Mathematics

http://www.aimspress.com/journal/Math

AIMS Mathematics, 8(2): 2648–2658.
DOI: 10.3934/math.2023138
Received: 17 September 2022
Revised: 13 October 2022
Accepted: 25 October 2022
Published: 08 November 2022

Research article

Optical soliton solutions for Lakshmanan-Porsezian-Daniel equation with parabolic law nonlinearity by trial function method

Chen Peng[1,2*] **and Zhao Li**[1]

[1] College of Computer Science, Chengdu University, Chengdu 610106, China
[2] V. C. & V. R. Key Lab of Sichuan Province, Sichuan Normal University, Chengdu 610068, China

* **Correspondence:** Email: pengchen@cdu.edu.cn.

Abstract: In this paper, the trial function method is used to address the Lakshmanan-Porsezian-Daniel (LPD) equation with parabolic law nonlinearity. Implementing the traveling wave hypothesis reduces the LPD equation to an ordinary differential equation (ODE). From this ODE, many exact solutions, such as kink solitary wave solutions, bell shaped solitary wave solutions, triangular function solutions, periodic function solutions, singular solutions and Jacobian elliptic function solutions, are retrieved. Among them, some solutions are new. By suitable choice of parameters, we also draw 3D surface and 2D graphs of density, contour and level curves of some precise solutions for intuitive investigation.

Keywords: solitons; trial function method; Lakshmanan-Porsezian-Daniel equation
Mathematics Subject Classification: 35C05, 35C07, 35R11

1. Introduction

Optical soliton dynamics is mainly described by non-linear Schrödinger equations (NLSEs) [1–5]. The primary purposes of research on optical solitons are to develop the telecommunications industry and optimize communications network technology. NLSEs consist of many well-known ones, like the Schrödinger-Hirota equation, Gerdjikov-Ivanov equation, Radhakrishnan-Kundu-Lakshmanan equation, Kundu-Eckhaus equation, Maxwell-Bloch equation, Biswas-Milovic equation, Chen-Lee-Liu equation and so on. Among NLSEs, the Lakshmanan-Porsezian-Daniel (LPD) equation, proposed in the category of Heisenberg spin chains [6–9], governs the transmission of solitons through a variety of waveguides and has been widely studied by many authors [10–12]. It is worth mentioning that the LPD equation, which follows from the so-called AKNS hierarchy and has non-autonomous generalized reductions and integrated models, has gained huge interest recently [13–15]. The studies on optical solitons with LPD have a great effect on modern electronic communication systems and reflect the propagation rules for nonlinear optics.

The trial function method was proposed by Liu [16] for finding certain types of solutions for nonlinear evolution equations (NLEEs). This method has been reliably and effectively used to seek the solitary traveling wave solutions from various NLEE models [17–21]. Generally speaking, although many advanced techniques, such as the undetermined coefficients method [22], the modified simple equation method [23], the improved tan $\frac{\psi(\eta)}{2}$-expansion method [24], the modified auxiliary equation method [25] and the modified extended direct algebraic method [26], have been adopted to study the solutions of the governing LPD model exhibited in the following, some of its soliton solutions have not been explored. Our purpose is to retrieve new optical soliton solutions for the model hereunder by the trial function method.

In this paper, we consider the dimensionless form of the LPD model in the presence of spatio-temporal dispersion (STD) with full nonlinearity and higher order dispersion [22]:

$$iu_t + au_{xx} + bu_{xt} + \mathcal{F}(|u|^2)u = \sigma u_{xxxx} + \alpha(u_x)^2 u^* + \beta|u_x|^2 u + \gamma|u|^2 u_{xx} + \lambda u^2 u^*_{xx} + \delta|u|^4 u. \tag{1.1}$$

In (1.1), the independent variables x and t denote the space and time, respectively, while the independent variable $u(x,t)$ represents the complex-valued wave profile. The coefficients of group-velocity dispersion and STD are represented by parameters a and b, respectively. The real-valued functional $\mathcal{F}$, which is decided by the refractive index of the propagation medium, such as optical fiber, provides the general form of nonlinearity, including Kerr law, parabolic law, power law, dual-power law and log law nonlinearities. In particular, $\mathcal{F}(|u|^2)u$ is a k-times continuously differentiable function [1].

$$\mathcal{F}(|u|^2)u \in \bigcup_{n,m=1}^{\infty} \mathbb{C}^k((-n,n)\times(-m,m):\mathbb{R}^2).$$

Then, for the terms on the right hand side of (1.1), σ is the coefficient of fourth-order dispersion; α, β, γ and λ indicate the other perturbation terms with nonlinear effects. The two-photon absorption is represented by parameter δ. For $b = 0$ and removing $\mathcal{F}(|u|^2)u$, the model (1.1) becomes the original LPD model [3,4] that is extensively studied. Therefore, we consider the LPD model with functional $\mathcal{F}$ and $b \neq 0$.

