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Research article

Research on open and shared data from government-enterprise cooperation based on a stochastic differential game

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Abstract: Based on the perspective of government and enterprises, we explore the cooperative strategy and cost-sharing problem of cooperative open sharing of data between government and enterprises. In order to accurately analyze the data-opening strategies of government and enterprises, stochastic differential game theory is applied to construct the Nash non-cooperative game, Stackelberg master-slave game and cooperative game models with government and enterprises as game subjects to obtain the optimal open data effort, the optimal trajectory of social data open sharing level and the optimal benefit function of government and enterprises in three scenarios. Combined with numerical simulations to analyze the sensitivity of the relevant parameters affecting the level of social data openness, the results of the study revealed the following: ① When the government's income distribution ratio is greater than 1/3, the benefits of the government and the enterprises under the Stackelberg master-slave game and the effort to open and share data are greater than in the Nash non-cooperative situation; in the case of a cooperative game, the degree of effort and total revenue of both parties reach the Pareto optimal state. ② When the government's income distribution ratio is greater than 1/3, the expectation and variance of the open data and shared stock under the cost-sharing situation and the corresponding limit value are all greater than the value in the Nash non-cooperative situation, and in the cooperative game, the expectation and variance of open data and shared stock and its corresponding limit value are the greatest. ③ The government and enterprises coexist with profit and risk under the influence of random interference factors, and high profit means high risk. This research provides a theoretical basis and practical guidance for promoting the open sharing of government and enterprise data.

Keywords: open data sharing; open data sharing stock; stochastic differential game; government-enterprise cooperation; benefit distribution

Mathematics Subject Classification: 91A23,91A65

1. Introduction

In the era of a digital economy, data have become a valuable resource for which various industries, fields and countries compete, constituting an important production factor that drives global economic development. Undoubtedly, the circulation of data elements is closely related to the two major data holders, i.e., the government and enterprises. The government currently holds over 80% of the country's public data resources, covering various fields such as transportation, healthcare and ecology [1], while enterprises have huge amounts of commercial data, such as consumer transaction behavior and Internet access behavior, which have potential exploitation value. According to a report released by McKinsey, open data can bring more than $3 trillion in additional value each year in seven fields, including education, transportation and consumer goods [2]. The United States of America launched an early exploration of open data sharing. In 2009, it established an open government data platform. Since 2013, it has successively issued a series of open data policies, such as the "Administrative Orders on the Disclosure of Government Information and Machine Readability", the "U.S. Open Data Action Plan", the "Open Shared Data Act" and so on. The United Kingdom, Germany, Australia and other countries have all undertaken effective measures to promote the process of data openness, including the establishment of specialized agencies to promote data openness, the establishment of e-government platforms to provide shared data to the public and the establishment of a sound data security system. In 2020, China issued the "Opinions on Building a More Complete Factor Market Allocation System and Mechanism", which clearly proposed to “accelerate the cultivation of the data factor market" [3] to expedite the process of open sharing of government and enterprise data. China's 14th Five-Year Plan clearly proposes to "accelerate digital development", expand public data sharing and opening and activate the potential of data elements. The words further point out the direction for data opening and sharing, and the promotion of data opening and sharing between government and enterprises is the primary key measure to realize the circulation of data elements.

The application value of government data and enterprise data has been widely recognized by all sectors of society. For example, the outbreak of COVID-19 has caused shortages and uneven distribution of supplies around the world, especially medical supplies. In order to solve the shortage of frontline medical supplies, the Ministry of Industry and Information Technology of China has played a leading role in establishing a national key medical supply dispatching platform. The platform collects, summarizes, analyzes and monitors the data of key medical supplies providers in all regions of the country, providing a reasonable and effective basis for the dispatch of medical supplies throughout China. The platform mainly monitors key medical supplies that are in short supply, such as medical masks, goggles, rubber gloves and protective suits. By March 2022, the national key medical supply dispatching platform had mastered the data of 10 categories and 169 categories of key medical supplies and cooperated with more than 4,200 medical enterprises. The government has provided strong support for epidemic prevention and control by opening and sharing data with medical enterprises. The government uses the production capacity, output, inventory, transportation and logistics data of medical supplies provided by enterprises to improve the supply efficiency of emergency resources. By combining the data of medical supply distribution and the demand provided by the government, the enterprise can reasonably plan and arrange the production scale of the enterprise, avoid unnecessary costs and achieve a win-win situation of mutual benefit.

However, there are still many problems with the open sharing of government and enterprise data, mainly from two aspects. First, regarding internal factors, the government and enterprises are not strong in their willingness to open up and share data, which leads to a lack of data opening and sharing, and the quality is difficult to guarantee; for example, the docking between the open platform of public data resources and the data platform of enterprises is relatively outdated, the efficiency and benefits of data integration and integrated use are not fully reflected and data resources are not extended to the terminal. Second, regarding external factors, relevant laws and regulations are not perfect, data management and other technologies are not sufficient and the public is concerned about data privacy. As a result of both internal and external pressures, the willingness of the government and enterprises to cooperate is low, and the level of open data sharing is low. Therefore, under the conditions that state secrets, enterprise privacy and personal privacy data are not leaked, it is important to study how to improve the willingness of government and enterprises to cooperate and share openly, maximize the open sharing of data between the government and enterprises and promote the two-way flow of data.

From the perspective of open data sharing, scholars at home and abroad have explored the definition of open data sharing, the international experience of open data sharing and the value of open data sharing, with government data open sharing as the main focus and enterprises and research institutions data open sharing as a supplement. In terms of the definition of open data sharing, some scholars believe that open data is the opening of the data chain, including the opening of data and information, and the opening of raw data [4]. Other scholars define open data sharing as "storing and organizing a large amount of Internet data according to the specific needs of users and existing Internet protocols, rules and frameworks, while the data used may come from different data sources or different data types, with the ultimate goal of achieving openness, sharing and reuse of information in cyberspace, in order to seek the greatest possible infinite access and reuse" [5]. In terms of international experience in open data sharing, Chu and Wang [6,7] analyzed open data sharing websites from four perspectives, i.e., data sharing, website services, data innovation interpretation and interactive communication, on the topic of government data open sharing in Ireland and the United States of America; their results indicated that open data sharing can help promote high-quality development at the levels of health support, science and technology innovation and economic development. Sun et al. [8] conducted a comparative analysis of open government data sharing in the USA, the UK and China on four levels: organizational system, policy and legal system, implementation and supervision and feedback platform construction; they found that China was lagging behind on all four levels compared to developed countries such as the USA and UK. Chen [9] analyzed the advantages and disadvantages of French open data sharing in the field of anti-corruption data on four levels: organizational mechanism, policy mechanism, legal mechanism and guarantee mechanism to provide inspiration for the construction of China's anti-corruption system. In terms of the value of open data sharing, open shared data is currently used in a large number of ways in various fields, such as finance and insurance, education, healthcare, transportation and ecology, providing a scientific basis for judgment in development decisions in each field. For example, Blesa et al. [10] used open data on the number of vehicles, climate and socio-economics in different provinces of Spain to supplement insurance premium estimation models, helping insurance companies to adjust premium ratings and predict market opportunities. Sullivan et al. [11] applied open data to the field of biological conservation, using public behavioral data to study the spatial and temporal distribution of birds and explore the impact of anthropogenic changes on ecosystems. Gilmore et al. [12] applied open data to the storage, transmission and management of educational online videos. Pencina et al. [13] promoted open data sharing in the research field by establishing a review board mechanism to motivate researchers to openly share their clinical trial data. Huettmann et al. [14] explored the impediments to the open sharing of digital geographic information system data from the Ross Sea ecosystem and suggested effective preventive management measures. Boschmann et al. [15] used openly shared data to measure critical chloride content (C crit values) in concrete, overcoming limitations in sample experiments. Fan et al. [16] combined deep learning methods to use openly shared public data to accurately detect and identify drug side effects to help doctors solve problems faced when prescribing drugs. Smith et al. [17] used Global Biodiversity Information Facility open-share data to identify the rarest species.

Some scholars have also used game theory to study the issue of open data sharing. The main focus is on government data openness, with the government as the data provider and enterprises or the public as the data user, using models such as the Nash equilibrium, evolutionary game and labor game to explore how to improve the degree of government data openness and sharing. Wang and Li [18] used the evolutionary game model to construct a government open-data quality control mechanism and found that regulatory incentives above a certain threshold could effectively control the quality of open government data. Cui et al. [19] used a peer-level government as the game subject and found that increasing the learning and imitation ability of the peer-level government could effectively promote the level of information disclosure with the help of an evolutionary game model. Li and Jiang [19] constructed a dynamic evolutionary process between government open data and enterprise use of data, and they found that factors such as reputation benefits, government data opening costs and enterprise data application capabilities can all influence government-enterprise decisions. Wei et al. [21] established a three-way evolutionary game model with the government and two enterprises as the main players; they found that the enterprises' data sharing behavior is influenced by factors such as the nature of data and default payments, while government regulation has low influence. Xu et al. [22] constructed a tripartite evolutionary game with data providers, users and management organizations from the perspective of improving data quality, studied the key factors that hinder government data sharing and proposed a "quality-trust-transformation" multi-entity collaboration strategy.

Through a literature review, it was found that open data sharing has become a project vigorously carried out and implemented by countries around the world, and it is gradually receiving attention from academics. Although scholars have achieved some milestones in their research, the following shortcomings still exist. First, in terms of research subjects, most of the existing literature only focuses on single subjects such as government or enterprises when studying the issue of open data sharing, ignoring the fact that the level of social data opening and sharing is jointly determined by multiple subjects, while governments are the generators and holders of massive government data and enterprises are the collectors of massive social data; so, it is important to explore how to open and share the massive data owned by both the government and enterprises to promote the process of economic digitization. Second, in terms of research methods, existing research mainly focuses on case and empirical analysis, supplemented by inductive evolutionary, econometric, empirical and game models, and the research on games mostly focus on the Nash equilibrium, evolutionary games, labor management games and other model methods, which mostly restrict the behavior of participants to a certain fixed choice, i.e., the government and other participating subjects can only choose fixedly. In other words, the government can only choose a certain strategy, such as "open" or "not open". In reality, the degree of open data sharing by the government and other participants is constantly changing over time, while the differential game model takes into account the dynamics of time and is a dynamic game model that analyzes the cooperative decision-making problem of the game parties in continuous time. Therefore, it is widely used in the research of dynamic game problems, such as low carbon technology sharing [23], supply chains [24,25], collaborative innovation [26,27], government-enterprise collaboration [28], financial markets [29] and other issues in random dynamic game research.

