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Research article

Implementation of Yang residual power series method to solve fractional non-linear systems

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Abstract: In this study, we implemented the Yang residual power series (YRPS) methodology, a unique analytical treatment method, to estimate the solutions of a non-linear system of fractional partial differential equations. The RPS approach and the Yang transform are togethered in the YRPS method. The suggested approach to handle fractional systems is explained along with its application. With fewer calculations and greater accuracy, the limit idea is used to solve it in Yang space to produce the YRPS solution for the proposed systems. The benefit of the new method is that it requires less computation to get a power series form solution, whose coefficients should be established in a series of algebraic steps. Two attractive initial value problems were used to test the technique's applicability and performance. The behaviour of the approximative solutions is numerically and visually discussed, along with the effect of fraction order ς. It was observed that the proposed method's approximations and exact solutions were completely in good agreement. The YRPS approach results highlight and show that the approach may be utilized to a variety of fractional models of physical processes easily and with analytical efficiency.

Keywords: Yang transform; Caputo operator; residual power series; systems of fractional differential equations
Mathematics Subject Classification: 32B15, 34A34, 35A22, 35A24, 45A10

1. Introduction

In the modelling of numerous real-world events in control theory, chemistry, physics and other branches of engineering and science, fractional derivatives can be used successfully [1–5]. The basic reason for this is that fractional calculus can be used to model real-world issues successfully because realistic models not only depend on the present time but also on the past historical time [6–8]. As a result, the fractional differential equations (FDEs) have drawn the interest of numerous scientific and engineering academics due to their significant role in understanding a number of real-life process that happen in the natural sciences, such as mechanical systems, wave propagation phenomena, earthquake modelling, image processing and control theory. Fractional calculus can be used to describe and reformulate these processes as FDEs. The use of FDEs in the aforementioned and other phenomena is notable for their nonlocality feature. Similarly, the differential operators give an excellent way to describe the memory and inherited characteristics of distinct processes and materials [9, 10].

In the framework of fractional derivatives, partial differential equations (PDEs) are regarded as a potential tool in mathematical modelling to explain and understand some physical processes structures that are complicated due to outside influences. Because of this reason, researchers utilized them to both construct a natural problem that is easily accessible and to simplify the regulating plan without losing some inherited material or memorial impact [11–13]. The development of trustworthy numerical methods to handle the fraction PDEs of physical concern has also been the subject of countless efforts, many of which have been successful [14–19]. Many real-world issues, such as earthquakes, gas dynamics, traffic flow and oscillation, can be formulated as nonlinear PDEs in the framework of fractional derivative, and the solutions of fractional PDEs provide remarkable insight into the behaviour of particular dynamic systems [20, 21]. Therefore, it is important to develop a practical and useful method for identifying analytical solutions to these and other issues. To examine and develop analytical-approximate solutions of fractional FDEs and PDEs, researchers have recently used a variety of analytical and numerical methodologies, such as the Homotopy perturbation transform method [22], residual power series (RPS) method [23], Adomian decomposition method [24], variational iteration transform method [25, 26], Iterative laplace transform method [27], Elzaki transform decomposition method [28, 29] and Natural transform decomposition method [30, 31].

A numerical analytic method for solving many forms of ordinary, partial, integro-differential equations and fractional fuzzy differential equations is known as the residual power series (RPS) technique. The RPSM was introduced by Jordanian mathematician Omar Abu Arqub in 2013 [32]. Due to the fact that it offers closed-form solutions of well-known functions, it is an efficient optimization strategy [33, 34]. With the use of the fractional residual power series (FRPS) technique, fuzzy FDEs, a variety of FDEs, and integral equations having fractional-order are solved. Such as, Coupled fractional resonant Schrödinger equations [35], time-fractional Fokker-Planck models [36], fractional fredholm integro-differential equations [37], fractional Newell-hitehead-Segel equation [38], singular initial value problems [39], Fractional partial differential equations [40], fractional Kundu-Eckhaus and massive Thirring equations [41] and several classes of fractional fuzzy differential equations [42, 43]. The Yang transform (YT) is a powerful tool for resolving numerous complex models that are appearing in various branches of the natural sciences. When analytical methods are combined with the YT operator, non-linear problems can be solved more quickly and with more precision.

Researchers introduced a new approach to solve fractional differential equations by combining two well-known approaches. Several of these groups include a combination of the homotopy analysis method and the natural transform [44], also homotopy perturbation approach and the Sumudu transform [45], the Yang transform and the Adomian decomposition method [46], and the Laplace transform with RPSM [47]. The main goal of this work is to examine the approximate solution of nonlinear systems by implementing the Yang residual power series (YRPS) method. The YT and RPS approaches are combined in the YRPS approach, which provides both approximate and accurate solutions as quickly fractional power series (FPS) solutions. The proposed system is converted to Yang space, and then the solutions in the form of algebraic equations are created. Lastly, the Yang inverse is applied to the proposed problem results. In contrast to the FRPS approach, which depends on the fractional derivative and consume time to compute the various derivatives of fractional-order in steps of determining the solutions, the unknown coefficients in a modified Yang expansion can be identified by employing the limit idea. The YRPS approach take less time and provide higher accuracy with minor computational requirements.

In this article, we consider fractional nonlinear systems as:

$$\begin{aligned} &D_t^{\varsigma}u + v_{\vartheta}w_{\psi} - v_{\psi}w_{\vartheta} + u = 0, \quad 0 < \varsigma \leq 1,\\ &D_t^{\varsigma}v + w_{\vartheta}u_{\psi} - u_{\psi}w_{\vartheta} - v = 0,\\ &D_t^{\varsigma}w + u_{\vartheta}v_{\psi} - u_{\psi}v_{\vartheta} - w = 0, \end{aligned} \tag{1.1}$$

having initial sources

$$\begin{aligned} u(\vartheta,\psi,0) &= f_0(\vartheta,\psi),\\ v(\vartheta,\psi,0) &= g_0(\vartheta,\psi),\\ w(\vartheta,\psi,0) &= h_0(\vartheta,\psi), \end{aligned} \tag{1.2}$$

and

$$\begin{aligned} &D_t^{\varsigma}u - u_{\vartheta\vartheta} - 2uu_{\vartheta} + (uv)_{\vartheta} = 0, \quad 0 < \varsigma \leq 1,\\ &D_t^{\varsigma}v - v_{\vartheta\vartheta} - 2vv_{\vartheta} + (uv)_{\vartheta} = 0, \end{aligned} \tag{1.3}$$

having initial sources

$$\begin{aligned} u(\vartheta,\psi,0) &= f_0(\vartheta,\psi),\\ v(\vartheta,\psi,0) &= g_0(\vartheta,\psi). \end{aligned} \tag{1.4}$$

Following is the breakdown of our study. Section 2 reviews the YT, as well as some fundamental definitions and theorem pertaining to fractional calculus. Section 3 describes the idea behind the suggested technique for creating the approximation of the fractional model taken into account in Eq (1.1). In Section 4, the YRPS methodology is applied to solve fractional nonlinear systems in order to show the applicability and efficacy of the method in analysing the solutions of time-PDEs of fractional order. Section 5 concludes with a summary of our results.