The current paper consists of the following parts. In Section 2, we will demonstrate the algorithm of the trial function method. In Section 3, Eq (1.1) with parabolic law nonlinearity will be reduced to an ODE via the traveling wave hypothesis. In order to retrieve the exact traveling wave solutions from this ODE, we will employ the trial function methodology united with the complete discrimination system method [27]. In Section 4, kink, periodic, bell shaped and triangle solitons are extracted for specific parameter values and depicted through graphical demonstration. Section 5 is the conclusion of this paper.

2. The trial function method

The basic steps of the trial function method [16] are itemized as follows.

Step 1: Assume equation

$$N(u, u_t, u_x, u_{xt}, u_{xx}, u_{xxx}, \cdots) = 0 \tag{2.1}$$

denotes the nonlinear evolution equation (NLEE) for unknown function $u = u(t, x)$. Implementing complex traveling wave transformation $u(x, t) = U(\xi)e^{i\phi(x,t)}$, $\xi = x - ct$, converts (2.1) to an ODE,

$$M(U, U', U'', \cdots) = 0, \tag{2.2}$$

where $U' = \frac{du}{d\xi}$ and M denotes the polynomial of U and its derivatives with respect to ξ.

Step 2: In this paper, the simple trial function is considered as

$$(U')^2 = F(U) = \sum_{i=0}^{K} a_i U^i, \tag{2.3}$$

where integer K is determined by the balance principle, and the coefficients $a_i (i = 1, \cdots K)$ will be retrieved later. A polynomial equation $G(U) = 0$ is obtained by substituting (2.3) into (2.2). By making all the coefficients of $G(U)$ equal to zero, one can get the algebraic equation system for $a_i (i = 1, \cdots K)$ and solve it. Then, $F(U)$ becomes fixed.

Step 3: By separating variables and integrating, (2.2) can be reduced to

$$\pm(\xi - \xi_0) = \int \frac{\mathrm{d}U}{\sqrt{F(U)}}, \tag{2.4}$$

where ξ_0 is an integral constant. The classification of the roots for polynomial $F(U)$, which are extracted via the polynomial complete discriminant system method [27], eventually addresses exact traveling wave solutions for Eq (2.1).

It is worth mentioning that the trial equation can be chosen in several forms besides (2.3) to deal with different NLEEs.

3. Applications

The assumption is that the exact solutions for Eq (1.1) satisfy the traveling wave hypothesis [1]

$$u(x, t) = U(\xi)e^{i\zeta(x,t)}, \quad \xi = x - ct, \tag{3.1}$$

where U is the shape of the wave profile with c representing the wave velocity, and $\zeta(x, t) = -kx + \omega t + \varphi_0$ is the phase component. The parameter k is frequency, ω is the wave number, and φ_0 represents the phase constant. By plugging the hypothesis (3.1) into (1.1) and decomposing the real and imaginary parts, we obtain the real part as

$$\begin{aligned} &\sigma U'''' - (a - bc + 6\sigma k^2)U'' + (\omega - \omega bk + ak^2 + \sigma k^4)U - \mathcal{F}(U^2)U \\ &\quad - (\alpha - \beta + \gamma + \lambda)k^2 U^3 + \delta U^5 + (\alpha + \beta)U(U')^2 + (\gamma + \lambda)U^2 U'' = 0, \end{aligned} \tag{3.2}$$

while the imaginary portion reads as

$$(bck - c + b\omega - 2ak - 4\sigma k^3)U' + 2(\alpha + \gamma - \lambda)kU^2U' + 4\sigma kU''' = 0. \tag{3.3}$$

Equating the linearly independent coefficients in (3.3) to zero imposes the constraint conditions

$$\sigma = 0, \tag{3.4}$$

$$\alpha + \gamma - \lambda = 0, \tag{3.5}$$

and the velocity of the soliton is attained as

$$c = \frac{2ak - b\omega}{bk - 1}, \text{ for } bk \neq 1. \tag{3.6}$$

The parabolic law nonlinearity is presented by

$$\mathcal{F}(u) = c_1 u + c_2 u^2, \tag{3.7}$$

where c_1 and c_2 are real parameters. Inserting (3.4) and (3.7) into (3.2) yields

$$\begin{aligned}(bc - a)U'' + (\omega - \omega bk + ak^2)U - (c_1 U^2 + c_2 U^4)U - (2\alpha - \beta + 2\gamma)k^2 U^3 \\ + \delta U^5 + (\alpha + \beta)U(U')^2 + (\alpha + 2\gamma)U^2 U'' = 0.\end{aligned} \tag{3.8}$$