Therefore, based on the stochastic differential game model, this paper establishes Nash non-cooperative game, Stackelberg master-slave game and collaborative cooperative game models with government and enterprises as the main body, and it clarifies in depth the behavioral changes of government and enterprise open data sharing in the three game situations, as combined with Ito. The process explores the expectation and variance of open data sharing stock in different situations, identifies the influence of random factor interference in the model and provides a theoretical basis for promoting government-enterprise cooperation to improve the level of open data sharing under the interference of random factors.

The rest of the paper is organized as follows. The stochastic differential model is constructed and solved in Section 2. Comparative analysis of the results and simulation analysis are provided in Section 3. Finally, conclusions and policy recommendations are given in Section 4.

2. Model construction and solution

2.1. Basic assumptions

The random interference factors in the process of open data sharing are complex and changeable. They mainly include the external environment, such as the political and cultural environment, industry background, humanistic factors, etc., as well as the character, preference, corporate culture and information acquisition capabilities of the government and corporate decision-makers, in addition to differences in the characteristics of the enterprises itself. These random interference factors are difficult to capture by the government and enterprises, but they have a greater impact on the equilibrium results. This article considers that the open data system is composed of the government (G) and the enterprises (E). The government mainly refers to non-profit sector organizations, such as government organizations and public institutions that participate in the construction of open and shared government data. Enterprises include two major categories: digital economy enterprises and traditional real economy enterprises. In the era of the digital economy, both digital economy companies and traditional real economy companies that are in urgent need of digital transformation have great demand for data and are willing to open and share data. They hope to use data to perform data-driven production and business activities, and to improve production and business activities. Regarding the effectiveness and scientific nature of the system, both the government and enterprises are rational subjects; the basic assumptions are as follows.

Hypothesis 1. In the context of the digital economy, the efforts of the government and enterprises to improve the level of open sharing of social data are as follows: $X(t)$ and $Y(t)$ ($X(t) \geq 0$, $Y(t) \geq 0$), which represent the degree of manpower, capital, time, etc. [30], that the government and enterprises have spent opening their own data. The open and shared level of social data is characterized by the open and shared stock of social data, which is determined by the efforts of the government to open and share data and the extent of the efforts of enterprises to open and share data [31]. The open and shared stock of social data is a dynamic variable that changes with time t. $K(t)$ represents the open and shared stock of social data at time t and is included in the system random variable $\sigma(K(t))dz(t)$; we simulated random interference factors as the standard Wiener process, also known as standard Brownian motion. Therefore, $\sigma(K(t))dz(t) = \sigma\sqrt{K(t)}dz(t)$, where σ is the fluctuation coefficient of open data and shared stock and $Z(t)$ is the standard Wiener process.

At this time, the change law of open and shared data satisfies the following differential equation:

$$\begin{cases} dK(t) = [\alpha_G X(t) + \alpha_E Y(t) - \delta K(t)]dt + \sigma\sqrt{K(t)}dz(t) \\ K(0) = K_0 \geq 0 \end{cases} \quad (2.1)$$

Among them, α_G and α_E respectively represent the influence coefficient of the open and shared data activities of the government and enterprises at all times on the open and shared stock of social data; $\delta(\delta > 0)$ represents when the efforts of the government and enterprises to open and share data are all 0, and the open and shared stock of social data is due to the stock decay coefficient, which is caused by timeliness.

Hypothesis 2. The cost C_G for the government to open shared data is positively related to its degree of effort $X(t)$, and the marginal cost increases as the degree of effort to open shared data increases, namely, $C_G'(X(t)) > 0$ and $C_G''(X(t)) > 0$; therefore, the cost of the government's open and shared data is $C_G = \frac{1}{2}\beta_G X^2(t)$, where $\beta_G > 0$ represents the cost coefficient of the government's effort to open and share data. In the same way, the cost of the enterprises' open sharing of data is obtained as $C_E = \frac{1}{2}\beta_E Y^2(t)$, where $\beta_E > 0$ represents the cost coefficient of the enterprises' effort to open and share data. In the process of open data sharing, the government acts as the leader and the enterprises act as the follower. Due to the constraints of capital, manpower, technology and other conditions, the technical level of open sharing of government data is greater than that of the enterprises. With the aim of improving the level of social open data sharing, the government uses incentives such as sharing open-data sharing technology to share the cost of enterprise open data sharing [23], assuming that the cost-sharing ratio is $\psi(t)$ and $1 - \psi(t)$ $(0 < \psi(t) < 1)$.

Hypothesis 3. The government and enterprises provide high-quality open data; enterprises and the public are more satisfied with the government, enhance the credibility of the government and release data dividends in the process of data value re-creation [22]. High-quality data can be widely used in various fields, such as urban transportation, medical care, enterprises production and operation and product development, to give full play to their maximum value, optimize business operations and improve the convenience of residents' daily lives. Data openness can bring enormous social welfare effects to social development [1]. Assume that the social welfare effects brought about by the open sharing of data by the government and enterprises are as follows:

$$Q(t) = Q_0 + \mu_G X(t) + \mu_E Y(t) + \tau K(t), \quad (2.2)$$

where $Q_0 > 0$ represents the initial state of social welfare, μ_G and μ_E respectively represent the influence coefficient of the government and enterprises' efforts to open and share data on the social welfare effect and τ represents the influence coefficient of the open social data and shared stock on the social welfare effect. The open sharing of data by the government and enterprises causes an increase in social welfare, greatly increasing the public's satisfaction with the government and enterprises. At the same time, products developed using these data can be used to bring their own value into play and bring benefits to the government and enterprises. Let the coefficient of the social welfare effect on the benefits of open sharing of data by the government and enterprises be $\lambda(\lambda > 0)$; then, the total benefits brought by the open sharing of data by the government and enterprises are $\lambda Q(t)$.

Hypothesis 4. The total revenue of the open sharing of data between the government and enterprises is distributed between the two [31], and the distribution ratios are ω and $1-\omega$, with $0 \le \omega(t) \le 1$. In an infinite time zone, the government and the enterprises have the same discount rate $r(r > 0)$, and both have the goal of seeking the optimal data opening strategy for maximizing their own benefits in an infinite time zone.

Therefore, the objective functions obtained by the government and the enterprises are as follows:

$$\max_{X,\psi} J_G = E\int_0^\infty e^{-rt}\left\{\omega\lambda[Q_0+\mu_G X(t)+\mu_E Y(t)+\tau K(t)]-\frac{1}{2}\beta_G X^2(t)-\frac{1}{2}\psi(t)\beta_E Y^2(t)]\right\}dt,$$

$$\max_{Y} J_E = E\int_0^\infty e^{-rt}\left\{(1-\omega)\lambda[Q_0+\mu_G X(t)+\mu_E Y(t)+\tau K(t)]-\frac{1}{2}(1-\psi(t))\beta_E Y^2(t)]\right\}dt.$$

To clarify the behavioral strategies of non-cooperative and cooperative games between the government and enterprises in the process of open data sharing under random interference factors, this chapter explores three modes, i.e., the Nash non-cooperative game, Stackelberg master-slave game and cooperative game. To facilitate this description, the following text omits "(t) " of the state variables and control variables and uses N, S and C to respectively mark the Nash non-cooperative game, Stackelberg master-slave game and cooperative game under random interference factors.

2.2. Nash non-cooperative game

In the context of non-cooperation, the government and enterprises independently choose their own degree of effort to open and share data to maximize their own profits. Therefore, at this time, the government will not open shared data to enterprises to share the cost of enterprises open data sharing, that is $\psi(t) = 0$. In the Nash non-cooperative game, the Markovian Nash equilibrium between government and enterprises is as follows:

$$J_G^N = \int_0^\infty e^{-rt}\left\{\omega\lambda[Q_0+\mu_G X(t)+\mu_E Y(t)+\tau K(t)]-\frac{1}{2}\beta_G X^2(t)]\right\}dt,$$

$$J_E^N = \int_0^\infty e^{-rt}\left\{(1-\omega)\lambda[Q_0+\mu_G X(t)+\mu_E Y(t)+\tau K(t)]-\frac{1}{2}\beta_E Y^2(t)]\right\}dt,$$

$$s.t.\ \begin{cases} dK(t) = [\alpha_G X(t)+\alpha_E Y(t)-\delta K(t)]dt+\sigma\sqrt{K(t)}dz(t) \\ K(0)=K_0 \ge 0 \end{cases}.$$

Since the open-loop control strategy can only observe the system state, it cannot describe the dynamic change process of the decision-making structure with the open sharing of social data. The closed-loop control strategy can synchronize the state and time and has a better control effect than the open-loop control strategy. Therefore, this paper describes the use of the Hamilton-Jacobi-

Bellman (HJB) equation to solve the Markovian closed-loop Nash equilibrium of non-cooperative games. Assuming that other parameters are not related to time, for the convenience of describing them below, the time t is omitted, and Proposition 1 is obtained as follows.

