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Research article

Fractional evaluation of Kaup-Kupershmidt equation with the exponential-decay kernel

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Abstract: In this paper, we investigate the semi-analytical solution of Kaup-Kupershmidt equations with the help of a modified method known as the new iteration transformation technique. This method combines the Yang transform and the new iteration technique. The nonlinear terms can be calculated straightforwardly by a new iteration method. The numerical simulation results have been presented to demonstrate the reliability and validity of the proposed approach. The result confirms that the suggested technique is the best tool for dealing with any nonlinear problems arising in technology and science. In addition, in terms of figures for varying fractional order, the physical behavior of new iteration transformation technique solutions has been shown and the numerical simulation is also exhibited. The solutions of the new iteration transformation technique reveal that the projected technique is reliable, competitive and powerful for studying complex nonlinear fractional type models.

Keywords: Kaup-Kupershmidt equation; Yang transform; new iterative transform method; Caputo-Fabrizio operator
Mathematics Subject Classification: 33B15, 34A34, 35A20, 35A22, 44A10

1. Introduction

Due to its well-established applications in various scientific and technical fields, fractional calculus has gained prominence during the last three decades. Many pioneers have shown that when adjusted by integer-order models, fractional-order models may accurately represent complex events [1,2]. The Caputo fractional derivatives are nonlocal in contrast to the integer-order derivatives, which are local in nature. In other words, the integer-order derivative may be used to analyze changes in the area around a point, but the Caputo fractional derivative can be used to analyze changes in the whole interval [3, 4]. Senior mathematicians including Riemann [5], Caputo [6], Podlubny [7], Ross [8], Liouville [9], Miller and others, collaborated to create the fundamental foundation for fractional order integrals and derivatives. The theory of fractional-order calculus has been related to real-world projects, and it has been applied to chaos theory [10], signal processing [11], electrodynamics [12], human diseases [13,14], and other areas [15,16].

Due to the numerous applications of fractional differential equations in engineering and science such as chaos theory [17], fluid and continuum mechanics [18], electrodynamics [19], signal processing [20], biological population models [21], finance [22], and others, fractional differential equations are now more widely known. Unfortunately, discovering precise solutions to nonlinear fractional differential equations has proved to be quite challenging. For such issues to be resolved, efficient tools are needed. Because of this, we will attempt to apply an efficient analytical technique to solve nonlinear arbitrary order differential equations [23–25]. Many strategies in collaboration fields may be delightfully and even more accurately analyzed using fractional differential equations. Various strategies have been developed in this regard, some of them are as follows, such as the fractional Adomian decomposition method (FADM) [26], Elzaki transform Decomposition method (ETDM) [27, 28], the fractional Variational iteration method (FVIM) [29], the fractional natural decomposition method (FNDM) [30], Iterative Laplace transform method (ILTM) [31], Yang transform decomposition method (YTDM) [32], and the fractional Homotopy perturbation method (FHPM) [33] and so on [34–36].

The renowned dispersive classical Kaup-Kupershmidt equation was first proposed by Kaup in 1980 and updated by Kupershmidt in 1994 [37]. The time-fractional modified Kaup-Kupeshmidt (KK) equation will be examined in this paper. The fractional-order KK equation is applied to analyze the behaviour of nonlinear dispersive waves and capillary gravity waves. The nonlinear fifth-order evolution equation has the following form:

$$D_\eta^\wp \psi(\xi,\eta) + j\psi\psi_{\xi\xi\xi} + kp\psi_\xi\psi_{\xi\xi} + l\psi^2\psi_\xi + \psi_{\xi\xi\xi\xi\xi} = 0, \tag{1.1}$$

where $0 < \wp \leq 1$ denotes the order time-fractional derivative and j, k and l are constants. By altering the values of j, k and l, the fifth-order nonlinear evolution equation mentioned above may be transformed into the fifth-order time-fractional Kaup-Kupershmidt equation. The following equation is reduced to by choosing $j = -15, k = -15$ and $l = 45$,

$$D_\eta^\wp \psi(\xi,\eta) - 15\psi\psi_{\xi\xi\xi} - 15p\psi_\xi\psi_{\xi\xi} + 45\psi^2\psi_\xi + \psi_{\xi\xi\xi\xi\xi} = 0, \tag{1.2}$$

with the initial condition

$$\psi(\xi,0) = g(\xi).$$

The famous Kaup-Kupershmidt equation has been the subject of many studies in recent years. At $p = \frac{5}{2}$, the classical KK equation is integrable [38] and has bilinear forms [39]. Soliton and solitary wave solutions for broad nonlinear evolution equations may be found separately, implementing four alternative methods.

The mathematical models are significant techniques in order to understand the behavior of some implementation in many areas such as engineering, fluid dynamic and physics. The important of mathematical equations for real application are built as non-linear differential equations. Thus, several efforts have been exerted to improve powerful and useful techniques for finding the exact solutions of nonlinear partial differential equations. Nonlinear evolution equations model many complex phenomena in physics including solid state, plasma, optical fibers and chemical, fluid mechanics, non-linear optics, etc. Exploring exact traveling wave solutions plays a significant role in nonlinear physics. In order to examine nonlinear equations with physical ramifications, Ablowitz and Clarkson used the inverse scattering method to produce soliton solutions [40]. Hu and Tam utilized Hirota's method and Mathematica to get a comparable solution [41]. One of the integrable instances of the Henon-Heiles system was the fifth order Kaup-Kupershmidt equation, which Musette and Verhoeven published.

2. Basic preliminaries

Definition 2.1. The Caputo-Fabrizio (CF) fractional derivative is given as [42–44]:

$$ {}^{CF}D_{\eta}^{\wp}[g(\eta)] = \frac{N(\wp)}{1-\wp}\int_0^{\eta} g^{'}(\varrho)K(\eta,\varrho)d\varrho, \; n-1 < \wp \leq n. \tag{2.1}$$

The normalisation function is $N(\wp)$ with $N(0) = N(1) = 1$,

$$ {}^{CF}D_{\eta}^{\wp}[g(\eta)] = \frac{N(\wp)}{1-\wp}\int_0^{\eta} [g(\eta) - g(\varrho)]K(\eta,\varrho)d\varrho. \tag{2.2}$$

Definition 2.2. The Laplace transform of CF derivative for $N(\wp) = 1$ is defined as [43]:

$$ L\left[{}^{CF}D_{\eta}^{\wp}[g(\eta)]\right] = \frac{\omega L[g(\eta) - g(0)]}{\omega + \wp(1-\omega)}. \tag{2.3}$$

Definition 2.3. The fractional CF integral is given as [43]:

$$ {}^{CF}I_{\eta}^{\wp}[g(\eta)] = \frac{1-\wp}{N(\wp)}g(\eta) + \frac{\wp}{N(\wp)}\int_0^{\eta} g(\varrho)d\varrho, \; \eta \geq 0, \;\; \wp \in (0,1]. \tag{2.4}$$