2. Preliminaries

In this part, we review some definitions and fractional derivatives theorems in Caputo manner along with YT properties.

Definition 2.1. In Caputo sense the fractional derivative of a function $u(\vartheta, t)$ is given as [48]

$$ {}^{C}D_t^{\varsigma}u(\vartheta, t) = J_t^{m-\varsigma}u^{m}(\vartheta, t), \quad m-1 < \varsigma \leq m, \quad t > 0, \tag{2.1} $$

with $m \in N$ and J_t^{ς} represents the fractional integral in Riemann-Liouville manner of $u(\vartheta, t)$ as

$$ J_t^{\varsigma}u(\vartheta, t) = \frac{1}{\Gamma(\varsigma)}\int_0^t (t-\tau)^{\varsigma-1}u(\vartheta, \tau)d\tau. \tag{2.2} $$

Definition 2.2. The YT of a function $\phi(\kappa)$ is stated by $\mathcal{Y}\{u(t)\}$ or M(s) as [49, 50]

$$ \mathcal{Y}\{u(t)\} = M(s) = \int_0^{\infty} e^{\frac{-t}{s}}u(t)dt, \quad t > 0, \quad s \in (-t_1, t_2). \tag{2.3} $$

The inverse YT is given as

$$ \mathcal{Y}^{-1}\{M(s)\} = u(t). \tag{2.4} $$

Definition 2.3. The nth derivatives YT is given [49, 50]

$$ \mathcal{Y}\{u^{n}(t)\} = \frac{M(s)}{s^{n}} - \sum_{k=0}^{n-1}\frac{u^{k}(0)}{s^{n-k-1}}, \quad \forall \ n = 1, 2, 3, \cdots \tag{2.5} $$

Definition 2.4. The YT of derivatives having order fraction is as [49, 50]

$$ \mathcal{Y}\{u^{\varsigma}(t)\} = \frac{M(s)}{s^{\varsigma}} - \sum_{k=0}^{n-1}\frac{u^{k}(0)}{s^{\varsigma-(k+1)}}, \quad 0 < \varsigma \leq n. \tag{2.6} $$

Theorem 2.5. *The fractional power series $\sum_{m-0}^{\infty} a_m(t-\varepsilon)^{mp}$ can converge within just three of the following ways:*
(1) The series converges only when the radius of convergence equals zero, or $t = \varepsilon$.
(2) The series converges with a radius of convergence equal to ∞ for all $t \geq \varepsilon$.
(3) The series diverges for $t > c + R$ and converges for $\varepsilon \leq t < \varepsilon + R$ and some real positive integer R. In this context, R refers to the fractional power series radius of convergence.

3. Idea of YRPS

In this part, we will present the general implementation of YRPS for solving fractional nonlinear systems of PDEs.

By employing YT to Eq (1.1), we have

$$ \begin{aligned} &U(\vartheta, \psi, s) - sf_0(\vartheta, \psi) + s^{\varsigma}\mathcal{Y}_t\left[\mathcal{Y}_t^{-1}[V_\vartheta]\mathcal{Y}_t^{-1}[W_\psi] - \mathcal{Y}_t^{-1}[V_\psi]\mathcal{Y}_t^{-1}[W_\vartheta] + \mathcal{Y}_t^{-1}[U]\right] = 0, \\ &V(\vartheta, \psi, s) - sg_0(\vartheta, \psi) + s^{\varsigma}\mathcal{Y}_t\left[\mathcal{Y}_t^{-1}[W_\vartheta]\mathcal{Y}_t^{-1}[U_\psi] - \mathcal{Y}_t^{-1}[U_\psi]\mathcal{Y}_t^{-1}[W_\vartheta] - \mathcal{Y}_t^{-1}[V]\right] = 0, \\ &W(\vartheta, \psi, s) - sh_0(\vartheta, \psi) + s^{\varsigma}\mathcal{Y}_t\left[\mathcal{Y}_t^{-1}[U_\vartheta]\mathcal{Y}_t^{-1}[V_\psi] - \mathcal{Y}_t^{-1}[U_\psi]\mathcal{Y}_t^{-1}[V_\vartheta] - \mathcal{Y}_t^{-1}[W]\right] = 0. \end{aligned} \tag{3.1} $$

Considering that the solution of Eq (3.1) has the appropriate expansion

$$\begin{aligned} U(\vartheta,\psi,s) &= \sum_{n=0}^{\infty} s^{n\varsigma+1} f_n(\vartheta,\psi,s), \quad V(\vartheta,\psi,s) = \sum_{n=0}^{\infty} s^{n\varsigma+1} g_n(\vartheta,\psi,s), \\ W(\vartheta,\psi,s) &= \sum_{n=0}^{\infty} s^{n\varsigma+1} h_n(\vartheta,\psi,s). \end{aligned} \tag{3.2}$$

The k^{th}-truncated series are

$$\begin{aligned} U(\vartheta,\psi,s) &= sf_0(\vartheta,\psi,s) + \sum_{n=1}^{k} s^{n\varsigma+1} f_n(\vartheta,\psi,s), \quad V(\vartheta,\psi,s) = sg_0(\vartheta,\psi,s) + \sum_{n=1}^{k} s^{n\varsigma+1} g_n(\vartheta,\psi,s), \\ W(\vartheta,\psi,s) &= sh_0(\vartheta,\psi,s) + \sum_{n=1}^{k} s^{n\varsigma+1} h_n(\vartheta,\psi,s). \quad k = 1,2,3,4\cdots \end{aligned} \tag{3.3}$$

