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Research article

Blowup for C^1 solutions of Euler equations in R^N with the second inertia functional of reference

Manwai Yuen*

Department of Mathematics and Information Technology, The Education University of Hong Kong, Hong Kong

* **Correspondence:** Email: yuenmw@eduhk.hk.

Abstract: The compressible Euler equations are an elementary model in mathematical fluid mechanics. In this article, we combine the Sideris and Makino-Ukai-Kawashima's classical functional techniques to study the new second inertia functional of reference:

$$H_{ref}(t) = \frac{1}{2}\int_{\Omega(t)} (\rho - \bar{\rho}) \left|\vec{x}\right|^2 dV,$$

for the blowup phenomena of C^1 solutions $(\rho, \vec{u})$ with the support of $(\rho - \bar{\rho}, \vec{u})$, and with a positive constant $\bar{\rho}$ for the adiabatic index $\gamma > 1$. We find that if the total reference mass

$$M_{ref}(0) = \int_{\mathbf{R}^N} (\rho_0(\vec{x}) - \bar{\rho}) dV \geq 0,$$

and the total reference energy

$$E_{ref}(0) = \int_{\mathbf{R}^N} \left(\frac{1}{2}\rho_0(\vec{x}) \left|\vec{u}_0(\vec{x})\right|^2 + \frac{K}{\gamma - 1}\left(\rho_0^{\gamma}(\vec{x}) - \bar{\rho}^{\gamma}\right)\right) dV,$$

with a positive constant K is sufficiently large, then the corresponding solution blows up on or before any finite time $T > 0$.

Keywords: compressible Euler equations; initial value problems; blowup; second inertia functional of reference; energy method
Mathematics Subject Classification: 35B30, 35B40, 35B44, 35Q31, 76N10

1. Introduction

The following system in mathematical fluid mechanics is the compressible Euler equations in $\vec{x} = (x_1, x_2, \cdots, x_N) \in \mathbf{R}^N$:

$$\begin{cases} \rho_t + \nabla\cdot(\rho\vec{u}) = 0, \\ (\rho\vec{u})_t + \nabla\cdot(\rho\vec{u}\otimes\vec{u}) + \nabla P = \vec{0}. \end{cases} \tag{1.1}$$

As usual, $\rho = \rho(t,\vec{x}) \geq 0$, $\vec{u} = \vec{u}(t,\vec{x}) = (u_1, u_2, \cdots, u_N) \in \mathbf{R}^N$ and $P = P(\rho)$ are the density, velocity and pressure function respectively. The γ-law for the pressure term is usually coupled, as

$$P = K\rho^\gamma, \tag{1.2}$$

with a constant $K > 0$ and the adiabatic index $\gamma > 1$.

The compressible Euler equations (1.1) and (1.2) are fundamental to studies in the physical sciences, such as of plasma, the atmosphere of the Earth and condensed matter [1–4]. Readers can find excellent reviews in [5–8]. Notably, the shallow water equations in fluid mechanics coincide with the Euler equations (1.1) and (1.2) when $\gamma = 2$ in the mathematical structure [4, 9].

The local well-posedness of the compressible Euler equations (1.1) and (1.2) is seen in [5, 7, 10, 11]. If the smooth solutions of the compressible Euler equations (1.1) and (1.2) are not global in time, we regard these phenomena as the blowing up of solutions. The blowup phenomena of the compressible Euler equations (1.1) and (1.2) may represent the shock formation or turbulence in real physical applications. Research on the blowup phenomena of the compressible Euler equations is a very active field, as it is intimately related to the incompressible Navier-Stokes equations, which are related to one of the Millennium Prize Problems, posed by the Clay Mathematics Institute [12].

In 1985, Sideris [13] probed the functional

$$F(t) = \int_{\mathbf{R}^3} \vec{x}\cdot\rho\vec{u}\,dV, \tag{1.3}$$

where $dV = dx_1dx_2dx_3$, to obtain the singularity formation for the three-dimensional compressible Euler equations (1.1) and (1.2). Sideris' important work proved that if the initial conditional $F(0)$ is appropriately large, then the C^1 solutions blow up in a finite time with the following lemma.

