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Research article

(α1, 2, β1, 2)-complex intuitionistic fuzzy subgroups and its algebraic structure

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Abstract: A complex intuitionistic fuzzy set is a generalization framework to characterize several applications in decision making, pattern recognition, engineering, and other fields. This set is considered more fitting and coverable to Intuitionistic Fuzzy Sets (IDS) and complex fuzzy sets. In this paper, the abstraction of $(\alpha_{1,2}, \beta_{1,2})$ complex intuitionistic fuzzy sets and $(\alpha_{1,2}, \beta_{1,2})$-complex intuitionistic fuzzy subgroups were introduced regarding to the concept of complex intuitionistic fuzzy sets. Besides, we show that $(\alpha_{1,2}, \beta_{1,2})$-complex intuitionistic fuzzy subgroup is a general form of every complex intuitionistic fuzzy subgroup. Also, each of $(\alpha_{1,2}, \beta_{1,2})$-complex intuitionistic fuzzy normal subgroups and cosets are defined and studied their relationship in the sense of the commutator of groups and the conjugate classes of group, respectively. Furthermore, some theorems connected the $(\alpha_{1,2}, \beta_{1,2})$-complex intuitionistic fuzzy subgroup of the classical quotient group and the set of all $(\alpha_{1,2}, \beta_{1,2})$-complex intuitionistic fuzzy cosets were studied and proved. Additionally, we expand the index and Lagrange's theorem to be suitable under $(\alpha_{1,2}, \beta_{1,2})$-complex intuitionistic fuzzy subgroups.

Keywords: complex intuitionistic fuzzy set; $(\alpha_{1,2}, \beta_{1,2})$-complex intuitionistic fuzzy set; $(\alpha_{1,2}, \beta_{1,2})$-complex intuitionistic fuzzy subgroup; $(\alpha_{1,2}, \beta_{1,2})$-complex intuitionistic fuzzy normal subgroup
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1. Introduction

Intuitionistic Fuzzy Sets (IFS) [1] is a generalization of Fuzzy Sets (FS) [2]. Many problems have been successfully solved using IFS compared to FS. IFS is characterized by representing information that depends on human subjectivity with answers of "yes", "No", and "I am not sure, I do not know, …

etc. i.e. IFS has membership, non-membership, and hesitant functions, respectively, which is help to represent human information in medical application, multi-attribute decision making, renewable energy, manufacturing industry, and other fields [3–7].

Two approaches in the literature have been generated from the idea of combining FS and the mathematical field of algebra. In 1971, the first approach was given by Rosenfeld [8] under the concept of fuzzy subgroup. Many researchers studied, generalized, and discussed Rosenfeld approach [9–15]. In 1989, Biswas introduced intuitionistic fuzzy subgroup (IFSG) and its properties [16]. The notion of (α, β)-cut of IFSG was presented by Sharma [17]. Sharma and other researcher have been widely studied IFSG and its properties [18–23]. In 1994, the second approach was introduced by Dip [24], named fuzzy group based on fuzzy space. Before started his new approach, Dib and Youssef established a new structure of fuzzy cartesian product, relations and functions [25]. Dib approach can be summarized in replacing fuzzy space and fuzzy binary operation instead of the universal set and binary operation, respectively, at traditional algebra. The fuzzy normal subgroup is defined using fuzzy space by Dib and Hassan [26]. Marashdeh and Salleh generalized the concept of fuzzy space to intuitionistic fuzzy space to create the theory of Intuitionistic Fuzzy Group (IFG) [27] and then Intuitionistic Fuzzy Normal Subgroup (IFNSG) [28].

Moreover, complicated information involving periodicity and changeable meaning for the same data gives a new type of uncertainty. This type of complicated information comes from the rapid development of our daily life and modern technology. Therefore, we need a proper mathematical tool that has the ability to represent the uncertainty and periodicity semantics of information at the same time. So, the expected tool should be helpful and easily used by decision-makers to decrease the difficulties of giving a proper solution to the decision-making problems. To beat this obstacle, Ramot et al. [29,30] introduced the concept of Complex Fuzzy Sets (CFS) and logic. Also, Alkouri and Salleh [31–33] introduced Complex Intuitionistic Fuzzy Set (CIFS), relations and its operations. Both CFS and CIFS generated an extra range which lies within the unit disk in the complex plane. This extension helps to represent complicated information that carries time-periodic problems and two-dimension data simultaneously in a single set. Differences between CFS and CIFS appear in the ability of CIFS to represent information in more detail by using an extra complex non-membership function. Besides CIFS is considered a generalization of CFS that can solve a problem that seems too much roughs in CF [30,31]. Several enhancement and development applications using CIFS and its generalizations have increased rapidly appeared in different fields [34–40], for example, decision-making process, information security management, graph and group, cellular network and etc.

Thus, it is greatly necessary to create additional notions of IFS and CIFS relating to complex set members. In 2016, Alhusbann and Salleh [41] introduced the notion of complex fuzzy group based on Dib approach. One year Later, Alsarahead and Ahmed [42–44] produced different notions named complex fuzzy subgroup, complex fuzzy subring and complex fuzzy soft subgroups from Rosenfeld and Liu approach [8,45]. In 2021, (α, β)-complex fuzzy sets, subgroups, and their properties were introduced by Alolaiyan et al. [46]. Furthermore, they introduced the notion of (α, β)-complex fuzzy cosets to formulate (α, β)-complex fuzzy normal subgroup. Besides, (α, β)-complex fuzzy quotient ring induced by (α, β)-CFNSG and (α, β)-complex fuzzification of Lagrange's Theorem were also derived in [46]. Concurrently, with developing the fuzzy subgroup to the complex fuzzy subgroup. IFS has been clearly developed into IFG and IFNSG [27,28]. Also, in the realm of complex numbers, the notion of Complex Intuitionistic Fuzzy Group (CIFG) and Complex Intuitionistic Fuzzy normal Subgroup were produced by Alhusban et al. in 2016 and 2017 [47,48].

Recently, Xiao and other researchers studied, generalized and applied the Complex evidence theory in the field of quantum mechanics and decisions. Also, they designed several algorithms, model and methods using quantum information in complex plane to predicting and describing human decision-making behaviors to be applied in pattern classification [49–52].

In this study, our motivations are, 1- to introduce more generalized notion under CIFS, i.e. $(\alpha_{1,2},\beta_{1,2})$-Complex Intuitionistic Fuzzy Subgroups (CIFSG). 2- to define the reduced relations between our concept and each of (α, β)-CFS, and CIFS. 3- to study the algebraic structure of $(\alpha_{1,2},\beta_{1,2})$-complex intuitionistic fuzzy normal subgroup (CIFNSG) and some algebraic notions as coset, quotient group, and Lagrange theorem under $(\alpha_{1,2},\beta_{1,2})$ CIFSG. Therefore, the main purpose of this study is to introduce a powerful extension of CIFS and (α, β)-complex fuzzy set and subgroup. Besides, we show the relation between each of $(\alpha_{1,2},\beta_{1,2})$-CIFNSG and CIF cosets. Also, we prove some theorems connected the $(\alpha_{1,2},\beta_{1,2})$-CIFSG of the classical quotient group and the set of all $(\alpha_{1,2},\beta_{1,2})$-CIF cosets. Finally, we expand the index and Lagrange's theorem to be suitable under $(\alpha_{1,2},\beta_{1,2})$-CIFSG.

2. Preliminaries

(Alpha 1,2, Beta 1,2) CIF- complex intuitionistic fuzzy cosets is a cornerstone in the structure of Lagrange's theorem under CIFSG. Also, Cosets is a particular type of CIF subgroup named by a CIF normal subgroup. Its can be used as the elements of another group called a quotient group or factor group under CIFS. As a future research, Cosets OF CIFS also may appear in other areas of mathematics such as vector spaces and error-correcting codes.

In this section, we recall some useful definitions to produce our work successfully.

Definition 2.1. [2] A complex intuitionistic fuzzy set *A*, defined by $A=\{\langle x,\mu_A(x),\gamma_A(x)\rangle : x\epsilon X\}$, where $\mu_A(x): X\to\{a|a\in \mathbb{C},\ |a\le 1\}$, $\gamma_A(x): X\to\{\acute{a}|\acute{a}\in \mathbb{C},\ |\acute{a}\le 1\}$, and $|\mu_A(x)+\gamma_A(x)|\le 1$, and $i=\sqrt{-1}$, each of $r_A(x)$, $k_A(x)$ belong to the interval [0, 1] such that $0\le r_A(x)+k_A(x)\le 1$, also $w^r{}_A(x)$ and $w^k{}_A(x)$ are real-valued.

Definition 2.2 [2]. Let $A=\{\langle x,\mu_A(o),\gamma_A(o)\rangle : o\epsilon O\}$ be a complex intuitionistic fuzzy set. Define the complement of $A, c(A)$, as $c(A)=\ddot{A}=\{\langle x,\gamma_A(o),\mu_A(o)\rangle : x\epsilon X\}=\left\{\langle x,k_A(o).e^{i(w^k{}_A(o))},r_A(o)e^{i(w^r{}_A(o))}\rangle : o\epsilon O\right\}$, where $w^k{}_A(o)=w_A(o), 2\pi-w_A(o)$ or $w_A(o)+\pi$.

Definition 2.3. [2] Let $\mu_A(o)=r_A(o)e^{i\,\omega^r{}_A(o)}$, and $\nu_A(o)=k_A(o)e^{i\,\omega^k{}_A(o)}$, and $\mu_B(o)=r_B(o)e^{i\,\omega_B(o)}$, and $\nu_B(o)=k_B(o)e^{i\,\omega^k{}_B(o)}$. Be two membership and non-membership functions of complex intuitionistic fuzzy sets *A* and *B* respectively, on *O*.

a. *B* is *subset of A*, " $A\supseteq B$ or $B\subseteq A$ ", if for any $o\in O$, $r_A(o)\le r_B(o)$, $k_A(o)\ge k_B(o)$, $\omega^r{}_A(o)\le\omega^r{}_B(o)$, and $\omega^k{}_A(o)\ge\omega^k{}_B(o)$.

b. *A* union *B*, $A\cup B$, as
$$A\cup B=\left\{x,\max(r_A(o),r_B(o))e^{i\max(\omega^r{}_A(o),\omega^r{}_B(o))},\min(k_A(o),k_B(o))e^{i\min(\omega^k{}_A(o),\omega^k{}_B(o))}\right\}.$$

c. *A* intersection *B*, denoted by $A\cap B$, as
$$A\cap B=\left\{x,\min(r_A(o),r_B(o))e^{i\min(\omega^r{}_A(o),\omega^r{}_B(o))},\max(k_A(o),k_B(o))e^{i\max(\omega^k{}_A(o),\omega^k{}_B(o))}\right\}.$$

Some definitions related to (α,β)-CFS and (α,β)-CFSG of a group *G* are selected from the reference of Alolaiyan et al. [46], as follow:

Definition 2.4. [46] Let $A=\left\{<a,S(a)e^{i\alpha w_A^{S(a)}}>: a\epsilon G\right\}$, be CFS of group G, for any $\alpha\epsilon[0,1], and\ \beta\ \epsilon[0,2\pi]$, such that $S(a)\le\alpha, w_A^S\le\beta$, or ($S(a)\ge\alpha,\ w_A^s\ge\beta$). Then, the set $A_{(\alpha,\beta)}$is called an (α,β)-CFS and is defined as:

$$S_{A_\alpha}e^{iw_{A_\beta}^{S(a)}}=\min\{S(a)e^{iw_A^{S(a)}},\alpha e^{i\beta}\}=\min\{S_A(a),\alpha\}e^{i\min(w_A^{S(a)},\ \beta)},$$

where, $S_{A_\alpha}e^{iw_{A_\beta}^{S(a)}}$ is called a complex membership function of (α,β)-complex fuzzy sets.

Definition 2.5. [46]. Let $A_{(\alpha,\beta)}$ be an (α,β)-CFS of group G for $\alpha,\beta\epsilon[0,1]$. Then, $A_{(\alpha,\beta)}$ is called **(**α,β**)**-CFSG of group G if it satisfies the following axioms:

$$SA_\alpha(pq)e^{iw_{A_\beta}^{S(pq)}}\ge\min\{SA_\alpha(p)e^{i\alpha w_{A_\beta}^{S(p)}},SA_\alpha(q)e^{i\alpha w_{A_\beta}^{S(q)}}\}$$

$$SA_\alpha(p^{-1})e^{i\alpha w_{A_\beta}^{S(p^{-1})}}\ge SA_\alpha(p)e^{i\alpha w_{A_\beta}^{S(p)}},\forall p,q\in G.$$

Definition 2.6. [46]. Let $A_{(\alpha,\beta)}$ be an $(\alpha,\beta)-$ CFSG of group G, where $\alpha\in[0,1]$ and $\beta\in[0,2\pi]$. Then the $(\alpha,\beta)-$ CFS $gA_{(\alpha,\beta)}(a)=\left\{\left(a,S_{gA_\alpha}(a)e^{iw_{gA_\beta}^{S_{gA_\alpha}}(a)}\right),a\in G\right\}$ of G is called a (α,β)-complex fuzzy left coset of G determined by $A_{(\alpha,\beta)}$ and g and is described as:

$$S_{gA_\alpha}(o)e^{i\alpha w_{gA_\beta}^{S_{gA_\alpha}}(o)}=S_{A_\alpha}(g^{-1}o)e^{i\alpha w_{A_\beta}^{S_{A_\alpha}}(g^{-1}o)}=\min\left\{S_A(g^{-1}o)e^{iw_A^{S_A}(g^{-1}o)},\alpha\ e^{i\beta}\right\}.$$

Similarly, they defined (α,β)-complex fuzzy right coset [45].

Definition 2.7. [46]. Let $A_{(\alpha,\beta)}$ be an $(\alpha,\beta)-$ CFSG of group G, where $\alpha\in[0,1]$ and $\beta\in[0,2\pi]$. Then, $A_{(\alpha,\beta)}$ is called a (α,β)-CFNSG if $A_{(\alpha,\beta)}(gh)=A_{(\alpha,\beta)}(hg)$. Equivalently (α,β)-CFSG $A_{(\alpha,\beta)}$ is (α,β)-CFNSG of group G if: $A_{(\alpha,\beta)}g(h)=gA_{(\alpha,\beta)}(h)$, for all $g,h\in G$.

Definition 2.8. [45]. Let $A_{(\alpha,\beta)}$ be an (α,β)-CFSG of finite group$G$. Then, the cardinality of the set $G/A_{(\alpha,\beta)}$ of all (α,β)-complex fuzzy left cosets of G by $A_{(\alpha,\beta)}$ is called the index of (α,β)-CFSG and is denoted by$\left[G:A_{(\alpha,\beta)}\right]$.

3. $(\alpha_{1,2},\beta_{1,2})$-complex intuitionistic fuzzy subgroups

In this section, we define the hybrid models of $(\alpha_{1,2},\beta_{1,2})$-CIFSs and $(\alpha_{1,2},\beta_{1,2})$-CIFSGs. We prove that every CIFSG is also $(\alpha_{1,2},\beta_{1,2})$-CIFSG but the converse may not be true generally, and we discuss some basic characterization of this phenomenon.

Definition 3.1. Let $A=\left\{<a,S(a)e^{i\alpha w_A^{S(a)}},L(a)e^{i\alpha w_A^{L(a)}}>:a\epsilon G\right\}$, be CIFS of group G, for any $\alpha_{1,2},\beta_{1,2}\epsilon[0,1]$, $S,L,w_A^S,w_A^L\in[0,1]$ such that $S\le\alpha_1,L\ge\alpha_2\ w_A^S\le\beta_1,w_A^L\ge\beta_2$ or ($S\ge\alpha_1,L\le\alpha_2,\ w_A^s\ge\beta_1,w_A^l\le\beta_2$), Then, the set $A_{(\alpha_{1,2},\beta_{1,2})}$is called an $(\alpha_{1,2},\beta_{1,2})$-CIFS and is defined as:

$$S_{A_{\alpha_1}}e^{i\alpha w_{A_{\beta_1}}^{S_{A_{\alpha_1}}(a)}}=\min\{S(a)e^{i\alpha w_A^{S(a)}},\alpha_1e^{i\beta_1}\}=\min\{S_A(a),\alpha_1\}e^{i\alpha\min(w_A^{S(a)},\ \beta_1)}\}$$

$$L_{A_{\alpha_2}} e^{i\alpha w_{A_{\beta_2}}^{L_{A_{\alpha_2}}(a)}} = \max\{L(a)e^{i\alpha w_A^{L(a)}}, \alpha_2 e^{i\beta_2}\} = \max\{L_A(a), \alpha_2\}e^{i\alpha \max(w_A^{L(a)}, \beta_2)}\},$$

where

$$S_{A_{\alpha_1}} = \min\{S(a), \alpha_1\}, w_{A_{\beta_1}}^{S(a)} = \min\{w_A^{S(a)}, \beta_1\}, L_{A_{\alpha_2}} = \max\{\mathrm{L}(\mathrm{a}), \alpha_2\}, w_{A_{\beta_2}}^{L(a)} = \max\{w_A^{L(a)}, \beta_2\},$$

$$S + L \leq 1,\ w^S + w^L \leq 1, \alpha_1 + \alpha_2 \leq 1, \text{and } \beta_1 + \beta_2 \leq 1.$$

In this paper we shall use $SA_{\alpha_1} e^{i\alpha w_{A_{\beta_1}}^{SA_{\alpha_1}(a)}}$, $LA_{\alpha_2} e^{i\alpha w_{A_{\beta_2}}^{LA_{\alpha_2}(a)}}$ as a membership function and non-membership function of $(\alpha_{1,2}, \beta_{1,2})$-CIFSs, $A_{(\alpha_{1,2},\beta_{1,2})}$ *and* $B_{(\alpha_{1,2},\beta_{1,2})}$ respectively.

Definition Let $A_{(\alpha_{1,2},\beta_{1,2})}$, $B_{(\alpha_{1,2},\beta_{1,2})}$ be a two $(\alpha_{1,2}, \beta_{1,2})$-CIFSs of G. Then

(1) A $(\alpha_{1,2}, \beta_{1,2})$–CIFS $A_{(\alpha_{1,2},\beta_{1,2})}$ is homogeneous $(\alpha_{1,2}, \beta_{1,2})$–CIFS if, for all $p, q \in G$, we have $S_{A_{\alpha_1}}(p) \leq S_{A_{\alpha_1}}(q)$, $L_{A_{\alpha_2}}(p) \leq L_{A_{\alpha_2}}(q)$ if and only if $w_{A_{\beta_1}}^{S_{A_{\alpha_1}}(a)}(p) \leq w_{A_{\beta_1}}^{S_{A_{\alpha_1}}(a)}(q)$, $w_{A_{\beta_2}}^{L_{A_{\alpha_2}}(a)}(p) \leq w_{A_{\beta_2}}^{L_{A_{\alpha_2}}(a)}(q)$.