Definition 2.4. The Yang transform of $g(\eta)$ is define as [42]:

$$ \mathcal{Y}[g(\eta)] = \chi(\omega) = \int_0^{\infty} g(\eta)e^{-\frac{\eta}{\omega}}d\eta, \; \eta > 0. \tag{2.5}$$

Remark 2.1. The Yang transform of few terms formulaes are given as:

$$ \begin{aligned} Y[1] &= \omega, \\ Y[\eta] &= \omega^2, \\ Y[\eta^i] &= \Gamma(i+1)\omega^{i+1}. \end{aligned} \tag{2.6}$$

Lemma 2.1. *Let the Laplace transform of $g(\eta)$ is $F(\omega)$, then $\chi(\omega) = F(1/\omega)$.*

Proof. From (2.5), we can obtain another form of the Yang transform by substituting $\eta/\omega = \xi$ as:

$$L\left[g(\eta)\right] = \chi(\omega) = \omega \int_0^\infty g(\omega\xi)e^{\xi}d\xi,\ \xi > 0. \tag{2.7}$$

Since $L\left[g(\eta)\right] = F(\omega)$, then we have

$$F(\omega) = L\left[g(\eta)\right] = \int_0^\infty g(\eta)e^{-\omega\eta}d\eta. \tag{2.8}$$

Put $\eta = \xi/\omega$ in (2.8), we get

$$F(\omega) = \frac{1}{\omega}\int_0^\infty g\left(\frac{\xi}{\omega}\right)e^{\xi}d\xi. \tag{2.9}$$

Thus, from (2.7), we have

$$F(\omega) = \chi\left(\frac{1}{\omega}\right). \tag{2.10}$$

Also from (2.5) and (2.8), we have

$$F\left(\frac{1}{\omega}\right) = \chi\left(\omega\right). \tag{2.11}$$

The links (2.10) and (2.11) represent the duality connection between the Laplace and Yang transforms. □

Lemma 2.2. *Let $g(\eta)$ be a continue function; then, the Yang transformation CF derivative of $g(\eta)$ is defined as [42–44]:*

$$Y\left[g(\eta)\right] = \frac{Y[g(\eta) - \omega g(0)]}{1 + \wp(\omega - 1)}. \tag{2.12}$$

Proof. The Laplace transformation of fractional CF operator is defined as:

$$L\left[g(\eta)\right] = \frac{L[\omega g(\eta) - g(0)]}{\omega + \wp(1 - \omega)}. \tag{2.13}$$

Also, we get that the connection between Yang and Laplace properties, i.e., $\chi(\omega) = F(1/\omega)$. To obtained the necessary result, we put ω by $1/\omega$ in (2.13), and we get

$$\begin{aligned} Y\left[g(\eta)\right] &= \frac{\frac{1}{\omega}Y[g(\eta) - g(0)]}{\frac{1}{\omega} + \wp(1 - \frac{1}{\omega})}, \\ Y\left[g(\eta)\right] &= \frac{Y[g(\eta) - \omega g(0)]}{1 + \wp(\omega - 1)}. \end{aligned} \tag{2.14}$$

The proof is completed. □

3. The propose technique general implementation

Consider the fractional partial differential equation:

$$D_\eta^\wp \psi(\xi, \eta) + M\psi(\xi, \eta) + N\psi(\xi, \eta) = h(\xi, \eta),\ n - 1 < \wp \leq n, \tag{3.1}$$

where $n \in \mathbb{N}$, M is linear and N nonlinear terms and h is a source term.

With the initial condition,

$$\psi^k(\xi, 0) = g_k(\xi),\ k = 0, 1, 2, \ldots, n-1. \tag{3.2}$$

Using the Yang transformation of (3.1), we achieve as:

$$Y[D_\eta^\wp \psi(\xi,\eta)] + Y[M\psi(\xi,\eta) + N\psi(\xi,\eta)] = Y[h(\xi,\eta)]. \tag{3.3}$$

Applying the differentiation property is given as:

$$Y[\psi(\xi,\eta)] = \omega u(\xi,0) + \frac{1}{(1+\wp(\omega-1))}Y[h(\xi,\eta)] - \frac{1}{(1+\wp(\omega-1))}Y[M\psi(\xi,\eta) + N\psi(\xi,\eta)], \tag{3.4}$$

applying inverse Yang transform of (3.4) into

$$\begin{aligned}\psi(\xi,\eta) = Y^{-1}&\left[\left(\omega u(\xi,0) + \frac{1}{(1+\wp(\omega-1))}Y[h(\xi,\eta)]\right)\right]\\ &- Y^{-1}\left[\frac{1}{(1+\wp(\omega-1))}Y[M\psi(\xi,\eta) + N\psi(\xi,\eta)]\right].\end{aligned} \tag{3.5}$$

As, through iterative method, we get

$$\psi(\xi,\eta) = \sum_{m=0}^{\infty} \psi_m(\xi,\eta). \tag{3.6}$$

Here, M is linear operator,

$$M\left(\sum_{m=0}^{\infty} \psi_m(\xi,\eta)\right) = \sum_{m=0}^{\infty} M\left[\psi_m(\xi,\eta)\right], \tag{3.7}$$

and N is the nonlinear term, we have

$$N\left(\sum_{m=0}^{\infty} \psi_m(\xi,\eta)\right) = \psi_0(\xi,\eta) + M\left(\sum_{k=0}^{m} \psi_k(\xi,\eta)\right) - N\left(\sum_{k=0}^{m} \psi_k(\xi,\eta)\right). \tag{3.8}$$

Putting (3.6), (3.7) and (3.8) in (3.5), we achieve the following result:

$$\begin{aligned}\sum_{m=0}^{\infty} \psi_m(\xi,\eta) = Y^{-1}&\left[\frac{1}{(1+\wp(\omega-1))}\left(\omega\psi(\xi,0) + Y[h(\xi,\eta)]\right)\right]\\ &- Y^{-1}\left[\frac{1}{(1+\wp(\omega-1))}E\left[M\left(\sum_{k=0}^{m}\psi_k(\xi,\eta)\right) - N\left(\sum_{k=0}^{m}\psi_k(\xi,\eta)\right)\right]\right].\end{aligned} \tag{3.9}$$

Implementing the iterative technique, we have

$$\psi_0(\xi,\eta) = Y^{-1}\left[\left(\omega\psi(\xi,0) + \frac{1}{(1+\wp(\omega-1))}Y(g(\xi,\eta))\right)\right], \tag{3.10}$$

$$\psi_1(\xi,\eta) = -Y^{-1}\left[\frac{1}{(1+\wp(\omega-1))}Y[M[\psi_0(\xi,\eta)] + N[\psi_0(\xi,\eta)]\right], \tag{3.11}$$

$$\psi_{m+1}(\xi,\eta) = -Y^{-1}\left[\frac{1}{(1+\wp(\omega-1))}Y\left[-M\left(\sum_{k=0}^{m}\psi_k(\xi,\eta)\right) - N\left(\sum_{k=0}^{m}\psi_k(\xi,\eta)\right)\right]\right],\ m \geq 1. \tag{3.12}$$

