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Research article

Data-driven two-stage sparse distributionally robust risk optimization model for location allocation problems under uncertain environment

Zhimin Liu*

School of Mathematics Science, Liaocheng University, Shandong, China

* **Correspondence:** Email: lzmliuzhimin@163.com.

Abstract: Robust optimization is a new modeling method to study uncertain optimization problems, which is to find a solution with good performance for all implementations of uncertain input. This paper studies the optimal location allocation of processing plants and distribution centers in uncertain supply chain networks under the worst case. Considering the uncertainty of the supply chain and the risk brought by the uncertainty, a data-driven two-stage sparse distributionally robust risk mixed integer optimization model is established. Based on the complexity of the model, a distribution-separation hybrid particle swarm optimization algorithm (DS-HPSO) is proposed to solve the model, so as to obtain the optimal location allocation scheme and the maximum expected return under the worst case. Then, taking the fresh-food supply chain under the COVID-19 as an example, the impact of uncertainty on location allocation is studied. This paper compares the data-driven two-stage sparse distributionally robust risk mixed integer optimization model with the two-stage sparse risk optimization model, and the data results show the robustness of this model. Moreover, this paper also discusses the impact of different risk weight on decision-making. Different decision makers can choose different risk weight and obtain corresponding benefits and optimal decisions. In addition, the DS-HPSO is compared with distribution-separation hybrid genetic algorithm and distributionally robust L-shaped method to verify the effectiveness of the algorithm.

Keywords: supply chain management; two-stage sparse distributionally robust risk mixed integer optimization model; location allocation; uncertainty; DS-HPSO algorithm
Mathematics Subject Classification: 90B06, 90C90

1. Introduction

Location allocation is one of the most critical and strategic issues in the design and management of supply chain networks, and has always been the focus of attention and research by many scholars [1–6]. According to statista database, the global supply chain management market maintained a stable growth from 2020 to 2026. In 2020, the global supply chain management market was valued at 15.85 billion U.S. dollars and is expected to reach almost 31 billion U.S. dollars by 2026 (www.statista.com). Location decisions have a long-term impact on supply chain operations. The importance of location allocation is that once the location is selected and the distribution mode is determined, it will directly affect service mode, service quality, service cost and service efficiency, thus affecting enterprise profits and market competitiveness. Therefore, research of location allocation has significant economic and social significance.

In most studies of location problems, they tend to concentrate on the location of facilities [7–10], warehouses [11] and distribution centers [5, 12], and assume that uncertain probability information is known. However, in many cases, there may not be enough historical data or accurate prediction methods to obtain or forecast accurate probability information. Soyster proposes a robust optimization method to solve the problem of data uncertainty and the inability to obtain probabilistic information [13].

In supply chain management, there are many uncertain factors, many problems need robustness, and distributed robust optimization method can solve this problem [14–16]. The difference between distributed robust optimization method and stochastic programming is that distributed robust optimization method usually assumes that the probability distribution of random parameters is unknown in advance, and constructs an uncertain set with the real probability as the reference center.

In real life, due to market fluctuations, we can not get accurate information about transportation costs, demand and other parameters. Two-stage stochastic programming is an effective method to solve location allocation problem in an uncertain environment [7]. And the two-stage stochastic programming problems have been widely studied [11, 17–27]. Solving the two-stage stochastic optimization model is usually complex, because it needs to calculate the expected value of a multivariable random variable. Scenario based stochastic optimization method is a common method to solve two-stage stochastic optimization problems [28, 29]. However, the solution of scenario based stochastic optimization model largely depends on the defined scenario and its occurrence probability, and the increase of the number of scenarios, the solution of the model is easy to fall into the problem of dimension disaster.

In addition, most location problems are risk neutral, that is, taking the expected cost (or expected profit) as the objective function without considering the risk [5, 8–11, 20, 24, 26, 27, 30, 31]. However, compared with the risk neutral method, the risk aversion method considers the influence of the variability of random results, and provides a more effective solution to the decision-making problem in an uncertain environment. Decision makers can evaluate strategies based on their risk preferences. Therefore, in view of these problems in the actual market fluctuations, this paper considers the relevant risks in order to get a more effective solution.

Recently, sparse optimization methods have attracted a lot of interest in computer vision, image processing, bioinformatics and portfolio [32–38]. According to the data structure and sparse form under the background of practical application, choosing the appropriate sparse optimization model and design-related algorithm can achieve the purpose of solving the problem quickly and effectively. The advantage of sparse representation is to reduce the complexity of representation, and the more straightforward reason is to reduce coefficient parameters. Through sparse representation, the information contained in the data can be given full play, and redundant data information can be removed to maximize the utilization of data. As far as we know, there is still a lack of research on sparse location allocation of the supply chain. Therefore, in order to fill this gap, sparse representation is applied in the modeling of this paper.

Based on the above discussion, this paper studies how to construct a data-driven two-stage sparse distributionally robust risk mixed integer optimization model to discuss the location optimization problem under the uncertainty of transportation cost and demand. In the first stage, the location variable is the decision variable in the supply chain network. These decision variables are determined according to the realization of different scenarios in the second stage; Therefore, the decision-making process is called "here and now". The variable in the second stage is the transportation volume between each link in the supply chain network. Different from the classical two-stage random location problem, the model in this paper does not assume the known probability distribution of random transportation cost and demand. Instead, using the historical data of sales and transportation costs, a data-driven approach is adopted to construct an uncertain set with the real probability distribution centered on the reference distribution.

Note that the data-driven two-stage sparse distributionally robust risk mixed integer optimization model is at least as hard as the two-stage stochastic mixed binary programming, which is a NP-hard problem [39]. Many scholars have implemented the idea of distribution separation to solve robust optimization problems [40, 41]. In addition, Medsker proposed many ideas for designing hybrid intelligent algorithms, which laid a foundation for solving mixed integer programming problems [42]. Moreover, to the best of our knowledge, intelligent algorithms to solve data-driven two-stage sparse distributionally robust risk mixed integer optimization problems are lacking. Therefore, in order to fill this gap, this paper designs a distributed separation-hybrid particle swarm optimization algorithm to solve the data-driven two-stage sparse distributionally robust risk mixed integer optimization model based on the idea of distributed separation and hybrid intelligence algorithm. The main contributions of this paper are as following:

• Considering the uncertainty of supply chain, the data-driven two-stage sparse distributionally robust risk mixed integer optimization model is established.

• The data-driven two-stage sparse distributionally robust risk mixed integer optimization model is solved by combining the distributed separation algorithm with the hybrid intelligence algorithm. The effectiveness of the algorithm is verified by comparison.

• Based on numerical results, management insights are proposed.

The rest of this paper is set as follows. Section 2 we review the literature and identify research gaps compared to our contributions. Section 3 describes the concepts related to risk measurement and moment uncertainty. Data-driven two-stage sparse distributionally robust risk mixed integer optimization model is established in Section 4. In Section 5, an algorithm for solving the model is proposed. Section 6 gives an example to verify the effectiveness of the model and algorithm. Section 7 conclusion.

2. Literature review

The location problem has a very wide range of applications in production and life, logistics, and even military, such as the location of factories, warehouses, emergency centers, fire stations, garbage disposal centers, logistics centers, missile warehouses, etc. This problem has been the focus of many scholars' attention and research. Nick et al. established a model of distribution points and inventory allocation in post-disaster humanitarian logistics. They also stressed the importance of considering the temporal component of the system and the social costs incurred in facility location [3]. Ricciardi et al. considered the randomness of the throughput cost of facilities, derived the mixed deterministic nonlinear problem of the optimal location of facilities, and proposed two heuristic algorithms to solve the problem [8]. Liu et al. studied the optimal location of processing plants and distribution centers in a supply chain network with stochastic transportation costs and demand. Considering the uncertainty and risk of the supply chain, a two-stage mean risk stochastic mixed integer optimization model is established. An improved hybrid binary particle swarm optimization algorithm is proposed to solve the model [12]. Rahman et al. studied and developed a model to determine the optimal spatial distribution of emergency evacuation centers to improve the flood emergency planning in Sylhet region in northeastern Bangladesh. The proposed model can be used to improve the planning of the distribution of emergency evacuation centers. The application of the model will help reduce casualties and property losses, and improve emergency operations [43]. Qi et al. studied a competitive facility location problem and established a bilevel mixed integer nonlinear program model. They proposed an approximation algorithm to find high-quality solutions with constant approximation guarantees [44].

With the rapid development of economy, as well as the uncertainty factors in real life, we can not get accurate information about transportation costs, demand and other parameters. Noyan proposed that two-stage stochastic programming can effectively solve the location allocation problem under uncertain environment [7]. And two-stage stochastic programming problem has been widely studied. Chen et al. studied the two-stage stochastic and distributionally robust linear complementarity problems [17, 18]. Ricoramirez et al. studied the optimal location of booster disinfection stations in water distribution networks. To solve the problem, they established a two-stage stochastic mixed integer linear programming model and proposed a simplified stochastic method [24]. Dillon et al. studied the network optimization of the blood supply chain using a two-stage stochastic optimization model [26]. Liu et al. used a two-stage distributionally robust optimization model to study the size of medical service stations with chance constraints [27]. Misagh et al. studied the location-inventory-routing problem of the red meat supply chain in emerging economies. Considering variable costs, they established a two-stage mixed integer linear programming model, and gave operational decisions [45].

In supply chain management, due to the existence of many uncertain factors, many problems need robustness. The distributionally robust is a powerful tool to solve the problem of how to make decisions in an uncertain environment. Liu et al. studies the worst-case optimal location allocation of processing plants and distribution centers in supply chain networks under uncertain transportation costs and customer demands. Considering the uncertainty of the supply chain, they established a two-stage mixed integer optimization model with split blue bar, and proposed hybrid intelligent algorithm to solve the obtained model to obtain the optimal location and maximum expected revenue of the supply chain in the worst case [16]. Yang et al. studied the multi-period location allocation problem with multiple resource and capability levels under the condition of uncertain emergency demand and resource fulfillment time, established mixed integer linear programming, and proposed a branch-and-Benders-cut algorithm to solve it [46]. Liu et al. studies a broad class of facility location problems in the context of adaptive robust stochastic optimization. Considering the demand uncertainty, a robust optimization model is established, and a nested benders decomposition algorithm is proposed to solve the model [47]. Shehadeh studies the problem of fleet-sizing, routing and scheduling of mobile facilities with time-dependent and random demands, proposes two distributionally robust optimization models, and gives a decomposition-based algorithm to solve these models [48].

