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Research article

An MADM-based fuzzy parameterized framework for solar panels evaluation in a fuzzy hypersoft expert set environment

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Abstract: The selection of parameters plays a vital role in the multi-attribute decision-making process. In some situations, it is observed that the nature of parameters is ambiguous and a multi-decisive opinion is necessary for managing such parametric uncertainty. In the literature, there is no suitable model that can cope with such situations. This study was purposed to develop a novel context called the fuzzy parameterized fuzzy hypersoft expert set (FPFHSE-set), which is capable of managing the uncertain nature of parameters and the multi-decisive opinion of experts collectively in one model. In this way, the proposed model may be described as the generalization of the existing model fuzzy parameterized fuzzy soft expert set (FPFSE-set). Theoretic, axiomatic and algorithmic approaches have been employed for the characterization of the basic notions of the FPFHSE-set. In order to handle multi-attribute decision-making, two algorithms are proposed and then validated by applying them to some real-world scenarios in the FPFHSE-set environment. The merits and superiority of the new algorithms are presented by comparing them with some existing fuzzy decision-making models. According to the proposed FPFHSE-set-based decision-making approaches, the experts have more freedom in specifying their preferences and thoughts according to their expertise, and they can process new types of data. Therefore, this paper presents a state-of-the-art improvement that provides a holistic view to understand and handle the multi-attribute decision-making issues focused on the objective of classifying alternatives according to multiple attributes by multiple experts.

Keywords: fuzzy set; soft set; hypersoft set; hypersoft expert set; fuzzy parameterized fuzzy hypersoft expert set; decision making
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1. Introduction

Zadeh [1] presented the idea of a fuzzy set (F-set) to represent systems with unclear data. By using these set models, several issues that could not be solved until 1965 were resolved. The soft set (S-set), which enables the parametric categorization of options, was first introduced by Molodtsov [2] in 1999. Different angles of the S-sets were explored, and certain S-set hybrids [3–5] were created. Fuzzy soft sets (FS-sets), an extension of F-sets and S-sets, were established by Maji et al. [6] to address the shortcomings of F-sets in terms of the supply of parameterization tools. The FS-set meets the requirements of the S-set in addition to validating the F-set. Instead of using a power set, it essentially employs the collection of F-subsets as a set of single-argument approximation functions throughout the universe of discourse. Numerous writers investigated some of the extended types [7, 8] and the matrices [9–11] of FS-sets. Huang et al. [12] discussed the noise-tolerant fuzzy β covering by integrating multi-granulation rough set and feature subset selection. Later on, Huang & Li [13] investigated the discernibility measure of a fuzzy β covering and discussed its application. They [14] also explored the concept of noise-tolerant discrimination indexes for fuzzy γ covering and feature subset selection. The S-set models place emphasis on a single expert's viewpoint inside a single model. But there are many circumstances where it is necessary to have distinct viewpoints on multiple models. Alkhazaleh and Salleh [15] developed the idea of the soft expert set (SE-set) to address the shortcomings of the S-set with reference to the opinions of various experts in various models. Convexity-cum-concavity on SE-sets was theorized, and its specific qualities were described by Ihsan et al. [16] By introducing the usage of the fuzzy soft expert set (FSE-set) in decision-making problems (DMPs), Alkhazaleh and Salleh [17] expanded on their previous work. Ihsan et al. [18] once again described the convexity on the FSE-set and its specific characteristics. By substituting a multi-argument approximate function for a single argument approximation function in [19], S-sets were generalized to become a hypersoft set (HS-set) in 2018. Saeed et al. [20] outlined the fundamentals of HS-set and clarified its features with the help of examples. Composite mappings on a fuzzy hypersoft set (FHS-set) were approached analytically and theoretically by Ahsan et al. [21] who also examined the set's specific characteristics. Additionally, they used examples to validate some of the results. By introducing the concept of bipolarity into the HS-sets, Musa and Asaad [22] established the bipolar HS-sets and researched its properties. The hybrids of the hypersoft graph were presented by Saeed et al. [23], who also studied its theoretical operations and provided generalized findings. For intuitionistic fuzzy hypersoft sets (IFHS-sets), Zulqarnain et al. [24] deduced various resilient aggregation operations and used them to solve the DMPs. Neutrosophic hypersoft mappings (NHS-mappings) and NHS semi-opens were proposed by Saeed et al. [25] and Ajay et al. [26] respectively. The writers of the article [27] looked at a few NHS-graph operations and products. Rahman et al. [28] introduced a unique method to NHS-graphs and analyzed some of its characteristics. Rahman et al. [29] described the different FHS-set, IFHS-set, NHS-set and HS-set structures. Rahman et al. [30] provided the concept of convexity on the HS-set and demonstrated some of its characteristics. The authors of the works [31, 32] created the rough forms of HS-sets and provided an application for the optimal chemical material selection in decision-making (DM). The authors of [33, 34], and other researchers looked into the complicated types and bijective types of the HS-set and used them to apply to the DM. In the environment of attributed sets being further divided into disjoint attribute valued sets, Ihsan et al. [35–38] extended the work of HS-sets to hypersoft expert sets (HSE-sets), fuzzy HSE-sets

(FHSE-sets), single-valued neutrosophic HSE-sets, and bijective HSE-sets to know the opinions of various experts in various models. Kamacı and Saqlain [39] researched the FHSE-set, which displays all expert views in an FHS-set model without any operations, and the n-ary FHSE-set, which displays all expert opinions in an n-ary FHS-set model without any operations.

The information below explains the research gap and the motivation for the selection of the suggested structure. Çağman et al. [40] conceived the fuzzy parameterized soft set (FPS-set) based on the notion of extension for Molodtsov's S-set and as a result gave relevance degrees to the parameters in the S-set structure. They provided an application for the optimum product selection and a solution to the DMPs. The fuzzy parameterized fuzzy soft set (FPFS-set) was introduced by Tella et al. [41] and used to address the multi-attribute decision-making (MADM) issue, which included multi-expert evaluation. A brand-new DM model built on the FPFS-set and its application was introduced by Zhu and Zhan [42]. It is clear that the S-set models only consider one expert's viewpoint. However, there are several instances in real life where we require the advice of other specialists. The SE-set was created to solve this problem without the need for any additional operations. This set was expanded to include the FSE-set in the same year. The hybridized FSE-set structures with use in DM models were researched by Bashir and Salleh [43]. Additionally, they contrasted their findings and explored the use of the generalized form of the method. The fuzzy parameterized fuzzy soft expert set (FPFSE-set), developed by Hazaymeh et al. [44], was applied in DM. The additional classification of attributes into their corresponding attribute-valued disjoint sets is possible in some circumstances. To deal with these circumstances involving several conclusive opinions in a multi-argument S-set environment, new structures must be devised. As a result, the HS-set and FHS-set were created. By converting the set of attributes into multi-disjoint attribute valued sets in 2021, Rahman et al. [45] upgraded the concept of the FPS-set to the fuzzy parameterized hypersoft set (FPHS-set) and described its implementations in DM. The authors expanded on their work and presented the idea of NHS-sets in [46]. Ihsan et al. expanded the HS-set to form the HSE-set and provided a real-world application. The fuzzy parameterized kind of FHSE-set is a gap in the literature that has to be addressed in light of some real-world instances of HSE-sets with fuzzy graded parameters. The fuzzy parameterized fuzzy hypersoft expert set (FPFHSE-set) is created as a result and has certain characteristics. Two DM methods are suggested and implemented in MADM by utilizing the aggregate operations of FPFHSE-sets. The following is a list of some of the planned study's major contributions:

(1) The FPFHSE-set, which is the fuzzy parameterized type of the FHSE-set, is conceptualized.
(2) The introduction of several axiomatic characteristics, set-theoretic procedures and laws for the FPFHSE-sets is backed by numerical examples.
(3) A DM-based daily-life scenario is used to test two DM algorithms that have been newly developed.
(4) To assess the benefits of the suggested DM models, several of the current models are compared with them.

The pattern for the rest of the paper is presented as follows. Section 2 reviews some fundamental concepts for the S-sets, FS-sets and HSE-sets. Section 3 is devoted to the FPFHSE-sets and their operations such as the complement, union, intersection and products. Section 4 introduces a DM algorithm based on the FPFHSE-sets and presents its application in the MADM in an FPFHSE-set environment. Section 5 describes the weighted kind of FPFHSE-set and gives a weighted FPFHSE-set based algorithm with its implementation. Section 6 presents the comparative analysis and discusses the results. To encourage readers to request more extensions, Section 7 describes the paper's goals and future objectives.

2. Elementary definitions

This section of the paper reviews the elementary notions on the S-sets, FS-sets, HSE-sets and their operations. In this paper, $\mathcal{I}$, $\mathcal{Q}$ and $\mathcal{U} = \{0 = disagree, 1 = agree\}$ denotes the sets of experts (or specialists), parameters and opinions (or conclusions), respectively. Also, $\mathcal{P} = \mathcal{Q} \times \mathcal{I} \times \mathcal{U}$ and $\mathcal{S} \subseteq \mathcal{P}$. The notation $\hat{\triangle}$ represents the universal set where hits power set is $P(\hat{\triangle})$ and $\mathbb{I}$ =[0, 1]. To handle the uncertain nature of the information, Zadeh [1] proposed the F-set in 1965 as an extension of the classical set (also known as the crisp set). The membership function used by this set maps the set of elements to $\mathbb{I}$.

Definition 2.1. [1] A set $\aleph$ written as $\aleph = \{(\mu_{\aleph}(\delta)/\delta) | \ \delta \in \hat{\triangle}\}$ is called an F-set, where $\mu_{\aleph} : \hat{\triangle} \to \mathbb{I}$ and $\mu_{\aleph}(\delta)$ represents the membership value of $\delta \in \hat{\triangle}$ into $\aleph$.

Definition 2.2. [1] Let $\aleph$ and $\Im$ be two F-sets; then, the following operations are described:

(1) $\aleph \cup \Im = \{(max\{\mu_{\aleph}(\delta), \mu_{\Im}(\delta)\}/\delta) | \ \delta \in \hat{\triangle}\}$,
(2) $\aleph \cap \Im = \{(min\{\mu_{\aleph}(\delta), \mu_{\Im}(\delta)\}/\delta) | \ \delta \in \hat{\triangle}\}$,
(3) $\aleph^c = \{(1 - \mu_{\aleph}(\delta)/\delta) | \ \delta \in \hat{\triangle}\}$.

Note 1. For the union operation $\cup$ and the intersection operation $\cap$ on the F-sets, they are said to have commutative, associative, distributive, and De-Morgan laws. The F-set contains a kind of lack in terms of the parameterization tool. To remedy this deficiency, Molodtsov [2] introduced the S-set as a mathematical tool to classify the uncertain or vagueness data.

Definition 2.3. [2] A pair $(\Lambda, \mathcal{A})$ is named as an S-set on $\hat{\triangle}$ if $\Lambda : \mathcal{A} \to P(\hat{\triangle})$, where $\mathcal{A} \subseteq \mathcal{Q}$.

Definition 2.4. [6] A pair $(\Gamma, \mathcal{A})$ is named as an FS-set on $\hat{\triangle}$ if $\Gamma : \mathcal{A} \to FP(\hat{\triangle})$, where $\mathcal{A} \subseteq \mathcal{Q}$ and $FP(\hat{\triangle})$ is a collection of fuzzy subsets of $\hat{\triangle}$.

The characteristics must be organized into groups of sub-attributive qualities in some actual circumstances. The HS-set, which addresses the insufficiency of the S-set and treats the conditions with a multi-argument approximation function, was created by Smarandache [19] because the existing concept of the S-set is insufficient and incompatible with such circumstances.

Definition 2.5. [19] Suppose $\ell_1, \ell_2, \ell_3, ..., \ell_\alpha$ $(\alpha \geq 1)$ are α distinct attributes and the sets $\mathcal{L}_1, \mathcal{L}_2, \mathcal{L}_3, ..., \mathcal{L}_\alpha$ are corresponding attribute values with $\mathcal{L}_m \cap \mathcal{L}_n = \emptyset$ for $m \neq n$, given $m, n \in \{1, 2, 3, ..., \alpha\}$. The pair $(\Theta, \mathfrak{G})$ is named as an HS-set over $\hat{\triangle}$ if $\Theta : \mathfrak{G} \to P(\hat{\triangle})$, where $\mathfrak{G} = \mathcal{L}_1 \times \mathcal{L}_2 \times \mathcal{L}_3 \times ... \times \mathcal{L}_\alpha$. An HS-set $(\Theta, \mathfrak{G})$ is called an FHS-set if $P(\hat{\triangle})$ is replaced with $FP(\hat{\triangle})$.

The following definitions are recalled from Ihsan et al. [35]:

Definition 2.6. An HSE-set $(\Omega, \breve{S})$ is described by the mapping Ω as $\Omega : \breve{S} \to P(\hat{\triangle})$ where $\breve{S} \subseteq \breve{P} = \mathcal{Q} \times \mathcal{I} \times \mathcal{U}$ and $\mathcal{Q} = \mathcal{Q}_1 \times \mathcal{Q}_2 \times \mathcal{Q}_3 \times ... \times \mathcal{Q}_r$ such that $\mathcal{Q}_1, \mathcal{Q}_2, \mathcal{Q}_3, ..., \mathcal{Q}_r$ are different parameter sets corresponding to r different parameters $q_1, q_2, q_3, ..., q_r$.

Definition 2.7. The HSE-set $(\Omega_1, \breve{S}_1)$ is a subset of the HSE-set $(\Omega_2, \breve{S}_2)$, denoted by $(\Omega_1, \breve{S}_1) \subseteq (\Omega_2, \breve{S}_2)$ if $\breve{S}_1 \subseteq \breve{S}_2$ and $\forall \ \Upsilon \in \breve{S}_1, \ \Omega_1(\Upsilon) \subseteq \Omega_2(\Upsilon)$.

Definition 2.8. The relative complement of the HSE-set $(\Omega, \breve{S})$, denoted by $(\Omega, \breve{S})^r = (\Omega^r, \breve{S})$, is defined by $\Omega^r : \breve{S} \rightarrow P(\hat{\Delta})$ such that $\Omega^r(\Upsilon) = \hat{\Delta} - \Omega(\Upsilon) = (\Omega(\Upsilon))^c$ for all $\Upsilon \in \breve{S}$.

Definition 2.9. The restricted union of HSE-sets $(\Omega_1, \breve{S}_1)$ and $(\Omega_2, \breve{S}_2)$ is again HSE-set $(\Omega_3, \breve{S}_3) = (\Omega_1, \breve{S}_1) \Cup (\Omega_2, \breve{S}_2)$, where $\breve{S}_3 = \breve{S}_1 \cap \breve{S}_2$ and $\Omega_3(\Upsilon) = \Omega_1(\Upsilon) \cup \Omega_2(\Upsilon)$ for all $\Upsilon \in \breve{S}_3$.