Proposition 1. In the Nash non-cooperative game, the Markovian Nash equilibrium between the government and enterprises is as follows.
(1) The best efforts of the government and enterprises to open and share data are as follows:

$$X_N^* = \frac{\omega\lambda[\mu_G(r+\delta)+\tau\alpha_G]}{\beta_G(r+\delta)} \quad Y_N^* = \frac{(1-\omega)\lambda[\mu_E(r+\delta)+\tau\alpha_E]}{\beta_E(r+\delta)} . \tag{2.3}$$

(2) The expectation and expectation limits of the open and shared stock of data under random interference factors in the Nash non-cooperative game are as follows:

$$E\left[K_N(t)\right] = e^{-\delta t}(K_0 - \frac{A_N}{\delta} + \frac{A_N}{\delta}e^{\delta t}) , \quad \lim_{t\to\infty} E\left[K_N(t)\right] = \frac{A_N}{\delta} . \tag{2.4}$$

(3) The variance and variance limits of the open and shared stock K of data under random interference factors are as follows:

$$D\left[K_N(t)\right] = \frac{\sigma^2\left[(A_N - 2\delta K_0)e^{-2\delta t} - 2e^{-\delta t}(A_N - \delta K_0) + A_S\right]}{2\delta^2} , \tag{2.5}$$

$$\lim_{t\to\infty} D\left[K_N(t)\right] = \frac{\sigma^2 A_N}{2\delta^2} .$$

Among them, $A_N = \alpha_G \frac{\omega\lambda[\mu_G(r+\delta)+\tau\alpha_G]}{\beta_G(r+\delta)} + \alpha_E \frac{(1-\omega)\lambda[\mu_E(r+\delta)+\tau\alpha_E]}{\beta_E(r+\delta)}$.
(4) The optimal income expressions of government and enterprises are as follows:

$$V_{GN}^*(K) = \frac{\omega\lambda\tau}{r+\delta}K + \frac{\omega\lambda Q_0}{r} + \frac{\omega^2\lambda^2[\mu_G(r+\delta)+\tau\alpha_G]^2}{2r\beta_G(r+\delta)^2} + \frac{\omega(1-\omega)\lambda^2[\mu_E(r+\delta)+\tau\alpha_E]^2}{r\beta_E(r+\delta)^2} ,$$

$$V_{EN}^*(K) = \frac{(1-\omega)\lambda\tau}{r+\delta}K + \frac{(1-\omega)\lambda Q_0}{r} + \frac{(1-\omega)^2\lambda^2[\mu_E(r+\delta)+\tau\alpha_E]^2}{2r\beta_E(r+\delta)^2} + \frac{\omega(1-\omega)\lambda^2[\mu_G(r+\delta)+\tau\alpha_G]^2}{r\beta_G(r+\delta)^2} . \tag{2.6}$$

Proof. According to the optimal control problem solving method [33–35], it is assumed that there are continuous bounded differential functions $V_{iN}(K)$ and $i \in (G,E)$, and the HJB equation is satisfied for $\forall K \geq 0$:

$$rV_{GN}(K) = \max_X[\omega\lambda(Q_0 + \mu_G X + \mu_E Y + \tau K) - \frac{1}{2}\beta_G X^2 + V_{GN}'(K)(\alpha_G X + \alpha_E Y - \delta K)] , \tag{2.7}$$

$$rV_{EN}(K) = \max_Y[(1-\omega)\lambda(Q_0 + \mu_G X + \mu_E Y + \tau K) - \frac{1}{2}\beta_E Y^2 + V_{EN}'(K)(\alpha_G X + \alpha_E Y - \delta K)] . \tag{2.8}$$

We find the first-order partial derivative of the control variable X in Eq (2.7) and the control variable Y in Eq (2.8) and set them equal to 0 to obtain the maximization condition:

$$X = \frac{\omega\lambda\mu_G + \alpha_G V_{GN}'(K)}{\beta_G} , \quad Y = \frac{(1-\omega)\lambda\mu_E + \alpha_E V_{EN}'(K)}{\beta_E} . \tag{2.9}$$

Substituting Eq (2.9) into HJB Eqs (2.7) and (2.8) eliminates the symbol max to obtain the following expression:

$$\begin{aligned} rV_{GN}(K) &= \omega\lambda Q_0 + [\omega\lambda\tau - \delta V'_{GN}(K)]K + \frac{[\omega\lambda\mu_G + \alpha_G V'_{GN}(K)]^2}{2\beta_G} \\ &+ \frac{[\omega\lambda\mu_E + \alpha_E V'_{GN}(K)][(1-\omega)\lambda\mu_E + \alpha_E V'_{EN}(K)]}{\beta_E}, \end{aligned} \tag{2.10}$$

$$\begin{aligned} rV_{EN}(K) &= (1-\omega)\lambda Q_0 + [(1-\omega)\lambda\tau - \delta V'_{EN}(K)]K + \frac{[(1-\omega)\lambda\mu_E + \alpha_E V'_{EN}(K)]^2}{2\beta_E} \\ &+ \frac{[\omega\lambda\mu_G + \alpha_G V'_{GN}(K)][(1-\omega)\lambda\mu_G + \alpha_G V'_{EN}(K)]}{\beta_G}. \end{aligned} \tag{2.11}$$

By examining the differential equations, Eqs (2.10) and (2.11), it can be inferred that the linear optimal return function of K is the solution of the HJB equation, so it is assumed that the expressions of $V_{GN}(K)$ and $V_{EN}(K)$ satisfy Eq (2.12):

$$V_{GN}(K) = p_1K + q_1,\quad V_{EN}(K) = p_2K + q_2, \tag{2.12}$$

where p_1, p_2, q_1 and q_2 are constants, and $V_{GN}(K)$ and $V_{EN}(K)$ are obtained by calculating the first-order reciprocal of the expressions of $\frac{dV_{GN}(K)}{dK} = p_1 = V'_{GN}(K)$ and $\frac{dV_{EN}(K)}{dK} = p_2 = V'_{EN}(K)$, respectively. Substituting them into Eqs (2.10) and (2.11), respectively, the method of undetermined coefficients is used to obtain the parameter values of the optimal return function as follows:

$$p_1 = \frac{\omega\lambda\tau}{r+\delta},\quad p_2 = \frac{(1-\omega)\lambda\tau}{r+\delta},$$

$$q_1 = \frac{\omega\lambda Q_0}{r} + \frac{\omega^2\lambda^2[\mu_G(r+\delta)+\tau\alpha_G]^2}{2r\beta_G(r+\delta)^2} + \frac{\omega(1-\omega)\lambda^2[\mu_E(r+\delta)+\tau\alpha_E]^2}{r\beta_E(r+\delta)^2},$$

$$q_2 = \frac{(1-\omega)\lambda Q_0}{r} + \frac{(1-\omega)^2\lambda^2[\mu_E(r+\delta)+\tau\alpha_E]^2}{2r\beta_E(r+\delta)^2} + \frac{\omega(1-\omega)\lambda^2[\mu_G(r+\delta)+\tau\alpha_G]^2}{r\beta_G(r+\delta)^2}.$$

Substituting the parameter values of p_1, p_2, q_1 and q_2 into Eq (2.12), the optimal income expression of the government and enterprises under the Nash non-cooperative equilibrium can be obtained as follows:

$$V^*_{GN}(K) = \frac{\omega\lambda\tau}{r+\delta}K + \frac{\omega\lambda Q_0}{r} + \frac{\omega^2\lambda^2[\mu_G(r+\delta)+\tau\alpha_G]^2}{2r\beta_G(r+\delta)^2} + \frac{\omega(1-\omega)\lambda^2[\mu_E(r+\delta)+\tau\alpha_E]^2}{r\beta_E(r+\delta)^2},$$

$$\begin{aligned} V^*_{EN}(K) &= \frac{(1-\omega)\lambda\tau}{r+\delta}K + \frac{(1-\omega)\lambda Q_0}{r} + \frac{(1-\omega)^2\lambda^2[\mu_E(r+\delta)+\tau\alpha_E]^2}{2r\beta_E(r+\delta)^2} \\ &+ \frac{\omega(1-\omega)\lambda^2[\mu_G(r+\delta)+\tau\alpha_G]^2}{r\beta_G(r+\delta)^2}. \end{aligned} \tag{2.13}$$

Using Eq (2.13) to find the first derivative and substituting it into Eq (2.9), the equilibrium solution of the degree of effort of the government and enterprises to open and share data is Eq (2.3).

Substituting the optimal effort of the government and enterprises to open and share data in Eq

(2.3) into Eq (2.1), we obtain

$$dK_N(t)=\left\{\begin{aligned}&-\delta K_N(t)+\alpha_G\frac{\omega\lambda[\mu_G(r+\delta)+\tau\alpha_G]}{\beta_G(r+\delta)}\\&+\alpha_E\frac{(1-\omega)\lambda[\mu_E(r+\delta)+\tau\alpha_E]}{\beta_E(r+\delta)}\end{aligned}\right\}dt+\sigma\sqrt{K_N(t)}dz(t). \tag{2.14}$$

Let $A_N=\alpha_G\frac{\omega\lambda[\mu_G(r+\delta)+\tau\alpha_G]}{\beta_G(r+\delta)}+\alpha_E\frac{(1-\omega)\lambda[\mu_E(r+\delta)+\tau\alpha_E]}{\beta_E(r+\delta)}$; then Eq (2.14) is simplified to

$$dK_N(t)=[-\delta K_N(t)+A_N]dt+\sigma\sqrt{K_N(t)}dz(t),\ K(0)=K_0. \tag{2.15}$$

Integrate both sides of Eq (2.15) at the same time and use the initial conditions to obtain

$$K_N(t)=K_0+\int_0^t[-\delta K_N(t)+A_N]dt+\int_0^t\sigma\sqrt{K_N(t)}dz(t). \tag{2.16}$$

At the same time, we take expectations on both sides of Eq (2.16) and use the zero-mean property of the standard Wiener process to obtain

$$E[K_N(t)]=K_0+\int_0^t\{-\delta E[K_N(t)]+A_N\}dt. \tag{2.17}$$

We further solve the integral to obtain the expectation of the open and shared stock of data in the Nash non-cooperative game under random interference factors:

$$E[K_N(t)]=e^{-\delta t}(K_0-\frac{A_N}{\delta}+\frac{A_N}{\delta}e^{\delta t}).$$

When t tends toward infinity, there is $\lim\limits_{t\to\infty}E[K_N(t)]=\lim\limits_{t\to\infty}e^{-\delta t}(K_0-\frac{A_N}{\delta}+\frac{A_N}{\delta}e^{\delta t})=\frac{A_N}{\delta}$.