Finally, (3.1) and (3.2) provides the series type result is given as:

$$\psi(\xi,\eta) \cong \psi_0(\xi,\eta) + \psi_1(\xi,\eta) + \psi_2(\xi,\eta) + \cdots + \psi_m(\xi,\eta),\ m \in \mathbb{N}. \tag{3.13}$$

4. Applications

Example 4.1. Consider the fractional-order KK equation,

$$D_\eta^\wp \psi(\xi,\eta) - 15\psi\psi_{\xi\xi\xi} - 15\varrho\psi_\xi\psi_{\xi\xi} + 45\psi^2\psi_\xi + \psi_{\xi\xi\xi\xi\xi} = 0, \tag{4.1}$$

with the initial condition,

$$\psi(\xi,0) = \frac{1}{4}\upsilon^2\lambda^2 \operatorname{sech}^2\left(\frac{\upsilon\xi\lambda}{2}\right) + \frac{\upsilon^2\lambda^2}{12}. \tag{4.2}$$

Using the Yang transform to (4.1), we obtain

$$Y[\psi(\xi,\eta)] = \omega\psi(\xi,0) + \frac{1}{(1+\wp(\omega-1))} Y\left[-15\psi\psi_{\xi\xi\xi} - 15\varrho\psi_\xi\psi_{\xi\xi} + 45\psi^2\psi_\xi + \psi_{\xi\xi\xi\xi\xi}\right]. \tag{4.3}$$

Applying inverse Yang transform of (4.3), we have

$$\psi(\xi,\eta) = Y^{-1}\left[\omega\psi(\xi,0)\right] + Y^{-1}\left[\frac{1}{(1+\wp(\omega-1))} Y\left(-15\psi\psi_{\xi\xi\xi} - 15\varrho\psi_\xi\psi_{\xi\xi} + 45\psi^2\psi_\xi + \psi_{\xi\xi\xi\xi\xi}\right)\right]. \tag{4.4}$$

Now, by using the suggested semi-analytical method, we have

$$\begin{aligned}
\psi_0(\xi,\eta) &= \frac{1}{4}\upsilon^2\lambda^2 \operatorname{sech}^2\left(\frac{\upsilon\xi\lambda}{2}\right) + \frac{\upsilon^2\lambda^2}{12},\\
\psi_1(\xi,\eta) &= Y^{-1}\left[\frac{1}{(1+\wp(\omega-1))} Y\left(-15\psi_{(0)}\psi_{(0)\xi\xi\xi} - 15\varrho\psi_{(0)\xi}\psi_{(0)\xi\xi} + 45\psi_{(0)}^2\psi_{(0)\xi} + \psi_{(0)\xi\xi\xi\xi\xi}\right)\right],\\
\psi_1(\xi,\eta) &= \Big(-\frac{1}{512}\upsilon^7\lambda^7(3843 + 480\varrho - 4(209+60\varrho)\cosh(\upsilon\xi\lambda)\\
&\qquad + \cosh(2\upsilon\xi\lambda)\operatorname{sech}^6\left(\frac{\upsilon\xi\lambda}{2}\right)\tanh\left(\frac{\upsilon\xi\lambda}{2}\right)\Big)(1+\wp\eta-\wp),\\
\psi_2(\xi,\eta) &= Y^{-1}\left[\frac{1}{(1+\wp(\omega-1))} Y\left(-15\psi_{(1)}\psi_{(1)\xi\xi\xi} - 15\varrho\psi_{(1)\xi}\psi_{(1)\xi\xi} + 45\psi_{(1)}^2\psi_{(1)\xi} + \psi_{(1)\xi\xi\xi\xi\xi}\right)\right],\\
\psi_2(\xi,\eta) &= (-733469760\varrho - 3947228724 + 6(148082560\varrho + 777305099 + 4358400\varrho^2)\cosh(\upsilon s\lambda)\\
&\quad - 20736000\varrho^2 - 48(3850520\varrho + 18859301 + 124800\varrho^2)\cosh(2\upsilon\xi\lambda)\\
&\quad + 46313277\cosh(3\upsilon\xi\lambda) + 10287360\varrho\cosh(3\upsilon\xi\lambda) + 345600\varrho^2\cosh(3\upsilon\xi\lambda)\\
&\quad - 305756\cosh(4\upsilon\xi\lambda) - 87360p\cosh(4\upsilon\xi\lambda) + \cosh(5\upsilon\xi\lambda))\operatorname{sech}^{12}\Big(\frac{\upsilon\xi\lambda}{2}\Big)\\
&\quad \times \frac{\upsilon^{12}\lambda^{12}}{524288}\Big((1-\wp)2\wp\eta + (1-\wp)^2 + \frac{\wp^2\eta^2}{2}\Big),\\
&\ \ \vdots\\
\psi_n(\xi,\eta) &= Y^{-1}\left[\frac{1}{(1+\wp(\omega-1))} Y\left(-15\psi_{(n)}\psi_{(n)\xi\xi\xi} - 15\varrho\psi_{(n)\xi}\psi_{(n)\xi\xi} + 45\psi_{(n)}^2\psi_{(n)\xi} + \psi_{(n)\xi\xi\xi\xi\xi}\right)\right].
\end{aligned}$$

The series form result is

$$\psi(\xi,\eta) = \psi_0(\xi,\eta) + \psi_1(\xi,\eta) + \psi_2(\xi,\eta) + \psi_3(\xi,\eta) + \cdots + \psi_n(\xi,\eta). \tag{4.5}$$

