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*Research article*

# Trapezoidal type-2 Pythagorean fuzzy TODIM approach for sensible decision-making with unknown weights in the presence of hesitancy

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**Abstract:** Motivated by the concept of type-2 fuzzy sets, we introduce a novel framework known as trapezoidal type-2 Pythagorean fuzzy sets (TRT-2-PFSs), an extension of triangular fuzzy sets. Basic operations like addition and scalar multiplication of two TRT-2-Pythagorean fuzzy numbers (TRT-2-PFNs) are defined. We also explore comparative analysis and distance measurements between two TRT-2-PFNs. A methodology for evaluating unknown weight vectors and criteria weights is proposed. Building upon TRT-2-PFSs, an extension of the TODIM (an acronym in Portuguese of interactive and multi-criteria decision-making) method is developed to address intricate decision-making challenges. Ultimately, the newly introduced TRT-2-PFS-based TODIM technique is employed to tackle multi-criteria decision-making (MCDM) problems.

**Keywords:** Pythagorean fuzzy set; sensible decision-making; TODIM; type-2 fuzzy set, MCDM
**Mathematics Subject Classification:** 03B52, 03E72, 08A72

## 1. Introduction

The primary objective of Multiple Criteria Decision Making (MCDM) is to identify the most suitable alternative from a set of options based on various criteria. When dealing with multiple criteria, the selection process becomes intricate. This falls within the domain of MCDM, a technique for choosing the best alternative considering multiple criteria. In numerous decision-making problems, data is often quantified with precise statistics [1,2]. However, practical situations sometimes prevent decision-makers from expressing preferences with exact numerical values [3,4]. To address this limitation, Atanassov [5] introduced intuitionistic fuzzy sets (IFS) as an improved version of fuzzy sets, using membership degree (MD) and non-membership degree (NMD) to represent information. However, the range of IFS is restricted due to the constraint that MD ($\varphi$) and NMD ($\psi$) cannot exceed one, i.e., $\varphi + \psi$. In complex decision-making scenarios, decision-makers may deviate from this requirement. For instance, if an expert uses IFS to express preferences with MD $0.8$ and NMD $0.7$, their sum is $1.5 \succ 1$, contradicting the constraint. Therefore, IFSs inadequately address such situations. To overcome this, Yager [6,7] proposed Pythagorean fuzzy sets (PFSs) with MD ($\varphi$) and NMD ($\psi$) such that $\varphi^2 + \psi^2 \preccurlyeq 1$. Subsequent studies have explored PFS applications extensively. Yager and Abbasov [8] examined the link between Pythagorean fuzzy numbers and complex numbers. Basic operational laws like division and subtraction for PFSs were introduced by Peng and Yang [9]. Zhang and Xu [10] extended the TOPSIS method under the Pythagorean fuzzy environment for MCDM problems. Further research includes Amin et al. [11] work on generalized cubic PFSs and Rahim et al. [12] presentation of basic operations for cubic PFSs. Huang et al. [13] developed an integrated design alternative assessment model by introducing Z-cloud rough numbers (ZCRNs) that combine cloud model, Z-numbers, and rough numbers to handle various uncertainties and introduced associated operating rules, comparison measures, correlation measures, and aggregation operators for ZCRNs. Xiao et al. [14] introduced a novel q-ROF score function for assessing q-ROF values and used it to develop q-ROF best-worst methods for determining fuzzy criteria weights. Huang et al. [15] used T-spherical fuzzy sets to represent expert preferences, introduce a maximizing deviation method to determine weights, developed a consensus mechanism and present a combined compromise solution method to rank failure modes effectively, as demonstrated in a case study.

### *1.1. Literature review*

The mentioned studies are widely utilized by scholars to address various applications. However, these studies predominantly focused on assessing objects using precise membership functions. Yet, real-world situations often introduce uncertainty in the form of imprecision or ambiguity, rather than probability. To tackle these challenges, Mendel and John [16] introduced Type-2 fuzzy sets (T-2FSs) as an extension of Type-1 fuzzy sets. T-2FSs include an additional membership function that offers experts greater flexibility in simulating uncertainty. A real number in the [0,1] range signifies a Type-1 fuzzy set, while the membership value of T-2FSs itself becomes a Type-1 fuzzy set. The foundational concepts of T-2FSs were presented by Mendel and We [17,18]. Extensive investigations into the operational laws of T-2FSs were undertaken by Chen and Lee [19]. Chen et al. [20] and Mitchell [21] provided levels for different Type-2 fuzzy numbers. Hung and Yang [22] introduced similarity measures for T-2FSs. Researchers have developed diverse techniques to address decision-making problems across various contexts. In this context, Singh and Garg introduced Type-2 intuitionistic fuzzy sets (T-2IFSs) to resolve MCDM problems. T-2IFSs represent a notable expansion of IFS, encompassing both primary and secondary MD and NMD elements. T-2IFSs effectively express decision-makers' preferences regarding their choices across various criteria. Den et al. [23] described the fundamental operational laws like union, complement, necessity operators, and possibility operators for T-2IFSs. Roy and Bhaumik [24] applied a game-theoretic approach to study intelligent water management problems within the triangular Type-2 intuitionistic fuzzy environment. Additionally, Mondal and Roy [25] introduced Type-2 Pythagorean fuzzy sets.

### 1.2. An extensive review of the TODIM method

The TODIM technique, initially introduced by Games and Lima in 1992 [26], offers an effective approach for addressing decision-making problems. This method considers the risk preferences of decision-makers, creating a versatile decision-making environment. Its adaptability has led to widespread applications in various decision-making scenarios. For instance, Fan et al. [27] utilized TODIM to tackle MCDM problems with diverse criteria values, including crisp numbers, interval numbers, linguistic variables, and fuzzy numbers. Wang et al. [28] integrated alpha-level sets with fuzzy information to develop a TODIM technique, while Wei et al. [29] introduced a hesitant fuzzy TODIM method with a score function. In a different context, Krohling et al. [30] applied the TODIM method to solve MCDM problems involving intuitionistic fuzzy information. Zhao et al. [31] proposed the Pythagorean fuzzy TODIM approach based on the cumulative prospect theory for multi-criteria group decision-making problems, and Kaur et al. [32] presented a Pythagorean fuzzy approach for sustainable supplier selection using TODIM. Zhao et al. [33] extended the TODIM method to interval-valued Pythagorean fuzzy sets, and Zhang et al. [34] expanded it through correlation coefficients of PFS. Additionally, Zhou and Chen combined AHP and TODIM for blockchain technology within a PFS framework [35]. Alternatively, Type-2 fuzzy sets, as introduced by Zadeh [36], provide a way to relax membership function constraints, commonly known as the footprint of uncertainty. Integrating Type-2 fuzzy sets into the existing TODIM [37] framework offers researchers greater flexibility in handling uncertainty. Castillo et al. [38] explored a unique class of IFSs capable of addressing specific types of uncertainty, drawing inspiration from the concepts of FS and generic T2-FSs.

In this context, building upon the concept of type-2 fuzzy sets and harnessing the advantages of Pythagorean fuzzy sets to express uncertainty, we introduce a novel structure called TRT-2-PFS. TRT-2-PFS combines elements of Type-2 fuzzy sets, creating a hybrid structure to address uncertainty. Furthermore, within our investigation, we deal with attribute values presented as TRT-2-PFS. Given the intricate and uncertain nature of the MCDM problem, decision-makers frequently encounter imprecise information that transcends the scope of crisp numerical representation. TRT-2-PFNs are a fundamental approach for conveying such imprecision. Consequently, it becomes imperative to explore MCDM in the context of emergency response, specifically when attribute values are expressed as TRT-2-PFNs. The conventional TODIM method is adept at handling multi attribute decision-making problems using crisp numerical data. However, we extend the TODIM method to address the unique challenges posed by the MCDM problem with attribute values in the form of TRT-2-PFNs.

### 1.3. Motivations

Triangular fuzzy sets, commonly used for modeling uncertainty in decision-making, exhibit limitations including lack of flexibility in representing asymmetric or complex uncertainties, difficulty in handling extreme values, and a limited ability to discriminate between different degrees of membership. They are constrained to a symmetrical triangular shape and may not adequately address situations with multimodal, or higher-order fuzzy data. To address this overcome, we introduce a novel concept known as TRT-2-PFS, which builds upon the foundation of trapezoidal membership functions. Trapezoidal fuzzy numbers, constituting the basis of our framework, are defined by two pivotal values that encapsulate the essence of uncertainty more effectively. This feature is especially pertinent in decision-making scenarios enriched with domain expertise. Consider a complex engineering project, where a range of completion times is plausible due to unforeseen challenges. TRT-2-PFS empowers decision-makers to represent these intricate uncertainties more aptly. Moreover, the trapezoidal membership function serves as a versatile tool for capturing the linguistic ambiguity inherent in decision-making. Often, stakeholders express their preferences in qualitative terms rather than precise numerical values. A trapezoidal membership function accommodates these varied expressions of uncertainty in a more generic manner.

This study delves into various facets of TRT-2-PFS. We explore its generation, operations, comparison, and the quantification of distances between two TRT-2-PFSs. This comprehensive analysis equips decision-makers with a toolkit to navigate uncertainties systematically and arrive at well-informed choices. To illustrate the practical implications of TRT-2-PFS, we present a groundbreaking technique: a new TODIM method. This innovative approach leverages the power of the TRT-2-PFS environment to address complex MCDM problems. By incorporating TRT-2-FFS into the TODIM framework, we offer decision-making an advanced tool to tackle intricate real-world challenges.

### *1.4. Objectives*

The following are the important contributions to this article:

1) This research introduces the novel concept of TRT-2-PFS, a cutting-edge framework for effectively modeling uncertainties in decision-making processes. This introduction provides a foundational understanding of a powerful new tool for addressing uncertainty.
2) By leveraging TRT-2-PFS, we provide decision-makers with a more sophisticated and versatile method for representing uncertainties affected by multiple factors, surpassing traditional numerical approaches. This advancement enhances the precision of uncertainty representation.
3) We develop an innovative TODIM technique based on TRT-2-PFS, empowering decision-makers to tackle intricate multi-criteria decision challenges with precision. This innovation advances the field of decision-making under uncertainty and illustrates the practical implications of TRT-2-PFS and the developed TODIM method through a real-world case study, demonstrating their effectiveness in solving complex decision-making problems and offering practical insights into their application.

The following is the article's structure: Section 2 introduces the idea Trapezoidal Type-2 Pythagorean fuzzy sets and describes the weight computation process for TODIM, which makes use of TRT-2-PFSs. Section 3 digs into the detailed process of the unique TODIM technique for informed decision-making, as well as a discussion of case scenario validation utilizing experimental data. Finally, Section 4 summarizes the article's conclusion.

## 2. TRT-2-PFS

In this section, some necessary definitions, and mathematical models for TRT-2-PFS are presented.