On the basis of Eq (2.15), using Ito's lemma, the change process of the square of the open shared stock of data is obtained:

$$\begin{cases}dK_N^2(t)=\left[-2\delta K_N^2(t)+(2A_N+\sigma^2)K_N(t)\right]dt+2\sigma K_N(t)\sqrt{K_N(t)}dz(t)\\K^2(0)=K_0^2\end{cases}. \tag{2.18}$$

Integrate both sides of Eq (2.18) at the same time and use the initial conditions to obtain

$$K_N^2(t)=\int_0^t\left[-2\delta K_N^2(t)+(2A_N+\sigma^2)K_N(t)\right]dt+\int_0^t2\sigma K_N(t)\sqrt{K_N(t)}dz(t)+K_0^2. \tag{2.19}$$

Similarly, take the expectation on both sides of Eq (2.19) and use the feature that the mean value of the standard Wiener process is zero to obtain

$$E\left[K_N^2(t)\right]=\int_0^T\left\{-2\delta E[K_N^2(t)]+(2A_N+\sigma^2)E[K_N(t)]\right\}dt+K_0^2. \tag{2.20}$$

Further solve the integral to obtain

$$E\left[K_N^2(t)\right]=e^{-2\delta t}\left[K_0^2+\frac{(A_{N'}-\delta K_0)(2A_N+\sigma^2)}{\delta^2}-\frac{A_N(2A_N+\sigma^2)}{2\delta^2}\right]$$
$$-e^{-\delta t}\frac{(A_N-\delta K_0)(2A_N+\sigma^2)}{\delta^2}+\frac{A_N(2A_N+\sigma^2)}{2\delta^2} \tag{2.21}$$

.

Combining equation $D[K_N(t)]=E\left[K_N^2(t)\right]-E[K_N(t)]^2$, we can obtain the variance of the open data shared stock K of the Nash non-cooperative game under random interference factors:

$$D\left[K_N(t)\right]=\frac{\sigma^2\left[(A_N-2\delta K_0)e^{-2\delta t}-2e^{-\delta t}(A_N-\delta K_0)+A_N\right]}{2\delta^2}.$$

When t tends toward infinity, $\lim_{t\to\infty}E\left[K_N^2(t)\right]=\frac{A_N(2A_N+\sigma^2)}{2\delta^2}$, so the variance limit of the open stock of data K in the Nash non-cooperative game under random interference factors is

$$\lim_{t\to\infty}D\left[K_N(t)\right]=\lim_{t\to\infty}E\left[K_N^2(t)\right]-\lim_{t\to\infty}E\left[K_N(t)\right]^2=\frac{\sigma^2A_N}{2\delta^2}.$$

When random interference factors do not exist, that is, $\sigma=0$, $\lim_{t\to\infty}E\left[K_N^2(t)\right]=\frac{A_N^2}{\delta^2}$; at this time, $D\left[K_N(t)\right]=E\left[K_N^2(t)\right]-E\left[K_N(t)\right]^2=0$. The proof is complete.

Assuming that the open and shared stock of data obeys a normal distribution, the confidence interval of the open and shared stock of data is $\left[E\left[K_N(t)\right]-1.96D\left[K_N(t)\right],E\left[K_N(t)\right]+1.96D\left[K_N(t)\right]\right]$ at a 95% confidence level. From Proposition 1, we know that the actual data open sharing stock may deviate from the expected value due to the influence of random interference factors. From the equilibrium results of the government and enterprises' open data sharing efforts, it can be seen that the government and enterprises make data open sharing decisions by examining their own costs of open data sharing, the social welfare effects brought by open data sharing and their own benefits to determine the optimal efforts. It can be seen that both the government and the enterprises determine the optimal level of effort by maximizing their own benefits, ignoring the level of open data sharing, and both parties' decisions are constrained by the cost coefficient.

2.3. Stackelberg master-slave game

In this case, the government, to encourage enterprises to increase the degree of data sharing, share part of the cost of open data sharing by free-sharing open data technology, financial subsidies, etc., to encourage enterprises to choose strategies in accordance with the government's wishes and promote open data sharing; the level achieves Pareto optimum, so the government and the enterprises sign a cost-sharing contract; that is, the government provides the enterprises with a cost-sharing ratio of $\psi(t)\neq 0$ $(0<\psi(t)<1)$. The game is divided into two stages. First, the government determines its own degree of effort to open shared data and the proportion of cost sharing to the enterprises, and then the enterprises determine its own optimal degree of effort to open shared data according to the government's strategy. Both parties to the game make independent decisions with the goal of maximizing their own profits. Based on the above assumptions, the objective functions of the government and enterprises are as follows:

$$J_G^S=\int_0^{\infty}e^{-rt}\left\{\omega\lambda[Q_0+\mu_G X(t)+\mu_E Y(t)+\tau K(t)]-\frac{1}{2}\beta_G X^2(t)-\frac{1}{2}\psi(t)\beta_E Y^2(t)]\right\}dt,$$

$$J_E^S=\int_0^{\infty}e^{-rt}\left\{(1-\omega)\lambda[Q_0+\mu_G X(t)+\mu_E Y(t)+\tau K(t)]-\frac{1}{2}(1-\psi(t))\beta_E Y^2(t)]\right\}dt,$$

$$s.t.\ \begin{cases} dK(t)=[\alpha_G X(t)+\alpha_E Y(t)-\delta K(t)]dt+\sigma\sqrt{K(t)}dz(t) \\ K(0)=K_0\geq 0 \end{cases}.$$

Using the HJB equation to solve the Stackelberg master-slave game equilibrium in the case of cost-sharing at this time, Proposition 2 is obtained as follows:

Proposition 2. In the Stackelberg master-slave game, the equilibrium between government and enterprises is as follows.

(1) The best efforts of the government and enterprises to open and share data are as follows:

$$X_S^*=\frac{\omega\lambda[\mu_G(r+\delta)+\tau\alpha_G]}{\beta_G(r+\delta)},$$

$$Y_S^*=\frac{(1+\omega)\lambda[\mu_E(r+\delta)+\tau\alpha_E]}{2\beta_E(r+\delta)},$$

$$\psi=\frac{3\omega-1}{\omega+1}\left(\frac{1}{3}<\omega\leq 1\right). \tag{2.22}$$

(2) The expectation and expectation limits of the open and shared stock of data in the Stackelberg master-slave game under random interference factors are

$$E\left[K_S(t)\right]=e^{-\delta t}\left(K_0-\frac{A_S}{\delta}+\frac{A_S}{\delta}e^{\delta t}\right),\quad \lim_{t\to\infty}E\left[K_S(t)\right]=\frac{A_S}{\delta}. \tag{2.23}$$

(3) The variance and variance limits of the open shared stock of data under random interference factors are

$$D\left[K_S(t)\right]=\frac{\sigma^2\left[(A_S-2\delta K_0)e^{-2\delta t}-2e^{-\delta t}(A_S-\delta K_0)+A_S\right]}{2\delta^2},$$

$$\lim_{t\to\infty}D\left[K_S(t)\right]=\frac{\sigma^2 A_S}{2\delta^2}. \tag{2.24}$$

Among them, $A_S=\alpha_G\dfrac{\omega\lambda[\mu_G(r+\delta)+\tau\alpha_G]}{\beta_G(r+\delta)}+\alpha_E\dfrac{(1+\omega)\lambda[\mu_E(r+\delta)+\tau\alpha_E]}{2\beta_E(r+\delta)}$.

(4) The optimal income expressions of government and enterprises are as follows:

$$V_{GS}^*(K)=\frac{\omega\lambda\tau}{r+\delta}K+\frac{\omega\lambda Q_0}{r}+\frac{\omega(1-\omega)\lambda^2\left[(r+\delta)\mu_E+\tau\alpha_E\right]^2}{r\beta_E(1-\psi)(r+\delta)^2}$$
$$+\frac{\omega^2\lambda^2\left[(r+\delta)\mu_G+\tau\alpha_G\right]^2}{2r\beta_G(r+\delta)^2}-\frac{\psi(1-\omega)^2\lambda^2\left[(r+\delta)\mu_E+\tau\alpha_E\right]^2}{2r\beta_E(1-\psi)^2(r+\delta)^2}$$

$$V_{ES}^*(K)=\frac{(1-\omega)\lambda\tau}{r+\delta}K+\frac{(1-\omega)\lambda Q_0}{r}+\frac{\omega(1-\omega)\lambda^2\left[(r+\delta)\mu_G+\tau\alpha_G\right]^2}{r\beta_G(r+\delta)^2}$$
$$+\frac{(1-\omega)^2\lambda^2\left[(r+\delta)\mu_E+\tau\alpha_E\right]^2}{2r\beta_E(1-\psi)(r+\delta)^2}. \tag{2.25}$$

Proof. Use the reverse induction method to solve the Stackelberg master-slave game equilibrium [36,37]. First, start with the optimal control problem of the enterprises. The optimal value function $V_{ES}(K)$ satisfies the following HJB equation:

$$rV_{ES}(K)=\max_Y[(1-\omega)\lambda(Q_0+\mu_G X+\mu_E Y+\tau K)-\frac{1}{2}(1-\psi)\beta_E Y^2+V_{ES}'(K)(\alpha_G X+\alpha_E Y-\delta K)]. \tag{2.26}$$

For Eq (2.26), find the first-order partial derivative of the enterprises' effort to open and share data Y, and make it equal to 0 according to the maximization condition to obtain

$$Y=\frac{(1-\omega)\lambda\mu_E+\alpha_E V_{ES}'(K)}{(1-\psi)\beta_E}. \tag{2.27}$$

