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Research article

New entanglement-assisted quantum codes constructed from Hermitian LCD codes

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Abstract: Hermitian linear complementary dual (LCD) codes are a class of linear codes that intersect with their Hermitian dual trivially. Each Hermitian LCD code can give an entanglement-assisted quantum error-correcting code (EAQECC) with maximal entanglement. Methods of constructing Hermitian LCD codes from known codes were developed, and seven new Hermitian LCD codes with parameters $[119, 4, 88]_4$, $[123, 4, 91]_4$, $[124, 4, 92]_4$, $[136, 4, 101]_4$, $[140, 4, 104]_4$, $[188, 4, 140]_4$ and $[212, 4, 158]_4$ were constructed. Seven families of Hermitian LCD codes and their related EAQECCs were derived from these codes. These new EAQECCs have better parameters than those known in the literature.

Keywords: quaternary Hermitian; linear complementary dual (LCD); linear code; entanglement-assisted quantum code
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1. Introduction

Linear complementary dual (LCD) codes were introduced by Massey in 1992 [1], and it was shown that LCD codes provide an optimum linear coding solution for the two-user binary adder channel. In 2016, Carlet and Guilley [2] investigated an interesting application of binary LCD codes against side-channel and fault injection attacks and presented several constructions of LCD codes. A Hermitian LCD code is also called *zero radical code* in [3], and each $[n, k, d]_4$ quaternary Hermitian LCD code gives a maximal entanglement-assisted quantum code with parameter $[[n, k, d; n - k]]_2$ [3, 4]. Carlet et al. [5] proved that any q-ary ($q > 3$) linear code is equivalent to an LCD code, and any q^2-ary ($q > 2$) linear code is equivalent to a q^2-ary Hermitian LCD code. Following these work, people pay much attention on investigating binary LCD codes and quaternary Hermitian LCD codes [6–10].

Entanglement-assisted stabilizer formalism was devised by Brun et al. in [11]. It has been proven that Hermitian LCD codes were used to construct maximal entanglement assisted quantum error-correcting codes (EAQECCs) by [3, 4]. According to [3, 4], an $[n, k, d]_4$ quaternary Hermitian LCD code gives a maximal entanglement-assisted quantum code with parameter $[[n, k, d; n-k]]_2$. Hence, it is important to study optimal quaternary Hermitian LCD codes for constructing $[[n, k, d; n-k]]_2$ EAQECCs. An $[n, k, d]_4$ Hermitian LCD code is *optimal* if it has the largest distance for a given n, k. Parameters of quaternary optimal $[n, k, d]_4$ Hermitian LCD codes with $k \leq 3$ are determined by [3, 6, 7]. There are some articles devoted to constructing $[n, k, d]_4$ Hermitian LCD codes with small n and $k \geq 4$ [8–10]. In [3], for each t with $4 \leq t \leq 88$, a $[t, 4, d_t]_4$ Hermitian LCD code with relatively large d_t was constructed. For $n = 85s + t$ with $s \geq 1$ and $4 \leq t \leq 84$, $[n, 4, d]_4$ Hermitian LCD codes are constructed by juxtaposing some simplex codes and $[t, 4, d_t]_4$ Hermitian LCD codes. In this paper, we will develop some methods for constructing new Hermitian LCD codes from known optimal codes and manage to improve some results on Hermitian LCD codes of [3], and then construct new maximal entanglement EAQECCs. All parameters of codes in this paper are calculated by Magma [12].

This paper is organized as follows. In section two, we propose some definitions and fundamental results on Hermitian LCD codes, self-orthogonal codes and entanglement-assisted codes. In section three, we provide two methods to construct Hermitian LCD codes and some examples. Finally, in section four, we conclude this paper.

2. Preliminaries

In this section, we prepare some definitions, notations and basic results used in this paper.