### *2.1 Trapezoidal type-2 Pythagorean fuzzy number*

A trapezoidal type-2 Pythagorean fuzzy number (TRT-2-PFN) is superior type-2 Pythagorean fuzzy sets on a real number set $\mathcal{R}$, indicated by

$$\hat{\mathcal{P}} = \left(\underline{p}, \underline{q}, \overline{q}, \overline{p}; \varphi_{\widehat{p_u}}, \psi_{\widehat{p_l}}\right)\left(\underline{q}, \underline{r}, \overline{r}, \overline{q}; \varphi_{\widehat{p_l}}, \psi_{\widehat{p_u}}\right).$$

The mathematical representation of upper membership (UM) and lower membership (LM) is represented in Eqs (1) and (2) respectively.

$$\varphi_{\widehat{\mathcal{P}_u}}(x) = \begin{cases} \dfrac{\varphi_u\left(x - \underline{p}\right)}{q_1 - \underline{p}} & \underline{p} \leqslant x \leqslant \underline{q} \\ \varphi_u & \underline{q} \leqslant x \leqslant \overline{q} \\ \dfrac{\varphi_u(\overline{q} - x)}{\overline{q} - q_2} & \overline{q} \leqslant x \leqslant \overline{p} \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

$$\varphi_{\widehat{\mathcal{P}_u}}(x) = \begin{cases} \dfrac{\varphi_u\left(x - \underline{q}\right)}{\underline{r} - \underline{q}} & \underline{q} \leqslant x \leqslant \underline{r} \\ \varphi_u & \underline{r} \leqslant x \leqslant \overline{r} \\ \dfrac{\varphi_u(\overline{q} - x)}{\overline{q} - \overline{r}} & \overline{r} \leqslant x \leqslant \overline{q} \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

Similarly, the mathematical representation of UM and LM are represented in Eqs (3) and (4) respectively.

$$\varphi_{\widehat{\mathcal{P}_u}}(x) = \begin{cases} \dfrac{\left(\underline{r} - x\right) + \left(x - \underline{q}\right)\psi_u}{\underline{r} - p_1} & \underline{q} \leqslant x \leqslant \underline{r} \\ \psi_u & \underline{r} \leqslant x \leqslant \overline{r} \\ \dfrac{(x - \overline{r}) + (\overline{q} - x)\psi_u}{\overline{q} - \overline{r}} & \overline{r} \leqslant x \leqslant \overline{q} \\ 1 & \text{otherwise} \end{cases} \tag{3}$$

$$\varphi_{\widehat{\mathcal{P}_u}}(x) = \begin{cases} \dfrac{\left(\underline{q} - x\right) + \left(x - \underline{p}\right)\psi_u}{\underline{r} - p_1} & \underline{p} \leqslant x \leqslant \underline{q} \\ \psi_u & \underline{q} \leqslant x \leqslant \overline{q} \\ \dfrac{(x - \overline{q}) + (\overline{p} - x)\psi_u}{\overline{p} - \overline{q}} & \overline{q} \leqslant x \leqslant \overline{p} \\ 1 & \text{otherwise} \end{cases} \tag{4}$$

where $0 \leqslant \varphi_{\widehat{p_u}}, \psi_{\widehat{p_l}}, \varphi_{\widehat{p_l}}, \psi_{\widehat{p_u}} \leqslant 1$, $\left(\varphi_{\widehat{p_u}}\right)^2 + \left(\psi_{\widehat{p_l}}\right)^2 \leqslant 1$, and $\left(\varphi_{\widehat{p_l}}\right)^2 + \psi_{\widehat{p_u}}{}^2 \leqslant 1$. The upper hesitancy of the term $x$ to $\hat{\rho}$ is defined as:

$$H_{\widehat{p}_u} = \sqrt{1 - \left(\varphi_{\widehat{p_u}}(x)\right)^2 + \left(\psi_{\widehat{p_l}}(x)\right)^2}, \tag{5}$$

similarly, lower hesitancy is defined as:

$$H_{\widehat{p}_l} = \sqrt{1 - \left(\varphi_{\widehat{p_l}}(x)\right)^2 + \left(\psi_{\widehat{p_u}}(x)\right)^2}. \tag{6}$$

*2.2. Generation of TRT-2-PFN*

Let $\alpha_{ij}^k = \left(\left(\underline{\alpha}_{ij}^k, \alpha_{ij}^k, \overline{\alpha}_{ij}^k\right); \varphi_{\alpha_{ij}}^k, \psi_{\alpha_{ij}}^k\right)$ be the estimation value given by $k^{th}$ consultants for $i^{th}$ alternative with respect to $j^{th}$ criterion. In addition, $d^{th}$ decision-maker construct a decision matrix of TRT-2-PFNs in the form of

$$\widehat{\mathcal{P}}^k = \left[\widehat{\mathcal{D}}_{ij}^k\right]_{m\times n} \tag{7}$$

where $\widehat{\mathcal{D}}_{ij}^k = \left(\left(\underline{p}_{ij}^e, \underline{q}_{ij}^e, \overline{q}_{ij}^e, \overline{p}_{ij}^e\right); \varphi_{u_{\widehat{\mathcal{D}}_{ij}^k}}, \psi_{l_{\widehat{\mathcal{D}}_{ij}^k}}\right)\left(\left(\underline{q}_{ij}^e, \underline{r}_{ij}^e, \overline{r}_{ij}^e, \overline{q}_{ij}^e\right); \varphi_{l_{\widehat{\mathcal{D}}_{ij}^k}}, \psi_{u_{\widehat{\mathcal{D}}_{ij}^k}}\right)$,

$\underline{p}_{ij}^e = min\left(\underline{p}_{ij}^1, \underline{p}_{ij}^2, \dots, \underline{p}_{ij}^k\right)$, $\underline{q}_{ij}^e = min\left(p_{ij}^k\right)$, $\overline{q}_{ij}^e = max\left(p_{ij}^k\right)$, $\overline{p}_{ij}^e = max\left(\underline{p}_{ij}^1, \underline{p}_{ij}^2, \dots, \underline{p}_{ij}^k\right)$,

$\underline{r}_{ij}^e = \underline{q}_{ij}^k + \frac{\sigma_{\widehat{\mathcal{D}}_{ij}^k}}{2}$, $\overline{r}_{ij}^e = \overline{q}_{ij}^e - \frac{\sigma_{\widehat{\mathcal{D}}_{ij}^k}}{2}$, where $\sigma_{\widehat{\mathcal{D}}_{ij}^k}$ stranded for deviation of $p_{ij}^k$

$\varphi_{u_{\widehat{\mathcal{D}}_{ij}^k}} = max\left(\varphi_{p_{ij}}^1, \varphi_{p_{ij}}^2, \dots, \varphi_{p_{ij}}^k\right)$, $\varphi_{l_{\widehat{\mathcal{D}}_{ij}^k}} = min\left(\varphi_{p_{ij}}^1, \varphi_{p_{ij}}^2, \dots, \varphi_{p_{ij}}^k\right)$, $\psi_{u_{\widehat{\mathcal{D}}_{ij}^k}} = max\left(\psi_{p_{ij}}^1, \psi_{p_{ij}}^2, \dots, \psi_{p_{ij}}^k\right)$, and $\psi_{l_{\widehat{\mathcal{D}}_{ij}^k}} = min\left(\psi_{p_{ij}}^1, \psi_{p_{ij}}^2, \dots, \psi_{p_{ij}}^k\right)$.

*2.3. Operational laws of TRT-2-PFNs*

In this section, some basic operational laws such as addition and scalar multiplication of TRT-2-PFNs are discussed.

2.3.1. Sum of two TRT-2-PFNs

**Definition 1.** Let $\hat{a} = \left(\underline{a}, \underline{b}, \overline{b}, \overline{a}; \varphi_{\widehat{a_u}}, \psi_{\widehat{a_l}}\right)\left(\underline{b}, \underline{c}, \overline{c}, \overline{b}; \varphi_{\widehat{a_l}}, \psi_{\widehat{a_u}}\right)$,

and $\hat{d} = \left(\underline{d}, \underline{e}, \overline{e}, \overline{d}; \varphi_{\widehat{d_u}}, \psi_{\widehat{d_l}}\right)\left(\underline{e}, \underline{f}, \overline{f}, \overline{e}; \varphi_{\widehat{d_l}}, \psi_{\widehat{d_u}}\right)$ are two TRT-2-PFNs, then

$$\hat{a}+\hat{d}=\begin{pmatrix}\underline{a}+\underline{d},\underline{b}+\underline{e},\overline{b}+\overline{e},\overline{a}+\overline{d};\sqrt{1-\left(1-\left(\varphi_{\widehat{a_u}}\right)^2\right)\left(1-\left(\varphi_{\widehat{d_u}}\right)^2\right)},\\ \sqrt{\left(\left(1-\left(\varphi_{\widehat{a_u}}\right)^2\right)\left(1-\left(\varphi_{\widehat{d_u}}\right)^2\right)-\left(1-\left(\left(\varphi_{\widehat{a_u}}\right)^2+\left(\psi_{\widehat{a_l}}\right)^2\right)\left(1-\left(\left(\varphi_{\widehat{d_u}}\right)^2+\left(\psi_{\widehat{d_l}}\right)^2\right)\right)\right)\right)}\end{pmatrix}$$
$$\begin{pmatrix}\underline{b}+\underline{e},\underline{c}+\underline{f},\overline{c}+\overline{f},\overline{b}+\overline{e};\sqrt{1-\left(1-\left(\varphi_{\widehat{a_l}}\right)^2\right)\left(1-\left(\varphi_{\widehat{d_l}}\right)^2\right)},\\ \left(\sqrt{\left(1-\left(\varphi_{\widehat{a_l}}\right)^2\right)\left(1-\left(\varphi_{\widehat{d_l}}\right)^2\right)-\left(1-\left(\left(\varphi_{\widehat{a_l}}\right)^2+\left(\psi_{\widehat{a_u}}\right)^2\right)\left(1-\left(\left(\varphi_{\widehat{d_l}}\right)^2+\left(\psi_{\widehat{d_u}}\right)^2\right)\right)\right)}\right)\end{pmatrix}. \quad (8)$$

2.3.2. Scalar multiplication of TRT-2-PFNs

**Definition 2.** Let $\hat{a}=\left(\underline{a},\underline{b},\overline{b},\overline{a};\varphi_{\widehat{a_u}},\psi_{\widehat{a_l}}\right)\left(\underline{b},\underline{c},\overline{c},\overline{b};\varphi_{\widehat{a_l}},\psi_{\widehat{a_u}}\right)$ be a TRT-2-PFNs and $\xi$ be any positive real number then

$$\xi\hat{a}=\begin{pmatrix}\xi\underline{a},\xi\underline{b},\xi\overline{b},\xi\overline{a};\sqrt{1-\left(1-\left(\varphi_{\widehat{a_u}}\right)^2\right)^{\xi}},\\ \sqrt{\left(\left(1-\left(\varphi_{\widehat{a_u}}\right)^2\right)^{\xi}-\left(1-\left(\left(\varphi_{\widehat{a_u}}\right)^2+\left(\psi_{\widehat{a_l}}\right)^2\right)\right)^{\xi}\right)}\end{pmatrix}$$
$$\begin{pmatrix}\xi\underline{b},\xi\underline{c},\xi\overline{c},\xi\overline{b};\sqrt{1-\left(1-\left(\varphi_{\widehat{a_l}}\right)^2\right)^{\xi}},\\ \left(\sqrt{\left(1-\left(\varphi_{\widehat{a_l}}\right)^2\right)^{\xi}-\left(1-\left(\left(\varphi_{\widehat{a_l}}\right)^2+\left(\psi_{\widehat{a_u}}\right)^2\right)\right)^{\xi}}\right)\end{pmatrix}.$$