Considering that the government will rationally predict that the enterprises will choose the best degree of effort to open and share data based on the above reaction function, Eq (2.27) is substituted into the government's HJB equation to obtain

$$rV_{GS}(K)=\max_{X,\psi}\left\{\begin{array}{l}\omega\lambda(Q_0+\mu_G X+\mu_E\dfrac{(1-\omega)\lambda\mu_E+\alpha_E V_{ES}'(K)}{(1-\psi)\beta_E}+\tau K)\\ -\dfrac{1}{2}\beta_G X^2-\dfrac{1}{2}\psi\beta_E\left[\dfrac{(1-\omega)\lambda\mu_E+\alpha_E V_{ES}'(K)}{(1-\psi)\beta_E}\right]^2\\ +V_{GS}'(K)(\alpha_G X+\alpha_E\dfrac{(1-\omega)\lambda\mu_E+\alpha_E V_{ES}'(K)}{(1-\psi)\beta_E}-\delta K)\end{array}\right\}. \tag{2.28}$$

This process solves the first-order partial derivatives of Eq (2.28) with respect to X and ψ and sets them equal to 0 to obtain the condition that maximizes the right side of the equation:

$$X=\frac{\omega\lambda\mu_G+\alpha_G V_{GS}'(K)}{\beta_G},\quad \psi=\frac{(3\omega-1)\lambda\mu_E-\alpha_E V_{ES}'(K)+2\alpha_E V_{GS}'(K)}{(\omega+1)\mu_E+\alpha_E V_{ES}'(K)+2\alpha_E V_{GS}'(K)}. \tag{2.29}$$

Substitute the optimal strategy equations of the enterprises and the government, i.e., Eqs (2.27) and (2.29), into their respective HJB equations to eliminate the symbols "max" and simplify them to obtain

$$rV_{GS}(K)=\omega\lambda Q_0+\left[\omega\lambda\tau-\delta V_{GS}'(K)\right]K+\frac{\left[\omega\lambda\mu_G+\alpha_G V_{GS}'(K)\right]^2}{2\beta_G}$$
$$+\frac{\left[\omega\lambda\mu_E+\alpha_E V_{GS}'(K)\right]\left[(1-\omega)\lambda\mu_E+\alpha_E V_{ES}'(K)\right]}{\beta_E(1-\psi)}-\frac{\psi\left[(1-\omega)\lambda\mu_E+\alpha_E V_{ES}'(K)\right]^2}{2\beta_E(1-\psi)^2},$$

$$rV_{ES}(K)=(1-\omega)\lambda Q_0+\left[(1-\omega)\lambda\tau-\delta V'_{ES}(K)\right]K+\frac{\left[(1-\omega)\lambda\mu_E+\alpha_E V'_{ES}(K)\right]^2}{2\beta_E(1-\psi)}$$
$$+\frac{\left[(1-\omega)\lambda\mu_G+\alpha_G V'_{ES}(K)\right]\left[\omega\lambda\mu_G+\alpha_G V'_{GS}(K)\right]}{\beta_G}. \quad (2.30)$$

In the same way, by observing the structure of the above differential equation, Eq (2.30), the linear optimal return function of K is the solution of the HJB equation, so it is assumed that the expressions of $V_{GS}(K)$ and $V_{ES}(K)$ satisfy Eq (2.31):

$$V_{GS}(K)=h_1K+g_1,\ V_{ES}(K)=h_2K+g_2, \quad (2.31)$$

where h_1, h_2, g_1 and g_2 are constants, $\frac{dV_{GS}(K)}{dK}=h_1=V'_{GS}(K)$ and $\frac{dV_{ES}(K)}{dK}=h_2=V'_{ES}(K)$. Substituting the above assumptions into Eq (2.29) and merging similar terms, the parameter values obtained are as follows:

$$h_1=\frac{\omega\lambda\tau}{r+\delta},\ h_2=\frac{(1-\omega)\lambda\tau}{r+\delta},$$
$$g_1=\frac{\omega\lambda Q_0}{r}+\frac{\omega^2\lambda^2\left[(r+\delta)\mu_G+\tau\alpha_G\right]^2}{2r\beta_G(r+\delta)^2}+\frac{\omega(1-\omega)\lambda^2\left[(r+\delta)\mu_E+\tau\alpha_E\right]^2}{r\beta_E(1-\psi)(r+\delta)^2}-\frac{\psi(1-\omega)^2\lambda^2\left[(r+\delta)\mu_E+\tau\alpha_E\right]^2}{2r\beta_E(1-\psi)^2(r+\delta)^2},$$
$$g_2=\frac{(1-\omega)\lambda Q_0}{r}+\frac{\omega(1-\omega)\lambda^2\left[(r+\delta)\mu_G+\tau\alpha_G\right]^2}{r\beta_G(r+\delta)^2}+\frac{(1-\omega)^2\lambda^2\left[(r+\delta)\mu_E+\tau\alpha_E\right]^2}{2r\beta_E(1-\psi)(r+\delta)^2}.$$

Substituting the solved h_1, h_2, g_1 and g_2 into Eqs (2.27) and (2.29) of the first-order maximization condition of the government and the enterprises, respectively, the optimal degree of effort for the government and the enterprises to open and share data under the Stackelberg master-slave game is obtained, namely Eq (2.22). Substituting h_1, h_2, g_1 and g_2 into Eq (2.31), the optimal income expression for the government and the enterprises is obtained, that is, Eq (2.25).

Since the proof process for the expectation, expectation limit, variance and variance limit of the open data shared stock K in Proposition 2 are similar to those in Proposition 1, we do not repeat too much here, and part of the proof process is omitted.

Assuming that the open and shared stock of data follows a normal distribution, the confidence interval for the open and shared stock of data is $\left[E[K_S(t)]-1.96D[K_S(t)],E[K_S(t)]+1.96D[K_S(t)]\right]$ at a 95% confidence level. From Proposition 2, it can be seen that the actual data open-sharing stock may deviate from the expected value due to random disturbance factors, but the actual value is not completely uncontrollable, and it always fluctuates up and down from the expected expectation given a certain confidence level, which is manageable within a certain range. From the Stackelberg equilibrium, it can be seen that the factors and relationships affecting the optimal effort of the government and enterprises in the respective open shared data in the cost-sharing scenario are the same as those in the Nash non-cooperative game scenario, where both parties determine the optimal effort only with the goal of maximizing their own benefits. At the same time, the optimal effort level of enterprises to open shared data is positively correlated with the cost-sharing ratio given by the government, indicating that the government's cost-sharing measures have an incentive effect, and that the total income distribution ratio reaches $\frac{1}{3}<\omega\le 1$, which is the primary prerequisite for the government to provide cost-sharing.

2.4. Cooperative game

In the case of government-enterprise collaboration, the government and companies no longer decide their own optimal strategies based on the goal of maximizing their own benefits, but instead use collaborative methods to reduce the impact of negative externalities, such as information asymmetry, and they work to improve social data, open the sharing level and determine the optimal strategy based on the premise of maximizing overall benefits. In summary, the objective function is as follows:

$$J^C = J_G^C + J_E^C = \int_0^\infty e^{-rt}\left\{\lambda[Q_0 + \mu_G X(t) + \mu_E Y(t) + \tau K(t)] - \frac{1}{2}\beta_G X^2(t) - \frac{1}{2}\beta_E Y^2(t)\right\}dt$$

$$s.t.\ \begin{cases} dK(t) = [\alpha_G X(t) + \alpha_E Y(t) - \delta K(t)]dt + \sigma\sqrt{K(t)}dz(t) \\ K(0) = K_0 \geq 0 \end{cases}.$$

Proposition 3. In the cooperative game, the equilibrium between the government and the enterprises is as follows.

(1) The best efforts of the government and enterprises to open and share data are as follows:

$$X_C^* = \frac{\lambda[\mu_G(r+\delta) + \tau\alpha_G]}{\beta_G(r+\delta)},\quad Y_C^* = \frac{\lambda[\mu_E(r+\delta) + \tau\alpha_E]}{(r+\delta)\beta_E}. \tag{2.32}$$

(2) The expectation and expectation limits of the open and shared stock of data under random interference factors in the collaborative game are

$$E\left[K_C(t)\right] = e^{-\delta t}(K_0 - \frac{A_C}{\delta} + \frac{A_C}{\delta}e^{\delta t}),\quad \lim_{t\to\infty} E\left[K_C(t)\right] = \frac{A_C}{\delta}. \tag{2.33}$$

(3) The variance and variance limits of the open data shared stock K under random interference factors in the collaborative cooperative game are

$$D\left[K_C(t)\right] = \frac{\sigma^2\left[(A_C - 2\delta K_0)e^{-2\delta t} - 2e^{-\delta t}(A_C - \delta K_0) + A_C\right]}{2\delta^2},$$

$$\lim_{t\to\infty} D\left[K_C(t)\right] = \frac{\sigma^2 A_C}{2\delta^2}. \tag{2.34}$$

Among them, $A_C = \alpha_G \frac{\lambda[\mu_G(r+\delta) + \tau\alpha_G]}{\beta_G(r+\delta)} + \alpha_E \frac{\lambda[\mu_E(r+\delta) + \tau\alpha_E]}{(r+\delta)\beta_E}$.