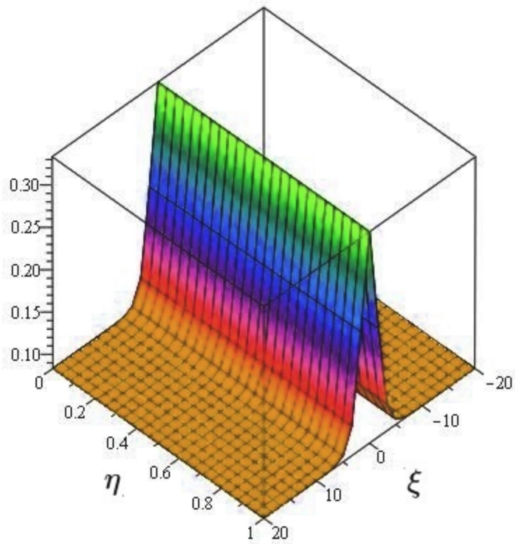
Therefore, we have

$$\psi(\xi,\eta)=\frac{1}{4}\upsilon^2\lambda^2\text{sech}^2\left(\frac{\upsilon\xi\lambda}{2}\right)+\frac{\upsilon^2\lambda^2}{12}+\left(-\frac{1}{512}\upsilon^7\lambda^7(480p+3843-4(60\varrho+209)\cosh(\upsilon\xi\lambda)\right.$$
$$+\cosh(2\upsilon\xi\lambda)\text{sech}^6\left(\frac{\upsilon\xi\lambda}{2}\right)\tanh\left(\frac{\upsilon\xi\lambda}{2}\right)\Big)(1+\wp\eta-\wp)+\Big(-733469760\varrho-3947228724$$
$$-20736000\varrho^2+6(1480925060\varrho+778300098+3358400\varrho^2)\cosh(\upsilon s\lambda)-48(3850520\varrho$$
$$+18859301+124800\varrho^2)\cosh(2\upsilon\xi\lambda)+46313277\cosh(3\upsilon\xi\lambda)+10287360\varrho\cosh(3\upsilon\xi\lambda)$$
$$+345600\varrho^2\cosh(3\upsilon\xi\lambda)-305756\cosh(4\upsilon\xi\lambda)-87360\varrho\cosh(4\upsilon\xi\lambda)$$
$$+\cosh(5\upsilon\xi\lambda)\Big)\text{sech}^{12}\left(\frac{\upsilon\xi\lambda}{2}\right)\frac{\upsilon^{12}\lambda^{12}}{524288}\left((1-\wp)2\wp\eta+(1-\wp)^2+\frac{\wp^2\eta^2}{2}\right)+\cdots.$$

For $\wp=1$, the exact results of (4.1) is given by

$$\psi(\xi,\eta)=\frac{1}{4}\upsilon^2\lambda^2\text{sech}^2\left(\frac{\lambda}{2}\left(\frac{-\upsilon^5(-8\lambda^2\nu+16\nu^2+\lambda^4)}{16\Gamma(1+\wp)}\eta^{\wp}+\upsilon\xi\right)\right)+\frac{\upsilon^2\lambda^2}{12}.$$

The suggested technique provides approximate analytical answers with a number of free parameters. The analytical results are incredibly helpful for interpreting the internal components of natural phenomena. Depending on the physical conditions, explicit answers reflected various approximations. Figure 1 compares the result produced using the proposed technique and the exact and analytical answer for the fractional-order KK equation. Figure 2 demonstrates that distinct fractional-orders of $\wp$ with respect to ξ and η comparison reveal that they are in close proximity. In Table 1, the various fractional-order of $\wp, \upsilon=0.5$ and $\lambda=1$ of Example 4.1.



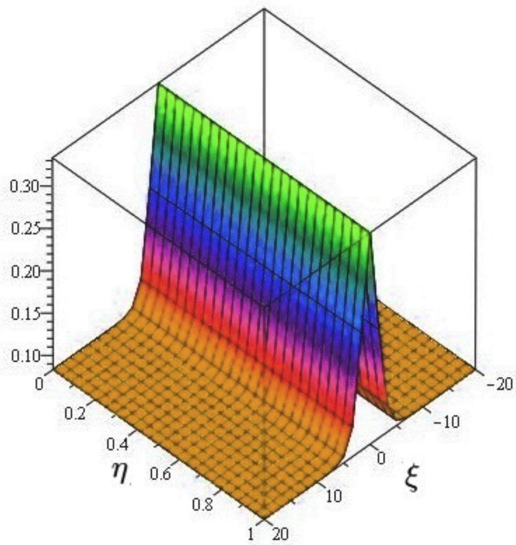


Figure 1. The exact and analytic solution of $\upsilon=0.5$ and $\lambda=1$ of Example 4.1.

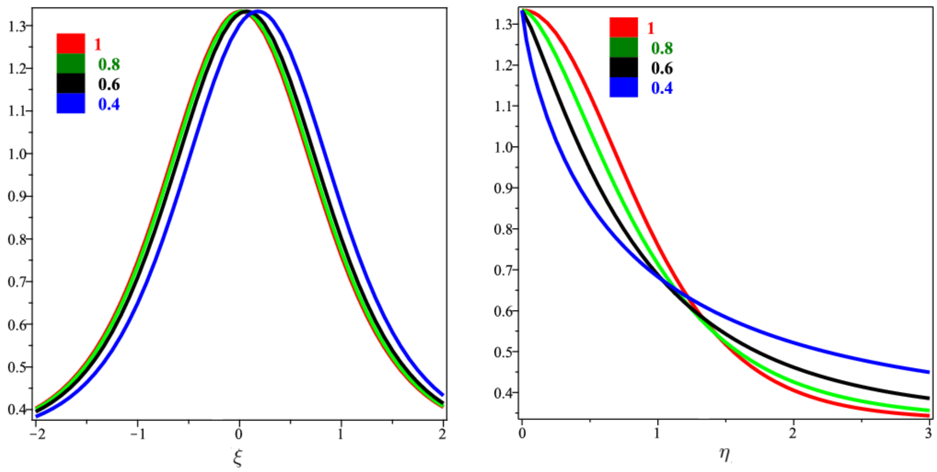


Figure 2. The fractional-order $\wp$ of Example 4.1 with respect to ξ and $\eta = 1$.

Table 1. The various fractional-order of $\wp, \upsilon = 0.5$ and $\lambda = 1$ of Example 4.1.

η	ξ	$\wp = 0.4$	$\wp = 0.6$	$\wp = 0.8$	$\wp = 1(NITM)$	$\wp = 1(q-HATM)$ [45]
	0.2	7.7794000000E-08	5.9046000000E-08	3.1881000000E-08	1.5379000000E-08	1.5379000000E-08
	0.4	1.5668400000E-07	1.1893800000E-07	6.4232000000E-08	3.0990000000E-08	3.0990000000E-08
0.1	0.6	2.3529200000E-07	1.7864200000E-07	9.6509000000E-08	4.6575000000E-08	4.6575000000E-08
	0.8	3.1347800000E-07	2.3807000000E-07	1.2867500000E-07	6.2123000000E-08	6.2123000000E-08
	1	3.9110400000E-07	2.9712500000E-07	1.6069300000E-07	7.7622000000E-08	7.7622000000E-08
	0.2	1.5316700000E-07	1.1624400000E-07	6.2757000000E-08	3.0269000000E-08	3.0269000000E-08
	0.4	3.1100100000E-07	2.3605500000E-07	1.2746600000E-07	6.1491000000E-08	6.1491000000E-08
0.2	0.6	4.6827700000E-07	3.5549900000E-07	1.9202900000E-07	9.2664000000E-08	9.2664000000E-08
	0.8	6.2471400000E-07	4.7438600000E-07	2.5637000000E-07	1.2376100000E-07	1.2376100000E-07
	1	7.8003300000E-07	5.9253400000E-07	3.2041800000E-07	1.5476200000E-07	1.5476200000E-07
	0.2	2.2606900000E-07	1.7156700000E-07	9.2620000000E-08	4.4671000000E-08	4.4671000000E-08
	0.4	4.6285600000E-07	3.5130300000E-07	1.8968900000E-07	9.1506000000E-08	9.1506000000E-08
0.3	0.6	6.9881100000E-07	5.3049100000E-07	2.8654200000E-07	1.3826500000E-07	1.3826500000E-07
	0.8	9.3351200000E-07	7.0884900000E-07	3.8306300000E-07	1.8491500000E-07	1.8491500000E-07
	1	1.1665440000E-06	8.8610300000E-07	4.7914600000E-07	2.3141700000E-07	2.3141700000E-07
	0.2	2.9650200000E-07	2.2501400000E-07	1.2147100000E-07	5.8586000000E-08	5.8586000000E-08
	0.4	6.1224700000E-07	4.6468100000E-07	2.5090400000E-07	1.2103200000E-07	1.2103200000E-07
0.4	0.6	9.2689100000E-07	7.0362200000E-07	3.8004700000E-07	1.8338100000E-07	1.8338100000E-07
	0.8	1.2398730000E-06	9.4146100000E-07	5.0875300000E-07	2.4558200000E-07	2.4558200000E-07
	1	1.5506360000E-06	1.1778340000E-06	6.3687500000E-07	3.0758800000E-07	3.0758800000E-07
	0.2	3.6446500000E-07	2.7658900000E-07	1.4931100000E-07	7.2012000000E-08	7.2012000000E-08
	0.4	7.5917400000E-07	5.7618900000E-07	3.1110700000E-07	1.5007100000E-07	1.5007100000E-07
0.5	0.6	1.1525180000E-06	8.7488900000E-07	4.7254400000E-07	2.2800900000E-07	2.2800900000E-07
	0.8	1.5437950000E-06	1.1722200000E-06	6.3343900000E-07	3.0576500000E-07	3.0576500000E-07
	1	1.9323090000E-06	1.4677220000E-06	7.9360600000E-07	3.8327700000E-07	3.8327700000E-07