(2) A $(\alpha_{1,2}, \beta_{1,2})$–CIFS $A_{(\alpha_{1,2},\beta_{1,2})}$ is homogeneous $(\alpha_{1,2}, \beta_{1,2})$–CIFS with $B_{(\alpha_{1,2},\beta_{1,2})}$ for all $p, q \in G$, we have $S_{A_{\alpha_1}}(p) \leq S_{A_{\alpha_1}}(q)$, $L_{A_{\alpha_2}}(p) \leq L_{A_{\alpha_2}}(q)$ if and only if $w_{A_{\beta_1}}^{S_{A_{\alpha_1}}(a)}(p) \leq w_{A_{\beta_1}}^{S_{A_{\alpha_1}}(a)}(q)$, $w_{A_{\beta_2}}^{L_{A_{\alpha_2}}(a)}(p) \leq w_{A_{\beta_2}}^{L_{A_{\alpha_2}}(a)}(q)$.

In this paper, we take $(\alpha_{1,2}, \beta_{1,2})$-CIFS as homogeneous $(\alpha_{1,2}, \beta_{1,2})$-CIFS.

Remark 1. Let , $M_{(\alpha_{1,2},\beta_{1,2})}$ and $N_{(\alpha_{1,2},\beta_{1,2})}$ be two $(\alpha_{1,2}, \beta_{1,2})$-CIFSs of G. Then $(M \cap N)_{(\alpha_{1,2},\beta_{1,2})} = M_{(\alpha_{1,2},\beta_{1,2})} \cap N_{(\alpha_{1,2},\beta_{1,2})}$

Example 3.1. Consider a group Z_4={0, 1, 2, 3} is a group. Let $\alpha_1 = 0.7, \alpha_2 = 0.2, \beta_1 = 0.4, \textit{and } \beta_2 = 0.5$, and a CIFS A $= \{< 0, 0.8e^{i\alpha 0.4}, 0.1e^{i\alpha 0.3} >, < 1, 0.4e^{i\alpha 0.1}, 0.5e^{i\alpha 0.6} >, < 2, 0.3e^{i0.6}, 0.7e^{i\alpha 0.3} >, < 3, 0.7e^{i\alpha 0.4}, 0.1e^{i\alpha 0.5} >\}$ of a group Z_4, Then, the set $A_{(\alpha_{1,2},\beta_{1,2})}$ is called an $(\alpha_{1,2}, \beta_{1,2})$-CIFS and is defined as:

$$A_{(\alpha_{1,2},\beta_{1,2})} = \{< 0, 0.7e^{i\alpha 0.4}, 0.2e^{i\alpha 0.5} >, < 1, 0.4e^{i\alpha 0.1}, 0.5e^{i\alpha 0.6} >, < 2, 0.3e^{i0.4}, 0.7e^{i\alpha 0.5} >,$$

$$< 3, 0.7e^{i\alpha 0.4}, 0.2e^{i\alpha 0.5} >\}.$$

Definition 3.2. Let $A_{(\alpha_{1,2},\beta_{1,2})}$ be an $(\alpha_{1,2}, \beta_{1,2})$-CIFS of group G for $\alpha_{1,2}, \beta_{1,2} \in [0,1]$. Then, $A_{(\alpha_{1,2},\beta_{1,2})}$ is called $(\alpha_{1,2}, \beta_{1,2})$-Complex Intuitionistic Fuzzy subgroupoid of group G if the following axioms hold:

$$S_{A_{\alpha_1}}(pq)e^{i\alpha w_{A_{\beta_1}}^{S_{A_{\alpha_1}}(pq)}} \geq \min\{S_{A_{\alpha_1}}(p)e^{i\alpha w_{A_{\beta_1}}^{S_{A_{\alpha_1}}(p)}}, S_{A_{\alpha_1}}(q)e^{i\alpha w_{A_{\beta_1}}^{S_{A_{\alpha_1}}(q)}}\}$$

$$L_{A_{\alpha_2}}(pq)e^{i\alpha w_{A_{\beta_2}}^{L_{A_{\alpha_2}}(pq)}} \leq \max\{L_{A_{\alpha_2}}(p)e^{i\alpha w_{A_{\beta_2}}^{L_{A_{\alpha_2}}(p)}}, L_{A_{\alpha_2}}(q)e^{i\alpha w_{A_{\beta_2}}^{L_{A_{\alpha_2}}(q)}}\}.$$

Example 3.2. Recalling that $A_{(\alpha_{1,2},\beta_{1,2})}=\{<0,0.7e^{i\alpha 0.4},0.2e^{i\alpha 0.5}>,<1,0.4e^{i\alpha 0.1},0.5e^{i\alpha 0.6}>,<2,0.3e^{i0.4},0.7e^{i\alpha 0.5}>,\ <3,0.7e^{i\alpha 0.4},0.2e^{i\alpha 0.5}>\}$ from Example 3.1. satisfies all axioms of Definition 3.3. for all elements in the group Z_4. For example (Let *p*=1, *q*= 2), so pq=3, $p^{-1}=3$ in Z_4. Then:

Axiom 1:

$$SA_{0.7}(3)e^{i\alpha w_{A_{0.4}}^{S(3)}}=0.7e^{i\alpha 0.4}\geq \min SA_{0.7}(1)e^{i\alpha w_{A_{0.4}}^{S(1)}}=0.4e^{i\alpha 0.1}, SA_{0.7}(2)e^{i\alpha w_{A_{0.4}}^{S(1)}}=0.3e^{i0.4}$$
$$=\min\{0.4e^{i\alpha 0.1},0.3e^{i0.4}\}=\min\{0.4,0.3\}\,e^{i\alpha min\{0.1,0.4\}}=0.3e^{i\alpha 0.1}.$$

Axiom 2:

$$SA_{0.7}(p^{-1}=3)e^{i\alpha w_{A_{0.4}}^{S(p^{-1}=3)}}=0.7e^{i\alpha 0.4}\geq SA_{0.7}(p=1)e^{i\alpha w_{A_{0.4}}^{S(p=1)}}=0.4e^{i\alpha 0.1}.$$

Axiom 3:

$$LA_{0.2}(pq=3)e^{i\alpha w_{A_{0.5}}^{L(pq=3)}}=0.2e^{i\alpha 0.5}\leq \max\{LA_{0.2}(p=1)e^{i\alpha w_{A_{0.5}}^{L(p=1)}}, LA_{0.2}(q=2)e^{i\alpha w_{A_{0.5}}^{L(q=2)}}\}$$
$$=\max\{0.5e^{i\alpha 0.6},0.7e^{i\alpha 0.5}\}=\max\{0.5,0.7\}\,e^{i\alpha max\{0.6,0.5\}}=0.7e^{i\alpha 0.6}.$$

Axiom 4:

$$LA_{0.2}(p^{-1}=3)e^{i\alpha w_{A_{0.5}}^{L(p^{-1}=3)}}=0.2e^{i\alpha 0.5}\leq LA_{0.2}(p=1)e^{i\alpha w_{A_{0.5}}^{L(p=1)}}=0.5e^{i\alpha 0.6}.$$

Definition 3.3. Let $A_{(\alpha_{1,2},\beta_{1,2})}$ be an $(\alpha_{1,2},\beta_{1,2})$-CIFS of group G for $\alpha_{1,2},\beta_{1,2}\epsilon[0,1]$. Then, $A_{(\alpha_{1,2},\beta_{1,2})}$ is named.

($\alpha_{1,2},\beta_{1,2}$**)**-CIFSG of group *G* if the following axioms hold:

(1) $S_{A_{\alpha_1}}(pq)e^{i\alpha w_{A_{\beta_1}}^{S_{A_{\alpha_1}}(pq)}}\geq \min\{S_{A_{\alpha_1}}(p)e^{i\alpha w_{A_{\beta_1}}^{S(p)}}, S_{A_{\alpha_1}}(q)e^{i\alpha w_{A_{\beta_1}}^{S_{A_{\alpha_1}}(q)}}\}.$

(2) $S_{A_{\alpha_1}}(p^{-1})e^{i\alpha w_{A_{\beta_1}}^{S_{A_{\alpha_1}}(p^{-1})}}\geq S_{A_{\alpha_1}}(p)e^{i\alpha w_{A_{\beta_1}}^{S_{A_{\alpha_1}}(p)}}.$

(3) $L_{A_{\alpha_2}}(pq)e^{i\alpha w_{A_{\beta_2}}^{L_{A_{\alpha_2}}(pq)}}\leq \max\{L_{A_{\alpha_2}}(p)e^{i\alpha w_{A_{\beta_2}}^{L_{A_{\alpha_2}}(p)}}, L_{A_{\alpha_2}}(q)e^{i\alpha w_{A_{\beta_2}}^{L_{A_{\alpha_2}}(q)}}\}.$

(4) $L_{A_{\alpha_2}}(p^{-1})e^{i\alpha w_{A_{\beta_2}}^{L_{A_{\alpha_2}}(p^{-1})}}\leq L_{A_{\alpha_2}}(p)e^{i\alpha w_{A_{\beta_2}}^{L_{A_{\alpha_2}}(p)}}$, for all $p,q\epsilon G$.

Remark 2. $A_{(\alpha_{1,2},\beta_{1,2})}$ be an $(\alpha_{1,2},\beta_{1,2})$-CIFS of group G, for $\alpha_{1,2},\beta_{1,2}\ \epsilon[0,1]$. Then,

(1) $S_{A_{\alpha_1}}(p^{-1}q)e^{i\alpha w_{A_{\beta_1}}^{S_{A_{\alpha_1}}(p^{-1}q)}}\geq \min\{S_{A_{\alpha_1}}(p)e^{i\alpha w_{A_{\beta_1}}^{S_{A_{\alpha_1}}(p)}}, S_{A_{\alpha_1}}(q)e^{i\alpha w_{A_{\beta_1}}^{S_{A_{\alpha_1}}(q)}}\}.$

(2) $L_{A_{\alpha_2}}(p^{-1}q)e^{i\alpha w_{A_{\beta_2}}^{L_{A_{\alpha_2}}(p^{-1}q)}}\leq \max\{LA_{\alpha_2}(p)e^{i\alpha w_{A_{\beta_2}}^{L_{A_{\alpha_2}}(p)}}, LA_{\alpha_2}(q)e^{i\alpha w_{A_{\beta_2}}^{L_{A_{\alpha_2}}(q)}}\}.$

Theorem 3.1. If $A_{(\alpha_{1,2},\beta_{1,2})}$ be an $(\alpha_{1,2},\beta_{1,2})$-CIFS of group G, for all $p, q\epsilon G$. Then,

(1) $\mathrm{S}_{A_{\alpha_1}}(q)e^{i\alpha w_{A_{\beta_1}}^{\mathrm{S}_{A_{\alpha_1}}(q)}} \leq \mathrm{S}_{A_{\alpha_1}}(e)e^{i\alpha w_{A_{\beta_1}}^{\mathrm{S}_{A_{\alpha_1}}(e)}}$, (2) $\mathrm{S}_{A_{\alpha_1}}(p^{-1}q)e^{i\alpha w_{A_{\beta_1}}^{\mathrm{S}_{A_{\alpha_1}}(p^{-1}q)}} = \mathrm{S}_{A_{\alpha_1}}(e)e^{i\alpha w_{A_{\beta_1}}^{\mathrm{S}_{A_{\alpha_1}}(e)}}$,

(3) $\mathrm{L}_{A_{\alpha_2}}(q)e^{i\alpha w_{A_{\beta_2}}^{\mathrm{L}_{A_{\alpha_2}}(q)}} \geq \mathrm{L}_{A_{\alpha_2}}(e)e^{i\alpha w_{A_{\beta_2}}^{\mathrm{L}_{A_{\alpha_2}}(e)}}$, (4) $\mathrm{L}_{A_{\alpha_2}}(p^{-1}q)e^{i\alpha w_{A_{\beta_2}}^{\mathrm{L}_{A_{\alpha_2}}(p^{-1}q)}} = \mathrm{L}_{A_{\alpha_2}}(e)e^{i\alpha w_{A_{\beta_2}}^{\mathrm{L}_{A_{\alpha_2}}(e)}}$,

which implies that

$$\mathrm{S}_{A_{\alpha_1}}(p)e^{i\alpha w_{A_{\beta_1}}^{\mathrm{S}_{A_{\alpha_1}}(p)}} = \mathrm{S}_{A_{\alpha_1}}(q)e^{i\alpha w_{A_{\beta_1}}^{\mathrm{S}_{A_{\alpha_1}}(q)}}$$

and

$$\mathrm{L}_{A_{\alpha_2}}(p)e^{i\alpha w_{A_{\beta_2}}^{\mathrm{L}_{A_{\alpha_2}}(p)}} = \mathrm{L}_{A_{\alpha_2}}(q)e^{i\alpha w_{A_{\beta_2}}^{\mathrm{L}_{A_{\alpha_2}}(q)}}.$$

Theorem 3.2. Let $A_{(\alpha_{1,2},\beta_{1,2})}$ be an $(\alpha_{1,2},\beta_{1,2})$ complex intuitionistic fuzzy subgroupoid of a finite *G*, and then $A_{(\alpha_{1,2},\beta_{1,2})}$ is $(\alpha_{1,2},\beta_{1,2})$-CIFSG of finite *G*.

Proof. Let $p\epsilon G$ and *G* is a finite group; therefore, $p^n=e$, where *p* has finite order *n*., where *e* is the natural element of group *G*. Then, we have $p^{-1} = p^{n-1}$, now, by using the Definition 3.3 twice, we get

$$\mathrm{S}_{A_{\alpha_1}}(p^{-1})e^{i\alpha w_{A_{\beta_1}}^{\mathrm{S}_{A_{\alpha_1}}(p^{-1})}} = \mathrm{S}_{A_{\alpha_1}}(p^{n-1})e^{i\alpha w_{A_{\beta_1}}^{\mathrm{S}_{A_{\alpha_1}}(p^{n-1})}}$$

$$= \mathrm{S}_{A_{\alpha_1}}(p^{n-2}p)e^{i\alpha w_{A_{\beta_1}}^{\mathrm{S}_{A_{\alpha_1}}(p^{n-2}p)}}$$

$$\geq \mathrm{S}_{A_{\alpha_1}}(p)e^{i\alpha w_{A_{\beta_1}}^{\mathrm{S}_{A_{\alpha_1}}(p)}},$$

$$(p^{-1})e^{i\alpha w_{A_{\beta_2}}^{\mathrm{L}_{A_{\alpha_2}}(p^{-1})}} = \mathrm{L}_{A_{\alpha_2}}(p^{n-1})e^{i\alpha w_{A_{\beta_2}}^{\mathrm{L}_{A_{\alpha_2}}(p^{n-1})}}$$

$$=\mathrm{L}_{A_{\alpha_2}}(p^{n-2}p)e^{i\alpha w_{A_{\beta_2}}^{\mathrm{L}_{A_{\alpha_2}}(p^{n-2p})}}$$

$$\leq \mathrm{L}_{A_{\alpha_2}}(p)e^{i\alpha w_{A_{\beta_2}}^{\mathrm{L}_{A_{\alpha_2}}(p)}}.$$

Theorem3.3. Let $A_{(\alpha_{1,2},\beta_{1,2})}$ be an $(\alpha_{1,2},\beta_{1,2})$-CIFSG of a group G, Let $p\epsilon G$ and

$$\mathrm{S}_{A_{\alpha_1}}(p)e^{i\alpha w_{A_{\beta_1}}^{\mathrm{S}_{A_{\alpha_1}}(p)}} = \mathrm{S}_{A_{\alpha_1}}(e)e^{i\alpha w_{A_{\beta_1}}^{\mathrm{S}_{A_{\alpha_1}}(e)}}, \mathrm{L}_{A_{\alpha_2}}(p)e^{i\alpha w_{A_{\beta_2}}^{\mathrm{L}_{A_{\alpha_2}}(p)}} = \mathrm{L}_{A_{\alpha_2}}(e)e^{i\alpha w_{A_{\beta_2}}^{\mathrm{L}_{A_{\alpha_2}}(e)}},$$

then

$$\mathrm{S}_{A_{\alpha_1}}(pq)e^{i\alpha w_{A_{\beta_1}}^{\mathrm{S}_{A_{\alpha_1}}(pq)}} = \mathrm{S}_{A_{\alpha_1}}(q)e^{i\alpha w_{A_{\beta_1}}^{\mathrm{S}_{A_{\alpha_1}}(q)}},$$

and

$$\mathrm{L}_{A_{\alpha_2}}(pq)e^{i\alpha w_{A_{\beta_2}}^{\mathrm{L}_{A_{\alpha_2}}(pq)}} = \mathrm{L}_{A_{\alpha_2}}(q)e^{i\alpha w_{A_{\beta_2}}^{\mathrm{L}_{A_{\alpha_2}}(q)}}, \text{ for all } q\epsilon G.$$

Proof.

(1) Given that $\mathrm{S}_{A_{\alpha_1}}(p)e^{i\alpha w_{A_{\beta_1}}^{\mathrm{S}_{A_{\alpha_1}}(p)}} = \mathrm{S}_{A_{\alpha_1}}(e)e^{i\alpha w_{A_{\beta_1}}^{\mathrm{S}_{A_{\alpha_1}}(e)}}$. Then form Theorem we have that

$$\mathrm{S}_{A_{\alpha_1}}(q)e^{i\alpha w_{A_{\beta_1}}^{\mathrm{S}_{A_{\alpha_1}}(q)}} \leq \mathrm{S}_{A_{\alpha_1}}(p)e^{i\alpha w_{A_{\beta_1}}^{\mathrm{S}_{A_{\alpha_1}}(p)}}, \text{ for all } q\epsilon G.$$

Let

$$\mathrm{S}_{A_{\alpha_1}}(pq)e^{i\alpha w_{A_{\beta_1}}^{\mathrm{S}_{A_{\alpha_1}}(pq)}} \geq \min\left\{\mathrm{S}_{A_{\alpha_1}}(p)e^{i\alpha w_{A_{\beta_1}}^{\mathrm{S}_{A_{\alpha_1}}(p)}}, \mathrm{S}_{A_{\alpha_1}}(q)e^{i\alpha w_{A_{\beta_1}}^{\mathrm{S}_{A_{\alpha_1}}(q)}}\right\}$$

$$\mathrm{S}_{A_{\alpha_1}}(pq)e^{i\alpha w_{A_{\beta_1}}^{\mathrm{S}_{A_{\alpha_1}}(pq)}} \geq \mathrm{S}_{A_{\alpha_1}}(q)e^{i\alpha w_{A_{\beta_1}}^{\mathrm{S}_{A_{\alpha_1}}(q)}}. \quad (1)$$

Now, assume that

$$\mathrm{S}_{A_{\alpha_1}}(q)e^{i\alpha w_{A_{\beta_1}}^{\mathrm{S}_{A_{\alpha_1}}(q)}} = \mathrm{S}_{A_{\alpha_1}}(p^{-1}pq)e^{i\alpha w_{A_{\beta_1}}^{\mathrm{S}_{A_{\alpha_1}}(p^{-1}pq)}}$$

$$\geq \min\left\{\mathrm{S}_{A_{\alpha_1}}(p)e^{i\alpha w_{A_{\beta_1}}^{\mathrm{S}_{A_{\alpha_1}}(p)}}, \mathrm{S}_{A_{\alpha_1}}(pq)e^{i\alpha w_{A_{\beta_1}}^{\mathrm{S}_{A_{\alpha_1}}(pq)}}\right\}.$$