Let $F_4 = \{0, 1, \omega, \omega^2\}$ be the Galois field with four elements, where $\omega^2 = 1 + \omega, \omega^3 = 1$. The n-dimensional space over F_4 is denoted as F_4^n. The Hamming weight $wt(x)$ of a vector $x \in F_4^n$ is the number of nonzero components of x. The distance $d(x, y)$ between $x, y \in F_4^n$ $(x \neq y)$ is $wt(x - y)$. A quaternary $[n, k]_4$ code C is a k-dimensional vector subspace of F_4^n, $C = [n, k, d]_4$ if the minimum distance of two differerent codewords $x, y \in C$ is d. A $k \times n$ matrix G is a generator matrix of C if its rows form a basis for C [13].

The conjugation of $x \in F_4$ is defined by $\bar{x} = x^2$. Let $u = (u_1, u_2 \cdots u_n)$ and $v = (v_1, v_2 \cdots v_n)$ be vectors of F_4^n; their Hermitian inner product is

$$(u, v)_h = \sum_{i=1}^{n} u_i \cdot \overline{v_i} = \sum_{i=1}^{n} u_i \cdot v_i^2.$$

If C is a linear code over F_4, then its Hermitian dual code is

$$C^{\perp_h} = \{u \in F_4^n \mid (u, v)_h = 0, \forall v \in C\}.$$

If $C \subseteq C^{\perp_h}$, then C is called a Hermitian self-orthogonal code, and if $C \cap C^{\perp_h} = \{0\}$, then C is called a Hermitian LCD code. C is a Hermitian self-orthogonal code if and only if $GG^\dagger = 0$, and C is a Hermitian LCD code if and only if $k = rank(GG^\dagger)$, where G is a generator matrix of C and $G^\dagger$ is the conjugate transpose of G.

If G is a generator matrix of $C = [n, k]_4$ and $G = [I_k \mid A]$, where I_k is the $k \times k$ identity matrix, C is called a system code. We say that two $[n, k]_4$ codes C_1 and C_2 are equivalent, provided there is a monomial matrix M and an automorphism σ of the field F_4 such that $C_2 = \sigma(C_1 M)$. Each $[n, k]_4$ is equivalent to a system code [13].

A code is called *optimal* when its minimum distance is maximal for a given length and dimension, or when its length is minimal for the given dimension and minimum distance [14]. Let $n_q(k,d)$ be the smallest value of n, for which there exists an $[n,k,d]_q$ code. A lower bound on $n_q(k,d)$ is the Griesmer bound given by $n_q(k,d) \geqslant \sum_{i=0}^{k-1}\lceil d/q^i \rceil$. For $C = [n,k,d]_4$, if $n_4(k,d) = \sum_{i=0}^{k-1}\lceil d/4^i \rceil$, C is an optimal linear code. If $n_4(k,d) = \sum_{i=0}^{k-1}\lceil d/4^i \rceil + 1$, C is a near-optimal linear code.

We will use two lemmas to construct Hermitian LCD codes from known codes.

Lemma 1. *Suppose $G_1 = G_{k\times n_1}$ generates an $[n_1,k,d_1]$ Hermitian self-orthogonal code C_1, and $G_2 = G_{k\times n_2}$ generates an $[n_2,k,d_2]$ code C_2.*

(1) If C_2 is a Hermitian LCD code, then there is an $[n_1+n_2,k,d_1+d_2]$ Hermitian LCD code.

(2) If C_2 is a Hermitian self-orthogonal code, then there is an $[n_1+n_2,k,d_1+d_2]$ Hermitian self-orthogonal code and an $[n_1+n_2-k,k,d_1+d_2-k]$ Hermitian LCD code.

Proof. Let $G_3 = [G_1 \mid G_2]$, then G_3 generates an $[n_1+n_2,k,d_1+d_2]$ code C_3 according to [15], and

$$G_3G_3^{\dagger} = G_1G_1^{\dagger} + G_2G_2^{\dagger} = G_2G_2^{\dagger}$$

according to C_1 is a Hermitian self-orthogonal code.