Definition 2.10. The union (so called extended) of HSE-sets $(\Omega_1, \breve{S}_1)$ and $(\Omega_2, \breve{S}_2)$ is again HSE-set $(\Omega_3, \breve{S}_3) = (\Omega_1, \breve{S}_1) \cup (\Omega_2, \breve{S}_2)$, where $\breve{S}_3 = \breve{S}_1 \cup \breve{S}_2$, $\forall \Upsilon \in \breve{S}_3$ and

$$\Omega_3(\Upsilon) = \begin{cases} \Omega_1(\Upsilon), & \Upsilon \in \breve{S}_1 - \breve{S}_2; \\ \Omega_2(\Upsilon), & \Upsilon \in \breve{S}_2 - \breve{S}_1; \\ \Omega_1(\Upsilon) \cup \Omega_2(\Upsilon), & \Upsilon \in \breve{S}_1 \cap \breve{S}_2. \end{cases}$$

Definition 2.11. The restricted intersection of HSE-sets $(\Omega_1, \breve{S}_1)$ and $(\Omega_2, \breve{S}_2)$ is again HSE-set $(\Omega_3, \breve{S}_3) = (\Omega_1, \breve{S}_1) \Cap (\Omega_2, \breve{S}_2)$, where $\breve{S}_3 = \breve{S}_1 \cap \breve{S}_2$ and $\Omega_3(\Upsilon) = \Omega_1(\Upsilon) \cap \Omega_2(\Upsilon)$ for all $\Upsilon \in \breve{S}_3$.

Definition 2.12. The extended intersection of HSE-sets $(\Omega_1, \breve{S}_1)$ and $(\Omega_2, \breve{S}_2)$ on $\hat{\Delta}$ is an HSE-set $(\Omega_3, \breve{S}_3) = (\Omega_1, \breve{S}_1) \cap_e (\Omega_2, \breve{S}_2)$, where $\breve{S}_3 = \breve{S}_1 \cup \breve{S}_2$, $\forall \Upsilon \in \breve{S}_3$ and

$$\Omega_3(\Upsilon) = \begin{cases} \Omega_1(\Upsilon), & \Upsilon \in \breve{S}_1 - \breve{S}_2; \\ \Omega_2(\Upsilon), & \Upsilon \in \breve{S}_2 - \breve{S}_1; \\ \Omega_1(\Upsilon) \cap \Omega_2(\Upsilon), & \Upsilon \in \breve{S}_1 \cap \breve{S}_2. \end{cases}$$

Note 2. For the restricted union operation $\Cup$, the extended union $\cup$, the restricted intersection $\Cap$ and the extended intersection operation $\cap_e$ on the HSE-sets, they are said to have commutative, associative, distributive, and De-Morgan laws.

3. Notions of an FPFHSE-set

In this section, a completely new structure of an FPFHSE-set is established by extending the existing concepts of an FPFS-set, FPFSE-set, HSE-set and FHSE-set.

Definition 3.1. Let $\mathfrak{A} = \{\mathfrak{A}_1, \mathfrak{A}_2, \mathfrak{A}_3, ..., \mathfrak{A}_n\}$ be a non-overlapping sub-parametric valued set for different parameters $a_i, i = 1, 2, ..., n$ respectively. An FPFHSE-set $(\mho, \mathbb{S})$ (or $\mho_{\mathbb{S}}$) over $\hat{\Delta}$ is defined as $(\mho, \mathbb{S}) = \{((\zeta(q)/q, x, u), \mho(\imath)) : (\zeta(q)/q, x, u) = \imath \in \mathbb{S}$ and $\mho(\imath) \in FP(\hat{\Delta})\}$ where $Q = \mathfrak{A}_1 \times \mathfrak{A}_2 \times \mathfrak{A}_3 \times ... \times \mathfrak{A}_n$ and $\mathcal{P} = Q \times \mathcal{I} \times \mathcal{U}$ such that $(q, x, u) \in \mathcal{P}$, $\mathcal{H} = \{(\zeta(q)/q, x, u) : (q, x, u) \in \mathcal{P},\ \zeta(q) \in \mathbb{I}\}$ and $\mathbb{S} \subseteq \mathcal{H}$ and $\mho : \mathbb{S} \rightarrow FP(\hat{\Delta})$ is called an approximate function.

Example 3.1. Imagine that a chain of colleges is looking for a construction business to renovate the campus to keep up with globalization and requires the advice of certain professionals. Let $\hat{\Delta} = \{\delta_1, \delta_2, \delta_3, \delta_4\}$ be a set consisting of construction companies and $\check{\mathcal{J}}_1 = \{p_{11}, p_{12}\}$, $\check{\mathcal{J}}_2 = \{p_{21}, p_{22}\}$ and $\check{\mathcal{J}}_3 = \{p_{31}, p_{32}\}$ be different parameter sets for the parameters p_1= quality characteristics, p_2= cheap and p_3= quality of design, respectively. Now $\check{\mathcal{J}} = \check{\mathcal{J}}_1 \times \check{\mathcal{J}}_2 \times \check{\mathcal{J}}_3$ such that

$$\check{\mathcal{J}} = \begin{Bmatrix} q_1 = (p_{11}, p_{21}, p_{31}),\ q_2 = (p_{11}, p_{21}, p_{32}),\ q_3 = (p_{11}, p_{22}, p_{31}),\ q_4 = (p_{11}, p_{22}, p_{32}) \\ q_5 = (p_{12}, p_{21}, p_{31}),\ q_6 = (p_{12}, p_{21}, p_{32}),\ q_7 = (p_{12}, p_{22}, p_{31}),\ q_8 = (p_{12}, p_{22}, p_{32}) \end{Bmatrix},$$

and assume that the fuzzy subset of $\check{\mathcal{J}}$ is

$$\mathcal{G} = \left\{0.2/q_1, 0.4/q_2, 0.5/q_3, 0.8/q_4, 0.7/q_5, 0.1/q_6, 0.3/q_7, 0.9/q_8\right\}.$$

Also, let $\mathcal{I} = \{x, y, z\}$ be a set of experts and $\mathcal{U} = \{0 = disagree,\ 1 = agree\}$. Now $\mathcal{H} = \mathcal{G} \times \mathcal{I} \times \mathcal{U}$, i.e.,

$$\mathcal{H} = \left\{\begin{matrix} (0.2/q_1, x, 0), (0.2/q_1, x, 1), (0.2/q_1, y, 0), (0.2/q_1, y, 1), (0.2/q_1, z, 0), (0.2/q_1, z, 1) \\ (0.4/q_2, x, 0), (0.4/q_2, x, 1), (0.4/q_2, y, 0), (0.4/q_2, y, 1), (0.4/q_2, z, 0), (0.4/q_2, z, 1) \\ (0.5/q_3, x, 0), (0.5/q_3, x, 1), (0.5/q_3, y, 0), (0.5/q_3, y, 1), (0.5/q_3, z, 0), (0.5/q_3, z, 1) \\ (0.8/q_4, x, 0), (0.8/q_4, x, 1), (0.8/q_4, y, 0), (0.8/q_4, y, 1), (0.8/q_4, z, 0), (0.8/q_4, z, 1) \\ (0.7/q_5, x, 0), (0.7/q_5, x, 1), (0.7/q_5, y, 0), (0.7/q_5, y, 1), (0.7/q_5, z, 0), (0.7/q_5, z, 1) \\ (0.1/q_6, x, 0), (0.1/q_6, x, 1), (0.1/q_6, y, 0), (0.1/q_6, y, 1), (0.1/q_6, z, 0), (0.1/q_6, z, 1) \\ (0.3/q_7, x, 0), (0.3/q_7, x, 1), (0.3/q_7, y, 0), (0.3/q_7, y, 1), (0.3/q_7, z, 0), (0.3/q_7, z, 1) \\ (0.9/q_8, x, 0), (0.9/q_8, x, 1), (0.9/q_8, y, 0), (0.9/q_8, y, 1), (0.9/q_8, z, 0), (0.9/q_8, z, 1) \end{matrix}\right\},$$

and

$$\mathbb{S} = \left\{\begin{matrix} (0.2/q_1, x, 0), (0.2/q_1, x, 1), (0.2/q_1, y, 0), (0.2/q_1, y, 1), (0.2/q_1, z, 0), (0.2/q_1, z, 1) \\ (0.4/q_2, x, 0), (0.4/q_2, x, 1), (0.4/q_2, y, 0), (0.4/q_2, y, 1), (0.4/q_2, z, 0), (0.4/q_2, z, 1) \\ (0.5/q_3, x, 0), (0.5/q_3, x, 1), (0.5/q_3, y, 0), (0.5/q_3, y, 1), (0.5/q_3, z, 0), (0.5/q_3, z, 1) \end{matrix}\right\}$$

is a subset of $\mathcal{H}$. The following survey depicts the choices of three experts:

$$\mho_1 = \mho(0.2/q_1, x, 1) = \left\{\frac{\delta_1}{0.2}, \frac{\delta_2}{0.7}, \frac{\delta_3}{0.5}, \frac{\delta_4}{0.1}\right\},\ \mho_2 = \mho(0.2/q_1, y, 1) = \left\{\frac{\delta_1}{0.4}, \frac{\delta_2}{0.8}, \frac{\delta_3}{0.4}, \frac{\delta_4}{0.2}\right\},$$

$$\mho_3 = \mho(0.2/q_1, z, 1) = \left\{\frac{\delta_1}{0.7}, \frac{\delta_2}{0.5}, \frac{\delta_3}{0.6}, \frac{\delta_4}{0.3}\right\},\ \mho_4 = \mho(0.4/q_2, x, 1) = \left\{\frac{\delta_1}{0.9}, \frac{\delta_2}{0.4}, \frac{\delta_3}{0.7}, \frac{\delta_4}{0.3}\right\},$$

$$\mho_5 = \mho(0.4/q_2, y, 1) = \left\{\frac{\delta_1}{0.4}, \frac{\delta_2}{0.8}, \frac{\delta_3}{0.3}, \frac{\delta_4}{0.2}\right\},\ \mho_6 = \mho(0.4/q_2, z, 1) = \left\{\frac{\delta_1}{0.5}, \frac{\delta_2}{0.3}, \frac{\delta_3}{0.6}, \frac{\delta_4}{0.8}\right\},$$

$$\mho_7 = \mho(0.5/q_3, x, 1) = \left\{\frac{\delta_1}{0.2}, \frac{\delta_2}{0.9}, \frac{\delta_3}{0.4}, \frac{\delta_4}{0.5}\right\},\ \mho_8 = \mho(0.5/q_3, y, 1) = \left\{\frac{\delta_1}{0.4}, \frac{\delta_2}{0.6}, \frac{\delta_3}{0.7}, \frac{\delta_4}{0.9}\right\},$$

$$\mho_9 = \mho(0.5/q_3, z, 1) = \left\{\frac{\delta_1}{0.7}, \frac{\delta_2}{0.3}, \frac{\delta_3}{0.5}, \frac{\delta_4}{0.2}\right\},\ \mho_{10} = \mho(0.2/q_1, x, 0) = \left\{\frac{\delta_1}{0.3}, \frac{\delta_2}{0.2}, \frac{\delta_3}{0.4}, \frac{\delta_4}{0.1}\right\},$$

$$\mho_{11} = \mho(0.2/q_1, y, 0) = \left\{\frac{\delta_1}{0.1}, \frac{\delta_2}{0.9}, \frac{\delta_3}{0.6}, \frac{\delta_4}{0.2}\right\},\ \mho_{12} = \mho(0.2/q_1, z, 0) = \left\{\frac{\delta_1}{0.2}, \frac{\delta_2}{0.1}, \frac{\delta_3}{0.3}, \frac{\delta_4}{0.5}\right\},$$

$$\mho_{13} = \mho(0.4/q_2, x, 0) = \left\{\frac{\delta_1}{0.8}, \frac{\delta_2}{0.3}, \frac{\delta_3}{0.5}, \frac{\delta_4}{0.7}\right\},\ \mho_{14} = \mho(0.4/q_2, y, 0) = \left\{\frac{\delta_1}{0.7}, \frac{\delta_2}{0.2}, \frac{\delta_3}{0.9}, \frac{\delta_4}{0.4}\right\},$$

$$\mho_{15} = \mho(0.4/q_2, z, 0) = \left\{\frac{\delta_1}{0.6}, \frac{\delta_2}{0.7}, \frac{\delta_3}{0.3}, \frac{\delta_4}{0.2}\right\},\ \mho_{16} = \mho(0.5/q_3, x, 0) = \left\{\frac{\delta_1}{0.1}, \frac{\delta_2}{0.4}, \frac{\delta_3}{0.7}, \frac{\delta_4}{0.8}\right\},$$

$$\mho_{17} = \mho(0.5/q_3, y, 0) = \left\{\frac{\delta_1}{0.2}, \frac{\delta_2}{0.9}, \frac{\delta_3}{0.8}, \frac{\delta_4}{0.3}\right\},\ \mho_{18} = \mho(0.5/q_3, z, 0) = \left\{\frac{\delta_1}{0.5}, \frac{\delta_2}{0.3}, \frac{\delta_3}{0.6}, \frac{\delta_4}{0.1}\right\}.$$

The FPFHSE-set can be described as

$$(\mho, \mathbb{S}) = \left\{ \begin{array}{l} \left((0.2/q_1, x, 1), \left\{\frac{\delta_1}{0.2}, \frac{\delta_2}{0.7}, \frac{\delta_3}{0.5}, \frac{\delta_4}{0.1}\right\}\right), \left((0.2/q_1, y, 1), \left\{\frac{\delta_1}{0.4}, \frac{\delta_2}{0.8}, \frac{\delta_3}{0.4}, \frac{\delta_4}{0.2}\right\}\right) \\ \left((0.2/q_1, z, 1), \left\{\frac{\delta_1}{0.7}, \frac{\delta_2}{0.5}, \frac{\delta_3}{0.6}, \frac{\delta_4}{0.3}\right\}\right), \left((0.4/q_2, x, 1), \left\{\frac{\delta_1}{0.9}, \frac{\delta_2}{0.4}, \frac{\delta_3}{0.7}, \frac{\delta_4}{0.3}\right\}\right) \\ \left((0.4/q_2, y, 1), \left\{\frac{\delta_1}{0.4}, \frac{\delta_2}{0.8}, \frac{\delta_3}{0.3}, \frac{\delta_4}{0.2}\right\}\right), \left((0.4/q_2, z, 1), \left\{\frac{\delta_1}{0.5}, \frac{\delta_2}{0.3}, \frac{\delta_3}{0.6}, \frac{\delta_4}{0.8}\right\}\right) \\ \left((0.5/q_3, x, 1), \left\{\frac{\delta_1}{0.2}, \frac{\delta_2}{0.9}, \frac{\delta_3}{0.4}, \frac{\delta_4}{0.5}\right\}\right), \left((0.5/q_3, y, 1), \left\{\frac{\delta_1}{0.4}, \frac{\delta_2}{0.6}, \frac{\delta_3}{0.7}, \frac{\delta_4}{0.9}\right\}\right) \\ \left((0.5/q_3, z, 1), \left\{\frac{\delta_1}{0.7}, \frac{\delta_2}{0.3}, \frac{\delta_3}{0.5}, \frac{\delta_4}{0.2}\right\}\right), \left((0.2/q_1, x, 0), \left\{\frac{\delta_1}{0.3}, \frac{\delta_2}{0.2}, \frac{\delta_3}{0.4}, \frac{\delta_4}{0.1}\right\}\right) \\ \left((0.2/q_1, y, 0), \left\{\frac{\delta_1}{0.1}, \frac{\delta_2}{0.9}, \frac{\delta_3}{0.6}, \frac{\delta_4}{0.2}\right\}\right), \left((0.2/q_1, z, 0), \left\{\frac{\delta_1}{0.2}, \frac{\delta_2}{0.1}, \frac{\delta_3}{0.3}, \frac{\delta_4}{0.5}\right\}\right) \\ \left((0.4/q_2, x, 0), \left\{\frac{\delta_1}{0.8}, \frac{\delta_2}{0.3}, \frac{\delta_3}{0.5}, \frac{\delta_4}{0.7}\right\}\right), \left((0.4/q_2, y, 0), \left\{\frac{\delta_1}{0.7}, \frac{\delta_2}{0.2}, \frac{\delta_3}{0.9}, \frac{\delta_4}{0.4}\right\}\right) \\ \left((0.4/q_2, z, 0), \left\{\frac{\delta_1}{0.6}, \frac{\delta_2}{0.7}, \frac{\delta_3}{0.3}, \frac{\delta_4}{0.2}\right\}\right), \left((0.5/q_3, x, 0), \left\{\frac{\delta_1}{0.1}, \frac{\delta_2}{0.4}, \frac{\delta_3}{0.7}, \frac{\delta_4}{0.8}\right\}\right) \\ \left((0.5/q_3, y, 0), \left\{\frac{\delta_1}{0.2}, \frac{\delta_2}{0.9}, \frac{\delta_3}{0.8}, \frac{\delta_4}{0.3}\right\}\right), \left((0.5/q_3, z, 0), \left\{\frac{\delta_1}{0.5}, \frac{\delta_2}{0.3}, \frac{\delta_3}{0.6}, \frac{\delta_4}{0.1}\right\}\right) \end{array} \right\}.$$

Note 3. Considering the FPFHSE-set $(\mho, \mathbb{S})$ in Example 3.1, it can be said that $\mho_{(\delta_1)}(0.2/q_1, x, 1) = 0.2$, $\mho_{(\delta_3)}(0.5/q_3, z, 1) = 0.5$, $\mho_{(\delta_1)}(0.2/q_1, x, 0) = 0.3$ and $\mho_{(\delta_2)}(0.4/q_2, y, 0) = 0.2$. The others can be interpreted similarly.