In order to apply the extended trial function algorithm to tackle (3.8), we shall assume that the structure of solutions is in accord with the form

$$(U')^2 = \sum_{i=0}^{n} a_i U^i. \tag{3.9}$$

Balancing the orders between $U(U')^2$ and U^5 yields $n = 4$. Then, it is easy to compute

$$U'' = 2a_4 U^3 + \frac{3}{2} a_3 U^2 + a_2 U + \frac{a_1}{2}. \tag{3.10}$$

By substituting (3.9) and (3.10) in (3.8) and solving the obtained algebraic equation system about the coefficients of each power of U, the following results are received:

$$\begin{aligned}a_4 &= \frac{c_2 - \delta}{3\alpha + \beta + 4\gamma}, \qquad a_3 = 0, \\ a_2 &= \frac{(3\alpha + \beta + 4\gamma)[c_1 + (2\alpha - \beta + 2\gamma)k^2] - 2(bc - a)(c_2 - \delta)}{(3\alpha + \beta + 4\gamma)(2\alpha + \beta + 2\gamma)}, \quad a_1 = 0, \\ a_0 &= \frac{2(bc - a)^2(c_2 - \delta) - (bc - a)(3\alpha + \beta + 4\gamma)[c_1 + (2\alpha - \beta + 2\gamma)k^2]}{(\alpha + \beta)(3\alpha + \beta + 4\gamma)(2\alpha + \beta + 2\gamma)} - \frac{\omega - \omega bk + ak^2}{\alpha + \beta},\end{aligned} \tag{3.11}$$

and (3.9) consequently becomes $(U')^2 = a_4 U^4 + a_2 U^2 + a_0$. Implementing the transformation

$$U = \pm\sqrt{(4a_4)^{-\frac{1}{3}}\phi}, \ \xi_1 = (4a_4)^{\frac{1}{3}}\xi, \ q = 4a_2(4a_4)^{-\frac{2}{3}}, \ r = 4a_0(4a_4)^{-\frac{1}{3}}, \tag{3.12}$$

reduces the Eq (3.9) to

$$(\phi_{\xi_1})^2 = \phi(\phi^2 + q\phi + r). \tag{3.13}$$

Then, after separating variables and integrating, (3.13) becomes

$$\pm(\xi_1 - \xi_0) = \int \frac{\mathrm{d}\phi}{\sqrt{\phi F(\phi)}} = \int \frac{\mathrm{d}\phi}{\sqrt{\phi(\phi^2 + q\phi + r)}}, \tag{3.14}$$

where ξ_0, an arbitrary constant, is the integration constant.

Now, the second-order complete discrimination system

$$F(\phi) = \phi^2 + q\phi + r, \quad \Delta = q^2 - 4r, \tag{3.15}$$

is used to classify the roots of $F(U)$. Four cases of the solutions for Eq (3.14) are studied, as follows:

Case 1: $\Delta = 0$, so that $a_2^2 = 4a_0a_4$. In this case, $\phi > 0$ is valid, and (3.14) is reduced to

$$\pm(\xi_1 - \xi_0) = \int \frac{d\phi}{\sqrt{\phi}\left(\phi + \frac{q}{2}\right)}. \tag{3.16}$$

When $q < 0$, the solutions of Eq (1.1) are obtained as

$$u_1(x,t) = \pm\sqrt{\frac{2(bc-a)(c_2-\delta) - (3\alpha+\beta+4\gamma)[c_1 + (2\alpha-\beta+2\gamma)k^2]}{2(2\alpha+\beta+2\gamma)(c_2-\delta)}}e^{i(-kx+\omega t+\varphi_0)} \tag{3.17}$$
$$\times\tanh\left[\sqrt{\frac{2(bc-a)(c_2-\delta) - (3\alpha+\beta+4\gamma)[c_1 + (2\alpha-\beta+2\gamma)k^2]}{2(3\alpha+\beta+4\gamma)(2\alpha+\beta+2\gamma)}}\left(x - \frac{2ak-b\omega}{bk-1}t - \xi_0\right)\right],$$

$$u_2(x,t) = \pm\sqrt{\frac{2(bc-a)(c_2-\delta) - (3\alpha+\beta+4\gamma)[c_1 + (2\alpha-\beta+2\gamma)k^2]}{2(2\alpha+\beta+2\gamma)(c_2-\delta)}}e^{i(-kx+\omega t+\varphi_0)} \tag{3.18}$$
$$\times\coth\left[\sqrt{\frac{2(bc-a)(c_2-\delta) - (3\alpha+\beta+4\gamma)[c_1 + (2\alpha-\beta+2\gamma)k^2]}{2(3\alpha+\beta+4\gamma)(2\alpha+\beta+2\gamma)}}\left(x - \frac{2ak-b\omega}{bk-1}t - \xi_0\right)\right].$$

When $q > 0$, the solutions of Eq (1.1) are obtained as

$$u_3(x,t) = \pm\sqrt{\frac{(3\alpha+\beta+4\gamma)[c_1 + (2\alpha-\beta+2\gamma)k^2] - 2(bc-a)(c_2-\delta)}{2(2\alpha+\beta+2\gamma)(c_2-\delta)}}e^{i(-kx+\omega t+\varphi_0)} \tag{3.19}$$
$$\times\tan\left[\sqrt{\frac{(3\alpha+\beta+4\gamma)[c_1 + (2\alpha-\beta+2\gamma)k^2] - 2(bc-a)(c_2-\delta)}{2(3\alpha+\beta+4\gamma)(2\alpha+\beta+2\gamma)}}\left(x - \frac{2ak-b\omega}{bk-1}t - \xi_0\right)\right].$$