2.3.3. Comparative study of Two TRT-2-PFNs

**Definition 3.** Let $\hat{a}=\left(\underline{a},\underline{b},\overline{b},\overline{a};\varphi_{\widehat{a_u}},\psi_{\widehat{a_l}}\right)\left(\underline{b},\underline{c},\overline{c},\overline{b};\varphi_{\widehat{a_l}},\psi_{\widehat{a_u}}\right)$ be a TRT-2-PFN. The score function is defined as follows:

$$Sc(\hat{a})=\frac{(\underline{a}+\underline{b}+\overline{b}+\overline{a})\left(\left(\varphi_{\widehat{a_u}}\right)^2-\left(\psi_{\widehat{a_l}}\right)^2\right)+(\underline{b}+\underline{c}+\overline{c}+\overline{b})\left(\left(\varphi_{\widehat{a_l}}\right)^2-\left(\psi_{\widehat{a_u}}\right)^2\right)}{4}, \quad (9)$$

and accuracy function is defined as follows:

$$Ac(\hat{a})=\frac{(\underline{a}+\underline{b}+\overline{b}+\overline{a})\left(\left(\varphi_{\widehat{a_u}}\right)^2+\left(\psi_{\widehat{a_l}}\right)^2\right)+(\underline{b}+\underline{c}+\overline{c}+\overline{b})\left(\left(\varphi_{\widehat{a_l}}\right)^2+\left(\psi_{\widehat{a_u}}\right)^2\right)}{4}. \quad (10)$$

**Definition 4.** Let $\hat{a_\iota}=\left(\underline{a_i},\underline{b_i},\overline{b_i},\overline{a_i};\varphi_{\widehat{a_{\iota u}}},\psi_{\widehat{a_{\iota l}}}\right)\left(\underline{b_i},\underline{c_i},\overline{c_i},\overline{b_i};\varphi_{\widehat{a_{\iota l}}},\psi_{\widehat{a_{\iota u}}}\right)$ $(i=1,2)$ be the collection of TRT-2-PFNs. then we have

i. If $Sc(\hat{a}_1) \succ Sc(\hat{a}_2)$ then $\hat{a}_1 \succ \hat{a}_2$,
ii. If $Sc(\hat{a}_1) \prec Sc(\hat{a}_2)$ then $\hat{a}_1 \prec \hat{a}_2$,
iii. If $Sc(\hat{a}_1) = Sc(\hat{a}_2)$,
  (a) If $Ac(\hat{a}_1) \succ Ac(\hat{a}_2)$ then $\hat{a}_1 \succ \hat{a}_2$,
  (b) If $Ac(\hat{a}_1) \succ Ac(\hat{a}_2)$ then $\hat{a}_1 \prec \hat{a}_2$,
  (c) If $Ac(\hat{a}_1) = Ac(\hat{a}_2)$ then $\hat{a}_1 \sim \hat{a}_2$.

**Definition 5.** Let $\widehat{a_l} = \left(\underline{a_i}, \underline{b_i}, \overline{b_i}, \overline{a_i}; \varphi_{\widehat{a_{i_u}}}, \psi_{\widehat{a_{i_l}}}\right)\left(\underline{b_i}, \underline{c_i}, \overline{c_i}, \overline{b_i}; \varphi_{\widehat{a_{i_l}}}, \psi_{\widehat{a_{i_u}}}\right)$ $(i = 1,2,3,4)$ be the collection of TRT-2-PFNs. then we have

1) If $\hat{a}_1 \subseteq \hat{a}_2$ and $\hat{a}_2 \subseteq \hat{a}_3$ then $\hat{a}_1 \subseteq_P \hat{a}_3$.
2) If $\hat{a}_1 \subseteq \hat{a}_2$ then $\hat{a}_2^c \subseteq \hat{a}_1^c$.
3) If $\hat{a}_1 \subseteq \hat{a}_2$ and $\hat{a}_1 \subseteq \hat{a}_3$ then $\hat{a}_1 \subseteq \hat{a}_2 \cap \hat{a}_3$.
4) If $\hat{a}_1 \subseteq \hat{a}_2$ and $\hat{a}_3 \subseteq \hat{a}_4$ then $\hat{a}_1 \cup \hat{a}_3 \subseteq_P \hat{a}_2 \cup \hat{a}_4$ and $\hat{a}_1 \cap \hat{a}_3 \subseteq_P \hat{a}_2 \cap \hat{a}_4$.
5) If $\hat{a}_1 \subseteq \hat{a}_2$ and $\hat{a}_3 \subseteq \hat{a}_2$ then $\hat{a}_1 \cup \hat{a}_3\beta_2$.

*2.4. Distance between two TRT-2-PFNs*

The distance measure between two TRT-2-PFNs is a real function $\Phi\colon \mathrm{TRT-2-PFN} \times \mathrm{TRT-2-PFN} \to [0,1]$ if $\Phi$ has the characteristics listed below:

a) $\Phi(\widehat{\mathcal{D}}, \widehat{\mathcal{D}}) = 0$;
b) $\Phi(\widehat{\mathcal{D}}, \hat{\mathcal{E}}) = \Phi(\hat{\mathcal{E}}, \widehat{\mathcal{D}})$;
c) For three TRT-2-PFNs $\widehat{\mathcal{D}}$, $\hat{\mathcal{E}}$ and $\hat{\mathcal{F}}$ then $\Phi(\widehat{\mathcal{D}}, \hat{\mathcal{F}}) = \Phi(\widehat{\mathcal{D}}, \hat{\mathcal{E}}) + \Phi(\hat{\mathcal{E}}, \hat{\mathcal{F}})$.

2.4.1. Hamming distance

Let $\hat{\alpha} = (\underline{a}, \underline{b}, \overline{b}, \overline{a}; \varphi_{\widehat{a_u}}, \psi_{\widehat{a_l}})(\underline{b}, \underline{c}, \overline{c}, \overline{b}; \varphi_{\widehat{a_l}}, \psi_{\widehat{a_u}})$ and $\hat{\beta} = \left(\underline{d}, \underline{e}, \overline{e}, \overline{d}; \varphi_{\widehat{d_u}}, \psi_{\widehat{d_l}}\right)\left(\underline{e}, \underline{f}, \overline{f}, \overline{e}; \varphi_{\widehat{d_l}}, \psi_{\widehat{d_u}}\right)$ be two TRT-2-PFNs. then Hamming distance between them is defined as

$$\mathrm{H}(\hat{\alpha}, \hat{\beta}) = \frac{1}{16}\left|(\varphi_{\widehat{a_u}})^2\underline{a} - (\varphi_{\widehat{d_u}})^2\underline{d}\right| + \left|(\varphi_{\widehat{a_u}})^2\underline{b} - (\varphi_{\widehat{d_u}})^2\underline{e}\right| + \left|(\varphi_{\widehat{a_u}})^2\overline{b} - (\varphi_{\widehat{d_u}})^2\overline{e}\right| +$$
$$\left|(\varphi_{\widehat{a_u}})^2\overline{a} - (\varphi_{\widehat{d_u}})^2\overline{d}\right| + \left|(\psi_{\widehat{a_l}})^2\underline{a} - (\psi_{\widehat{d_l}})^2\underline{d}\right| + \left|(\psi_{\widehat{a_l}})^2\underline{b} - (\psi_{\widehat{d_l}})^2\underline{e}\right| + \left|(\psi_{\widehat{a_l}})^2\overline{b} - (\psi_{\widehat{d_l}})^2\overline{e}\right| +$$
$$\left|(\psi_{\widehat{a_l}})^2\overline{a} - (\psi_{\widehat{d_l}})^2\overline{d}\right| + |\underline{a} - \underline{d}| + 2|\underline{b} - \underline{e}| + 2|\overline{b} - \overline{e}| + |\overline{a} - \overline{d}| + \left|(\varphi_{\widehat{a_l}})^2\underline{b} - (\varphi_{\widehat{d_l}})^2\underline{e}\right| +$$
$$\left|(\varphi_{\widehat{a_l}})^2\underline{c} - (\varphi_{\widehat{d_l}})^2\underline{f}\right| + \left|(\varphi_{\widehat{a_l}})^2\overline{c} - (\varphi_{\widehat{d_l}})^2\overline{f}\right| + \left|(\varphi_{\widehat{a_l}})^2\overline{b} - (\varphi_{\widehat{d_l}})^2\overline{e}\right| + \left|(\psi_{\widehat{a_l}})^2\underline{b} - (\psi_{\widehat{d_l}})^2\underline{e}\right| +$$
$$\left|(\psi_{\widehat{a_l}})^2\underline{c} - (\psi_{\widehat{d_l}})^2\underline{f}\right| + \left|(\psi_{\widehat{a_l}})^2\overline{c} - (\psi_{\widehat{d_l}})^2\overline{f}\right| + \left|(\psi_{\widehat{a_l}})^2\overline{b} - (\psi_{\widehat{d_l}})^2\overline{e}\right| + |\underline{c} - \underline{f}| + |\overline{f} - \overline{c}|.$$

*2.5. Decision-maker weights and criteria weights*

The classic fuzzy TODIM method is used to determine decision-maker weights and criteria weights. Suppose $\widehat{\mathcal{D}}_{ij}^{k}$ $(i = 1,2, \ldots, m; j = 1,2, \ldots, n)$ is the assessment values of alternative $X_i$ under the criterion $\mathcal{C}_i$ given by the decision-maker $D_t$ $(t = 1,2, \ldots, l)$. The assessment values of each alternative can be expressed as:

$$\hat{g}_{ij}^{*} = \left(\underline{a}_{ij}^{*}, \underline{b}_{ij}^{*}, \overline{b}_{ij}^{*}, \overline{a}_{ij}^{*}; \varphi_{u_{\hat{g}_{ij}}}^{*}, \psi_{l_{\hat{g}_{ij}}}^{*}\right)\left(\underline{b}_{ij}^{*}, \underline{c}_{ij}^{*}, \overline{c}_{ij}^{*}, \overline{b}_{ij}^{*}; \varphi_{l_{\hat{g}_{ij}}}^{*}, \psi_{u_{\hat{g}_{ij}}}^{*}\right) \tag{11}$$

and it may be calculated using the following equation:

$$\hat{g}_{ij}^{*} = \frac{1}{t}\left(\hat{g}_{ij}^{1}, \hat{g}_{ij}^{2}, \ldots, \hat{g}_{ij}^{t}\right). \tag{12}$$