Example 4.2. Consider the fractional-order KK equation,

$$D_{\eta}^{\wp}\psi(\xi,\eta) - 15\psi\psi_{\xi\xi\xi} - 15\varrho\psi_{\xi}\psi_{\xi\xi} + 45\psi^2\psi_{\xi} + \psi_{\xi\xi\xi\xi\xi} = 0, \tag{4.6}$$

with the initial condition,

$$\psi(\xi,0) = \frac{4}{3}c - \frac{4}{p}\operatorname{csch}^2\left(\sqrt{c}\xi\right). \tag{4.7}$$

Using the Yang transform to (4.6), we get

$$Y[\psi(\xi,\eta)] = \omega\psi(\xi,0) + \frac{1}{(1+\wp(\omega-1))}Y\left[-15\psi\psi_{\xi\xi\xi} - 15\varrho\psi_{\xi}\psi_{\xi\xi} + 45\psi^2\psi_{\xi} + \psi_{\xi\xi\xi\xi\xi}\right]. \tag{4.8}$$

Applying inverse Yang transform of (4.8), we have

$$\psi(\xi,\eta) = Y^{-1}\left[\omega\psi(\xi,0)\right] + Y^{-1}\left[\frac{1}{(1+\wp(\omega-1))}Y\left(-15\psi\psi_{\xi\xi\xi} - 15\varrho\psi_{\xi}\psi_{\xi\xi} + 45\psi^2\psi_{\xi} + \psi_{\xi\xi\xi\xi\xi}\right)\right]. \tag{4.9}$$

Now, by using the suggested semi-analytical method, we have

$$\begin{aligned}
\psi_0(\xi,\eta) &= \frac{4}{3}c - \frac{4}{\varrho}\operatorname{csch}^2\left(\sqrt{c}\xi\right),\\
\psi_1(\xi,\eta) &= Y^{-1}\left[\frac{1}{(1+\wp(\omega-1))}Y\left(-15\psi_{(0)}\psi_{(0)\xi\xi\xi} - 15\varrho\psi_{(0)\xi}\psi_{(0)\xi\xi} + 45\psi_{(0)}^2\psi_{(0)\xi} + \psi_{(0)\xi\xi\xi\xi\xi}\right)\right].\\
\psi_1(\xi,\eta) &= (63\varrho^2 + 360 - 420\varrho + 4\varrho(16\varrho - 15)\cosh\left(2\sqrt{c}\xi\right)\\
&\quad + \varrho^2\cosh\left(4\sqrt{cx}\right)\operatorname{sech}^6\left(\sqrt{c}\xi\right)\tanh\left(\sqrt{c}\xi\right)\frac{16c^{7/2}}{\varrho^3}(1+\wp\eta-\wp),\\
\psi_2(\xi,\eta) &= Y^{-1}\left[\frac{1}{(1+\wp(\omega-1))}Y\left(-15\psi_{(1)}\psi_{(1)\xi\xi\xi} - 15\varrho\psi_{(1)\xi}\psi_{(1)\xi\xi} + 45\psi_{(1)}^2\psi_{(1)\xi} + \psi_{(1)\xi\xi\xi\xi\xi}\right)\right].\\
\psi_2(\xi,\eta) &= \Big\{\Big(-306084\varrho^4 - 3110400 + 14515200\varrho - 26369280\varrho^3 - 6(2217600\varrho - 432000\\
&\quad +2656400\varrho^3 - 4451160\varrho^2 + 9181\varrho^4\Big)\cosh\left(2\sqrt{c}\xi\right) + 48\varrho\Big(41590\varrho^2 + 14400 - 60780\varrho\\
&\quad +4789\varrho^3\Big)\cosh\left(4\sqrt{c}\xi\right) - 59040\varrho^3\cosh\left(6\sqrt{c}\xi\right) + 79920\varrho^2\cosh\left(6\sqrt{c}\xi\right)\\
&\quad - 20883\varrho^4\cosh\left(6\sqrt{c}\xi\right) - 240\varrho^3\cosh\left(8\sqrt{c}\xi\right) + \varrho^4\cosh\left(10\sqrt{c}\xi\right) + 244\varrho^4\cosh\left(8\sqrt{c}\xi\right)\Big\}\\
&\quad \times\frac{8c^2\operatorname{sech}^{12}\sqrt{c}\xi}{\varrho^5}\left((1-\wp)2\wp\eta + (1-\wp)^2 + \frac{\wp^2\eta^2}{2}\right),\\
&\quad \vdots\\
\psi_n(\xi,\eta) &= Y^{-1}\left[\frac{1}{(1+\wp(\omega-1))}Y\left(-15\psi_{(n)}\psi_{(n)\xi\xi\xi} - 15\varrho\psi_{(n)\xi}\psi_{(n)\xi\xi} + 45\psi_{(n)}^2\psi_{(n)\xi} + \psi_{(n)\xi\xi\xi\xi\xi}\right)\right].
\end{aligned}$$

The series form result is

$$\psi(\xi,\eta) = \psi_0(\xi,\eta) + \psi_1(\xi,\eta) + \psi_2(\xi,\eta) + \psi_3(\xi,\eta) + \cdots + \psi_n(\xi,\eta). \tag{4.10}$$