When $p = 0$, so that solutions will appear for $a_2 = a_0 = 0$, the solutions of Eq (1.1) are

$$u_4(x,t) = \pm\sqrt{\frac{3\alpha+\beta+4\gamma}{4(c_2-\delta)}}\left(x - \frac{2ak-b\omega}{bk-1}t - \xi_0\right)^{-1} \times e^{i(-kx+\omega t+\varphi_0)}. \tag{3.20}$$

Case 2: For $\Delta > 0$ and $r = 0$ ($a_0 = 0$), Eq (3.14) is reduced to

$$\pm(\xi_1 - \xi_0) = \int \frac{d\phi}{\phi\sqrt{\phi + q}}. \tag{3.21}$$

If $q > 0$, provided $-q < \phi < 0$ is valid, the solutions of Eq (1.1) are procured as

$$u_5(x,t) = \pm\sqrt{\frac{2(bc-a)(c_2-\delta)-(3\alpha+\beta+4\gamma)[c_1+(2\alpha-\beta+2\gamma)k^2]}{(2\alpha+\beta+2\gamma)(c_2-\delta)}}e^{i(-kx+\omega t+\varphi_0)} \tag{3.22}$$
$$\times\text{sech}\Big[\sqrt{\frac{(3\alpha+\beta+4\gamma)[c_1+(2\alpha-\beta+2\gamma)k^2]-2(bc-a)(c_2-\delta)}{(3\alpha+\beta+4\gamma)(2\alpha+\beta+2\gamma)}}\Big(x-\frac{2ak-b\omega}{bk-1}t-\xi_0\Big)\Big];$$

and when $\phi > 0$, the solutions of Eq (1.1) are

$$u_6(x,t) = \pm\sqrt{\frac{(3\alpha+\beta+4\gamma)[c_1+(2\alpha-\beta+2\gamma)k^2]-2(bc-a)(c_2-\delta)}{(2\alpha+\beta+2\gamma)(c_2-\delta)}}e^{i(-kx+\omega t+\varphi_0)} \tag{3.23}$$
$$\times\text{csch}\Big[\sqrt{\frac{(3\alpha+\beta+4\gamma)[c_1+(2\alpha-\beta+2\gamma)k^2]-2(bc-a)(c_2-\delta)}{(3\alpha+\beta+4\gamma)(2\alpha+\beta+2\gamma)}}\Big(x-\frac{2ak-b\omega}{bk-1}t-\xi_0\Big)\Big].$$

If $q < 0$, the solutions of Eq (1.1) are obtained as

$$u_7(x,t) = \pm\sqrt{\frac{2(bc-a)(c_2-\delta)-(3\alpha+\beta+4\gamma)[c_1+(2\alpha-\beta+2\gamma)k^2]}{(2\alpha+\beta+2\gamma)(c_2-\delta)}}e^{i(-kx+\omega t+\varphi_0)} \tag{3.24}$$
$$\times\sec\Big[\sqrt{\frac{2(bc-a)(c_2-\delta)-(3\alpha+\beta+4\gamma)[c_1+(2\alpha-\beta+2\gamma)k^2]}{(3\alpha+\beta+4\gamma)(2\alpha+\beta+2\gamma)}}\Big(x-\frac{2ak-b\omega}{bk-1}t-\xi_0\Big)\Big].$$

Case 3: $\Delta > 0$ and $r \neq 0$, which leads to $F(U)$ having two distinct roots. For convenience, by assuming $\lambda_1 < \lambda_2 < \lambda_3$, satisfying that one of these equals 0, Eq (3.14) can be denoted by

$$\pm(\xi_1-\xi_0) = \int\frac{\mathrm{d}\phi}{\sqrt{(\phi-\lambda_1)(\phi-\lambda_2)(\phi-\lambda_3)}}. \tag{3.25}$$

When $\phi > \lambda_3$, by using the transformation $\phi = \frac{-\lambda_2\sin^2\psi+\lambda_3}{\cos^2\psi}$, we can attain the solutions of Eq (1.1), as

$$u_8(x,t) = \pm\sqrt{\Big(\frac{4c_2-4\delta}{3\alpha+\beta+4\gamma}\Big)^{-\frac{1}{3}}\frac{\lambda_3-\lambda_2\text{sn}^2\Big(\frac{\sqrt{\lambda_3-\lambda_1}}{2}\big(\frac{4c_2-4\delta}{3\alpha+\beta+4\gamma}\big)^{\frac{1}{3}}(x-\frac{2ak-b\omega}{bk-1}t-\xi_0),m\Big)}{\lambda_3\text{cn}^2\Big(\frac{\sqrt{\lambda_3-\lambda_1}}{2}\big(\frac{4c_2-4\delta}{3\alpha+\beta+4\gamma}\big)^{\frac{1}{3}}(x-\frac{2ak-b\omega}{bk-1}t-\xi_0),m\Big)}}\times e^{i(-kx+\omega t+\varphi_0)}. \tag{3.26}$$