By Yang residual functions (YRFs)

$$\begin{aligned} \mathcal{Y}_t Res_u(\vartheta,\psi,s) =& U(\vartheta,\psi,s) - sf_0(\vartheta,\psi,s) + s^{\varsigma}\mathcal{Y}_t\left[\mathcal{Y}_t^{-1}[V_\vartheta]\mathcal{Y}_t^{-1}[W_\psi] - \mathcal{Y}_t^{-1}[V_\psi]\mathcal{Y}_t^{-1}[W_\vartheta] + \mathcal{Y}_t^{-1}[U]\right], \\ \mathcal{Y}_t Res_v(\vartheta,\psi,s) =& V(\vartheta,\psi,s) - sg_0(\vartheta,\psi,s) + s^{\varsigma}\mathcal{Y}_t\left[\mathcal{Y}_t^{-1}[W_\vartheta]\mathcal{Y}_t^{-1}[U_\psi] - \mathcal{Y}_t^{-1}[U_\psi]\mathcal{Y}_t^{-1}[W_\vartheta] - \mathcal{Y}_t^{-1}[V]\right], \\ \mathcal{Y}_t Res_\rho(\vartheta,\psi,s) =& W(\vartheta,\psi,s) - sh_0(\vartheta,\psi,s) + s^{\varsigma}\mathcal{Y}_t\left[\mathcal{Y}_t^{-1}[U_\vartheta]\mathcal{Y}_t^{-1}[V_\psi] - \mathcal{Y}_t^{-1}[U_\psi]\mathcal{Y}_t^{-1}[V_\vartheta] - \mathcal{Y}_t^{-1}[W]\right]. \end{aligned} \tag{3.4}$$

And the k^{th}-YRFs as:

$$\begin{aligned} \mathcal{Y}_t Res_{u,k}(\vartheta,\psi,s) =& U_k(\vartheta,\psi,s) - sf_0(\vartheta,\psi,s) + s^{\varsigma}\mathcal{Y}_t\left[\mathcal{Y}_t^{-1}[V_{\vartheta,k}]\mathcal{Y}_t^{-1}[W_{\psi,k}] - \mathcal{Y}_t^{-1}[V_{\psi,k}]\mathcal{Y}_t^{-1}[W_{\vartheta,k}] + \mathcal{Y}_t^{-1}[U_k]\right], \\ \mathcal{Y}_t Res_{v,k}(\vartheta,\psi,s) =& V_k(\vartheta,\psi,s) - sg_0(\vartheta,\psi,s) + s^{\varsigma}\mathcal{Y}_t\left[\mathcal{Y}_t^{-1}[W_{\vartheta,k}]\mathcal{Y}_t^{-1}[U_{\psi,k}] - \mathcal{Y}_t^{-1}[U_{\psi,k}]\mathcal{Y}_t^{-1}[W_{\vartheta,k}] - \mathcal{Y}_t^{-1}[V_k]\right], \\ \mathcal{Y}_t Res_{w,k}(\vartheta,\psi,s) =& W_k(\vartheta,\psi,s) - sh_0(\vartheta,\psi,s) + s^{\varsigma}\mathcal{Y}_t\left[\mathcal{Y}_t^{-1}[U_{\vartheta,k}]\mathcal{Y}_t^{-1}[V_{\psi,k}] - \mathcal{Y}_t^{-1}[U_{\psi,k}]\mathcal{Y}_t^{-1}[V_{\vartheta,k}] - \mathcal{Y}_t^{-1}[W_k]\right]. \end{aligned} \tag{3.5}$$

To highlight some facts, the YRPSM contains the following characteristics:

- $\mathcal{Y}_t Res(\vartheta,\psi,s) = 0$ and $\lim_{j\to\infty} \mathcal{Y}_t Res_{u,k}(\vartheta,\psi,s) = \mathcal{Y}_t Res_u(\vartheta,\psi,s)$ for each $s > 0$.
- $\lim_{s\to\infty} s\mathcal{Y}_t Res_u(\vartheta,\psi,s) = 0 \Rightarrow \lim_{s\to\infty} s\mathcal{Y}_t Res_{u,k}(\vartheta,\psi,s) = 0$.
- $\lim_{s\to\infty} s^{k\varsigma+1}\mathcal{Y}_t Res_{u,k}(\vartheta,\psi,s) = \lim_{s\to\infty} s^{k\varsigma+1}\mathcal{Y}_t Res_{u,k}(\vartheta,\psi,s) = 0, \quad 0 < \varsigma \leq 1, \quad k = 1,2,3,\cdots.$

Now to determine the coefficients $f_n(\vartheta,\psi,s)$, $g_n(\vartheta,\psi,s)$, $h_n(\vartheta,\psi,s)$ and $l_n(\vartheta,\psi,s)$, we resolve the below system recursively as

$$\begin{aligned} \lim_{s\to\infty} s^{k\varsigma+1}\mathcal{Y}_t Res_{u,k}(\vartheta,\psi,s) = 0, \quad k = 1,2,\cdots, \\ \lim_{s\to\infty} s^{k\varsigma+1}\mathcal{Y}_t Res_{v,k}(\vartheta,\psi,s) = 0, \quad k = 1,2,\cdots, \\ \lim_{s\to\infty} s^{k\varsigma+1}\mathcal{Y}_t Res_{\rho,k}(\vartheta,\psi,s) = 0, \quad k = 1,2,\cdots. \end{aligned} \tag{3.6}$$

Finally by employing inverse YT to Eq (3.3), to obtain the k^{th} analytical solutions of $u_k(\vartheta,\psi,t)$, $v_k(\vartheta,\psi,t)$ and $\rho_k(\vartheta,\psi,t)$

4. Applications

In this part, we find the solution of fractional nonlinear systems by implementing the suggested approach.