The degree of similarity $s\left(\hat{g}_{ij}^{k}, \hat{g}_{ij}^{*}\right)$ between $\hat{g}_{ij}^{*}$ and $\hat{g}_{ij}^{k}$ is defined as follows:

$$s\left(\hat{g}_{ij}^{k}, \hat{g}_{ij}^{*}\right) = 1 - \frac{\mathcal{D}\left(\hat{g}_{ij}^{k}, \hat{g}_{ij}^{*}\right)}{\sum_{t=1}^{l} \mathcal{D}\left(\hat{g}_{ij}^{k}, \hat{g}_{ij}^{*}\right)} \tag{13}$$

where $i = 1,2, \ldots, m; j = 1,2, \ldots, n;$ $k = 1,2, \ldots, l.$

The weight of decision-makers can be calculated as follows:

$$\omega_{ij}^{t} = 1 - \frac{s\left(\hat{g}_{ij}^{k}, \hat{g}_{ij}^{*}\right)}{\sum_{t=1}^{l} s\left(\hat{g}_{ij}^{k}, \hat{g}_{ij}^{*}\right)} \tag{14}$$

where $i = 1,2, \ldots, m; j = 1,2, \ldots, n;$ $t = 1,2, \ldots, l.$

Combination of different decision matrices $\hat{\mathcal{P}}^{k} = \left[\widehat{\mathcal{D}}_{ij}^{k}\right]_{m \times n}$ into group decision matrix $\mathbb{G} = \left[\hat{\mathbb{g}}_{ij}^{k}\right]_{m \times n}$ as follows:

$$\hat{\mathbb{g}}_{ij} = \left(\underline{x_{ij}}, \underline{y_{ij}}, \overline{y_{ij}}, \overline{x_{ij}}; \varphi_{u_{\hat{\mathbb{g}}_{ij}}}, \psi_{u_{\hat{\mathbb{g}}_{ij}}}\right)\left(\underline{y_{ij}}, \underline{z_{ij}}, \overline{z_{ij}}, \overline{y_{ij}}; \varphi_{u_{\hat{\mathbb{g}}_{ij}}}, \psi_{u_{\hat{\mathbb{g}}_{ij}}}\right)$$

$$= \sum_{t=1}^{l} \omega_{ij}^{t} g_{ij}^{t} = \begin{pmatrix} \left(\sum_{t=1}^{l} \omega_{ij}^{t} \underline{a}_{ij}^{t}, \sum_{t=1}^{l} \omega_{ij}^{t} \underline{b}_{ij}^{t}, \sum_{t=1}^{l} \omega_{ij}^{t} \overline{b}_{ij}^{t}, \sum_{t=1}^{l} \omega_{ij}^{t} \overline{a}_{ij}^{t}\right); 1 - \prod_{t=1}^{l}\left(1 - \varphi_{u_{g_{ij}^{t}}}\right)^{\omega}, \\ \prod_{t=1}^{l}\left(1 - \varphi_{u_{g_{ij}^{t}}}\right)^{\omega} - \prod_{t=1}^{l}\left(1 - \left(\varphi_{u_{g_{ij}^{t}}} + \psi_{l_{g_{ij}^{t}}}\right)\right)^{\omega} \end{pmatrix}$$

$$\begin{pmatrix} \left(\sum_{t=1}^{l} \omega_{ij}^{t} \underline{b}_{ij}^{t}, \sum_{t=1}^{l} \omega_{ij}^{t} \underline{c}_{ij}^{t}, \sum_{t=1}^{l} \omega_{ij}^{t} \overline{c}_{ij}^{t}, \sum_{t=1}^{l} \omega_{ij}^{t} \overline{b}_{ij}^{t}\right); 1 - \prod_{t=1}^{l}\left(1 - \varphi_{l_{g_{ij}^{t}}}\right)^{\omega}, \\ \prod_{t=1}^{l}\left(1 - \varphi_{l_{g_{ij}^{t}}}\right)^{\omega} - \prod_{t=1}^{l}\left(1 - \left(\varphi_{l_{g_{ij}^{t}}} + \psi_{u_{g_{ij}^{t}}}\right)\right)^{\omega} \end{pmatrix}.$$

To estimate the criteria weight for the group decision matrix $\mathbb{G} = \left[\widehat{\mathbb{g}}_{ij}^{k}\right]_{m\times n}$, the mean of the analysis for the set of criteria $\mathcal{C}_j$ is determined as follows:

$$\widehat{\mathbb{g}}_{ij}^{*} = \left(\underline{x}_{ij}^{*}, \underline{y}_{ij}^{*}, \overline{y}_{ij}^{*}, \overline{x}_{ij}^{*}; \varphi_{u_{\hat{g}_{ij}}}^{*}, \psi_{l_{\hat{g}_{ij}}}^{*}\right)\left(\underline{y}_{ij}^{*}, \underline{z}_{ij}^{*}, \overline{z}_{ij}^{*}, \overline{y}_{ij}^{*}; \varphi_{l_{\hat{g}_{ij}}}^{*}, \psi_{u_{\hat{g}_{ij}}}^{*}\right),$$

$$\widehat{\mathbb{g}}_{ij}^{*} = \frac{1}{m}\left(\widehat{\mathbb{g}}_{1j}, \widehat{\mathbb{g}}_{2j}, \dots, \widehat{\mathbb{g}}_{mj}\right), \text{ where } i = 1,2,\dots,m; j = 1,2,\dots,n;\ k = 1,2,\dots,l.$$

The weight for the criteria $\mathcal{C}_j$ will then be calculated as follows:

$$w_j = 1 - \frac{d\left(\widehat{\mathbb{g}}_{ij}^{k}, \widehat{\mathbb{g}}_{ij}^{*}\right)}{\sum_{j}^{n}\sum_{i=1}^{n} d\left(\widehat{\mathbb{g}}_{ij}^{k}, \widehat{\mathbb{g}}_{ij}^{*}\right)} \tag{15}$$

where $d\left(\widehat{\mathbb{g}}_{ij}^{k}, \widehat{\mathbb{g}}_{ij}^{*}\right)$ represents the distance between $\widehat{\mathbb{g}}_{ij}^{k}$ and $\widehat{\mathbb{g}}_{ij}^{*}$.

## 3. TRT-2 Pythagorean fuzzy TODIM for MCDM

The method for the planned TRT-2 Pythagorean fuzzy sets based on TODIM for sensible decisions in MCDM is presented.

Step 1. Collect the information and assessment values of each alternative $X_i$ concerning each criterion $\mathcal{C}_i$ given by the decision-makers $\mathcal{D}^t$ in the form of TRT-2-PFNs. the collective decision matrix for decision-maker $\mathcal{D}^t (t = 1,2,\dots,l)$ is given by:

$$\mathbb{G}^k = \left[\widehat{\mathbb{g}}_{ij}^{k}\right]_{m\times n}\left(\widehat{\mathbb{g}}_{ij} = \left(\underline{x_{ij}^{k}}, \underline{y_{ij}^{k}}, \overline{y_{ij}^{k}}, \overline{x_{ij}^{k}}; \varphi_{u_{\widehat{\mathbb{g}}_{ij}^{k}}}, \psi_{u_{\widehat{\mathbb{g}}_{ij}^{k}}}\right)\left(\underline{y_{ij}^{k}}, \underline{z_{ij}^{k}}, \overline{z_{ij}^{k}}, \overline{y_{ij}^{k}}; \varphi_{u_{\widehat{\mathbb{g}}_{ij}^{k}}}, \psi_{u_{\widehat{\mathbb{g}}_{ij}^{k}}}\right)\right). \tag{16}$$

Step 2. Convert the collective decision matrix $\mathbb{G}^k = \left[\widehat{\mathbb{g}}_{ij}^{k}\right]_{m\times n}$ into normalized decision matrix $\mathcal{R}^k = \left[\hat{r}_{ij}^{k}\right]_{m\times n}$.

$$\mathcal{R}^k = \left[\hat{r}_{ij}^{k}\right]_{m\times n}\left(\widehat{\mathbb{g}}_{ij} = \left(\underline{\hat{x}_{ij}^{k}}, \underline{\hat{y}_{ij}^{k}}, \overline{\hat{y}_{ij}^{k}}, \overline{\hat{x}_{ij}^{k}}; \varphi_{u_{\hat{r}_{ij}^{k}}}, \psi_{u_{\hat{r}_{ij}^{k}}}\right)\left(\underline{\hat{y}_{ij}^{k}}, \underline{\hat{z}_{ij}^{k}}, \overline{\hat{z}_{ij}^{k}}, \overline{\hat{y}_{ij}^{k}}; \varphi_{u_{\hat{r}_{ij}^{k}}}, \psi_{u_{\hat{r}_{ij}^{k}}}\right)\right). \tag{17}$$

Step 3. Calculate the weight vector of decision-makers $\mathcal{D}^t$ with respect to criteria $\mathcal{C}_j$ by using Eqs (12) and (13).

$$\omega_{ij}^{t} = \left(\omega_{ij}^{1}, \omega_{ij}^{2}, \dots, \omega_{ij}^{l}\right). \tag{18}$$

Step 4. Convert the individual decision matrices $\widehat{\mathcal{P}}^k = \left[\widehat{\mathcal{D}}_{ij}^{k}\right]_{m\times n}$ into group decision matrix $\mathbb{G} = \left[\widehat{\mathbb{g}}_{ij}^{k}\right]_{m\times n}$.

Step 5. Determine the weight vector $w = (w_1, w_2, \dots, w_n)$ of each criterion $\mathcal{C}_j$ $(j = 1,2, \dots, n)$ using Eq (15).

Step 6. Determine the relative weight $w_r$ of each relative criterion $\mathcal{C}_r$.

$$w_{rj} = \frac{w_j}{w_r} \tag{19}$$

where $w_r = max(w_j)$ $(j = 1,2, \dots, n)$.

Step 7. Based on the existing TODIM approach, the dominance of alternative $X_i$ over the alternative $X_k$ can be calculated as follows:

$$\Delta_j(X_i, X_k) = \begin{cases} \sqrt{\frac{w_{rj}}{\sum_{j=1}^{n} d(\widehat{\mathbb{g}}_{ij}, \widehat{\mathbb{g}}_{kj})}} & \text{if} \quad \widehat{\mathbb{g}}_{ij} \succ \widehat{\mathbb{g}}_{kj} \\ 0 & \text{if} \quad \widehat{\mathbb{g}}_{ij} = \widehat{\mathbb{g}}_{kj} \\ \frac{-1}{\rho}\sqrt{\frac{\sum_{j=1}^{n} d(\widehat{\mathbb{g}}_{ij}, \widehat{\mathbb{g}}_{kj})}{w_{rj}}} & \text{if} \quad \widehat{\mathbb{g}}_{ij} \prec \widehat{\mathbb{g}}_{kj} \end{cases}. \tag{20}$$

Step 8. The dominance degree matrix concerning each criterion $\mathcal{C}_j$ can be calculated as:

$$\Delta_j = \left[\Delta_{ij}^{k}\right]_{m \times n} = \begin{matrix} X_1 \\ X_2 \\ \vdots \\ X_m \end{matrix} \begin{pmatrix} \Delta_{11}^{j} & \Delta_{12}^{j} & \cdots & \Delta_{1m}^{j} \\ \Delta_{21}^{j} & \Delta_{22}^{j} & \cdots & \Delta_{2m}^{j} \\ \vdots & \vdots & \ddots & \vdots \\ \Delta_{n1}^{j} & \Delta_{n2}^{j} & \cdots & \Delta_{nm}^{j} \end{pmatrix} \tag{21}$$

where $\Delta_{11}^{j} = \Delta_{22}^{j} = \cdots = \Delta_{tt}^{j} = 0$.