(4) The optimal total income expression of the government and enterprises is:

$$V_C^*(K) = \frac{\lambda\tau}{r+\delta}K + \frac{\lambda Q_0}{r} + \frac{\lambda^2[\mu_G(r+\delta) + \tau\alpha_G]^2}{2r\beta_G(r+\delta)^2} + \frac{\lambda^2[\mu_E(r+\delta) + \tau\alpha_E]^2}{2r\beta_E(r+\delta)^2}. \tag{2.35}$$

Proof. In the case of collaborative cooperation, the total optimal value function of the government and enterprises $V_C(K)$ satisfies the following HJB equation:

$$rV_C(K) = \max_{X,Y}[\lambda(Q_0 + \mu_G X + \mu_E Y + \tau K) - \frac{1}{2}\beta_G X^2 - \frac{1}{2}\beta_E Y^2 + V_C'(K)(\alpha_G X + \alpha_E Y - \delta K)]. \tag{2.36}$$

To obtain the conditions for maximizing the right side of Eq (2.36), find the first-order partial derivatives of the control variables X and Y and set them equal to 0 so that

$$X=\frac{\lambda\mu_G+\alpha_G V_C'(K)}{\beta_G},\quad Y=\frac{\lambda\mu_E+\alpha_E V_C'(K)}{\beta_E}. \tag{2.37}$$

Similarly, the maximization condition Eq (2.37) is substituted into HJB Eq (2.36); thus, the symbol" max " is eliminated and the result is eliminated and simplified:

$$rV_C(K)=[\lambda\tau-\delta V_C'(K)]K+\lambda Q_0+\frac{[\lambda\mu_G+\alpha_G V_C'(K)]^2}{2\beta_G}+\frac{[\lambda\mu_E+\alpha_E V_C'(K)]^2}{2\beta_E}. \tag{2.38}$$

According to the structure of the differential equation, i.e., Eq (2.38), it is inferred that the linear optimal return function for K is the solution of the HJB equation, so the hypothetical expression satisfies $V_C(K)=mK+n$. Among them, m and n are constants to be solved and $dV_C(K)/dK=m=V_C'(K)$. Substituting the expression $V_C(K)$ and its first derivative into Eq (2.38) to solve the parameter value gives

$$m,\ n:\ m=\frac{\lambda\tau}{r+\delta},\quad n=\frac{\lambda Q_0}{r}+\frac{\lambda^2[\mu_G(r+\delta)+\tau\alpha_G]^2}{2r\beta_G(r+\delta)^2}+\frac{\lambda^2[\mu_E(r+\delta)+\tau\alpha_E]^2}{2r\beta_E(r+\delta)^2}.$$

Substituting m and n into the first-order maximization conditional Eq (2.37) of the government and the enterprises, respectively, the optimal degree of effort for the government and the enterprises to open and share data under the cooperative game is obtained, which is Eq (2.32). Substituting m and n into the optimal total income expression of the government and the enterprises, respectively, gives Eq (2.35).

Since the proof process for the expectation, expectation limit, variance and variance limit of the open data shared stock K in Proposition 3 is similar to that of Proposition 1, we do not repeat too much here, and part of the proof process is omitted.

Assuming that the open and shared stock of data follows a normal distribution, the confidence interval for the open and shared stock of data is $\left[E[K_C(t)]-1.96D[K_C(t)],E[K_C(t)]+1.96D[K_C(t)]\right]$ at a 95% confidence level. From Proposition 3, it can be seen that, in the cooperative game under random interference factors, although the government and enterprises cannot accurately determine the true state of the open and shared stock of data, they can accurately grasp the expectations of the true state. Within the allowable range of error, the government and enterprises can combine their expectations of the expected goals to make corresponding strategic choices.

From the analysis of game equilibrium in the case of collaborative cooperation, it can be seen that the optimal effort of the government and enterprises to open and share data is no longer affected by the total income distribution ratio; that is, the two parties no longer consider the maximization of their own income, but consider overall income maximization as the goal to determine the best decision-making. Other influencing factors and influence relationships are the same as in the previous two game conclusions.

3. Results and discussion

3.1. Comparative analysis of equilibrium results

Comparing Propositions 1–3 in the three situations of the Nash non-cooperative game, Stackelberg master-slave game and cooperative game, the following relevant inferences are obtained.

Corollary 1. When the government's open and shared data total revenue distribution ratio reaches $\frac{1}{3} < \omega \leq 1$, the relationships between the efforts of the government and enterprises to open and share data in the three game situations are $X_N^* = X_S^* < X_C^*$, $Y_N^* < Y_S^* < Y_C^*$, $Y_S^* - Y_N^* = \frac{3\omega-1}{2}\frac{\lambda[\mu_E(r+\delta)+\tau\alpha_E]}{\beta_E(r+\delta)} > 0$ and $\frac{Y_S^*-Y_N^*}{Y_S^*} = \psi$.

Proof. From the comparison of Eqs (2.3), (2.22) and (2.32),

$$X_C^* - X_S^* = \frac{(1-\omega)\lambda[\mu_G(r+\delta)+\tau\alpha_G]}{\beta_G(r+\delta)} > 0,$$

$$Y_S^* - Y_N^* = \frac{(3\omega-1)\lambda[\mu_E(r+\delta)+\tau\alpha_E]}{2\beta_E(r+\delta)} > 0 \quad \left(\frac{1}{3} < \omega \leq 1\right),$$

$$Y_C^* - Y_S^* = \frac{(1-\omega)\lambda[\mu_E(r+\delta)+\tau\alpha_E]}{2\beta_E(r+\delta)} > 0.$$

$\frac{Y_S^*-Y_N^*}{Y_S^*} = \psi = \frac{3\omega-1}{\omega+1}$ ($\frac{1}{3} < \omega \leq 1$). The proof is completed.

From Corollary 1, it can be seen that, when the government's open data sharing ratio reaches $\frac{1}{3} < \omega \leq 1$, compared with the Nash non-cooperative game equilibrium in the case of no cost sharing, under the Stackelberg master-slave game with cost sharing, the government undertakes the optimal effort to open shared data. Although there is no change, according to the equilibrium difference of enterprises in these two situations, the optimal effort of enterprises to open and share data is significantly improved, and the increase ratio is equal to the cost-sharing ratio provided by the government to enterprises; when the government and enterprises collaborate, the best efforts of both parties to open and share data are greater than in the other two situations.

Corollary 2. When the total revenue distribution ratio of open shared data by the government reaches $\frac{1}{3} < \omega \leq 1$, the expectation and variance of the open shared stock of data under the Stackelberg master-slave game model and the corresponding limit value are greater than the values under the Nash non-cooperative game model. When $\frac{1}{3} < \omega \leq 1$ is established, we can obtain

$$\begin{cases} E[K_S(t)] > E[K_N(t)], \lim_{t\to\infty} E[K_S(t)] > \lim_{t\to\infty} E[K_N(t)] \\ D[K_S(t)] > D[K_N(t)], \lim_{t\to\infty} D[K_S(t)] > \lim_{t\to\infty} D[K_N(t)] \end{cases}.$$

Proof. When $\frac{1}{3} < \omega \leq 1$, comparing the sizes of A_N and A_S, we can obtain

$$A_S - A_N = \frac{(3\omega-1)\lambda\alpha_E[\mu_E(r+\delta)+\tau\alpha_E]}{2\beta_E(r+\delta)} > 0.$$

Thus, for $\forall t \in (0, \infty)$, we can obtain

$$E\left[K_S(t)\right]-E\left[K_N(t)\right]=\frac{A_S-A_N}{\delta}(1-e^{-\delta t})>0,$$

$$\lim_{t\to\infty}E\left[K_S(t)\right]-\lim_{t\to\infty}E\left[K_N(t)\right]=\frac{A_S-A_N}{\delta}>0,$$

$$\lim_{t\to\infty}D\left[K_S(t)\right]-\lim_{t\to\infty}D\left[K_N(t)\right]=\frac{\sigma^2(A_S-A_N)}{2\delta^2}>0,$$

$$D\left[K_S(t)\right]-D\left[K_N(t)\right]=\frac{\sigma^2(A_S-A_N)(1-2e^{-\delta t}+e^{-2\delta t})}{2\delta^2}.$$

For $t\in(0,\infty)$, there is $\frac{d(1-2e^{-\delta t}+e^{-2\delta t})}{dt}=2\delta(e^{-\delta t}-e^{-2\delta t})>0$, that is, the first derivative of $1-2e^{-\delta t}+e^{-2\delta t}$ with respect to t is zero, and when $t=0$, $1-2e^{-\delta t}+e^{-2\delta t}=0$; thus, for $\forall t\in(0,\infty)$, $1-2e^{-\delta t}+e^{-2\delta t}>0$, and therefore, $D\left[K_S(t)\right]-D\left[K_N(t)\right]>0$. The proof is completed.

It can be seen from Corollary 2 that the cost-sharing mechanism is an effective incentive mechanism. Similar to certain situations, when the total income distribution ratio reaches $\frac{1}{3}<\omega\le 1$, compared to the Nash non-cooperative game, the Stackelberg master-slave game achieves a higher level of open data sharing, but, due to the existence of uncertain random interference factors, such as external factors, the variance under the Stackelberg master-slave game is greater than the variance under the Nash non-cooperative game, indicating that benefits and risks coexist. Although the Stackelberg master-slave game can obtain greater returns, its open data and shared stock is also highly uncertain.

