BIS	SUM2	SUM
T11(M1 vs. M2)	(0,0,1)	(0,0,1)	(0,0,**2**)	(0,0,1)	(0,1,0)	(1,0,0)	(**1**,**1**,**1**)	(0,2,**3**)
T12(M1 vs. M3)	(0,1,0)	(0,0,1)	(0,**1**,**1**)	(1,0,0)	(0,0,1)	(1,0,0)	(**2**,0,1)	(**2**,1,**2**)
T13(M1 vs. M4)	(0,1,0)	(0,1,0)	(0,**2**,0)	(0,0,1)	(0,1,0)	(1,0,0)	(**1**,**1**,**1**)	(1,**3**,1)
T14(M1 vs. M5)	(0,1,0)	(0,1,0)	(0,**2**,0)	(0,0,1)	(0,1,0)	(1,0,0)	(**1**,**1**,**1**)	(1,**3**,1)
T15(M2 vs. M3)	(0,1,0)	(0,1,0)	(0,**2**,0)	(0,0,1)	(1,0,0)	(0,1,0)	(**1**,**1**,**1**)	(1,**3**,1)
T16(M2 vs. M4)	(0,0,1)	(0,0,1)	(0,0,**2**)	(0,0,1)	(1,0,0)	(0,1,0)	(**1**,**1**,**1**)	(1,1,**3**)
T17(M2 vs. M5)	(0,0,1)	(0,1,0)	(0,**1**,**1**)	(0,0,1)	(1,0,0)	(0,1,0)	(**1**,**1**,**1**)	(1,**2**,**2**)
T18(M3 vs. M4)	(0,0,1)	(1,0,0)	(**1**,0,**1**)	(0,1,0)	(0,0,1)	(0,1,0)	(**1**,**1**,**1**)	(**2**,1,**2**)
T19(M3 vs. M5)	(0,0,1)	(0,0,1)	(0,0,**2**)	(0,1,0)	(0,1,0)	(0,1,0)	(0,**3**,0)	(0,**3**,2)
T20(M4 vs. M5)	(1,0,0)	(0,1,0)	(**1**,**1**,0)	(0,0,1)	(0,1,0)	(0,1,0)	(0,**2**,1)	(1,**3**,1)

Notes: (1) M1, M2, M3, M4 and M5 are explained in Table 2. (2) There are ten groups (T11−T20) of models that are used to perform the MDM tests based on *RMAE* loss function. For each group, the models inside the bracket, respectively, are the first and the second models in the corresponding MDM test. (3) According to Choudhry and Wu [18], each MDM test generates two statistics, S_1 and S_2. The null hypothesis of both S_1 and S_2 is that no statistical difference exists between two sets of forecast errors, whereas the alternative hypothesis of S_1 (or, respectively, S_2) is that the first (or, respectively, second) set of forecast errors is significantly smaller than the second (or, respectively, first). (4) If S_1 is significant, then the first forecasting model in the bracket beside the corresponding test outperforms the second model in that bracket. If S_2 is significant, then the second forecasting model in the bracket beside the corresponding test outperforms the first model in that bracket. If neither of S_1 and S_2 is significant, then the two models produce equally accurate forecasts. (5) The first (or, respectively, second) number in parenthesis of each cell takes the value of 1 if statistic S_1 (or, respectively, S_2) is significant at 10% level and 0 otherwise whereas the third number takes the value of 1 if neither of statistic S_1 and S_2 is significant at 10% level. (6) The first (or, respectively, second) number in parenthesis of column 'SUM' denotes the total number of all variables that the first (or, respectively, second) model outperforms the second (or, respectively, first) whereas the third number represents the total number of all variables that the two models produce equally accurate forecasts. The first (or, respectively, second) number in parenthesis of column 'SUM1' denotes the total number of two exchange rates that the first (or, respectively, second) model outperforms the second (or, respectively, first) whereas the third number represents the total number of two exchange rates that the two models produce equally accurate forecasts. The first (or, respectively, second) number in parenthesis of column 'SUM2' denotes the total number of three exchange rate indices that the first (or, respectively, second) model outperforms the second (or, respectively, first) whereas the third number represents the total number of three exchange rate indices that the two models produce equally accurate forecasts. (7) The bold font denotes the largest value among three numbers inside the parenthesis for each cell.