In addition, most location allocation problems are risk neutral, that is, risk is not considered [4–6, 16, 27, 44, 46–49]. However, considering risk provides a more effective solution to the decision-making problem under uncertain environment. Kim and Weber proposed an efficient algorithm for solving the worst-case risk averages for tail uncertainty. This method is applied in the context of financial markets and cyber risk [50]. Embrechts et al. studied robustness in the context of quantitative risk management and optimization. This paper mainly studies two risk measures, value-at-risk and expected shortfall. Through the application in banking and insurance supervision, additional insights are proposed [51]. Liu et al. proposed a distributionally robust optimization model based on kernel density estimation and mean entropic value-at-risk, which can be applied to portfolio selection, newsboy problem and linear two-stage stochastic programming. The numerical results show that the model has good application prospects [52]. Therefore, in view of the uncertainty of the actual problem, this paper considers the relevant risks in order to expect a more effective solution.

In the era of data, sparse model is widely used because of its excellent performance [35–38]. Liu et al. proposed a tracking algorithm based on local sparse model [35]. Zhang and Patel proposed a new cartoon and texture decomposition method based on convolutional sparse and low-rank coding for image processing. Numerical experiments show that the proposed method performs well compared with the existing image separation methods [37]. Bertsimas and Wright studied the problem of sparse portfolio selection, constructed a portfolio with maximum expected return and minimum variance, and proposed a scalable method to solve the problem. Numerical results show that this method can significantly improve the speed of sparse portfolio selection problem [38]. There are many problems in the supply chain that can be sparsely represented. To the best of our knowledge, there is still a lack of research on sparse location allocation in supply chains. Therefore, in order to fill this gap, sparse representation is applied in modeling in this paper.

To analyze the above studies, Table 1 creates a survey based on different criteria to determine the gap analysis of each research focus in the supply chain. Our criteria include location type, uncertain parameters, stochastic approach method, robustness, risk consideration and solution method. According to the research results in Table 1, the following research gaps are identified:

• Most recent studies mainly consider facility location to establish optimization models, and most of them consider the uncertainty of the supply chain.

• Two-stage, distributionally robust and risk aversion are recent literature. However, the simultaneous consideration of these factors in the location allocation problem has not been studied.

• Most recent studies put forward corresponding algorithms to solve the established model.

• Sparse representation has been applied in image processing and portfolio selection, but has not been studied in location allocation.

• Most recent studies have proposed a realistic case study to prove the applicability of the model.

In order to fill these research gaps, this paper proposes a data-driven two-stage sparse distributionally robust risk optimization model for location allocation problems under uncertain environment. The location of processing factories and distribution centers, and the uncertainty of demand and transportation costs are considered. Our supply chain network consists of four parts: suppliers, processing factories, distribution centers and consumers. The final contribution of this research is to propose a distributed separation hybrid particle swarm optimization algorithm for solving the case of fresh-food supply chain location allocation in the context of COVID-19.

Table 1. Gap analysis of various research focus of supply chain.

References	Location type	Uncertain parameters	Stochastic approach	Robust	Risk	Solving algorithm
[4]	Relief centers	Demand, supply	Two-stage	None-robust	None-risk	Heuristic
[5]	Distribution center	–	Two-stage	None-robust	None-risk	Matheuristic
[6]	Facility	Demand, transportation cost	Two-stage	None-Robust	None-risk	HIA
[7]	Emergency facility	Demand	Two-stage	None-robust	None-risk	Bender decomposition
[8]	Facility	Transportation cost	One-stage	None-robust	None-risk	Heuristic
[9]	Facility	Demand	Two-stage	None-robust	None-risk	L-shaped
[10]	Facility	Demand	One-stage	None-robust	None-risk	Combined simulated annealing
[11]	Factory, warehouse	–	Two-stage	None-robust	None-risk	Heuristic
[12]	Distribution center	Demand, transportation cost	Two-stage	None-robust	Risk	Hybrid intelligent algorithm
[16]	Facility, distribution center	Demand, transportation cost	Two-stage	Robust	None-Risk	Hybrid intelligent algorithm
[20]	Factory	–	Two-stage	None-robust	None-risk	Heuristic
[23]	–	Aquatic products	Two-stage	None-Robust	Risk	Cutting decomposition algorithm
[24]	Disinfection stations	Water quality	Two-stage	None-robust	None-risk	Stochastic decomposition
[26]	–	Demand	Two-stage	None-robust	None-risk	CPLEX
[27]	Medical service station	Demand	One-stage	Robust	None-risk	Outer approximation
[28]	–	Demand, buying cost	Two-stage	Robust	None-risk	CPLEX
[30]	Facility	Quantity of waste	Two-stage	Robust	None-risk	ϵ-constraint method
[31]	Facility	Transportation cost	Two-stage	Robust	None-risk	Memetic algorithm
[44]	Facility	–	Bilevel	None-Robust	None-risk	Approximation algorithm
[46]	Relief facility	Demand, fulfillment time	One-stage	Robust	None-risk	Branch-and-Benders-cut
[47]	Facility	Demand	Two-stage	Robust	None-risk	Benders decomposition
[48]	–	Demand	One-stage	Robust	None-Risk	Decomposition-based algorithm
[49]	Distribution center	–	One-stage	None-Robust	None-risk	Heuristic
[53]	–	Demand, prices	One-stage	None-Robust	None-Risk	Hybrid meta-heuristic algorithm
This paper	Factory, distribution center	Demand, transportation cost	Two-stage	Robust	Risk	DS-HPSO

3. Preliminaries

In this section, we establish a data-driven two-stage sparse distributionally robust risk mixed integer optimization model under the uncertainty of customer demand and transportation cost. Firstly, we review some basic concepts of risk measures and uncertainty sets to help readers understand the proposed data-driven two-stage sparse distributionally robust risk mixed integer optimization model

3.1. Conditional value-at-risk

Risk measures are functions that use scalar values to represent risks associated with random variables, and they provide a way to compare outcomes based on decision makers' appetite for risk [54]. CVaR is a risk measure with regular invariance and consistency, which is the basis of other risk measures [55, 56]. Rockafellar et al. introduced it into the optimization field, studied mathematical models of CVaR metrics, and transformed the corresponding mathematical models into equivalent convex stochastic optimization models [57]. Based on the nature of CVaR, we use CVaR to measure the supply chain cost risk caused by the uncertainty of demand and transportation cost. In order to solve the problem conveniently, we use some properties of CVaR to express CVaR as linear programming of discrete distribution. Next, we give some related definitions and relationships.

Definition 1. [57] For a random loss variable $\mathcal{X}$ and for any given confidence level $\alpha \in [0, 1)$, CVaR is defined as:

$$\text{CVaR}_{\alpha}(\mathcal{X}) = \min_{\eta \in \mathbb{R}}\{\eta + \frac{1}{1-\alpha}E([\mathcal{X} - \eta]_{+})\}, \tag{3.1}$$

where $[x]_+ = \max(0, x)$.

The random variable $\mathcal{X}$ is defined on a probability space $(\Omega, \mathcal{F}, \mathcal{P})$, where Ω denotes the sample space for $\mathcal{X}$, $\mathcal{F}$ denoting a $\sigma-$algebra on Ω, and $\mathcal{P}$ is a known probability distribution, respectively.

A risk-averse decision-maker could choose a larger α value e.g., $\alpha = 0.98$ for the confidence level. Besides this, the ability to specify the confidence level allows for greater flexibility to capture a wide range of risk preferences, including risk neutral ($\alpha = 0$) and pessimistic worst-case ($\alpha \rightarrow 1$) preferences.

Suppose $\mathcal{X}$ is a stochastic variable with realization $\chi_1, \cdots, \chi_j, \cdots, \chi_n$ and corresponding probabilities $p_1, \cdots, p_n$. Then CVaR can be equivalently expressed as the following linear programming [54],

$$\min\{\eta + \frac{1}{1-\alpha} \sum_{j \in [n]} p_j w_j : w_j \geq \chi_j - \eta \ \ \forall j \in [n], \mathbf{w} \in \mathbb{R}^n_+, \eta \in \mathbb{R}\}, \tag{3.2}$$

where $[n] = \{1, \cdots, n\}$, $\mathbf{w} = [w_1, \cdots, w_n]$.

In modeling, since it is impossible to know in advance the order of decision related implementations, the representation of Eq (3.2) is important for the development of mathematical programs.

3.2. Moment uncertainty

Let the random vector ξ be defined on $(\Omega, \mathcal{F}, \mathcal{P})$. we assume $\Omega = \mathbb{R}^n$, and let the first and second moment information of the random vector $\xi \in \Omega$ be known. Let $\mu = \mathbb{E}_p[\xi] \in \mathbb{R}^n$ and $\Upsilon = \mathbb{E}_p[(\xi - \mu)(\xi - \mu)^T] \in \mathbb{S}^n$ be the expected vector and the covariance matrix of random vector ξ, respectively. The ambiguity set that have the same first and second moments is defined as

$$\mathcal{P} := \{p : \int_{\xi \in \Omega} p(\xi) d\xi = 1, \int_{\xi \in \Omega} \xi p(\xi) d\xi = \mu, \int_{\xi \in \Omega} \xi \xi^T p(\xi) d\xi = \Upsilon + \mu\mu^T\}.$$

4. Two-stage sparse distributionally robust risk mixed integer optimization model

In this section, considering the uncertainty of transportation cost and customer demand in the supply chain, we establish a data-driven two-stage sparse distributionally robust risk mixed integer optimization model. The location variables of processing factories and distribution centers are the decision-making variables of the first stage. The second stage decision is related to transportation costs, inventory costs, and demand, and the goal is to minimize the worst-case cost function. At the same time, the CVaR of the whole cost function is also considered in the objective function. The structure of the location-allocation supply chain network is shown in Figure 1.