When $\lambda_1 < \phi < \lambda_2$, implementing the transformation $\phi = \lambda_1 + (\lambda_2-\lambda_1)\sin^2\psi$, the Jacobian elliptic function solutions of Eq (1.1) are given by

$$u_9(x,t) = \pm e^{i(-kx+\omega t+\varphi_0)} \tag{3.27}$$
$$\times\sqrt{\Big(\frac{4c_2-4\delta}{3\alpha+\beta+4\gamma}\Big)^{-\frac{1}{3}}\Big[\lambda_1+(\lambda_2-\lambda_1)\text{sn}^2\Big(\frac{\sqrt{\lambda_3-\lambda_1}}{2}\big(\frac{4c_2-4\delta}{3\alpha+\beta+4\gamma}\big)^{\frac{1}{3}}(x-\frac{2ak-b\omega}{bk-1}t-\xi_0),m\Big)\Big]},$$

where $m^2 = \frac{\lambda_2-\lambda_1}{\lambda_3-\lambda_1}$.

Case 4: $\Delta < 0$. Implementing the transformation $\phi = \sqrt{r}\tan^2\frac{\psi}{2}$ converts Eq (3.14) to

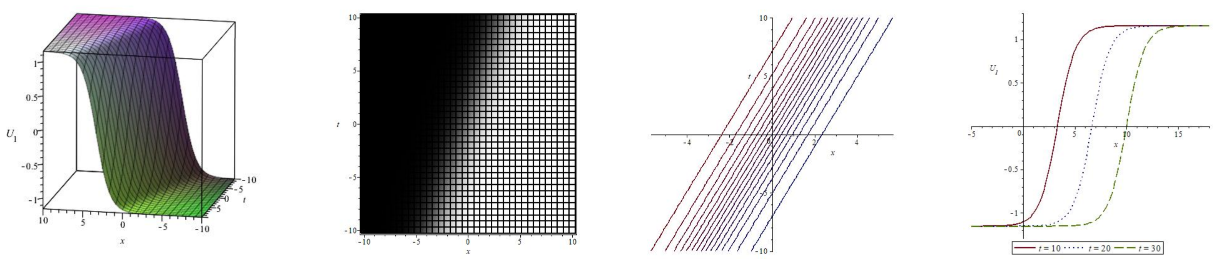
$$\pm(\xi_1-\xi_0) = r^{-\frac{1}{4}}\int\frac{\mathrm{d}\psi}{\sqrt{1-m^2\sin^2\psi}}, \tag{3.28}$$

where $m^2 = \frac{1}{2}(1-\frac{q}{2\sqrt{r}})$. In this case, the Jacobian elliptic function solutions of Eq (1.1) are given by

$$\begin{aligned} u_{10}(x,y) = \pm\Big\{&\frac{2(bc-a)^2(c_2-\delta)^2-(bc-a)(3\alpha+\beta+4\gamma)[c_1+(2\alpha-\beta+2\gamma)k^2](c_2-\delta)}{(\alpha+\beta)(2\alpha+\beta+2\gamma)} \\ &-\frac{(\omega-\omega bk+ak^2)(c_2-\delta)}{(\alpha+\beta)(3\alpha+\beta+4\gamma)}\Big\}^{\frac{1}{4}}e^{i(-kx+\omega t+\theta)}\times\Big[\frac{2}{1+\mathrm{cn}\Big(2\Big\{\frac{2(bc-a)^2(c_2-\delta)^2}{(\alpha+\beta)(3\alpha+\beta+4\gamma)^2(2\alpha+\beta+2\gamma)} -\frac{(bc-a)[c_1+(2\alpha-\beta+2\gamma)k^2](c_2-\delta)}{(\alpha+\beta)(3\alpha+\beta+4\gamma)(2\alpha+\beta+2\gamma)}-\frac{(\omega-\omega bk+ak^2)(c_2-\delta)}{(\alpha+\beta)(3\alpha+\beta+4\gamma)}\Big\}^{\frac{1}{4}}\big(x-\frac{2ak-b\omega}{bk-1}t-\xi_0\big),m\Big)}-1\Big]^{\frac{1}{2}}. \end{aligned} \tag{3.29}$$