Step 9. The global dominance of alternative $X_i$ over alternative $X_k$ can be calculated as follows:

$$\hat{r}_{ij}^{k} = \left( \begin{matrix} \left( \frac{\underline{x}_{ij}^{t} - min_j(\underline{x}_{ij}^{t})}{max_j(\overline{x_{ij}^{t}}) - min_j(\underline{x}_{ij}^{t})}, \frac{\underline{y}_{ij}^{t} - min_j(x_{ij}^{t})}{max_j(\overline{x_{ij}^{t}}) - min_j(\underline{x}_{ij}^{t})}, \frac{\overline{\underline{y}_{ij}^{t}} - min_j(\underline{x}_{ij}^{t})}{max_j(\overline{x_{ij}^{t}}) - min_j(\underline{x}_{ij}^{t})}, \frac{\overline{x_{ij}^{t}} - min_j(\underline{x}_{ij}^{t})}{max_j(\overline{x_{ij}^{t}}) - min_j(\underline{x}_{ij}^{t})} \right); \\ \varphi_{u_{\hat{r}_{ij}^{k}}}, \psi_{u_{\hat{r}_{ij}^{k}}} \end{matrix} \right)$$

$$\left( \begin{matrix} \left( \frac{\underline{y}_{ij}^{t} - min_j(\underline{x}_{ij}^{t})}{max_j(\overline{x_{ij}^{t}}) - min_j(\underline{x}_{ij}^{t})}, \frac{\underline{z}_{ij}^{t} - min_j(x_{ij}^{t})}{max_j(\overline{x_{ij}^{t}}) - min_j(\underline{x}_{ij}^{t})}, \frac{\overline{\underline{z}_{ij}^{t}} - min_j(\underline{x}_{ij}^{t})}{max_j(\overline{x_{ij}^{t}}) - min_j(\underline{x}_{ij}^{t})}, \frac{\overline{y_{ij}^{t}} - min_j(\underline{x}_{ij}^{t})}{max_j(\overline{x_{ij}^{t}}) - min_j(\underline{x}_{ij}^{t})} \right); \\ \varphi_{u_{\hat{r}_{ij}^{k}}}, \psi_{u_{\hat{r}_{ij}^{k}}} \end{matrix} \right) \tag{22}$$

$$\sigma(X_i, X_k) = \sum_{j=1}^{n} \Delta_j(X_i, X_k). \tag{23}$$

Step 10. To calculate the global value for alternative $X_i$, we can normalize the global dominance degree matrix using the following method:

$$\mu_i = \frac{\sum_{k=1}^{m} \sigma(X_i, X_k) - min_{i \in m}(\sum_{k=1}^{m} \sigma(X_i, X_k))}{max_{i \in m}(\sum_{k=1}^{m} \sigma(X_i, X_k)) - min_{i \in m}(\sum_{k=1}^{m} \sigma(X_i, X_k))}. \quad (24)$$

Step 11. Rank the alternative and select the optimal one.

*3.1. Case study*

The experiments are carried out about selecting resources for renewable energy. Because of the massive consumption of fossil fuels in recent decades, an unprecedented quantity of chemicals has been released into the environment. Renewable sources of energy (RES) do not emit such chemicals and are thus particularly successful in reducing environmental harm. As a result, it is important to develop such energy production, which will also aid in alleviating renewable power difficulties. For instance, the Chinese “long-term renewable energy development plan” states that they would make significant investments in renewable sources of energy such as hydropower, wind, biomass, solar, and geothermal energy [37].

In this scenario, we selected four renewable energy resources (RES) which are Solar energy $(X_1)$, wind energy $(X_2)$, geothermal energy $(X_3)$ and ocean energy $X_4$ as alternative solutions. Each form of RES has advantages and disadvantages in defending on the local environment. Thus, it is critical to choose the optimal source among them to optimize advantage. Four factors (criteria), including energy source superiority $(\mathcal{C}_1)$, socio-political $(\mathcal{C}_2)$, economic $(\mathcal{C}_3)$ and environmental $(\mathcal{C}_4)$, are used to assess the best alternative. Three experts $(\mathcal{D}^1, \mathcal{D}^2, \mathcal{D}^3)$ be responsible to provide their performance for each RES after combining the assessment given by the three advisors. The advisors will assess each alternative with concerning criterion and provide a decision matrix within the form of TRT-2-PFN. The criteria values are expressed as triangular Pythagorean fuzzy numbers (TPFN) with performance ratings ranging from 1 to 5. Superior efficiency for this criterion is shown the substantially greater values. There are six advisors $(\Psi_1, \Psi_2, \Psi_3, \Psi_4, \Psi_5, \Psi_6)$. Advisor $\Psi_1$ and $\Psi_2$ will be under $\mathcal{D}^1$, $\Psi_3$ and $\Psi_4$ will be under $\mathcal{D}^2$ and $\Psi_5$ and $\Psi_6$ will be under $\mathcal{D}^3$. The number of advisors is a personal preference that may be modified by the organizers.

Tables 1, 2, 3, 4, 5 and 6 indicate the assessment ratings supplied by advisors $\Psi_1$, $\Psi_2$, $\Psi_3$, $\Psi_4$, $\Psi_5$ and $\Psi_6$.

**Table 1.** Decision matrix of $\Psi_1$.

| Alternatives | $\mathcal{C}_1$ | $\mathcal{C}_2$ | $\mathcal{C}_3$ | $\mathcal{C}_4$ |
|---|---|---|---|---|
| $X_1$ | $((2,1,3); 0.3,0.3)$ | $((2,3,5); 0.1,0.2)$ | $((3,3,4); 0.3,0.4)$ | $((2,2,1); 0.2,0.7)$ |
| $X_2$ | $((1,1,2); 0.5,0.7)$ | $((1,4,3); 0.4,0.6)$ | $((1,1,3); 0.2,0.5)$ | $((5,2,2); 0.7,0.1)$ |
| $X_3$ | $((2,1,3); 0.6,0.4)$ | $((5,2,4); 0.3,0.5)$ | $((4,3,1); 0.6,0.2)$ | $((4,2,3); 0.4,0.5)$ |
| $X_4$ | $((3,1,4); 0.3,0.3)$ | $((3,1,2); 0.6,0.2)$ | $((3,2,3); 0.5,0.5)$ | $((2,1,5); 0.4,0.6)$ |

**Table 2.** decision matrix of $\Psi_2$.

| Alternatives | $\mathcal{C}_1$ | $\mathcal{C}_2$ | $\mathcal{C}_3$ | $\mathcal{C}_4$ |
|---|---|---|---|---|
| $X_1$ | $((1,1,4);0.5,0.4)$ | $((4,4,2);0.4,0.3)$ | $((3,1,4);0.4,0.1)$ | $((2,2,3);0.8,0.1)$ |
| $X_2$ | $((4,2,3);0.2,0.4)$ | $((1,5,4);0.2,0.8)$ | $((4,2,5);0.2,0.5)$ | $((3,1,5);0.5,0.3)$ |
| $X_3$ | $((5,1,2);0.6,0.3)$ | $((2,3,1);0.1,0.4)$ | $((5,3,3);0.6,0.5)$ | $((4,3,2);0.3,0.7)$ |
| $X_4$ | $((2,1,3);0.1,0.7)$ | $((3,1,2);0.5,0.2)$ | $((2,2,1);0.3,0.7)$ | $((1,5,4);0.5,0.6)$ |

**Table 3.** decision matrix of $\Psi_3$.

| Alternatives | $\mathcal{C}_1$ | $\mathcal{C}_2$ | $\mathcal{C}_3$ | $\mathcal{C}_4$ |
|---|---|---|---|---|
| $X_1$ | $((4,1,3);0.7,0.6)$ | $((1,4,2);0.2,0.3)$ | $((3,1,4);0.8,0.4)$ | $((3,2,4);0.5,0.3)$ |
| $X_2$ | $((3,2,1);0.4,0.5)$ | $((3,1,5);0.5,0.2)$ | $((4,2,3);0.7,0.3)$ | $((1,1,5);0.5,0.2)$ |
| $X_3$ | $((4,1,2);0.7,0.2)$ | $((5,2,2);0.3,0.4)$ | $((2,1,3);0.5,0.7)$ | $((2,1,2);0.7,0.4)$ |
| $X_4$ | $((5,3,3);0.5,0.3)$ | $((4,3,1);0.5,0.2)$ | $((4,3,5);0.4,0.1)$ | $((4,3,3);0.3,0.5)$ |

**Table 4.** decision matrix of $\Psi_4$.

| Alternatives | $\mathcal{C}_1$ | $\mathcal{C}_2$ | $\mathcal{C}_3$ | $\mathcal{C}_4$ |
|---|---|---|---|---|
| $X_1$ | $((3,5,1);0.5,0.2)$ | $((3,1,1);0.2,0.3)$ | $((2,1,3);0.2,0.7)$ | $((1,5,3);0.5,0.6)$ |
| $X_2$ | $((4,3,2);0.3,0.4)$ | $((4,3,3);0.3,0.5)$ | $((3,5,4);0.3,0.4)$ | $((4,1,4);0.7,0.2)$ |
| $X_3$ | $((2,4,1);0.6,0.4)$ | $((2,4,3);0.5,0.4)$ | $((1,3,5);0.2,0.6)$ | $((5,4,2);0.3,0.4)$ |
| $X_4$ | $((5,1,2);0.2,0.3)$ | $((2,2,3);0.7,0.4)$ | $((5,1,2);0.4,0.3)$ | $((4,3,5);0.4,0.5)$ |

**Table 5.** decision matrix of $\Psi_5$.

| Alternatives | $\mathcal{C}_1$ | $\mathcal{C}_2$ | $\mathcal{C}_3$ | $\mathcal{C}_4$ |
|---|---|---|---|---|
| $X_1$ | $((4,3,4);0.6,0.7)$ | $((5,2,4);0.4,0.3)$ | $((3,4,1);0.1,0.5)$ | $((2,1,4);0.5,0.3)$ |
| $X_2$ | $((5,2,3);0.3,0.2)$ | $((1,1,5);0.3,0.5)$ | $((2,2,3);0.4,0.7)$ | $((3,4,3);0.3,0.6)$ |
| $X_3$ | $((3,5,1);0.1,0.4)$ | $((3,3,2);0.2,0.8)$ | $((4,5,4);0.3,0.5)$ | $((4,5,5);0.2,0.8)$ |
| $X_4$ | $((2,1,2);0.5,0.3)$ | $((2,5,2);0.1,0.3)$ | $((5,3,2);0.2,0.4)$ | $((1,3,1);0.1,0.7)$ |

**Table 6.** decision matrix of $\Psi_6$.

| Alternatives | $\mathcal{C}_1$ | $\mathcal{C}_2$ | $\mathcal{C}_3$ | $\mathcal{C}_4$ |
|---|---|---|---|---|
| $X_1$ | $((3,1,3); 0.4,0.2)$ | $((5,3,3); 0.5,0.2)$ | $((5,4,5); 0.6,0.4)$ | $((5,1,5); 0.5,0.2)$ |
| $X_2$ | $((4,2,2); 0.2,0.3)$ | $((4,5,5); 0.7,0.3)$ | $((4,3,3); 0.7,0.2)$ | $((4,2,1); 0.4,0.3)$ |
| $X_3$ | $((2,4,5); 0.7,0.4)$ | $((2,4,3); 0.4,0.2)$ | $((1,2,4); 0.5,0.3)$ | $((4,3,3); 0.6,0.4)$ |
| $X_4$ | $((5,3,1); 0.1,0.8)$ | $((3,1,4); 0.3,0.5)$ | $((2,1,1); 0.3,0.4)$ | $((2,4,5); 0.2,0.6)$ |

The next step is to develop a decision matrix for the decision-makers $\mathcal{D}^1$, $\mathcal{D}^2$, and $\mathcal{D}^3$, which are presented in Tables 7, 8, and 9.