Lemma 1.1 (Proposition in [13], Lemma 2 in [14])**.** *Let $(\rho,\vec{u})$ be a C^1 solution of the Euler equations (1.1) and (1.2) in $\mathbf{R}^N$, with a lifespan $T > 0$ and the following initial condition,*

$$\begin{cases} (\rho(0,\vec{x}), \vec{u}(0,\vec{x})) = (\bar{\rho} + \rho_0(\vec{x}), \vec{u}_0(\vec{x})), \\ Supp(\rho_0(\vec{x}), \vec{u}_0(\vec{x})) \subseteq \{\vec{x} : |\vec{x}| \leq R\}, \end{cases} \tag{1.4}$$

for some positive constants $\bar{\rho}$ and R. We have

$$(\rho,\vec{u}) = (\bar{\rho},\vec{0}), \tag{1.5}$$

for $t \in [0,T)$ and $|\vec{x}| \geq R + \sigma t$, where $\sigma = \sqrt{K\gamma\bar{\rho}^{\gamma-1}} > 0$.

In 1986, Makino, Ukai and Kawashima [11] extended the idea of Sideris' functional (1.3) to study the secondary inertia functional as follows

$$H(t) = \frac{1}{2}\int_{\mathbf{R}^N} \rho\left|\vec{x}\right|^2 dV, \tag{1.6}$$

to show that there is no global non-trivial C^1 solution for the regular solutions with initial compact support. For additional blowup analysis, readers are referred to [15–23].

2. Materials and methods

In this article, we combine the Sideris and Makino-Ukai-Kawashima classical functional techniques in [11] and [13], to study the new second inertia functional of reference,

$$H_{ref}(t) = \frac{1}{2}\int_{\mathbf{R}^N} (\rho - \bar{\rho}) \left|\vec{x}\right|^2 dV, \tag{2.1}$$

for the C^1 solutions with the non-vacuum state in Lemma 1.1.

3. Results

3.1. Main theorems

Consequently, we obtain blowup results for the following theorem with the total reference energy and the total reference mass:

Theorem 3.1. *Suppose that* $(\rho, \vec{u})$ *is the solution in Lemma* 1.1*. If the total reference mass*

$$M_{ref}(0) = \int_{\mathbf{R}^N} (\rho_0(\vec{x}) - \bar{\rho}) dV \geq 0, \tag{3.1}$$

and the total reference energy

$$E_{ref}(0) = \int_{\mathbf{R}^N} \left(\frac{1}{2}\rho_0(\vec{x}) \left|\vec{u}_0(\vec{x})\right|^2 + \frac{K}{\gamma - 1}\left(\rho_0^{\gamma}(\vec{x}) - \bar{\rho}^{\gamma}\right)\right) dV, \tag{3.2}$$

is sufficiently large, the corresponding solution blows up on or before any finite time $T > 0$*.*

Remark 3.1. *To the best of the author's knowledge, this represents the first study of the second inertia functional of reference (2.1).*

3.2. Blowup with the second inertia functional of reference

The following two lemmas for the conservation laws, the total reference mass and the total reference energy of the solutions in Lemma 1.1 with the non-vacuum state are well-known.

Lemma 3.1. *For the solution in Lemma* 1.1*, we have*

$$M_{ref}(t) = \int_{\mathbf{R}^N} (\rho - \bar{\rho}) dV = M_{ref}(0); \tag{3.3}$$

that is, the total reference mass $M_{ref}(t)$ *is conserved.*

Lemma 3.2. *For the solution in Lemma* 1.1*, we have*

$$E_{ref}(t) = \int_{\mathbf{R}^N} \left(\frac{1}{2}\rho \left|\vec{u}\right|^2 + \frac{K}{\gamma - 1}\left(\rho^{\gamma} - \bar{\rho}^{\gamma}\right)\right) dV = E_{ref}(0); \tag{3.4}$$

that is, the total reference energy $E_{ref}(t)$ *is conserved.*

Then, the second derivative of the second inertia functional of reference in Lemma 1.1, is obtained by the following lemma.