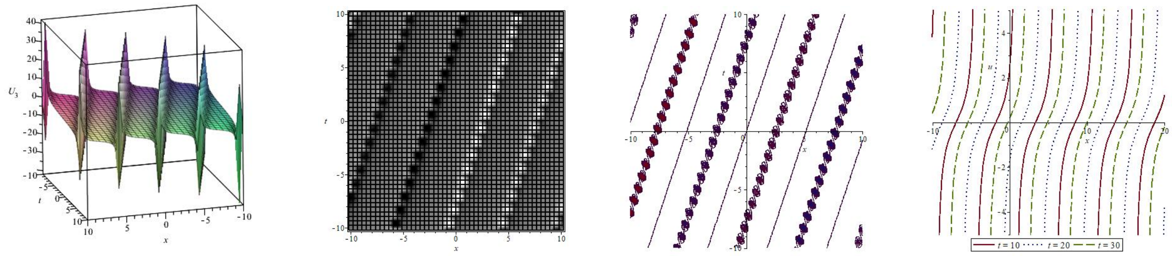
4. Graphical illustrations

This section includes the graphical presentations for some obtained solutions. We select suitable values of parameters to simulate exact solutions of Eq (1.1) and choose + instead of $\pm$ to plot all the graphs using Maple software. The envelope of kink solitary wave solution u_1, periodic function solution u_3, bell shaped solitary wave solution u_5 and triangle analytical solution u_7 are represented in Figures 1(a)–4(a), respectively. Contour plots and density plots of these solutions are depicted in Figures 1(b), 1(c)–4(b), 4(c). 2D plots in Figures 1(d)–4(d) capture the level curves at different times $t = 10, 20, 30$. In recently published works, researchers mainly describe the structures for the absolute values of the obtained solutions and illustrated some interesting results. In this paper, we present the envelope of the obtained solutions to help in understanding properties of the LPD model, which describes many phenomena in nonlinear science.



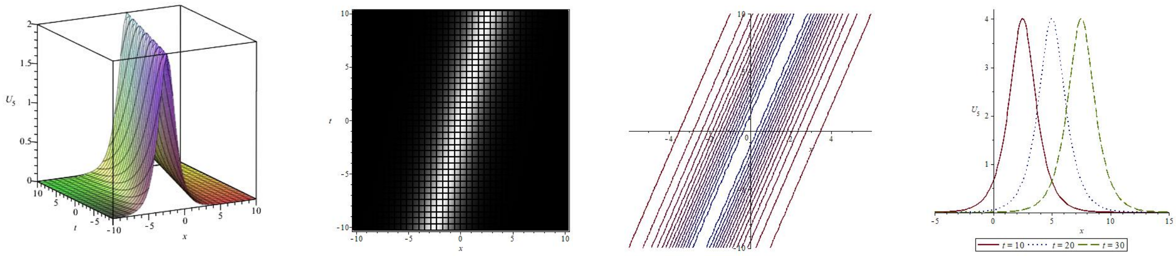
(a) 3D surface plot of (3.17) (b) Contour plot of (3.17) (c) Density plot of (3.17) (d) 2D plot at $t = 10, 20, 30$

Figure 1. The profile of kink soliton solution U_1 with $k = 4$, $\omega = 3$ and $c = \frac{1}{3}$. The parameters are: $a = \frac{1}{2}$, $b = 1$, $c_1 = -51\frac{5}{12}$, $c_2 = 3$, $\alpha = \beta = \gamma = \delta = 1$ and $\xi_0 = 0$.



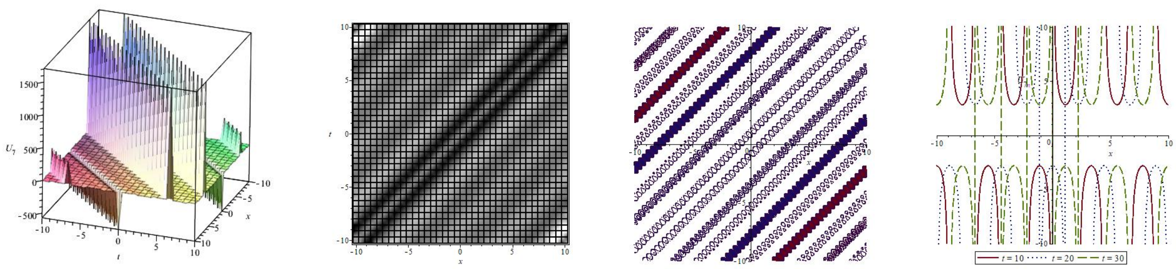
(a) 3D surface plot of (3.19) (b) Density plot of (3.19) (c) Contour plot of (3.19) (d) 2D plot at $t = 10, 20, 30$

Figure 2. The profile of periodic soliton solution U_3 with $k = 4$, $\omega = 3$ and $c = \frac{1}{3}$. The parameters are: $a = \frac{1}{2}$, $b = 1$, $c_1 = -44\frac{1}{3}$, $c_2 = 3$, $\alpha = \beta = \gamma = \delta = 1$ and $\xi_0 = 0$.