**Table 7.** TRT-2-PFN decision-matrix for $\mathcal{D}^1$.

| $X_i$ | $\mathcal{C}_1$ | $\mathcal{C}_2$ | $\mathcal{C}_3$ | $\mathcal{C}_4$ |
|---|---|---|---|---|
| $X_1$ | $((4,2,3,1); 0.3,0.4)$ | $((3,4,5,4); 0.7,0.5)$ | $((2,1,5,1); 0.4,0.6)$ | $((2,3,5,4); 0.3,0.6)$ |
| | $((5,2,1,4); 0.5,0.6)$ | $((2,2,3,3); 0.1,0.6)$ | $((4,3,3,1); 0.3,0.7)$ | $((3,2,1,4); 0.2,0.7)$ |
| $X_2$ | $((2,3,5,4); 0.5,0.4)$ | $((3,3,2,1); 0.4,0.7)$ | $((4,5,5,2); 0.5,0.6)$ | $((2,3,5,4); 0.3,0.4)$ |
| | $((3,2,1,4); 0.2,0.3)$ | $((5,3,1,3); 0.5,0.2)$ | $((2,3,1,2); 0.3,0.1)$ | $((3,2,1,4); 0.2,0.5)$ |
| $X_3$ | $((1,3,3,1); 0.4,0.7)$ | $((4,2,2,5); 0.2,0.3)$ | $((2,3,3,1); 0.7,0.4)$ | $((2,3,5,4); 0.2,0.4)$ |
| | $((2,3,5,1); 0.1,0.3)$ | $((3,4,1,1); 0.3,0.5)$ | $((1,2,1,5); 0.4,0.2)$ | $((3,2,1,4); 0.5,0.7)$ |
| $X_4$ | $((3,2,2,3); 0.6,0.4)$ | $((4,1,2,4); 0.4,0.5)$ | $((5,3,1,3); 0.3,0.5)$ | $((2,3,5,4); 0.5,0.4)$ |
| | $((5,2,1,1); 0.5,0.1)$ | $((3,2,5,3); 0.2,0.6)$ | $((1,3,5,2); 0.5,0.6)$ | $((3,1,5,5); 0.2,0.3)$ |

**Table 8.** TRT-2-PFN decision-matrix for $\mathcal{D}^2$.

| $X_i$ | $\mathcal{C}_1$ | $\mathcal{C}_2$ | $\mathcal{C}_3$ | $\mathcal{C}_4$ |
|---|---|---|---|---|
| $X_1$ | $((2,3,5,4); 0.5,0.4)$ | $((2,2,1,3); 0.4,0.7)$ | $((1,3,3,4); 0.3,0.4)$ | $((2,3,3,2); 0.5,0.4)$ |
| | $((3,2,1,4); 0.2,0.5)$ | $((3,4,5,4); 0.5,0.3)$ | $((3,5,4,4); 0.2,0.5)$ | $((5,2,4,2); 0.1,0.3)$ |
| $X_2$ | $((1,2,1,5); 0.2,0.4)$ | $((1,3,1,5); 0.7,0.6)$ | $((5,3,5,2); 0.1,0.3)$ | $((2,3,3,5); 0.7,0.5)$ |
| | $((2,2,1,3); 0.3,0.6)$ | $((2,4,1,5); 0.2,0.5)$ | $((3,2,3,4); 0.5,0.6)$ | $((2,2,1,3); 0.2,0.2)$ |
| $X_3$ | $((5,3,5,2); 0.6,0.5)$ | $((3,3,1,3); 0.3,0.5)$ | $((4,3,5,1); 0.3,0.2)$ | $((1,3,1,4); 0.1,0.4)$ |
| | $((3,5,1,1); 0.3,0.4)$ | $((5,2,3,4); 0.2,0.3)$ | $((3,4,1,3); 0.2,0.5)$ | $((1,2,4,5); 0.3,0.5)$ |
| $X_4$ | $((4,2,3,1); 0.2,0.6)$ | $((1,5,5,4); 0.5,0.4)$ | $((1,3,4,4); 0.3,0.4)$ | $((3,3,1,4); 0.4,0.3)$ |
| | $((4,3,1,4); 0.4,0.3)$ | $((3,5,1,3); 0.2,0.7)$ | $((3,3,5,3); 0.1,0.8)$ | $((5,1,1,3); 0.7,0.5)$ |

**Table 9.** TRT-2-PFN decision-matrix for $\mathcal{D}^3$.

| $X_i$ | $\mathcal{C}_1$ | $\mathcal{C}_2$ | $\mathcal{C}_3$ | $\mathcal{C}_4$ |
|---|---|---|---|---|
| $X_1$ | $((1,2,5,3);0.3,0.4)$ | $((3,3,4,4);0.2,0.4)$ | $((1,4,5,2);0.1,0.9)$ | $((4,1,2,2);0.3,0.5)$ |
| | $((3,2,1,4);0.2,0.5)$ | $((1,2,4,3);0.3,0.6)$ | $((2,2,5,4);0.7,0.3)$ | $((5,3,3,4);0.7,0.2)$ |
| $X_2$ | $((5,3,2,1);0.5,0.3)$ | $((1,4,5,5);0.3,0.7)$ | $((4,2,1,1);0.7,0.1)$ | $((5,3,5,2);0.4,0.6)$ |
| | $((2,4,1,3);0.3,0.2)$ | $((4,1,2,3);0.1,0.4)$ | $((3,2,1,2);0.6,0.4)$ | $((3,2,2,1);0.3,0.8)$ |
| $X_3$ | $((5,4,1,1);0.8,0.2)$ | $((4,1,2,2);0.1,0.2)$ | $((1,3,3,3);0.4,0.5)$ | $((2,2,5,2);0.5,0.7)$ |
| | $((3,1,5,3);0.4,0.3)$ | $((3,5,2,4);0.4,0.3)$ | $((4,5,1,5);0.3,0.2)$ | $((1,2,5,4);0.4,0.5)$ |
| $X_4$ | $((4,5,3,3);0.2,0.6)$ | $((2,3,5,4);0.2,0.3)$ | $((1,3,1,2);0.9,0.3)$ | $((4,3,5,3);0.6,0.4)$ |
| | $((3,2,1,4);0.1,0.4)$ | $((3,3,1,5);0.6,0.1)$ | $((2,4,5,3);0.8,0.2)$ | $((2,2,1,5);0.6,0.7)$ |

In the next step, we will normalize these three TRT-2-PFNs by using Eq (22). The normalized decision matrices are in Tables 10, 11 and 12.

**Table 10.** normalized decision matrix $\mathcal{D}^1$.

| $\mathcal{C}_1$ | $\mathcal{C}_2$ |
|---|---|
| $X_1$: ((0.212,0.331,0.214,0.534); 0.3,0.4) ((0.335,0.435,0.421,0.324); 0.5,0.6) | $X_1$: ((0.301,0.336,0.317,0.398); 0.7,0.5) ((0.355,0.405,0.414,0.386); 0.1,0.6) |
| $X_2$: ((0435,0.543,0215,0.342); 0.5,0.4) ((0.311,0.234,0.463,0.581); 0.2,0.3) | $X_2$: ((0.302,0.332,0.294,0.418); 0.4,0.7) ((0.352,0.387,0.416,0.367); 0.5,0.2) |
| $X_3$: ((0.421,0.531,0.632,0.356); 0.4,0.7) ((0.281,0.489,0.557,0.243); 0.1,0.3) | $X_3$: ((0.284,0.301,0.295,0312); 0.2,0.3) ((0.331,0.391,0.389,0.307); 0.3,0.5) |
| $X_4$: ((0.327,0.341,0.427,0.487); 0.6,0.4) ((0.367,0.536,0.439,0.307); 0.5,0.1) | $X_4$: ((0.387,0.373,0.451,0.510); 0.4,0.5) ((0.211,0.283,0.314,0.332); 0.2,0.6) |
| $\mathcal{C}_3$ | $\mathcal{C}_4$ |
| $X_1$: ((0.292,0.303,0.289,0.339); 0.4,0.6) ((0.321,0.340,0.341,0.375); 0.3,0.7) | $X_1$: ((0.346,0.326,0.337,0.318); 0.3,0.6) ((0.332,0.355,0.382,0.348); 0.2,0.7) |
| $X_2$: ((0.402,0.381,0.395,0.418); 0.5,0.6) ((0.373,0.400,0.416,0.385); 0.3,0.1) | $X_2$: ((0.285,0.293,0.279,0.231); 0.3,0.4) ((0.308,0.317,0.327,0.303); 0.2,0.5) |
| $X_3$: ((0.320,0.353,0.414,0.434); 0.7,0.4) ((0.387,0.404,0.411,0.424); 0.4,0.2) | $X_3$: ((0.405,0.432,0.458,0467); 0.2,0.4) ((0.411,0.433,0.429,0.404); 0.5,0.7) |
| $X_4$: ((0.412,0.371,0.342,0.383); 0.3,0.5) ((0.393,0.405,0.413,0.371); 0.5,0.6) | $X_4$: ((0.321,0.351,0.346,0.352); 0.5,0.4) ((0.315,0.335,0.322,0.317); 0.2,0.3) |

**Table 11.** normalized decision matrix $\mathcal{D}^2$.

| $\mathcal{C}_1$ | $\mathcal{C}_2$ |
|---|---|
| $X_1$: ((0.480,0.391,0.387,0.419); 0.5,0.2) ((0.383,0.432,0.404,0.373); 0.4,0.5) | $X_1$: ((0.293,0.295,0.302,0.317); 0.4,0.7) ((0.314,0.323,0.352,0.367); 0.5,0.3) |
| $X_2$: ((0.356,0.345,0.356,0.362); 0.2,0.4) ((0.331,0.384,0.390,0.379); 0.3,0.6) | $X_2$: ((0.332,0.371,0.311,0.343); 0.7,0.6) ((0.453,0.416,0.401,0.423); 0.2,0.5) |
| $X_3$: ((0.412,0.398,0.421,0.491); 0.6,0.5) ((0.432,0.431,0.424,0.320); 0.3,0.4) | $X_3$: ((0.415,0.401,0.432,0.401); 0.3,0.5) ((0.426,0.408,0.429,0.425); 0.2,0.3) |
| $X_4$: ((0.293,0.300,0.312,0.311); 0.2,0.6) ((0.301,0.354,0.376,0.381); 0.4,0.3) | $X_4$: ((0.298,0.304,0.272,0.324); 0.5,0.4) ((0.353,0.350,0.341,0.324); 0.2,0.7) |
| $\mathcal{C}_3$ | $\mathcal{C}_4$ |
| $X_1$: ((0.430,0.392,0.404,0.417); 0.3,0.4) ((0.385,0.396,0.406,0.383); 0.2,0.5) | $X_1$: ((0.293,0.311,0.282,0.332); 0.5,0.4) ((0.335,0.435,0.321,0.324); 0.1,0.3) |
| $X_2$: ((0.345,0.334,0.290,0.343); 0.1,0.3) ((0.332,0.316,0.337,0.351); 0.5,0.6) | $X_2$: ((0.312,0.302,0.293,0.331); 0.7,0.5) ((0.335,0.435,0.421,0.321); 0.2,0.2) |
| $X_3$: ((0.261,0.306,0.236,0.240); 0.3,0.2) ((0.301,0.305,0.321,0.352); 0.2,0.5) | $X_3$: ((0.299,0.309,0.283,0.339); 0.1,0.4) ((0.305,0.334,0.327,0.327); 0.3,0.5) |
| $X_4$: ((0.234,0.233,0.274,0.234); 0.3,0.4) ((0.305,0.335,0.321,0.322); 0.1,0.8) | $X_4$: ((0.212,0.331,0.214,0.534); 0.4,0.3) ((0.323,0.315,0.421,0.325); 0.7,0.5) |