(1) If C_2 is a Hermitian LCD code, then $rank(G_3G_3^{\dagger}) = rank(G_2G_2^{\dagger}) = k$, hence, (1) holds.

(2) If C_2 is a Hermitian self-orthogonal code, then $G_3G_3^{\dagger} = 0$ and G_3 generates an $[n_1+n_2,k,d_1+d_2]$ Hermitian self-orthogonal code. This code is equivalent to a Hermitian self-orthogonal code with generator matrix $G_3' = [I_k \mid A]$. Deleting the first k-columns of $G_3' = [I_k \mid A]$, one can obtain a $k\times(n_1+n_2-k)$ matrix A. From $G_3'(G_3')^{\dagger} = 0$, we know $AA^{\dagger} = I_k$, hence, A generates a Hermitian LCD code with parameters $[n_1+n_2-k,k,d_1+d_2-k]$ according to [13]. □

Lemma 2. *Suppose $C_1 = [n_1,k,d_1]$ is a Hermitian self-orthogonal code and it has a codeword of largest weight $w_{max}(w_{max} > 1)$. If there is an $[n_2,k-1,d_2]$ Hermitian LCD code C_2, then there is an $[n_1+n_2-1,k,d]$ Hermitian LCD code, where $d \geq min\{d_1+d_2-1, w_{max}-1\}$.*

Proof. Let α be a codeword in C_1 with weight w_{max} and its first nonzero coordinate is 1, i.e., $\alpha = (0,\cdots,0,1,x_{k+1},\cdots,x_{n_1})$, where 1 is the j-th coordinate. Therefore, we can choose a generator matrix G_1 of C_1 such that

$$G_1 = \begin{pmatrix} M_{(k-1)\times n_1} \\ a \end{pmatrix},$$

where M is a matrix whose j-th column is zero vector. Delet the j-th column of G_1 and denote the obtained matrix as G_1'. Since C_1 is a Hermitian self-orthogonal code, one can deduce that

$$G_1'(G_1')^{\dagger} = \begin{pmatrix} 0_{(k-1)\times(k-1)} & \mathbf{0}^{\mathbf{T}}_{\mathbf{(k-1)}} \\ \mathbf{0}_{\mathbf{(k-1)}} & 1 \end{pmatrix}.$$

Let G_2 be a generator matrix of C_2 and $G_2' = \begin{pmatrix} G_2 \\ \mathbf{0} \end{pmatrix}$. Construct $G_3 = [G_1 \mid G_2']$, $G_3' = [G_1' \mid G_2']$. According to [16], G_3 generates an $[n_1+n_2,k,d_3]$ code with $d_3 \geq min\{d_1+d_2, w_{max}\}$. Thus G_3 generates an $[n_1+n_2-1,k,d]$ code with $d \geq min\{d_1+d_2-1, w_{max}-1\}$. It is not difficult to check that

$$G_3'(G_3')^{\dagger} = \begin{pmatrix} G_2G_2^{\dagger} & \mathbf{0}^{\mathbf{T}}_{\mathbf{(k-1)}} \\ \mathbf{0}_{\mathbf{(k-1)}} & 1 \end{pmatrix}.$$

From this, we can derive that the lemma holds. □

To construct Hermitian LCD code $C = [n, k]_4$ with larger minimum distance, we need some special Hermitian self-orthogonal codes and two Hermitian LCD codes.