(a) 3D surface plot of (3.22) (b) Density plot of (3.22) (c) Contour plot of (3.22) (d) 2D plot at $t = 10, 20, 30$

Figure 3. The profile of bell shaped soliton solution U_5 with $k = 2$, $\omega = \frac{7}{4}$ and $c = \frac{1}{4}$. The parameters are: $a = \frac{1}{2}$, $b = 1$, $c_1 = -6\frac{31}{32}$, $c_2 = -1$, $\alpha = \beta = \gamma = \delta = 1$ and $\xi_0 = 0$.



(a) 3D surface plot of (3.24) (b) Density plot of (3.24) (c) Contour plot of (3.24) (d) 2D plot at t=10, 20, 30

Figure 4. The profile of triangle function solution U_7 with $k = 2$, $\omega = 1$ and $c = 1$. The parameters are: $a = \frac{1}{2}$, $b = 1$, $c_1 = -21\frac{3}{4}$, $c_2 = 3$, $\alpha = \beta = \gamma = \delta = 1$ and $\xi_0 = 0$.

5. Conclusions

In this work, the Lakshmanan-Porsezian-Daniel equation with parabolic law nonlinearity is studied by applying the trial function methodology and complete discrimination system for polynomials method. Many new complex exact solutions of the LPD model are obtained, such as kink soliton solutions, bell shaped soliton solutions, triangular function solutions, periodic function solutions, singular solutions and Jacobian function solutions, which are not obtained by other techniques. In [23–25], the solutions are obtained by setting the constrain conditions $\alpha + \beta = 0$ and $\gamma + \lambda = 0$ for the coefficients in Eq (1.1). Comparing our results with the previous works, we address the model without the constrain conditions above and attain some new results. Also, Figures 1–4 depict the solutions in (3.17), (3.19), (3.22) and (3.24) with suitable values of parameters, which can be expected to reveal more dynamic behaviors and physical properties of the LPD model. We think the methodology used in this work is able to solve more complicated and various nonlinear partial differential equations.

Acknowledgments

This project is supported by funding of V.C. & V.R. Key Lab of Sichuan Province. This work was funded by the Scientific Research Funds of Chengdu University under grant No. 2081920034.

Conflict of interest

The authors declare no conflicts of interest.

References

1. A. Biswas, S. Konar, *Introduction to non-Kerr law optical solitons*, Boca Raton: Chapman Hall/CRC Press, 2006.
2. A. Biswas, Soliton solutions of the perturbed resonant nonlinear Schrödinger's equation with full nonlinearity by semi-inverse variational principle, *Quant. Phys. Lett.*, **1** (2013), 79–83.
3. A. Biswas, D. Milovic, M. Savescu, M. F. Mahmood, R. Kohl, Optical soliton perturbation in nanofibers with improved nonlinear Schrödinger's equation by semi-inverse variational principle, *J. Nonlinear Opt. Phys.*, **21** (2012), 500543. https://doi.org/10.1142/S0218863512500543
4. A. Ankiewicz, D. J. Kedziora, A. Chowdury, U. Bandelow, N. Akhmediev, Infinite hierarchy of nonlinear Schrödinger equations and their solutions, *Phys. Rev. E*, **93** (2016), 012206. https://doi.org/10.1103/PhysRevE.93.012206
5. E. M. E. Zayed, E. M. M. Elshater, Jacobi elliptic solutions, soliton solutions and other solutions to four higher-order nonlinear Schrödinger equations using two mathematical methods, *Optik*, **1** (2017), 1044–1062. https://doi.org/10.1016/j.ijleo.2016.11.120
6. M. Lakshmanan, Continuum spin system as an exactly solvable dynamical system, *Phys. Lett. A*, **61** (1977), 53–58. https://doi.org/10.1016/0375-9601(77)90262-6
7. M. Lakshmanan, K. Porsezian, M. Daniel, Effect of discreteness on the continuum limit of the Heisenberg spin chain, *Phys. Lett. A*, **133** (1988), 483–488. https://doi.org/10.1016/0375-9601(88)90520-8
8. K. Porsezian, M. Daniel, M. Lakshmanan, On the integrability aspects of the one-dimensional classical continuum isotropic biquadratic Heisenberg spin chain, *J. Math. Phys.*, **33** (1992), 1807–1816. https://doi.org/10.1063/1.529658