**Table 12.** normalized decision matrix $\mathcal{D}^3$.

| $\mathcal{C}_1$ | $\mathcal{C}_2$ |
|---|---|
| $X_1$: ((0.341,0.312,0.294,0.335); 0.3,0.4) ((0.314,0.326,0.340,0.314); 0.2,0.5) | $X_1$: ((0.401,0.431,0.455,0.431); 0.2,0.4) ((0.395,0.475,0.412,0.426); 0.3,0.6) |
| $X_2$: ((0.407,0.332,0.367,0.398); 0.5,0.3) ((0.315,0.350,0.421,0.323); 0.3,0.2) | $X_2$: ((0.432,0.413,0.445,0.423); 0.3,0.7) ((0.389,0.437,0.431,0.427); 0.1,0.4) |
| $X_3$: ((0.432,0.416,0.404,0.434); 0.8,0.2) ((0.398,0.452,0.430,0.421); 0.4,0.3) | $X_3$: ((0.503,0.471,0.442,0.514); 0.1,0.2) ((0.438,0.415,0.429,0.434); 0.4,0.3) |
| $X_4$: ((0.512,0.431,0.514,0.501); 0.2,0.6) ((0.433,0.437,0.429,0.424); 0.1,0.4) | $X_4$: ((0.452,0.433,0.457,0.507); 0.2,0.3) ((0.504,0.453,0.441,0.457); 0.6,0.1) |
| $\mathcal{C}_3$ | $\mathcal{C}_4$ |
| $X_1$: ((0.244,0.302,0.295,0.431); 0.1,0.9) ((0.350,0.375,0.420,0.358); 0.7,0.3) | $X_1$: ((0.348,0.343,0.414,0.434); 0.3,0.5) ((0.313,0.405,0.452,0.424); 0.7,0.2) |
| $X_2$: ((0.282,0.331,0.314,0.354); 0.7,0.1) ((0.395,0.408,0.441,0.454); 0.6,0.4) | $X_2$: ((0.432,0.431,0.455,0.345); 0.4,0.6) ((0.335,0.407,0.323,0.421); 0.3,0.8) |
| $X_3$: ((0.401,0.394,0.424,0.431); 0.4,0.5) ((0.452,0.430,0.423,0.426); 0.3,0.2) | $X_3$: ((0.312,0.302,0.374,0.327); 0.5,0.7) ((0.335,0.435,0.421,0.326); 0.4,0.5) |
| $X_4$: ((0.422,0.431,0.351,0.432); 0.9,0.2) ((0.335,0.435,0.421,0.324); 0.8,0.2) | $X_4$: ((0.324,0.319,0.344,0.331); 0.6,0.4) ((0.405,0.435,0.423,0.332); 0.6,0.7) |

Now using Eqs (12), (13), and (14) to determine the weight of each decision-maker. The results are summarized in Tables 13, 14, and 15.

**Table 13.** weight vector of decision-maker $\mathcal{D}^1$.

| Alternatives | $\mathcal{C}_1$ | $\mathcal{C}_2$ | $\mathcal{C}_3$ | $\mathcal{C}_4$ |
|---|---|---|---|---|
| $X_1$ | 0.3412 | 0.3387 | 0.3577 | 0.3365 |
| $X_2$ | 0.3313 | 0.3154 | 0.3298 | 0.3411 |
| $X_3$ | 0.2916 | 0.3034 | 0.3412 | 0.3217 |
| $X_4$ | 0.3361 | 0.3437 | 0.3266 | 0.3045 |

**Table 14.** weight vector of decision-maker $\mathcal{D}^2$.

| Alternatives | $\mathcal{C}_1$ | $\mathcal{C}_2$ | $\mathcal{C}_3$ | $\mathcal{C}_4$ |
|---|---|---|---|---|
| $X_1$ | 0.3452 | 0.2376 | 0.3675 | 0.3298 |
| $X_2$ | 0.2564 | 0.3427 | 0.3354 | 0.3019 |
| $X_3$ | 0.3156 | 0.3678 | 0.3198 | 0.3144 |
| $X_4$ | 0.3126 | 0.3421 | 0.3156 | 0.3367 |

**Table 15.** weight vector of decision-maker $\mathcal{D}^3$.

| Alternatives | $\mathcal{C}_1$ | $\mathcal{C}_2$ | $\mathcal{C}_3$ | $\mathcal{C}_4$ |
|---|---|---|---|---|
| $X_1$ | 0.2143 | 0.3421 | 0.2341 | 0.2354 |
| $X_2$ | 0.3425 | 0.2312 | 0.3411 | 0.3123 |
| $X_3$ | 0.3216 | 0.3564 | 0.2342 | 0.3178 |
| $X_4$ | 0.3125 | 0.3265 | 0.2436 | 0.3275 |

Using Eq (17) to combine the group decision matrix. The rating values of the alternatives concerning each criterion are summarized in Table 16.

**Table 16.** The group decision matrix.

| $\mathcal{C}_1$ | $\mathcal{C}_2$ |
|---|---|
| $X_1$: $\begin{pmatrix}(0.191,0.423,0.697,0.874);\\0.701,0.310\end{pmatrix}\begin{pmatrix}(0.413,0.493,0.628,0.697);\\0.470,0.539\end{pmatrix}$ | $X_1$: $\begin{pmatrix}(0.149,0.379,0.656,0.900);\\0.699,0.310\end{pmatrix}\begin{pmatrix}(0.379,0.449,0.586,0.367);\\0.490,0.514\end{pmatrix}$ |
| $X_2$: $\begin{pmatrix}(0.150,0.379,0.656,0.900);\\0.699,0.300\end{pmatrix}\begin{pmatrix}(0.379,0.449,0.586,0.656);\\0.495,0.514\end{pmatrix}$ | $X_2$: $\begin{pmatrix}(0.150,0.410,0.739,0.996);\\0.799,0.210\end{pmatrix}\begin{pmatrix}(0.410,0.494,0.656,0.739);\\0.533,0.475\end{pmatrix}$ |
| $X_3$: $\begin{pmatrix}(0.138,0.310,0.693,0.911);\\0.729,0.280\end{pmatrix}\begin{pmatrix}(0.310,0.407,0.597,0.693);\\0.488,0.521\end{pmatrix}$ | $X_3$: $\begin{pmatrix}(0.032,0.258,0.622,0.943);\\0.694,0.315\end{pmatrix}\begin{pmatrix}(0.258,0.350,0.530,0.623);\\0.530,0.479\end{pmatrix}$ |
| $X_4$: $\begin{pmatrix}(0.089,0.293,0.633,0.921);\\0.720,0.259\end{pmatrix}\begin{pmatrix}(0.293,0.378,0.548,0.463);\\0.546,0.463\end{pmatrix}$ | $X_4$: $\begin{pmatrix}(0.185,0.322,0.702,0.945);\\0.697,0.312\end{pmatrix}\begin{pmatrix}(0.322,0.423,0.601,0.702);\\0.449,0.560\end{pmatrix}$ |
| $\mathcal{C}_3$ | $\mathcal{C}_4$ |
| $X_1$: $\begin{pmatrix}(0.112,0.295,0.631,0.824);\\0.680,0.329\end{pmatrix}\begin{pmatrix}(0.295,0.379,0.547,0.631);\\0.417,0.592\end{pmatrix}$ | $X_1$: $\begin{pmatrix}(0.056,0.265,0.453,0.733);\\0.586,0.423\end{pmatrix}\begin{pmatrix}(0.265,0.315,0.403,0.443);\\0.395,0.614\end{pmatrix}$ |
| $X_2$: $\begin{pmatrix}(0.239,0.417,0.749,0.994);\\0.705,0.304\end{pmatrix}\begin{pmatrix}(0.417,0.501,0.665,0.749);\\0.475,0.533\end{pmatrix}$ | $X_2$: $\begin{pmatrix}(0.079,0.320,0.693,0.919);\\0.766,0.243\end{pmatrix}\begin{pmatrix}(0.320,0.417,0.595,0.691);\\0.471,0.537\end{pmatrix}$ |
| $X_3$: $\begin{pmatrix}(0.105,0.266,0.652,0.873);\\0.690,0.319\end{pmatrix}\begin{pmatrix}(0.267,0.366,0.552,0.652);\\0.461,0.547\end{pmatrix}$ | $X_3$: $\begin{pmatrix}(0.108,0.341,0.698,0.974);\\0.714,0.295\end{pmatrix}\begin{pmatrix}(0.341,0.437,0.602,0.698);\\0.565,0.444\end{pmatrix}$ |
| $X_4$: $\begin{pmatrix}(0.059,0.268,0.,0.623.824);\\0.758,0.251\end{pmatrix}\begin{pmatrix}(0.268,0.360,0.531,0.623);\\0.487,0.522\end{pmatrix}$ | $X_4$: $\begin{pmatrix}(0.143,0.352,0.580,0.808);\\0.726,0.283\end{pmatrix}\begin{pmatrix}(0.353,0.410,0.523,0.580);\\0.496,0.512\end{pmatrix}$ |

To calculate the relative criteria weight $w_{rj}$ by using Eq (19). The relative weight of each criterion is listed in Table 17.

**Table 17.** Relative weight of criterion.

| | $C_1$ | $C_2$ | $C_3$ | $C_4$ |
|---|---|---|---|---|
| $w_{rj}$ | 0.9347 | 1.0000 | 0.6800 | 0.7520 |

The dominance of each alternative with respect to each criterion is summarized in Tables 18, 19, 20, and 21.

**Table 18.** Dominance degree matrix for $C_1$.

| $\Delta_1$ | $X_1$ | $X_2$ | $X_3$ | $X_4$ |
|---|---|---|---|---|
| $X_1$ | 0.000 | −0.086 | −0.129 | 0.052 |
| $X_2$ | 0.086 | 0.000 | −0.087 | 0.035 |
| $X_3$ | 0.0.32 | 0.022 | 0.000 | 0.025 |
| $X_4$ | −0.186 | −0.112 | −0.80 | 0.000 |

**Table 19.** Dominance degree matrix for $C_2$.

| $\Delta_2$ | $X_1$ | $X_2$ | $X_3$ | $X_4$ |
|---|---|---|---|---|
| $X_1$ | 0.000 | −0.326 | −0.086 | −0.190 |
| $X_2$ | 0.086 | 0.000 | −0.251 | −0.142 |
| $X_3$ | 0.035 | 0.074 | 0.000 | 0.036 |
| $X_4$ | 0.061 | 0.048 | −0.159 | 0.000 |

**Table 20.** Dominance degree matrix for $C_3$.

| $\Delta_3$ | $X_1$ | $X_2$ | $X_3$ | $X_4$ |
|---|---|---|---|---|
| $X_1$ | 0.000 | 0.050 | 0.016 | 0.018 |
| $X_2$ | −0.310 | 0.000 | −0.305 | 0.060 |
| $X_3$ | −0.066 | 0.049 | 0.000 | 0.013 |
| $X_4$ | −0.074 | −0.308 | −0.042 | 0.000 |

**Table 21.** Dominance degree matrix for $C_4$.

| $\Delta_4$ | $X_1$ | $X_2$ | $X_3$ | $X_4$ |
|---|---|---|---|---|
| $X_1$ | 0.000 | −0.396 | 0.068 | −0.298 |
| $X_2$ | 0.060 | 0.000 | 0.018 | −0.178 |
| $X_3$ | −0.409 | −0.122 | 0.000 | −0.194 |
| $X_4$ | 0.045 | 0.032 | 0.029 | 0.000 |

The global dominance of alternative $X_i$ over $X_k$ can be determined using Eq (23), which is summarized in Table 22.