Lemma 3.3. *For the solution in Lemma* 1.1*, we consider the second inertia functional of reference,*

$$H_{ref}(t) = \frac{1}{2}\int_{\mathbf{R}^N} (\rho - \bar{\rho}) \left|\vec{x}\right|^2 dV. \tag{3.5}$$

We have

$$\ddot{H}_{ref}(t) = \int_{\mathbf{R}^N} \left(\rho \left|\vec{u}\right|^2 + NK(\rho^\gamma - \bar{\rho}^\gamma)\right) dV. \tag{3.6}$$

Proof. We take a derivative with respect to t of the second inertia functional of reference (3.5) to obtain

$$\dot{H}_{ref}(t) = \frac{1}{2}\int_{|\vec{x}|<R+\sigma t} (\rho - \bar{\rho})_t \left|\vec{x}\right|^2 dV = \frac{1}{2}\int_{|\vec{x}|<R+\sigma t} \rho_t \left|\vec{x}\right|^2 dV, \tag{3.7}$$

with the support of $\rho - \bar{\rho}$, to remove the surface integral.

From the mass equation $(1.1)_1$, we have

$$\dot{H}_{ref}(t) = -\frac{1}{2}\int_{|\vec{x}|<R+\sigma t} \nabla \cdot (\rho\vec{u}) \left|\vec{x}\right|^2 dV = \int_{|\vec{x}|<R+\sigma t} \vec{x} \cdot \rho\vec{u} dV. \tag{3.8}$$

We then consider the further derivative of functional (3.8), that is

$$\ddot{H}_{ref}(t) = \int_{|\vec{x}|<R+\sigma t} \vec{x} \cdot (\rho\vec{u})_t \, dV \tag{3.9}$$

and

$$\ddot{H}_{ref}(t) = \int_{|\vec{x}|<R+\sigma t} \vec{x} \cdot [-\nabla \cdot (\rho\vec{u} \otimes \vec{u}) - \nabla K (\rho^\gamma - \bar{\rho}^\gamma)] \, dV \tag{3.10}$$

by the momentum equations $(1.1)_2$.

We can calculate Eq (3.10) by splitting it into two parts as follows.

For the first term on the right side of Eq (3.10), by the integration by parts with the boundary condition for $\vec{u}$, we have

$$\begin{aligned} -\int_{|\vec{x}|<R+\sigma t} \vec{x} \cdot [\nabla \cdot (\rho\vec{u} \otimes \vec{u})] \, dV &= -\sum_{h=1}^{N} \int_{|\vec{x}|<R+\sigma t} x_h \sum_{i=1}^{N} \partial_i (\rho u_i u_h) \, dV \\ &= \int_{|\vec{x}|<R+\sigma t} \rho\vec{u} \cdot \vec{u} dV \\ &= \int_{|\vec{x}|<R+\sigma t} \rho \left|\vec{u}\right|^2 dV. \end{aligned} \tag{3.11}$$

For the second term, we obtain

$$-\int_{|\vec{x}|<R+\sigma t} \vec{x} \cdot \nabla K (\rho^\gamma - \bar{\rho}^\gamma) \, dV = \int_{|\vec{x}|<R+\sigma t} NK (\rho^\gamma - \bar{\rho}^\gamma) \, dV. \tag{3.12}$$

Thus,

$$\ddot{H}_{ref}(t) = \int_{|\vec{x}|<R+\sigma t} (\rho \left|\vec{u}\right|^2 + NK (\rho^\gamma - \bar{\rho}^\gamma)) dV. \tag{3.13}$$

The proof is complete. □

We require an additional lemma to control the positivity of the final term in Equation (3.13).