As shown in Figure 1, the supply chain consists of S types of suppliers, I types of processing factories, J types of distribution centers, and K types of customers. Suppliers supply V kinds of raw materials, processing plants produce L kinds of products, and then transport to the distribution center, which completes the distribution task, in order to meet the random needs of customers as much as possible. We assume that customer's demand cannot be overserved, but it is possible that they are not fully satisfied.

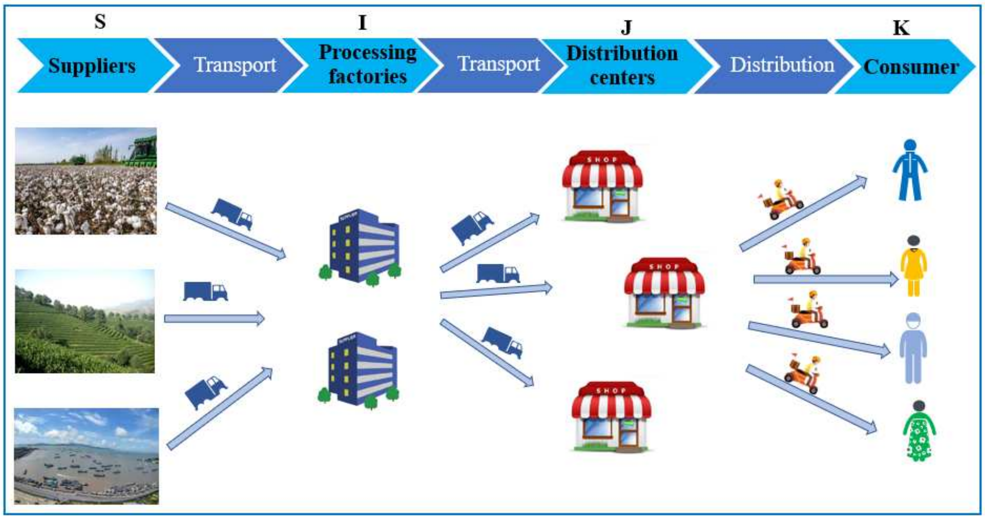


Figure 1. Network structure of location-allocation supply chain based on data-driven approach.

In the location allocation problem, we consider the supply capacity of the supplier and the cost of raw materials. The fixed cost, production cost and production capacity of the processing factory are considered. The fixed cost and distribution capacity of distribution center are considered, and the inventory cost of distribution center is also considered. Transportation costs between nodes in the supply chain network, as well as customer demand, are uncertain and generated by a data-driven approach. The enterprise's goal is to choose the optimal number and location of processing factories and distribution centers in the market area on the premise of satisfying customers' random demand as much as possible, so that the supply chain can obtain the maximum expected profit in the worst-case.

For simplicity, we apply the following symbols for this model. All the vectors used in this paper are assumed to be column vectors.

4.1. Notation

S The set of suppliers;
I The set of processing factories;
J The set of distribution centers;
K The set of customers;
V The set of raw materials;
L The set of products;
Parameters
f_i Fixed cost of operating processing factory i, $i \in I$;
g_j Fixed cost of operating distribution center j, $j \in J$;
a_{vs} The ability of supplier s to provide raw material v, $s \in S, v \in V$;
r_{vs} The cost of raw materials v provided by the supplier s, $s \in S, v \in V$;

n_{vl} The quantity of raw material v required for processing product l, $l \in L, v \in V$;
b_{il} The ability of processing factory i to produce product l, $i \in I, l \in L$;
q_{il} The cost of producing product l in the processing factory i, $i \in I, l \in L$;
τ_{lj} The ability of distribution center j to store product l, $j \in J, l \in L$;
w_{lj} The inventory cost of product l stored in distribution center j, $l \in L$, $j \in J$;
h_l Retail price of unit product l, $l \in L$;
$\tilde{t}_{vsi}$ Transportation cost of supplier s transporting raw material v to processing factory i, (Random Variable), $s \in S, v \in V, i \in I$;
$\tilde{m}_{lij}$ Transportation cost of processing factory i transporting product l to distribution center j, (Random Variable), $l \in L, i \in I, j \in J$;
$\tilde{u}_{ljk}$ Distribution cost of distribution center j transporting product l to consumer k, (Random Variable), $l \in L, j \in J, k \in K$;
$\tilde{d}_{lk}$ Demand of consumer k for product l, (Random Variable), $k \in K, l \in L$;
$\xi(\omega)$ Demand and transportation cost vector $\xi(\omega) = (\tilde{t}_{vsi}(\omega), \tilde{m}_{lij}(\omega), \tilde{u}_{ljk}(\omega), \tilde{d}_{lk}(\omega))$, $\omega \in \Omega$, (Random Variable);
λ Weight of CVaR;
$E(\cdot)$ Expectation;
First-stage decision variables
e_i 0-1 decision variable, 1 means open processing factory i, otherwise 0, $i \in I$;
c_j 0-1 decision variable, 1 means open distribution center j, otherwise 0, $j \in J$;
Second-stage decision variables
x_{vsi} The quantity of raw materials v transported by the supplier s to the processing factory i, $v \in V, s \in S, i \in I$;
y_{lij} The quantity of product l transported by the processing factory i to the distribution center j, $l \in L, i \in I, j \in J$;
z_{ljk} The quantity of product l transported by the distribution center j to the consumer k, $l \in L$, $j \in J, k \in K$.

4.2. The data-driven two-stage sparse distributionally robust risk mixed integer optimization model

Based on the above description, we establish a data-driven two-stage sparse distributionally robust risk mixed integer optimization model,

$$\begin{aligned} \min_{\mathbf{e},\mathbf{c}} \quad & \sum_I f_i e_i + \sum_J g_j c_j + \sup_{p \in \mathcal{P}} \{(1-\lambda) E_p[Q(\mathbf{e}, \mathbf{c}, \xi(\omega))] + \lambda \mathrm{CVaR}_\alpha(Q(\mathbf{e}, \mathbf{c}, \xi(\omega)))\} \\ \text{s.t.} \quad & \|\mathbf{e}\|_0 \le \tau_{\mathbf{e}}, \|\mathbf{c}\|_0 \le \tau_{\mathbf{c}}, \\ & e_i \in \{0,1\}, c_j \in \{0,1\}, \forall i \in I, j \in J, \end{aligned} \tag{4.1}$$

where $\mathbf{e} = (e_1, \cdots, e_I)$, $\mathbf{c} = (c_1, \cdots, c_J)$, $\alpha \in [0, 1)$, $\lambda \in [0, 1]$, $\|\mathbf{e}\|_0$ denotes the cardinality of the vector $\mathbf{e}$, that is, the number of nonzero elements in $\mathbf{e}$, $Q(\mathbf{e}, \mathbf{c}, \xi(\omega))$ is the optimal value of the second-stage problem

$$Q(\mathbf{e},\mathbf{c},\xi(\omega)) = \min_{\mathbf{x},\mathbf{y},\mathbf{z}} \sum_V \sum_S (r_{vs} \sum_I x_{vsi}) + \sum_I \sum_L (q_{il} \sum_J y_{lij}) \tag{4.2}$$
$$+ \sum_V \sum_S \sum_I \tilde{t}_{vsi}(\omega) x_{vsi} + \sum_L \sum_I \sum_J \tilde{m}_{lij}(\omega) y_{lij} + \sum_L \sum_J \sum_K \tilde{u}_{ljk}(\omega) z_{ljk}$$
$$+ \sum_L \sum_J (w_{lj}(\sum_I y_{lij} - \sum_K z_{ljk})) - \sum_L (h_l \sum_J \sum_K z_{ljk})$$

$$\text{s. t.} \quad \sum_I x_{vsi} \le a_{vs} \ \ \forall v,\ s, \tag{4.3a}$$
$$\sum_L (n_{vl} \sum_J y_{lij}) \le \sum_S x_{vsi} \ \ \forall v,\ i, \tag{4.3b}$$
$$\sum_J y_{lij} \le e_i b_{il} \ \ \forall i,\ l, \tag{4.3c}$$
$$\sum_K z_{ljk} \le \sum_I y_{lij} \ \ \forall l,\ j, \tag{4.3d}$$
$$\sum_K z_{ljk} \le c_j \tau_{lj} \ \ \forall l,\ j, \tag{4.3e}$$
$$\sum_J z_{ljk} \le \tilde{d}_{lk}(\omega) \ \ \forall l,\ k, \tag{4.3f}$$
$$x_{vsi}, y_{lij}, z_{ljk} \ge 0, \ \ \forall v,\ s,\ i,\ j,\ k,\ l. \tag{4.3g}$$

In the data-driven two-stage sparse distributionally robust risk mixed integer optimization model, the objective function (4.1) is to minimize the expected cost and risk of the supply chain in the worst case. The second stage problem (4.2) consists of the cost function and revenue from selling products. In particular, if $\lambda = 0$, (4.1) is a risk-neutral two-stage sparse distributionally robust mixed integer optimization model. Constraints (4.3a), (4.3c), and (4.3e) represent the constraints on the raw material supply capacity of suppliers, the production capacity of processing plants and distribution capacity of distribution centers, respectively. Constraints (4.3b) and (4.3d) represent the balance conditions for raw materials and products, respectively. Constraint (4.3f) ensure that consumer's demand are not overserved. Constraint (4.3g) ensures the nonnegative nature of the decision variable.

Next, we give a theorem to show that the worst-case distribution exists.

Theorem 1. If the uncertain set $\mathcal{P}$ is convex and closed, and the second stage objective function $Q(\mathbf{e},\mathbf{c},\xi(\omega))$ is concave and lower semi-continuous, then (4.1) there is a worst-case distribution.