9. K. Porsezian, *On the discrete and continuum integrable Heisenberg spin chain models*, Boston: Springer, 1993. https://doi.org/10.1007/978-1-4899-1609-9-42
10. J. Manafian, M. Foroutan, A. Guzali, Applications of the ETEM for obtaining optical soliton solutions for the Lakshmanan-Porsezian-Daniel model, *Eur. Phys. J. Plus*, **132** (2017), 494. https://doi.org/10.1140/epjp/i2017-11762-7
11. W. Liu, D. Q. Qiu, Z. W. Wu, J. S. He, Dynamical behavior of solution in integrable nonlocal Lakshmanan-Porsezian-Daniel equation, *Commun. Theor. Phys.*, **65** (2016), 671–676. https://doi.org/10.1088/0253-6102/65/6/671
12. A. Biswas, M. Ekici, A. Sonmezoglu, H. Triki, F. B. Majid, Q. Zhou, et al., Optical solitons with Lakshmanan-Porsezian-Daniel model using a couple of integration schemes, *Optik*, **158** (2018), 705–711. https://doi.org/10.1016/j.ijleo.2017.12.190
13. V. N. Serkin, T. L. Belyaeva, Novel soliton breathers for the higher-order Ablowitz-Kaup-Newell-Segur hierarchy, *Optik*, **174** (2018), 259–265. https://doi.org/10.1016/j.ijleo.2018.08.034
14. V. N. Serkin, T. L. Belyaeva, Optimal control for soliton breathers of the Lakshmanan-Porsezian-Daniel, Hirota, and cmKdV models, *Optik*, **175** (2018), 17–27. https://doi.org/10.1016/j.ijleo.2018.08.140
15. S. Kumar, A. Biswas, Q. Zhou, Y. Yıldırım, H. M. Alshehri, M. R. Belic, Straddled optical solitons for cubic-quartic Lakshmanan-Porsezian-Daniel model by Lie symmetry, *Phys. Lett. A*, **417** (2021), 127706. https://doi.org/10.1016/j.physleta.2021.127706
16. C. S. Liu, Trial equation method to nonlinear evolution equations with rank inhomogeneous: mathematical discussions and its applications, *Commun. Theor. Phys.*, **45** (2006), 219–223. https://iopscience.iop.org/article/10.1088/0253-6102/45/2/005
17. A. Biswas, Y. Yıldırım, E. Yasar, Q. Zhou, S. P. Moshokoa, M. Belic, Optical soliton perturbation with quadratic-cubic nonlinearity using a couple of strategic algorithms, *Chin. J. Phys.*, **56** (2018), 1990–1998. https://iopscience.iop.org/article/10.1016/J.CJPH.2018.09.009
18. M. Ekici, A. Sonmezoglu, A. Biswas, M. R. Belic, Optical solitons in (2+1)-dimensions with Kundu-Mukherjee-Naskar equation by extended trial function scheme, *Chin. J. Phys.*, **57** (2018), 72–77. https://doi.org/10.1016/j.cjph.2018.12.011
19. X. Xiao, Z. X. Yin, Exact single travelling wave solutions to the fractional perturbed Gerdjikov-Ivanov equation in nolinear optics, *Mod. Phys. Lett. B*, **35** (2021), 2150337. https://doi.org/10.1142/S0217984921503772
20. J. Y. Hu, X. B. Feng, Y. F. Yang, Optical envelope patterns perturbation with full nonlinearity for Gerdjikov-Ivanov equation by trial equation method, *Optik*, **240** (2021), 166877. https://doi.org/10.1016/J.IJLEO.2021.166877
21. Z. Li, P. Li, T. Y. Han, Bifurcation, traveling wave solutions, and stability analysis of the fractional generalized Hirota-Satsuma coupled KdV equations, *Discrete Dyn. Nat. Soc.*, **2021** (2021), 5303295. https://doi.org/10.1155/2021/5303295
22. J. V. Guzman, R. T. Alqahtani, Q. Zhou, M. F. Mahmood, S. P. Moshokoa, M. Z. Ullah, et al., Optical solitons for Lakshmanan-Porsezian-Danielmodel with spatio-temporal dispersion using the method of undetermined coefficients, *Optik*, **144** (2017), 115–123. https://doi.org/10.1016/J.IJLEO.2017.06.102

23. A. Biswas, Y. Yıldırım, E. Yasar, Q. Zhou, S. P. Moshokoa, M. Belic, Optical solitons for Lakshmanan-Porsezian-Daniel model by modified simple equation method, *Optik*, **160** (2018), 24–32. https://doi.org/10.1016/J.IJLEO.2018.01.100
24. G. Akram, M. Sadaf, M. Dawood, D. Baleanu, Optical solitons for Lakshmanan-Porsezian-Daniel equation with Kerr law non-linearity using improved $\tan\frac{\psi(\eta)}{2}$-expansion technique, *Results Phys.*, **29** (2021), 104758. https://doi.org/10.1016/J.RINP.2021.104758
25. G. Akram, M. Sadaf, M. A. Khan, Abundant optical solitons for Lakshmanan-Porsezian-Daniel model by the modified auxiliary equation method, *Optik*, **251** (2022), 168163. https://doi.org/10.1016/j.ijleo.2021.168163
26. M. B. Hubert, G. Betchewe, M. Justin, S. Y. Doka, K. T. Crepin, A. Biswas, et al., Optical solitons with Lakshmanan-Porsezian-Daniel model by modified extended direct algebraic method, *Optik*, **162** (2018), 228–236. https://doi.org/10.1016/j.ijleo.2018.02.091
27. C. S. Liu, Exact travelling wave solutions for (1+1)-dimensional dispersive long wave equation, *Chin. Phys.*, **14** (2005), 1710–1715. https://doi.org/10.1088/1009-1963/14/9/005