4.1. Problem

Let us assume the system of fractional PDEs:

$$\begin{aligned} &D_t^{\varsigma} u + v_{\vartheta} w_{\psi} - v_{\psi} w_{\vartheta} + u = 0, \\ &D_t^{\varsigma} v + w_{\vartheta} u_{\psi} - u_{\psi} w_{\vartheta} - v = 0, \\ &D_t^{\varsigma} w + u_{\vartheta} v_{\psi} - u_{\psi} v_{\vartheta} - w = 0. \end{aligned} \tag{4.1}$$

By considering Eq (4.1), having below initial sources:

$$\begin{aligned} &u(\vartheta, \psi, 0) = e^{\vartheta+\psi}, \\ &v(\vartheta, \psi, 0) = e^{\vartheta-\psi}, \\ &w(\vartheta, \psi, 0) = e^{\psi-\vartheta}. \end{aligned} \tag{4.2}$$

By employing YT to Eq (4.1) and using Eq (4.2), we have

$$\begin{aligned} &U(\vartheta, \psi, s) - se^{\vartheta+\psi} + s^{\varsigma} \mathcal{Y}_t \left[\mathcal{Y}_t^{-1}[V_{\vartheta}] \mathcal{Y}_t^{-1}[W_{\psi}] - \mathcal{Y}_t^{-1}[V_{\psi}] \mathcal{Y}_t^{-1}[W_{\vartheta}] + \mathcal{Y}_t^{-1}[U] \right] = 0, \\ &V(\vartheta, \psi, s) - se^{\vartheta-\psi} + s^{\varsigma} \mathcal{Y}_t \left[\mathcal{Y}_t^{-1}[W_{\vartheta}] \mathcal{Y}_t^{-1}[U_{\psi}] - \mathcal{Y}_t^{-1}[U_{\psi}] \mathcal{Y}_t^{-1}[W_{\vartheta}] - \mathcal{Y}_t^{-1}[V] \right] = 0, \\ &W(\vartheta, \psi, s) - se^{\psi-\vartheta} + s^{\varsigma} \mathcal{Y}_t \left[\mathcal{Y}_t^{-1}[U_{\vartheta}] \mathcal{Y}_t^{-1}[V_{\psi}] - \mathcal{Y}_t^{-1}[U_{\psi}] \mathcal{Y}_t^{-1}[V_{\vartheta}] - \mathcal{Y}_t^{-1}[W] \right] = 0. \end{aligned} \tag{4.3}$$

The k^{th}-truncated term series are

$$\begin{aligned} &U(\vartheta, \psi, s) = se^{\vartheta+\psi} + \sum_{n=1}^{k} s^{n\varsigma+1} f_n(\vartheta, \psi, s), \quad V(\vartheta, \psi, s) = se^{\vartheta-\psi} + \sum_{n=1}^{k} s^{n\varsigma+1} g_n(\vartheta, \psi, s), \\ &W(\vartheta, \psi, s) = se^{\psi-\vartheta} + \sum_{n=1}^{k} s^{n\varsigma+1} h_n(\vartheta, \psi, s), \quad k = 1, 2, 3, 4 \cdots \end{aligned} \tag{4.4}$$

and the k^{th}-YRFs as:

$$\begin{aligned} \mathcal{Y}_t Res_{u,k}(\vartheta, \psi, s) =& U_k(\vartheta, \psi, s) - se^{\vartheta+\psi} + s^{\varsigma} \mathcal{Y}_t \left[\mathcal{Y}_t^{-1}[V_{\vartheta,k}] \mathcal{Y}_t^{-1}[W_{\psi,k}] - \mathcal{Y}_t^{-1}[V_{\psi,k}] \mathcal{Y}_t^{-1}[W_{\vartheta,k}] + \mathcal{Y}_t^{-1}[U_k] \right], \\ \mathcal{Y}_t Res_{v,k}(\vartheta, \psi, s) =& V_k(\vartheta, \psi, s) - se^{\vartheta-\psi} + s^{\varsigma} \mathcal{Y}_t \left[\mathcal{Y}_t^{-1}[W_{\vartheta,k}] \mathcal{Y}_t^{-1}[U_{\psi,k}] - \mathcal{Y}_t^{-1}[U_{\psi,k}] \mathcal{Y}_t^{-1}[W_{\vartheta,k}] - \mathcal{Y}_t^{-1}[V_k] \right], \\ \mathcal{Y}_t Res_{w,k}(\vartheta, \psi, s) =& W_k(\vartheta, \psi, s) - se^{\psi-\vartheta} + s^{\varsigma} \mathcal{Y}_t \left[\mathcal{Y}_t^{-1}[U_{\vartheta,k}] \mathcal{Y}_t^{-1}[V_{\psi,k}] - \mathcal{Y}_t^{-1}[U_{\psi,k}] \mathcal{Y}_t^{-1}[V_{\vartheta,k}] - \mathcal{Y}_t^{-1}[W_k] \right]. \end{aligned} \tag{4.5}$$

To find $f_k(\vartheta, \psi, s)$, $g_k(\vartheta, \psi, s)$ and $h_k(\vartheta, \psi, s)$ $k = 1, 2, 3, \cdots$, the kth-truncated series equation Eq (4.4) will be inserted into the kth-Yang residual function equation Eq (4.5), which will then be multiplied by $s^{k\varsigma+1}$ to solve the relation recursively $\lim_{s\to\infty}(s^{k\varsigma+1} \mathcal{Y}_t Res_{u,k}(\vartheta, \psi, s)) = 0$, $\lim_{s\to\infty}(s^{k\varsigma+1} \mathcal{Y}_t Res_{v,k}(\vartheta, \psi, s)) = 0$, and $\lim_{s\to\infty}(s^{k\varsigma+1} \mathcal{Y}_t Res_{w,k}(\vartheta, \psi, s)) = 0$, $k = 1, 2, 3, \cdots$.