Proof. From the question,

$$\mathcal{P} := \{p : \int_{\xi\in\Omega} p(\xi)d\xi = 1, \int_{\xi\in\Omega} \xi p(\xi)d\xi = \mu, \int_{\xi\in\Omega} \xi\xi^T p(\xi)d\xi = \Upsilon + \mu\mu^T\}.$$

Let $\forall p_1, p_2 \in \mathcal{P}, \beta \in (0,1)$, then

$$\int_{\xi\in\Omega} \beta p_1(\xi) + (1-\beta)p_2(\xi)d\xi = \beta \int_{\xi\in\Omega} p_1(\xi)d\xi + (1-\beta)\int_{\xi\in\Omega} p_2(\xi)d\xi = 1,$$

$$\int_{\xi\in\Omega} \xi(\beta p_1(\xi) + (1-\beta)p_2(\xi))d\xi = \beta \int_{\xi\in\Omega} \xi p_1(\xi)d\xi + (1-\beta) \int_{\xi\in\Omega} \xi p_2(\xi)d\xi = \beta\mu + (1-\beta)\mu = \mu,$$

$$\int_{\xi\in\Omega} \xi\xi^T(\beta p_1(\xi) + (1-\beta)p_2(\xi))d\xi = \beta(\Upsilon + \mu\mu^T) + (1-\beta)(\Upsilon + \mu\mu^T) = \Upsilon + \mu\mu^T,$$

get $\beta p_1(\xi) + (1-\beta)p_2(\xi) \in \mathcal{P}$, so $\mathcal{P}$ is convex, and by definition $\mathcal{P}$ is closed. In addition, $Q(\mathbf{e}, \mathbf{c}, \xi(\omega))$ is a linear function, so $Q(\mathbf{e}, \mathbf{c}, \xi(\omega))$ is concave, and the function $Q(\mathbf{e}, \mathbf{c}, \xi(\omega))$ is a continuous function under the constraints (4.3a)–(4.3g), so $Q(\mathbf{e}, \mathbf{c}, \xi(\omega))$ is a lower semi-continuous function. According to Peyman [58] corollary 4.6, the distribution in the worst-case must exist.

Note that the data-driven two-stage sparse distributionally robust risk mixed integer optimization model is at least as hard as the two-stage stochastic mixed binary program, which is an NP-hard problem. Moreover, the problem (4.2) has $|V||S|+|V||I|+|I||L|+2|L||J|+|L||K|+|V||S||I|+|L||I||J|+|L||J||K|$ constraints for each $\xi(\omega)$. When the random variable $\xi(\omega)$ or the supply chain network has a vast number of members, this makes models (4.1) and (4.2) very complicated and difficult to solve. Based on the complexity of the models (4.1) and (4.2), in the following part, we propose a DS-HPSO that can effectively solve it.

5. Distribution-separation hybrid particle swarm optimization algorithm for data-driven two-stage sparse distributionally robust risk mixed integer optimization model

This section focuses on the computation of the data-driven two-stage sparse distributionally robust risk mixed integer optimization models (4.1) and (4.2). Without loss of generality, in the following discussion, we denote by $\xi_n(\omega)$ the $n-th$ scenario, and the corresponding nominal probability is p_n (unknown), $n \in N$, where N is the set of scenarios.

The two-stage sparse distributionally robust risk process of location allocation problem is shown in Figure 2. The decision vector $\mathbf{e}$ and $\mathbf{c}$ are the first-stage decision which must be considered before the realizations of random transportation cost and demand vector $\xi(\omega)$ coming out. In the second-stage, the random transportation cost and demand $\xi(\omega)$, $\mathbf{e}$ and $\mathbf{c}$ are known, the decision variables are $\mathbf{x}$, $\mathbf{y}$, $\mathbf{z}$. The goal is to minimize the expected value of the cost function in the worst-case sparse distribution.

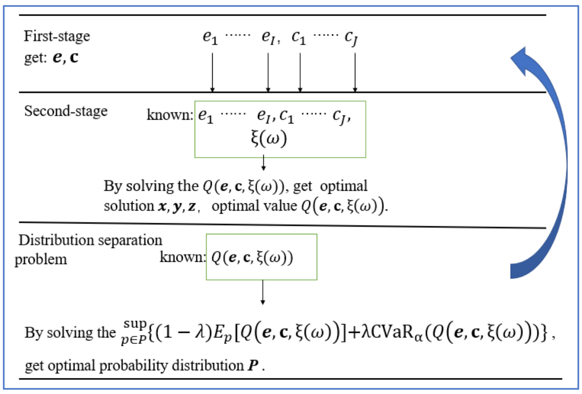


Figure 2. Two-stage sparse distributionally robust process of supply chain problem.

The optimization problem

$$Q := \sup_{p\in\mathcal{P}}\{(1-\lambda)E_p[Q(\mathbf{e},\mathbf{c},\xi(\omega))] + \lambda\mathrm{CVaR}_\alpha(Q(\mathbf{e},\mathbf{c},\xi(\omega)))\} \tag{5.1}$$

is called the distribution separation problem, and the algorithm to solve it is called the distribution separation algorithm [40, 41]. Some scholars have applied the idea of distribution separation to solve robust optimization problems [40, 41]. Moreover, Medsker proposed many ideas of designing hybrid intelligent algorithms, which laid a foundation for solving hybrid integer programming problems [42]. In addition, as far as we know, research of hybrid intelligence algorithm to solve data-driven two-stage sparse distributionally robust risk mixed integer optimization problem is still scarce. Therefore, in order to fill this gap, this paper designs a distributed separation-hybrid particle swarm optimization algorithm to solve the data-driven two-stage sparse distributionally robust risk mixed integer optimization model based on the idea of distributed separation and hybrid intelligence algorithm.

Among many intelligent algorithms, PSO has the advantages of fast search speed and high efficiency. Therefore, based on the above discussion and combined with the complexity of the problem considered in this paper, a DS-HPSO algorithm is proposed to solve the data-driven two-stage sparse distributionally robust risk mixed integer optimization model.

In order to use DS-HPSO to solve data-driven two-stage sparse distributionally robust risk mixed integer optimization model more effectively, we make the following modifications to DS-HPSO:

(a) Combine the synchronous dynamic learning factor with the compression factor. It is added into binary particle swarm optimization algorithm (B-PSO) to improve the convergence speed and ensure the global convergence of the algorithm.

(b) The boundary conditions are introduced in B-PSO to improve the search efficiency of particles and avoid the expansion and diffusion of the particle swarm.

(c) Combining the idea of distribution separation with hybrid B-PSO to form DS-HPSO.

Next, we introduce the detailed solution process of the algorithm.

Let $\mathbf{X} = (e_1, \cdots, e_I, c_1, \cdots, c_J)$, $D = I + J$. Suppose that in the D dimensional objective search space, there are N particles forming a community, in which the $i' - th$ particle is represented as a D-dimensional vector $\mathbf{X}_{i'} = (\chi_{i'1}, \cdots, \chi_{i'd}, \cdots, \chi_{i'D})$, $i' = 1, 2, \cdots, N'$. The flight velocity of the $i' - th$ particle is also a D-dimensional vector, which is written as $\mathscr{V}_{i'} = (v_{i'1}, v_{i'2}, \cdots, v_{i'D})$, $i' = 1, 2, \cdots, N'$. The personal best position and global best position of particle i' in the $k' - th$ iteration denoted as $P^{k'}_{bi'}$ and $P^{k'}_{gi'}$, respectively.

Denote $Fit(\cdot)$ as the fitness function,

$$Fit(\mathbf{X}) = -\{\sum_I f_i e_i + \sum_J g_j c_j + Q\},$$

where, $Q := \sup_{p\in\mathcal{P}}\{(1-\lambda)E[Q(\mathbf{X}, \xi(\omega))] + \lambda\mathrm{CVaR}_\alpha(Q(\mathbf{X}, \xi(\omega)))\}$.

Therefore, the smaller the target value of the first stage, the higher the fitness of the particle. Among them, for each $\xi(\omega)$, the second stage function value $Q(\mathbf{X}, \xi(\omega))$ is calculated by simplex algorithm.

In the k'-th iteration of the calculation, the particle updates its velocity vector according to the following formula,

$$\mathscr{V}_{i'}^{k'+1} = \varrho \mathscr{V}_{i'}^{k'} + c_1 \cdot rand \cdot (P_{bi'}^{k'} - \mathbf{X}_{i'}^{k'}) + c_2 \cdot rand \cdot (P_{gi'}^{k'} - \mathbf{X}_{i'}^{k'}), \tag{5.2}$$

where $c_1, c_2 \in [c_{min}, c_{max}]$ are the dynamic learning factors, the learning factor at the k'-th iteration is $c_1 = c_2 = c_{min} + \frac{(c_{max}-c_{min})k'}{T}$, and satisfies $c_1 + c_2 \geq 4$, T is the maximum number of iterations. ϱ is the compression factor which is set by the following expression [59, 60] $\varrho = \frac{2}{|2-\varsigma-\sqrt{\varsigma^2-4\varsigma}|}$, $\varsigma = c_1 + c_2$.

In Eq (5.2), the first term describes the velocity of the previous iteration of the particle to ensure the global convergence of the algorithm. The second and third terms guarantee the local convergence of the algorithm. Because as the number of iterations increases, c_1 and c_2 increase linearly while ϱ decreases. This ensures that each particle can detect a better region of the global scope at a faster rate when the algorithm starts. In subsequent iterations, the smaller ϱ ensures that the particle can be searched finely near the extreme point, thus giving the algorithm a greater probability of converging to the optimal global solution.

In order to improve the particle search efficiency, avoid the expansion and diffusion of the particle swarm, and avoid the blind search of particle swarm in a large range, similar to [6], we introduce the following boundary conditions:

$$(\mathscr{V}_{i'}^{k'+1})_d = \begin{cases} \mathscr{V}_{max}, & (\mathscr{V}_{i'}^{k'+1})_d > \mathscr{V}_{max} \\ \mathscr{V}_{min}, & (\mathscr{V}_{i'}^{k'+1})_d < \mathscr{V}_{min} \\ (\mathscr{V}_{i'}^{k'+1})_d, & otherwise \end{cases}. \tag{5.3}$$

In (5.3), $\mathscr{V}_{max}$ is the maximum velocity limit and $\mathscr{V}_{min}$ is the minimum velocity limit.