Definition 3.2. Let $(\mho_1, \mathbb{S}_1)$ and $(\mho_2, \mathbb{S}_2)$ be two FPFHSE-sets on $\hat{\Delta}$. Then, $(\mho_1, \mathbb{S}_1)$ is an FPFHSE-subset of $(\mho_2, \mathbb{S}_2)$, shown by $(\mho_1, \mathbb{S}_1)\widetilde{\subseteq}(\mho_2, \mathbb{S}_2)$, if (i) $\mathbb{S}_1 \subseteq \mathbb{S}_2$ and (ii) $\mho_1(\imath) \subseteq \mho_2(\imath)$ for all $\imath \in \mathbb{S}_1$.

Example 3.2. Considering the Example 3.1. Also, we suppose

$$\mathbb{S}_1 = \left\{(0.2/q_1, x, 1), (0.2/q_1, y, 1), (0.5/q_3, y, 1), (0.5/q_3, y, 0), (0.2/q_1, z, 0), (0.5/q_3, z, 1)\right\}$$

and

$$\mathbb{S}_2 = \left\{ \begin{array}{l} (0.2/q_1, x, 1), (0.5/q_3, x, 0), (0.5/q_3, x, 1), (0.2/q_1, y, 1), (0.5/q_3, y, 1) \\ (0.2/q_1, y, 0), (0.5/q_3, y, 0), (0.2/q_1, z, 0), (0.5/q_3, z, 1), (0.2/q_1, z, 1) \end{array} \right\}.$$

It is clear that $\mathbb{S}_1 \subset \mathbb{S}_2$. Suppose $(\mho_1, \mathbb{S}_1)$ and $(\mho_2, \mathbb{S}_2)$ are respectively defined as

$$(\mho_1, \mathbb{S}_1) = \left\{ \begin{array}{l} \left((0.2/q_1, x, 1), \left\{\frac{\delta_1}{0.1}, \frac{\delta_2}{0.6}, \frac{\delta_3}{0.4}, \frac{\delta_4}{0.1}\right\}\right), \left((0.2/q_1, t, 1), \left\{\frac{\delta_1}{0.3}, \frac{\delta_2}{0.6}, \frac{\delta_3}{0.2}, \frac{\delta_4}{0.1}\right\}\right) \\ \left((0.5/q_3, y, 1), \left\{\frac{\delta_1}{0.2}, \frac{\delta_2}{0.5}, \frac{\delta_3}{0.6}, \frac{\delta_4}{0.8}\right\}\right), \left((0.5/q_3, u, 1), \left\{\frac{\delta_1}{0.6}, \frac{\delta_2}{0.2}, \frac{\delta_3}{0.4}, \frac{\delta_4}{0.1}\right\}\right) \\ \left((0.2/q_1, z, 0), \left\{\frac{\delta_1}{0.1}, \frac{\delta_2}{0.1}, \frac{\delta_3}{0.2}, \frac{\delta_4}{0.4}\right\}\right), \left((0.5/q_3, y, 0), \left\{\frac{\delta_1}{0.1}, \frac{\delta_2}{0.8}, \frac{\delta_3}{0.7}, \frac{\delta_4}{0.2}\right\}\right) \end{array} \right\},$$

and

$$(\mho_2, \mathbb{S}_2) = \left\{ \begin{array}{l} \left((0.2/q_1, x, 1), \left\{\frac{\delta_1}{0.2}, \frac{\delta_2}{0.7}, \frac{\delta_3}{0.5}, \frac{\delta_4}{0.1}\right\}\right), \left((0.2/q_1, y, 1), \left\{\frac{\delta_1}{0.4}, \frac{\delta_2}{0.8}, \frac{\delta_3}{0.4}, \frac{\delta_4}{0.2}\right\}\right) \\ \left((0.5/q_3, x, 1), \left\{\frac{\delta_1}{0.2}, \frac{\delta_2}{0.9}, \frac{\delta_3}{0.4}, \frac{\delta_4}{0.5}\right\}\right), \left((0.5/q_3, y, 1), \left\{\frac{\delta_1}{0.4}, \frac{\delta_2}{0.6}, \frac{\delta_3}{0.7}, \frac{\delta_4}{0.9}\right\}\right) \\ \left((0.2/q_1, z, 1), \left\{\frac{\delta_1}{0.7}, \frac{\delta_2}{0.5}, \frac{\delta_3}{0.6}, \frac{\delta_4}{0.3}\right\}\right), \left((0.5/q_3, z, 1), \left\{\frac{\delta_1}{0.7}, \frac{\delta_2}{0.3}, \frac{\delta_3}{0.5}, \frac{\delta_4}{0.2}\right\}\right) \\ \left((0.2/q_1, z, 0), \left\{\frac{\delta_1}{0.2}, \frac{\delta_2}{0.1}, \frac{\delta_3}{0.3}, \frac{\delta_4}{0.5}\right\}\right), \left((0.2/q_1, y, 0), \left\{\frac{\delta_1}{0.1}, \frac{\delta_2}{0.9}, \frac{\delta_3}{0.6}, \frac{\delta_4}{0.2}\right\}\right) \\ \left((0.5/q_3, x, 0), \left\{\frac{\delta_1}{0.1}, \frac{\delta_2}{0.4}, \frac{\delta_3}{0.7}, \frac{\delta_4}{0.8}\right\}\right), \left((0.5/q_3, y, 0), \left\{\frac{\delta_1}{0.2}, \frac{\delta_2}{0.9}, \frac{\delta_3}{0.8}, \frac{\delta_4}{0.3}\right\}\right) \end{array} \right\},$$

then, we have $(\mho_1, \mathbb{S}_1)\widetilde{\subseteq}(\mho_2, \mathbb{S}_2)$.

Definition 3.3. Two FPFHSE-sets $(\mho_1, \mathbb{S}_1)$ and $(\mho_2, \mathbb{S}_2)$ over $\hat{\Delta}$ are equal, shown by $(\mho_1, \mathbb{S}_1) = (\mho_2, \mathbb{S}_2)$, if $(\mho_1, \mathbb{S}_1)\widetilde{\subseteq}(\mho_2, \mathbb{S}_2)$ and $(\mho_2, \mathbb{S}_2)\widetilde{\subseteq}(\mho_1, \mathbb{S}_1)$.

Definition 3.4. Let $\mathbb{S}^* = \{(\zeta(q)/q, x, 1) : (\zeta(q)/q, x, 1) \in \mathbb{S}\}$ and $\mathbb{S}^\dagger = \{(\zeta(q)/q, x, 0) : (\zeta(q)/q, x, 0) \in \mathbb{S}\}$ such that $\mathbb{S} = \mathbb{S}^* \cup \mathbb{S}^\dagger$.

(1) An agree-FPFHSE-set $(\mho, \mathbb{S})_{ag}$ on $\hat{\Delta}$ is an FPFHSE-subset of $(\mho, \mathbb{S})$ and characterized as

$$(\mho, \mathbb{S})_{ag} = \{(\imath, \mho_{ag}(\imath)) : \imath \in \mathbb{S}^{*} \text{ and } \mho_{ag}(\imath) = \mho(\imath)\}.$$

(2) A disagree-FPFHSE-set $(\mho, \mathbb{S})_{dag}$ is an FPFHSE-subset of $(\mho, \mathbb{S})$ on $\hat{\Delta}$ and characterized as

$$(\mho, \mathbb{S})_{dag} = \{(\imath, \mho_{dag}(\imath)) : \imath \in \mathbb{S}^{\dagger} \text{ and } \mho_{dag}(\imath) = \mho(\imath)\}.$$

Example 3.3. The agree-FPFHSE-set for the FPFHSE-set in Example 3.1 is found as

$$(\mho, \mathbb{S})_{\text{ag}} = \left\{ \begin{array}{l} \left((0.2/q_1, x, 1), \left\{\frac{\delta_1}{0.2}, \frac{\delta_2}{0.7}, \frac{\delta_3}{0.5}, \frac{\delta_4}{0.1}\right\}\right), \left((0.2/q_1, y, 1), \left\{\frac{\delta_1}{0.4}, \frac{\delta_2}{0.8}, \frac{\delta_3}{0.4}, \frac{\delta_4}{0.2}\right\}\right) \\ \left((0.2/q_1, z, 1), \left\{\frac{\delta_1}{0.7}, \frac{\delta_2}{0.5}, \frac{\delta_3}{0.6}, \frac{\delta_4}{0.3}\right\}\right), \left((0.4/q_2, x, 1), \left\{\frac{\delta_1}{0.9}, \frac{\delta_2}{0.4}, \frac{\delta_3}{0.7}, \frac{\delta_4}{0.3}\right\}\right) \\ \left((0.4/q_2, y, 1), \left\{\frac{\delta_1}{0.4}, \frac{\delta_2}{0.8}, \frac{\delta_3}{0.3}, \frac{\delta_4}{0.2}\right\}\right), \left((0.4/q_2, z, 1), \left\{\frac{\delta_1}{0.5}, \frac{\delta_2}{0.3}, \frac{\delta_3}{0.6}, \frac{\delta_4}{0.8}\right\}\right) \\ \left((0.5/q_3, x, 1), \left\{\frac{\delta_1}{0.2}, \frac{\delta_2}{0.9}, \frac{\delta_3}{0.4}, \frac{\delta_4}{0.5}\right\}\right), \left((0.5/q_3, y, 1), \left\{\frac{\delta_1}{0.4}, \frac{\delta_2}{0.6}, \frac{\delta_3}{0.7}, \frac{\delta_4}{0.9}\right\}\right) \\ \left((0.5/q_3, z, 1), \left\{\frac{\delta_1}{0.7}, \frac{\delta_2}{0.3}, \frac{\delta_3}{0.5}, \frac{\delta_4}{0.2}\right\}\right) \end{array} \right\}.$$

Also, the disagree-FPFHSE-set for this FPFHSE-set is

$$(\mho, \mathbb{S})_{\text{dag}} = \left\{ \begin{array}{l} \left((0.2/q_1, x, 0), \left\{\frac{\delta_1}{0.3}, \frac{\delta_2}{0.2}, \frac{\delta_3}{0.4}, \frac{\delta_4}{0.1}\right\}\right), \left((0.2/q_1, y, 0), \left\{\frac{\delta_1}{0.1}, \frac{\delta_2}{0.9}, \frac{\delta_3}{0.6}, \frac{\delta_4}{0.2}\right\}\right) \\ \left((0.2/q_1, z, 0), \left\{\frac{\delta_1}{0.2}, \frac{\delta_2}{0.1}, \frac{\delta_3}{0.3}, \frac{\delta_4}{0.5}\right\}\right), \left((0.4/q_2, x, 0), \left\{\frac{\delta_1}{0.8}, \frac{\delta_2}{0.3}, \frac{\delta_3}{0.5}, \frac{\delta_4}{0.7}\right\}\right) \\ \left((0.4/q_2, y, 0), \left\{\frac{\delta_1}{0.7}, \frac{\delta_2}{0.2}, \frac{\delta_3}{0.9}, \frac{\delta_4}{0.4}\right\}\right), \left((0.4/q_2, z, 0), \left\{\frac{\delta_1}{0.6}, \frac{\delta_2}{0.7}, \frac{\delta_3}{0.3}, \frac{\delta_4}{0.2}\right\}\right) \\ \left((0.5/q_3, x, 0), \left\{\frac{\delta_1}{0.1}, \frac{\delta_2}{0.4}, \frac{\delta_3}{0.7}, \frac{\delta_4}{0.8}\right\}\right), \left((0.5/q_3, y, 0), \left\{\frac{\delta_1}{0.2}, \frac{\delta_2}{0.9}, \frac{\delta_3}{0.8}, \frac{\delta_4}{0.3}\right\}\right) \\ \left((0.5/q_3, z, 0), \left\{\frac{\delta_1}{0.5}, \frac{\delta_2}{0.3}, \frac{\delta_3}{0.6}, \frac{\delta_4}{0.1}\right\}\right) \end{array} \right\}.$$

Definition 3.5. The relative complement of an FPFHSE-set $(\mho, \mathbb{S})$, shown by $(\mho, \mathbb{S})^{\widetilde{r}}$, is characterized by $(\mho, \mathbb{S})^{\widetilde{r}} = (\mho^{\widetilde{r}}, \mathbb{S})$, where $\mho^{\widetilde{r}} : \mathbb{S} \to FP(\hat{\Delta})$ is defined by $\mho^{\widetilde{r}}(\imath) = \hat{\Delta} - \mho(\imath) = (\mho(\imath))^{c}$ for all $\imath \in \mathbb{S}$.