**Table 22.** Global dominance degree.

| | $X_1$ | $X_2$ | $X_3$ | $X_4$ |
|---|---|---|---|---|
| $X_1$ | 0.000 | −1.081 | −0.257 | −0.698 |
| $X_2$ | −0.271 | 0.000 | −0.838 | −0.285 |
| $X_3$ | −0.582 | −0.017 | 0.000 | −0.121 |
| $X_4$ | −0.334 | −0.501 | −0.545 | 0.000 |

Finally, by using Eq (24) to calculate the global value of each alternative. the global values and rank of presented in Table 23.

**Table 23.** Global values and order ranking.

| $\sigma$ | $\mu$ | ranking |
|---|---|---|
| $X_1$ | 0.001 | 4 |
| $X_2$ | 0.3532 | 2 |
| $X_3$ | 0.991 | 1 |
| $X_4$ | 0.216 | 3 |

As we can see, after redefining our resource selection problem and using a new method to defend TRT-2-PFSs, we concluded that alternative $X_3$ has the highest global dominance value, making it the optimal alternative out of the four. We set the loss $\rho$ attenuation factor to 1. Its range is 1 to 2.5, but it is entirely dependent on the performance of decision-makers.

*3.2. Validity test*

We used a set of testing techniques developed by Wang and Trianaphyllou [39] that included the following phases to verify the robustness and adaptability of the suggested method over a wide variety of circumstances.

**Stability Test (Step 1):** For this assessment, rating values connected with fewer desirable alternatives were substituted for those associated with non-optimal alternatives. The top alternative should not change, keeping its status as the greatest option notwithstanding this modification. By highlighting the ranking's stability under constant relative criterion weights, this test highlights the method's robustness against alterations in alternative ratings.

**Transitivity test (Step 2):** Transitivity is a crucial characteristic of decision-making processes; it states that if alternative A is chosen over alternative B and alternative B is preferred over alternative C, then alternative A should be preferred over alternative C. Through this test, the suggested method's adherence to the transitive property was confirmed, as well as the obtained rankings' logical coherence.

**Aggregation Test (Step 3):** Sometimes it is required to break large issues down into smaller subproblems while using the same decision-making strategy in complex decision-making settings. The aggregation test checks to see if the final ranking of solutions to these subproblems is identical to the initial rating. By consistently producing correct rankings across various issue subdivisions, this verifies the suggested approach's capacity to tackle complicated problems.

**Validity test using criterion 1**

The ranking order achieved through the proposed approach is $X_3 \succ X_2 \succ X_4 \succ X_1$. To evaluate the robustness of our approach according to test criterion 1, we conducted a sensitivity analysis by replacing the non-optimal alternative $X_1$ with a subpar alternative $X_1^*$. The rating values of $X_1^*$ were defined as $\big((1,2,1,2);0.1,0.5\big)\big((1,3,2,2);0.2,0.2\big)$, $\big((1,3,1,1);0.1,0.2\big)\big((1,3,2,2);0.1,0.2\big)$, $\big((1,1,1,2);0.3,0.4\big)\big((1,3,2,1);0.1,0.6\big)$ and $\big((1,2,1,1);0.1,0.4\big)\big((1,1,2,2);0.3,0.7\big)$.

Utilizing our approach, we computed the aggregated score values for the alternatives: $\mu(X_1) = 0.001$, $\mu(X_1^*) = 0.0003$, $\mu(X_2) = 0.3532$ and $\mu(X_3) = 0.991$. Consequently, the revised ranking order emerged as $X_3 \succ X_2 \succ X_1 \succ X_1^*$, with the best alternative maintaining its position consistent with the original approach. This outcome underscores the reliability and consistency of our proposed approach, particularly with respect to test criterion 1.

To assess the validity based on criteria 2 and 3, we consider fragmented decision-making subcases, namely $\{X_1,,X_3,X_4\}$, $\{X_2,X_3,X_4\}$ and $\{X_2,X_3,X_1\}$. Implementing the outlined procedure, their rank orders are obtained as follows: $X_3 \succ X_4 \succ X_1$, $X_3 \succ X_4 \succ X_2$ and $X_3 \succ X_2 \succ X_1$, respectively. Upon aggregating all results, the overall ranking emerges as $X_3 \succ X_4 \succ X_2 \succ X_1$, which aligns with the original outcomes of our decision-making approach. Therefore, we can confirm that our proposed approach remains valid according to test criteria 2 and 3.

**Table 24.** Different types of FSs and their attributes.

| Characteristics | Different types of fuzzy sets | | | | | |
|---|---|---|---|---|---|---|
| | FS | IFS | Type-2 FS | Type-2 IFS | PFS | Type-2 PFS |
| membership value | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| Describe ambiguity | MD | MD and NMD | Both primary and secondary membership | Primary and secondary membership, non-membership, and uncertainty | Membership and non-membership grade | Primary and secondary membership, non-membership, and uncertainty |
| unknown parameters | × | × | ✓ | ✓ | ✓ | ✓ |
| The ability of Multi-Attribute modelling | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| modelling of increasing uncertainty | × | × | ✓ | ✓ | × | ✓ |
| Taking reluctance into account while making decisions | × | ✓ | × | ✓ | × | ✓ |

### 3.3. *Comparative analysis*

In the existing literature, a variety of fuzzy sets have been developed to address specific scenarios, utilizing their unique properties. Notable examples include fuzzy sets, type-2 fuzzy sets, intuitionistic fuzzy sets, and Pythagorean fuzzy sets, which have gained recognition in the field of fuzzy set theory. Our study introduces a novel addition to this landscape: the trapezoidal type-2 Pythagorean fuzzy set. To facilitate a clear comparison, Table 24 provides a comprehensive overview of these fuzzy sets based on various attributes. All these fuzzy sets share the common feature of using graded membership values, allowing them to effectively represent uncertainty across multiple attributes. However, type-2 fuzzy sets, type-2 intuitionistic fuzzy sets, and type-2 Pythagorean fuzzy sets excel in modeling scenarios with high levels of uncertainty, especially when dealing with hesitancy, a nuanced form of uncertainty. Pythagorean fuzzy sets stand out for their ability to comprehensively address parameter uncertainty through primary and secondary memberships. Among these options, the type-2 Pythagorean fuzzy set is a standout choice due to its versatility in handling complex challenges and capturing the essence of reluctance. This unique capability positions it as a powerful tool for addressing multifaceted complexities and accommodating scenarios where decision-makers exhibit hesitation in their preferences or judgments.

### *3.4. Advantages*

- The utilization of Trapezoidal Type-2 Pythagorean fuzzy sets provides an extended range of uncertainty representation, enabling decision-makers to express their preferences more accurately, especially in situations involving complex and hesitant judgments.
- By combining the characteristics of trapezoidal fuzzy and type-2 fuzzy sets, our methods effectively address multifaceted uncertainties and provide a more comprehensive framework for decision-making.
- The method presents a systemic process for evaluating unknown weights of decision-makers and criteria, contributing to a more objective and precise weighting of factors in the decision-making process.
- The novel Trapezoidal Pythagorean fuzzy TODIM method integrates the strengths of both the TODIM technique and the trapezoidal type-2 Pythagorean fuzzy sets. This integration enhances the accuracy and reliability of the decision outcomes.
- Through rigorous testing and comparison with existing decision-making approaches, the validity and robustness of the proposed method have been demonstrated, ensuring its practical applicability in real-world scenarios.
- The method facilitates the identification of the optimal alternatives by considering global dominance values. This assists decision-makers in selecting the best option, aligning with their preferences and hesitancy levels.
- The loss $\rho$ attenuation factor can be adjusted within a range of 1 to 2.5, offering decision-makers the flexibility to fine-tune method according to their requirements and preferences.
- By integrating hesitancy and advanced fuzzy set concepts, the proposed approach aids in more efficient allocation of resources, leading to informed and rational decision outcomes.

### *3.5. Limitations*

- The method involves multiple steps for evaluating unknown weights, conducting comparisons, and aggregating scores. This complexity might require additional computational resources and time, especially for larger decision-making problems.
- Accurate weight assignment and preference evaluations rely on expert input. If experts provide incomplete or inaccurate information, the quality of the decision outcomes could be compromised.

- The method's effectiveness is contingent on the subjectivity of decision-makers and their ability to express hesitancy accurately. Biases or inconsistent judgment could lead to unreliable results.
- While the proposed method is designed to handle hesitancy and uncertainty effectively, its optimal performance may be limited to specific decision contexts. Its applicability might vary across different domains and industries.
- The loss $\rho$ attenuation factor's range of adjustment (1 to 2.5) offers flexibility, but determining the optimal value requires a deep understanding of the problem and may necessitate trial and error.
- While the proposed method has been validated through comparisons and testing, real-world scenarios might present unforeseen challenges that could impact the method's performance.

## 4. Conclusions

In this study, we introduce an innovative structure known as trapezoidal type-2 Pythagorean fuzzy sets, by synergizing the concepts of trapezoidal fuzzy sets and type-2 fuzzy sets. Additionally, we present fundamental operational laws, including addition, scalar multiplication, distance measurement, and comparison, specifically tailored for trapezoidal type-2 Pythagorean fuzzy numbers. To tackle the issue of unknown weights, we put forth a method for assessing the weights associated with each decision-maker and criterion. Furthermore, we devise a novel TOMID method, operating within the trapezoidal type-2 Pythagorean fuzzy environment, which effectively addresses multi-criteria decision-making (MCDM) problems with uncertain weights. Additionally, we contribute a method that facilitates the transformation of triangular Pythagorean fuzzy numbers into the proposed trapezoidal type-2 Pythagorean fuzzy framework. To demonstrate the practicality and versatility of our proposed TODIM method, we present a detailed example, showcasing its rationality and adaptability. For empirical validation, we benchmark the outcomes of our proposed approach against various existing decision-making methods. Through this comparative analysis, we establish the efficacy and robustness of our TODIM approach in resolving complex MCDM scenarios. This comprehensive investigation underscores the valuable contributions of our work in enhancing decision-making methodologies and addressing uncertainty within a trapezoidal type-2 Pythagorean fuzzy framework.

There is an exciting avenue for future research to expand upon our proposed work by integrating Z-fuzzy clouds [40] and best-worst entropy methods [41]. By doing so, we can further enhance the depth and scope of our investigations, providing a richer and more comprehensive perspective in our upcoming research endeavors. This promising direction opens opportunities for exploring and advancing the capabilities of our study.

## Use of AI tools declaration

The authors declare that they have not used Artificial Intelligence (AI) tools in the creation of this article.

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**Conflict of interest**

The authors declare no conflicts of interest.

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