Again, from Theorem 3.1, we have

$$\min\left\{\mathrm{S}_{A_{\alpha_1}}(p)e^{i\alpha w_{A_{\beta_1}}^{\mathrm{S}_{A_{\alpha_1}}(p)}}, \mathrm{S}_{A_{\alpha_1}}(pq)e^{i\alpha w_{A_{\beta_1}}^{\mathrm{S}_{A_{\alpha_1}}(pq)}}\right\} = \mathrm{S}_{A_{\alpha_1}}(pq)e^{i\alpha w_{A_{\beta_1}}^{\mathrm{S}_{A_{\alpha_1}}(pq)}}.$$

Therefore, we obtain

$$\mathrm{S}_{A_{\alpha_1}}(q)e^{i\alpha w_{A_{\beta_1}}^{\mathrm{S}_{A_{\alpha_1}}(q)}} \geq \mathrm{S}_{A_{\alpha_1}}(pq)e^{i\alpha w_{A_{\beta_1}}^{\mathrm{S}_{A_{\alpha_1}}(pq)}}. \quad (2)$$

From Eqs (1) and (2), we have

$$S_{A_{\alpha_1}}(pq)e^{i\alpha w_{A_{\beta_1}}^{S_{A_{\alpha_1}}(pq)}} = S_{A_{\alpha_1}}(q)e^{i\alpha w_{A_{\beta_1}}^{S_{A_{\alpha_1}}(q)}}.$$

(2) Assume that $L_{A_{\alpha_2}}(p)e^{i\alpha w_{A_{\beta_2}}^{L_{A_{\alpha_2}}(p)}} = L_{A_{\alpha_2}}(e)e^{i\alpha w_{A_{\beta_2}}^{L_{A_{\alpha_2}}(e)}}$. Then from Theorem 3.3 we have that

$$L_{A_{\alpha_2}}(p)e^{i\alpha w_{A_{\beta_2}}^{L_{A_{\alpha_2}}(p)}} \le L_{A_{\alpha_2}}(q)e^{i\alpha w_{A_{\beta_2}}^{L_{A_{\alpha_2}}(q)}}, \text{ for all } q\epsilon G.$$

Let

$$L_{A_{\alpha_2}}(pq)e^{i\alpha w_{A_{\beta_2}}^{L_{A_{\alpha_2}}(pq)}} \le \max\{L_{A_{\alpha_2}}(p)e^{i\alpha w_{A_{\beta_2}}^{L_{A_{\alpha_2}}(p)}}, LA_{\alpha_2}(q)e^{i\alpha w_{A_{\beta_2}}^{L_{A_{\alpha_2}}(q)}}\}$$

$$L_{A_{\alpha_2}}(pq)e^{i\alpha w_{A_{\beta_2}}^{L_{A_{\alpha_2}}(pq)}} \le L_{A_{\alpha_2}}(q)e^{i\alpha w_{A_{\beta_2}}^{L_{A_{\alpha_2}}(q)}}. \quad (3)$$

Now, assume that

$$L_{A_{\alpha_2}}(q)e^{i\alpha w_{A_{\beta_2}}^{L_{A_{\alpha_2}}(q)}} = L_{A_{\alpha_2}}(p^{-1}pq)e^{i\alpha w_{A_{\beta_2}}^{L_{A_{\alpha_2}}(p^{-1}pq)}}$$

$$\le \max\{L_{A_{\alpha_2}}(p)e^{i\alpha w_{A_{\beta_2}}^{L_{A_{\alpha_2}}(p)}}, L_{A_{\alpha_2}}(pq)e^{i\alpha w_{A_{\beta_2}}^{L_{A_{\alpha_2}}(pq)}}\}.$$

Again, from Theorem 3.1, we have

$$\max\{L_{A_{\alpha_2}}(p)e^{i\alpha w_{A_{\beta_2}}^{L_{A_{\alpha_2}}(p)}}, L_{A_{\alpha_2}}(pq)e^{i\alpha w_{A_{\beta_2}}^{L_{A_{\alpha_2}}(pq)}}\} = L_{A_{\alpha_2}}(pq)e^{i\alpha w_{A_{\beta_2}}^{L_{A_{\alpha_2}}(pq)}}.$$

Therefore, we obtain

$$L_{A_{\alpha_2}}(q)e^{i\alpha w_{A_{\beta_2}}^{L_{A_{\alpha_2}}(q)}} \le L_{A_{\alpha_2}}(pq)e^{i\alpha w_{A_{\beta_2}}^{L_{A_{\alpha_2}}(pq)}}. \quad (4)$$

From Eqs (3) and (4), we have

$$L_{A_{\alpha_2}}(pq)e^{i\alpha w_{A_{\beta_2}}^{L_{A_{\alpha_2}}(pq)}} = L_{A_{\alpha_2}}(q)e^{i\alpha w_{A_{\beta_2}}^{L_{A_{\alpha_2}}(q)}}.$$

Theorem 3.4 Every CIFSG of group *G* is also $(\alpha_{1,2}, \beta_{1,2})$-CIFSG of *G*.

Proof. Assume A be CIFSG of group G, and p, qϵG. then

$$S_{A_{\alpha_1}}(pq)e^{i\alpha w_{A_{\beta_1}}^{S_{A_{\alpha_1}}(pq)}} = \min\{S_{A_{\alpha_1}}(pq)e^{i\alpha w_{A_{\beta_1}}^{S_{A_{\alpha_1}}(pq)}}, \alpha e^{i\beta}\}$$

$$\ge \min\{\min\{S_{A_{\alpha_1}}(p)e^{i\alpha w_{A_{\beta_1}}^{S_{A_{\alpha_1}}(p)}}, SA_{\alpha_1}(q)e^{i\alpha w_{A_{\beta_1}}^{S_{A_{\alpha_1}}(q)}}\}, \alpha e^{i\beta}\}$$

$$= \min\{\min\left\{S_{A_{\alpha_1}}(p)e^{i\alpha w_{A_{\beta_1}}^{S_{A_{\alpha_1}}(p)}}, \alpha e^{i\beta}\right\}, \min\left\{S_{A_{\alpha_1}}(q)e^{i\alpha w_{A_{\beta_1}}^{S_{A_{\alpha_1}}(q)}}, \alpha e^{i\beta}\right\}\}$$

$$= \min\{S_{A_{\alpha_1}}(p)e^{i\alpha w_{A_{\beta_1}}^{S_{A_{\alpha_1}}(p)}}, S_{A_{\alpha_1}}(q)e^{i\alpha w_{A_{\beta_1}}^{S_{A_{\alpha_1}}(q)}}\}.$$

Further, we assume that

$$S_{A_{\alpha_1}}(p^{-1})e^{i\alpha w_{A_{\beta_1}}^{S_{A_{\alpha_1}}(p^{-1})}} = \min\left\{S_{A_{\alpha_1}}(p^{-1})e^{i\alpha w_{A_{\beta_1}}^{S_{A_{\alpha_1}}(p^{-1})}}, \alpha e^{i\beta}\right\}$$

$$\geq \min\{S_{A_{\alpha_1}}(p)e^{i\alpha w_{A_{\beta_1}}^{S_{A_{\alpha_1}}(p)}}, \alpha e^{i\beta}\}$$

$$= S_{A_{\alpha_1}}(p)e^{i\alpha w_{A_{\beta_1}}^{S_{A_{\alpha_1}}(p)}}.$$

Consider

$$L_{A_{\alpha_2}}(pq)e^{i\alpha w_{A_{\beta_2}}^{L_{A_{\alpha_2}}(pq)}} = \max L_{A_{\alpha_2}}(pq)e^{i\alpha w_{A_{\beta_2}}^{L_{A_{\alpha_2}}(pq)}}, \alpha e^{i\beta}$$

$$\leq \max\{\max\{L_{A_{\alpha_2}}(p)e^{i\alpha w_{A_{\beta_2}}^{L_{A_{\alpha_2}}(p)}}, L_{A_{\alpha_2}}(q)e^{i\alpha w_{A_{\beta_2}}^{L_{A_{\alpha_2}}(q)}}\}, \alpha e^{i\beta}\}$$

$$= \max\{\max\left\{L_{A_{\alpha_2}}(p)e^{i\alpha w_{A_{\beta_2}}^{L_{A_{\alpha_2}}(p)}}, \alpha e^{i\beta}\right\}, \min\left\{L_{A_{\alpha_2}}(q)e^{i\alpha w_{A_{\beta_2}}^{L_{A_{\alpha_2}}(q)}}, \alpha e^{i\beta}\right\}\}$$

$$= \max\left\{L_{A_{\alpha_2}}(p)e^{i\alpha w_{A_{\beta_2}}^{L_{A_{\alpha_2}}(p)}}, L_{A_{\alpha_2}}(q)e^{i\alpha w_{A_{\beta_2}}^{L_{A_{\alpha_2}}(q)}}\right\}.$$

Further, we assume that

$$L_{A_{\alpha_2}}(p^{-1})e^{i\alpha w_{A_{\beta_2}}^{L_{A_{\alpha_2}}(p^{-1})}} = \max\left\{L_{A_{\alpha_2}}(p^{-1})e^{i\alpha w_{A_{\beta_2}}^{L_{A_{\alpha_2}}(p^{-1})}}, \alpha e^{i\beta}\right\}$$

$$\leq \max\{L_{A_{\alpha_2}}(p)e^{i\alpha w_{A_{\beta_2}}^{L_{A_{\alpha_2}}(p)}}, \alpha e^{i\beta}\}$$

$$= L_{A_{\alpha_2}}(p)e^{i\alpha w_{A_{\beta_2}}^{L_{A_{\alpha_2}}(p)}}.$$

Theorem 3.5. Suppose A is a CIFS of group G where

$$S_{A_{\alpha_1}}(p^{-1})e^{i\alpha w_{A_{\beta_1}}^{S_{A_{\alpha_1}}(p^{-1})}} = S_{A_{\alpha_1}}(p)e^{i\alpha w_{A_{\beta_1}}^{S_{A_{\alpha_1}}(p)}}, \forall p \in G.\ \text{Let}\ \alpha_1 e^{i\beta_1} \leq r_1 e^{iw_1},$$

$$L_{A_{\alpha_2}}(p^{-1})e^{i\alpha w_{A_{\beta_2}}^{L_{A_{\alpha_2}}(p^{-1})}} =, L_{A_{\alpha_2}}(p)e^{i\alpha w_{A_{\beta_2}}^{L_{A_{\alpha_2}}(p)}}, \forall p \in G.\ \text{Let}\ \alpha_2 e^{i\beta_2} \geq r_2 e^{iw_2},$$

such that $\alpha_1 \leq r_1, \beta_1 \leq w_1, \alpha_2 \leq r_2, \beta_2 \leq w_2$.

Where

$$r_1 e^{iw_1} = \min\{\mathrm{S}_{A_{\alpha_1}}(p) e^{i\alpha w_{A_{\beta_1}}^{\mathrm{S}_{A_{\alpha_1}}(p)}} : p \in G\}, r_2 e^{iw_2} = \max\{\mathrm{L}_{A_{\alpha_2}}(p) e^{i\alpha w_{A_{\beta_2}}^{\mathrm{L}_{A_{\alpha_2}}(p)}} : p \in G\}$$

$$\alpha_1, r_1, \beta_1, w_1, \alpha_2, r_2, \beta_2, w_2 \in [0,1].$$

Then, $A_{(\alpha_{1,2}, \beta_{1,2})}$ is an $(\alpha_{1,2}, \beta_{1,2})$-CIFSG of G.

Proof. Let $\alpha_1 e^{i\beta_1} \leq r_1 e^{iw_1}$, implies that $\min\{\mathrm{S}_{A_{\alpha_1}}(p) e^{i\alpha w_{A_{\beta_1}}^{\mathrm{S}_{A_{\alpha_1}}(p)}} : p \in G \geq \alpha_1 e^{i\beta_1}$, which implies that $\min\{\mathrm{S}_{A_{\alpha_1}}(p) e^{i\alpha w_{A_{\beta_1}}^{\mathrm{S}_{A_{\alpha_1}}(p)}}, \alpha_1 e^{i\beta_1}\} = \alpha_1 e^{i\beta_1}, \forall p \in G$, which implies that $\mathrm{S}_{A_{\alpha_1}}(p) e^{i\alpha w_{A_{\beta_1}}^{\mathrm{S}_{A_{\alpha_1}}(p)}} = \alpha_1 e^{i\beta_1}$.

$$\mathrm{S}_{A_{\alpha_1}}(pq) e^{i\alpha w_{A_{\beta_1}}^{\mathrm{S}_{A_{\alpha_1}}(pq)}} \geq \min\{\mathrm{S}_{A_{\alpha_1}}(p) e^{i\alpha w_{A_{\beta_1}}^{\mathrm{S}_{A_{\alpha_1}}(p)}}, \mathrm{S}_{A_{\alpha_1}}(q) e^{i\alpha w_{A_{\beta_1}}^{\mathrm{S}_{A_{\alpha_1}}(q)}}\}.$$

Moreover,

$$\mathrm{S}_{A_{\alpha_1}}(p^{-1}) e^{i\alpha w_{A_{\beta_1}}^{\mathrm{S}_{A_{\alpha_1}}(p^{-1})}} = \mathrm{S}_{A_{\alpha_1}}(p) e^{i\alpha w_{A_{\beta_1}}^{\mathrm{S}_{A_{\alpha_1}}(p)}}.$$

This implies that

$$\mathrm{S}_{A_{\alpha_1}}(p^{-1}) e^{i\alpha w_{A_{\beta_1}}^{\mathrm{S}_{A_{\alpha_1}}(p^{-1})}} = \mathrm{S}_{A_{\alpha_1}}(p) e^{i\alpha w_{A_{\beta_1}}^{\mathrm{S}_{A_{\alpha_1}}(p)}}.$$

Let $\alpha_2 e^{i\beta_2} \leq r_2 e^{iw_2}$, implies that $\max\{\mathrm{L}_{A_{\alpha_2}}(p) e^{i\alpha w_{A_{\beta_2}}^{\mathrm{L}_{A_{\alpha_2}}(p)}} : p \in G \leq \alpha_2 e^{i\beta_2},\}$ which implies that $\max\{\mathrm{L}_{A_{\alpha_2}}(p) e^{i\alpha w_{A_{\beta_2}}^{\mathrm{L}_{A_{\alpha_2}}(p)}} : p \in G, \alpha_2 e^{i\beta_2}\} = \alpha_2 e^{i\beta_2}, \forall p \in G$, which implies that

$$\mathrm{L}_{A_{\alpha_2}}(p) e^{i\alpha w_{A_{\beta_2}}^{\mathrm{L}_{A_{\alpha_2}}(p)}} = \alpha_2 e^{i\beta_2}$$

$$\mathrm{L}_{A_{\alpha_2}}(pq) e^{i\alpha w_{A_{\beta_2}}^{\mathrm{L}_{A_{\alpha_2}}(pq)}} \leq \max\{\mathrm{L}_{A_{\alpha_2}}(p) e^{i\alpha w_{A_{\beta_2}}^{\mathrm{L}_{A_{\alpha_2}}(p)}}, \mathrm{L}A_{\alpha_2}(q) e^{i\alpha w_{A_{\beta_2}}^{\mathrm{L}_{A_{\alpha_2}}(q)}}\}$$

$$\mathrm{L}_{A_{\alpha_2}}(p^{-1}) e^{i\alpha w_{A_{\beta_2}}^{\mathrm{L}_{A_{\alpha_2}}(p^{-1})}} = \mathrm{L}_{A_{\alpha_2}}(p) e^{i\alpha w_{A_{\beta_2}}^{\mathrm{L}_{A_{\alpha_2}}(p)}}$$

$$\mathrm{L}_{A_{\alpha_2}}(p^{-1}) e^{i\alpha w_{A_{\beta_2}}^{\mathrm{L}_{A_{\alpha_2}}(p^{-1})}} = \mathrm{L}_{A_{\alpha_2}}(p) e^{i\alpha w_{A_{\beta_2}}^{\mathrm{L}_{A_{\alpha_2}}(p)}}.$$

Therefore, $A_{(\alpha_{1,2},\beta_{1,2})}$ is an $(\alpha_{1,2},\beta_{1,2})$-CIFSG of G.

Theorem 3.6. If $M_{(\alpha_{1,2},\beta_{1,2})}$ and $N_{(\alpha_{1,2},\beta_{1,2})}$ are two $(\alpha_{1,2},\beta_{1,2})$-CIFSGs of G, then $M_{(\alpha_{1,2},\beta_{1,2})} \cap N_{(\alpha_{1,2},\beta_{1,2})}$ is also $(\alpha_{1,2},\beta_{1,2})$- CIFSG of G.

Proof.

$$\mathrm{S}_{(M\cap N)_{\alpha_1}}(pq)e^{i\alpha w_{(M\cap N)_{\beta_1}}^{(M\cap N)_{\alpha_1}(pq)}} = SM_{\alpha_1} \cap N_{\alpha_1}(pq)e^{i\alpha w_{M_{\beta_1}\cap N_{\beta_1}}^{SM_{\alpha_1}\cap N_{\alpha_1}(pq)}}$$

$$= \min\{\mathrm{S}_{M_{\alpha_1}}(pq)e^{i\alpha w_{M_{\beta_1}}^{\mathrm{S}_{M_{\alpha_1}}(pq)}}, \mathrm{S}_{N_{\alpha_1}}(pq)e^{i\alpha w_{N_{\beta_1}}^{\mathrm{S}_{N_{\alpha_1}}(pq)}}\}$$

$$\geq \min\{\min\left\{\mathrm{S}_{M_{\alpha_1}}(p)e^{i\alpha w_{M_{\beta_1}}^{\mathrm{S}_{M_{\alpha_1}}(p)}}, \mathrm{S}_{M_{\alpha_1}}(q)e^{i\alpha w_{M_{\beta_1}}^{\mathrm{S}_{M_{\alpha_1}}(q)}}\right\}, \min\left\{SN_{\alpha_1}(p)e^{i\alpha w_{N_{\beta_1}}^{S(p)}}, SN_{\alpha_1}(q)e^{i\alpha w_{N_{\beta_1}}^{S(q)}}\right\}\},$$

$$\min\{\min\left\{\mathrm{S}_{M_{\alpha_1}}(p)e^{i\alpha w_{M_{\beta_1}}^{\mathrm{S}_{M_{\alpha_1}}(p)}}, \mathrm{S}_{N_{\alpha_1}}(p)e^{i\alpha w_{N_{\beta_1}}^{\mathrm{S}_{N_{\alpha_1}}(p)}}\right\}, \min\left\{\mathrm{S}_{M_{\alpha_1}}(q)e^{i\alpha w_{M_{\beta_1}}^{\mathrm{S}_{M_{\alpha_1}}(q)}}, \mathrm{S}_{N_{\alpha_1}}(q)e^{i\alpha w_{N_{\beta_1}}^{\mathrm{S}_{N_{\alpha_1}}(q)}}\right\}\}$$

$$= \min\left\{S_{M_{\alpha_1}\cap N_{\alpha_1}}(p)e^{i\alpha w_{M_{\beta_1}\cap N_{\beta_1}}^{S_{M_{\alpha_1}\cap N_{\alpha_1}}(p)}}, S_{M_{\alpha_1}\cap N_{\alpha_1}}(q)e^{i\alpha w_{M_{\beta_1}\cap N_{\beta_1}}^{S_{M_{\alpha_1}\cap N_{\alpha_1}}(q)}}\right\}$$

$$= \min\left\{\mathrm{S}_{(M\cap N)_{\alpha_1}}(p)e^{i\alpha w_{(M\cap N)_{\beta_1}}^{\mathrm{S}_{(M\cap N)_{\alpha_1}}(p)}}, \mathrm{S}_{(M\cap N)_{\alpha_1}}(q)e^{i\alpha w_{(M\cap N)_{\beta_1}}^{\mathrm{S}_{(M\cap N)_{\alpha_1}}(q)}}\right\}.$$