Corollary 3. Under the cooperative game model, the expectation and variance of open data and shared stock and its corresponding limit value are the highest, namely,

$$\begin{cases}E\left[K_C(t)\right]>E\left[K_S(t)\right],E\left[K_C(t)\right]>E\left[K_N(t)\right]\\ \lim\limits_{t\to\infty}E\left[K_C(t)\right]>\lim\limits_{t\to\infty}E\left[K_S(t)\right],\lim\limits_{t\to\infty}E\left[K_C(t)\right]>\lim\limits_{t\to\infty}E\left[K_N(t)\right]\\ D\left[K_C(t)\right]>D\left[K_S(t)\right],D\left[K_C(t)\right]>D\left[K_N(t)\right]\\ \lim\limits_{t\to\infty}D\left[K_C(t)\right]>\lim\limits_{t\to\infty}D\left[K_S(t)\right],D\left[K_C(t)\right]>D\left[K_N(t)\right]\end{cases}.$$

Proof. Comparing the sizes of A_C, A_S and A_N, respectively, we can obtain

$$A_C-A_S=\frac{(1-\omega)\lambda\alpha_G[\mu_G(r+\delta)+\tau\alpha_G]}{\beta_G(r+\delta)}+\frac{(1-\omega)\lambda\alpha_E[\mu_E(r+\delta)+\tau\alpha_E]}{2\beta_E(r+\delta)}>0,$$

$$A_C-A_N=\alpha_G\frac{(1-\omega)\lambda[\mu_G(r+\delta)+\tau\alpha_G]}{\beta_G(r+\delta)}+\alpha_E\frac{\omega\lambda[\mu_E(r+\delta)+\tau\alpha_E]}{\beta_E(r+\delta)}>0.$$

It can be seen that $A_C>A_S$ and $A_C-A_N>0$. Thus, for $\forall t\in(0,\infty)$, we can obtain

$$E\left[K_C(t)\right]-E\left[K_S(t)\right]=\frac{A_C-A_S}{\delta}(1-e^{-\delta t})>0,$$

$$E\left[K_C(t)\right]-E\left[K_N(t)\right]=\frac{A_C-A_N}{\delta}(1-e^{-\delta t})>0,$$

$$\lim_{t\to\infty}E\left[K_C(t)\right]-\lim_{t\to\infty}E\left[K_S(t)\right]=\frac{A_C-A_S}{\delta}>0,$$

$$\lim_{t\to\infty} E\left[K_C(t)\right] - \lim_{t\to\infty} E\left[K_N(t)\right] = \frac{A_C - A_N}{\delta} > 0,$$

$$\lim_{t\to\infty} D\left[K_C(t)\right] - \lim_{t\to\infty} D\left[K_S(t)\right] = \frac{\sigma^2 (A_C - A_S)}{2\delta^2} > 0,$$

$$\lim_{t\to\infty} D\left[K_C(t)\right] - \lim_{t\to\infty} D\left[K_N(t)\right] = \frac{\sigma^2 (A_C - A_N)}{2\delta^2} > 0,$$

$$D\left[K_C(t)\right] - D\left[K_S(t)\right] = \frac{\sigma^2 (A_C - A_S)(1 - 2e^{-\delta t} + e^{-2\delta t})}{2\delta^2} > 0,$$

$$D\left[K_C(t)\right] - D\left[K_N(t)\right] = \frac{\sigma^2 (A_C - A_N)(1 - 2e^{-\delta t} + e^{-2\delta t})}{2\delta^2} > 0.$$

The proof is completed.

It can be seen from Corollary 3 that, under the cooperative game, the government and enterprises have made greater efforts in the process of data opening and sharing, thus achieving a higher level of data opening and sharing than in the non-cooperative game. However, also due to the existence of uncertain random interference factors, such as the outside, the variance under the cooperative game is greater than the variance under the non-cooperative game, indicating that the return and the risk coexist, and high return means high risk. If the government and enterprises want to obtain higher system returns in the cooperative game, they must also bear greater risks. Therefore, in actual situations, the choice of government-enterprise collaboration is not always the best choice. The government and enterprises have different risk preferences, and the choice tendencies that they make will also be different. The choice of game mode will also be biased; that is, if the government and enterprises have a higher risk appetite, they are more willing to choose cooperative games to obtain higher returns; if the government and enterprises have a relatively conservative attitude toward risk, they are more inclined to choose the Stackelberg master-slave game or Nash non-cooperative game to avoid incurring too high a risk.

Corollary 4. The overall optimal benefits of the two parties under the collaborative cooperation situation of the government and the enterprises are always at the highest level compared to the other two situations; when $\frac{1}{3} < \omega \le 1$, there is a cost-sharing situation. The optimal benefits obtained by the government and the enterprises openly sharing data are both higher than the optimal return in the Nash non-cooperative situation. They are $V_{GS}^*(K) > V_{GN}^*(K)$ and $V_{ES}^*(K) > V_{EN}^*(K)$.

Proof.

$$V_{GS}^*(K) - V_{GN}^*(K) = \frac{(3\omega - 1)^2 \lambda^2 \left[(r+\delta)\mu_E + \tau\alpha_E\right]^2}{8r\beta_E (r+\delta)^2} > 0,$$

$$V_{ES}^*(K) - V_{EN}^*(K) = \frac{(3\omega - 1)(1-\omega)\lambda^2 \left[(r+\delta)\mu_E + \tau\alpha_E\right]^2}{4r\beta_E (r+\delta)^2} \quad \left(\frac{1}{3} < \omega \le 1\right).$$

The proof is completed.

It can be seen from Corollary 4 that, from the perspective of overall income, the government and enterprise collaboration are both in a Pareto optimal state. On the one hand, the government and enterprises cooperate to solve the "free rider" problem of one party and reduce unnecessary economic losses in the process of open data sharing; on the other hand, the cooperation between the two parties results in "1+1>2" synergies of the company, resulting in additional benefits. When the government's income distribution ratio reaches a certain level, the government is willing to share a certain amount of data opening and sharing costs for enterprises, prompting enterprises to contribute a greater level of effort so that the overall income will be Pareto improved. In the case of Nash non-cooperation, the government and enterprises only proceed from the perspective of maximizing their own benefits, ignoring the level of openness and sharing of data to determine their optimal decisions so that their efforts, the benefits of both parties and the overall benefits are all at the lowest level. The efficiency is the worst.

3.2. Simulation analysis

When random interference factors do not exist, that is, $\sigma = 0$, $\lim\limits_{t\to\infty} E[K_N^2(t)] = \frac{A_N^2}{\delta^2}$ at this time.

From Propositions 1–3 and Corollaries 1–4, it can be seen that, when random factors are considered, the open and shared stock of data, government-enterprise revenue and total system revenue in the three models are all affected by multiple parameters in the model, which is not easy to observe, so we numerically simulated it with the help of MATLAB software. We set the benchmark parameters to $\alpha_G = 0.6$, $\alpha_E = 0.6$, $K_0 = 5$, $\delta = 0.1$, $\beta_G = 2$, $\beta_E = 2$, $r = 0.9$, $Q_0 = 1$, $\mu_G = 0.5$, $\mu_E = 0.5$, $\tau = 0.5$, $\lambda = 10$, $\omega = 0.6$ and $\sigma = 0.4$.

For the convenience of the simulation, we discretized Eq (2.1), which is the state equation of open data and shared stock to obtain

$$K(t+\Delta t) = K(t) + [\alpha_G X(t) + \alpha_E Y(t) - \delta K(t)]\Delta t + \sigma\sqrt{K(t)}\sqrt{\Delta t}\zeta(t) \quad (3.1)$$

Among them, $\zeta(t)$ obeys an independent and identically distributed standard normal distributed random variable, namely, $\zeta(t) \sim N(0,1)$, and the sampling step was set to $\Delta t = 0.001$. Combining Eq (2.1) and Propositions 1–3, we obtain Figures 1–4.

Figure 1 describes the evolution process of open and shared stock data in the three cases of cost-sharing mechanisms and collaboration when considering random interference. It can be seen from the figure that, under the influence of uncertain random interference factors, such as the outside, the open and shared stock of data in the three situations is in a state of constant change, but it always fluctuates around the expected value within a certain range. In reality, it is difficult for the government and enterprises to obtain a clear open and shared stock of data. Therefore, an approximate value of the open and shared stock of data within the allowable error range was selected for evaluation. We used the confidence interval to estimate the changes in the open and shared stock of data and provide suggestions for governments and enterprises to make corresponding strategic choices. It can also be seen from the figure that, in the three cases, the open and shared stock of data has gradually increased over time and converged to the open and shared stock of data in a stable state. Among the cases, the Stackelberg case is better than the Nash non-cooperative case, and the cooperative cooperation situation achieves Pareto optimality. When $\frac{1}{3} < \omega \leq 1$, because the government is willing to share part of the cost of open and shared data for enterprises, the cost of open and shared data is reduced, stimulating enterprises to increase their efforts to open and share data, leading to an increase in the stock of open and shared data. At the same time, in the case of non-cooperation, whether it is a no cost-sharing mechanism or a cost-sharing mechanism, the government and enterprises only consider maximizing their own benefits and ignoring the overall benefits, making the stock of open data sharing in non-cooperative situations lower than collaborative cooperation. The circumstances verify Corollaries 2 and 3.

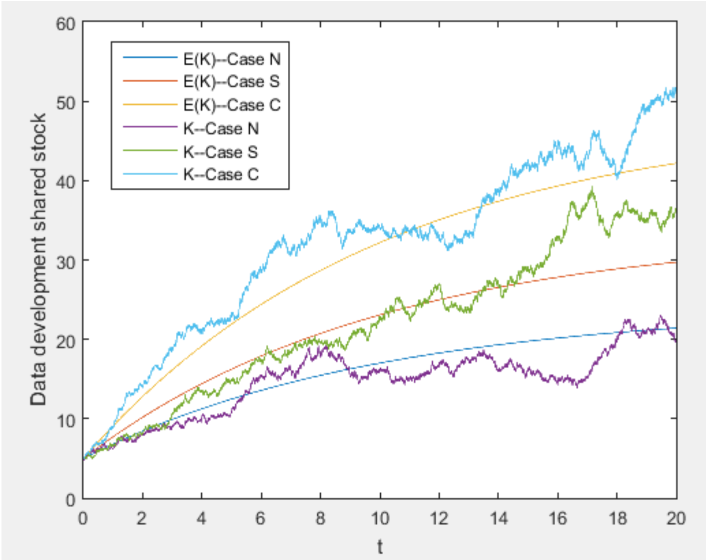

Figure 1. Comparison of the open data and shared stock.

Figures 2 and 3 describe the comparison of government and enterprise income changes with or without cost sharing under the interference of random factors. It can be seen from the figures that, even if there are uncertainties, before and after cost sharing, the revenue of the government and enterprises increases and stabilizes over time, and for the government and enterprises, there is a Stackelberg situation with cost sharing. The respective benefits of both parties are greater than the benefits of non-cooperative Nash without cost sharing. Since the government has reduced part of the cost pressure for the open sharing of corporate data, it has encouraged companies to invest more effort and indirectly promotes the overall level of open data sharing in the system. The social welfare effect has been significantly improved, leading to a Pareto improvement in the benefits of both government and enterprises. The cost-sharing mechanism has an effective incentive effect, which verifies Corollary 4.