For the sake of simplicity, we use 2 and 3 to represent ω and ω^2. Let $\mathbf{1_n}=(1, 1, \ldots, 1)_{1\times n}$ and $\mathbf{0_n}=(0, 0, \ldots, 0)_{1\times n}$ denote the all-one vector and the all-zero vector of length n, respectively. Construct

$$S_2 = \begin{pmatrix} 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 2 & 3 \end{pmatrix} = (\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5),$$

$$S_3 = \begin{pmatrix} S_2 & \mathbf{0}_{2\times 1} & S_2 & S_2 & S_2 \\ \mathbf{0}_5 & 1 & \mathbf{1}_5 & 2\cdot\mathbf{1}_5 & 3\cdot\mathbf{1}_5 \end{pmatrix} = (\beta_1, \beta_2, \ldots, \beta_{21}),$$

$$S_4 = \begin{pmatrix} S_3 & \mathbf{0}_{3\times 1} & S_3 & S_3 & S_3 \\ \mathbf{0}_{21} & 1 & \mathbf{1}_{21} & 2\cdot\mathbf{1}_{21} & 3\cdot\mathbf{1}_{21} \end{pmatrix} = (\gamma_1, \gamma_2, \ldots, \gamma_{85}),$$

where $\alpha_i(1 \leqslant i \leqslant 5), \beta_j(1 \leqslant j \leqslant 21)$, $\gamma_k(1 \leqslant k \leqslant 85)$ are column vectors over F_4.

The matrices S_2, S_3 and S_4 generate $C_{2,5} = [5, 2, 4]_4$, $C_{3,21} = [21, 3, 16]_4$ and $C_{4,85} = [85, 4, 64]_4$ simplex codes, respectively. Let $A_{4,80} = (\gamma_6, \gamma_7, \ldots, \gamma_{85})$, $A_{4,64} = (\gamma_{22}, \gamma_{23}, \ldots, \gamma_{85})$. It is self-evident to see $A_{4,80}$ and $A_{4,64}$ generate $[80, 4, 60]_4$ and $[64, 4, 48]_4$ codes, respectively. Therefore, all these five codes are Hermitian self-orthogonal codes [14, 15].

It is shown that there are Hermitian LCD codes $[72, 4, 53]_4$ in [3] and $[26, 3, 19]_4$ in [7], and these two codes have generator matrices $G_{4,72}$ and $G_{3,26}$ as follows:

$$G_{4,72} = \begin{pmatrix} 11111111110111010111101111011110111101110101111011110111101110101111011110111 \\ 130230123011230011230112301123011230011230112301123011230011230112301123 \\ 1122223333300000111111222222333330000011111122222233333 \\ 00000000011111111111111111111222222222222222222222222333333333333333333333 \end{pmatrix},$$

$$G_{3,26} = \begin{pmatrix} 10033322001112233330011131 \\ 01020112032103133103102333 \\ 00102321211233123100220102 \end{pmatrix}.$$

3. Construction of Hermitian LCD codes and EAQECCs

We first construct seven Hermitian LCD codes by employing lemmas and known codes in the previous section.

Theorem 1. *There are Hermitian LCD codes with parameters* $[119, 4, 88]_4$, $[123, 4, 91]_4$, $[124, 4, 92]_4$, $[136, 4, 101]_4$, $[140, 4, 104]_4$, $[188, 4, 140]_4$ *and* $[212, 4, 158]_4$.

Proof. (1) Let $G_1 = G_{4,64}$ as follows:

$$\begin{pmatrix} 1000113302213100332313002322003011131212022132001132223310032233 \\ 0100030001102010103130132311323201203223222232222330333331111110 \\ 0010331322021113021200311320311220022133323210101102023232033110 \\ 0001313130320112301133321212030130211013032011233210023302101322 \end{pmatrix},$$

$G_2 = A_{4,64}$. Both of them can generate $[64, 4, 48]_4$ Hermitian self-orthogonal codes.