Lemma 3.4. *For the solution in Lemma* 1.1*, if* $M_{ref}(0) \geq 0$*, then*

$$\int_{|\vec{x}|<R+\sigma t} (\rho^\gamma - \bar{\rho}^\gamma)\, dV \geq 0. \tag{3.14}$$

Proof. For the solution in Lemma 1.1, by the reverse Holder's inequality and $M_{ref}(0) \geq 0$, we have

$$\begin{aligned} \int_{|\vec{x}|<R+\sigma t} \rho^\gamma dV &\geq \left(\int_{|\vec{x}|<R+\sigma t} 1 dV \right)^{-\gamma+1} \left(\int_{|\vec{x}|<R+\sigma t} (\rho^\gamma)^{\frac{1}{\gamma}} dV \right)^\gamma \\ &= (B(t))^{-\gamma+1} \left(\int_{|\vec{x}|<R+\sigma t} \rho dV \right)^\gamma, \end{aligned} \tag{3.15}$$

where the volume function,

$$\begin{aligned} B(t) &= \frac{\pi^{\frac{N}{2}} (R+\sigma t)^N}{\Gamma\left(\frac{N}{2}+1\right)},\ for\ |\vec{x}| < R+\sigma t, \\ &= (B(t))^{-\gamma+1} \left(M_{ref}(0) + \int_{|\vec{x}|<R+\sigma t} \bar{\rho} dV \right)^\gamma \qquad \text{by Lemma 3.1}, \\ &\geq (B(t))^{-\gamma+1} (0 + B(t)\bar{\rho})^\gamma \\ &= B(t)\bar{\rho}^\gamma = \int_{|\vec{x}|<R+\sigma t} \bar{\rho}^\gamma dV. \end{aligned} \tag{3.16}$$

Inequality (3.14) is thus proven. □

At this stage, we can present the proof of Theorem 3.1.

Proof of Theorem 3.1. Our method for the novel second inertia functional of reference, that is

$$H_{ref}(t) = \frac{1}{2} \int_{\mathbf{R}^N} (\rho - \bar{\rho}) \left|\vec{x}\right|^2 dV, \tag{3.17}$$

utilizes the functional techniques in the seminal papers of Makino, Ukai and Kawashima [11] and Sideris [13].

By Lemma 3.3, we have

$$\begin{aligned} \ddot{H}_{ref}(t) &= \int_{|\vec{x}|<R+\sigma t} \left(\frac{2}{2}\rho \left|\vec{u}\right|^2 + \frac{N(\gamma-1)K}{\gamma-1} (\rho^\gamma - \bar{\rho}^\gamma) \right) dV \\ &\geq \int_{|\vec{x}|<R+\sigma t} \left(\frac{\min(2, N(\gamma-1))}{2} \rho \left|\vec{u}\right|^2 + \frac{\min(2, N(\gamma-1))K}{\gamma-1} (\rho^\gamma - \bar{\rho}^\gamma) \right) dV \\ &= \min(2, N(\gamma-1)) E_{ref}(0), \end{aligned} \tag{3.18}$$

with the total reference mass $M_{ref}(0) \geq 0$ by Lemma 3.4 and with the total reference energy $E_{ref}(0) = \int_{\mathbf{R}^N} \left(\frac{1}{2}\rho_0(\vec{x}) \left|\vec{u}_0(\vec{x})\right|^2 + \frac{K}{\gamma-1} \left(\rho_0^\gamma(\vec{x}) - \bar{\rho}^\gamma \right) \right) dV$ by Lemma 3.2.