Few terms are as:

$$
\begin{aligned}
&f_1(\vartheta,\psi,s) = e^{\vartheta+\psi}, \quad g_1(\vartheta,\psi,s) = e^{\vartheta-\psi}, \quad h_1(\vartheta,\psi,s) = e^{\psi-\vartheta},\\
&f_2(\vartheta,\psi,s) = -e^{\vartheta+\psi}, \quad g_2(\vartheta,\psi,s) = e^{\vartheta-\psi}, \quad h_2(\vartheta,\psi,s) = e^{\psi-\vartheta},\\
&f_3(\vartheta,\psi,s) = e^{\vartheta+\psi}, \quad g_3(\vartheta,\psi,s) = e^{\vartheta-\psi}, \quad h_3(\vartheta,\psi,s) = e^{\psi-\vartheta},\\
&f_4(\vartheta,\psi,s) = -e^{\vartheta+\psi}, \quad g_4(\vartheta,\psi,s) = e^{\vartheta-\psi}, \quad h_4(\vartheta,\psi,s) = e^{\psi-\vartheta},\\
&f_5(\vartheta,\psi,s) = e^{\vartheta+\psi}, \quad g_5(\vartheta,\psi,s) = e^{\vartheta-\psi}, \quad h_5(\vartheta,\psi,s) = e^{\psi-\vartheta},\\
&f_6(\vartheta,\psi,s) = -e^{\vartheta+\psi}, \quad g_6(\vartheta,\psi,s) = e^{\vartheta-\psi}, \quad h_6(\vartheta,\psi,s) = e^{\psi-\vartheta},
\end{aligned}
\tag{4.6}
$$

and go on.

By substituting the values of $f_k(\vartheta,\psi,s)$, $g_k(\vartheta,\psi,s)$ and $h_k(\vartheta,\psi,s)$, $k = 1, 2, 3, \cdots$, in Eq (4.4), we have

$$
\begin{aligned}
U(\vartheta,\psi,s) &= se^{\vartheta+\psi} - s^{\varsigma+1}e^{\vartheta+\psi} + s^{2\varsigma+1}e^{\vartheta+\psi} - s^{3\varsigma+1}e^{\vartheta+\psi} + s^{4\varsigma+1}e^{\vartheta+\psi} - s^{5\varsigma+1}e^{\vartheta+\psi} + s^{6\varsigma+1}e^{\vartheta+\psi} + \cdots,\\
V(\vartheta,\psi,s) &= se^{\vartheta-\psi} + s^{\varsigma+1}e^{\vartheta-\psi} + s^{2\varsigma+1}e^{\vartheta-\psi} + s^{3\varsigma+1}e^{\vartheta-\psi} + s^{4\varsigma+1}e^{\vartheta-\psi} + s^{5\varsigma+1}e^{\vartheta-\psi} + s^{6\varsigma+1}e^{\vartheta-\psi} + \cdots,\\
W(\vartheta,\psi,s) &= se^{\psi-\vartheta} + s^{\varsigma+1}e^{\psi-\vartheta} + s^{2\varsigma+1}e^{\psi-\vartheta} + s^{3\varsigma+1}e^{\psi-\vartheta} + s^{4\varsigma+1}e^{\psi-\vartheta} + s^{5\varsigma+1}e^{\psi-\vartheta} + s^{6\varsigma+1}e^{\psi-\vartheta} + \cdots.
\end{aligned}
\tag{4.7}
$$

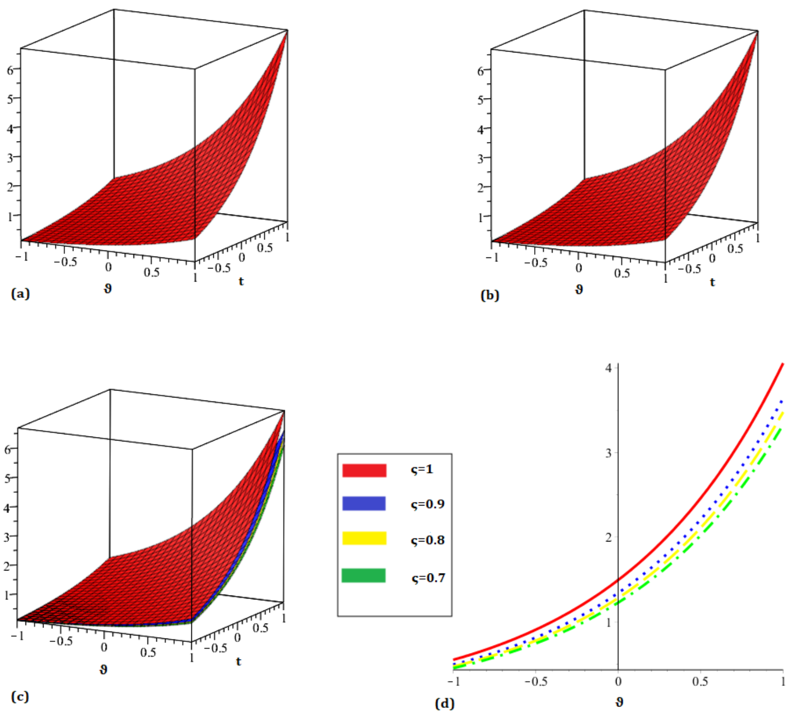

Figure 1. 3-D and 2-D behavior for $u(\vartheta,\psi,t)$ of Problem 1.

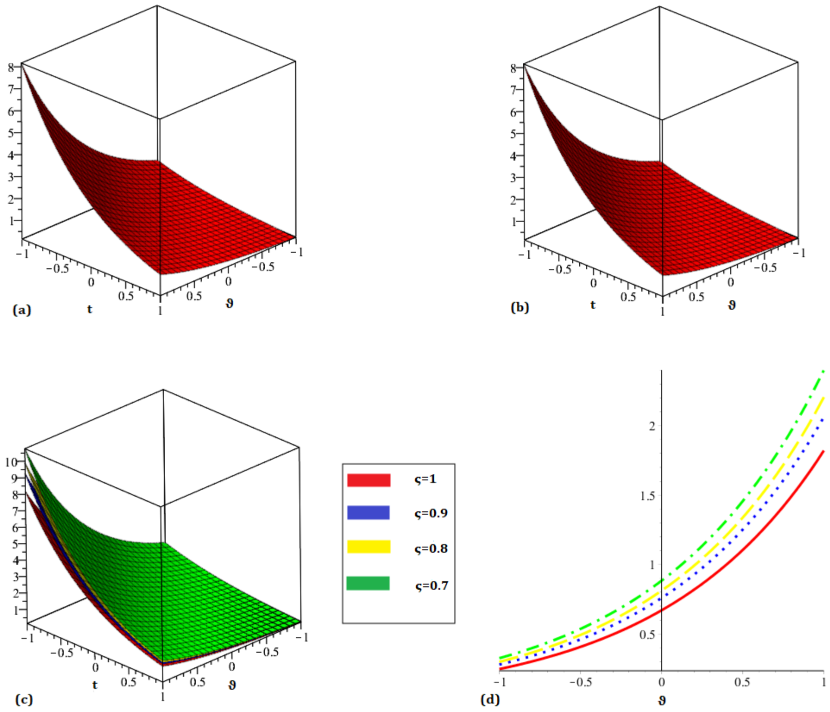

Figure 2. 3-D and 2-D behavior for $v(\vartheta, \psi, t)$ of Problem 1.