Example 3.4. The relative complement of the FPFHSE-set given in Example 3.1 is obtained as

$$(\mho, \mathbb{S})^{\widetilde{\text{r}}} = \left\{ \begin{array}{l} \left((0.2/q_1, x, 1), \left\{\frac{\delta_1}{0.8}, \frac{\delta_2}{0.3}, \frac{\delta_3}{0.5}, \frac{\delta_4}{0.9}\right\}\right), \left((0.2/q_1, y, 1), \left\{\frac{\delta_1}{0.6}, \frac{\delta_2}{0.2}, \frac{\delta_3}{0.6}, \frac{\delta_4}{0.8}\right\}\right) \\ \left((0.2/q_1, z, 1), \left\{\frac{\delta_1}{0.3}, \frac{\delta_2}{0.5}, \frac{\delta_3}{0.4}, \frac{\delta_4}{0.7}\right\}\right), \left((0.5/q_3, x, 1), \left\{\frac{\delta_1}{0.8}, \frac{\delta_2}{0.1}, \frac{\delta_3}{0.6}, \frac{\delta_4}{0.5}\right\}\right) \\ \left((0.5/q_3, y, 1), \left\{\frac{\delta_1}{0.6}, \frac{\delta_2}{0.4}, \frac{\delta_3}{0.3}, \frac{\delta_4}{0.1}\right\}\right), \left((0.5/q_3, z, 1), \left\{\frac{\delta_1}{0.3}, \frac{\delta_2}{0.7}, \frac{\delta_3}{0.5}, \frac{\delta_4}{0.8}\right\}\right) \\ \left((0.4/q_2, x, 1), \left\{\frac{\delta_1}{0.1}, \frac{\delta_2}{0.6}, \frac{\delta_3}{0.3}, \frac{\delta_4}{0.7}\right\}\right), \left((0.4/q_2, y, 1), \left\{\frac{\delta_1}{0.6}, \frac{\delta_2}{0.2}, \frac{\delta_3}{0.7}, \frac{\delta_4}{0.8}\right\}\right) \\ \left((0.4/q_2, z, 1), \left\{\frac{\delta_1}{0.5}, \frac{\delta_2}{0.7}, \frac{\delta_3}{0.4}, \frac{\delta_4}{0.2}\right\}\right), \left((0.2/q_1, x, 0), \left\{\frac{\delta_1}{0.7}, \frac{\delta_2}{0.3}, \frac{\delta_3}{0.6}, \frac{\delta_4}{0.9}\right\}\right) \\ \left((0.2/q_1, y, 0), \left\{\frac{\delta_1}{0.9}, \frac{\delta_2}{0.1}, \frac{\delta_3}{0.4}, \frac{\delta_4}{0.1}\right\}\right), \left((0.2/q_1, z, 0), \left\{\frac{\delta_1}{0.8}, \frac{\delta_2}{0.9}, \frac{\delta_3}{0.7}, \frac{\delta_4}{0.5}\right\}\right) \\ \left((0.5/q_3, x, 0), \left\{\frac{\delta_1}{0.9}, \frac{\delta_2}{0.6}, \frac{\delta_3}{0.3}, \frac{\delta_4}{0.2}\right\}\right), \left((0.5/q_3, y, 0), \left\{\frac{\delta_1}{0.8}, \frac{\delta_2}{0.1}, \frac{\delta_3}{0.2}, \frac{\delta_4}{0.7}\right\}\right) \\ \left((0.5/q_3, z, 0), \left\{\frac{\delta_1}{0.5}, \frac{\delta_2}{0.7}, \frac{\delta_3}{0.4}, \frac{\delta_4}{0.9}\right\}\right), \left((0.4/q_2, x, 0), \left\{\frac{\delta_1}{0.2}, \frac{\delta_2}{0.7}, \frac{\delta_3}{0.5}, \frac{\delta_4}{0.3}\right\}\right) \\ \left((0.4/q_2, y, 0), \left\{\frac{\delta_1}{0.3}, \frac{\delta_2}{0.8}, \frac{\delta_3}{0.1}, \frac{\delta_4}{0.4}\right\}\right), \left((0.4/q_2, z, 0), \left\{\frac{\delta_1}{0.4}, \frac{\delta_2}{0.3}, \frac{\delta_3}{0.7}, \frac{\delta_4}{0.8}\right\}\right) \end{array} \right\}.$$

Definition 3.6. Let $(\mho_1, \mathbb{S}_1)$ and $(\mho_2, \mathbb{S}_2)$ be two FPFHSE-sets on $\hat{\Delta}$. The restricted union of two FPFHSE-sets $(\mho_1, \mathbb{S}_1)$ and $(\mho_2, \mathbb{S}_2)$ is denoted and defined by $(\mho_3, \mathbb{S}_3) = (\mho_1, \mathbb{S}_1)\widetilde{\mathbb{U}}(\mho_2, \mathbb{S}_2)$ where $\mathbb{S}_3 = \mathbb{S}_1 \cap \mathbb{S}_2$ and $\mho_3(\imath) = \mho_1(\imath) \cup \mho_2(\imath)$ for all $\imath \in \mathbb{S}_3$.

Definition 3.7. Let $(\mho_1, \mathbb{S}_1)$ and $(\mho_2, \mathbb{S}_2)$ be two FPFHSE-sets on $\hat{\Delta}$. The extended union of two FPFHSE-sets $(\mho_1, \mathbb{S}_1)$ and $(\mho_2, \mathbb{S}_2)$ is denoted and defined by $(\mho_3, \mathbb{S}_3) = (\mho_1, \mathbb{S}_1)\widetilde{\cup}(\mho_2, \mathbb{S}_2)$ where $\mathbb{S}_3 = \mathbb{S}_1 \cup \mathbb{S}_2$ and for all $\imath \in \mathbb{S}_3$,

$$\mho_3(\imath) = \begin{cases} \mho_1(\imath), & \imath \in \mathbb{S}_1 - \mathbb{S}_2; \\ \mho_2(\imath), & \imath \in \mathbb{S}_2 - \mathbb{S}_1; \\ \mho_1(\imath) \cup \mho_2(\imath), & \imath \in \mathbb{S}_1 \cap \mathbb{S}_2. \end{cases}$$

Definition 3.8. Let $(\mho_1, \mathbb{S}_1)$ and $(\mho_2, \mathbb{S}_2)$ be two FPFHSE-sets on $\hat{\Delta}$. The restricted intersection of two FPFHSE-sets $(\mho_1, \mathbb{S}_1)$ and $(\mho_2, \mathbb{S}_2)$ is denoted and defined by $(\mho_3, \mathbb{S}_3) = (\mho_1, \mathbb{S}_1)\widetilde{\Cap}(\mho_2, \mathbb{S}_2)$ where $\mathbb{S}_3 = \mathbb{S}_1 \cap \mathbb{S}_2$ and $\mho_3(\imath) = \mho_1(\imath) \cap \mho_2(\imath)$ for all $\imath \in \mathbb{S}_3$.

Definition 3.9. Let $(\mho_1, \mathbb{S}_1)$ and $(\mho_2, \mathbb{S}_2)$ be two FPFHSE-sets on $\hat{\Delta}$. The extended intersection of two FPFHSE-sets $(\mho_1, \mathbb{S}_1)$ and $(\mho_2, \mathbb{S}_2)$ is denoted and defined by $(\mho_3, \mathbb{S}_3) = (\mho_1, \mathbb{S}_1)\widetilde{\cap}(\mho_2, \mathbb{S}_2)$ where $\mathbb{S}_3 = \mathbb{S}_1 \cup \mathbb{S}_2$ and for all $\imath \in \mathbb{S}_3$,

$$\mho_3(\imath) = \begin{cases} \mho_1(\imath), & \imath \in \mathbb{S}_1 - \mathbb{S}_2; \\ \mho_2(\imath), & \imath \in \mathbb{S}_2 - \mathbb{S}_1; \\ \mho_1(\imath) \cap \mho_2(\imath), & \imath \in \mathbb{S}_1 \cap \mathbb{S}_2. \end{cases}$$

Example 3.5. Consider Example 3.1 and the following two sets:

$$\mathbb{S}_1 = \left\{(0.2/q_1, s, 1), (0.5/q_3, s, 0), (0.2/q_1, t, 1), (0.5/q_3, t, 1), (0.5/q_3, t, 0), (0.2/q_1, u, 0), (0.5/q_3, u, 1)\right\},$$

$$\mathbb{S}_2 = \begin{Bmatrix} (0.2/q_1, s, 1), (0.5/q_3, s, 0), (0.5/q_3, s, 1), (0.2/q_1, t, 1), (0.5/q_3, t, 1) \\ (0.2/q_1, u, 1), (0.5/q_3, t, 0), (0.2/q_1, u, 0), (0.5/q_3, u, 1), (0.2/q_1, t, 0) \end{Bmatrix}.$$

Suppose $(\mho_1, \mathbb{S}_1)$ and $(\mho_2, \mathbb{S}_2)$ are two FPFHSE-sets on $\hat{\Delta}$ such that

$$(\mho_1, \mathbb{S}_1) = \left\{ \begin{array}{l} \left((0.2/q_1, s, 1), \left\{\frac{\delta_1}{0.1}, \frac{\delta_2}{0.6}, \frac{\delta_3}{0.4}, \frac{\delta_4}{0.1}\right\}\right), \left((0.2/q_1, t, 1), \left\{\frac{\delta_1}{0.3}, \frac{\delta_2}{0.6}, \frac{\delta_3}{0.2}, \frac{\delta_4}{0.1}\right\}\right) \\ \left((0.5/q_3, t, 1), \left\{\frac{\delta_1}{0.2}, \frac{\delta_2}{0.5}, \frac{\delta_3}{0.6}, \frac{\delta_4}{0.8}\right\}\right), \left((0.5/q_3, u, 1), \left\{\frac{\delta_1}{0.6}, \frac{\delta_2}{0.2}, \frac{\delta_3}{0.4}, \frac{\delta_4}{0.1}\right\}\right) \\ \left((0.2/q_1, u, 0), \left\{\frac{\delta_1}{0.1}, \frac{\delta_2}{0.1}, \frac{\delta_3}{0.2}, \frac{\delta_4}{0.4}\right\}\right), \left((0.5/q_3, s, 0), \left\{\frac{\delta_1}{0.1}, \frac{\delta_2}{0.3}, \frac{\delta_3}{0.6}, \frac{\delta_4}{0.7}\right\}\right) \\ \left((0.5/q_3, t, 0), \left\{\frac{\delta_1}{0.1}, \frac{\delta_2}{0.8}, \frac{\delta_3}{0.7}, \frac{\delta_4}{0.2}\right\}\right) \end{array} \right\},$$

and

$$(\mho_2, \mathbb{S}_2) = \left\{ \begin{array}{l} \left((0.2/q_1, s, 1), \left\{\frac{\delta_1}{0.2}, \frac{\delta_2}{0.7}, \frac{\delta_3}{0.5}, \frac{\delta_4}{0.1}\right\}\right), \left((0.2/q_1, t, 1), \left\{\frac{\delta_1}{0.4}, \frac{\delta_2}{0.8}, \frac{\delta_3}{0.4}, \frac{\delta_4}{0.2}\right\}\right) \\ \left((0.5/q_3, s, 1), \left\{\frac{\delta_1}{0.2}, \frac{\delta_2}{0.9}, \frac{\delta_3}{0.4}, \frac{\delta_4}{0.5}\right\}\right), \left((0.5/q_3, t, 1), \left\{\frac{\delta_1}{0.4}, \frac{\delta_2}{0.6}, \frac{\delta_3}{0.7}, \frac{\delta_4}{0.9}\right\}\right) \\ \left((0.2/q_1, u, 1), \left\{\frac{\delta_1}{0.7}, \frac{\delta_2}{0.5}, \frac{\delta_3}{0.6}, \frac{\delta_4}{0.3}\right\}\right), \left((0.5/q_3, u, 1), \left\{\frac{\delta_1}{0.7}, \frac{\delta_2}{0.3}, \frac{\delta_3}{0.5}, \frac{\delta_4}{0.2}\right\}\right) \\ \left((0.2/q_1, u, 0), \left\{\frac{\delta_1}{0.2}, \frac{\delta_2}{0.1}, \frac{\delta_3}{0.3}, \frac{\delta_4}{0.5}\right\}\right), \left((0.2/q_1, t, 0), \left\{\frac{\delta_1}{0.1}, \frac{\delta_2}{0.9}, \frac{\delta_3}{0.6}, \frac{\delta_4}{0.2}\right\}\right) \\ \left((0.5/q_3, s, 0), \left\{\frac{\delta_1}{0.1}, \frac{\delta_2}{0.4}, \frac{\delta_3}{0.7}, \frac{\delta_4}{0.8}\right\}\right), \left((0.5/q_3, t, 0), \left\{\frac{\delta_1}{0.2}, \frac{\delta_2}{0.9}, \frac{\delta_3}{0.8}, \frac{\delta_4}{0.3}\right\}\right) \end{array} \right\}.$$

Then, we have

$$(\mho_1, \mathbb{S}_1)\widetilde{\cup}(\mho_2, \mathbb{S}_2) = \left\{ \begin{array}{l} \left((0.2/q_1, s, 1), \left\{\frac{\delta_1}{0.2}, \frac{\delta_2}{0.7}, \frac{\delta_3}{0.5}, \frac{\delta_4}{0.1}\right\}\right), \left((0.2/q_1, t, 1), \left\{\frac{\delta_1}{0.4}, \frac{\delta_2}{0.8}, \frac{\delta_3}{0.4}, \frac{\delta_4}{0.2}\right\}\right) \\ \left((0.5/q_3, s, 1), \left\{\frac{\delta_1}{0.2}, \frac{\delta_2}{0.9}, \frac{\delta_3}{0.4}, \frac{\delta_4}{0.5}\right\}\right), \left((0.5/q_3, t, 1), \left\{\frac{\delta_1}{0.4}, \frac{\delta_2}{0.6}, \frac{\delta_3}{0.7}, \frac{\delta_4}{0.9}\right\}\right) \\ \left((0.2/q_1, u, 1), \left\{\frac{\delta_1}{0.7}, \frac{\delta_2}{0.5}, \frac{\delta_3}{0.6}, \frac{\delta_4}{0.3}\right\}\right), \left((0.5/q_3, u, 1), \left\{\frac{\delta_1}{0.7}, \frac{\delta_2}{0.3}, \frac{\delta_3}{0.5}, \frac{\delta_4}{0.2}\right\}\right) \\ \left((0.2/q_1, u, 0), \left\{\frac{\delta_1}{0.2}, \frac{\delta_2}{0.1}, \frac{\delta_3}{0.3}, \frac{\delta_4}{0.5}\right\}\right), \left((0.2/q_1, t, 0), \left\{\frac{\delta_1}{0.1}, \frac{\delta_2}{0.9}, \frac{\delta_3}{0.6}, \frac{\delta_4}{0.2}\right\}\right) \\ \left((0.5/q_3, s, 0), \left\{\frac{\delta_1}{0.1}, \frac{\delta_2}{0.4}, \frac{\delta_3}{0.7}, \frac{\delta_4}{0.8}\right\}\right), \left((0.5/q_3, t, 0), \left\{\frac{\delta_1}{0.2}, \frac{\delta_2}{0.9}, \frac{\delta_3}{0.8}, \frac{\delta_4}{0.3}\right\}\right) \end{array} \right\}$$

and

$$(\mho_1, \mathbb{S}_1)\widetilde{\Cap}(\mho_2, \mathbb{S}_2) = \left\{ \begin{array}{l} \left((0.2/q_1, s, 1), \left\{\frac{\delta_1}{0.1}, \frac{\delta_2}{0.6}, \frac{\delta_3}{0.4}, \frac{\delta_4}{0.1}\right\}\right), \left((0.2/q_1, t, 1), \left\{\frac{\delta_1}{0.3}, \frac{\delta_2}{0.6}, \frac{\delta_3}{0.2}, \frac{\delta_4}{0.1}\right\}\right) \\ \left((0.5/q_3, t, 1), \left\{\frac{\delta_1}{0.2}, \frac{\delta_2}{0.5}, \frac{\delta_3}{0.6}, \frac{\delta_4}{0.8}\right\}\right), \left((0.5/q_3, u, 1), \left\{\frac{\delta_1}{0.6}, \frac{\delta_2}{0.2}, \frac{\delta_3}{0.4}, \frac{\delta_4}{0.1}\right\}\right) \\ \left((0.2/q_1, u, 0), \left\{\frac{\delta_1}{0.1}, \frac{\delta_2}{0.1}, \frac{\delta_3}{0.2}, \frac{\delta_4}{0.4}\right\}\right), \left((0.5/q_3, s, 0), \left\{\frac{\delta_1}{0.1}, \frac{\delta_2}{0.3}, \frac{\delta_3}{0.6}, \frac{\delta_4}{0.7}\right\}\right) \\ \left((0.5/q_3, t, 0), \left\{\frac{\delta_1}{0.1}, \frac{\delta_2}{0.8}, \frac{\delta_3}{0.7}, \frac{\delta_4}{0.2}\right\}\right) \end{array} \right\}.$$

Definition 3.10. Let $(\mho_1, \mathbb{S}_1)$ and $(\mho_2, \mathbb{S}_2)$ be two FPFHSE-sets on $\hat{\Delta}$; then, the AND-product of $(\mho_1, \mathbb{S}_1)$ and $(\mho_2, \mathbb{S}_2)$, denoted by $(\mho_1, \mathbb{S}_1)\widetilde{\wedge}(\mho_2, \mathbb{S}_2)$, is defined by $(\mho_1, \mathbb{S}_1)\widetilde{\wedge}(\mho_2, \mathbb{S}_2) = (\mho_3, \mathbb{S}_1 \times \mathbb{S}_2)$, where $\mho_3(\imath_1, \imath_2) = \mho_1(\imath_1) \cap \mho_2(\imath_2)$, $\forall(\imath_1, \imath_2) \in \mathbb{S}_1 \times \mathbb{S}_2$.