Since the value and change of the particle in the state space are limited to the two values of 0 and 1, and the $v_{i'd}^{k'+1}$ of each dimension of velocity represents the possibility that the value of position $\chi_{i'd}^{k'+1}$ is 1. The location update equation is expressed as follows [61, 62]:

$$\mathscr{S}(\mathscr{V}_{i'}^{k'+1}) = \frac{1}{1 + exp(-\mathscr{V}_{i'}^{k'+1})}, \tag{5.4}$$

$$\chi_{i'd}^{k'+1} = \begin{cases} 1, & \mathscr{S}(\mathscr{V}_{i'}^{k'+1})_d > rand \\ 0, & otherwise \end{cases}. \tag{5.5}$$

In (5.4), $\mathscr{S}(\mathscr{V}_{i'}^{k'+1})$ is a sigmoid function. In (5.5), $\mathbf{X}_{i'}^{k'+1} = (\chi_{i'1}^{k'+1}, \cdots, \chi_{i'd}^{k'+1}, \cdots, \chi_{i'D}^{k'+1})$ denotes the position of the i' particle after the k'-th iteration.

Based on the above description, we give the detailed calculation process of the DS-HPSO algorithm.

Algorithm 1 : Distribution separation-hybrid particle swarm optimization algorithm (DS-HPSO).

Initialize the particle swarm. The population size is N', maximum number of iterations T, $\mathscr{V}_{max} > \mathscr{V}_{min}$, $c_{max} > c_{min} > 0$, $P_b^0 = ones(N, 1)$, $P_g^0 = eps$, $\mathbf{X}_{best} = ones(1, I + J)$, $k' = 0$, $i' = 1$;

Initializes particle velocity and position. Randomly obtain the initial population of binary code $\mathbf{X}^0 = round(rand(N', I + J))$, $P = \mathbf{X}^0$, $\mathscr{V}^0 = rand(I', I + J) \cdot (\mathscr{V}_{max} - \mathscr{V}_{min}) + \mathscr{V}_{min}$;

if $k' > T$ **then**

  Stop.

else

  if $i' > N'$ **then**

    $k' = k' + 1, i' = 1$;

  else

    Solve the second stage optimization problem (4.2) through the simplex algorithm;

    Solve the distribution separation problem (5.1), to get Q;

    Calculate the fitness $Fit(\mathbf{X}_{i'}^{k'})$;

    Update the individual's optimal position and value.

    if $Fit(\mathbf{X}_{i'}^{k'}) > P_{bi'}^{k'}$ **then**

      $P_{i'} = \mathbf{X}_{i'}^{k'}, P_{bi'}^{k'} = Fit(\mathbf{X}_{i'}^{k'})$;

    else

      $P_{bi'}^{k'} = P_{bi'}^{k'}$;

    end if

    Update the global optimal position and value.

    if $P_{bi'}^{k'} > P_g^{k'}$ **then**

      $\mathbf{X}_{best} = P_{i'}, P_g^{k'} = P_{bi'}^{k'}$;

    else

      $P_g^{k'} = P_g^{k'}$;

    end if

    Update velocity particles by (5.2);

    Boundary condition treatment by (5.3);

    Update location particle by (5.4)-(5.5);

    $i' = i' + 1$;

  end if

end if

Return the particle $\mathbf{X}_{best}$ as the optimal solution, and $-Fit(\mathbf{X}_{best})$ as the optimal value.

Theorem 2. Algorithm 1 satisfies the convergence condition and converges to the optimal solution.

Proof. According to [63], the convergence region of particle swarm optimization algorithm is

$$\begin{cases} \varrho < 1, \\ c > 0, \\ 2\varrho - c + 2 > 0. \end{cases}$$

According to (5.2), $\varrho = \frac{2}{|2-\varsigma-\sqrt{\varsigma^2-4\varsigma}|}$, $\varsigma = c_1 + c_2 \geq 4$, $c_1 = c_2 = c_{min} + \frac{(c_{max}-c_{min})k'}{T}$, $c = c_1 \cdot rand + c_2 \cdot rand$, the convergence region is not empty, so the convergence condition is satisfied. It is also known from [60] that algorithm 1 converges to the optimal solution.

Theorem 3. The complexity of Algorithm 1 in solving two-stage sparse distributionally robust risk mixed integer optimization models (4.1) and (4.2) is $O(TN' \cdot \max\{|\mathcal{P}|, M\})$, where, $|\mathcal{P}|$ represents the number of elements in the uncertain set $\mathcal{P}$, M represents the number of constraints in the optimization model (4.2), that is, $M = |V||S|+|V||I|+|I||L|+2|L||J|+|L||K|+|V||S||I|+|L||I||J| +|L||J||K|$.

Proof. According to the Algorithm 1, we know that the maximum iteration is T. In each iteration, in order to calculate the fitness function, it is necessary to apply the $N'-$degree simplex and distribution separation algorithm to calculate the fitness function $Fit(\mathbf{X})$. According to [64], the complexity of simplex algorithm is $O(M)$. In addition, the complexity of the distribution separation algorithm is $O(|\mathcal{P}|)$. Therefore, the complexity of Algorithm 1 is $O(TN' \cdot \max\{|\mathcal{P}|, M\})$.

6. Applications

In this section, a numerical example is used to study the location allocation problem of a fresh food in Shanghai by using the data-driven two-stage sparse distributionally robust risk mixed integer optimization model. All algorithm program code are written in MATLAB R2014a, and run in Intel(R) Core(TM) i7-8565U CPU @ 1.80 GHz, memory 8.00-GB Lenovo computer.

In the calculation, parameters in DS-HPSO are: $I' = 100$, $\mathcal{V}_{max} = 10, \mathcal{V}_{min} = -10$, $c_{min} = 2$, $c_{max} = 2.1$. The algorithm stop condition is $T = 1000$.

In the numerical results, "Pro" represents the total profit of the supply chain, and $\mathbf{X}_{best}$ represents the optimal solution of problem (4.1). $x^*_{vsi} = \sum_N p_n x^n_{vsi}$ represents the quantity of raw material v transported from supplier s to processing factory i, $y^*_{lij} = \sum_N p_n y^n_{lij}$ represents the quantity of product l transported from processing factory i to distribution center j, $z^*_{ljk} = \sum_N p_n z^n_{ljk}$ represents the quantity of product l transported from distribution center j to consumer k, where, $(x^n_{vmi}, y^n_{lij}, z^n_{ljk})$ is the optimal solution of the TSSDRRMIO model, p is the optimal solution of (5.1). "TI" represents the calculation time of the algorithm, in seconds.

In numerical calculation, the ambiguity set is defined by the first and second moment information of transportation cost and demand. Assume that the first and second moment information of random variable $\xi(\omega)$ is given. Let $\mu = E[\xi(\omega)]$ be the mean vector and $\Upsilon = E[(\xi(\omega) - \mu)(\xi(\omega) - \mu)^T]$ be the covariance matrix of the random variable $\xi(\omega)$.

6.1. Problem description

As a result of the COVID-19 epidemic, more and more consumers are actively or passively opting for contactless consumption. People's consumption behavior and patterns are gradually changing, and online consumption will become more popular [65]. In this context, relevant enterprises need to more accurately anticipate and respond to the long-term and far-reaching impact of COVID-19 on consumer behavior patterns in order to more effectively create opportunities and win markets.

Next, we use the two-stage sparse distributionally robust risk optimization model mentioned above to study the location allocation of cold storage and distribution center of a famous fresh chain brand in Shanghai. Because of the special nature of fresh goods, they need to be stored in a refrigerator and quickly distributed to consumers. How to rationally select refrigerated warehouses and distribution centers to deal with the uncertainties brought by the post COVID-19 epidemic world, reduce risks and storage costs, and realize timely delivery from cold storage to distribution centers and rapid supply from distribution centers to consumers is a problem worthy of in-depth study.

We have selected 20 retail stores in this chain brand as the distribution centers to be selected, and 15 large-scale and well-reputed cold storage companies in Shanghai as the cold storage to be selected. These suppliers, cold storage, distribution centers, and consumer area locations are shown in Figure 3. For the sake of discussion, we marked four suppliers, numbered from A1–A4, and also marked 15 cold storage locations to be selected, numbered from B1–B15. Marked 20 distribution centers to be selected, numbered from C1–C20. The consumer demand area is marked by four minions, which are defined as “Demand area 1 (green), 2 (gray), 3 (red) and 4 (blue)”. Demand area 1 consists of Jiading, Baoshan, Putuo, Changning, Xuhui, Huangpu, Jingan, Hongkou, and Yangpu district. Demand area 2 consists of Qingpu, Songjiang, and Minhang district. Demand area 3 consists of Jinshan and Fengxian district. Demand area 4 consists of Pudong new district.

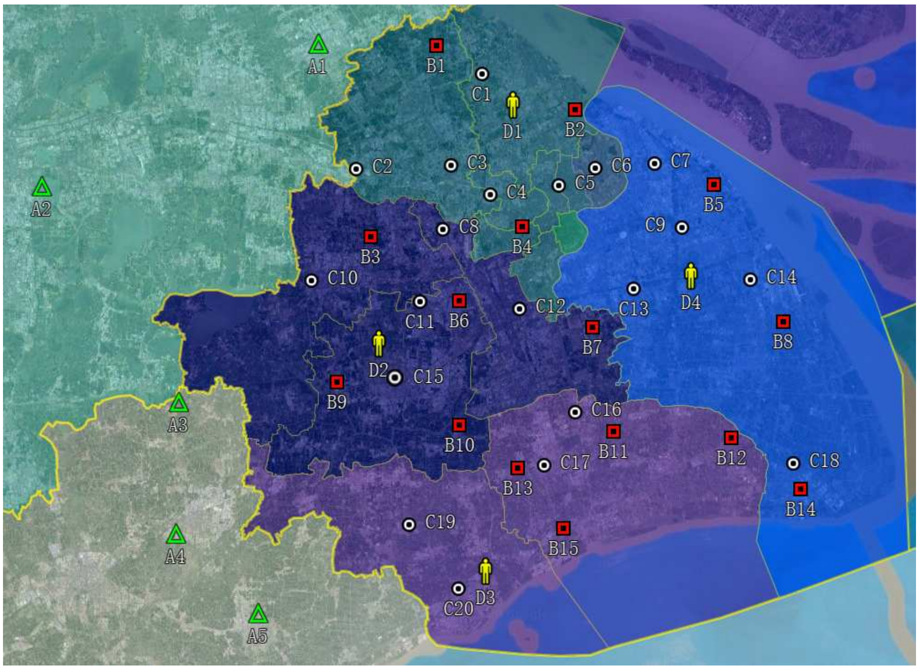


Figure 3. A fresh-food supply chain in Shanghai.