Therefore, we have

$$H_{ref}(t) \geq H_{ref}(0) + \dot{H}_{ref}(0)t + \frac{\min(2, N(\gamma-1))E_{ref}(0)}{2}t^2. \tag{3.19}$$

Then, from

$$\frac{1}{2}(R+\sigma t)^2 \int_{|\vec{x}|<R+\sigma t} \rho dV - \frac{\bar{\rho}}{2} \int_{|\vec{x}|<R+\sigma t} \left|\vec{x}\right|^2 dV \geq \frac{1}{2} \int_{|\vec{x}|<R+\sigma t} (\rho - \bar{\rho}) \left|\vec{x}\right|^2 dV = H_{ref}(t), \tag{3.20}$$

we obtain

$$\frac{1}{2}(R+\sigma t)^2 \int_{|\vec{x}|<R+\sigma t} \rho dV > H_{ref}(t), \tag{3.21}$$

$$\frac{1}{2}(R+\sigma t)^2 \left[M_{ref}(0) + \bar{\rho}B(t)\right] > H_{ref}(0) + \dot{H}_{ref}(0)t + \frac{\min(2, N(\gamma-1))E_{ref}(0)}{2}t^2, \tag{3.22}$$

where the volume function $B(t) = \frac{\pi^{\frac{N}{2}}(R+\sigma t)^N}{\Gamma(\frac{N}{2}+1)}$ for $|\vec{x}| < R + \sigma t$, by Lemma 3.1 and Inequality (3.19).

If the total reference energy $E_{ref}(0)$ is sufficiently large, there is a contradiction to inequality (3.22) on or before T. Thus, the corresponding C^1 solution blows up on or before any finite time $T > 0$. The proof is complete. □

Remark 3.2. *We can choose the total reference energy such that*

$$\frac{\min(2, N(\gamma-1))E_{ref}(0)}{2} >> \max(\frac{1}{2}\left|(R+\sigma T)^2 (M_{ref}(0) + \bar{\rho}B(T))\right|, \left|H_{ref}(0)\right|, \left|\dot{H}_{ref}(0)\right| T), \tag{3.23}$$

$$E_{ref}(0) >> \frac{\max\left(\left|(R+\sigma T)^2 (M_{ref}(0) + \bar{\rho}B(T))\right|, \left|H_{ref}(0)\right|, \left|\dot{H}_{ref}(0)\right| T\right)}{N(\gamma-1)T^2}, \tag{3.24}$$

for any finite time $T > 0$, to fulfill the sufficiently large condition in Theorem 3.1.

Remark 3.3. *In Theorem 3.1, the initial velocity $\vec{u}_0(\vec{x})$ can be sufficiently large such that the kinetic energy $\int_{\mathbf{R}^N} \frac{1}{2}\rho_0(\vec{x}) \left|\vec{u}_0(\vec{x})\right|^2 dV$ is sufficiently large in the sense that the total reference energy $E_{ref}(0)$ is sufficiently large, such that*

$$\begin{aligned} E_{ref}(0) &= \int_{\mathbf{R}^N} \left(\frac{1}{2}\rho_0(\vec{x}) \left|\vec{u}_0(\vec{x})\right|^2 + \frac{K}{\gamma-1}\left(\rho_0^{\gamma}(\vec{x}) - \bar{\rho}^{\gamma}\right)\right) dV \\ &>> \frac{\max\left((R+\sigma T)^2 (\int_{\mathbf{R}^N} (\rho_0(\vec{x}) - \bar{\rho})dV + \bar{\rho}B(T)), -\frac{1}{2}\int_{\mathbf{R}^N} (\rho_0(\vec{x}) - \bar{\rho}) \left|\vec{x}\right|^2 dV - T\int_{\mathbf{R}^N} \vec{x}\cdot\rho_0(\vec{x})\vec{u}_0(\vec{x})dV\right)}{N(\gamma-1)T^2}, \end{aligned} \tag{3.25}$$

to meet the requirement for showing blowup phenomenon on or before any finite time $T > 0$.

Then, it is simple to obtain the following corollary.