Further,

$$\begin{aligned}\mathrm{S}_{(M\cap N)_{\alpha_1}}(p^{-1})e^{i\alpha w_{(M\cap N)_{\beta_1}}^{\mathrm{S}_{(M\cap N)_{\alpha_1}}(p^{-1})}} &= \mathrm{S}_{M_{\alpha_1}\cap N_{\alpha_1}}(p^{-1})e^{i\alpha w_{M_{\beta_1}\cap N_{\beta_1}}^{\mathrm{S}_{M_{\alpha_1}\cap N_{\alpha_1}}(p^{-1})}} \\ &= \min\{\mathrm{S}_{M_{\alpha_1}}(p^{-1})e^{i\alpha w_{M_{\beta_1}}^{\mathrm{S}_{M_{\alpha_1}}(p^{-1})}}, S_{N_{\alpha_1}}(p^{-1})e^{i\alpha w_{N_{\beta_1}}^{S_{N_{\alpha_1}}(p^{-1})}}\} \\ &\geq \min\{S_{M_{\alpha_1}}(p)e^{i\alpha w_{M_{\beta_1}}^{S_{M_{\alpha_1}}(p^{-1})}}, \mathrm{S}_{N_{\alpha_1}}(p)e^{i\alpha w_{N_{\beta_1}}^{\mathrm{S}_{N_{\alpha_1}}(p^{-1})}}\} \\ &= \mathrm{S}_{(M\cap N)_{\alpha_1}}(p)e^{i\alpha w_{(M\cap N)_{\beta_1}}^{\mathrm{S}_{(M\cap N)_{\alpha_1}}(p)}}.\end{aligned}$$

And

$$\begin{aligned}\mathrm{L}_{(M\cap N)_{\alpha_2}}(pq)e^{i\alpha w_{(M\cap N)_{\beta_2}}^{\mathrm{L}_{(M\cap N)_{\alpha_2}}(pq)}} &= \mathrm{L}_{M_{\alpha_2}\cap N_{\alpha_2}}(pq)e^{i\alpha w_{M_{\beta_2}\cap N_{\beta_2}}^{\mathrm{L}_{M_{\alpha_2}\cap N_{\alpha_2}}(pq)}} \\ &= \max\{\mathrm{L}_{M_{\alpha_2}}(pq)e^{i\alpha w_{M_{\beta_2}}^{\mathrm{L}_{M_{\alpha_2}}(pq)}}, \mathrm{L}_{N_{\alpha_2}}(pq)e^{i\alpha w_{N_{\beta_2}}^{\mathrm{L}_{N_{\alpha_2}}(pq)}}\}\end{aligned}$$

$$\leq \max\{\max\left\{\mathrm{L}_{M_{\alpha_2}}(p)e^{i\alpha w_{M_{\beta_2}}^{\mathrm{L}_{M_{\alpha_2}}(p)}}, \mathrm{L}_{M_{\alpha_2}}(q)e^{i\alpha w_{M_{\beta_2}}^{\mathrm{L}_{M_{\alpha_2}}(q)}}\right\},$$

$$\max\left\{\mathrm{L}_{N_{\alpha_2}}(p)e^{i\alpha w_{N_{\beta_2}}^{\mathrm{L}_{N_{\alpha_2}}(p)}}, \mathrm{L}_{N_{\alpha_2}}(q)e^{i\alpha w_{N_{\beta_2}}^{\mathrm{L}_{N_{\alpha_2}}(q)}}\right\} = \max\{\max\left\{\mathrm{L}_{M_{\alpha_2}}(p)e^{i\alpha w_{M_{\beta_2}}^{\mathrm{L}_{M_{\alpha_2}}(p)}}, \mathrm{L}_{N_{\alpha_2}}(p)e^{i\alpha w_{N_{\beta_2}}^{L(p)}}\right\},$$

$$\max\left\{\mathrm{L}_{M_{\alpha_2}}(q)e^{i\alpha w_{M_{\beta_2}}^{\mathrm{L}_{M_{\alpha_2}}(q)}}, \mathrm{L}_{N_{\alpha_2}}(q)e^{i\alpha w_{N_{\beta_2}}^{\mathrm{L}_{N_{\alpha_2}}(q)}}\right\} = \max\left\{L_{M_{\alpha_2}\cap N_{\alpha_2}}(p)e^{i\alpha w_{M_{\beta_2}\cap N_{\beta_2}}^{L_{M_{\alpha_2}\cap N_{\alpha_2}}(p)}}, L_{M_{\alpha_2}\cap N_{\alpha_2}}(q)e^{i\alpha w_{M_{\beta_2}\cap N_{\beta_2}}^{L_{M_{\alpha_2}\cap N_{\alpha_2}}(q)}}\right\}$$

$$= \max\left\{\mathrm{L}_{(M\cap N)_{\alpha_2}}(p)e^{i\alpha w_{(M\cap N)_{\beta_2}}^{\mathrm{L}_{(M\cap N)_{\alpha_2}}(p)}}, \mathrm{L}_{(M\cap N)_{\alpha_2}}(q)e^{i\alpha w_{(M\cap N)_{\beta_2}}^{\mathrm{L}_{(M\cap N)_{\alpha_2}}(q)}}\right\}.$$

Further,

$$\mathrm{L}_{(M\cap N)_{\alpha_2}}(p^{-1})e^{i\alpha w_{(M\cap N)_{\beta_2}}^{(M\cap N)_{\alpha_2}(p^{-1})}} = \mathrm{L}_{M_{\alpha_2}\cap N_{\alpha_2}}(p^{-1})e^{i\alpha w_{M_{\beta_2}\cap N_{\beta_2}}^{M_{\alpha_2}\cap N_{\alpha_2}(p^{-1})}}$$

$$= \max\{L_{M_{\alpha_2}}(p^{-1})e^{i\alpha w_{M_{\beta_2}}^{L_{M_{\alpha_2}}(p^{-1})}}, \mathrm{L}_{N_{\alpha_2}}(p^{-1})e^{i\alpha w_{N_{\beta_2}}^{\mathrm{L}_{N_{\alpha_2}}(p^{-1})}}\}$$

$$= \max\{L_{M_{\alpha_2}}(p)e^{i\alpha w_{M_{\beta_2}}^{L_{M_{\alpha_2}}(p)}}, \mathrm{L}_{N_{\alpha_2}}(p)e^{i\alpha w_{N_{\beta_2}}^{\mathrm{L}_{N_{\alpha_2}}(p)}}\}$$

$$= \mathrm{L}_{(M\cap N)_{\alpha_2}}(p)e^{i\alpha w_{(M\cap N)_{\beta_2}}^{\mathrm{L}_{(M\cap N)_{\alpha_2}}(p)}}.$$

Consequently, $M_{(\alpha_{1,2},\beta_{1,2})} \cap N_{(\alpha_{1,2},\beta_{1,2})}$ is also $(\alpha_{1,2}, \beta_{1,2})$- CIFSG of G.

Remark 3. The union of two $(\alpha_{1,2}, \beta_{1,2})$ −CIFSGs may not be $(\alpha_{1,2}, \beta_{1,2})$ −CIFSGs.

Example 3.3. Let S_3 be a symmetric group of all permutation of 3 elements. Suppose $M_{(\alpha_{1,2},\beta_{1,2})}$ and $N_{(\alpha_{1,2},\beta_{1,2})}$ are two sets of $(\alpha_{1,2}, \beta_{1,2})$ −CIFSGs of S_4, and $\alpha_1 = 0.6, \alpha_2 = 0.3, \beta_1 = 0.4, and\ \beta_2 = 0.5$, given as:

$$M_{(\alpha_{1,2},\beta_{1,2})}(\rho) = \begin{cases} < \rho, 0.6e^{i\alpha 0.2}, 0.3e^{i\alpha 0.7} > & \text{if } \rho = \rho_1 \\ < \rho, 0.2e^{i\alpha 0.1}, 0.5e^{i\alpha 0.7} > & \text{otherwise} \end{cases},$$

$$N_{(\alpha_{1,2},\beta_{1,2})}(\rho) = \begin{cases} < \rho, 0.5e^{i\alpha 0.1}, 0.5e^{i\alpha 0.5} > & \text{if } \rho = \rho_2 \\ < \rho, 0.3e^{i\alpha 0.1}, 0.7e^{i\alpha 0.6} > & \text{otherwise} \end{cases}.$$

So,

$$M_{(\alpha_{1,2},\beta_{1,2})} \cup N_{(\alpha_{1,2},\beta_{1,2})} = \begin{cases} < \rho, 0.6e^{i\alpha 0.2}, 0.3e^{i\alpha 0.6} > & \text{if } \rho = \rho_1 \\ < \rho, 0.5e^{i\alpha 0.1}, 0.5e^{i\alpha 0.5} > & \text{if } \rho = \rho_2 \\ < \rho, 0.3e^{i\alpha 0.1}, 0.5e^{i\alpha 0.6} > & \text{otherwise} \end{cases}.$$

Take $M_{(\alpha_{1,2},\beta_{1,2})} \cup N_{(\alpha_{1,2},\beta_{1,2})}(\rho_1) =< 0.6e^{i\alpha 0.2}, 0.3e^{i\alpha 0.6} >$ and $M_{(\alpha_{1,2},\beta_{1,2})} \cup N_{(\alpha_{1,2},\beta_{1,2})}(\rho_2) =< 0.5e^{i\alpha 0.1}, 0.5e^{i\alpha 0.5} >$, and $M_{(\alpha_{1,2},\beta_{1,2})} \cup N_{(\alpha_{1,2},\beta_{1,2})}(\rho_1\rho_2 = \rho_0) =< 0.3e^{i\alpha 0.1}, 0.5e^{i\alpha 0.6} >$.
Clearly, Axiom 1 from definition 3.3 does not hold

$$SM \cup N_{\alpha_1}(\rho_1\rho_2 = \rho_0)e^{i\alpha w^{S(\rho_1\rho_2=\rho_0)}_{M\cup N_{\beta_1}}} = 0.3e^{i\alpha 0.1}$$

$$\ngeq \min\left\{SM \cup N_{\alpha_1}(\rho_1)e^{i\alpha w^{S(\rho_1)}_{M\cup N_{\beta_1}}}, SM \cup N_{\alpha_1}(\rho_2)e^{i\alpha w^{S(\rho_2)}_{M\cup N_{\beta_1}}}\right\}$$

$$= \min\left\{0.6e^{i\alpha 0.2}, 0.5e^{i\alpha 0.1}\right\} = 0.5e^{i\alpha 0.1}.$$

4. CIFNSG and lagrange's theorem under $(\alpha_{1,2}, \beta_{1,2})$-CIFSG

We get started by introducing the notation of $(\alpha_{1,2}, \beta_{1,2})$-complex intuitionistic fuzzy cosets of $(\alpha_{1,2}, \beta_{1,2})$-CIFSG. Hence, a quotient group induced by CIDNSGs generalized. After that we study Lagrange's theorem under $(\alpha_{1,2}, \beta_{1,2})$-CIFSG.

Definition 13. Let $A_{(\alpha_{1,2},\beta_{1,2})}$ be an $(\alpha_{1,2}, \beta_{1,2}) -$ CIFSG of group G, where $\alpha_{1,2}, \beta_{1,2} \in [0,1]$. Then the $(\alpha_{1,2}, \beta_{1,2}) -$CIFS $gA_{(\alpha_{1,2},\beta_{1,2})}(a) = \left\{\left(a, S_{gA_{\alpha_1}}(a)e^{i\alpha w^{S_{gA_{\alpha_1}}(a)}_{gA_{\beta_1}}},\ L_{gA_{\alpha_2}}(a)e^{i\alpha w^{L_{gA_{\alpha_2}}(a)}_{gA_{\beta_2}}}\right), a \in G\right\}$ of G is called a $(\alpha_{1,2}, \beta_{1,2})$- complex intuitionistic fuzzy left coset of G determined by $A_{(\alpha_{1,2},\beta_{1,2})}$ and g and is describe as:

$$S_{gA_{\alpha_1}}(o)e^{i\alpha w^{S_{gA_{\alpha_1}}(o)}_{gA_{\beta_1}}} = S_{A_{\alpha_1}}(g^{-1}o)e^{i\alpha w^{S_{A_{\alpha_1}}(g^{-1}o)}_{A_{\beta_1}}} = \min\left\{S_A(g^{-1}o)e^{i\alpha w^{S_A(g^{-1}o)}_A}, \alpha_1 e^{i\beta_1}\right\},$$

and

$$L_{gA_{\alpha_2}}(o)e^{i\alpha w^{L_{gA_{\alpha_2}}(a)}_{gA_{\beta_2}}} = L_{A_{\alpha_2}}(g^{-1}o)e^{i\alpha w^{L_{A_{\alpha_2}}(g^{-1}a)}_{A_{\beta_2}}} = max\left\{L_A(g^{-1}o)e^{i\alpha w^{L_A(g^{-1}o)}_A}, \alpha_2 e^{i\beta_2}\right\},$$

for all $o, g \in G$.

Similarly we can define $(\alpha_{1,2}, \beta_{1,2})$ - complex intuitionistic fuzzy right coset is described as:

$$S_{gA_{\alpha_1}}(o)e^{i\alpha w^{S_{gA_{\alpha_1}}(o)}_{gA_{\beta_1}}} = S_{A_{\alpha_1}}(og^{-1})e^{i\alpha w^{S_{A_{\alpha_1}}(og^{-1})}_{A_{\beta_1}}} = \min\left\{S_A(og^{-1})e^{i\alpha w^{S_A(og^{-1})}_A}, \alpha_1 e^{i\beta_1}\right\},$$

and

$$L_{gA_{\alpha_2}}(o)e^{i\alpha w^{L_{gA_{\alpha_2}}(o)}_{gA_{\beta_2}}} = L_{A_{\alpha_2}}(ag^{-1})e^{i\alpha w^{L_{A_{\alpha_2}}(ag^{-1})}_{A_{\beta_2}}} = max\left\{L_A(og^{-1})e^{i\alpha w^{L_A(og^{-1})}_A}, \alpha_2 e^{i\beta_2}\right\},$$

for all $o, g \in G$.

Definition 14. Let $A_{(\alpha_{1,2},\beta_{1,2})}$ be an $(\alpha_{1,2}, \beta_{1,2})$-CIFSG of group G, where $\alpha_{1,2}, \beta_{1,2} \in [0,1]$. Then, $A_{(\alpha_{1,2},\beta_{1,2})}$ is called a $(\alpha_{1,2}, \beta_{1,2})$ -CIFNSG if $A_{(\alpha_{1,2},\beta_{1,2})}(gh) = A_{(\alpha_{1,2},\beta_{1,2})}(hg)$. Equivalently $(\alpha_{1,2}, \beta_{1,2})$-CIFSG $A_{(\alpha_{1,2},\beta_{1,2})}$ is $(\alpha_{1,2}, \beta_{1,2})$-CIFNSG of group G if: $A_{(\alpha_{1,2},\beta_{1,2})}g(h) = gA_{(\alpha_{1,2},\beta_{1,2})}(h)$, for all $g, h \in G$.

Remark 4. Let $A_{(\alpha_{1,2},\beta_{1,2})}$ be an $(\alpha_{1,2},\beta_{1,2})$-CIFNSG of group G. Then, $A_{(\alpha_{1,2},\beta_{1,2})}(h^{-1}gh) = A_{(\alpha_{1,2},\beta_{1,2})}(g)$, for all $g,h \in G$.

Theorem 7. If A is CIFNSG of group G, then $A_{(\alpha_{1,2},\beta_{1,2})}$ is an $(\alpha_{1,2},\beta_{1,2})$-CIFNSG of G.

Proof. Suppose o,g are elements of G. Then, for the membership function we have

$$S_A(g^{-1}o)e^{i\alpha w_A^S(g^{-1}o)} = S_A(og^{-1})e^{i\alpha w_A^S(og^{-1})}.$$

This implies that

$$min\left\{S_A(g^{-1}o)e^{i\alpha w_A^S(g^{-1}o)}, \alpha_1 e^{i\beta_1}\right\} = \min\left\{S_A(og^{-1})e^{i\alpha w_A^S(og^{-1})}, \alpha_1 e^{i\beta_1}\right\},$$

which implies that $S_{gA_{\alpha_1}}(o)e^{i\alpha w_{gA_{\beta_1}}^{S_{gA_{\alpha_1}}}(o)} = S_{A_{\alpha_1}g}(o)e^{i\alpha w_{A_{\beta_1}g}^{S_{A_{\alpha_1}g}}(o)}$.This implies that $gA_{(\alpha_1,\beta_1)}(o) = A_{(\alpha_1,\beta_1)}g(o)$. Now, for the non-membership function we have:

$$L_A(g^{-1}o)e^{i\alpha w_A^L(g^{-1}o)} = L_A(og^{-1})e^{i\alpha w_A^L(og^{-1})}.$$

This implies that

$$max\left\{L_A(g^{-1}o)e^{i\alpha w_A^l(g^{-1}o)}, \alpha_2 e^{i\beta_2}\right\} = \max\left\{L_A(og^{-1})e^{i\alpha w_A^L(og^{-1})}, \alpha_2 e^{i\beta_2}\right\},$$

which implies that $L_{gA_{\alpha_2}}(o)e^{i\alpha w_{gA_{\beta_2}}^{L_{gA_{\alpha_2}}}(o)} = L_{A_{\alpha_2}g}(o)e^{i\alpha w_{A_{\beta_2}g}^{L_{A_{\alpha_2}g}}(o)}$. This implies that $gA_{(\alpha_2,\beta_2)}(o) = A_{(\alpha_2,\beta_2)}g(o)$.

Therefore, $A_{(\alpha_{1,2},\beta_{1,2})}$ is $(\alpha_{1,2},\beta_{1,2})$-CIFNSG of G.

Theorem 8. Let $A_{(\alpha_{1,2},\beta_{1,2})}$ be $(\alpha_{1,2},\beta_{1,2})$-CIFSG of group G such that for membership we have $\alpha_1 e^{i\beta_1} < re^{iw^r}$ such that $\alpha_1 \le r$ and $\beta_1 \le w^r$, where $re^{iw^r} = \min\left\{S_A(o)e^{i\alpha w_A^S(o)}, \forall o \in G\right\}$ and for non-membership we have $\alpha_2 e^{i\beta_2} < ke^{iw^k}$ such that $\alpha_2 \ge k$ and $\beta_2 \ge w^k$, where $ke^{iw^k} = \max\left\{L_A(o)e^{i\alpha w_A^L(o)}, \forall o \in G\right\}$ and $\alpha_{1,2}, \beta_{1,2}, r, k, w^r, w^k \in [0,1]$. with CIFS conditions: $0 \le \alpha_1 + \alpha_2 \le 1$, $0 \le \beta_1 + \beta_2 \le 1$, $0 \le r + k \le 1$, and $0 \le w^r + w^k \le 1$. Then, $A_{(\alpha_{1,2},\beta_{1,2})}$ is a $(\alpha_{1,2},\beta_{1,2})$-complex intuitionistic fuzzy normal subgroup of group G.