Definition 3.11. Let $(\mho_1, \mathbb{S}_1)$ and $(\mho_2, \mathbb{S}_2)$ be two FPFHSE-sets on $\hat{\Delta}$; then, the OR-product of $(\mho_1, \mathbb{S}_1)$ and $(\mho_2, \mathbb{S}_2)$, denoted by $(\mho_1, \mathbb{S}_1)\widetilde{\vee}(\mho_2, \mathbb{S}_2)$, is defined by $(\mho_1, \mathbb{S}_1)\widetilde{\vee}(\mho_2, \mathbb{S}_2) = (\mho_3, \mathbb{S}_1 \times \mathbb{S}_2)$, where $\mho_3(\imath_1, \imath_2) = \mho_1(\imath_1) \cup \mho_2(\imath_2)$, $\forall(\imath_1, \imath_2) \in \mathbb{S}_1 \times \mathbb{S}_2$.

Example 3.6. Reconsider the Example 3.1 and take the sets $\mathbb{S}_1 = \{(0.2/q_1, s, 1), (0.2/q_1, t, 1), (0.5/q_3, s, 0)\}$ and $\mathbb{S}_2 = \{(0.2/q_1, s, 1), (0.5/q_3, s, 0)\}$. Suppose $(\mho_1, \mathbb{S}_1)$ and $(\mho_2, \mathbb{S}_2)$ are two FPFHSE-sets on $\hat{\Delta}$ such that

$$(\mho_1, \mathbb{S}_1) = \left\{ \begin{array}{l} \left((0.2/q_1, x, 1), \left\{\frac{\delta_1}{0.1}, \frac{\delta_2}{0.6}, \frac{\delta_3}{0.4}, \frac{\delta_4}{0.1}\right\}\right), \left((0.2/q_1, y, 1), \left\{\frac{\delta_1}{0.3}, \frac{\delta_2}{0.6}, \frac{\delta_3}{0.2}, \frac{\delta_4}{0.1}\right\}\right) \\ \left((0.5/q_3, x, 0), \left\{\frac{\delta_1}{0.1}, \frac{\delta_2}{0.3}, \frac{\delta_3}{0.6}, \frac{\delta_4}{0.7}\right\}\right) \end{array} \right\}$$

and

$$(\mho_2, \mathbb{S}_2) = \left\{ \left((0.2/q_1, x, 1), \left\{\frac{\delta_1}{0.2}, \frac{\delta_2}{0.7}, \frac{\delta_3}{0.5}, \frac{\delta_4}{0.1}\right\}\right), \left((0.5/q_3, x, 0), \left\{\frac{\delta_1}{0.1}, \frac{\delta_2}{0.4}, \frac{\delta_3}{0.7}, \frac{\delta_4}{0.8}\right\}\right) \right\}.$$

Then, we obtain the following:

$$(\mho_1, \mathbb{S}_1)\widetilde{\wedge}(\mho_2, \mathbb{S}_2) = \left\{ \begin{array}{l} \left(((0.2/q_1, x, 1), (0.2/q_1, x, 1)), \left\{\frac{\delta_1}{0.1}, \frac{\delta_2}{0.6}, \frac{\delta_3}{0.4}, \frac{\delta_4}{0.1}\right\}\right) \\ \left(((0.2/q_1, y, 1), (0.2/q_1, x, 1)), \left\{\frac{\delta_1}{0.2}, \frac{\delta_2}{0.6}, \frac{\delta_3}{0.2}, \frac{\delta_4}{0.1}\right\}\right) \\ \left(((0.5/q_3, x, 0), (0.2/q_1, x, 1)), \left\{\frac{\delta_1}{0.1}, \frac{\delta_2}{0.3}, \frac{\delta_3}{0.5}, \frac{\delta_4}{0.1}\right\}\right) \\ \left(((0.2/q_1, x, 1), (0.5/q_3, x, 0)), \left\{\frac{\delta_1}{0.1}, \frac{\delta_2}{0.4}, \frac{\delta_3}{0.4}, \frac{\delta_4}{0.1}\right\}\right) \\ \left(((0.2/q_1, y, 1), (0.5/q_3, x, 0)), \left\{\frac{\delta_1}{0.1}, \frac{\delta_2}{0.4}, \frac{\delta_3}{0.2}, \frac{\delta_4}{0.1}\right\}\right) \\ \left(((0.5/q_3, x, 0), (0.5/q_3, x, 0)), \left\{\frac{\delta_1}{0.1}, \frac{\delta_2}{0.3}, \frac{\delta_3}{0.6}, \frac{\delta_4}{0.7}\right\}\right) \end{array} \right\}$$

and

$$(\mho_1, \mathbb{S}_1)\widetilde{\vee}(\mho_2, \mathbb{S}_2) = \left\{ \begin{array}{l} \left(((0.2/q_1, x, 1), (0.2/q_1, x, 1)), \left\{\frac{\delta_1}{0.2}, \frac{\delta_2}{0.7}, \frac{\delta_3}{0.5}, \frac{\delta_4}{0.1}\right\}\right) \\ \left(((0.2/q_1, y, 1), (0.2/q_1, x, 1)), \left\{\frac{\delta_1}{0.3}, \frac{\delta_2}{0.7}, \frac{\delta_3}{0.5}, \frac{\delta_4}{0.1}\right\}\right) \\ \left(((0.5/q_3, x, 0), (0.2/q_1, x, 1)), \left\{\frac{\delta_1}{0.2}, \frac{\delta_2}{0.7}, \frac{\delta_3}{0.6}, \frac{\delta_4}{0.7}\right\}\right) \\ \left(((0.2/q_1, x, 1), (0.5/q_3, x, 0)), \left\{\frac{\delta_1}{0.1}, \frac{\delta_2}{0.6}, \frac{\delta_3}{0.7}, \frac{\delta_4}{0.8}\right\}\right) \\ \left(((0.2/q_1, y, 1), (0.5/q_3, x, 0)), \left\{\frac{\delta_1}{0.3}, \frac{\delta_2}{0.6}, \frac{\delta_3}{0.7}, \frac{\delta_4}{0.8}\right\}\right) \\ \left(((0.5/q_3, x, 0), (0.5/q_3, x, 0)), \left\{\frac{\delta_1}{0.1}, \frac{\delta_2}{0.4}, \frac{\delta_3}{0.7}, \frac{\delta_4}{0.8}\right\}\right) \end{array} \right\}.$$

4. An MADM decision support model based on FPFHSE-sets

This section aims to describe an MADM decision support model that was designed by using set theoretic aggregation operations of an FPFHSE-set and a weighted FPFHSE-set. Note that from now on, $|\cdot|$ represents the cardinality of a set and $[|\cdot|]$ represents the absolute value of a real number.

4.1. Statement of the problem

In some developing countries like Pakistan, the electricity is produced through thermal and hydro energy sources which are costly as compared to other sources of energy. Therefore, all kinds of customers prefer to use alternative sources of electricity with comparatively less investment. Solar panels are the most significant type of alternative source in this regard therefore its usage has been increased tremendously in Pakistan. Several consumers belonging to all sectors have switched to solar panel solutions. It is being used in industries, agriculture, offices and in homes. These sustainable solar systems ensure energy conservation which makes them a smart option for users. Solar solutions are a mechanical choice for clean and renewable energy in Pakistan due to the availability of a rich amount of sunlight throughout the year. There are many kinds of solar panels available on the market. Due to the availability of sub standard and low quality solar panels, the consumers are very much cautious and concerned regarding its purchase. The selection of good and durable solar panels is a challenging task which involves various criteria. The MADM technique is very helpful for decision-makers to select a suitable solar panel product by considering appropriate parameters under the conditions of an uncertain algebraic setting. In the following subsection, an algorithm (Algorithm 1) is proposed to assist the consumers in purchasing a suitable solar panel product with the help of some decision-makers.

The step-wise elaboration of the proposed algorithm is hereby presented below.

As the Algorithm 1 consists of only three stages with seven steps, the input stage is based on simple sets and the construction and computation stages are composed of mathematical formulations that can easily be instructed to any mathematical cum computational software. Due to the smaller size (few steps) and convenient computations, the algorithm is expected to execute in a faster way as compared to the usual execution speed preserving the reliability and consistency.

Note that the agree-oriented decision value ρ_i, the disagree-oriented decision value η_i and the overall decision value o_i for the FPFHSE-set $(\mho, \mathcal{K})$ are between 0 and 1, that is, $0 \leq \rho_i \leq 1$, $0 \leq \eta_i \leq 1$ and $0 \leq o_i \leq 1$. Figure 1 presents the flowchart of Algorithm 1.

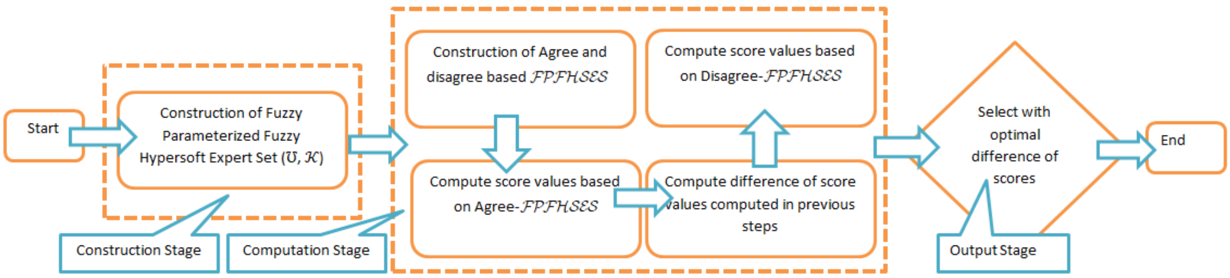

Figure 1. Flow diagram of Algorithm 1.

Algorithm 1. MADM-based optimum selection of solar panels by using aggregations of FPFHSE-Sets.

Start Construction

(1) Construct FPFHSE-set $(\mho, \mathcal{K})$ on $\hat{\Delta} = \{\hbar_i : i \in \{1, 2, ..., p\}\}$.
(2) $\mathcal{K} = \Bbbk \times \mathcal{I} \times \mathcal{U}$, where $\Bbbk$ is the set of fuzzy-valued multi disjoint attributes, $\mathcal{I}$ is the set of experts.

Computation

(3) Determine an Agree FPFHSE-set $(\mho, \mathcal{K})_{ag}$ and a Disagree FPFHSE-set $(\mho, \mathcal{K})_{dag}$ for the FPFHSE-set $(\mho, \mathcal{K})$.
(4) Compute the agree-oriented decision value

$$\rho_i = \frac{1}{|\Bbbk| \times |\mathcal{I}|} \times \sum_{s=1}^{|\Bbbk|\times|\mathcal{I}|} \hbar_{si} \times \zeta(\wp_{([|1+\frac{s-1}{|\mathcal{I}|}|])})$$

for each alternative $\hbar_i$ considering the agree FPFHSE-set $(\mho, \mathcal{K})_{ag}$. Here, the value $\hbar_{si}$ is obtained as $\hbar_{si} = \mho_{(\hbar_i)}(\zeta(\wp_j)/\wp_j, \varepsilon_k, 1)$, where $s = (j-1) \times |I| + k$ for $j \in \{1, 2, ..., |\Bbbk|\}$ and $k \in \{1, 2, ..., |\mathcal{I}|\}$. It is clear that $1 \leq s \leq |\Bbbk| \times |\mathcal{I}|$.
(5) Compute the disagree-oriented decision value

$$\eta_i = \frac{1}{|\Bbbk| \times |\mathcal{I}|} \times \sum_{t=1}^{|\Bbbk|\times|\mathcal{I}|} \hbar_{ti} \times \zeta(\wp_{([|1+\frac{t-1}{|\mathcal{I}|}|])})$$

for each alternative $\hbar_i$ considering the disagree FPFHSE-set $(\mho, \mathcal{K})_{dag}$. Here, the value $\hbar_{ti}$ is obtained as $\hbar_{ti} = \mho_{(\hbar_i)}(\zeta(\wp_j)/\wp_j, \varepsilon_k, 0)$, where $t = (j-1) \times |I| + k$ for $j \in \{1, 2, ..., |\Bbbk|\}$ and $k \in \{1, 2, ..., |\mathcal{I}|\}$. It is clear that $1 \leq t \leq |\Bbbk| \times |\mathcal{I}|$.
(6) Find the overall decision value

$$o_i = \frac{1}{2} \times (1 + \frac{\rho_i \times (\rho_i - \eta_i)}{\rho_i + \eta_i})$$

for each alternative $\hbar_i$.

Output

(7) Determine v for selection of best alternative, where $o_v = max\{o_1, o_2, ...\}$, and make decision.