Therefore, we have

$$\psi(\xi,\eta) = \frac{4}{3}c - \frac{4}{\varrho}\text{csch}^2\left(\sqrt{c\xi}\right) + (63\varrho^2 + 360 - 420\varrho + 4\varrho(16\varrho - 15)\cosh\left(2\sqrt{c\xi}\right)$$
$$+ \varrho^2\cosh\left(4\sqrt{cx}\right)\text{sech}^6\left(\sqrt{c\xi}\right)\tanh\left(\sqrt{c\xi}\right)\frac{16c^{7/2}}{\varrho^3}(1 + \wp\eta - \wp)$$
$$+ \Big\{14515200\varrho - 3110400 - 306084\varrho^4 - 26369280\varrho^3$$
$$- 6(2656400\varrho^3 + 2217600\varrho - 4451160\varrho^2 - 432000 + 9181\varrho^4)\cosh\left(2\sqrt{c\xi}\right)$$
$$+ 48\varrho(41590\varrho^2 + 14400 + 4789\varrho^3 - 60780\varrho)\cosh\left(4\sqrt{c\xi}\right) + 79920\varrho^2\cosh\left(6\sqrt{c\xi}\right)$$
$$- 59040\varrho^3\cosh\left(6\sqrt{c\xi}\right) - 20883\varrho^4\cosh\left(6\sqrt{c\xi}\right) - 240\varrho^3\cosh\left(8\sqrt{c\xi}\right)$$
$$+ \varrho^4\cosh\left(10\sqrt{c\xi}\right) + 244\varrho^4\cosh\left(8\sqrt{c\xi}\right)\Big\}\frac{8c^2\text{sech}^{12}\sqrt{c\xi}}{\varrho^5}\left((1-\wp)2\wp\eta + (1-\wp)^2 + \frac{\wp^2\eta^2}{2}\right)$$
$$+ \cdots.$$

For $\wp = 1$, the exact results of (4.6) is given by

$$\psi(\xi,\eta) = \frac{4}{3}c - \frac{4}{\varrho}c\text{sech}^2\left(\sqrt{c}\left(\xi + 8(3c^2 - 5\varrho c)\eta\right)\right).$$

The suggested technique provides approximate analytical answers with a number of free parameters. The analytical results are incredibly helpful for interpreting the internal components of natural phenomena. Depending on the physical conditions, explicit answers reflected various approximations. Figure 3 compares the result produced using the proposed technique and the exact and analytical answer for the fractional-order KK equation. Figure 4 demonstrates that distinct fractional orders of $\wp$ with respect to ξ and η comparison reveal that they are in close proximity.

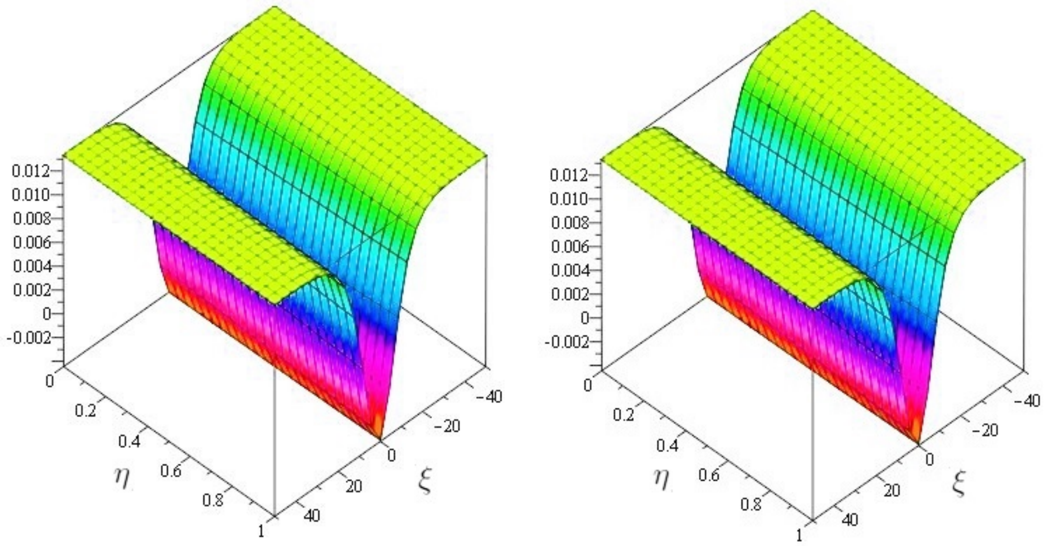


Figure 3. The exact and analytic solution of of Example 4.2.

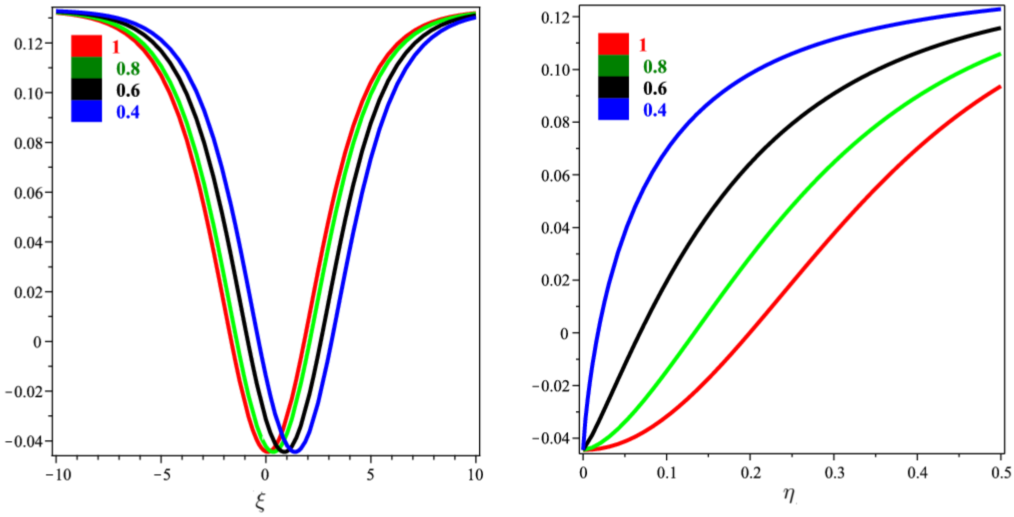


Figure 4. The fractional-order $\wp$ of Example 4.2 with respect to ξ and η.