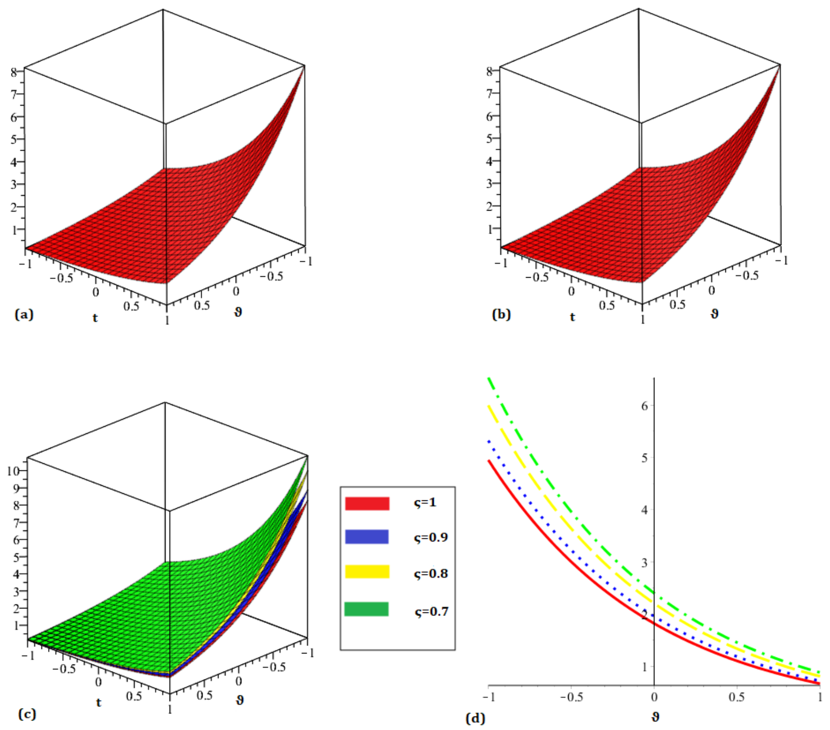

Figure 3. 3-D and 2-D behavior for $w(\vartheta, \psi, t)$ of Problem 1.

By employing inverse YT, we have

$$u(\vartheta,\psi,t)=e^{\vartheta+\psi}\left(1-\frac{t^{\varsigma}}{\Gamma(\varsigma+1)}+\frac{t^{2\varsigma}}{\Gamma(2\varsigma+1)}-\frac{t^{3\varsigma}}{\Gamma(3\varsigma+1)}+\frac{t^{4\varsigma}}{\Gamma(4\varsigma+1)}-\frac{t^{5\varsigma}}{\Gamma(5\varsigma+1)}+\cdots\right),$$
$$v(\vartheta,\psi,t)=e^{\vartheta-\psi}\left(1+\frac{t^{\varsigma}}{\Gamma(\varsigma+1)}+\frac{t^{2\varsigma}}{\Gamma(2\varsigma+1)}+\frac{t^{3\varsigma}}{\Gamma(3\varsigma+1)}+\frac{t^{4\varsigma}}{\Gamma(4\varsigma+1)}+\frac{t^{5\varsigma}}{\Gamma(5\varsigma+1)}+\cdots\right),$$
$$\rho(\vartheta,\psi,t)=e^{\psi-\vartheta}\left(1+\frac{t^{\varsigma}}{\Gamma(\varsigma+1)}+\frac{t^{2\varsigma}}{\Gamma(2\varsigma+1)}+\frac{t^{3\varsigma}}{\Gamma(3\varsigma+1)}+\frac{t^{4\varsigma}}{\Gamma(4\varsigma+1)}+\frac{t^{5\varsigma}}{\Gamma(5\varsigma+1)}+\cdots\right).$$

Putting $\varsigma=1$

$$\begin{aligned}u(\vartheta,\psi,t)&=e^{\vartheta+\psi}\left(1-t+\frac{t^2}{2!}-\frac{t^3}{3!}+\frac{t^4}{4!}-\frac{t^5}{5!}+\cdots\right),\\ v(\vartheta,\psi,t)&=e^{\vartheta-\psi}\left(1+t+\frac{t^2}{2!}+\frac{t^3}{3!}+\frac{t^4}{4!}+\frac{t^5}{5!}+\cdots\right),\\ w(\vartheta,\psi,t)&=e^{\psi-\vartheta}\left(1+t+\frac{t^2}{2!}+\frac{t^3}{3!}+\frac{t^4}{4!}+\frac{t^5}{5!}+\cdots\right).\end{aligned}\tag{4.8}$$

Thus we get exact solutions as

$$\begin{aligned}u(\vartheta,\psi,t)&=e^{\vartheta+\psi-t},\\ v(\vartheta,\psi,t)&=e^{\vartheta-\psi+t},\\ \rho(\vartheta,\psi,t)&=e^{\psi-\vartheta+t}.\end{aligned}\tag{4.9}$$

Table 1. Behavior on the basis of Error by implementing YRPSM for the suggested problem at different values of t and ϑ with $\varsigma=1$, $\psi=1$.