Constructing $G_3 = [G_1 \mid G_2]$, $G_4 = [A_{4,80} \mid G_2]$, $G_5 = [G_1 \mid G_2 \mid G_2]$, then G_3, G_4 and G_5 generate $[128, 4, 96]_4$, $[144, 4, 108]_4$ and $[192, 4, 144]_4$ Hermitian self-orthogonal codes, respectively. From these three codes, one can obtain $[124, 4, 92]_4$, $[140, 4, 104]_4$ and $[188, 4, 140]_4$ Hermitian LCD codes according to Lemma 1. Deleting five columns, $(1, 0, 0, 0)^T$, $(0, 1, 0, 0)^T$, $(0, 0, 1, 0)^T$, $(0, 0, 0, 1)^T$ and $(1, 1, 1, 1)^T$, from G_3, one can obtain a 4×123 matrix. This matrix generates a $[123, 4, 91]_4$ Hermitian LCD code.

Deleting five columns from G_3, $(1, 2, 3, 1)^T$, $(1, 1, 3, 2)^T$, $(0, 1, 0, 1)^T$, $(1, 0, 3, 3)^T$ and $(1, 3, 3, 0)^T$, one obtains a 4×123 matrix $A_{4,123}$ and a $[123, 4, 92]_4$ Hermitian self-orthogonal code; hence, there is a $[119, 4, 88]_4$ Hermitian LCD code.

(2) Let $G_{4,72}$ be the generator matrix of a $[72, 4, 53]_4$ Hermitian LCD code given in the previous section. Constructing $G_{4,136} = (G_2 \mid G_{4,72})$, this matrix generates a $[136, 4, 101]_4$ Hermitian LCD code.

Using the generator matrix $A_{4,123}$ of the $[123, 4, 92]_4$ Hermitian self-orthogonal code given in (1), we can construct $G_{4,187} = [A_{4,123} \mid G_2]$, with $G_{4,187}$ generating a $[187, 4, 140]_4$ Hermitian self-orthogonal code. This code has a codeword of weight 172. From this $[187, 4, 140]_4$ code and a $[26, 3, 19]_4$ Hermitian LCD code, we can construct a $[212, 4, 158]_4$ Hermitian LCD code by Lemma 2.

Summarizing the previous discussions, we complete the proof. □

Notation 1. *The Hermitian LCD codes* $[119, 4, 88]_4$, $[124, 4, 92]_4$, $[136, 4, 101]_4$, $[140, 4, 104]_4$ *and* $[188, 4, 140]_4$ *are also optimal linear codes, while* $[123, 4, 91]_4$ *and* $[212, 4, 158]_4$ *are near-optimal linear codes [17]. All these Hermitian LCD codes have distances larger than those in [3]. For* $N = 119, 123, 124, 136, 140$, *denote* $N = 85 + n_1$ *with* $n_1 = 34, 38, 39, 51, 55$. *For* $N = 188, 212$, *denote* $N = 2 \times 85 + n_1$ *with* $n_1 = 18, 42$. *These seven codes can be denoted as* $[N, 4, 64a + d_{n_1}]_4$, *where* $a = [N/85]$ *and* $d_{n_1} = 24, 27, 28, 37, 40, 12, 30$ *for* $n_1 = 34, 38, 39, 51, 55, 18, 42$, *respectively. From these seven Hermitian LCD codes, one can deduce seven families of Hermitian LCD codes and their related maximal entanglement-assisted quantum codes.*

Theorem 2. *These are the following maximal entanglement-assisted quantum codes:*

(1) If $s \geq 1$, *there are* $[[85s + 34, 4, 64s + 24; 85s + 30]]$, $[[85s + 38, 4, 64s + 27; 85s + 34]]$, $[[85s + 39, 4, 64s + 28; 85s + 35]]$, $[[85s + 51, 4, 64s + 37; 85s + 47]]$, $[[85s + 55, 4, 64s + 40; 85s + 51]]$.

(2) If $s \geq 2$, *there are* $[[85s + 18, 4, 64s + 12; 85s + 14]]$, $[[85s + 42, 4, 64s + 30; 85s + 38]]$.