Proof. Having $\alpha_1 e^{i\beta_1} < re^{iw^r}$ then we get $\min\left\{S_A(o)e^{i\alpha w_A^S(o)}, \forall o \in G\right\} \ge \alpha_1 e^{i\beta_1}$, which implies that $S_A(o)e^{i\alpha w_A^S(o)} \ge \alpha_1 e^{i\beta_1}$, for all $o \in G$. And $\alpha_2 e^{i\beta_2} > ke^{iw^k}$ implies that $\max\left\{L_A(o)e^{i\alpha w_A^L(o)}, \forall o \in G\right\} \le \alpha_2 e^{i\beta_2}$, which implies that $L_A(o)e^{i\alpha w_A^L(o)} \le \alpha_2 e^{i\beta_2}$, for all $o \in G$.

Thus,

$$S_{gA_{\alpha_1}}(o)e^{i\alpha w_{gA_{\beta_1}}^{S_{gA_{\alpha_1}}}(o)} = \min\left\{S_A(g^{-1}o)e^{i\alpha w_A^S(g^{-1}o)}, \alpha_1 e^{i\beta_1}\right\} = \alpha_1 e^{i\beta_1}$$

and

$$L_{gA_{\alpha_2}}(o)e^{i\alpha w_{gA_{\beta_2}}^{L_{gA_{\alpha_2}}}(o)} = \max\left\{L_A(g^{-1}o)e^{i\alpha w_A^L(g^{-1}o)}, \alpha_2 e^{i\beta_2}\right\} = \alpha_2 e^{i\beta_2}, \text{ for any } a \in G.$$

Similarly,

$$S_{A_{\alpha_1}g}(o)e^{i\alpha w_{A_{\beta_1}g}^{S_{A_{\alpha_1}g}}(o)} = \min\left\{S_A(og^{-1})e^{i\alpha w_A^S(og^{-1})}, \alpha_1 e^{i\beta_1}\right\} = \alpha_1 e^{i\beta_1}$$

and

$$L_{A_{\alpha_2}g}(o)e^{i\alpha w_{A_{\beta_2}g}^{L_{A_{\alpha_2}g}}(o)} = \max\left\{L_A(og^{-1})e^{i\alpha w_A^L(og^{-1})}, \alpha_2 e^{i\beta_2}\right\} = \alpha_2 e^{i\beta_2}.$$

This implies that

$$S_{gA_{\alpha_1}}(o)e^{i\alpha w_{gA_{\beta_1}}^{S_{gA_{\alpha_1}}}(o)} = S_{A_{\alpha_1}g}(o)e^{i\alpha w_{A_{\beta_1}g}^{S_{A_{\alpha_1}g}}(o)}$$

and

$$L_{gA_{\alpha_2}}(o)e^{i\alpha w_{gA_{\beta_2}}^{L_{gA_{\alpha_2}}}(o)} = L_{A_{\alpha_2}g}(o)e^{i\alpha w_{A_{\beta_2}g}^{L_{A_{\alpha_2}g}}(o)}.$$

Theorem 9. Let $A_{(\alpha_{1,2},\beta_{1,2})}$ be an $(\alpha_{1,2},\beta_{1,2})$-CIFSG of a group G, then $A_{(\alpha_{1,2},\beta_{1,2})}$ is an $(\alpha_{1,2},\beta_{1,2})$-CIFNSG if and only if $A_{(\alpha_{1,2},\beta_{1,2})}$ is constant in the in the conjugacy class of group G.

Proof. Suppose that $A_{(\alpha_{1,2},\beta_{1,2})}$ is an $(\alpha_{1,2},\beta_{1,2})$-CIFNSG. Then, so we get

$$\begin{aligned} S_{A_{\alpha_1}}(h^{-1}gh)e^{i\alpha w_{A_{\beta_1}}^{S_{A_{\alpha_1}}}(h^{-1}gh)} &= S_{A_{\alpha_1}}(ghh^{-1})e^{i\alpha w_{A_{\beta_1}}^{S_{A_{\alpha_1}}}(ghh^{-1})} \\ &= S_{A_{\alpha_1}}(g)e^{i\alpha w_{A_{\beta_1}}^{S_{A_{\alpha_1}}}(g)}, \forall g,h \in G, \end{aligned}$$

and

$$\begin{aligned} L_{A_{\alpha_2}}(h^{-1}gh)e^{i\alpha w_{A_{\beta_2}}^{L_{A_{\alpha_2}}}(h^{-1}gh)} &= L_{A_{\alpha_2}}(ghh^{-1})e^{i\alpha w_{A_{\beta_2}}^{L_{A_{\alpha_2}}}(ghh^{-1})} \\ &= L_{A_{\alpha_2}}(g)e^{i\alpha w_{A_{\beta_2}}^{L_{A_{\alpha_2}}}(g)}, \forall g,h \in G. \end{aligned}$$

Conversely, suppose that $A_{(\alpha_{1,2},\beta_{1,2})}$ is constant in all conjugate classes of group G. Then,

$$\begin{aligned} S_{A_{\alpha_1}}(gh)e^{i\alpha w_{A_{\beta_1}}^{S_{A_{\alpha_1}}}(gh)} &= S_{A_{\alpha_1}}(ghgg^{-1})e^{i\alpha w_{A_{\beta_1}}^{S_{A_{\alpha_1}}}(ghgg^{-1})} \\ &= S_{A_{\alpha_1}}(g(hg)g^{-1})e^{i\alpha w_{A_{\beta_1}}^{S_{A_{\alpha_1}}}(g(hg)g^{-1})} \\ &= S_{A_{\alpha_1}}(hg)e^{i\alpha w_{A_{\beta_1}}^{S_{A_{\alpha_1}}}(hg)}, \forall g,h \in G, \end{aligned}$$

and

$$L_{A_{\alpha_2}}(gh)e^{i\alpha w_{A_{\beta_2}}^{L_{A_{\alpha_2}}}(pq)} = L_{A_{\alpha_2}}(ghgg^{-1})e^{i\alpha w_{A_{\beta_2}}^{L_{A_{\alpha_2}}}(pqpp^{-1})}$$

$$= L_{A_{\alpha_2}}(g(hg)g^{-1})e^{i\alpha w_{A_{\beta_2}}^{L_{A_{\alpha_2}}}(g(hg)g^{-1})}$$

$$= L_{A_{\alpha_2}}(hg)e^{i\alpha w_{A_{\beta_2}}^{L_{A_{\alpha_2}}}(hg)}, \forall g,h \in G.$$

Theorem 10. If $A_{(\alpha_{1,2},\beta_{1,2})}$ is an $(\alpha_{1,2},\beta_{1,2})$-CIFSG of a group G, then $A_{(\alpha_{1,2},\beta_{1,2})}$ is an $(\alpha_{1,2},\beta_{1,2})$-complex intuitionistic fuzzy normal subgroup if and only if $S_{A_{\alpha_1}}([g,h])e^{i\alpha w_{A_{\beta_1}}^{S_{A_{\alpha_1}}}([g,h])} \geq S_{A_{\alpha_1}}(g)e^{i\alpha w_{A_{\beta_1}}^{S_{A_{\alpha_1}}}(g)}$, and $L_{A_{\alpha_2}}([g,h])e^{i\alpha w_{A_{\beta_2}}^{L_{A_{\alpha_2}}}([g,h])} \leq L_{A_{\alpha_2}}(g)e^{i\alpha w_{A_{\beta_2}}^{L_{A_{\alpha_2}}}(g)}$, $\forall g,h \in G$.

Proof. Suppose that $A_{(\alpha_{1,2},\beta_{1,2})}$ is an $(\alpha_{1,2},\beta_{1,2})$- CIFNSG. Let $x,y \in G$ be element of group. So,

$$S_{A_{\alpha_1}}(g^{-1}h^{-1}gh)e^{i\alpha w_{A_{\beta_1}}^{S_{A_{\alpha_1}}}(g^{-1}h^{-1}gh)} \geq \min\left\{S_{A_{\alpha_1}}(h^{-1}gh)e^{i\alpha w_{A_{\beta_1}}^{S_{A_{\alpha_1}}}(h^{-1}gh)}, S_{A_{\alpha_1}}(g^{-1})e^{i\alpha w_{A_{\beta_1}}^{S_{A_{\alpha_1}}}(g^{-1})}\right\}$$

$$= \min\left\{S_{A_{\alpha_1}}(\mathrm{g})e^{i\alpha w_{A_{\beta_1}}^{S_{A_{\alpha_1}}}(\mathrm{g})}, S_{A_{\alpha_1}}(\mathrm{g})e^{i\alpha w_{A_{\beta_1}}^{S_{A_{\alpha_1}}}(\mathrm{g})}\right\}$$

$$S_{A_{\alpha_1}}([g,h])e^{i\alpha w_{A_{\beta_1}}^{S_{A_{\alpha_1}}}([g,h])} \geq S_{A_{\alpha_1}}(\mathrm{g})e^{i\alpha w_{A_{\beta_1}}^{S_{A_{\alpha_1}}}(\mathrm{g})},$$

and

$$L_{A_{\alpha_2}}(g^{-1}h^{-1}gh)e^{i\alpha w_{A_{\beta_2}}^{L_{A_{\alpha_2}}}(g^{-1}h^{-1}gh)}$$

$$\leq max\left\{L_{A_{\alpha_2}}(h^{-1}gh)e^{i\alpha w_{A_{\beta_2}}^{L_{A_{\alpha_2}}}(h^{-1}gh)}, L_{A_{\alpha_2}}(g^{-1})e^{i\alpha w_{A_{\beta_2}}^{L_{A_{\alpha_2}}}(g^{-1})}\right\}$$

$$= \min\left\{L_{A_{\alpha_2}}(\mathrm{g})e^{i\alpha w_{A_{\beta_2}}^{L_{A_{\alpha_2}}}(\mathrm{g})}, L_{A_{\alpha_2}}(\mathrm{g})e^{i\alpha w_{A_{\beta_2}}^{L_{A_{\alpha_2}}}(\mathrm{g})}\right\}$$

$$L_{A_{\alpha_2}}([g,h])e^{i\alpha w_{A_{\beta_2}}^{L_{A_{\alpha_2}}}([g,h])} \leq L_{A_{\alpha_2}}(\mathrm{g})e^{i\alpha w_{A_{\beta_2}}^{L_{A_{\alpha_2}}}(\mathrm{g})}.$$

Conversely, suppose that

$$S_{A_{\alpha_1}}([g,h])e^{i\alpha w_{A_{\beta_1}}^{S_{A_{\alpha_1}}}([g,h])} \geq S_{A_{\alpha_1}}(g)e^{i\alpha w_{A_{\beta_1}}^{S_{A_{\alpha_1}}}(g)},$$

and

$$L_{A_{\alpha_2}}([g,h])e^{i\alpha w_{A_{\beta_2}}^{L_{A_{\alpha_2}}([g,h])}} \leq L_{A_{\alpha_2}}(g)e^{i\alpha w_{A_{\beta_2}}^{L_{A_{\alpha_2}}(g)}}.$$

Let $g, o \in G$ be an element. Consider

$$S_{A_{\alpha_1}}(g^{-1}og)e^{i\alpha w_{A_{\beta_1}}^{S_{A_{\alpha_1}}(g^{-1}og)}} = S_{A_{\alpha_1}}(oo^{-1}g^{-1}og)e^{i\alpha w_{A_{\beta_1}}^{S_{A_{\alpha_1}}(oo^{-1}g^{-1}og)}}$$

$$\geq \min\left\{S_{A_{\alpha_1}}(o)e^{i\alpha w_{A_{\beta_1}}^{S_{A_{\alpha_1}}(o)}}, S_{A_{\alpha_1}}([o,g])e^{i\alpha w_{A_{\beta_1}}^{S_{A_{\alpha_1}}([o,g])}}\right\}$$

$$= S_{A_{\alpha_1}}(o)e^{i\alpha w_{A_{\beta_1}}^{S_{A_{\alpha_1}}(o)}} \tag{5}$$

and,

$$L_{A_{\alpha_2}}(g^{-1}og)e^{i\alpha w_{A_{\beta_2}}^{L_{A_{\alpha_2}}(g^{-1}og)}} = L_{A_{\alpha_2}}(oo^{-1}g^{-1}og)e^{i\alpha w_{A_{\beta_2}}^{L_{A_{\alpha_2}}(oo^{-1}g^{-1}og)}}$$

$$\leq \max\left\{L_{A_{\alpha_2}}(o)e^{i\alpha w_{A_{\beta_2}}^{L_{A_{\alpha_2}}(o)}}, L_{A_{\alpha_2}}([o,g])e^{i\alpha w_{A_{\beta_2}}^{L_{A_{\alpha_2}}([o,g])}}\right\}$$

$$= L_{A_{\alpha_2}}(o)e^{i\alpha w_{A_{\beta_2}}^{L_{A_{\alpha_2}}(o)}} \tag{5*}$$

Thus,

$$S_{A_{\alpha_1}}(g^{-1}og)e^{i\alpha w_{A_{\beta_1}}^{S_{A_{\alpha_1}}(g^{-1}og)}} \geq S_{A_{\alpha_1}}(o)e^{i\alpha w_{A_{\beta_1}}^{S_{A_{\alpha_1}}(o)}}, \forall o, g \in G. \tag{6}$$

and,

$$L_{A_{\alpha_2}}(g^{-1}og)e^{i\alpha w_{A_{\beta_2}}^{L_{A_{\alpha_2}}(g^{-1}og)}} \leq L_{A_{\alpha_2}}(o)e^{i\alpha w_{A_{\beta_2}}^{L_{A_{\alpha_2}}(o)}}, \forall o, g \in G. \tag{6*}$$

Now,

$$S_{A_{\alpha_1}}(o)e^{i\alpha w_{A_{\beta_1}}^{S_{A_{\alpha_1}}(o)}} = S_{A_{\alpha_1}}(gg^{-1}ogg^{-1})e^{i\alpha w_{A_{\beta_1}}^{S_{A_{\alpha_1}}(gg^{-1}ogg^{-1})}}$$

$$\geq \min\left\{S_{A_{\alpha_1}}(g)e^{i\alpha w_{A_{\beta_1}}^{S_{A_{\alpha_1}}(g)}}, S_{A_{\alpha_1}}(g^{-1}og)e^{i\alpha w_{A_{\beta_1}}^{S_{A_{\alpha_1}}(g^{-1}og)}}\right\} \tag{7}$$

And,

$$L_{A_{\alpha_2}}(o)e^{i\alpha w_{A_{\beta_2}}^{L_{A_{\alpha_2}}(o)}} = L_{A_{\alpha_2}}(gg^{-1}ogg^{-1})e^{i\alpha w_{A_{\beta_2}}^{L_{A_{\alpha_2}}(gg^{-1}ogg^{-1})}}$$

$$\leq \max\left\{L_{A_{\alpha_2}}(g)e^{i\alpha w_{A_{\beta_2}}^{L_{A_{\alpha_2}}(g)}}, L_{A_{\alpha_2}}(g^{-1}og)e^{i\alpha w_{A_{\beta_2}}^{L_{A_{\alpha_2}}(g^{-1}og)}}\right\}. \quad (7^*)$$

Here. Two cases have to be proved.

Case 1. If

$$\min\left\{S_{A_{\alpha_1}}(g)e^{i\alpha w_{A_{\beta_1}}^{S_{A_{\alpha_1}}(g)}}, S_{A_{\alpha_1}}(g^{-1}og)e^{i\alpha w_{A_{\beta_1}}^{S_{A_{\alpha_1}}(g^{-1}og)}}\right\} = S_{A_{\alpha_1}}(g)e^{i\alpha w_{A_{\beta_1}}^{S_{A_{\alpha_1}}(g)}}.$$

Then, we obtain

$$S_{A_{\alpha_1}}(o)e^{i\alpha w_{A_{\beta_1}}^{S_{A_{\alpha_1}}(o)}} \geq S_{A_{\alpha_1}}(g)e^{i\alpha w_{A_{\beta_1}}^{S_{A_{\alpha_1}}(g)}}, \forall o, g \in G.$$

And if

$$\max\left\{L_{A_{\alpha_2}}(g)e^{i\alpha w_{A_{\beta_2}}^{L_{A_{\alpha_2}}(g)}}, L_{A_{\alpha_2}}(g^{-1}og)e^{i\alpha w_{A_{\beta_2}}^{L_{A_{\alpha_2}}(g^{-1}og)}}\right\} = L_{A_{\alpha_2}}(g)e^{i\alpha w_{A_{\beta_2}}^{L_{A_{\alpha_2}}(g)}}.$$

Then, we obtain

$$L_{A_{\alpha_2}}(o)e^{i\alpha w_{A_{\beta_2}}^{L_{A_{\alpha_2}}(o)}} \leq L_{A_{\alpha_2}}(g)e^{i\alpha w_{A_{\beta_1}}^{S_{A_{\alpha_1}}(g)}}, \forall o, g \in G.$$

This implies that $A_{(\alpha_{1,2},\beta_{1,2})}$ is a constant mapping.

Case 2. If

$$\min\left\{S_{A_{\alpha_1}}(g)e^{i\alpha w_{A_{\beta_1}}^{S_{A_{\alpha_1}}(g)}}, S_{A_{\alpha_1}}(g^{-1}og)e^{i\alpha w_{A_{\beta_1}}^{S_{A_{\alpha_1}}(g^{-1}og)}}\right\} = S_{A_{\alpha_1}}(g^{-1}og)e^{i\alpha w_{A_{\beta_1}}^{S_{A_{\alpha_1}}(g^{-1}og)}}.$$

Then, from Eq (7) we have

$$S_{A_{\alpha_1}}(o)e^{i\alpha w_{A_{\beta_1}}^{S_{A_{\alpha_1}}(o)}}\Bigg\} \geq S_{A_{\alpha_1}}(g^{-1}og)e^{i\alpha w_{A_{\beta_1}}^{S_{A_{\alpha_1}}(g^{-1}og)}}, \quad (8)$$

and if

$$\max\left\{L_{A_{\alpha_2}}(g)e^{i\alpha w_{A_{\beta_2}}^{L_{A_{\alpha_2}}(g)}}, L_{A_{\alpha_2}}(g^{-1}og)e^{i\alpha w_{A_{\beta_2}}^{L_{A_{\alpha_2}}(g^{-1}og)}}\right\} = L_{A_{\alpha_2}}(g^{-1}og)e^{i\alpha w_{A_{\beta_2}}^{L_{A_{\alpha_2}}(g^{-1}og)}}.$$

Then, from Eq (7) we have

$$L_{A_{\alpha_2}}(o)e^{i\alpha w_{A_{\beta_2}}^{L_{A_{\alpha_2}}(o)}}\Bigg\} \leq L_{A_{\alpha_2}}(g^{-1}og)e^{i\alpha w_{A_{\beta_2}}^{L_{A_{\alpha_2}}(g^{-1}og)}}. \quad (8^*)$$

In the view of Equations (6,6*) and (8,8*) we have

$$S_{A_{\alpha_1}}(o)e^{i\alpha w_{A_{\beta_1}}^{S_{A_{\alpha_1}}}(o)}\Big\} = S_{A_{\alpha_1}}(g^{-1}og)e^{i\alpha w_{A_{\beta_1}}^{S_{A_{\alpha_1}}}(g^{-1}og)}$$

and

$$L_{A_{\alpha_2}}(o)e^{i\alpha w_{A_{\beta_2}}^{L_{A_{\alpha_2}}}(o)}\Big\} = L_{A_{\alpha_2}}(g^{-1}og)e^{i\alpha w_{A_{\beta_2}}^{L_{A_{\alpha_2}}}(g^{-1}og)}.$$

Hence, $A_{(\alpha_{1,2},\beta_{1,2})}$ is constant.