End

Example 4.1. Assume that there are five models of solar panels forming the universe of discourse $\hat{\Delta} = \{\hbar_1, \hbar_2, \hbar_3, \hbar_4, \hbar_5\}$ and $\mathcal{I}$=$\{\mathcal{E}_1$=$Shane, \mathcal{E}_2$=$Lee, \mathcal{E}_3$= $Akbar\}$ is a set of experts for this purchase. The attribute-valued sets for prescribed attributes “power in watts”, “number of cells”, “maximum voltage in volts”, “cell type” and “number of diods in junction box” are given as

$$\top_1 = \{430 = \curlyvee_1, 530 = \curlyvee_2\},$$
$$\top_2 = \{144 = \curlyvee_3, 155 = \curlyvee_4\},$$

$$\mathsf{T}_3 = \{1000 = \curlyvee_5, 1500 = \curlyvee_6\},$$
$$\mathsf{T}_4 = \{mono = \curlyvee_7, poly = \curlyvee_8\},$$
$$\mathsf{T}_5 = \{3 = \curlyvee_9, 6 = \curlyvee_{10}\}.$$

Then, we have

$$\mathsf{T} = \mathsf{T}_1 \times \mathsf{T}_2 \times \mathsf{T}_3 \times \mathsf{T}_4 \times \mathsf{T}_5 = \left\{ \begin{array}{l} (\curlyvee_1, \curlyvee_3, \curlyvee_5, \curlyvee_7, \curlyvee_9), (\curlyvee_1, \curlyvee_3, \curlyvee_5, \curlyvee_7, \curlyvee_{10}), (\curlyvee_1, \curlyvee_3, \curlyvee_5, \curlyvee_8, \curlyvee_9), (\curlyvee_1, \curlyvee_3, \curlyvee_5, \curlyvee_8, \curlyvee_{10}) \\ (\curlyvee_1, \curlyvee_3, \curlyvee_6, \curlyvee_7, \curlyvee_9), (\curlyvee_1, \curlyvee_3, \curlyvee_6, \curlyvee_7, \curlyvee_{10}), (\curlyvee_1, \curlyvee_3, \curlyvee_6, \curlyvee_8, \curlyvee_9), (\curlyvee_1, \curlyvee_3, \curlyvee_6, \curlyvee_8, \curlyvee_{10}) \\ (\curlyvee_1, \curlyvee_4, \curlyvee_5, \curlyvee_7, \curlyvee_9), (\curlyvee_1, \curlyvee_4, \curlyvee_5, \curlyvee_7, \curlyvee_{10}), (\curlyvee_1, \curlyvee_4, \curlyvee_5, \curlyvee_8, \curlyvee_9), (\curlyvee_1, \curlyvee_4, \curlyvee_5, \curlyvee_8, \curlyvee_{10}) \\ (\curlyvee_1, \curlyvee_4, \curlyvee_6, \curlyvee_7, \curlyvee_9), (\curlyvee_1, \curlyvee_4, \curlyvee_6, \curlyvee_7, \curlyvee_{10}), (\curlyvee_1, \curlyvee_4, \curlyvee_6, \curlyvee_8, \curlyvee_9), (\curlyvee_1, \curlyvee_4, \curlyvee_6, \curlyvee_8, \curlyvee_{10}) \\ (\curlyvee_2, \curlyvee_3, \curlyvee_5, \curlyvee_7, \curlyvee_9), (\curlyvee_2, \curlyvee_3, \curlyvee_5, \curlyvee_7, \curlyvee_{10}), (\curlyvee_2, \curlyvee_3, \curlyvee_5, \curlyvee_8, \curlyvee_9), (\curlyvee_2, \curlyvee_3, \curlyvee_5, \curlyvee_8, \curlyvee_{10}) \\ (\curlyvee_2, \curlyvee_3, \curlyvee_6, \curlyvee_7, \curlyvee_9), (\curlyvee_2, \curlyvee_3, \curlyvee_6, \curlyvee_7, \curlyvee_{10}), (\curlyvee_2, \curlyvee_3, \curlyvee_6, \curlyvee_8, \curlyvee_9), (\curlyvee_2, \curlyvee_3, \curlyvee_6, \curlyvee_8, \curlyvee_{10}) \\ (\curlyvee_2, \curlyvee_4, \curlyvee_5, \curlyvee_7, \curlyvee_9), (\curlyvee_2, \curlyvee_4, \curlyvee_5, \curlyvee_7, \curlyvee_{10}), (\curlyvee_2, \curlyvee_4, \curlyvee_5, \curlyvee_8, \curlyvee_9), (\curlyvee_2, \curlyvee_4, \curlyvee_5, \curlyvee_8, \curlyvee_{10}) \\ (\curlyvee_2, \curlyvee_4, \curlyvee_6, \curlyvee_7, \curlyvee_9), (\curlyvee_2, \curlyvee_4, \curlyvee_6, \curlyvee_7, \curlyvee_{10}), (\curlyvee_2, \curlyvee_4, \curlyvee_6, \curlyvee_8, \curlyvee_9), (\curlyvee_2, \curlyvee_4, \curlyvee_6, \curlyvee_8, \curlyvee_{10}) \end{array} \right\},$$

and we take $\Bbbk$ as a fuzzy subset of T by considering the preferences of decision-makers such that

$$\Bbbk = \left\{ \begin{array}{l} 0.2/\wp_1 = 0.2/(\curlyvee_1, \curlyvee_3, \curlyvee_5, \curlyvee_7, \curlyvee_9),\ 0.4/\wp_2 = 0.4/(\curlyvee_1, \curlyvee_3, \curlyvee_6, \curlyvee_7, \curlyvee_{10}) \\ 0.5/\wp_3 = 0.5/(\curlyvee_1, \curlyvee_4, \curlyvee_6, \curlyvee_8, \curlyvee_9),\ 0.7/\wp_4 = 0.7/(\curlyvee_2, \curlyvee_3, \curlyvee_6, \curlyvee_8, \curlyvee_9) \\ 0.6/\wp_5 = 0.6/(\curlyvee_2, \curlyvee_4, \curlyvee_6, \curlyvee_7, \curlyvee_{10}) \end{array} \right\}.$$

Now $\mathcal{K} = \Bbbk \times \mathcal{I} \times \mathcal{U}$, i.e.,

$$\mathcal{K} = \left\{ \begin{array}{l} (0.2/\wp_1, \varepsilon_1, 1), (0.2/\wp_1, \varepsilon_2, 1), (0.2/\wp_1, \varepsilon_3, 1), (0.4/\wp_2, \varepsilon_1, 1), (0.4/\wp_2, \varepsilon_2, 1), (0.4/\wp_2, \varepsilon_3, 1) \\ (0.5/\wp_3, \varepsilon_1, 1), (0.5/\wp_3, \varepsilon_2, 1), (0.5/\wp_3, \varepsilon_3, 1), (0.7/\wp_4, \varepsilon_1, 1), (0.7/\wp_4, \varepsilon_2, 1), (0.7/\wp_4, \varepsilon_3, 1) \\ (0.6/\wp_5, \varepsilon_1, 1), (0.6/\wp_5, \varepsilon_2, 1), (0.6/\wp_5, \varepsilon_2, 1), (0.2/\wp_1, \varepsilon_1, 0), (0.2/\wp_1, \varepsilon_2, 0), (0.2/\wp_1, \varepsilon_3, 0) \\ (0.4/\wp_2, \varepsilon_1, 0), (0.4/\wp_2, \varepsilon_2, 0), (0.4/\wp_2, \varepsilon_3, 0), (0.5/\wp_3, \varepsilon_1, 0), (0.5/\wp_3, \varepsilon_2, 0), (0.5/\wp_3, \varepsilon_3, 0) \\ (0.7/\wp_4, \varepsilon_1, 0), (0.7/\wp_4, \varepsilon_2, 0), (0.7/\wp_4, \varepsilon_3, 0), (0.6/\wp_5, \varepsilon_1, 0), (0.6/\wp_5, \varepsilon_2, 0), (0.6/\wp_5, \varepsilon_2, 0) \end{array} \right\}.$$

The hierarchical structure of this MADM problem based on solar panel selection is shown in Figure 2.

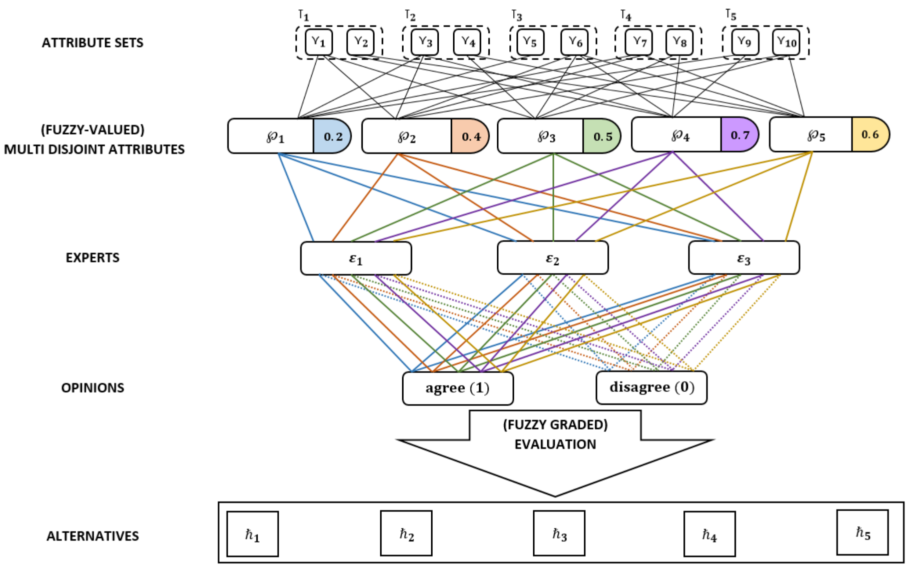

Figure 2. Hierarchical structure of decision problem.

Step 1. By evaluating (with fuzzy grading) alternatives according to the (fuzzy-valued) multi disjoint attributes, they (Shane, Lee and Akbar) construct an FPFHSE-set $(\mho, \mathcal{K})$ as follows:

$$(\mho,\mathcal{K}) = \left\{ \begin{array}{l}
\left((0.2/\wp_1,\varepsilon_1,1),\left\{\frac{\hbar_1}{0.9},\frac{\hbar_2}{0.3},\frac{\hbar_3}{0.6},\frac{\hbar_4}{0.3},\frac{\hbar_5}{0.3}\right\}\right),\left((0.2/\wp_1,\varepsilon_2,1),\left\{\frac{\hbar_1}{0.8},\frac{\hbar_2}{0.1},\frac{\hbar_3}{0.6},\frac{\hbar_4}{0.6},\frac{\hbar_5}{0.9}\right\}\right)\\
\left((0.2/\wp_1,\varepsilon_3,1),\left\{\frac{\hbar_1}{0.7},\frac{\hbar_2}{0.3},\frac{\hbar_3}{0.3},\frac{\hbar_4}{0.3},\frac{\hbar_5}{0.6}\right\}\right),\left((0.4/\wp_2,\varepsilon_1,1),\left\{\frac{\hbar_1}{0.6},\frac{\hbar_2}{0.4},\frac{\hbar_3}{0.7},\frac{\hbar_4}{0.5},\frac{\hbar_5}{0.2}\right\}\right)\\
\left((0.4/\wp_2,\varepsilon_2,1),\left\{\frac{\hbar_1}{0.5},\frac{\hbar_2}{0.6},\frac{\hbar_3}{0.3},\frac{\hbar_4}{0.4},\frac{\hbar_5}{0.7}\right\}\right),\left((0.4/\wp_2,\varepsilon_3,1),\left\{\frac{\hbar_1}{0.4},\frac{\hbar_2}{0.3},\frac{\hbar_3}{0.3},\frac{\hbar_4}{0.8},\frac{\hbar_5}{0.3}\right\}\right)\\
\left((0.5/\wp_3,\varepsilon_1,1),\left\{\frac{\hbar_1}{0.2},\frac{\hbar_2}{0.5},\frac{\hbar_3}{0.6},\frac{\hbar_4}{0.7},\frac{\hbar_5}{0.5}\right\}\right),\left((0.5/\wp_3,\varepsilon_2,1),\left\{\frac{\hbar_1}{0.2},\frac{\hbar_2}{0.4},\frac{\hbar_3}{0.5},\frac{\hbar_4}{0.3},\frac{\hbar_5}{0.8}\right\}\right)\\
\left((0.5/\wp_3,\varepsilon_3,1),\left\{\frac{\hbar_1}{0.3},\frac{\hbar_2}{0.6},\frac{\hbar_3}{0.4},\frac{\hbar_4}{0.3},\frac{\hbar_5}{0.3}\right\}\right),\left((0.7/\wp_4,\varepsilon_1,1),\left\{\frac{\hbar_1}{0.9},\frac{\hbar_2}{0.1},\frac{\hbar_3}{0.5},\frac{\hbar_4}{0.4},\frac{\hbar_5}{0.1}\right\}\right)\\
\left((0.7/\wp_4,\varepsilon_2,1),\left\{\frac{\hbar_1}{0.8},\frac{\hbar_2}{0.4},\frac{\hbar_3}{0.2},\frac{\hbar_4}{0.7},\frac{\hbar_5}{0.6}\right\}\right),\left((0.7/\wp_4,\varepsilon_3,1),\left\{\frac{\hbar_1}{0.6},\frac{\hbar_2}{0.1},\frac{\hbar_3}{0.3},\frac{\hbar_4}{0.6},\frac{\hbar_5}{0.3}\right\}\right)\\
\left((0.6/\wp_5,\varepsilon_1,1),\left\{\frac{\hbar_1}{0.5},\frac{\hbar_2}{0.2},\frac{\hbar_3}{0.1},\frac{\hbar_4}{0.1},\frac{\hbar_5}{0.7}\right\}\right),\left((0.6/\wp_5,\varepsilon_2,1),\left\{\frac{\hbar_1}{0.4},\frac{\hbar_2}{0.6},\frac{\hbar_3}{0.4},\frac{\hbar_4}{0.6},\frac{\hbar_5}{0.8}\right\}\right)\\
\left((0.6/\wp_5,\varepsilon_3,1),\left\{\frac{\hbar_1}{0.2},\frac{\hbar_2}{0.6},\frac{\hbar_3}{0.3},\frac{\hbar_4}{0.5},\frac{\hbar_5}{0.8}\right\}\right),\left((0.2/\wp_1,\varepsilon_1,0),\left\{\frac{\hbar_1}{0.1},\frac{\hbar_2}{0.9},\frac{\hbar_3}{0.8},\frac{\hbar_4}{0.6},\frac{\hbar_5}{0.2}\right\}\right)\\
\left((0.2/\wp_1,\varepsilon_2,0),\left\{\frac{\hbar_1}{0.7},\frac{\hbar_2}{0.6},\frac{\hbar_3}{0.2},\frac{\hbar_4}{0.8},\frac{\hbar_5}{0.7}\right\}\right),\left((0.2/\wp_1,\varepsilon_3,0),\left\{\frac{\hbar_1}{0.2},\frac{\hbar_2}{0.6},\frac{\hbar_3}{0.1},\frac{\hbar_4}{0.1},\frac{\hbar_5}{0.4}\right\}\right)\\
\left((0.4/\wp_2,\varepsilon_1,0),\left\{\frac{\hbar_1}{0.1},\frac{\hbar_2}{0.4},\frac{\hbar_3}{0.2},\frac{\hbar_4}{0.3},\frac{\hbar_5}{0.8}\right\}\right),\left((0.4/\wp_2,\varepsilon_2,0),\left\{\frac{\hbar_1}{0.2},\frac{\hbar_2}{0.7},\frac{\hbar_3}{0.6},\frac{\hbar_4}{0.5},\frac{\hbar_5}{0.6}\right\}\right)\\
\left((0.4/\wp_2,\varepsilon_3,0),\left\{\frac{\hbar_1}{0.7},\frac{\hbar_2}{0.3},\frac{\hbar_3}{0.1},\frac{\hbar_4}{0.3},\frac{\hbar_5}{0.2}\right\}\right),\left((0.5/\wp_3,\varepsilon_1,0),\left\{\frac{\hbar_1}{0.9},\frac{\hbar_2}{0.4,},\frac{\hbar_3}{0.5},\frac{\hbar_4}{0.3},\frac{\hbar_5}{0.9}\right\}\right)\\
\left((0.5/\wp_3,\varepsilon_2,0),\left\{\frac{\hbar_1}{0.8},\frac{\hbar_2}{0.2},\frac{\hbar_3}{0.3},\frac{\hbar_4}{0.5},\frac{\hbar_5}{0.7}\right\}\right),\left((0.5/\wp_3,\varepsilon_3,0),\left\{\frac{\hbar_1}{0.6},\frac{\hbar_2}{0.6},\frac{\hbar_3}{0.5},\frac{\hbar_4}{0.4},\frac{\hbar_5}{0.1}\right\}\right)\\
\left((0.7/\wp_4,\varepsilon_1,0),\left\{\frac{\hbar_1}{0.5},\frac{\hbar_2}{0.3},\frac{\hbar_3}{0.2},\frac{\hbar_4}{0.3},\frac{\hbar_5}{0.5}\right\}\right),\left((0.7/\wp_4,\varepsilon_2,0),\left\{\frac{\hbar_1}{0.9},\frac{\hbar_2}{0.5},\frac{\hbar_3}{0.3},\frac{\hbar_4}{0.6},\frac{\hbar_5}{0.4}\right\}\right)\\
\left((0.7/\wp_4,\varepsilon_3,0),\left\{\frac{\hbar_1}{0.4},\frac{\hbar_2}{0.7},\frac{\hbar_3}{0.6},\frac{\hbar_4}{0.9},\frac{\hbar_5}{0.5}\right\}\right),\left((0.6/\wp_5,\varepsilon_1,0),\left\{\frac{\hbar_1}{0.2},\frac{\hbar_2}{0.6},\frac{\hbar_3}{0.9},\frac{\hbar_4}{0.4},\frac{\hbar_5}{0.8}\right\}\right)\\
\left((0.6/\wp_5,\varepsilon_2,0),\left\{\frac{\hbar_1}{0.3},\frac{\hbar_2}{0.2},\frac{\hbar_3}{0.3},\frac{\hbar_4}{0.2},\frac{\hbar_5}{0.9}\right\}\right),\left((0.6/\wp_5,\varepsilon_3,0),\left\{\frac{\hbar_1}{0.1},\frac{\hbar_2}{0.5},\frac{\hbar_3}{0.5},\frac{\hbar_4}{0.1},\frac{\hbar_5}{0.2}\right\}\right)
\end{array} \right\}.$$

Step 2. The agree FPFHSE-set $(\mho, \mathcal{K})_{ag}$ for the FPFHSE-set $(\mho, \mathcal{K})$ is presented in Table 1. Each component $\hbar_{si}$ in Table 1 is obtained as $\hbar_{si} = \mho_{(\hbar_i)}(\zeta(\wp_j)/\wp_j, \varepsilon_k, 1)$, where $s = (j-1) \times 3 + k$ for $j \in \{1, 2, 3, 4, 5\}$ and $k \in \{1, 2, 3\}$. Then, we can say that $1 \leq s \leq 5 \times 3$.