t	ϑ	$\lvert u_{exact}-u_{YRPSM}\rvert$	$\lvert v_{exact}-v_{YRPSM}\rvert$	$\lvert w_{exact}-w_{YRPSM}\rvert$
0.2	0.2	2.671×10^{-8}	9.927×10^{-8}	1.8080×10^{-8}
	0.4	3.263×10^{-8}	1.212×10^{-8}	1.4810×10^{-8}
	0.6	3.986×10^{-7}	1.481×10^{-8}	1.2126×10^{-7}
	0.8	4.868×10^{-7}	1.808×10^{-7}	9.9270×10^{-7}
	1	5.945×10^{-6}	2.209×10^{-7}	8.1280×10^{-6}
0.4	0.2	2.1267×10^{-7}	7.981×10^{-8}	1.4544×10^{-8}
	0.4	2.5975×10^{-7}	9.748×10^{-7}	1.1908×10^{-7}
	0.6	3.1727×10^{-6}	1.190×10^{-7}	9.7489×10^{-7}
	0.8	3.8752×10^{-6}	1.454×10^{-6}	7.9817×10^{-6}
	1	4.7330×10^{-5}	1.776×10^{-6}	6.5349×10^{-6}
0.6	0.2	7.142×10^{-6}	2.707×10^{-7}	4.9333×10^{-6}
	0.4	8.7233×10^{-6}	3.3068×10^{-6}	4.0390×10^{-6}
	0.6	1.0654×10^{-5}	4.0390×10^{-5}	3.3068×10^{-5}
	0.8	1.3013×10^{-5}	4.9333×10^{-5}	2.7074×10^{-5}
	1	1.5894×10^{-4}	6.0254×10^{-5}	2.2166×10^{-5}

4.2. Problem

Let us assume the system of fractional Burger's equations:

$$\begin{aligned} D_t^{\varsigma} u - u_{\vartheta\vartheta} - 2uu_{\vartheta} + (uv)_{\vartheta} &= 0, \\ D_t^{\varsigma} v - v_{\vartheta\vartheta} - 2vv_{\vartheta} + (uv)_{\vartheta} &= 0. \end{aligned} \tag{4.10}$$

By considering Eq (4.10), having below initial sources:

$$\begin{aligned} u(\vartheta, 0) &= \sin(\vartheta), \\ v(\vartheta, 0) &= \sin(\vartheta). \end{aligned} \tag{4.11}$$

By employing YT to Eq (4.10) and using Eq (4.11), we get

$$\begin{aligned} U(\vartheta, s) - s\sin(\vartheta) + s^{\varsigma}\mathcal{Y}_t\left[-\mathcal{Y}_t^{-1}[U_{\vartheta\vartheta}] - 2\mathcal{Y}_t^{-1}[U]\mathcal{Y}_t^{-1}[U_{\vartheta}] + \mathcal{Y}_t^{-1}[(UV)_{\vartheta}]\right] &= 0, \\ V(\vartheta, s) - s\sin(\vartheta) + s^{\varsigma}\mathcal{Y}_t\left[-\mathcal{Y}_t^{-1}[V_{\vartheta\vartheta}] - 2\mathcal{Y}_t^{-1}[V]\mathcal{Y}_t^{-1}[V_{\vartheta}] + \mathcal{Y}_t^{-1}[(UV)_{\vartheta}]\right] &= 0. \end{aligned} \tag{4.12}$$

The k^{th}-truncated series are

$$U(\vartheta, s) = s\sin(\vartheta) + \sum_{n=1}^{k} s^{n\varsigma+1} f_n(\vartheta, s), V(\vartheta, s) = s\sin(\vartheta) + \sum_{n=1}^{k} s^{n\varsigma+1} g_n(\vartheta, s), k = 1, 2, 3, 4\cdots \tag{4.13}$$

and the k^{th}-YRFs as:

$$\begin{aligned} \mathcal{Y}_t Res_{u,k}(\vartheta, s) =& U_k(\vartheta, s) - s\sin(\vartheta) + s^{\varsigma}\mathcal{Y}_t\left[-\mathcal{Y}_t^{-1}[U_{\vartheta\vartheta,k}] - 2\mathcal{Y}_t^{-1}[U_k]\mathcal{Y}_t^{-1}[U_{\vartheta,k}] + \mathcal{Y}_t^{-1}[(UV)_{\vartheta,k}]\right], \\ \mathcal{Y}_t Res_{v,k}(\vartheta, s) =& V_k(\vartheta, s) - s\sin(\vartheta) + s^{\varsigma}\mathcal{Y}_t\left[-\mathcal{Y}_t^{-1}[V_{\vartheta\vartheta,k}] - 2\mathcal{Y}_t^{-1}[V_k]\mathcal{Y}_t^{-1}[V_{\vartheta,k}] + \mathcal{Y}_t^{-1}[(UV)_{\vartheta,k}]\right]. \end{aligned} \tag{4.14}$$

To find $f_k(\vartheta, s)$ and $g_k(\vartheta, s)$, $k = 1, 2, 3, \cdots$, the kth-truncated series equation Eq (4.13) will be inserted into the kth-Yang residual function equation Eq (4.14), which will then be multiplied by $s^{k\varsigma+1}$ to solve the relation recursively $\lim_{s\to\infty}(s^{k\varsigma+1}\mathcal{Y}_t Res_{u,k}(\vartheta, s)) = 0$ and $\lim_{s\to\infty}(s^{k\varsigma+1}\mathcal{Y}_t Res_{v,k}(\vartheta, s)) = 0$, $k = 1, 2, 3, \cdots$.

Few terms are as:

$$\begin{aligned} f_1(\vartheta, s) &= \sin(\vartheta), \quad g_1(\vartheta, s) = \sin(\vartheta), \\ f_2(\vartheta, s) &= -\sin(\vartheta), \quad g_2(\vartheta, s) = -\sin(\vartheta), \\ f_3(\vartheta, s) &= \sin(\vartheta), \quad g_3(\vartheta, s) = \sin(\vartheta), \\ f_4(\vartheta, s) &= -\sin(\vartheta), \quad g_4(\vartheta, s) = -\sin(\vartheta), \\ f_5(\vartheta, s) &= \sin(\vartheta), \quad g_5(\vartheta, s) = \sin(\vartheta), \\ f_6(\vartheta, s) &= -\sin(\vartheta), \quad g_6(\vartheta, s) = -\sin(\vartheta), \end{aligned} \tag{4.15}$$

and so on.