Theorem 11. Let $A_{(\alpha_{1,2},\beta_{1,2})}$ be $(\alpha_{1,2},\beta_{1,2})$-*CIFNSG* of group G. Then, the set $A_{(\alpha_{1,2},\beta_{1,2})}^{id} = \{x \in G: A_{(\alpha_{1,2},\beta_{1,2})}(x^{-1}) = A_{(\alpha_{1,2},\beta_{1,2})}(id)\}$ is a normal subgroup of group G.

Proof. Clearly that $A_{(\alpha_{1,2},\beta_{1,2})}^{id} \neq \emptyset$ because $id \in G$. Let $x, y \in A_{(\alpha_{1,2},\beta_{1,2})}^{id}$ be any elements. Consider

$$S_{A_{\alpha_1}}(xy)e^{i\alpha w_{A_{\beta_1}}^{S_{A_{\alpha_1}}}(xy)} \geq \min\left\{S_{A_{\alpha_1}}(x)e^{i\alpha w_{A_{\beta_1}}^{S_{A_{\alpha_1}}}(x)}, S_{A_{\alpha_1}}(y)e^{i\alpha w_{A_{\beta_1}}^{S_{A_{\alpha_1}}}(y)}\right\}$$

$$= \min\left\{S_{A_{\alpha_1}}(id)e^{i\alpha w_{A_{\beta_1}}^{S_{A_{\alpha_1}}}(id)}, S_{A_{\alpha_1}}(id)e^{i\alpha w_{A_{\beta_1}}^{S_{A_{\alpha_1}}}(id)}\right\}.$$

And

$$L_{A_{\alpha_2}}(xy)e^{i\alpha w_{A_{\beta_2}}^{L_{A_{\alpha_2}}}(xy)} \leq \max\left\{L_{A_{\alpha_2}}(x)e^{i\alpha w_{A_{\beta_2}}^{L_{A_{\alpha_2}}}(x)}, L_{A_{\alpha_2}}(y)e^{i\alpha w_{A_{\beta_2}}^{L_{A_{\alpha_2}}}(y)}\right\}$$

$$= \max\left\{L_{A_{\alpha_2}}(id)e^{i\alpha w_{A_{\beta_2}}^{L_{A_{\alpha_2}}}(id)}, L_{A_{\alpha_2}}(id)e^{i\alpha w_{A_{\beta_2}}^{L_{A_{\alpha_2}}}(id)}\right\}.$$

This implies that

$$S_{A_{\alpha_1}}(xy)e^{i\alpha w_{A_{\beta_1}}^{S_{A_{\alpha_1}}}(xy)} \geq S_{A_{\alpha_1}}(id)e^{i\alpha w_{A_{\beta_1}}^{S_{A_{\alpha_1}}}(id)}$$

and

$$L_{A_{\alpha_2}}(xy)e^{i\alpha w_{A_{\beta_2}}^{L_{A_{\alpha_2}}}(xy)} \leq L_{A_{\alpha_2}}(id)e^{i\alpha w_{A_{\beta_2}}^{L_{A_{\alpha_2}}}(id)}.$$

However,

$$S_{A_{\alpha_1}}(xy)e^{i\alpha w_{A_{\beta_1}}^{S_{A_{\alpha_1}}}(xy)} \leq S_{A_{\alpha_1}}(id)e^{i\alpha w_{A_{\beta_1}}^{S_{A_{\alpha_1}}}(id)},$$

and

$$L_{A_{\alpha_2}}(xy)e^{i\alpha w_{A_{\beta_2}}^{L_{A_{\alpha_2}}(xy)}} \geq L_{A_{\alpha_2}}(id)e^{i\alpha w_{A_{\beta_2}}^{L_{A_{\alpha_2}}(id)}}.$$

Therefore,

$$S_{A_{\alpha_1}}(xy)e^{i\alpha w_{A_{\beta_1}}^{S_{A_{\alpha_1}}(xy)}} = S_{A_{\alpha_1}}(id)e^{i\alpha w_{A_{\beta_1}}^{S_{A_{\alpha_1}}(id)}} \text{ and } L_{A_{\alpha_2}}(xy)e^{i\alpha w_{A_{\beta_2}}^{L_{A_{\alpha_2}}(xy)}} = L_{A_{\alpha_2}}(id)e^{i\alpha w_{A_{\beta_2}}^{L_{A_{\alpha_2}}(id)}}.$$

This implies that

$$A_{(\alpha_{1,2},\beta_{1,2})}(x^{-1}) = A_{(\alpha_{1,2},\beta_{1,2})}(id),$$

which implies that $xy \in A^{id}_{(\alpha_{1,2},\beta_{1,2})}$.

Further,

$$S_{A_{\alpha_1}}(y^{-1})e^{i\alpha w_{A_{\beta_1}}^{S_{A_{\alpha_1}}(y-1)}} \geq S_{A_{\alpha_1}}(y)e^{i\alpha w_{A_{\beta_1}}^{S_{A_{\alpha_1}}(y)}} = S_{A_{\alpha_1}}(id)e^{i\alpha w_{A_{\beta_1}}^{S_{A_{\alpha_1}}(id)}}$$

and

$$L_{A_{\alpha_2}}(y^{-1})e^{i\alpha w_{A_{\beta_2}}^{L_{A_{\alpha_2}}(y-1)}} \leq L_{A_{\alpha_2}}(y)e^{i\alpha w_{A_{\beta_2}}^{L_{A_{\alpha_2}}(y)}} = L_{A_{\alpha_2}}(\mathrm{id})e^{i\alpha w_{A_{\beta_2}}^{L_{A_{\alpha_2}}(id)}}.$$

However,

$$S_{A_{\alpha_1}}(x)e^{i\alpha w_{A_{\beta_1}}^{S_{A_{\alpha_1}}(x)}} \leq S_{A_{\alpha_1}}(id)e^{i\alpha w_{A_{\beta_1}}^{S_{A_{\alpha_1}}(id)}}$$

and

$$L_{A_{\alpha_2}}(x)e^{i\alpha w_{A_{\beta_2}}^{L_{A_{\alpha_2}}(x)}} \geq L_{A_{\alpha_2}}(id)e^{i\alpha w_{A_{\beta_2}}^{L_{A_{\alpha_2}}(id)}}.$$

Thus, $A^{id}_{(\alpha_{1,2},\beta_{1,2})}$ is subgroup of group G. Moreover, let $x \in A^{id}_{(\alpha_{1,2},\beta_{1,2})}$ and $y \in G$. We have

$$S_{A_{\alpha_1}}(y^{-1}xy)e^{i\alpha w_{A_{\beta_1}}^{S_{A_{\alpha_1}}(y^{-1}xy)}} = S_{A_{\alpha_1}}(x)e^{i\alpha w_{A_{\beta_1}}^{S_{A_{\alpha_1}}(x)}}$$

and

$$L_{A_{\alpha_2}}(y^{-1}xy)e^{i\alpha w_{A_{\beta_2}}^{L_{A_{\alpha_2}}(y^{-1}xy)}} = L_{A_{\alpha_2}}(x)e^{i\alpha w_{A_{\beta_2}}^{L_{A_{\alpha_2}}(x)}}.$$

This implies that $y^{-1}xy \in A^{id}_{(\alpha_{1,2},\beta_{1,2})}$. Hence, $A^{id}_{(\alpha_{1,2},\beta_{1,2})}$ is a normal subgroup.

Theorem 12. Let $A_{(\alpha_{1,2},\beta_{1,2})}$ be an $(\alpha_{1,2},\beta_{1,2})$-CIFNSG of group G. Then,

$$gA_{(\alpha_{1,2},\beta_{1,2})} = hA_{(\alpha_{1,2},\beta_{1,2})} \text{ if and if only } g^{-1}h \in A^{id}_{(\alpha_{1,2},\beta_{1,2})},$$

$$A_{(\alpha_{1,2},\beta_{1,2})}g = A_{(\alpha_{1,2},\beta_{1,2})}h \text{ if and if only } gh^{-1} \in A^{id}_{(\alpha_{1,2},\beta_{1,2})}.$$

Proof. (i) For any $g, h \in G$, we have $gA_{(\alpha_{1,2},\beta_{1,2})} = hA_{(\alpha_{1,2},\beta_{1,2})}$.Consider,

$$\begin{aligned} S_{A_{\alpha_1}}(g^{-1}h)e^{i\alpha w_{A_{\beta_1}}^{S_{A_{\alpha_1}}}(g^{-1}h)} &= \min\left\{S_A(g^{-1}h)e^{i\alpha w_A^{S_A}(g^{-1}h)}, \alpha_1 e^{i\beta_1}\right\} \\ &= \min\left\{S_{gA}(h)e^{i\alpha w_{gA}^{S_{gA}}(h)}, \alpha_1 e^{i\beta_1}\right\} \\ &= S_{gA_{\alpha_1}}(h)e^{i\alpha w_{gA_{\beta_1}}^{S_{gA_{\alpha_1}}}(h)} \\ &= S_{hA_{\alpha_1}}(h)e^{i\alpha w_{hA_{\beta_1}}^{S_{hA_{\alpha_1}}}(h)} \\ &= \min\left\{S_A(h^{-1}h)e^{i\alpha w_A^{S_A}(h^{-1}h)}, \alpha_1 e^{i\beta_1}\right\} \\ &= \min\left\{S_A(id)e^{i\alpha w_A^{S_A}(id)}, \alpha_1 e^{i\beta_1}\right\} \\ &= S_{A_{\alpha_1}}(id)e^{i\alpha w_{A_{\beta_1}}^{S_{A_{\alpha_1}}}(id)}. \end{aligned}$$

and

$$\begin{aligned} L_{A_{\alpha_2}}(g^{-1}h)e^{i\varphi_{A_\beta}(g^{-1}h)} &= \max\left\{L_A(g^{-1}h)e^{i\alpha w_A^{L_A}(g^{-1}h)}, \alpha_2 e^{i\beta_2}\right\} \\ &= \max\left\{L_{gA}(h)e^{i\alpha w_{gA}^{L_{gA}}(h)}, \alpha_2 e^{i\beta_2}\right\} \\ &= L_{gA_{\alpha_2}}(h)e^{i\alpha w_{gA_{\beta_2}}^{L_{gA_{\alpha_2}}}(h)} \\ &= L_{hA_{\alpha_2}}(h)e^{i\alpha w_{hA_{\beta_2}}^{L_{hA_{\alpha_2}}}((h)} \\ &= \max\left\{L_A(h^{-1}h)e^{i\alpha w_A^{L_A}(h^{-1}h)}, \alpha_2 e^{i\beta_2}\right\} \\ &= \max\left\{L_A(id)e^{i\alpha w_A^{L_A}(id)}, \alpha_2 e^{i\beta_2}\right\} \\ &= L_{A_{\alpha_2}}(id)e^{i\alpha w_{A_{\beta_2}}^{L_{A_{\alpha_2}}}(id)}. \end{aligned}$$

Therefore, $g^{-1}h \in A^{id}_{(\alpha_{1,2},\beta_{1,2})}$.

Conversely, let $g^{-1}h \in A^{id}_{(\alpha_{1,2},\beta_{1,2})}$ implies that

$$S_{A_{\alpha_1}}(g^{-1}h)e^{i\alpha w_{A_{\beta_1}}^{S_{A_{\alpha_1}}}(g^{-1}h)} = S_{A_{\alpha_1}}(id)e^{i\alpha w_{A_{\beta_1}}^{S_{A_{\alpha_1}}}(id)}$$

and

$$L_{A_{\alpha_2}}(g^{-1}h)e^{i\alpha w_{A_{\beta_2}}^{L_{A_{\alpha_2}}}(g^{-1}h)} = L_{A_{\alpha_2}}(id)e^{i\alpha w_{A_{\beta_2}}^{L_{A_{\alpha_2}}}(id)}.$$

Consider, $S_{gA_{\alpha_1}}(o)e^{i\alpha w_{A_{\beta_1}}^{S_{gA_{\alpha_1}}}(o)}$

$$\begin{aligned}
&= \min\left\{S_A(g^{-1}o)e^{i\alpha w_A^{S_A}(g^{-1}a)}, \alpha_1 e^{i\beta_1}\right\} \\
&= S_{A_{\alpha_1}}(g^{-1}a)e^{i\alpha w_{A_{\beta_1}}^{S_{A_{\alpha_1}}}(g^{-1}o)} \\
&= S_{A_{\alpha_1}}(g^{-1}h)(h^{-1}o)e^{i\alpha w_{A_{\beta_1}}^{S_{A_{\alpha_1}}}(g^{-1}h)(h^{-1}o)} \\
&\geq \min\left\{S_{A_{\alpha_1}}(g^{-1}h)e^{i\alpha w_{A_{\beta_1}}^{S_{A_{\alpha_1}}}(g^{-1}h)}, S_{A_{\alpha_1}}(h^{-1}o)e^{i\alpha w_{A_{\beta_1}}^{S_{A_{\alpha_1}}}(h^{-1}o)}\right\} \\
&= \min\left\{S_{A_{\alpha_1}}(id)e^{i\alpha w_{A_{\beta_1}}^{S_{A_{\alpha_1}}}(id)}, S_{A_{\alpha_1}}(h^{-1}o)e^{i\alpha w_{A_{\beta_1}}^{S_{A_{\alpha_1}}}(h^{-1}o)}\right\} \\
&= S_{A_{\alpha_1}}(h^{-1}o)e^{i\alpha w_{A_{\beta_1}}^{S_{A_{\alpha_1}}}(h^{-1}o)} \\
&= S_{hA_{\alpha_1}}(o)e^{i\alpha w_{hA_{\beta_1}}^{S_{hA_{\alpha_1}}}(o)}.
\end{aligned}$$

And, $L_{gA_{\alpha_2}}(o)e^{i\alpha w_{A_{\beta_2}}^{L_{gA_{\alpha_2}}}(o)}$

$$\begin{aligned}
&= \max\left\{L_A(g^{-1}o)e^{i\alpha w_A^{L_A}(g^{-1}o)}, \alpha_2 e^{i\beta_2}\right\} \\
&= L_{A_{\alpha_2}}(g^{-1}o)e^{i\alpha w_{A_{\beta_2}}^{L_{A_{\alpha_2}}}(g^{-1}o)} \\
&= L_{A_{\alpha_2}}(g^{-1}h)(h^{-1}o)e^{i\alpha w_{A_{\beta_2}}^{L_{A_{\alpha_2}}}(g^{-1}h)(h^{-1}o)} \\
&\leq \max\left\{L_{A_{\alpha_2}}(g^{-1}h)e^{i\alpha w_{A_{\beta_2}}^{L_{A_{\alpha_2}}}(g^{-1}h)}, S_{A_{\alpha_1}}(h^{-1}o)e^{i\alpha w_{A_{\beta_2}}^{L_{A_{\alpha_2}}}(h^{-1}o)}\right\} \\
&= \max\left\{L_{A_{\alpha_2}}(id)e^{i\alpha w_{A_{\beta_2}}^{L_{A_{\alpha_2}}}(id)}, L_{A_{\alpha_2}}(h^{-1}o)e^{i\alpha w_{A_{\beta_2}}^{L_{A_{\alpha_2}}}(h^{-1}o)}\right\} \\
&= L_{A_{\alpha_2}}(h^{-1}o)e^{i\alpha w_{A_{\beta_2}}^{L_{A_{\alpha_2}}}(h^{-1}o)} \\
&= L_{hA_{\alpha_2}}(o)e^{i\alpha w_{hA_{\beta_2}}^{L_{hA_{\alpha_2}}}(o)}.
\end{aligned}$$

Replace the position of g and h, and we gain

$$S_{hA_{\alpha_1}}(o)e^{i\alpha w_{hA_{\beta_1}}^{S_{hA_{\alpha_1}}}(o)} \geq S_{gA_{\alpha_1}}(o)e^{i\alpha w_{gA_{\beta_1}}^{S_{gA_{\alpha_1}}}(o)}$$

and

$$L_{hA_{\alpha_2}}(o)e^{i\alpha w_{hA_{\beta_2}}^{L_{hA_{\alpha_2}}}(o)} \leq L_{gA_{\alpha_2}}(o)e^{i\alpha w_{gA_{\beta_2}}^{L_{gA_{\alpha_2}}}(o)}.$$

Therefore,

$$S_{gA_{\alpha_1}}(o)e^{i\alpha w_{gA_{\beta_1}}^{S_{gA_{\alpha_1}}}(o)} = S_{hA_{\alpha_1}}(o)e^{i\alpha w_{hA_{\beta_1}}^{S_{hA_{\alpha_1}}}(o)}$$

and

$$L_{gA_{\alpha_2}}(o)e^{i\alpha w_{gA_{\beta_2}}^{L_{gA_{\alpha_2}}}(o)} = L_{hA_{\alpha_2}}(o)e^{i\alpha w_{hA_{\beta_2}}^{L_{hA_{\alpha_2}}}(o)}.$$

(ii) Similar to part (i).

Theorem 13. Let $A_{(\alpha_{1,2},\beta_{1,2})}$ be an $(\alpha_{1,2},\beta_{1,2})$-CIFNSG of group G and g, h, o, and f be any elements in G. If $gA_{(\alpha_{1,2},\beta_{1,2})} = oA_{(\alpha_{1,2},\beta_{1,2})}$ and $hA_{(\alpha_{1,2},\beta_{1,2})} = fA_{(\alpha_{1,2},\beta_{1,2})}$, then $ghA_{(\alpha_{1,2},\beta_{1,2})} = ofA_{(\alpha_{1,2},\beta_{1,2})}$.

Proof. Given that $gA_{(\alpha_{1,2},\beta_{1,2})} = oA_{(\alpha_{1,2},\beta_{1,2})}$ and $hA_{(\alpha_{1,2},\beta_{1,2})} = fA_{(\alpha_{1,2},\beta_{1,2})}$. This implies that $g^{-1}o, h^{-1}f \in A^{id}_{(\alpha_{1,2},\beta_{1,2})}$. Consider,

$$(gh)^{-1}(of) = h^{-1}(g^{-1}o)f = h^{-1}(g^{-1}o)(hh^{-1})f = [h^{-1}(g^{-1}o)(h)](h^{-1}f).$$

As $A^{id}_{(\alpha_{1,2},\beta_{1,2})}$ is normal subgroup of G. Therefor,

$$(gh)^{-1}(of) \in A^{id}_{(\alpha_{1,2},\beta_{1,2})}.$$

Consequently,

$$ghA_{(\alpha_{1,2},\beta_{1,2})} = ofA_{(\alpha_{1,2},\beta_{1,2})}.$$

Theorem 14. Let $G/A_{(\alpha_{1,2},\beta_{1,2})} = \left\{gA_{(\alpha_{1,2},\beta_{1,2})}: g \in G\right\}$ be the collection of all $(\alpha_{1,2},\beta_{1,2})$-complex intuitionistic fuzzy cosets of $(\alpha_{1,2},\beta_{1,2})$-CIFNSG $A_{(\alpha_{1,2},\beta_{1,2})}$ of G. Then, $(\star)$ is a well-defined binary operation under $G/A_{(\alpha_{1,2},\beta_{1,2})}$ and is defined as $gA_{(\alpha_{1,2},\beta_{1,2})} \star hA_{(\alpha_{1,2},\beta_{1,2})} = ghA_{(\alpha_{1,2},\beta_{1,2})}$ for all $g, h \in G$.