Proof. (1) For $s \geq 1$, if $n = 85s + n_1$ with $n_1 = 34, 38, 39, 51, 55$, denote $n = 85(s-1) + (85 + n_1) = 85(s-1) + N_1$, then $N_1 = 119, 123, 124, 136, 140$, respectively. Let G_{4,N_1} be generator matrices of Hermitian LCD codes $[N_1, 4, d_{N_1}]_4$ with $N_1 = 119, 123, 124, 136, 140$ given in Theorem 1. Constructing $G_{4,n} = [(s-1)S_4 \mid G_{4,N_1}]$, then $G_{4,n} = [(s-1)S_4 \mid G_{4,N_1}]$ generates $[n, 4, 64s + d_{n_1}]_4$ for $n_1 = 34, 38, 39, 51, 55$. Thus, (1) holds.

(2) For $s \geq 2$, $n = 85s + n_1$ with $n_1 = 18, 42$, denote $n = 85(s-2) + (170 + n_1) = 85(s-2) + N_1$ with $N_1 = 188, 212$. It is easy to see that (2) follows.

Using the Griesmer bound, one can check that the five families of Hermitian LCD codes $[85s + 34, 4, 64s + 24]_4$, $[85s + 39, 4, 64s + 28]_4$, $[85s + 51, 4, 64s + 37]_4$, $[85s + 55, 4, 64s + 40]_4$, $[85s + 18, 4, 64s + 12]_4$, are optimal codes, and $[85s + 38, 4, 64s + 27]_4$ and $[85s + 42, 4, 64s + 30]_4$ are near-optimal codes [17]. □

Comparing the above new obtained EAQECCs with those $[[n, 4]]$ of the same lengths in [3], it can be seen that our EAQECCs have larger minimum distances.

Table 1 shows the parameters of our maximal entanglement-assisted quantum codes and theirs.

Table 1. parameters of maximal entanglement-assisted quantum codes.

No.	$[[n, k, d; c]]$ in [3]	$[[n, k, d; c]]$ in Theorem 2
1	$[[85s + 34, 4, 64s + 23; 85s + 30]]$	$[[85s + 34, 4, 64s + 24; 85s + 30]](s \geq 1)$
2	$[[85s + 38, 4, 64s + 26; 85s + 34]]$	$[[85s + 38, 4, 64s + 27; 85s + 34]](s \geq 1)$
3	$[[85s + 39, 4, 64s + 27; 85s + 35]]$	$[[85s + 39, 4, 64s + 28; 85s + 35]](s \geq 1)$
4	$[[85s + 51, 4, 64s + 36; 85s + 47]]$	$[[85s + 51, 4, 64s + 37; 85s + 47]](s \geq 1)$
5	$[[85s + 55, 4, 64s + 39; 85s + 51]]$	$[[85s + 55, 4, 64s + 40; 85s + 51]](s \geq 1)$
6	$[[85s + 18, 4, 64s + 11; 85s + 14]]$	$[[85s + 18, 4, 64s + 12; 85s + 14]](s \geq 2)$
7	$[[85s + 42, 4, 64s + 29; 85s + 38]]$	$[[85s + 42, 4, 64s + 30; 85s + 38]](s \geq 2)$

4. Conclusions

In this paper, we discussed the construction of Hermitian LCD codes from two known codes and constructed seven $[n, 4, d]_4$ Hermitian LCD codes with larger minimum distances than the previously known $[n, 4]_4$ Hermitian LCD codes. From these seven codes, we derived seven families of Hermitian LCD codes. Five families of these Hermitian LCD codes were also optimal linear codes, and the other two families were near-optimal linear codes. From these seven families of Hermitian LCD codes, we gave seven families of entanglement-assisted quantum codes with maximal entanglement, and these newly obtained codes are better than those given in [3] of the same lengths.

The methods used in this work can be useful in studying Hermitian LCD codes with higher dimensions, and we will discuss such questions in the future.

Use of AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

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Conflict of interest

The authors declare no conflicts of interest.

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