6.2. Parameter settings

The fixed cost of cold storage mainly includes fixed rental fee, and the difference is mainly determined by the geographical location of cold storage. Generally speaking, the rent of remote cold storage is relatively low. The cost of refrigeration is mainly determined by the refrigerating equipment in the cold storage. The refrigerating capacity of a cold storage is determined by the size of the cold storage. Related parameters are shown in Table 2. Transportation cost is a key parameter of location-allocation problem. We estimate the transportation cost per unit of goods based on the actual distance, as shown in Tables 3–5. According to the market survey statistics, the expected daily demand of consumers is given in Table 6, where, $\varrho = 0.2$ represents the price elasticity of demand, $h_1 = 16$ CNY. In the calculation, $\tau_{\mathbf{e}} = 9$, $\tau_{\mathbf{c}} = 12$.

Table 2. Parameters for suppliers, cold storage companies, and distribution centers.

Index	**Suppliers**		**cold**	**storage**	**companies**	**Distribution**		**centers**
s, i, j	a_{1s}	r_{1s}	b_{i1}	q_{i1}	f_i	τ_{1j}	w_{1j}	g_j
1	15000	4.60	7500	0.80	550	6000	0.25	135
2	14000	4.70	7350	0.75	545	6480	0.25	130
3	14500	4.80	8450	0.85	580	6450	0.25	120
4	16000	4.65	7500	0.80	550	6470	0.25	125
5	15500	4.75	7320	0.74	545	6510	0.25	140
6	/	/	7830	0.83	555	6490	0.25	130
7	/	/	7480	0.79	548	6520	0.25	145
8	/	/	7350	0.74	550	6505	0.25	143
9	/	/	7850	0.82	560	6460	0.25	123
10	/	/	8210	0.83	570	6485	0.25	133
11	/	/	8300	0.84	575	6100	0.25	133
12	/	/	7500	0.80	550	6380	0.25	129
13	/	/	7950	0.81	560	6350	0.25	119
14	/	/	7560	0.80	554	6370	0.25	123
15	/	/	7500	0.80	550	6410	0.25	138
16	/	/	/	/	/	6590	0.25	128
17	/	/	/	/	/	6420	0.25	143
18	/	/	/	/	/	6405	0.25	142
19	/	/	/	/	/	6460	0.25	123
20	/	/	/	/	/	6485	0.25	133

Table 3. Expected transportation cost from supplier to cold storage company.

	B1	B2	B3	B4	B5	B6	B7	B8	B9	B10	B11	B12	B13	B14	B15
*A*1	0.18	0.20	0.19	0.20	0.21	0.20	0.21	0.23	0.21	0.22	0.23	0.24	0.23	0.25	0.23
*A*2	0.22	0.23	0.21	0.22	0.24	0.22	0.23	0.25	0.21	0.22	0.24	0.25	0.24	0.27	0.24
*A*3	0.22	0.22	0.20	0.21	0.23	0.20	0.22	0.24	0.19	0.20	0.22	0.23	0.21	0.24	0.22
*A*4	0.23	0.23	0.21	0.22	0.24	0.21	0.22	0.24	0.19	0.21	0.22	0.23	0.21	0.24	0.22
*A*5	0.24	0.24	0.21	0.22	0.24	0.21	0.22	0.24	0.20	0.20	0.22	0.23	0.21	0.24	0.21

Table 4. Expected transportation cost from cold storage company to distribution center.

	B1	B2	B3	B4	B5	B6	B7	B8	B9	B10	B11	B12	B13	B14	B15
*C*1	0.17	0.18	0.19	0.19	0.20	0.20	0.20	0.22	0.21	0.21	0.22	0.22	0.22	0.23	0.22
*C*2	0.19	0.20	0.18	0.19	0.21	0.19	0.20	0.22	0.20	0.20	0.21	0.22	0.21	0.23	0.22
*C*3	0.19	0.19	0.18	0.18	0.20	0.19	0.20	0.21	0.20	0.20	0.20	0.22	0.21	0.23	0.21
*C*4	0.19	0.19	0.19	0.17	0.19	0.19	0.19	0.21	0.20	0.20	0.20	0.21	0.20	0.22	0.21
*C*5	0.19	0.19	0.19	0.17	0.19	0.19	0.19	0.20	0.20	0.20	0.20	0.21	0.20	0.21	0.21
*C*6	0.19	0.18	0.20	0.18	0.18	0.19	0.19	0.20	0.21	0.20	0.20	0.21	0.21	0.22	0.21
*C*7	0.20	0.18	0.21	0.19	0.18	0.20	0.19	0.19	0.22	0.21	0.20	0.21	0.21	0.21	0.22
*C*8	0.19	0.19	0.18	0.18	0.20	0.18	0.19	0.21	0.19	0.19	0.20	0.21	0.20	0.22	0.21
*C*9	0.20	0.19	0.21	0.19	0.18	0.20	0.19	0.19	0.22	0.21	0.19	0.20	0.20	0.20	0.21
*C*10	0.20	0.21	0.18	0.20	0.22	0.19	0.21	0.23	0.18	0.20	0.21	0.22	0.20	0.23	0.21
*C*11	0.20	0.20	0.18	0.19	0.21	0.18	0.19	0.21	0.19	0.19	0.20	0.21	0.19	0.22	0.20
*C*12	0.19	0.20	0.19	0.18	0.20	0.18	0.18	0.20	0.19	0.19	0.19	0.20	0.19	0.21	0.20
*C*13	0.21	0.19	0.20	0.19	0.19	0.19	0.18	0.19	0.21	0.20	0.19	0.19	0.20	0.20	0.20
*C*14	0.22	0.20	0.22	0.20	0.18	0.21	0.19	0.18	0.22	0.21	0.20	0.19	0.21	0.20	0.21
*C*15	0.21	0.21	0.19	0.19	0.22	0.18	0.20	0.22	0.18	0.18	0.20	0.21	0.19	0.22	0.20
*C*16	0.22	0.21	0.20	0.19	0.20	0.19	0.18	0.20	0.20	0.18	0.18	0.19	0.18	0.20	0.18
*C*17	0.22	0.21	0.20	0.20	0.21	0.19	0.19	0.20	0.20	0.18	0.18	0.19	0.17	0.20	0.18
*C*18	0.23	0.22	0.23	0.21	0.20	0.21	0.20	0.19	0.22	0.21	0.19	0.18	0.20	0.17	0.20
*C*19	0.23	0.22	0.21	0.21	0.23	0.20	0.20	0.22	0.19	0.18	0.20	0.21	0.19	0.22	0.19
*C*20	0.23	0.23	0.21	0.21	0.22	0.21	0.21	0.22	0.20	0.19	0.20	0.21	0.19	0.21	0.19

Table 5. Expected transportation cost from distribution center to consumer.

	D1	D2	D3	D4
$C1$	1.18	1.21	1.23	1.21
$C2$	1.19	1.19	1.22	1.21
$C3$	1.18	1.20	1.22	1.22
$C4$	1.18	1.19	1.21	1.20
$C5$	1.18	1.20	1.22	1.19
$C6$	1.18	1.21	1.22	1.19
$C7$	1.19	1.21	1.22	1.19
$C8$	1.19	1.19	1.21	1.20
$C9$	1.19	1.21	1.22	1.18
$C10$	1.20	1.18	1.21	1.22
$C11$	1.19	1.18	1.20	1.20
$C12$	1.19	1.19	1.20	1.19
$C13$	1.20	1.20	1.21	1.18
$C14$	1.21	1.21	1.22	1.18
$C15$	1.21	1.17	1.20	1.21
$C16$	1.21	1.19	1.19	1.19
$C17$	1.21	1.19	1.19	1.20
$C18$	1.22	1.22	1.21	1.19
$C19$	1.22	1.19	1.18	1.21
$C20$	1.23	1.20	1.17	1.22

Table 6. Expected demand of consumer.

Demand area 1	**Demand area 2**	**Demand area 3**	**Demand area 4**
$18000 - \varrho h_1$	$17500 - \varrho h_1$	$17000 - \varrho h_1$	$16000 - \varrho h_1$

6.3. Results and discussions

In order to solve the location allocation problem, Monte Carlo method is used to generate 1000 scenarios, and each scenarios have 1000 random sampling points ($\xi_n(\omega)$, $n = 1, 2, \cdots, 1000,$) for random simulation. The first moment information is given in Tables 3–6. In addition, the random vectors are independent of each other, so the covariance matrix $\Upsilon = 0.004 * \mathbf{W}$, $\mathbf{W}$ is the identity matrix of 459−dimension.

We substituted the data in Table 2 and the sample points generated by the Monte Carlo method into the data-driven two-stage sparse distributionally robust risk mixed integer optimization model, and solved it using DS-HPSO. The numerical experimental results are shown in Figures 4–7 and Tables 7–11 respectively.

Table 7. Numerical optimal solution and value of the example.