Proof. We have $gA_{(\alpha_{1,2},\beta_{1,2})} = hA_{(\alpha_{1,2},\beta_{1,2})}$ and $oA_{(\alpha_{1,2},\beta_{1,2})} = fA_{(\alpha_{1,2},\beta_{1,2})}$, for any $o, f, g, h \in G$. Let $v \in G$ be any element, then

$$\left[gA_{(\alpha_{1,2},\beta_{1,2})} \star oA_{(\alpha_{1,2},\beta_{1,2})}\right](v) = \left(gaA_{(\alpha_{1,2},\beta_{1,2})}(v)\right) = \left(v, \mu_{paA_\alpha}(v)e^{i\varphi_{paA_\beta}(v)}\right).$$

Consider,

$$\begin{aligned}
S_{goA_{\alpha_1}}(v)e^{i\alpha w_{goA_{\beta_1}}^{S_{goA_{\alpha_1}}}(v)} &= \min\left\{S_{goA}(v)e^{i\alpha w_{goA}^{S_{goA}}(v)}, \alpha_1 e^{i\beta_1}\right\} \\
&= \min\left\{S_A((go)^{-1}v)e^{i\alpha w_A^{S_A}((go)^{-1}v)}, \alpha_1 e^{i\beta_1}\right\} \\
&= \min\left\{S_A\left(o^{-1}(g^{-1}v)\right)e^{i\alpha w_A^{S_A}\left(o^{-1}(g^{-1}v)\right)}, \alpha_1 e^{i\beta_1}\right\} \\
&= S_{oA_{\alpha_1}}(g^{-1}v)e^{i\alpha w_{oA_{\beta_1}}^{S_{oA_{\alpha_1}}}(g^{-1}v)}
\end{aligned}$$

$$= S_{fA_{\alpha_1}}(g^{-1}v)e^{i\alpha w_{fA_{\beta_1}}^{S_{fA_{\alpha_1}}}(g^{-1}v)}$$

$$= \min\left\{S_A\big(f^{-1}(g^{-1}v)\big)e^{i\alpha w_A^{S_A}\left(f^{-1}(g^{-1}v)\right)}, \alpha_1 e^{i\beta_1}\right\}$$

$$= \min\left\{S_A\big(g^{-1}(vf^{-1})\big)e^{i\alpha w_A^{S_A}\left(g^{-1}(vf^{-1})\right)}, \alpha_1 e^{i\beta_1}\right\}$$

$$= S_{gA_{\alpha_1}}(vf^{-1})e^{i\alpha w_{gA_{\beta_1}}^{S_{gA_{\alpha_1}}}(vf^{-1})}$$

$$= S_{hA_{\alpha_1}}(vf^{-1})e^{i\alpha w_{hA_{\beta_1}}^{S_{hA_{\alpha_1}}}(vf^{-1})}$$

$$= \min\left\{S_A\big(h^{-1}(vf^{-1})\big)e^{i\alpha w_A^{S_A}\left(h^{-1}(vf^{-1})\right)}, \alpha_1 e^{i\beta_1}\right\}$$

$$= \min\left\{S_A(h^{-1}v)f^{-1}e^{i\alpha w_A^{S_A}(h^{-1}v)f^{-1}}, \alpha_1 e^{i\beta_1}\right\}$$

$$= \min\left\{S_A(f^{-1}h^{-1}(v))e^{i\alpha w_A^{S_A}\left(f^{-1}h^{-1}(v)\right)}, \alpha_1 e^{i\beta_1}\right\}$$

$$= \min\left\{S_A((hf)^{-1}(v))e^{i\alpha w_A^{S_A}((hf)^{-1}(v))}, \alpha_1 e^{i\beta_1}\right\}$$

$$= S_{hfA_{\alpha_1}}(v)e^{i\alpha w_{hfA_{\beta_1}}^{S_{hfA_{\alpha_1}}}(v)}.$$

And,

$$L_{goA_{\alpha_2}}(v)e^{i\alpha w_{goA_{\beta_2}}^{L_{goA_{\alpha_2}}}(v)} = \max\left\{L_{goA}(v)e^{i\alpha w_{goA}^{L_{goA}}(v)}, \alpha_2 e^{i\beta_2}\right\}$$

$$= \max\left\{L_A((go)^{-1}v)e^{i\alpha w_A^{L_A}((go)^{-1}v)}, \alpha_2 e^{i\beta_2}\right\}$$

$$= \max\left\{L_A\big(o^{-1}(g^{-1}v)\big)e^{i\alpha w_A^{L_A}\left(o^{-1}(g^{-1}v)\right)}, \alpha_2 e^{i\beta_2}\right\}$$

$$= L_{oA_{\alpha_2}}(g^{-1}v)e^{i\alpha w_{oA_{\beta_2}}^{L_{oA_{\alpha_2}}}(g^{-1}v)}$$

$$= L_{fA_{\alpha_2}}(g^{-1}v)e^{i\alpha w_{fA_{\beta_2}}^{L_{fA_{\alpha_2}}}(g^{-1}v)}$$

$$= \max\left\{L_A\big(f^{-1}(g^{-1}v)\big)e^{i\alpha w_A^{L_A}\left(f^{-1}(g^{-1}v)\right)}, \alpha_2 e^{i\beta_2}\right\}$$

$$= \max\left\{L_A\big(g^{-1}(vf^{-1})\big)e^{i\alpha w_A^{L_A}\left(g^{-1}(vf^{-1})\right)}, \alpha_2 e^{i\beta_2}\right\}$$

$$= L_{gA_{\alpha_2}}(vf^{-1})e^{i\alpha w_{gA_{\beta_2}}^{L_{gA_{\alpha_2}}}(vf^{-1})}$$

$$= L_{hA_{\alpha_2}}(vf^{-1})e^{i\alpha w_{hA_{\beta_2}}^{L_{hA_{\alpha_2}}}(vf^{-1})}$$

$$= \max\left\{L_A\left(h^{-1}(vf^{-1})\right)e^{i\alpha w_A^{L_A}\left(h^{-1}(vf^{-1})\right)}, \alpha_2 e^{i\beta_2}\right\}$$

$$= \max\left\{L_A(h^{-1}v)f^{-1}e^{i\alpha w_A^{L_A}(h^{-1}v)f^{-1}}, \alpha_2 e^{i\beta_2}\right\}$$

$$= \max\left\{L_A(f^{-1}h^{-1}(v))e^{i\alpha w_A^{L_A}\left(f^{-1}h^{-1}(v)\right)}, \alpha_2 e^{i\beta_2}\right\}$$

$$= \max\left\{L_A((hf)^{-1}(v))e^{i\alpha w_A^{L_A}((hf)^{-1}(v))}, \alpha_2 e^{i\beta_2}\right\}$$

$$= S_{hfA_{\alpha_2}}(v)e^{i\alpha w_{hfA_{\beta_2}}^{S_{hfA_{\alpha_2}}}(v)}.$$

Hence, we concluded that the axiom of associative and closure under the presented binary operation * are satisfied for the set $/A_{(\alpha_{1,2},\beta_{1,2})}$.Further,

$$S_{A_{\alpha_1}}e^{i\alpha w_{A_{\beta_1}}^{S_{A_{\alpha_1}}}} \star S_{gA_{\alpha_1}}e^{\alpha w_{A_{\beta_1}}^{S_{gA_{\alpha_1}}}} = S_{idA_{\alpha_1}}e^{i\alpha w_{id_{\beta_1}}^{S_{id\alpha_1}}} \star S_{gA_{\alpha_1}}e^{i\alpha w_{A_{\beta_1}}^{S_{gA_{\alpha_1}}}} = S_{gA_{\alpha_1}}e^{i\alpha w_{gA_{\beta_1}}^{S_{gA_{\alpha_1}}}}$$
$$= S_{gA_{\alpha_1}}e^{i\alpha w_{gA_{\beta_1}}^{S_{gA_{\alpha_1}}}} \Longrightarrow S_{A_{\alpha_1}}e^{i\alpha w_{A_{\beta_1}}^{S_{A_{\alpha_1}}}}$$

and

$$L_{A_{\alpha_2}}e^{i\alpha w_{A_{\beta_2}}^{L_{A_{\alpha_2}}}} \star L_{gA_{\alpha_2}}e^{\alpha w_{A_{\beta_2}}^{L_{gA_{\alpha_2}}}} = L_{idA_{\alpha_2}}e^{i\alpha w_{id_{\beta_2}}^{L_{idA_{\alpha_2}}}} \star L_{gA_{\alpha_2}}e^{i\alpha w_{A_{\beta_2}}^{L_{gA_{\alpha_2}}}} = L_{gA_{\alpha_2}}e^{i\alpha w_{gA_{\beta_2}}^{L_{gA_{\alpha_2}}}} =$$
$$L_{gA_{\alpha_2}}e^{i\alpha w_{gA_{\beta_2}}^{L_{gA_{\alpha_2}}}} \Longrightarrow L_{A_{\alpha_2}},$$

an element of $G/A_{(\alpha_{1,2},\beta_{1,2})}$. So the inverse of every element of $G/A_{(\alpha_{1,2},\beta_{1,2})}$ exist if

$$S_{gA_{\alpha_1}}e^{i\alpha w_{gA_{\beta_1}}^{S_{gA_{\alpha_1}}}} \in G/A_{(\alpha_{1,2},\beta_{1,2})}.$$

And

$$L_{gA_{\alpha_2}}e^{i\alpha w_{A_{\beta_2}}^{L_{gA_{\alpha_2}}}} \in G/A_{(\alpha_{1,2},\beta_{1,2})},$$

and then there exists an element,

$$S_{g^{-1}A_{\alpha_1}}e^{i\alpha w_{gA_{\beta_1}}^{S_{g^{-1}A_{\alpha_1}}}} \in G/A_{(\alpha_{1,2},\beta_{1,2})}.$$

And

$$L_{h^{-1}A_{\alpha_2}} e^{i\alpha w_{h^{-1}A_{\beta_2}}^{h^{-1}A_{\alpha_2}}}$$

such that

$$S_{g^{-1}A_{\alpha_1}} e^{i\alpha w_{g^{-1}A_{\beta_1}}^{S_{g^{-1}A_{\alpha_1}}}} = S_{A_{\alpha_1}} e^{i\alpha w_{A_{\beta_1}}^{S_{A_{\alpha_1}}}}.$$

And

$$L_{g^{-1}A_{\alpha_2}} e^{i\alpha w_{g^{-1}A_{\beta_2}}^{L_{g^{-1}A_{\alpha_2}}}} = L_{A_{\alpha_2}} e^{i\alpha w_{A_{\beta_2}}^{L_{A_{\alpha_2}}}}.$$

As a result, $G/A_{(\alpha_{1,2},\beta_{1,2})}$ is a group. And is called the CIF quotient group of the G.

Lemma 1. Let $m: G \to G/A_{(\alpha_{1,2},\beta_{1,2})}$ be natural homomorphism and defined by the rule, $m(g) = gA_{(\alpha_{1,2},\beta_{1,2})}$ with the kernel $m = A^{id}_{(\alpha_{1,2},\beta_{1,2})}$.

Proof. Let g, h be any elements of group G, and then

$$m(gh) = ghA_{(\alpha_{1,2},\beta_{1,2})} = S_{ghA_{\alpha_1}} e^{i\alpha w_{ghA_{\beta_1}}^{S_{ghA_{\alpha_1}}}} = S_{gA_{\alpha_1}} e^{i\alpha w_{gA_{\beta_1}}^{S_{gA_{\alpha_1}}}} \star S_{hA_{\alpha_1}} e^{i\alpha w_{hA_{\beta_1}}^{S_{hA_{\alpha_1}}}}$$
$$= gA_{(\alpha_{1,2},\beta_{1,2})} \star hA_{(\alpha_{1,2},\beta_{1,2})} = m(g) \star m(h).$$

And

$$m(gh) = ghA_{(\alpha_{1,2},\beta_{1,2})} = L_{ghA_{\alpha_2}} e^{i\alpha w_{ghA_{\beta_2}}^{L_{ghA_{\alpha_2}}}} = L_{gA_{\alpha_2}} e^{i\alpha w_{gA_{\beta_2}}^{L_{gA_{\alpha_2}}}} \star L_{hA_{\alpha_2}} e^{i\alpha w_{hA_{\beta_2}}^{L_{hA_{\alpha_2}}}}$$
$$= gA_{(\alpha_{1,2},\beta_{1,2})} \star hA_{(\alpha_{1,2},\beta_{1,2})} = m(g) \star m(h).$$

Thus, m is homomorphism. Now,

$$\begin{aligned} \text{Kernal} &= \left\{g \in G: n(p) = idA_{(\alpha_{1,2},\beta_{1,2})}\right\} \\ &= \left\{g \in G: pA_{(\alpha_{1,2},\beta_{1,2})} = idA_{(\alpha,\beta)}\right\} \\ &= \left\{g \in G: p(id)^{-1} \in A^{id}_{(\alpha_{1,2},\beta_{1,2})}\right\} \\ &= \left\{g \in G: p \in A^{id}_{(\alpha_{1,2},\beta_{1,2})}\right\} \\ &= A^{id}_{(\alpha_{1,2},\beta_{1,2})}. \end{aligned}$$

Theorem 15. Let $A^{id}_{(\alpha_{1,2},\beta_{1,2})}$ be a normal subgroup of G. If

$$A_{(\alpha_{1,2},\beta_{1,2})} = \left\{\left(g, S_{A_{\alpha_1}}(g) e^{i\alpha w_{A_{\beta_1}}^{S_{A_{\alpha_1}}}(g)}, L_{A_{\alpha_2}}(g) e^{i\alpha w_{A_{\beta_2}}^{L_{A_{\alpha_2}}}(g)}\right): g \in G\right\}$$

is $(\alpha_{1,2}, \beta_{1,2})$-*CIFSG*, then, the $(\alpha_{1,2}, \beta_{1,2})$-*CIFS*.

$$\bar{A}_{(\alpha_{1,2},\beta_{1,2})} = \left\{\left(gA^{id}_{(\alpha_{1,2},\beta_{1,2})}, \bar{S}_{A_{\alpha_1}}\left(gA^{id}_{(\alpha_{1,2},\beta_{1,2})}\right)e^{i\alpha w_{A_{\beta_1}}^{\bar{S}_{A_{\alpha_1}}\left(gA^{id}_{(\alpha_{1,2},\beta_{1,2})}\right)}}, \bar{L}_{A_{\alpha_2}}\left(gA^{id}_{(\alpha_{1,2},\beta_{1,2})}\right)e^{i\alpha w_{A_{\beta_2}}^{\bar{L}_{A_{\alpha_2}}\left(gA^{id}_{(\alpha_{1,2},\beta_{1,2})}\right)}}\right) : g \in G\right\}$$

of $G/A^{id}_{(\alpha_{1,2},\beta_{1,2})}$ is also a (α,β)-CIFSG of $G/A^{id}_{(\alpha_{1,2},\beta_{1,2})}$ where

$$\bar{S}_{A_{\alpha_1}}\left(gA^{id}_{(\alpha_{1,2},\beta_{1,2})}\right)e^{i\alpha w_{A_{\beta_1}}^{\bar{S}_{A_{\alpha_1}}\left(gA^{id}_{(\alpha_{1,2},\beta_{1,2})}\right)}} = \max\left\{S_{A_{\alpha_1}}(go)e^{i\alpha w_{A_{\beta_1}}^{S_{A_{\alpha_1}}(go)}} : o \in A^{id}_{(\alpha_{1,2},\beta_{1,2})}\right\}.$$

And

$$\bar{L}_{A_{\alpha_2}}\left(gA^{id}_{(\alpha_{1,2},\beta_{1,2})}\right)e^{i\alpha w_{A_{\beta_2}}^{\bar{L}_{A_{\alpha_2}}\left(gA^{id}_{(\alpha_{1,2},\beta_{1,2})}\right)}} = \min\left\{\bar{L}_{A_{\alpha_2}}(go)e^{i\alpha w_{A_{\beta_2}}^{\bar{L}_{A_{\alpha_2}}(go)}} : o \in A^{id}_{(\alpha_{1,2},\beta_{1,2})}\right\}.$$

Proof. First, we shall prove that

$$\bar{S}_{A_{\alpha_1}}\left(gA^{id}_{(\alpha_{1,2},\beta_{1,2})}\right)e^{i\alpha w_{A_{\beta_1}}^{\bar{S}_{A_{\alpha_1}}\left(gA^{id}_{(\alpha_{1,2},\beta_{1,2})}\right)}}$$

and

$$\bar{L}_{A_{\alpha_2}}\left(gA^{id}_{(\alpha_{1,2},\beta_{1,2})}\right)e^{i\alpha w_{A_{\beta_2}}^{\bar{L}_{A_{\alpha_2}}\left(gA^{id}_{(\alpha_{1,2},\beta_{1,2})}\right)}}$$

is well-defined. Let $gA^{id}_{(\alpha_{1,2},\beta_{1,2})} = hA^{id}_{(\alpha_{1,2},\beta_{1,2})}$ then $h = go$, for some $o \in A^{id}_{(\alpha_{1,2},\beta_{1,2})}$.