In a word, the above findings further indicate that the CNY exchange rates and the CNY exchange rate indices have asymmetry and jump features. Therefore, the models with jump factors can better handle and capture the agglomeration and thick tail characteristics of return fluctuations in financial market than the models without jump factors. Our findings will provide valuable reference for investors in the Chinese foreign exchange market, and have great guiding significance to manage and control the exchange rate risk for policy-makers.

5. Conclusions

Exchange rate is an important lever for economic adjustment. Because it has important relationship with other economic factors, it affects the internal and external economies of the country and even the international economy, and plays an important role in macroeconomic regulations and economic stability. The exchange rate risk should not be overlooked in international financial market, and the exchange rate has an irreplaceable position for the development of China's economy.

With the development of global economic integration and the strengthening of the CNY internationalization, foreign transactions are becoming more frequent. In this process, the importance of the exchange rate is obvious. In this article, we mainly focus on the following two exchange rates, the Chinese yuan against the U.S. dollar (USD-CNY) and the Chinese yuan against the Euro (EUR-CNY). The return changes of these exchange rates are analyzed. In addition, in order to more comprehensively explore the change of exchange rate in China, three CNY exchange rate indices, which refer to relevant basket of currencies, are considered. They are the CNY exchange rate index referring the currency basket of China Foreign Exchange Trading System (CNYX-CFETS), the CNY exchange rate index referring the currency basket of Special Drawing Rights (CNYX-SDR) and the CNY exchange rate index referring the currency basket of Bank for International Settlements (CNYX-BIS). The CNY exchange rate index referring to a certain basket of currencies is beneficial to not only maintain the relative stability of the exchange rate of currency basket after the decoupling of the CNY from the U.S. dollar or the Euro, but also leaves enough space to make timely adjustments in the event of an economic imbalance. Compared with the reference to a single currency, the CNY exchange rate indices are able to better reflect the value change of the China's currency and the role of exchange rate to adjust import and export, international investment, and balance of payments.

In this paper, we take jump factors into account to study the forecast performance of each model in returns. The jump factors include the Poisson jumps and the double exponential jumps. Then the GARCH model with Poisson jump (M2), the GJR-GARCH model with Poisson jump (M3), the GARCH model with double exponential jump (M4) and the GJR-GARCH model with double exponential jump (M5) are presented in Section 3. The accuracy of the various models covering a range of exchange rate indices was measured using the modified Diebold–Mariano (MDM) test and two loss functions, *MAE* and *RMSE*. Our empirical results show that the fluctuations of exchange rates and exchange rate indices have features such as spikes, heavy tails and asymmetry. Models with jumps have better fittings than models without jumps, and can better characterize the agglomeration and heavy tail characteristics of the return fluctuations. It can be confirmed by the results that the GARCH(1,1) model with Poisson jumps (M2), the GJR-GARCH(1,1) model with Poisson jumps (M3), the GARCH(1,1) model with double exponential jumps (M4) and the GJR-GARCH(1,1) model with double exponential jumps (M5) are not worse than the GARCH(1,1) model (M1) (see T11–T14 based on column 'SUM' in Table 9). In addition, the GJR-GARCH(1,1) model with double exponential jumps (M5) is obviously superior to the GJR-GARCH(1,1) model with Poisson jumps (M3) (see T9 based on column 'SUM' in Table 8 and T19 based on column 'SUM' in Table 9). It can be explained that the GJR-GARCH(1,1) model with double exponential jumps contains both the asymmetric and the double exponential distribution which is more conducive in describing the leverage effect in the fluctuation process of return time-series. In addition, according to the results of column 'SUM1' about exchange rates and column 'SUM2' about exchange rate indices in Table 8 and Table 9, we find that the GARCH(1,1) model with double exponential jumps (M4) is the best choice for the USD-CNY exchange rate and EUR-CNY exchange rate, and the GARCH(1,1) model (M1) may be the best choice for the three exchange rate indices, the CNYX-CFETS, CNYX-SDR and CNYX-BIS. This also demonstrates that the CNY exchange rate indices referring to baskets of currencies will have stronger stability than the exchange rates only referring to a single currency.

Our research findings will be significant to provide practical reference and valuable information for government, financial institutions, or decision makers to effectively manage the exchange rate risk in China, and further improve the financial risk management mechanism. But this study is based on structured data information to build a model. In the actual market, with the development of financial big data, the factors that affect the exchange rate include both structured information and unstructured information. In our future research, the model can be further optimized based on unstructured information variables, which will have better description ability.

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Conflict of interest

The authors declare no conflicts of interest.

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