Table 1. Agree FPFHSE-set for $(\mho, \mathcal{K})$.

$(\mho,\mathcal{K})_{ag}$	$\hbar_1$	$\hbar_2$	$\hbar_3$	$\hbar_4$	$\hbar_5$
$(0.2/\wp_1,\varepsilon_1)$	0.9	0.3	0.6	0.3	0.3
$(0.2/\wp_1,\varepsilon_2)$	0.8	0.1	0.6	0.6	0.9
$(0.2/\wp_1,\varepsilon_3)$	0.7	0.3	0.3	0.3	0.6
$(0.4/\wp_2,\varepsilon_1)$	0.5	0.6	0.3	0.4	0.7
$(0.4/\wp_2,\varepsilon_2)$	0.5	0.6	0.3	0.4	0.7
$(0.4/\wp_2,\varepsilon_3)$	0.4	0.3	0.3	0.8	0.3
$(0.5/\wp_3,\varepsilon_1)$	0.2	0.5	0.6	0.7	0.5
$(0.5/\wp_3,\varepsilon_2)$	0.2	0.4	0.5	0.3	0.8
$(0.5/\wp_3,\varepsilon_3)$	0.3	0.6	0.4	0.3	0.3
$(0.7/\wp_4,\varepsilon_1)$	0.9	0.9	0.5	0.4	0.1
$(0.7/\wp_4,\varepsilon_2)$	0.8	0.4	0.2	0.7	0.6
$(0.7/\wp_4,\varepsilon_3)$	0.6	0.5	0.3	0.6	0.3
$(0.6/\wp_5,\varepsilon_1)$	0.5	0.2	0.9	0.1	0.7
$(0.6/\wp_5,\varepsilon_2)$	0.4	0.6	0.4	0.6	0.8
$(0.6/\wp_5,\varepsilon_3)$	0.2	0.6	0.3	0.5	0.8
$\rho_i = \frac{1}{5\times 3} \times \sum_{s=1}^{5\times 3} \hbar_{si} \times \zeta(\wp_{([[1+\frac{s-1}{3}]])})$	$\rho_1 = 0.244$	$\rho_2 = 0.244$	$\rho_3 = 0.2046$	$\rho_4 = 0.2293$	$\rho_5 = 0.2613$

The disagree FPFHSE-set $(\mho, \mathcal{K})_{dag}$ for the FPFHSE-set $(\mho, \mathcal{K})$ is presented in Table 2. Each component $\hbar_{ti}$ in Table 2 is obtained as $\hbar_{ti} = \mho_{(\hbar_i)}(\zeta(\wp_j)/\wp_j, \varepsilon_k, 0)$, where $t = (j-1) \times 3 + k$ for $j \in \{1, 2, 3, 4, 5\}$ and $k \in \{1, 2, 3\}$. Then, we can say that $1 \leq t \leq 5 \times 3$.

Table 2. Disagree FPFHSE-set for $(\mho, \mathcal{K})$.

$(\mho, \mathcal{K})_{dag}$	$\hbar_1$	$\hbar_2$	$\hbar_3$	$\hbar_4$	$\hbar_5$
$(0.2/\wp_1, \varepsilon_1)$	0.1	0.9	0.8	0.6	0.2
$(0.2/\wp_1, \varepsilon_2)$	0.7	0.6	0.2	0.8	0.7
$(0.2/\wp_1, \varepsilon_3)$	0.2	0.6	0.1	0.1	0.4
$(0.4/\wp_2, \varepsilon_1)$	0.1	0.4	0.2	0.3	0.8
$(0.4/\wp_2, \varepsilon_2)$	0.2	0.7	0.6	0.5	0.6
$(0.4/\wp_2, \varepsilon_3)$	0.7	0.3	0.1	0.3	0.2
$(0.5/\wp_3, \varepsilon_1)$	0.9	0.4	0.5	0.1	0.1
$(0.5/\wp_3, \varepsilon_2)$	0.8	0.2	0.3	0.5	0.7
$(0.5/\wp_3, \varepsilon_3)$	0.6	0.6	0.5	0.4	0.1
$(0.7/\wp_4, \varepsilon_1)$	0.5	0.3	0.2	0.3	0.5
$(0.7/\wp_4, \varepsilon_2)$	0.9	0.5	0.3	0.6	0.4
$(0.7/\wp_4, \varepsilon_3)$	0.4	0.7	0.6	0.9	0.5
$(0.6/\wp_5, \varepsilon_1)$	0.2	0.6	0.9	0.4	0.8
$(0.6/\wp_5, \varepsilon_2)$	0.3	0.2	0.3	0.2	0.9
$(0.6/\wp_5, \varepsilon_3)$	0.1	0.5	0.5	0.1	0.2
$\eta_i = \frac{1}{5\times3} \times \sum_{t=1}^{5\times3} \hbar_{ti} \times \zeta(\wp_{([\|1+\frac{t-1}{3}\|])})$	$\eta_1 = 0.2246$	$\eta_2 = 0.2273$	$\eta_3 = 0.2033$	$\eta_4 = 0.1946$	$\eta_5 = 0.23$

Steps 3–6. Considering ρ_i for the agree FPFHSE-set in Table 1 and η_i for the disagree FPFHSE-set in Table 2, we obtained the decision values o_i and ranked them as in Table 3.

Table 3. Ranking order of solar panels for selection (for FPFHSE-set).

$\rho_i = \frac{1}{15} \times \sum_{s=1}^{15} \hbar_{si} \times \zeta(\wp_{([\|1+\frac{s-1}{3}\|])})$	$\eta_i = \frac{1}{15} \times \sum_{t=1}^{15} \hbar_{ti} \times \zeta(\wp_{([\|1+\frac{t-1}{3}\|])})$	$o_i = \frac{1}{2} \times (1 + \frac{\rho_i \times (\rho_i - \eta_i)}{\rho_i + \eta_i})$	Ranking
$\rho_1 = 0.244$	$\eta_1 = 0.2246$	$o_1 = 0.5050$	3
$\rho_2 = 0.244$	$\eta_2 = 0.2273$	$o_2 = 0.5043$	4
$\rho_3 = 0.2046$	$\eta_3 = 0.2033$	$o_3 = 0.5003$	5
$\rho_4 = 0.2293$	$\eta_4 = 0.1946$	$o_4 = 0.5093$	1
$\rho_5 = 0.2613$	$\eta_5 = 0.23$	$o_5 = 0.5083$	2

The graphical representation of ranking of the alternatives is shown in Figure 3.

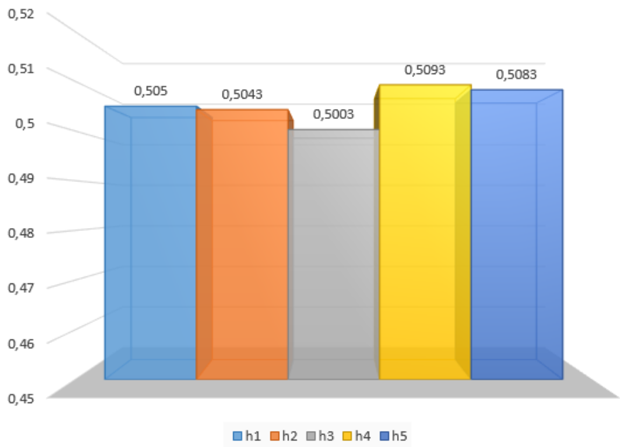

Figure 3. Ranking of alternatives for Algorithm 1.

Decision. Since $max\{o_1, o_2, o_3, o_4, o_5\} = \{0.5050, 0.5043, 0.5003, 0.5093, 0.5083\} = 0.5093 = o_4$, it can be said that the solar panel $\hbar_4$ is the best option to buy.

5. An MADM model based on a weighted FPFHSE-set

In this section, we present the concept of a weighted fuzzy parameterized fuzzy hypersoft expert set (WFPFHSE-set), and then propose an approach to MADM.

Definition 5.1. Let $(\mho, \mathbb{S})$ be an FPFHSE-set. A triplet $(\mho, \mathbb{S}, \varpi)$ is called a WFPFHSE-set for $(\mho, \mathbb{S})$, where the weight for $x_k \in \mathcal{I}$ is denoted by ϖ_k and $\varpi = (\varpi_1, \varpi_2, ..., \varpi_{|\mathcal{I}|})$.

When we consider the MADM problems in which experts may not be of equal importance at the evaluation stage, the WFPFHSE-set is an effective mathematical framework for modeling such problems. In other words, the WFPFHSE-set is an approach that takes into account the importance weights of experts in the evaluation in the FPFHSE-set environment. Considering the importance weights of experts in the evaluation in the FPFHSE-set environment, the following algorithm (Algorithm 2) is proposed for the selection.

The step-wise elaboration of the proposed algorithm is hereby presented below.

Note that the agree-oriented decision value ρ_i^{ϖ}, the disagree-oriented decision value η_i^{ϖ} and the overall decision value o_i^{ϖ} for the WFPFHSE-set $(\mho, \mathcal{K}, \varpi)$ are between 0 and 1, that is, $0 \leq \rho_i^{\varpi} \leq 1$, $0 \leq \eta_i^{\varpi} \leq 1$ and $0 \leq o_i^{\varpi} \leq 1$. The flowchart of Algorithm 2 is given in Figure 4.

As the Algorithm 1 consists of only three stages with seven steps, the input stage is based on simple sets and the construction and computation stages are composed of mathematical formulations that can easily be instructed to any mathematical cum computational software. Due to the smaller size (few steps) and convenient computations, the algorithm is expected to execute in a faster way as compared to the usual execution speed preserving the reliability and consistency.

Algorithm 2. MADM-based optimum selection of solar panels by using aggregations of weighted FPFHSE-sets.

Start Construction

(1) Construct WFPFHSE-set $(\mho, \mathcal{K}, \varpi)$ on $\hat{\Delta} = \{\hbar_i : \ i \in \{1, 2, ..., p\}\}$.

(2) $\mathcal{K} = \Bbbk \times \mathcal{I} \times \mathcal{U}$, where $\Bbbk$ is the set of fuzzy-valued multi disjoint attributes, $\mathcal{I}$ is the set of experts with the weight vector $\varpi = (\varpi_1, \varpi_2, ..., \varpi_{|\mathcal{I}|})$ and $\mathcal{U}$ is the set of opinions.

Computation

(3) Determine an Agree WFPFHSE-set $(\mho, \mathcal{K}, \varpi)_{ag}$ and a Disagree WFPFHSE-set $(\mho, \mathcal{K}, \varpi)_{dag}$ for $(\mho, \mathcal{K}, \varpi)$.

(4) Compute the agree-oriented decision value

$$\rho_i^{\varpi} = \frac{1}{|\Bbbk| \times |\mathcal{I}|} \times \sum_{s=1}^{|\Bbbk|\times|\mathcal{I}|} (\hbar_{si} \times \zeta(\wp_{([|1+\frac{s-1}{|\mathcal{I}|}|])}))^{\varpi(\lceil s \rceil_{|I|})}$$

for each alternative $\hbar_i$ considering the agree WFPFHSE-set $(\mho, \mathcal{K}, \varpi)_{ag}$. Here, the value $\hbar_{si}$ is obtained as $\hbar_{si} = \mho_{(\hbar_i)}(\zeta(\wp_j)/\wp_j, \varepsilon_k, 1)$, where $s = (j-1) \times |I| + k$ for $j \in \{1, 2, ..., |\Bbbk|\}$ and $k \in \{1, 2, ..., |\mathcal{I}|\}$. Also, $\lceil s \rceil_{|I|} = k$ means $k \equiv s \ mod(|I|)$ such that $k \le |I|$.

(5) Compute the disagree-oriented decision value

$$\eta_i^{\varpi} = \frac{1}{|\Bbbk| \times |\mathcal{I}|} \times \sum_{t=1}^{|\Bbbk|\times|\mathcal{I}|} (\hbar_{ti} \times \zeta(\wp_{([|1+\frac{t-1}{|\mathcal{I}|}|])}))^{\varpi(\lceil t \rceil_{|I|})}$$

for each alternative $\hbar_i$ considering the disagree WFPFHSE-set $(\mho, \mathcal{K}, \varpi)_{ag}$. Here, the value $\hbar_{ti}$ is obtained as $\hbar_{ti} = \mho_{(\hbar_i)}(\zeta(\wp_j)/\wp_j, \varepsilon_k, 1)$, where $t = (j-1) \times |I| + k$ for $j \in \{1, 2, ..., |\Bbbk|\}$ and $k \in \{1, 2, ..., |\mathcal{I}|\}$. Also, $\lceil t \rceil_{|I|} = k$ means $k \equiv t \ mod(|I|)$ such that $k \le |I|$.

(6) Find the overall decision value

$$o_i^{\varpi} = \frac{1}{2} \times (1 + \frac{\rho_i^{\varpi} \times (\rho_i^{\varpi} - \eta_i^{\varpi})}{\rho_i^{\varpi} + \eta_i^{\varpi}})$$

for each alternative $\hbar_i$.

Output

(7) Determine v for selection of best alternative, where $o_v^{\varpi} = max\{o_1^{\varpi}, o_2^{\varpi}, ...\}$, and make decision.

End

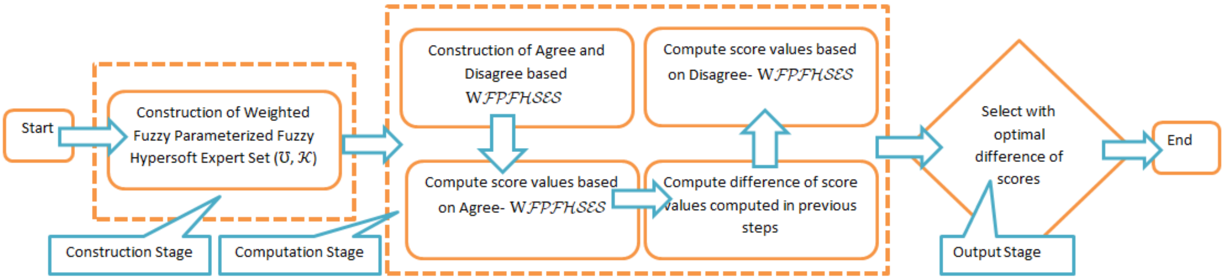

Figure 4. Flow diagram of Algorithm 2.