Now,

$$\begin{aligned}
\bar{S}_{A_{\alpha_1}}\left(hA^{id}_{(\alpha_{1,2},\beta_{1,2})}\right)e^{i\alpha w_{A_{\beta_1}}^{\bar{S}_{A_{\alpha_1}}\left(hA^{id}_{(\alpha_{1,2},\beta_{1,2})}\right)}} &= \max\left\{S_{A_{\alpha_1}}(hf)e^{i\alpha w_{A_{\beta_1}}^{S_{A_{\alpha_1}}(hf)}} : f \in A^{id}_{(\alpha_{1,2},\beta_{1,2})}\right\} \\
&= \max\left\{S_{A_{\alpha_1}}(gof)e^{i\alpha w_{A_{\beta_1}}^{S_{A_{\alpha_1}}(gof)}} : c = of \in A^{id}_{(\alpha_{1,2},\beta_{1,2})}\right\} \\
&= \max\left\{S_{A_{\alpha_1}}(gc)e^{i\alpha w_{A_{\beta_1}}^{S_{A_{\alpha_1}}(gc)}} : c \in A^{id}_{(\alpha_{1,2},\beta_{1,2})}\right\} \\
&= \bar{S}_{A_{\alpha_1}}\left(gA^{id}_{(\alpha_{1,2},\beta_{1,2})}\right)e^{i\alpha w_{A_{\beta_1}}^{\bar{S}_{A_{\alpha_1}}\left(gA^{id}_{(\alpha_{1,2},\beta_{1,2})}\right)}}.
\end{aligned}$$

And,

$$\bar{L}_{A_{\alpha_2}}\left(hA^{id}_{(\alpha_{1,2},\beta_{1,2})}\right)e^{i\alpha w_{A_{\beta_2}}^{\bar{L}_{A_{\alpha_2}}\left(hA^{id}_{(\alpha_{1,2},\beta_{1,2})}\right)}} = \min\left\{\bar{L}_{A_{\alpha_2}}(hf)e^{i\alpha w_{A_{\beta_2}}^{\bar{L}_{A_{\alpha_2}}(hf)}} : f \in A^{id}_{(\alpha_{1,2},\beta_{1,2})}\right\}$$

$$= \min\left\{L_{A_{\alpha_2}}(gof)e^{i\alpha w_{A_{\beta_2}}^{L_{A_{\alpha_2}}(gof)}} : c = of \in A^{id}_{(\alpha_{1,2},\beta_{1,2})}\right\}$$
$$= \min\left\{L_{A_{\alpha_2}}(gc)e^{i\alpha w_{A_{\beta_2}}^{L_{A_{\alpha_2}}(gc)}} : c \in A^{id}_{(\alpha_{1,2},\beta_{1,2})}\right\}$$
$$= \bar{L}_{A_{\alpha_2}}\left(gA^{id}_{(\alpha_{1,2},\beta_{1,2})}\right)e^{i\alpha w_{A_{\beta_2}}^{\bar{L}_{A_{\alpha_2}}\left(gA^{id}_{(\alpha_{1,2},\beta_{1,2})}\right)}}.$$

Therefore,

$$\bar{S}_{A_{\alpha_1}}\left(gA^{id}_{(\alpha_{1,2},\beta_{1,2})}\right)e^{i\alpha w_{A_{\beta_1}}^{\bar{S}_{A_{\alpha_1}}}\left(gA^{id}_{(\alpha_{1,2},\beta_{1,2})}\right)}$$

and

$$\bar{L}_{A_{\alpha_2}}\left(gA^{id}_{(\alpha_{1,2},\beta_{1,2})}\right)e^{i\alpha w_{A_{\beta_2}}^{\bar{L}_{A_{\alpha_2}}}\left(gA^{id}_{(\alpha_{1,2},\beta_{1,2})}\right)}$$

is will-defined.

Consider

$$\bar{S}_{A_{\alpha_1}}\left\{\left(gA^{id}_{(\alpha_{1,2},\beta_{1,2})}\right)\left(hA^{id}_{(\alpha_{1,2},\beta_{1,2})}\right)\right\}e^{i\alpha w_{A_{\beta_1}}^{\bar{S}_{A_{\alpha_1}}\left\{\left(gA^{id}_{(\alpha_{1,2},\beta_{1,2})}\right)\left(hA^{id}_{(\alpha_{1,2},\beta_{1,2})}\right)\right\}}}$$

$$= \bar{S}_{A_{\alpha_1}}\left(ghA^{id}_{(\alpha_{1,2},\beta_{1,2})}\right)e^{i\alpha w_{A_{\beta_1}}^{\bar{S}_{A_{\alpha_1}}\left(ghA^{id}_{(\alpha_{1,2},\beta_{1,2})}\right)}}$$

$$= \max\left\{S_{A_{\alpha_1}}(gho)e^{i\alpha w_{A_{\beta_1}}^{S_{A_{\alpha_1}}(gho)}} : o \in A^{id}_{(\alpha_{1,2},\beta_{1,2})}\right\}$$

$$\geq \max\left\{\min\left\{S_{A_{\alpha_1}}(gf)e^{i\alpha w_{A_{\beta_1}}^{S_{A_{\alpha_1}}(gf)}}, S_{A_{\alpha_1}}(hc)e^{i\alpha w_{A_{\beta_1}}^{S_{A_{\alpha_1}}(hc)}}\right\} : f, c \in A^{id}_{(\alpha_{1,2},\beta_{1,2})}\right\}$$

$$= \min\left\{\max\left\{S_{A_{\alpha_1}}(gf)e^{i\alpha w_{A_{\beta_1}}^{S_{A_{\alpha_1}}(gf)}}\right\}, \max\left\{S_{A_{\alpha_1}}(qc)e^{i\alpha w_{A_{\beta_1}}^{S_{A_{\alpha_1}}(qc)}}\right\} : f, c \in A^{id}_{(\alpha_{1,2},\beta_{1,2})}\right\}$$

$$= \min\left\{\bar{S}_{A_{\alpha_1}}\left(gA^{id}_{(\alpha_{1,2},\beta_{1,2})}\right)e^{i\alpha w_{A_{\beta_1}}^{\bar{S}_{A_{\alpha_1}}\left(gA^{id}_{(\alpha_{1,2},\beta_{1,2})}\right)}}, \bar{S}_{A_{\alpha_1}}\left(hA^{id}_{(\alpha_{1,2},\beta_{1,2})}\right)e^{i\alpha w_{A_{\beta_1}}^{\bar{S}_{A_{\alpha_1}}\left(hA^{id}_{(\alpha_{1,2},\beta_{1,2})}\right)}}\right\}.$$

And

$$\bar{L}_{A_{\alpha_2}}\left\{\left(gA^{id}_{(\alpha_{1,2},\beta_{1,2})}\right)\left(hA^{id}_{(\alpha_{1,2},\beta_{1,2})}\right)\right\}e^{i\alpha w^{\bar{L}_{A_{\alpha_2}}\left\{\left(gA^{id}_{(\alpha_{1,2},\beta_{1,2})}\right)\left(hA^{id}_{(\alpha_{1,2},\beta_{1,2})}\right)\right\}}_{A_{\beta_2}}}$$

$$=\bar{L}_{A_{\alpha_2}}\left(ghA^{id}_{(\alpha_{1,2},\beta_{1,2})}\right)e^{i\alpha w^{\bar{L}_{A_{\alpha_2}}\left(ghA^{id}_{(\alpha_{1,2},\beta_{1,2})}\right)}_{A_{\beta_2}}}$$

$$=\min\left\{\mathrm{L}_{A_{\alpha_2}}(gho)e^{i\alpha w^{\mathrm{L}_{A_{\alpha_2}}(gho)}_{A_{\beta_2}}}: o\in A^{id}_{(\alpha_{1,2},\beta_{1,2})}\right\}$$

$$\leq\min\left\{\max\left\{\mathrm{L}_{A_{\alpha_2}}(gf)e^{i\alpha w^{\mathrm{L}_{A_{\alpha_2}}(gf)}_{A_{\beta_2}}},\mathrm{L}_{A_{\alpha_2}}(hc)e^{i\alpha w^{\mathrm{L}_{A_{\alpha_2}}(hc)}_{A_{\beta_2}}}\right\}: f,c\in A^{id}_{(\alpha_{1,2},\beta_{1,2})}\right\}$$

$$=\max\left\{\min\left\{\mathrm{L}_{A_{\alpha_2}}(gf)e^{i\alpha w^{\mathrm{L}_{A_{\alpha_2}}(gf)}_{A_{\beta_2}}}\right\},\min\left\{\mathrm{L}_{A_{\alpha_2}}(qc)e^{i\alpha w^{\mathrm{L}_{A_{\alpha_2}}(qc)}_{A_{\beta_2}}}\right\}: f,c\in A^{id}_{(\alpha_{1,2},\beta_{1,2})}\right\}$$

$$=\max\left\{\bar{L}_{A_{\alpha_2}}\left(gA^{id}_{(\alpha_{1,2},\beta_{1,2})}\right)e^{i\alpha w^{\bar{L}_{A_{\alpha_2}}\left(gA^{id}_{(\alpha_{1,2},\beta_{1,2})}\right)}_{A_{\beta_2}}},\bar{L}_{A_{\alpha_2}}\left(hA^{id}_{(\alpha_{1,2},\beta_{1,2})}\right)e^{i\alpha w^{\bar{L}_{A_{\alpha_2}}\left(hA^{id}_{(\alpha_{1,2},\beta_{1,2})}\right)}_{A_{\beta_2}}}\right\}.$$

$$\bar{S}_{A_{\alpha_1}}\left(\left(gA^{id}_{(\alpha_{1,2},\beta_{1,2})}\right)^{-1}\right)e^{i\alpha w^{\bar{S}_{A_{\alpha_1}}\left(\left(gA^{id}_{(\alpha_{1,2},\beta_{1,2})}\right)^{-1}\right)}_{A_{\beta_1}}}=\bar{S}_{A_{\alpha_1}}\left(g^{-1}A^{id}_{(\alpha_{1,2},\beta_{1,2})}\right)e^{i\alpha w^{\bar{S}_{A_{\alpha_1}}\left(g^{-1}A^{id}_{(\alpha_{1,2},\beta_{1,2})}\right)}_{A_{\beta_1}}}$$

$$=\max\left\{S_{A_{\alpha_1}}(g^{-1}o)e^{i\alpha w^{S_{A_{\alpha_1}}(g^{-1}o)}_{A_{\beta_1}}}: o\in A^{id}_{(\alpha_{1,2},\beta_{1,2})}\right\}\geq\max\left\{S_{A_{\alpha_1}}(go)e^{i\alpha w^{S_{A_{\alpha_1}}(go)}_{A_{\beta_1}}}: o\in A^{id}_{(\alpha_{1,2},\beta_{1,2})}\right\}$$

$$=\bar{S}_{A_{\alpha_1}}\left(gA^{id}_{(\alpha_{1,2},\beta_{1,2})}\right)e^{i\alpha w^{\bar{S}_{A_{\alpha_1}}\left(gA^{id}_{(\alpha_{1,2},\beta_{1,2})}\right)}_{A_{\beta_1}}}.$$

And

$$\bar{L}_{A_{\alpha_2}}\left(\left(gA^{id}_{(\alpha_{1,2},\beta_{1,2})}\right)^{-1}\right)e^{i\alpha w^{\bar{L}_{A_{\alpha_2}}\left(\left(gA^{id}_{(\alpha_{1,2},\beta_{1,2})}\right)^{-1}\right)}_{A_{\beta_2}}}=\bar{L}_{A_{\alpha_2}}\left(g^{-1}A^{id}_{(\alpha_{1,2},\beta_{1,2})}\right)e^{i\alpha w^{\bar{L}_{A_{\alpha_2}}\left(g^{-1}A^{id}_{(\alpha_{1,2},\beta_{1,2})}\right)}_{A_{\beta_2}}}$$

$$=\min\left\{\mathrm{L}_{A_{\alpha_2}}(g^{-1}o)e^{i\alpha w^{\mathrm{L}_{A_{\alpha_2}}(g^{-1}o)}_{A_{\beta_2}}}: o\in A^{id}_{(\alpha_{1,2},\beta_{1,2})}\right\}\leq\min\left\{\mathrm{L}_{A_{\alpha_2}}(go)e^{i\alpha w^{\mathrm{L}_{A_{\alpha_2}}(go)}_{A_{\beta_2}}}: o\in A^{id}_{(\alpha_{1,2},\beta_{1,2})}\right\}$$

$$=\bar{\mathrm{L}}_{A_{\alpha_2}}\left(gA^{id}_{(\alpha_{1,2},\beta_{1,2})}\right)e^{i\alpha w^{\bar{\mathrm{L}}_{A_{\alpha_2}}\left(gA^{id}_{(\alpha_{1,2},\beta_{1,2})}\right)}_{A_{\beta_2}}}.$$

Remark 5. If $A_{(\alpha_{1,2},\beta_{1,2})}$ is an $(\alpha_{1,2},\beta_{1,2})$-*CIFSG* of a group G, let $g \in G$ and

$$S_{A_{\alpha_1}}(gh)e^{i\alpha w_{A_{\beta_1}}^{S_{A_{\alpha_1}}}(gh)} = S_{A_{\alpha_1}}(g)e^{i\alpha w_{A_{\beta_1}}^{S_{A_{\alpha_1}}}(g)}$$

and

$$L_{A_{\alpha_2}}(gh)e^{i\alpha w_{A_{\beta_2}}^{L_{A_{\alpha_2}}}(gh)} = L_{A_{\alpha_2}}(g)e^{i\alpha w_{A_{\beta_2}}^{L_{A_{\alpha_2}}}(g)},$$

for all $h \in G$ then

$$S_{A_{\alpha_1}}(p)e^{i\alpha w_{A_{\beta_1}}^{S_{A_{\alpha_1}}}(p)} = S_{A_{\alpha_1}}(id)e^{i\alpha w_{A_{\beta_1}}^{S_{A_{\alpha_1}}}(id)}$$

and

$$L_{A_{\alpha_2}}(p)e^{i\alpha w_{A_{\beta_2}}^{L_{A_{\alpha_2}}}(p)} = L_{A_{\alpha_2}}(id)e^{i\alpha w_{A_{\beta_2}}^{L_{A_{\alpha_2}}}(id)}.$$

Definition 15. Let $A_{(\alpha_{1,2},\beta_{1,2})}$ be a $(\alpha_{1,2},\beta_{1,2})$-CIFSG. Then, the cardinality of the set $G/A_{(\alpha_{1,2},\beta_{1,2})}$ of all $(\alpha_{1,2},\beta_{1,2})$-complex intuitionistic fuzzy left cosets of G by $A_{(\alpha_{1,2},\beta_{1,2})}$ is called the index of $(\alpha_{1,2},\beta_{1,2})$-complex intuitionistic fuzzy subgroup and is represented by $\left[G: A_{(\alpha_{1,2},\beta_{1,2})}\right]$.

Theorem 16. ($(\alpha_{1,2},\beta_{1,2})$-Complex intuitionistic Fuzzification of Lagrange's Theorem): Assume that a $(\alpha_{1,2},\beta_{1,2})$-complex intuitionistic fuzzy subgroup $A_{(\alpha_{1,2},\beta_{1,2})}$ of finite group G. Then, the index of $(\alpha_{1,2},\beta_{1,2})$-complex intuitionistic fuzzy subgroup of G divides the order of G.

Proof. By Lemma 1, Define a subgroup $\bar{M} = \left\{x \in G: xA_{(\alpha_{1,2},\beta_{1,2})} = idA_{(\alpha_{1,2},\beta_{1,2})}\right\}$. Using the Definition 13 $x \in \bar{M}$ and $v \in G$, we have $xA_{(\alpha_{1,2},\beta_{1,2})}(v) = idA_{(\alpha_{1,2},\beta_{1,2})}(v)$. This implies that $A_{(\alpha_{1,2},\beta_{1,2})}(x^{-1}v) = A_{(\alpha_{1,2},\beta_{1,2})}(v)$, by Remark 5, which shows that $x \in A^{id}_{(\alpha_{1,2},\beta_{1,2})}$. Therefore, $\bar{M}$ is contained in $A^{id}_{(\alpha_{1,2},\beta_{1,2})}$. Now, we take any element $x \in A^{id}_{(\alpha_{1,2},\beta_{1,2})}$ and using the fact $A^{id}_{(\alpha_{1,2},\beta_{1,2})}$ is subgroup of G, we have $A_{(\alpha_{1,2},\beta_{1,2})}(x^{-1}) = A_{(\alpha_{1,2},\beta_{1,2})}(id)$. From Theorem 13, the elements $x^{-1}, v \in A^{id}_{(\alpha_{1,2},\beta_{1,2})}$, which means that $xA_{(\alpha_{1,2},\beta_{1,2})} = idA_{(\alpha_{1,2},\beta_{1,2})}$, which implies that $x \in \bar{M}$. Hence, $A^{id}_{(\alpha_{1,2},\beta_{1,2})}$ is contained in $\bar{M}$. From this discussion, we can say that $\bar{M} = A^{id}_{(\alpha_{1,2},\beta_{1,2})}$.

Now, we define the partition of the group G into the disjoint union of right cosets, and this is defined as $G = s_1\bar{M}$

$$\cup\, s_2\bar{M} \cup \cdots \cup s_k\bar{m}. \ (i)$$

where $s_1\bar{M} = \bar{M}$. Now, we prove that, to each coset $s_j\bar{M}$ in relation (*i*), there exists an $(\alpha_{1,2},\beta_{1,2})$-complex intuitionistic fuzzy coset $s_jA_{(\alpha_{1,2},\beta_{1,2})}$ in $G/A^{id}_{(\alpha_{1,2},\beta_{1,2})}$, and this corresponding is injective.

Consider any coset $s_jA^{id}_{(\alpha_{1,2},\beta_{1,2})}$. Let $x \in A^{id}_{(\alpha_{1,2},\beta_{1,2})}$, then

$$
\begin{aligned}
m(s_j x) = s_j x A_{(\alpha_{1,2},\beta_{1,2})} &= s_j A_{(\alpha_{1,2},\beta_{1,2})} x A_{(\alpha_{1,2},\beta_{1,2})} \\
&= s_j A_{(\alpha_{1,2},\beta_{1,2})} id A_{(\alpha_{1,2},\beta_{1,2})} \\
&= s_j A_{(\alpha_{1,2},\beta_{1,2})}.
\end{aligned}
$$

Thus, m maps each element of $s_j A^{id}_{(\alpha_{1,2},\beta_{1,2})}$ into the $(\alpha_{1,2},\beta_{1,2})$-complex intuitionistic fuzzy coset $s_j A_{(\alpha_{1,2},\beta_{1,2})}$.

Between the set $G/A^{id}_{(\alpha_{1,2},\beta_{1,2})}$ and $s_j A_{(\alpha_{1,2},\beta_{1,2})}, 1 \le j \le k$ a natural correspondence $\bar{m}$ can be defined by

$$\bar{m}\left(s_j A^{id}_{(\alpha_{1,2},\beta_{1,2})}\right) = s_j A_{(\alpha_{1,2},\beta_{1,2})}, 1 \le j \le k.$$

The correspondence $\bar{m}$ is injective.

For this, let $s_i A_{(\alpha_{1,2},\beta_{1,2})} = s_l A_{(\alpha_{1,2},\beta_{1,2})}$, then $s_l^{-1} s_i A_{(\alpha_{1,2},\beta_{1,2})} = id A_{(\alpha_{1,2},\beta_{1,2})}$. By using (A), we have $s_l^{-1} s_i \in \bar{M}$, which means that $s_i A^{id}_{(\alpha_{1,2},\beta_{1,2})} = s_i A^{id}_{(\alpha_{1,2},\beta_{1,2})}$, and hence $\bar{m}$ is injective. It is quite clear from the above discussion that $\left[G: A^{id}_{(\alpha_{1,2},\beta_{1,2})}\right]$ and $\left[G: A_{(\alpha_{1,2},\beta_{1,2})}\right]$ are equal. Hence, $\left[G: A^{id}_{(\alpha_{1,2},\beta_{1,2})}\right]$ divides $O(G)$.

5. Conclusions

The concept of (α1,2, β1,2)-CIFSs was introduced as a generalization of (α1,2, β1,2)-CFS and classical CIFS. Then we defined (α1,2, β1,2)-CIF Subgroup and studied its algebraic structure. In order to establish Lagrange theorem under (α1,2, β1,2)-CIFSs, we introduced the notion of (α1,2, β1,2)−CIFS cosets. A special type of subgroup, named (α1,2, β1,2)-CIFNSGs, is created by using the notion of cosets under (α1,2, β1,2)-CIFSs. Moreover, we established an (α1,2, β1,2)-complex intuitionistic fuzzy quotient ring induced by (α1,2, β1,2)-CIFNSG. As future research, we may use our concepts to improve the assessment and prioritization method of key engineering characteristic for complex products [53]. Also can be employed current concepts in decision making problems and machine learning algorithm [54] to enhance the results in both references [53,54].

Conflict of interest

The authors declare there are no conflicts of interest.

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