Example 4.3. Consider the fractional-order KK equation,

$$D_\eta^\alpha \psi(\xi,\eta) = 5\psi\psi_{\xi\xi\xi} + \frac{25}{2}\psi_\xi\psi_{\xi\xi} + 5\psi^2\psi_\xi + \psi_{\xi\xi\xi\xi\xi} \tag{4.11}$$

with initial condition

$$\psi(\xi,0) = -2k^2 + \frac{24k^2}{1+e^{k\xi}}c - \frac{24k^2}{1+ek\xi}. \tag{4.12}$$

Using the Yang transform to (4.11), we get

$$Y[\psi(\xi,\eta)] = \omega\psi(\xi,0) + \frac{1}{(1+\wp(\omega-1))}Y\left[5\psi\psi_{\xi\xi\xi} + \frac{25}{2}\psi_\xi\psi_{\xi\xi} + 5\psi^2\psi_\xi + \psi_{\xi\xi\xi\xi\xi}\right]. \tag{4.13}$$

Applying inverse Yang transformation of (4.13), we have

$$\psi(\xi,\eta) = Y^{-1}\left[\omega\psi(\xi,0)\right] + Y^{-1}\left[\frac{1}{(1+\wp(\omega-1))}Y\left(5\psi\psi_{\xi\xi\xi} + \frac{25}{2}\psi_\xi\psi_{\xi\xi} + 5\psi^2\psi_\xi + \psi_{\xi\xi\xi\xi\xi}\right)\right]. \tag{4.14}$$

Now, by using the suggested semi-analytical method, we have

$$\psi_0(\xi,\eta) = -2k^2 + \frac{24k^2}{1+e^{k\xi}}c - \frac{24k^2}{1+ek\xi},$$

$$\psi_1(\xi,\eta) = Y^{-1}\left[\frac{1}{(1+\wp(\omega-1))}Y\left(5\psi_{(0)}\psi_{(0)\xi\xi\xi} + \frac{25}{2}\psi_{(0)\xi}\psi_{(0)\xi\xi} + 5\psi_{(0)}^2\psi_{(0)\xi} + \psi_{(0)\xi\xi\xi\xi\xi}\right)\right].$$

$$\psi_1(\xi,\eta) = \left(\frac{264e^{k\xi}(-1+e^{k\xi})k^7}{(1+e^{k\xi})^3}\right)(1+\wp\eta-\wp),$$

$$\psi_2(\xi,\eta) = Y^{-1}\left[\frac{1}{(1+\wp(\omega-1))}Y\left(5\psi_{(1)}\psi_{(1)\xi\xi\xi} + \frac{25}{2}\psi_{(1)\xi}\psi_{(1)\xi\xi} + 5\psi_{(1)}^2\psi_{(1)\xi} + \psi_{(1)\xi\xi\xi\xi\xi}\right)\right].$$

$$\psi_2(\xi,\eta) = \frac{2904e^{k\xi}(1-4e^{k\xi}+e^{2k\xi})k^{12}}{(1+e^{k\xi})^4}\left((1-\wp)2\wp\eta + (1-\wp)^2 + \frac{\wp^2\eta^2}{2}\right),$$

$$\psi_3(\xi,\eta)=Y^{-1}\left[\frac{1}{(1+\wp(\omega-1))}Y\left(5\psi_{(2)}\psi_{(2)\xi\xi\xi}+\frac{25}{2}\psi_{(2)\xi}\psi_{(2)\xi\xi}+5\psi_{(2)}^2\psi_{(2)\xi}+\psi_{(2)\xi\xi\xi\xi\xi}\right)\right].$$

$$\begin{aligned}\psi_3(\xi,\eta)=&\,2904e^{k\xi}\left(-1+e^{k\xi}\right)k^{17}\eta^{3\wp}\Bigg(\Big(11+54e^{k\xi}-4923e^{2k\xi}+10228e^{3k\xi}\\&-4923e^{4k\xi}+54e^{5k\xi}+11e^{6k\xi}\Big)\Gamma(1+\wp)^2-60e^{k\xi}\Big(1-38e^{k\xi}+90e^{2k\xi}\\&-38e^{3k\xi}+e^{4k\xi}\Big)\left(3\wp\left(-2\wp+1+\wp^2\right)\eta+\frac{\wp^3\eta^3}{6}-\frac{3\wp^2(\wp-1)\eta^2}{2}+3\wp^2-3\wp+1-\wp^3\right)\Bigg)\\&\div\left(1+e^{k\xi}\right)^9,\\&\vdots\end{aligned}$$

$$\psi_n(\xi,\eta)=Y^{-1}\left[\frac{1}{(1+\wp(\omega-1))}Y\left(5\psi_{(n)}\psi_{(n)\xi\xi\xi}+\frac{25}{2}\psi_{(n)\xi}\psi_{(n)\xi\xi}+5\psi_{(n)}^2\psi_{(n)\xi}+\psi_{(n)\xi\xi\xi\xi\xi}\right)\right].$$

The series form result is

$$\psi(\xi,\eta)=\psi_0(\xi,\eta)+\psi_1(\xi,\eta)+\psi_2(\xi,\eta)+\psi_3(\xi,\eta)+\cdots+\psi_n(\xi,\eta). \tag{4.15}$$

Therefore, we have

$$\begin{aligned}\psi(\xi,\eta)=&-2k^2+\frac{24k^2}{1+e^{k\xi}}c-\frac{24k^2}{1+ek\xi}+(1+\wp\eta-\wp)\left(\frac{264e^{k\xi}(-1+e^{k\xi})k^7}{(1+e^{k\xi})^3}\right)\\&+\frac{2904e^{k\xi}\left(1-4e^{k\xi}+e^{2k\xi}\right)k^{12}}{(1+e^{k\xi})^4}\times\left((1-\wp)2\wp\eta+(1-\wp)^2+\frac{\wp^2\eta^2}{2}\right)\\&+2904e^{k\xi}\left(-1+e^{k\xi}\right)k^{17}\eta^{3\wp}\Big(11+54e^{k\xi}-4923e^{2k\xi}\\&+10228e^{3k\xi}-4923e^{4k\xi}+54e^{5k\xi}+11e^{6k\xi}\Big)\Gamma(1+\wp)^2-60e^{k\xi}\Big(1-38e^{k\xi}+90e^{2k\xi}\\&-38e^{3k\xi}+e^{4k\xi}\Big)\left(3\wp(-2\wp+1+\wp^2)\eta+\frac{\wp^3\eta^3}{6}-\frac{3\wp^2(\wp-1)\eta^2}{2}+3\wp^2-3\wp+1-\wp^3\right)\\&\div\left(1+e^{k\xi}\right)^9+\cdots.\end{aligned}$$

For $\wp=1$, the exact results of (4.11) is given by

$$\psi(\xi,\eta)=-2k^2+\frac{24k^2}{1+e^{k\xi+11k^5\eta}}-\frac{24k^2}{(1+e^{k\xi+11k^5\eta})^2}.$$

The suggested technique provides approximate analytical answers with a number of free parameters. The analytical results are incredibly helpful for interpreting the internal components of natural phenomena. Depending on the physical conditions, explicit answers reflected various approximations. Figure 5 compares the result produced using the proposed technique and the exact and analytical answer for the fractional-order KK equation. Figure 6 demonstrates that distinct fractional orders of $\wp$ with respect to ξ and η comparison reveal that they are in close proximity. In Table 2, the various fractional-order of $\wp$ of Example 4.3.