Figure 4 is based on Figures 2 and 3 and further describes the comparison of the total revenue changes of the system under the interference of random factors, with or without cost sharing and collaboration. It can be seen from the figure that the total revenue of the system in the case of cost is significantly greater than the total revenue of the system in the case of no cost, once again demonstrating the effectiveness of the cost-sharing mechanism. At the same time, the total revenue of the system in the case of collaborative cooperation is significantly greater than the total revenue of the other two non-cooperative cases, indicating that the collaborative cooperation mechanism can comprehensively and effectively improve the overall efficiency of open data sharing and provide a basis for the open sharing of data between the government and enterprises. Combining Figures 2–4, it can be seen that, due to the influence of external uncertain factors, the actual value of the government and the enterprises' respective income and the total system income have a certain deviation from their expected values, but, within the allowable range of error, the true value always fluctuates up and down around the expected value. Therefore, although the actual return value may deviate from the expected value, the actual value is not completely uncontrollable. When faced with the complex process of the open sharing of data, the government and enterprises can combine expected values to provide reasonable estimates of the true value of decisions within a certain credible interval and then make scientific strategic choices.

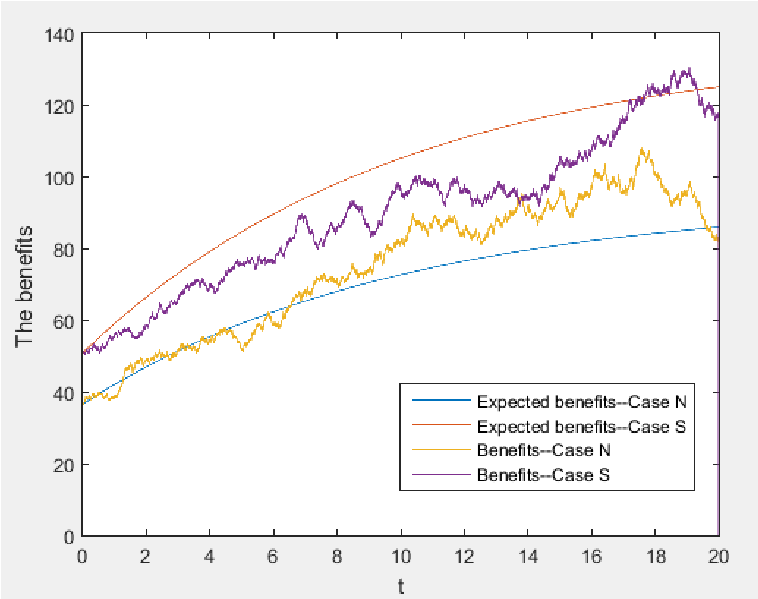

Figure 2. Comparison of the benefits of the government.

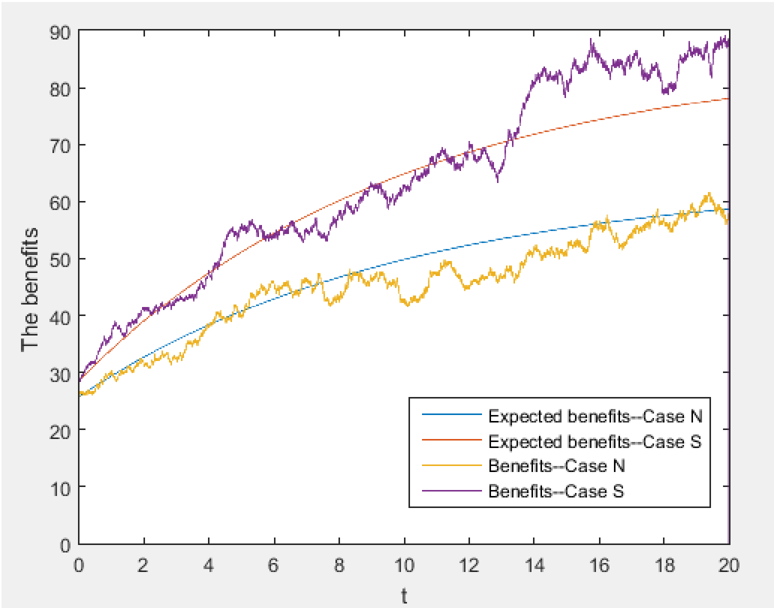

Figure 3. Comparison of the benefits of enterprises.

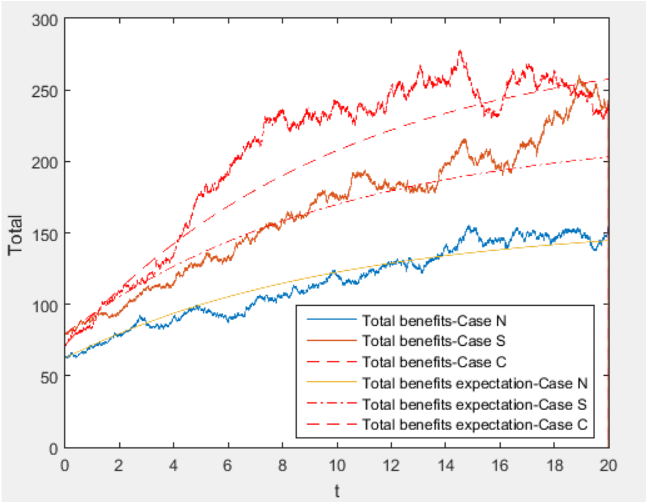

Figure 4. Comparison diagram of total system revenue under random factors.

4. Conclusions

Considering that the open sharing of data by the government and enterprises is closely related to the time factor, random interference factors were introduced; the stochastic differential game has been used to analyze the impact of the efforts of the government and enterprises to open and share data on the open and shared stock of social data. We constructed the Nash non-cooperative game, Stackelberg master-slave game and collaborative cooperation game model; and, through the evolution of the Ito process analyzed and compared the expectations and variance values in the three situations. Finally, MATLAB software was used to perform numerical simulations and comparative analysis of the open and shared stock, government and enterprise income and overall income data under random factors. Through the above research and analysis, the following was found.

(1) When the government's income distribution ratio is greater than 1/3, the benefits of the government and the enterprises in the case of cost sharing and the effort to open and share data are greater than those in the Nash non-cooperative case; in the case of collaborative cooperation, the total revenue of both parties reaches par, achieving an optimal state. In the actual situation, due to different factors, such as regions, production and operation models and customer types, there are enormous differences in the open sharing of data and the benefits obtained between enterprises. The government should formulate scientific and effective cost-sharing criteria according to the type of enterprises, establish multilevel cost sharing and compensation mechanisms such as manpower and technology, reduce the cost of enterprises data openness and sharing and provide a guarantee for promoting government-enterprise cooperation. At the same time, because the cost-sharing mechanism must be feasible under certain conditions, when the government and the enterprises have not yet reached a cooperative relationship, the government, as the leader, should use propaganda and encouragement to guide the enterprises in pursuing the maximization of its own interests while paying attention to the system. Overall profitability lays the foundation for promoting cooperation between the two parties.

(2) Through comparative analysis, it was found that data are open under collaborative cooperation. The expectation and variance of shared stock are the largest; when the government's income distribution ratio is greater than 1/3, the expectation and variance of open data shared stock under the Stackelberg master-slave game model are higher than those under the Nash non-

cooperative game model. The above conclusions show that, under the influence of random interference factors, benefits and risks coexist, and while high returns can be obtained, they also must bear higher risks. Therefore, the choice of strategies for participants with different risk preferences is also different. Through simulation, it was found that, due to the influence of random factors, the true value of the open and shared stock of data and government-enterprise income has a certain deviation from the expected value, but, within a certain range, it always fluctuates up and down around the expected value. Therefore, when faced with a complex environment, the government and enterprises can make decisions based on expectations and within a reasonable range of error.

(3) When the government and enterprises are faced with a complex external environment, they are affected by random interference factors, causing both benefits and risks to coexist. High returns mean high risks. Participants with different risk preferences make different strategic choices. For example, relatively conservative participants might be more willing to choose a cost-sharing mechanism to improve the efficiency of open data sharing, while participants with risk preferences are more willing to choose collaborative cooperation and earn high returns. At the same time, the study also shows that random interference factors lead to a certain deviation between the true value of the open and shared stock of data, government-enterprise revenue, etc., from the expected value, but, within a certain range, it always fluctuates up and down around the expected value. Therefore, in the face of a complex environment, the government and enterprises can also make decisions within a reasonable error range based on expected values. For example, when the open and shared stock of data obeys a normal distribution, the confidence interval at a 95% confidence level is $\left[E[K(t)]-1.96D[K(t)], E[K(t)]+1.96D[K(t)]\right]$.

The findings of the existing literature on open data sharing focus on the decision-making judgments of both the government and enterprises on whether to open and share data [22], the factors influencing the willingness of the government and enterprises to open and share data [37,21] and how to make use of the synergy effect to promote the degree of open data sharing between government and enterprises [30]. From the perspective of cost sharing and benefit distribution between the government and enterprises, the benefits of the government and enterprises in different cooperation scenarios were explored; the Pareto optimal state was found to be achieved when the government and enterprises cooperate. In addition, the conclusions of this paper focus on the impact of random disturbances on indicators such as the stock of open data sharing and the benefits to the government and enterprises.

The contribution of this paper is to broaden the field of application of differential game theory, enrich and develop the theoretical system of data open sharing research and explore the cost-sharing mechanism. An effective cost-sharing mechanism can provide a decision basis for the design of a government subsidy mechanism and the maximization of social welfare, thus laying the foundation for cooperation between government and enterprises in the area of data open sharing. The study also had certain limitations, such as the differential game model being based on the government and a single enterprise, when in fact, the process of open data sharing involves a game between the government and several enterprises, and even the participation of universities, the public and other subjects. Furthermore, there are many factors influencing open data sharing, and this work only considers the impact of a few important factors on the decision-making models of the government and enterprises; also, digital technology [39] can be taken into account in future research.

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Conflict of interest

The authors declare that they have no competing interests.

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