Corollary 3.1. *For the global solution in Lemma* 1.1*, we have the total reference mass*

$$M_{ref}(0) = \int_{\mathbf{R}^N} (\rho_0(\vec{x}) - \bar{\rho}) dV < 0, \tag{3.26}$$

or the total reference energy

$$E_{ref}(0) = \int_{\mathbf{R}^N} \left(\frac{1}{2}\rho_0(\vec{x}) \left|\vec{u}_0(\vec{x})\right|^2 + \frac{K}{\gamma - 1} \left(\rho_0^{\gamma}(\vec{x}) - \bar{\rho}^{\gamma} \right) \right) dV \tag{3.27}$$

is sufficiently small.

Remark 3.4. *Our method can also be applied to the non-isentropic Euler equations in* $\mathbf{R}^N$*,*

$$\begin{cases} \rho_t + \nabla \cdot (\rho \vec{u}) = 0, \\ (\rho \vec{u})_t + \nabla \cdot (\rho \vec{u} \otimes \vec{u}) + \nabla P = \vec{0}, \\ S_t + \vec{u} \cdot \nabla S = 0, \end{cases} \tag{3.28}$$

where $S = S(t, \vec{x}) \in \mathbf{R}$ *is the entropy and* $P = Ke^S \rho^{\gamma}$*.*

It is because by applying the same arguments in Makino-Ukai-Kawashima and Sideris's papers [11] and [13], the corresponding total energy is

$$E_{ref}(0) = \int_{\mathbf{R}^N} \left(\frac{1}{2}\rho_0(\vec{x}) \left|\vec{u}_0(\vec{x})\right|^2 + \frac{K}{\gamma - 1} \left(e^{S_0(\vec{x})} \rho_0^{\gamma}(\vec{x}) - e^{\bar{S}} \bar{\rho}^{\gamma} \right) \right) dV, \tag{3.29}$$

with the initial data

$$\begin{cases} (\rho(0, \vec{x}), \vec{u}(0, \vec{x}), S(0, \vec{x})) = \left(\bar{\rho} + \rho_0(\vec{x}), \vec{u}_0(\vec{x}), \bar{S} + S_0(\vec{x}) \right), \\ Supp(\rho_0(\vec{x}), \vec{u}_0(\vec{x}), S_0(\vec{x})) \subseteq \{\vec{x} : |\vec{x}| \leq R\}; \end{cases} \tag{3.30}$$

the corresponding second inertia functional of reference,

$$\ddot{H}_{ref}(t) = \int_{\mathbf{R}^N} \left(\rho \left|\vec{u}\right|^2 + NK(e^S \rho^{\gamma} - e^{\bar{S}} \bar{\rho}^{\gamma}) \right) dV; \tag{3.31}$$

and

$$\int_{|\vec{x}| < R + \sigma t} \left(e^S \rho^{\gamma} - e^{\bar{S}} \bar{\rho}^{\gamma} \right) dV \geq 0 \tag{3.32}$$

with $M_{ref}(0) \geq 0$*.*

4. Conclusions

In this article, we combine the Sideris and Makino-Ukai-Kawashima's classical functional techniques to study the new second inertia functional of reference

$$H_{ref}(t) = \frac{1}{2} \int_{\Omega(t)} (\rho - \bar{\rho}) \left|\vec{x}\right|^2 dV,$$

for the blowup phenomena of C^1 solutions $(\rho, \vec{u})$ with the support of $(\rho - \bar{\rho}, \vec{u})$, and with a positive constant $\bar{\rho}$ for the adiabatic index $\gamma > 1$. We find that if the total reference mass

$$M_{ref}(0) = \int_{\mathbf{R}^N} (\rho_0(\vec{x}) - \bar{\rho}) dV \geq 0,$$

and the total reference energy

$$E_{ref}(0) = \int_{\mathbf{R}^N} \left(\frac{1}{2} \rho_0(\vec{x}) \left| \vec{u}_0(\vec{x}) \right|^2 + \frac{K}{\gamma - 1} \left(\rho_0^\gamma(\vec{x}) - \bar{\rho}^\gamma \right) \right) dV,$$

with a positive constant K is sufficiently large, then the corresponding solution blows up on or before any finite time $T > 0$.

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Conflict of interest

The author declare no conflict of interest.

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