Table 2. The various fractional-order of $\wp$ of Example 4.3.

η	ξ	$\wp = 0.40$	$\wp = 0.60$	$\wp = 0.80$	$\wp = 1(NITM)$
	0.2	6.4600000000E-10	4.8300000000E-10	2.7900000000E-10	6.7000000000E-11
	0.4	6.4300000000E-10	4.8100000000E-10	2.7700000000E-10	6.6000000000E-11
0.1	0.6	6.4400000000E-10	4.8200000000E-10	2.8000000000E-10	6.9000000000E-11
	0.8	6.4500000000E-10	4.8400000000E-10	2.8200000000E-10	7.1000000000E-11
	1	6.4100000000E-10	4.8000000000E-10	2.7900000000E-10	6.9000000000E-11
	0.2	6.7100000000E-10	5.4000000000E-10	3.5400000000E-10	1.4300000000E-10
	0.4	6.6700000000E-10	5.3700000000E-10	3.5100000000E-10	1.4000000000E-10
0.2	0.6	6.5800000000E-10	5.2800000000E-10	3.4300000000E-10	1.3300000000E-10
	0.8	6.6300000000E-10	5.3300000000E-10	3.4900000000E-10	1.4000000000E-10
	1	6.5700000000E-10	5.2800000000E-10	3.4400000000E-10	1.3500000000E-10
	0.2	6.8100000000E-10	5.7600000000E-10	4.1200000000E-10	2.1000000000E-10
	0.4	6.8400000000E-10	5.8000000000E-10	4.1700000000E-10	2.1500000000E-10
0.3	0.6	6.7400000000E-10	5.7100000000E-10	4.0700000000E-10	2.0700000000E-10
	0.8	6.7700000000E-10	5.7400000000E-10	4.1100000000E-10	2.1100000000E-10
	1	6.6500000000E-10	5.6200000000E-10	4.0000000000E-10	2.0000000000E-10
	0.2	6.9600000000E-10	6.1600000000E-10	4.7400000000E-10	2.8700000000E-10
	0.4	6.8900000000E-10	6.0900000000E-10	4.6800000000E-10	2.8100000000E-10
0.4	0.6	6.8600000000E-10	6.0600000000E-10	4.6500000000E-10	2.7900000000E-10
	0.8	6.8900000000E-10	6.0900000000E-10	4.6900000000E-10	2.8300000000E-10
	1	6.7700000000E-10	5.9700000000E-10	4.5700000000E-10	2.7200000000E-10
	0.2	6.9800000000E-10	6.3800000000E-10	5.2000000000E-10	3.5100000000E-10
	0.4	7.0100000000E-10	6.4200000000E-10	5.2400000000E-10	3.5600000000E-10
0.5	0.6	6.9700000000E-10	6.3800000000E-10	5.2100000000E-10	3.5300000000E-10
	0.8	6.9300000000E-10	6.3400000000E-10	5.1700000000E-10	3.4900000000E-10
	1	6.9000000000E-10	6.3100000000E-10	5.1500000000E-10	3.4700000000E-10

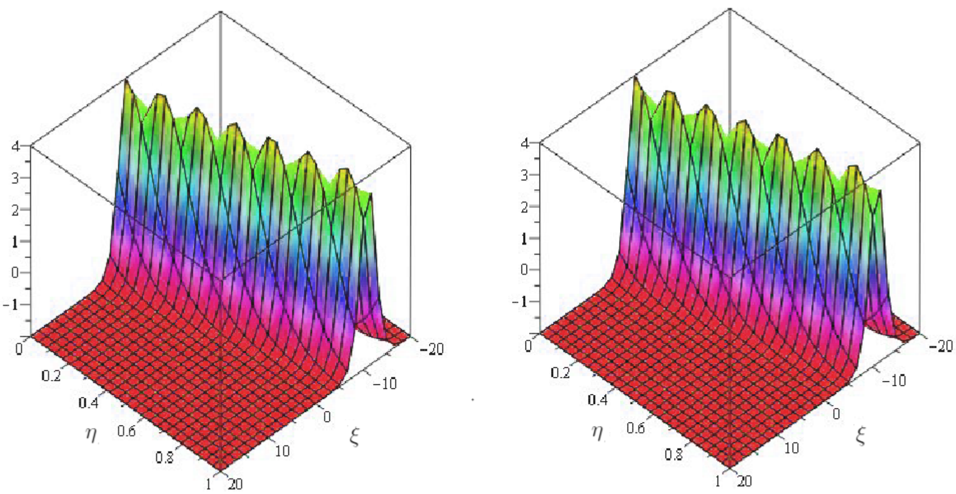


Figure 5. The exact and analytic solution of Example 4.3.

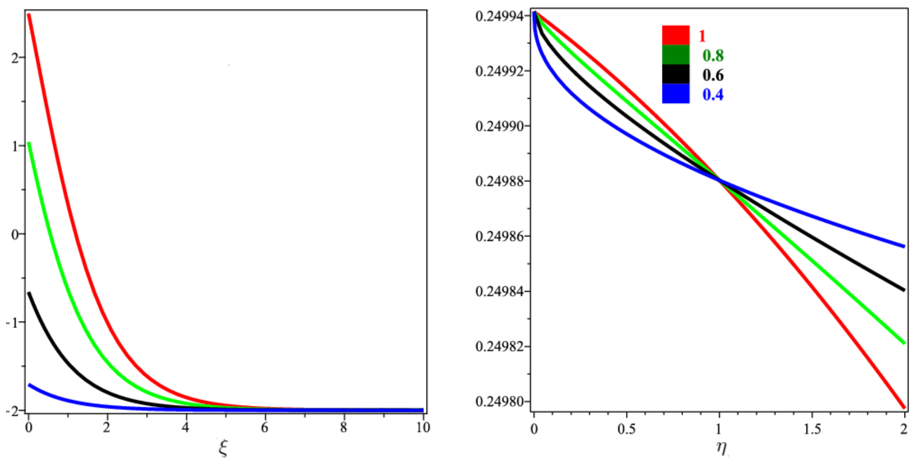


Figure 6. The fractional-order $\wp$ of Example 4.3 with respect to ξ and η.

5. Conclusions

In this article, we investigate the fractional-order solution of the Kaup-Kupershmidt equation by a new iterative transform method to show the accuracy of the suggested technique. The solutions we achieved by applying the suggested technique represent that the solution is in close contact with the exact result. We can conclude that the provided approach is eminently consistent and can be utilized to study a variety of fractional-order nonlinear mathematical models, allowing us to comprehend the behavior of highly nonlinear complex phenomena in relevant science and engineering domains. Moreover, the proposed technique provided the convergence series solutions with simple determinable components without perturbation, linearization or limiting assumption. The analytical and graphical results achieved by the proposed method were computationally very attractive and more accurate in finding the solutions to governing equations.

Conflict of interest

The authors declare no conflicts of interest.

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