Example 5.1. Reconsider the MADM problem given in Example 4.1 with following weights assigned to the experts. Suppose the weights of the experts ε_1, ε_2 and ε_3 are respectively 0.3, 0.5 and 0.8, i.e., $\varpi = (0.3, 0.5, 0.8)$. Now, Algorithm 2 is used to choose a solar panel product from a market. The agree WFPFHSE-set $(\mho, \mathcal{K}, \varpi)_{ag}$ and disagree WFPFHSE-set $(\mho, \mathcal{K}, \varpi)_{dag}$ for the WFPFHSE-set $(\mho, \mathcal{K}, \varpi)$ are presented in Tables 4 and 5, respectively. Considering ρ_i^{ϖ} for the agree WFPFHSE-set in Table 4 and η_i^{ϖ} for the disagree WFPFHSE-set in Table 5, we obtained the decision values o_i^{ϖ} and ranked them as in Table 6. Figure 5 provides a graphic depiction of the ranking of options.

Decision. Since $max\{o_1^{\varpi}, o_2^{\varpi}, o_3^{\varpi}, o_4^{\varpi}, o_5^{\varpi}\} = \{0.5085, 0.4872, 0.4950, 0.5087, 0.5109\} = 0.5109 = o_5^{\varpi}$, it can be said that the solar panel $\hbar_5$ is the best option to buy.

Table 4. Agree WFPFHSE-set for $(\mho, \mathcal{K}, \varpi)$.

$(\mho, \mathcal{K}, \varpi)_{ag}$	$\hbar_1$	$\hbar_2$	$\hbar_3$	$\hbar_4$	$\hbar_5$
$(0.2/\wp_1, 0.3/\varepsilon_1)$	0.9	0.3	0.6	0.3	0.3
$(0.2/\wp_1, 0.5/\varepsilon_2)$	0.8	0.1	0.6	0.6	0.9
$(0.2/\wp_1, 0.8/\varepsilon_3)$	0.7	0.3	0.3	0.3	0.6
$(0.4/\wp_2, 0.3/\varepsilon_1)$	0.5	0.6	0.3	0.4	0.7
$(0.4/\wp_2, 0.5/\varepsilon_2)$	0.5	0.6	0.3	0.4	0.7
$(0.4/\wp_2, 0.8/\varepsilon_3)$	0.4	0.3	0.3	0.8	0.3
$(0.5/\wp_3, 0.3/\varepsilon_1)$	0.2	0.5	0.6	0.7	0.5
$(0.5/\wp_3, 0.5/\varepsilon_2)$	0.2	0.4	0.5	0.3	0.8
$(0.5/\wp_3, 0.8/\varepsilon_3)$	0.3	0.6	0.4	0.3	0.3
$(0.7/\wp_4, 0.3/\varepsilon_1)$	0.9	0.1	0.5	0.4	0.1
$(0.7/\wp_4, 0.5/\varepsilon_2)$	0.8	0.4	0.2	0.7	0.6
$(0.7/\wp_4, 0.8/\varepsilon_3)$	0.6	0.1	0.3	0.6	0.3
$(0.6/\wp_5, 0.3/\varepsilon_1)$	0.5	0.2	0.1	0.1	0.7
$(0.6/\wp_5, 0.5/\varepsilon_2)$	0.4	0.6	0.4	0.6	0.8
$(0.6/\wp_5, 0.8/\varepsilon_3)$	0.2	0.6	0.3	0.5	0.8
$\rho_i^{\varpi} = \frac{1}{5\times3} \times \sum_{s=1}^{5\times3} (\hbar_{si} \times \zeta(\wp_{([\|1+\frac{s-1}{3}\|])}))^{\varpi(\lceil s \rceil_3)}$	$\rho_1^{\varpi} = 0.4683$	$\rho_2^{\varpi} = 0.4106$	$\rho_3^{\varpi} = 0.4051$	$\rho_4^{\varpi} = 0.4593$	$\rho_5^{\varpi} = 0.4899$

*Note: In this table, $\lceil s \rceil_3 = k$ means $k \equiv s \; mod(3)$ such that $k \leq 3$.

Table 5. Disagree WFPFHSE-set for $(\mho, \mathcal{K}, \varpi)$.

$(\mho, \mathcal{K}, \varpi)_{dag}$	$\hbar_1$	$\hbar_2$	$\hbar_3$	$\hbar_4$	$\hbar_5$
$(0.2/\wp_1, 0.3/\varepsilon_1)$	0.1	0.9	0.8	0.6	0.2
$(0.2/\wp_1, 0.5/\varepsilon_2)$	0.7	0.6	0.2	0.8	0.7
$(0.2/\wp_1, 0.8/\varepsilon_3)$	0.2	0.6	0.1	0.1	0.4
$(0.4/\wp_2, 0.3/\varepsilon_1)$	0.1	0.4	0.2	0.3	0.8
$(0.4/\wp_2, 0.5/\varepsilon_2)$	0.2	0.7	0.6	0.5	0.6
$(0.4/\wp_2, 0.8/\varepsilon_3)$	0.7	0.3	0.1	0.3	0.2
$(0.5/\wp_3, 0.3/\varepsilon_1)$	0.9	0.4	0.5	0.1	0.1
$(0.5/\wp_3, 0.5/\varepsilon_2)$	0.8	0.2	0.3	0.5	0.7
$(0.5/\wp_3, 0.8/\varepsilon_3)$	0.6	0.6	0.5	0.4	0.1
$(0.7/\wp_4, 0.3/\varepsilon_1)$	0.5	0.3	0.2	0.3	0.5
$(0.7/\wp_4, 0.5/\varepsilon_2)$	0.9	0.5	0.3	0.6	0.4
$(0.7/\wp_4, 0.8/\varepsilon_3)$	0.4	0.7	0.6	0.9	0.5
$(0.6/\wp_5, 0.3/\varepsilon_1)$	0.2	0.6	0.9	0.4	0.8
$(0.6/\wp_5, 0.5/\varepsilon_2)$	0.3	0.2	0.3	0.2	0.9
$(0.6/\wp_5, 0.8/\varepsilon_3)$	0.1	0.5	0.5	0.1	0.2
$\eta_i^{\varpi} = \frac{1}{5\times 3} \times \sum_{t=1}^{5\times 3} (\hbar_{ti} \times \zeta(\wp_{([\vert 1+\frac{t-1}{3}\vert])}))^{\varpi(\lceil t\rceil_3)}$	$\eta_1^{\varpi} = 0.4352$	$\eta_2^{\varpi} = 0.4652$	$\eta_3^{\varpi} = 0.4254$	$\eta_4^{\varpi} = 0.4256$	$\eta_5^{\varpi} = 0.4480$

*Note: In this table, $\lceil t\rceil_3 = k$ means $k \equiv t \ mod(3)$ such that $k \leq |I|$.

Table 6. Ranking order of solar panels for selection (for WFPFHSE-set).

$\rho_i^{\varpi} = \frac{1}{15} \times \sum_{s=1}^{15} (\hbar_{si} \times \zeta(\wp_{([\vert 1+\frac{s-1}{3}\vert])}))^{\varpi(\lceil s\rceil_{\vert I\vert})}$	$\eta_i^{\varpi} = \frac{1}{15} \times \sum_{t=1}^{15} (\hbar_{ti} \times \zeta(\wp_{([\vert 1+\frac{t-1}{3}\vert])}))^{\varpi(\lceil t\rceil_3)}$	$o_i^{\varpi} = \frac{1}{2} \times (1 + \frac{\rho_i^{\varpi}\times(\rho_i^{\varpi}-\eta_i^{\varpi})}{\rho_i^{\varpi}+\eta_i^{\varpi}})$	Ranking
$\rho_1^{\varpi} = 0.4683$	$\eta_1^{\varpi} = 0.4352$	$o_1^{\varpi} = 0.5085$	3
$\rho_2^{\varpi} = 0.4106$	$\eta_2^{\varpi} = 0.4652$	$o_2^{\varpi} = 0.4872$	5
$\rho_3^{\varpi} = 0.4051$	$\eta_3^{\varpi} = 0.4254$	$o_3^{\varpi} = 0.4950$	4
$\rho_4^{\varpi} = 0.4593$	$\eta_4^{\varpi} = 0.4256$	$o_4^{\varpi} = 0.5087$	2
$\rho_5^{\varpi} = 0.4899$	$\eta_5^{\varpi} = 0.4480$	$o_5^{\varpi} = 0.5109$	1

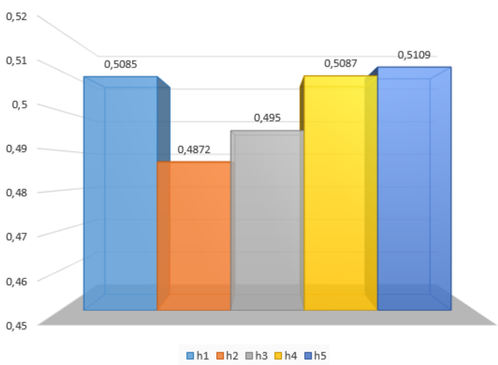

Figure 5. Ranking of alternatives for Algorithm 2.

By comparing the results in Tables 3 and 6 obtained by using Algorithm 1 and Algorithm 2, it can be said that the weights of the experts may affect the results. Consequently, $\hbar_4$ is selected without assigning weights to the experts and $\hbar_5$ is selected with weights assigning to experts. The ranking-based comparison of both algorithms is presented in Figure 6.

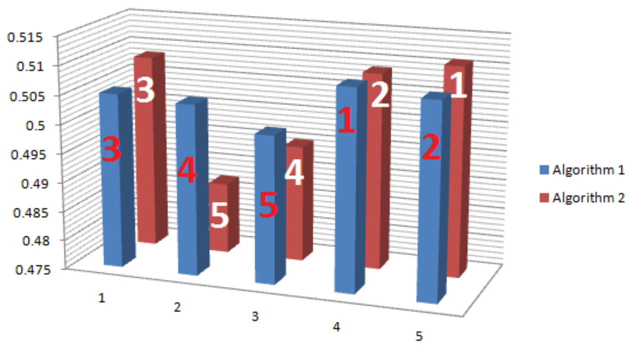

Figure 6. Comparison of both algorithms on the basis of computed ranking.

6. Comparison and discussion on FPFHSE-sets

This section is devoted to the comparison, discussion and merits for the proposed FPFHSE-sets.

6.1. Comparison

In comparison to many of the presently available S-set extensions, an FPFHSE model offers the highest rapport, accuracy, and agreeability. Comparing the FPFHSE-set to other models such as the FPFS-set, FPFSE-set and FPHS-set, makes it simple to observe this advantage. Because it incorporates the multi-argument approximate function (Maaf), which is very successful in MADM issues in a fuzzy environment, this suggested set model is more beneficial than the others. Table 7 illustrates a comparison between the proposed set model and other fuzzy models.

Table 7. Comparison analysis.

Methods	FPFS-set	FPFSE-set	FPFHS-set	FPFHSE-set
Multi Decisive Opinion	↓	↓	↓	↑
Multi Argument Appro. Function	↓	↓	↑	↑
Single Argument Appro. Function	↑	↑	↑	↑
Weight for Experts	↓	↓	↓	↑
Ranking	↓	↑	↓	↑

6.2. Discussion

In this part, a useful discussion is made about the proposed set structure, namely, the FPFHSE set.

(1) If the membership values of alternatives in the FPFHSE-set are not considered, then the FPHSE-set is obtained.

(2) If the expert set in the FPFHSE-set is excluded, it changes into the FPFHS-set.
(3) If the single argument approximate function is used instead of the Maaf, it reduces to the FPFSE-set.
(4) It becomes an FPFS-set if both the Maaf and the expert set are not considered.
(5) It takes the form of an FPS-set when the single argument approximate function is used and Maaf, the membership values of alternatives and the expert set are excluded.

6.3. Advantages of proposed model (FPFHSE-set)

The advantages of FPFHSE-sets are presented as follows:

(1) The created technique recognized the significance of the FHSE-set parameterization concept to address contemporary DM challenges. In addition to these points, this association has fantastic ability for real representation in the domain of computational invasions. The fuzzy parameterization mirrors the opportunity of the lifestyles to the degree of recognition and excusal.
(2) The proposed model gave several contradictory features under multiple deciding viewpoints their proper consideration, making the DM the best, most flexible, and most dependable.
(3) All of the elements and characteristics of current models like the FPHS-set, FPS-set, FPSE-set, FHSE-set, HSE-set, and S-set are incorporated into the suggested framework.

The benefits of the FPFHSE-set are illustrated in Table 8. The membership value (MV), degree of parameterization (DOP), single argument approximate function (SAAF), multi-argument approximate function (MAAF), and multi-decisive opinion (MDO) of current F-set extensions are some of the properties of the FPFHSE-set that are compared in this table.

Table 8. Advantages of FPFHSE-set under particular conditions.

Authors	Models	MV	DOP	SAAF	MAAF	MDO
Molodtsov [2]	S-set	↓	↓	↑	↓	↓
Maji et al. [6]	FS-set	↑	↓	↑	↓	↓
Ihsan et al. [35]	HSE-set	↓	↓	↑	↑	↑
Ihsan et al. [36]	FHSE-set	↑	↓	↑	↑	↑
Çağman et al. [40]	FPFS-set	↑	↑	↑	↓	↓
Bashir et al. [43]	FPFSE-set	↑	↑	↑	↓	↑
Rahman et al. [45]	FPHS-set	↑	↑	↑	↑	↓
Proposed Structure	FPFHSE-set	↑	↑	↑	↑	↑

In Tables 7 and 8, ↑ and ↓ mean *yes* and *no*, respectively. From Tables 7 and 8, it is clear that our proposed model is more generalized than some of the existing fuzzy models.

7. Conclusions

This paper focused on the fundamentals of FPFHSE-sets and thus presented their operations such as the union, intersection, complement, AND-product and OR-product with the basic properties like commutative, associative, distributive and De Morgan laws. Two new proposed algorithms were developed to cope with MADM problems regarding the selection of the best product. The FPFHSE-set models were compared with some of the existing F-set and S-set-like models, and the advantages of the newly proposed sets were discussed. The FPFHSE-set presented in this study is an extended type of many existing fuzzy soft models such as the FS-set, FPFS-set, FHS-set, etc. By using the developed algorithms, the solutions can be proposed to the MADM problems involving multi-disjoint attributes in a fuzzy environment. Thus, the MADM problems that current fuzzy and hypersoft models cannot cope with can be solved. Although the proposed model, i.e., the FPFHSE-set has many preferential features yet it has some limitations like it is unable to cope with the situations where the uncertainty of parameters is in terms of intuitionistic fuzzy or neutrosophic settings. Also it is not sufficient for the scenarios where parameters-based approximation of alternatives is in terms of intuitionistic fuzzy, Pythagorean fuzzy, spherical fuzzy, picture fuzzy or neutrosophic settings. Therefore future work may include the addressing of these limitations. Also the bipolarity, hesitant and dual types of FPFHSE-sets can be developed and their characteristics can be discussed in detail.

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Conflict of interest

The authors declare no conflicts of interest.

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