$\mathbf{e}_{best}$ =	(1, 1, 0, 1, 1, 0, 1, 0, 1, 0, 1, 0, 1, 1, 0)			
$\mathbf{c}_{best}$ =	(0, 1, 0, 1, 1, 1, 1, 1, 1, 1, 0, 1, 0, 1, 1, 0, 0, 0, 0, 1)			
$x^*_{112} = 6843.00$	$x^*_{119} = 180.00$	$x^*_{11\underline{11}} = 7977.00$	$x^*_{124} = 6440.00$	$x^*_{12\underline{14}} = 7560.00$
$x^*_{132} = 507.00$	$x^*_{137} = 7480.00$	$x^*_{141} = 7500.00$	$x^*_{149} = 550.00$	$x^*_{14\underline{13}} = 7950.00$
$x^*_{154} = 1060.00$	$x^*_{155} = 7320.00$	$x^*_{159} = 7120.00$	$y^*_{116} = 6393.00$	$y^*_{118} = 805.30$
$y^*_{11\underline{10}} = 301.70$	$y^*_{122} = 2450.00$	$y^*_{126} = 97.00$	$y^*_{12\underline{14}} = 4803.00$	$y^*_{142} = 2940.00$
$y^*_{147} = 4560.00$	$y^*_{157} = 1960.00$	$y^*_{159} = 5360.00$	$y^*_{179} = 1100.00$	$y^*_{17\underline{12}} = 6380.00$
$y^*_{195} = 6510.00$	$y^*_{19\underline{20}} = 1340.00$	$y^*_{1\underline{11}\ \underline{14}} = 1567.00$	$y^*_{1\underline{11}\ \underline{15}} = 6410.00$	$y^*_{1\underline{13}\ \underline{10}} = 2805.00$
$y^*_{1\underline{13}\ \underline{20}} = 5145.00$	$y^*_{1\underline{14}2} = 1090.00$	$y^*_{1\underline{14}4} = 6470.00$	$z^*_{123} = 6480.00$	$z^*_{141} = 2363.20$
$z^*_{143} = 4106.80$	$z^*_{151} = 6510.00$	$z^*_{161} = 6490.00$	$z^*_{174} = 6520.00$	$z^*_{181} = 805.40$
$z^*_{191} = 1828.20$	$z^*_{192} = 4631.80$	$z^*_{1\underline{10}4} = 3106.70$	$z^*_{1\underline{14}4} = 6370.00$	$z^*_{1\underline{15}2} = 6485.00$
$z^*_{1\underline{20}3} = 6410.00$	$val = -3.0630e + 05$	$pro = 6.1930e + 05$	$Others = 0$	

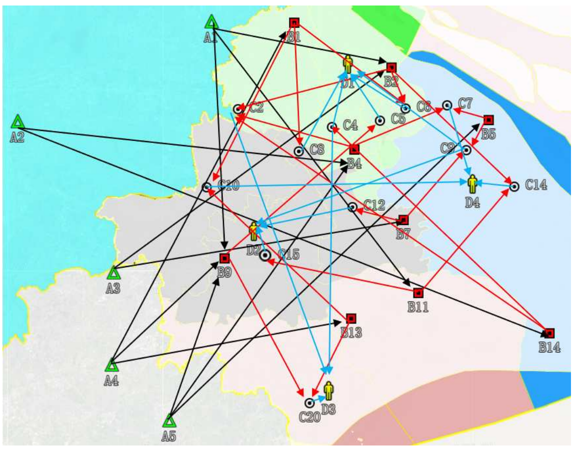


Figure 4. A famous fresh supply chain network in Shanghai.

In the worst case, in order to maximize the needs of customers, cold storage companies choose B1, B2, B4, B5, B7, B9, B11, B13, and B14 frozen products, and distribution centers choose C2, C4–C10, C12, C14, C15, and C20 to distribute products to customers, as shown in Figure 4. See Table 7 for the specific distribution scheme at this time. From Table 7, we can see that the maximum profit is $6.1930e + 05$CNY. The expected demand of customer D1 is $E(D1) = 17996.80$, which is provided by distribution centers C4, C5, C6, C8 and C9 with 2363.20, 6510.00, 6490.00, 805.40 and 1828.20 respectively. The expected demand of customer D2 is $E(D2) = 17496.80$, which is provided by distribution center C9, C12 and C15 with 4631.80, 6380.00 and 6485.00 respectively. The expected demand of customer D3 is $E(D3) = 16996.80$, which is provided by C2, C4 and C20 of distribution center 6480.00, 4106.80 and 6410.00 respectively. The expected demand of customer D4 is $E(D4) = 15996.80$, which is provided by distribution center C7, C10 and C14 with 6520.00, 3106.70 and 6370.00 respectively. Through the above data analysis, we found that in the worst case, this location allocation scheme can also meet the needs of customers.

We compare DS-HPSO algorithm with other discrete hybrid algorithms, DR L-shaped method [40] and DS-HGA [16], to illustrate the effectiveness and stability of the algorithm. Numerical results are given in Figure 5 and Table 8. In Table 8, the relative error is given by the following equation,

$$Error = \frac{Optimal\ \ Pro - Pro}{Optimal\ \ Pro} \times 100\%.$$

Table 8. Comparisons of different algorithms.

Algorithm	$\mathbf{e}_{best}$	$\mathbf{c}_{best}$	Pro	TI	Error(%)
DS-HPSO	(1, 1, 0, 1, 1, 0, 1, 0, 1, 0, 1, 0, 1, 1, 0)	(0, 1, 0, 1, 1, 1, 1, 1, 1, 1, 0, 1, 0, 1, 1, 0, 0, 0, 0, 1)	$6.1930e+05$	39355.1689	0.00
DS-HGA	(1, 1, 0, 1, 1, 0, 1, 0, 1, 0, 1, 0, 1, 1, 0)	(0, 1, 0, 1, 1, 1, 1, 1, 1, 1, 0, 1, 0, 1, 1, 0, 0, 0, 0, 1)	$6.1878e+05$	44893.7382	0.08
DR L-shaped	(1, 1, 0, 1, 1, 0, 1, 0, 1, 0, 1, 0, 1, 1, 0)	(0, 1, 0, 1, 1, 1, 1, 1, 1, 1, 0, 1, 0, 1, 1, 0, 0, 0, 0, 1)	$6.1906e+05$	41839.3739	0.04

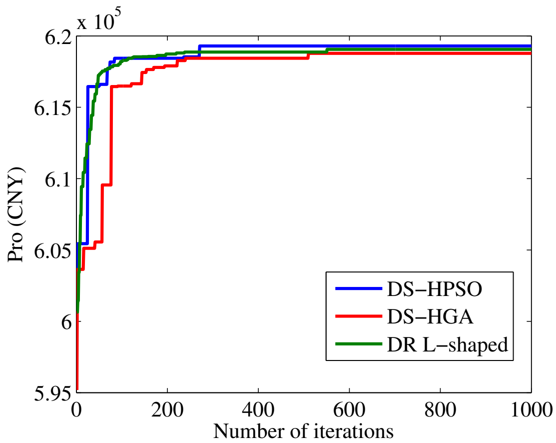


Figure 5. Comparisons of different algorithms.

Remark 1. In the computational experiments, the parameters in DS-HGA are taken as: $N = 100$, crossover rate $P_c = 0.8$, variation rate $P_m = 0.05$, maximum iterations $T = 1000$.

It can be seen from Table 8 and Figure 5 that the same optimal solution and similar optimal value are obtained no matter which algorithm is adopted when solving the location allocation problem. And the relative error of the optimal value is less than 0.08%, which is caused by the random simulation of Monte Carlo method. In addition, it can be seen from Table 8 that DS-HPSO algorithm have shorter computation time, and Figure 5 shows that DS-HPSO algorithm has faster convergence speed than other algorithms. These results show that DS-HPSO is more suitable for solving TSSDRRMIO model.

In order to better prove the performance of DS-HPSO, we verified the stability of the DS-HPSO by setting different iteration numbers, population size and different parameters. The numerical results are shown in Table 9.

Table 9. Results of DS-HPSO with different parameters.

System		Parameters		Results			
T	I'	c_{min}	c_{max}	$\mathbf{e}_{best}$	$\mathbf{c}_{best}$	Pro	Error(%)
200	10	2.0	2.1	(1, 1, 0, 1, 1, 0, 0, 1, 1, 1, 0, 0, 1, 0, 1)	(0, 0, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 0, 1, 0, 0, 1, 0)	6.1867e + 05	0.10
500	10	2.0	2.1	(1, 1, 0, 1, 1, 0, 1, 0, 1, 0, 1, 0, 1, 1, 0)	(0, 1, 0, 1, 1, 1, 1, 1, 1, 1, 0, 1, 0, 1, 1, 0, 0, 0, 0, 1)	6.1901e + 05	0.05
1000	10	2.0	2.1	(1, 1, 0, 1, 1, 0, 1, 0, 1, 0, 1, 0, 1, 1, 0)	(0, 1, 0, 1, 1, 1, 1, 1, 1, 1, 0, 1, 0, 1, 1, 0, 0, 0, 0, 1)	6.1907e + 05	0.04
2000	10	2.0	2.1	(1, 1, 0, 1, 1, 0, 1, 0, 1, 0, 1, 0, 1, 1, 0)	(0, 1, 0, 1, 1, 1, 1, 1, 1, 1, 0, 1, 0, 1, 1, 0, 0, 0, 0, 1)	6.1916e + 05	0.02
1000	100	2.0	2.1	(1, 1, 0, 1, 1, 0, 1, 0, 1, 0, 1, 0, 1, 1, 0)	(0, 1, 0, 1, 1, 1, 1, 1, 1, 1, 0, 1, 0, 1, 1, 0, 0, 0, 0, 1)	6.1915e + 05	0.02
1000	1000	2.0	2.1	(1, 1, 0, 1, 1, 0, 1, 0, 1, 0, 1, 0, 1, 1, 0)	(0, 1, 0, 1, 1, 1, 1, 1, 1, 1, 0, 1, 0, 1, 1, 0, 0, 0, 0, 1)	6.1930e + 05	0.00
1000	10	2.0	2.5	(1, 1, 0, 1, 1, 0, 1, 0, 1, 0, 1, 0, 1, 1, 0)	(0, 1, 0, 1, 1, 1, 1, 1, 1, 1, 0, 1, 0, 1, 1, 0, 0, 0, 0, 1)	6.1920e + 05	0.02
1000	10	2.0	3.0	(1, 1, 0, 1, 1, 0, 1, 0, 1, 0, 1, 0, 1, 1, 0)	(0, 1, 0, 1, 1, 1, 1, 1, 1, 1, 0, 1, 0, 1, 1, 0, 0, 0, 0, 1)	6.1917e + 05	0.02
1000	10	2.5	3.0	(1, 1, 0, 1, 1, 0, 1, 0, 1, 0, 1, 0, 1, 1, 0)	(0, 1, 0, 1, 1, 1, 1, 1, 1, 1, 0, 1, 0, 1, 1, 0, 0, 0, 0, 1)	6.1910e + 05	0.02

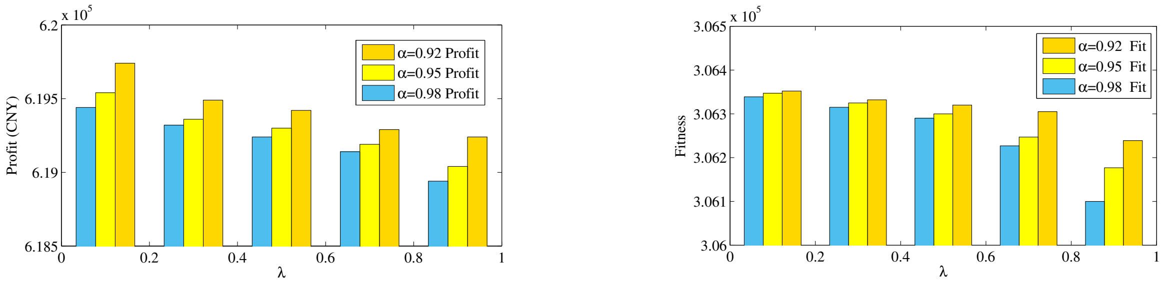
In Table 9, when we set different parameters, iteration numbers and population size in the DS-HPSO, the relative error is no more than 0.10%, which indicates that the DS-HPSO has strong robustness to parameters and can effectively solve the two-stage sparse distributionally robust risk mixed integer programming.