By substituting the values of $f_k(\vartheta, s)$ and $g_k(\vartheta, s)$, $k = 1, 2, 3, \cdots$, in Eq (4.13), we have

$$\begin{aligned} U(\vartheta, s) &= s\sin(\vartheta) - s^{\varsigma+1}\sin(\vartheta) + s^{2\varsigma+1}\sin(\vartheta) - s^{3\varsigma+1}\sin(\vartheta) + s^{4\varsigma+1}\sin(\vartheta) - s^{5\varsigma+1}\sin(\vartheta) + s^{6\varsigma+1}\sin(\vartheta) + \cdots, \\ V(\vartheta, s) &= s\sin(\vartheta) - s^{\varsigma+1}\sin(\vartheta) + s^{2\varsigma+1}\sin(\vartheta) - s^{3\varsigma+1}\sin(\vartheta) + s^{4\varsigma+1}\sin(\vartheta) - s^{5\varsigma+1}\sin(\vartheta) + s^{6\varsigma+1}\sin(\vartheta) + \cdots. \end{aligned} \tag{4.16}$$

By employing inverse YT, we have

$$u(\vartheta,t)=\sin(\vartheta)\left(1-\frac{t^{\varsigma}}{\Gamma(\varsigma+1)}+\frac{t^{2\varsigma}}{\Gamma(2\varsigma+1)}-\frac{t^{3\varsigma}}{\Gamma(3\varsigma+1)}+\frac{t^{4\varsigma}}{\Gamma(4\varsigma+1)}-\frac{t^{5\varsigma}}{\Gamma(5\varsigma+1)}+\cdots\right),$$
$$v(\vartheta,t)=\sin(\vartheta)\left(1-\frac{t^{\varsigma}}{\Gamma(\varsigma+1)}+\frac{t^{2\varsigma}}{\Gamma(2\varsigma+1)}-\frac{t^{3\varsigma}}{\Gamma(3\varsigma+1)}+\frac{t^{4\varsigma}}{\Gamma(4\varsigma+1)}-\frac{t^{5\varsigma}}{\Gamma(5\varsigma+1)}+\cdots\right).$$

Putting $\varsigma=1$

$$\begin{aligned}u(\vartheta,t)&=\sin(\vartheta)\left(1-t+\frac{t^2}{2!}-\frac{t^3}{3!}+\frac{t^4}{4!}-\frac{t^5}{5!}+\cdots\right),\\ v(\vartheta,t)&=\sin(\vartheta)\left(1-t+\frac{t^2}{2!}-\frac{t^3}{3!}+\frac{t^4}{4!}-\frac{t^5}{5!}+\cdots\right).\end{aligned}\tag{4.17}$$

Thus we get accurate solutions as

$$\begin{aligned}u(\vartheta,t)&=e^{-t}\sin(\vartheta),\\ v(\vartheta,t)&=e^{-t}\sin(\vartheta).\end{aligned}\tag{4.18}$$

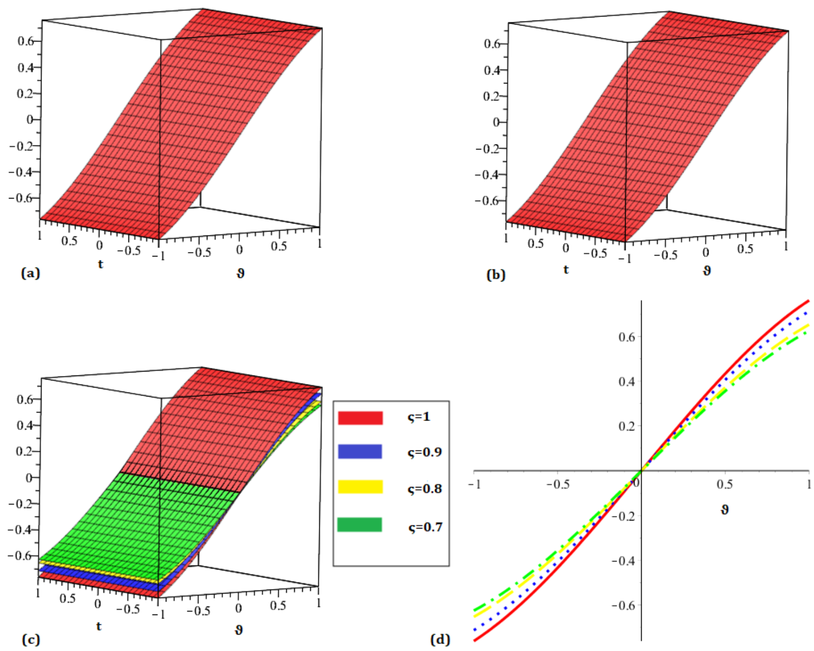

Figure 4. 3-D and 2-D behavior for $u(\vartheta,t)$ and $v(\vartheta,t)$ of Problem 2.

4.3. Results and discussion

In the present study, we obtained the closed form solution of two nonlinear fractional systems by means of YRPSM upto sixth order. The obtained results are describes with the help of graphs and table. The YRPSM results for system 1 on the basis of error is illustrated in Table 1. Figure 1a,b shows the exact and analytical behavior whereas Figure 1c,d shows the 3-D and 2-D behavior of approximate solution at different fractional-orders for $u(\vartheta, \psi, t)$. Figure 2a,b shows the exact and analytical behavior whereas Figure 2c,d shows the 3-D and 2-D behavior of approximate solution at different fractional-orders for $v(\vartheta, \psi, t)$. Figure 3a,b shows the exact and analytical behavior whereas Figure 3c,d shows the 3-D and 2-D behavior of approximate solution at different fractional-orders for $w(\vartheta, \psi, t)$ of system 1. Similarly, Figure 4a,b shows the exact and analytical behavior whereas Figure 4c,d shows the 3-D and 2-D behavior of approximate solution at different fractional-orders for $u(\vartheta, t)$ and $v(\vartheta, t)$ of system 2. The behavior of Table and Figures shows that our solution is in good agreement with the exact solutions of the problems.

5. Conclusions

To find the solutions of DEs and FDEs, a variety of numerical and analytical techniques are employed, some of which are regarded by the prospect of non-exact solutions. This work emphasised that the suggested strategy, YRPS, is an easy-to-use analytical technique for developing accurate and approximative solutions for certain systems of FDEs with suitable initial sources. The aforementioned method gave us the solutions in the Yang transform space by making it simple to calculate the expansion series constants with the aid of the limit concept at infinity. In contrast to the RPS approach, the YRPS method requires minimal calculations to obtain the series coefficients since it uses the limit idea rather than the fractional derivative. The calculated results demonstrate how closely the approximative solutions approach the accurate solution. This proves that the aforementioned method is an appropriate and extraordinarily effective method for obtaining the approximative and analytical solutions to a large number of linear and non-linear fractional problems that arise in engineering and applied physics.

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Conflict of interest

The authors declare that they have no competing interests.

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