In Table 10, we compare the influence of the two-stage sparse distributionally robust optimization model ($\lambda = 0$) and the two-stage sparse distributionally robust risk optimization model ($\lambda = 0.5$) on the location allocation problem. When the model does not consider risks, the profit of the supply chain is 6.2026e + 05 CNY. When the supply chain considers risks, the corresponding profit of the supply chain is 6.1930e + 05 CNY. The numerical results show that the profit is more conservative when the supply chain considers risk, which is consistent with the practice.

Table 10. Comparison of supply chain profit of model with $\lambda = 0.5$ and $\lambda = 0$.

λ	$\mathbf{e}_{best}$	$\mathbf{c}_{best}$	Pro
$\lambda = 0$	(0, 1, 0, 0, 1, 1, 0, 1, 1, 1, 0, 1, 1, 0, 1)	(1, 0, 1, 0, 1, 0, 1, 0, 1, 0, 1, 1, 1, 1, 1, 1, 1, 1, 0, 1)	6.2026e + 05
$\lambda = 0.5$	(1, 1, 0, 1, 1, 0, 1, 0, 1, 0, 1, 0, 1, 1, 0)	(0, 1, 0, 1, 1, 1, 1, 1, 1, 1, 0, 1, 0, 1, 1, 0, 0, 0, 0, 1)	6.1930e + 05

Different enterprises have different attitudes towards risk. Understanding the attitude of supply chain enterprises to risk is helpful to formulate appropriate risk management measures. In models (4.1) and (4.2), different λ values have different influences on decision making, and decision makers can obtain expected benefits by adjusting the weight of CVaR. Figure 6 shows the impact of different λ values on the profit and fitness function values of the supply chain.



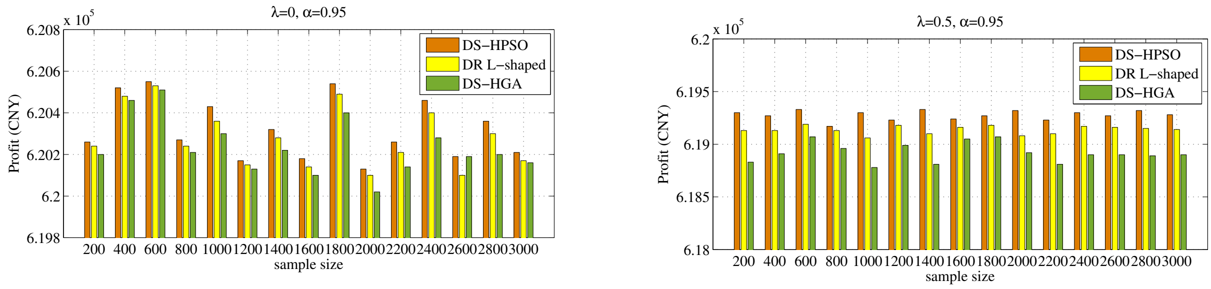
(a) (b)

Figure 6. Values of fitness functions and supply chain profit with different confidence.

The numerical results in Figure 6 show that with the increase of λ, the fitness function value and supply chain profit decrease correspondingly, which is consistent with the results discussed above.In other words, the more emphasis is placed on risk, the more conservative the fitness function value and supply chain profit are obtained.

In addition, when the value of λ is unchanged, a lower confidence level corresponds to a higher fitness function value and a higher supply chain profit. Therefore, decision makers can consider choosing a lower confidence level to obtain a higher return, that is, the high risk corresponds to the high expected return.

Figure 7 shows the comparison of the operational results of the DS-HPSO, DS-HGA and DR L-shaped methods to solve the problem of location-allocation. Obviously, the supply chain benefit obtained by the robust optimization model without considering the risk ($\lambda = 0$, $\alpha = 0.95$) is higher than that obtained by the robust risk optimization model ($\lambda = 0.5$, $\alpha = 0.95$), which indicates that the results of the robust optimization model without risk may overestimate the benefit in the problem of location allocation. In addition, as the sample data increases, the objective values obtained by the three methods tend to gradually tend to be determined values. In addition, the application of the method proposed in this paper to obtain supply chain benefits, significantly better than the other two robust optimization methods. Overall, the DS-HPSO method is better suited to solve the problem.



(a) (b)

Figure 7. Comparison of the results of solving the location allocation problem with different methods under different sample sizes.

Table 11 shows the location and profit of the supply chain in the worst-case and expected scenarios, in which the location decision and profit of the supply chain are different. In the robust case (i.e. worst-case), the supply chain's profit is $6.1930e + 05$ CNY, which is a conservative result. In other words, in the uncertain set, the profit of the supply chain in any situation is greater than $6.1930e + 05$CNY, and the optimization program can still be feasible and the model has certain robustness. In the expected case, the profit of the supply chain is $6.2034e + 05$CNY. At first glance, this solution appears to yield a better expected profit than the robust random problem solution. However, this prospective case problem paints an overly optimistic picture of the actual situation. Therefore, decision conclusions that decision makers can obtain through this method are more robust and reliable.

Table 11. Supply chain profit with robust and no robust.

$\xi(\omega)$	$\mathbf{e}_{best}$	$\mathbf{c}_{best}$	Pro
Robust	$(1,1,0,1,1,0,1,0,1,0,1,0,1,1,0)$	$(0,1,0,1,1,1,1,1,1,1,0,1,0,1,1,0,0,0,0,1)$	$6.1930e + 05$
No Robust	$(0,1,1,0,1,1,1,1,1,1,0,1,0,0,0)$	$(1,1,1,0,1,1,1,1,0,1,1,1,0,0,1,1,0,1,0,1)$	$6.2034e + 05$

6.4. Management insights

In this paper, we develop a data-driven two-stage sparse distributionally robust risk mixed integer optimization model considering the uncertainty of transportation costs and customer demand in the supply chain, as well as the risk measure of the total cost function. Compared with the risk neutral method (no risk), the risk aversion method takes into account the influence of random outcome variability and provides a more effective solution for decision making problems in uncertain environments. Decision makers can evaluate strategies based on their risk appetite. The numerical results in Table 10 and Figure 6 show that has different effects on decision making, and decision makers can obtain expected returns by adjusting the weight of CVaR. When the value of λ is unchanged, a lower confidence level corresponds to a higher fitness function value and a higher supply chain profit. Therefore, decision makers can consider choosing a lower confidence level to obtain a higher return, that is, the high risk corresponds to the high expected return.

Compared with the data-driven two-stage sparse risk mixed integer optimization model (no robust), the results obtained from the data-driven two-stage sparse distributionally robust risk mixed integer optimization model is more conservative (see Table 11). In other words, under any uncertain supply chain, the distributionally robust optimization scheme is still feasible, and the model has certain robustness. Although the non-robust optimization solution seems to yield better expected profits than the distributionally robust optimization solution. However, this non-robust optimization solution paints an overly optimistic picture of the actual situation. Therefore, decision makers can obtain more robust and reliable decision conclusions through this method.

7. Conclusions

Aiming at the location allocation problem with uncertain transportation cost and demand, this paper constructs a data-driven two-stage sparse distributionally robust risk mixed integer optimization model. Compared with the traditional stochastic optimization model, the proposed model is more robust in the numerical simulation. It is worth noting that the model constructed in this paper is not only applicable to the location allocation problem, but also can be extended to joint optimization problems such as emergency material scheduling and path optimization. In the study of supply chain optimization using two-stage sparse distributionally robust risk mixed integer optimization model, the construction of uncertainty set is the key. Based on some historical data, the uncertainty sets of the first and second moment structures of random variables are estimated. In addition to the moment-based constraint method used in this paper, uncertainty sets can also be constructed from some statistical metric theories, such as using Wasserstein distance to measure the difference between the truth and reference probability distribution. The difficulty of uncertainty set construction based on statistical measurement theory is how to determine the reference distribution function, which can be estimated from the data-driven method and non-parametric estimation method, which are also part of our future research.

This paper discusses the impact of different risk weight on decision-making. Different risk-loving decision-makers can choose different risk weight and obtain corresponding benefits and optimal decisions (see Table 10 and Figure 6). Moreover, this paper also compares the data-driven two-stage sparse distributionally robust risk mixed integer optimization model with the two-stage sparse risk optimization model, and the data results show the robustness of this model (see Table 11). The decision conclusions obtained by decision-makers through this method are more robust and reliable.

In addition, considering the complexity of the model. This paper proposes DS-HPSO to solve the problem studied. Moreover, we compare DS-HPSO with DS-HGA and DR L-Shaped, and the results show that our DS-HPSO is more suitable for the two-stage sparse distributionally robust risk programming.

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Conflict of interest

The author declares that there is no competing interest.

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